23

Alternating Current

IN THIS CHAPTER ....

Types of Current

There are two types of current which flows in any of the electrical appliances. These are as follows

Direct Current

Current whose direction does not change with time in a circuit is known as direct current (DC).

Alternating Current

Currents whose magnitude and direction changes continuously (periodically) with time is known as alternating currents and corresponding voltage as alternating voltage or emf.

The instantaneous value of time varying alternating voltage and current are given by

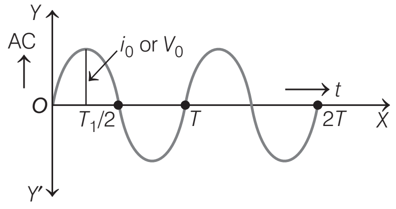
$$V = V_0 \sin \omega t$$

and $$i = i_0 \sin \omega t$$

where, V_0 and i_0 = peak value of voltage and current respectively

and ω is called angular frequency of AC.

where, $\omega = \frac{2\pi}{T} = 2\pi\nu$, here T is the time period or period of AC.



Mean or Average Value of AC

Mean or average value of an alternating current (AC) is the total charge flow for one complete cycle, divided by the time taken to complete the cycle. *i.e.* time period T.

The direction of AC changes after every half cycle. Hence, for a full cycle, the mean or average value is zero. Thus, we find average value for half cycle only.

It is given by $I_{\text{mean}} = \dfrac{1}{T/2}\displaystyle\int_0^{\pi/2} I\,dt$

Mean or average value of alternating current for first half cycle, $I_m = \dfrac{2I_0}{\pi} = 0.637\, I_0$

Mean or average value of alternating current for next half cycle, $I'_m = -\dfrac{2I_0}{\pi} = -\,0.637\, I_0$

So, mean or average value of alternating current for one complete cycle = 0.

In the same way, mean value of alternating voltage,

$$V_m = 0.637\, V_0$$

Root Mean Square Value of AC

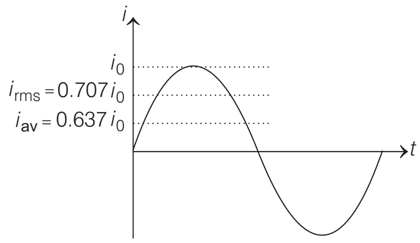
The steady current, which when passes through a resistance for a given time will produce the same amount of heat as the alternating current does in the same resistance and in the same time, is called rms value of alternating current. It is given by

$$i_{\text{rms}} = \frac{i_0}{\sqrt{2}} = 0.707\, i_0$$

Similarly, rms value of alternating emf,

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = 0.707\, V_0$$

The different values of i_0, i_{rms} and i_{av} are shown in figure



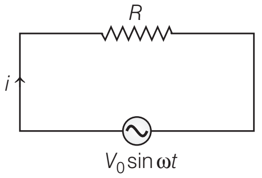
Resistive Circuit or *R*-Circuit

When an AC voltage source (*i.e.* $V = V_0 \sin \omega t$) is connected across a resistor in a circuit, then current in the circuit,

$$i = i_m \sin \omega t$$

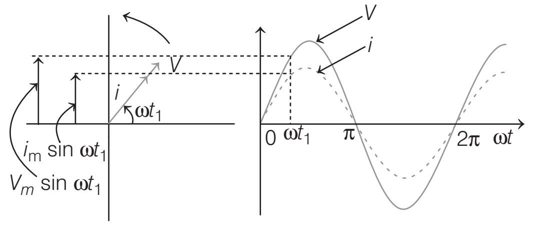
where, $i_m = \dfrac{V_m}{R}$.

The voltage and current are in phase with each other.



The **phasor diagram** and ***V* & *I* versus** ωt graph are as shown below.

A diagram representing alternating current and alternating emf (of same frequency) as rotating vectors (phasors) with the phase angle between them is called *phasor diagram*.

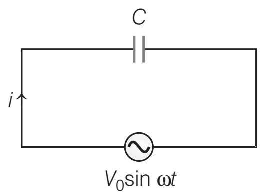


Capacitive Circuit or *C*-Circuit

When a capacitor is connected to an AC voltage source (*i.e.* $V = V_0 \sin \omega t$) in a circuit, then the current in the circuit,

$$i = i_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

where, $i_m = \dfrac{V_m}{1/\omega C}$.



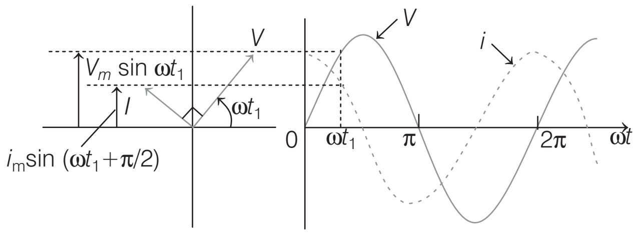
The term $\left(\dfrac{1}{\omega C}\right)$ is also analogous to resistance and it is called *capacitive reactance*. It is denoted by X_C, *i.e.*

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \nu C}$$

The dimension of capacitive reactance is the same as that of resistance and its SI unit is ohm (Ω).

The current in this circuit is ahead of voltage in phase by $\dfrac{\pi}{2}$ or one quarter cycle. It means, the current reaches its maximum value earlier than the voltage by one-fourth of a period.

The **phasor diagram** and ***V* & *I* *versus*** ω***t*** graph are as shown below

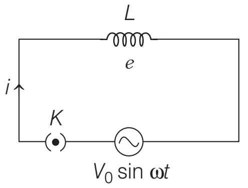


Inductive Circuit or *L*-Circuit

When an AC voltage source (*i.e.* $V = V_0 \sin \omega t$) is connected to a purely inductive circuit, then current in the circuit,

$$i = i_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

where, $i_m = \dfrac{V_m}{\omega L}$.



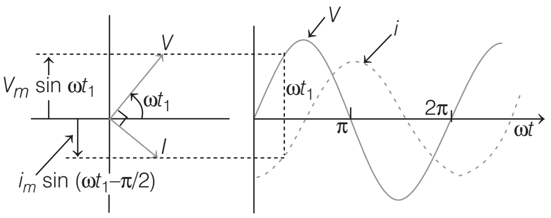
The quantity ωL is analogous to the resistance and is called *inductive reactance*. It is denoted by X_L, *i.e.*

$$X_L = \omega L = (2\pi\nu)L$$

The dimension of inductive reactance is same as that of resistance and its SI unit is ohm (Ω).

The current in this type of circuit lags the voltage in phase by $\dfrac{\pi}{2}$ or one-quarter $\left(\dfrac{1}{4}\right)$ cycle. It means the current reaches its maximum value later than the voltage by one-fourth of a period $\left(\dfrac{T}{4} = \dfrac{\pi/2}{\omega}\right)$.

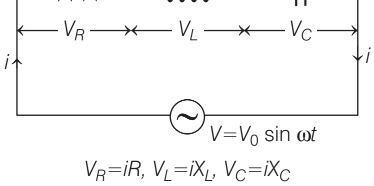
The **phasor diagram** and ***V* & *I versus* ω*t*** graph are as shown below



Note Capacitor blocks DC and acts as an open circuit while passing AC of high frequency, while inductor passes DC and blocks AC of very high frequency.

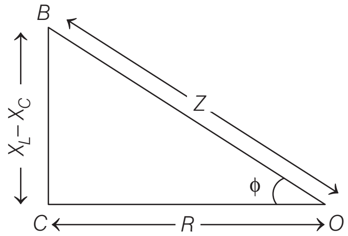
AC Voltage Applied to Series *R-L-C* Circuit

When a resistor, capacitor and an inductor are connected in series with an alternating voltage source as shown below



$V_R=iR$, $V_L=iX_L$, $V_C=iX_C$

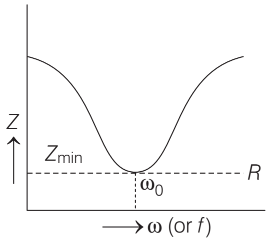
- **Current in the circuit** $i = i_0 \sin(\omega t \pm \phi)$, where $i_0 = \dfrac{V_0}{Z}$
- **Voltage across the circuit**
$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$
- **Impedance (*Z*)** The resistance offered by an AC circuit to the flow of alternating current through it, is called impedance of a given circuit.
It is given as $Z = \sqrt{R^2 + (X_L - X_C)^2}$
$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$
- **Phase angle** ϕ is the angle between total voltage V and total electric current and can be given as
$$\tan \phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$$
$$= \frac{\omega L - \dfrac{1}{\omega C}}{R}$$
$$= \frac{2\pi\nu L - \dfrac{1}{2\pi\nu C}}{R}$$



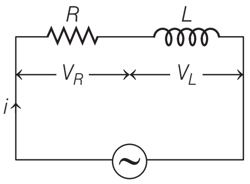
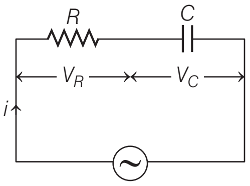
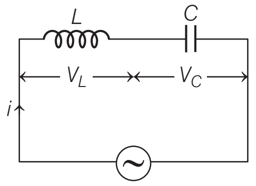
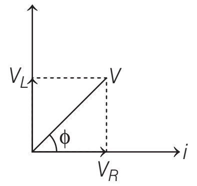
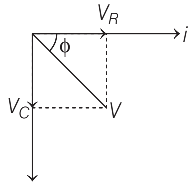
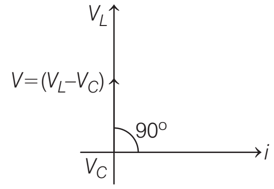
Special Cases

- When $X_L = X_C$, then $Z = R$ and $\tan \phi = 0\ (\because \phi = 0^\circ)$.
Hence, voltage and current are in the same phase. The AC circuit is non-inductive.
- When $X_L > X_C$, then $\tan \phi$ is positive.
Hence, voltage leads the current by a phase angle ϕ. The AC circuit is inductance dominated circuit.
- When $X_C > X_L$, then $\tan \phi$ is negative.
Hence, voltage lags behind the current by a phase angle ϕ. The AC circuit is capacitance dominated circuit.

The variation of *Z* with frequency ω is shown in figure below. At $\omega = \omega_0$, $X_L = X_C$, $Z = R$ = minimum.



Series *R-L-C* circuit can be further reduced into *R-L*, *R-C* and *L-C* circuits, whose circuit characteristics are given in the tabular form below

Characteristics	*R-L* circuit	*R-C* circuit	*L-C* circuit
Circuit	 $V_R=iR$, $V_L=iX_L$ $V=V_0 \sin \omega t$	 $V_R=iR$, $V_C=iX_C$ $V=V_0 \sin \omega t$	 $V_L=iX_L$, $V_C=iX_C$ $V=V_0 \sin \omega t$
Current	$i = i_0 \sin(\omega t - \phi)$	$i = i_0 \sin(\omega t + \phi)$	$i = i_0 \sin\left(\omega t \pm \frac{\pi}{2}\right)$
Peak current	$i_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + X_L^2}}$ $= \frac{V_0}{\sqrt{R^2 + 4\pi^2 v^2 L^2}}$	$i_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + X_C^2}}$ $= \frac{V_0}{\sqrt{R^2 + \frac{1}{4\pi^2 v^2 C^2}}}$	$i_0 = \frac{V_0}{Z} = \frac{V_0}{X_L - X_C}$ $= \frac{V_0}{\omega L - \frac{1}{\omega C}}$
Phasor diagram			
Applied voltage	$V = \sqrt{V_R^2 + V_L^2}$	$V = \sqrt{V_R^2 + V_C^2}$	$V = V_L - V_C$
Impedance	$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$ $= \sqrt{R^2 + 4\pi^2 v^2 L^2}$	$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$	$Z = X_L - X_C = X$
Phase difference	$\phi = \tan^{-1} \frac{X_L}{R}$ $= \tan^{-1} \frac{\omega L}{R}$	$\phi = \tan^{-1} \frac{X_C}{R}$ $= \tan^{-1} \frac{1}{\omega CR}$	$\phi = 90°$
Leading quantity	Voltage	Current	Either voltage or current

Example 1. *An alternating voltage $V(t) = 220 \sin 100\pi t$ volt is applied to a purely resistive load of 50 Ω. The time taken for the current to rise from half of the peak value to the peak value is* **[JEE Main 2019]**

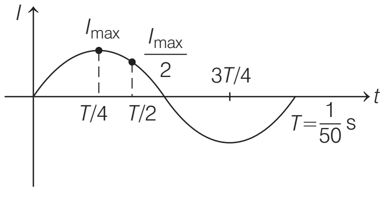
(a) 5 ms *(b)* 2.2 ms *(c)* 7.2 ms *(d)* 3.3 ms

Sol. (*d*) In an AC resistive circuit, current and voltage are in phase.

So, $$I = \frac{V}{R} \Rightarrow I = \frac{220}{50} \sin(100\pi t)$$

∴ Time period of one complete cycle of current is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{100\pi} = \frac{1}{50} \text{ s}$$



So, current reaches its maximum value at

$$t_1 = \frac{T}{4} = \frac{1}{200} \text{ s}$$

When current is half of its maximum value, then from Eq. (i), we have

$$I = \frac{I_{max}}{2} = I_{max} \sin(100\pi t_2)$$

$$\Rightarrow \quad \sin(100\pi t_2) = \frac{1}{2}$$

$$\Rightarrow \quad 100\pi t_2 = \frac{5\pi}{6}$$

So, instantaneous time at which current is half of maximum value is $t_2 = \frac{1}{120}$ s

Hence, time duration in which current reaches half of its maximum value after reaching maximum value is

$$\Delta t = t_2 - t_1 = \frac{1}{120} - \frac{1}{200} = \frac{1}{300} \text{ s} = 3.3 \text{ ms}$$

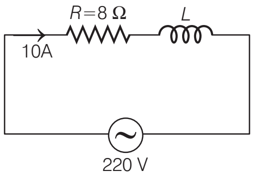
Example 2. *An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to* **[JEE Main 2016]**

(a) 80 H (b) 0.08 H
(c) 0.044 H (d) 0.065 H

Sol. (d) Given, $I = 10$ A, $V = 80$ V,

$$R = \frac{V}{I} = \frac{80}{10} = 8\ \Omega \quad \text{and} \quad \omega = 50 \text{ Hz}$$

For AC circuit, we have



$$I = \frac{V}{\sqrt{8^2 + X_L^2}}$$

$$\Rightarrow \quad 10 = \frac{220}{\sqrt{64 + X_L^2}} \Rightarrow \sqrt{64 + X_L^2} = 22$$

Squaring on both sides, we get

$$64 + X_L^2 = 484$$

$$\Rightarrow \quad X_L^2 = 484 - 64 = 420$$

$$\Rightarrow \quad X_L = \sqrt{420} \Rightarrow 2\pi \times \omega L = \sqrt{420}$$

Series inductor on an arc lamp, $L = \dfrac{\sqrt{420}}{(2\pi \times 50)} = 0.065$ H

Example 3. *When an alternating voltage of 220 V is applied across a device X, a current of 0.5 A flows through the circuit and is in phase with the applied voltage. When the same voltage is applied across another device Y, the same current again flows through the circuit, but it lags behind the applied voltage by (π /2) rad. Calculate the current flowing in the circuit when same voltage is applied across the series combination of X and Y.*

(a) 0.5 A (b) 0.3 A
(c) 2.5 A (d) 4.5 A

Sol. (b) The current and voltage are in phase with each other, when alternating voltage is applied across a resistor. Hence, the device X is resistor.

Obviously, $R = \dfrac{V_{\text{rms}}}{i_{\text{rms}}} = \dfrac{220}{0.5} = 440\ \Omega$

The current lags behind the voltage by phase angle $\pi/2$, when alternating voltage is applied across an inductor. Hence, the device Y is an inductor.

Obviously, $X_L = \dfrac{V_{\text{rms}}}{I_{\text{rms}}} = \dfrac{220}{0.5} = 440\ \Omega$

Here, $V_{\text{rms}} = 220$ V, $R = 440\ \Omega$, $X_L = 440\ \Omega$

If Z is impedance of L-R circuit, then $Z = \sqrt{R^2 + X_L^2}$

$$= \sqrt{440^2 + 440^2} = 400\sqrt{2}\ \Omega$$

Therefore, current in the L-R circuit,

$$i_{\text{rms}} = \frac{E_V}{Z} = \frac{220}{440\sqrt{2}} = 0.3535 \text{ A}$$

Resonance in Series *L-C-R* Circuit

It is a condition in a series L-C-R circuit, when at particular frequency ω_0, $X_L = X_C$ and the impedance is minimum ($Z = R$).

This frequency is called resonant frequency and given as

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{or} \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Resonant frequency is independent of resistance.

At resonating frequency,

$$Z = R = Z_{\min} \quad \text{and} \quad I = \frac{V}{Z} = I_{\max}$$

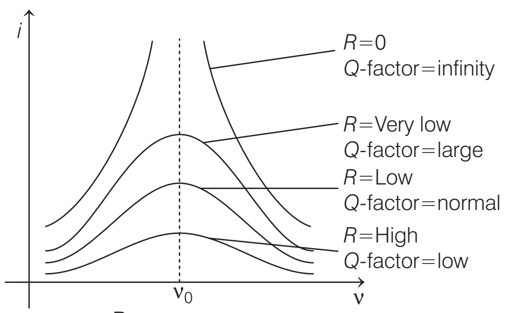
Q-Factor of Series Resonant Circuit

It is the measure of sharpness of the resonance of an L-C-R circuit. It is defined as the ratio of voltage developed across the inductance or capacitance at resonance to the impressed voltage, which is the voltage applied across R.

$$Q\text{-factor} = \frac{V_L \text{ or } V_C}{V_R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

It is also defined as

$$Q = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipation}}$$



Resonance curve

The quantity ($Q = \omega_0 / 2\Delta\omega$) is also the measure of sharpness of resonance, where $\omega_1 - \omega_2 = 2\Delta\omega$ is called the bandwidth of the circuit, such that $\omega_1 = \omega_0 + \Delta\omega$ and $\omega_2 = \omega_0 - \Delta\omega$.

Example 4. *In a series resonant L-C-R circuit, the voltage across R is 100 V and R = 1kΩ with C = 2 μF. The resonant frequency ω is 200 rad/s. At resonance, the voltage across L is*

(a) 2.5×10^{-2} V (b) 40 V
(c) 250 V (d) 4×10^{-3} V

Sol. (c) At resonance, $\omega L = 1/\omega C$

Current flowing through the circuit, $I = \frac{V_R}{R} = \frac{100}{1000} = 0.1$ A

So, voltage across L is given by $V_L = I X_L = I\omega L$.

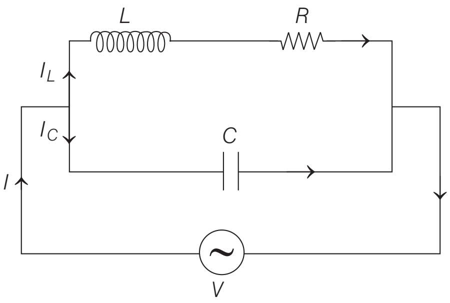
But $\omega L = 1/\omega C$

$\therefore \quad V_L = \frac{I}{\omega C} = V_C = \frac{0.1}{200 \times 2 \times 10^{-6}} = 250$ V

Parallel Circuit (Rejector Circuit)

Consider an alternating source connected across an inductance L in parallel with a capacitor C.

The resistance R is in series with the inductance.



Let the instantaneous value of emf applied be V and the corresponding current be I. Then,

$$I = I_L + I_C$$

$$\Rightarrow \quad \frac{1}{Z} = \frac{1}{R + j\omega L} + j\omega C$$

$\frac{1}{Z}$ is known as admittance (Y). Admittance is defined as the reciprocal of the impedance. Its unit is mho.

$\therefore$ The magnitude of the admittance,

$$|Y| = \left|\frac{1}{Z}\right| = \frac{\sqrt{R^2 + (\omega CR^2 + \omega^3 L^2 C - \omega L)^2}}{R^2 + \omega^2 L^2}$$

The admittance will be minimum, when

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

It gives the condition of resonance and the corresponding frequency, is known as *resonance frequency*. It is given as

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

At resonance frequency, admittance is minimum or the impedance is maximum.
Thus, the parallel circuit does not allow this frequency voltage or current from the source to pass in the circuit. Due to this reason the circuit with such a frequency is known as *rejector circuit*.

At resonance, the reactive component of **Y** vanishes or **Y** is real. The reciprocal of the admittance is called the *parallel resistor* or the *dynamic resistance*.

The dynamic resistance is thus, reciprocal of the real part of the admittance.

$\therefore$ Dynamic resistance $= \frac{R^2 + \omega^2 L^2}{R}$

$\therefore$ Peak current through the supply $= \frac{V_0}{L/CR} = \frac{V_0 CR}{L}$

The peak current through capacitor $= \frac{V_0}{1/\omega C} = \omega C V_0$.

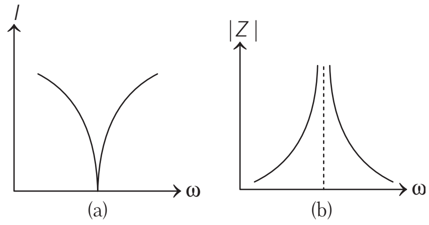
The ratio of the peak current through capacitor and through the supply is known as ***Q*-factor**.

Thus, Q-factor $= \frac{V_0 \omega C}{V_0 CR/L} = \frac{\omega L}{R}$

This is basically the measure of current magnification.

The rejector circuit at resonance exhibits current magnification of $\omega L/R$, similar to the voltage magnification of the same ratio exhibited by the series acceptor circuit at resonance.

Fig. (a) shows the variation of current I with angular frequency ω in parallel LC circuit (*i.e.* $R = 0$) and Fig. (b) shows the variation of impedance Z with ω.



Note At resonance, the current through the supply and voltage are in phase, while the current through the capacitor leads the voltage by 90°.

Example 5. *An AC circuit has $R = 100\ \Omega$, $C = 2\ \mu F$ and $L = 80$ mH connected in series. The quality factor of the circuit is* **[JEE Main 2020]**

(a) 2
(b) 0.5
(c) 20
(d) 400

Sol. (a) Given, $R = 100\ \Omega$, $C = 2\ \mu$F, $L = 80$ mH

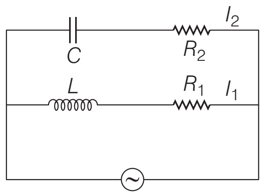
For a series L - C - R AC circuit,

Quality factor, $\phi = \frac{1}{R}\sqrt{\frac{L}{C}}$

$$= \frac{1}{100}\sqrt{\frac{80 \times 10^{-3}}{2 \times 10^{-6}}}$$

$\phi = 2$

Example 6. *In the below circuit, $C = \frac{\sqrt{3}}{2}\ \mu F$, $R_2 = 20\ \Omega$, $L = \frac{\sqrt{3}}{10}$ H and $R_1 = 10\ \Omega$. Current in L - R_1 path is I_1 and in C - R_2 path is I_2. The voltage of AC source is given by $V = 200\sqrt{2}\sin(100t)$ volts. The phase difference between I_1 and I_2 is*



[JEE Main 2019]

(a) 30° (b) 60° (c) 0° (d) 90°

Sol. (a) Phase difference between I_2 and V, *i.e.* C-R_2 circuit is given by

$$\tan\phi = \frac{X_C}{R_2}$$

$$\Rightarrow \quad \tan\phi = \frac{1}{C\omega R_2}$$

Substituting the given values, we get

$$\tan\phi = \frac{1}{\frac{\sqrt{3}}{2}\times 10^{-6}\times 100\times 20} = \frac{10^3}{\sqrt{3}}$$

$\therefore \phi_1$ is nearly 90°.

Phase difference between I_1 and V, *i.e.* in L-R_1 circuit is given by

$$\tan\phi_2 = -\frac{X_L}{R_1} = -\frac{L\omega}{R}$$

Substituting the given values, we get

$$\tan\phi_2 = -\frac{\frac{\sqrt{3}}{10}\times 100}{10} = -\sqrt{3}$$

As, $\tan\phi_2 = -\sqrt{3}$

$\therefore \quad \phi_2 = 120°$

Now, phase difference between I_1 and I_2 is

$$\Delta\phi = \phi_2 - \phi_1 = 120° - 90° = 30°$$

Power in an AC Circuit

It is defined as the rate at which work is being done in the circuit.

Instantaneous power of AC is given by

$$P = Vi = V_0 i_0 \sin\omega t \sin(\omega t + \phi)$$

Average power in an AC circuit,

$$P_{av} = V_{rms}\, i_{rms} \cos\phi = \frac{V_0}{\sqrt{2}}\frac{i_0}{\sqrt{2}}\cos\phi$$

where, $\cos\phi = \frac{\text{resistance } (R)}{\text{impedance } (Z)}$ is called the power factor of AC circuit.

Power factor is defined as cosine of the angle of lag or lead (*i.e.* $\cos\phi$). It is also defined as the ratio of resistance and impedance.

$$\cos\phi = \frac{P_{av}}{V_{rms}\, I_{rms}} = \frac{\text{True power}}{\text{Apparent power}}$$

The product of V_{rms} and I_{rms} gives the apparent power (or virtual power). While the true power is obtained by multiplying the apparent power by the power factor $\cos\phi$.

Thus, apparent power = $V_{rms} \times I_{rms}$

and true power = apparent power × power factor

Wattless Current

The current which consumes no power for its maintenance in the circuit is called wattless current or idle current.

or

If the resistance in an AC circuit is zero, its power factor will be zero. Although the current flows in the circuit, yet the average power remains zero, *i.e.* there is no energy dissipation in the circuit. Such a circuit is called the wattless circuit and the current flowing is called the wattless current.

If the circuit contains either inductance or capacitance only, then phase difference between current and voltage is 90°, *i.e.* $\phi = 90°$. The average power in such a circuit is

$$P_{av} = V_{rms} \times I_{rms} \times \cos\phi = V_{rms} \times I_{rms} \times \cos 90° = 0$$

The concept of wattless current is similar to that of a frictionless pendulum, where the total work done by gravity upon the pendulum in a cycle is zero.

Special Cases

***Case* I** When AC circuit contains R only. In this case, $\phi = 0$

$$\therefore \quad P_{av} = V_{rms} \times i_{rms} = \frac{V_{rms}^2}{R}$$

***Case* II** When AC circuit contains only capacitor.

In this case, $\phi = -\frac{\pi}{2}$

$$\therefore \quad P_{av} = 0$$

***Case* III** When AC circuit contains only inductance.

In this case, $\phi = \frac{\pi}{2}$

$$\therefore \quad P_{av} = 0$$

***Case* IV** When AC circuit contains resistance and capacitance both, then

$$\tan\phi = \frac{1/\omega C}{R}$$

and

$$\cos\phi = \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

$$P_{av} = \frac{V_{rms}^2 R}{\left(R^2 + \frac{1}{\omega^2 C^2}\right)}$$

***Case* V** When AC circuit contains resistance and inductance both.

Now, $\tan\theta = \dfrac{\omega L}{R}$ and $\cos\theta = \dfrac{R}{\sqrt{R^2 + \omega^2 L^2}}$

$$\therefore \quad P_{av} = \frac{V_{rms}^2 R}{(R^2 + \omega L^2)}$$

***Case* VI** When AC circuit contains inductance, capacitance and resistance, then

$$\tan\theta = \frac{\omega L - \dfrac{1}{\omega C}}{R}$$

$$\text{and} \quad \cos\theta = \frac{R}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$$

$$\therefore \quad P_{av} = V_{rms} \times i_{rms} \times \frac{R}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$$

Example 7. *In a series L-C-R circuit $R = 200\ \Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz, respectively. On taking out the capacitance from the circuit, the current lags behind the voltage by 30°. On taking out the inductor from the circuit, the current leads the voltage by 30°. The power dissipated in the L-C-R circuit is*

(a) 305 W *(b)* 210 W
(c) zero (d) 242 W

Sol. (*d*) The given circuit is under resonance as $X_L = X_C$.

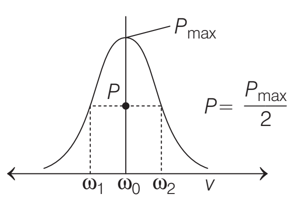
($\because$ same phase change in *L-R* and *C-R* circuits)

Hence, power dissipated in the circuit is

$$P = \frac{V^2}{R} = 242 \text{ W}$$

Half Power Points in Series *L-C-R* Circuit

The values of ω at which the power input is half of its maximum value are called half power points.
At these points,



$$\omega = \omega_0 \pm \Delta\omega = \omega_0 \pm \frac{1}{2}\left(\frac{2\Delta\omega}{\omega_0}\right)\omega_0 = \omega_0\left(1 \pm \frac{1}{2Q}\right)$$

The amplitude of the current falls by a factor of $\dfrac{1}{\sqrt{2}}$ and hence, the power ($P \propto I^2$) goes down by half.

The separation between these two half power points is $2\Delta\omega = \dfrac{\omega_0}{Q}$. This separation is called the *full-width* at half maximum and is $\dfrac{1}{Q}$ th fraction of the resonant frequency.

Example 8. *In an AC circuit, the instantaneous emf and current are given by*

$$e = 100 \sin 30t,\ i = 20 \sin\left(30t - \frac{\pi}{4}\right)$$

In one cycle of AC, the average power (in W) consumed by the circuit and the wattless current (in A) are, respectively

[JEE Main 2018]

(a) 50 , 10 *(b)* $\dfrac{1000}{\sqrt{2}}$, 10 *(c)* $\dfrac{50}{\sqrt{2}}$, 0 *(d)* 50 , 0

Sol. (*b*) Given, $e = 100 \sin 30t$ and $i = 20 \sin\left(30t - \dfrac{\pi}{4}\right)$

$\therefore$ Average power, $P_{av} = V_{rms} I_{rms} \cos\phi$

$$= \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \cos\frac{\pi}{4} = \frac{1000}{\sqrt{2}} \text{ W}$$

Wattless current, $I = I_{rms} \sin\phi = \dfrac{20}{\sqrt{2}} \times \sin\dfrac{\pi}{4} = \dfrac{20}{2} = 10$ A

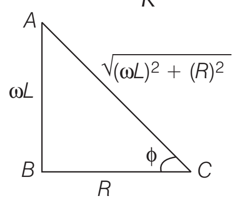
$$\therefore \quad P_{av} = \frac{1000}{\sqrt{2}} \text{ watt} \quad \text{and} \quad I_{wattless} = 10 \text{ A}$$

Example 9. *The power factor of an AC circuit having resistance R and inductance L (connected in series) and an angular velocity ω is*

(a) $\dfrac{R}{\omega L}$ *(b)* $\dfrac{R}{(R^2 + \omega^2 L^2)^{1/2}}$

(c) $\dfrac{\omega L}{R}$ *(d)* $\dfrac{R}{(R^2 - \omega^2 L^2)^{1/2}}$

Sol. (*b*) From the relation, $\tan\phi = \dfrac{\omega L}{R}$



Power factor, $\cos\phi = \dfrac{1}{\sqrt{1 + \tan^2\phi}}$

$$\Rightarrow \quad \cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

$$= \frac{1}{\sqrt{1 + (\omega L/R)^2}}$$

$$= \frac{1}{\sqrt{1 + \tan^2\phi}}$$

$$= \frac{1}{\sqrt{1 + (\omega L/R)^2}} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

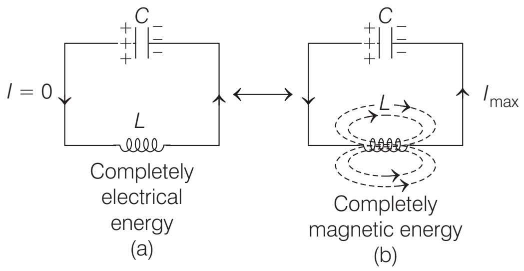
L-C Oscillations

When a charged capacitor is allowed to discharge through a non-resistive inductor, electrical oscillations of constant amplitude and frequency are produced. These oscillations are called *L-C* oscillations. The equation of *L-C* oscillations is given by

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$

and the charge oscillates with a frequency,

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$



Growth and Decay of Current in *L-R* Circuit

If a circuit containing a pure inductor L and a resistor R in series with a battery and a key is closed, then the current through the circuit rises exponentially and reaches upto a certain maximum value (steady state).

If circuit is opened from its steady state condition, battery is removed and it is closed again, then current through the circuit decreases exponentially.

In case of growth of current at time $t = 0$, inductor offers infinite resistance and at time $t = \infty$ (steady state) inductor offers zero resistance. This type of circuit is called *L-R* circuit.

Growth of Current in *L-R* Circuit

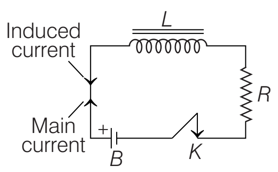
Growth of current in *L-R* circuit at any instant of time t is given by $I = I_0\,(1 - e^{-Rt/L})$

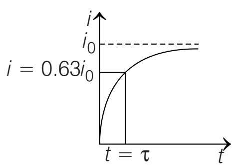
or $I = I_0(1 - e^{-t/5})$

where, I_0 = maximum current, L = self-inductance of the inductor and R = resistance of the circuit.

Here, $\frac{L}{R} = \tau$ is called *time constant* of an *L-R* circuit.

Time constant of an *L-R* circuit is the time in which current in the circuit grows to 63.2% of the maximum value of current.





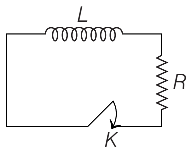
Growth of current

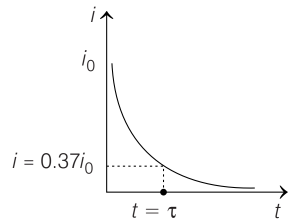
Decay of Current in *L-R* Circuit

Decay of current in *L-R* circuit at any time t is given by

$$I = I_0\, e^{-Rt/L} \text{ or } I = I_0 e^{-t/\tau}$$

Time constant (τ) of an *L-R* circuit is the time in which current decays to 36.8% of the maximum value of current (I_0).





Choke Coil

It is a device having high inductance and negligible resistance. It is used in AC circuits for the purpose of adjusting current to any required value in such a way that power loss in the circuit can be minimised. It is used in fluorescent tubes.

It is based on the principle of wattless current.

Transformer

The device used for converting low alternating voltage at high current into high voltage at low current and *vice-versa*.

Principle It works on the principle of mutual induction, *i.e.* if two coils are inductively coupled and when current or magnetic flux is changed through one of the two coils, then induced emf is produced in the other coil.

Types of Transformers

There are two types of transformers

Step-up Transformer The transformer used to change low voltage alternating emf to high voltage alternating emf (of same frequency) is called as step-up transformer.

In this transformer, $n_s > n_p, V_s > V_p$ and $i_p > i_s$

Step-down Transformer The transformer used to change high voltage alternating emf to low voltage alternating emf of same frequency is called a step-down transformer. In this transformer,

$$n_p > n_s,\, v_p > v_s \text{ and } i_p < i_s$$

where, n_p = number of turns in primary coil,

n_s = number of turns in secondary coil,

V_p = voltage drop across primary coil

and V_s = voltage induced across secondary coil.

Transformation ratio

If the number of turns in the primary coil is n_p, the number of turns in the secondary coil is n_s and the magnetic flux linked with each turns is ϕ, then for an ideal transformer when there is no leakage of flux, the magnetic flux linked with each turn of primary and secondary coil will be ϕ. Then,

$$\frac{V_s}{V_p} = \frac{n_s}{n_p}$$

This is known as transformation ratio.

Efficiency of Transformer

Efficiency of a transformer

$$= \frac{\text{Energy obtained from the secondary coil}}{\text{Energy given to the primary coil}}$$

$$= \frac{\text{Output power}}{\text{Input power}}$$

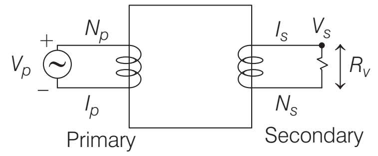
$$= \frac{V_s i_s}{V_p i_p}$$

Example 10. *A power transmission line feeds input power at 2300 V to a step-down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V by the transformer. If the current in the primary of the transformer is 5A and its efficiency is 90%, the output current would be* **[JEE Main 2019]**

(a) 45 A (b) 50 A

(c) 25 A (d) 35 A

Sol. (a) For a transformer, there are two circuits which have N_p and N_s (number of coil turns), I_p and I_s (currents) respectively as shown below



Here, input voltage, $V_p = 2300$ V

Number of turns in primary coil, $N_p = 4000$

Output voltage, $V_s = 230$ V

Output power, $P_s = V_s \cdot I_s$

Input power, $P_p = V_p I_p$

$\therefore$ The efficiency of the transformer is

$$\eta = \frac{\text{Output (secondary) power}}{\text{Input (primary) power}}$$

$$= \frac{V_s \cdot I_s}{V_p \cdot I_p} \times 100$$

$$\Rightarrow \quad \eta = \frac{(230)\,(I_s)}{(2300)\,(5)} \times 100$$

$$90 = \frac{230\, I_s}{(2300) \times 5} \times 100$$

$$\Rightarrow \quad I_s = 45 \text{ A}$$

Practice Exercise

ROUND I Topically Divided Problems

Peak Value, RMS Value and Average Value of AC

1. The voltage of an AC supply varies with time (t) as $V = 120\sin 100\pi t \cos 100\pi t$. The maximum voltage and frequency respectively are

(a) 120 V 100 Hz
(b) $\frac{120}{\sqrt{2}}$ V, 100 Hz
(c) 60 V, 200 Hz
(d) 60 V, 100 Hz

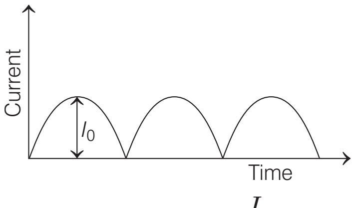
2. An alternating current having peak value 14 A used to heat a metal wire. To produce the same heating effect, a constant current I can be used, where I is

(a) 14 A
(b) 20 A
(c) 7 A
(d) 10 A

3. If the rms current in a 50 Hz AC circuit is 5 A, the value of the current (1/300) s after its value becomes zero is **[NCERT Exemplar]**

(a) $5\sqrt{2}$ A
(b) $5\sqrt{3/2}$ A
(c) (5/6) A
(d) $(5/\sqrt{2})$ A

4. The output sinusoidal current *versus* time curve of a rectifier is shown in the figure. The average value of output current in this case is



(a) 0
(b) $\frac{I_0}{2}$
(c) $\frac{2I_0}{\pi}$
(d) I_0

5. An alternating current is given by $I = I_1 \cos \omega t + I_2 \sin \omega t$. The root mean square current is given by

(a) $\frac{(I_1 + I_2)}{\sqrt{2}}$
(b) $\frac{(I_1 + I_2)^2}{2}$
(c) $\sqrt{\frac{I_1^2 + I_2^2}{2}}$
(d) $\frac{\sqrt{I_1^2 - I_2^2}}{2}$

6. An AC voltage source has an output of $V = (200\text{ V}) \sin 2\pi ft$. This source is connected to a 100 Ω resistor. Rms current in the resistance is

(a) 1.41 A
(b) 2.41 A
(c) 3.41 A
(d) 0.71 A

AC Circuits

7. A 60 μF capacitor is connected to a 110 V, 60 Hz AC supply. Determine the rms value of the current in the circuit. **[NCERT]**

(a) 2.5 A
(b) 2.1 A
(c) 3.1 A
(d) 3.5 A

8. A group of electric lamps having a total power rating of 1000 W is connected by an AC voltage $E = 200\sin(310t + 60°)$, then the rms value of the circuit current is

(a) 10 A
(b) $10\sqrt{2}$ A
(c) 20 A
(d) $20\sqrt{2}$ A

9. A circuit when connected to an AC source of 12 V gives a current of 0.2 A. The same circuit when connected to a DC source of 12 V, gives a current of 0.4 A. The circuit is

(a) series L-R
(b) series R-C
(c) series L-C
(d) series L-C-R

10. In AC series circuit, the resistance, inductive reactance and capacitive reactance are 3 Ω, 10 Ω and 14 Ω, respectively. The impedance of the circuit is

(a) 5 Ω
(b) 4 Ω
(c) 7 Ω
(d) 10 Ω

11. In an L-C-R circuit, $R = 100\ \Omega$. When capacitance C is removed, the current lags behind the voltage by $\pi/3$. When inductance L is removed, the current leads the voltage by $\pi/3$. The impedance of the circuit is

(a) 50 Ω
(b) 100 Ω
(c) 200 Ω
(d) 400 Ω

12. For a series R-L-C circuit, $R = X_L = 2X_C$. The impedance of the circuits and phase difference (between) V and i will be

(a) $\frac{\sqrt{5}R}{2}, \tan^{-1}(2)$
(b) $\frac{\sqrt{5}R}{2}, \tan^{-1}\left(\frac{1}{2}\right)$
(c) $\sqrt{5}X_C, \tan^{-1}(2)$
(d) $\sqrt{5}R, \tan^{-1}\left(\frac{1}{2}\right)$

13. An inductance coil has a reactance of 100 Ω. When an AC signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. The self-inductance of the coil is **[JEE Main 2020]**

(a) 1.1×10^{-2} H
(b) 1.1×10^{-1} H
(c) 5.5×10^{-5} H
(d) 6.7×10^{-7} H

14. A 100 μF capacitor in series with a 40 Ω resistance is connected to a 110 V-60 Hz supply. Calculate the maximum current in the circuit and the phase lag between the current maximum and voltage maximum?

(a) 1.5 A, 33° 33′
(b) 1.5 A, 60°
(c) 3.2 A, 33° 33′
(d) 3.2 A, 60°

15. An AC source of angular frequency ω is fed across a resistor R and a capacitor C in series. The current registered is I. If now the frequency of source is changed to $\omega/3$ (but maintaining the same voltage), the current in then circuit is found to be halved. Calculate the ratio of reactance to resistance at the original frequency ω.

(a) $\sqrt{\frac{3}{5}}$ (b) $\sqrt{\frac{2}{5}}$
(c) $\sqrt{\frac{1}{5}}$ (d) $\sqrt{\frac{4}{5}}$

16. A series of R-C circuit is connected to AC voltage source. Consider two cases : (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true? **[IIT JEE 2011]**

(a) $I_R^A > I_R^B$ (b) $I_R^A < I_R^B$
(c) $V_C^A = V_C^B$ (d) $V_C^A < V_C^B$

17. In a series L-C-R circuit, the inductive reactance X_L is 10 Ω and the capacitive reactance X_C is 4 Ω. The resistance R in the circuit is 6 Ω. The power factor of the circuit is **[JEE Main 2021]**

(a) $\frac{1}{2}$ (b) $\frac{1}{2\sqrt{2}}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

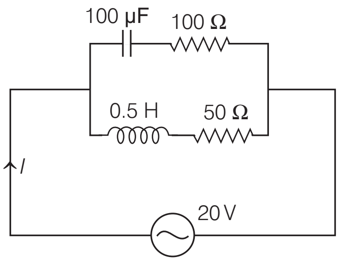
18. In L-C circuit, the inductance L = 40 mH and capacitance C = 100 μF. If a voltage $V(t) = 10\sin(314t)$ is applied to the circuit, the current in the circuit is given as **[JEE Main 2020]**

(a) $0.52\cos 314\,t$ (b) $0.52\sin 314\,t$
(c) $10\cos 314\,t$ (d) $5.2\cos 314\,t$

19. The phase difference between the alternating current and emf is $\pi/2$. Which of the following cannot be the constituent of the circuit?

(a) C alone (b) R, L (c) L, C (d) L alone

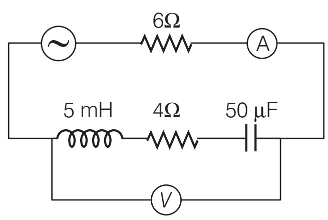
20. In the given circuit, the AC source has $\omega = 100$ rad/s Considering the inductor and capacitor to be ideal. The correct choice (s) is/are



(a) The current through the circuit I is 0.3 A
(b) The current through the circuit I is $0.3\sqrt{2}$ A
(c) The voltage across 100 Ω resistor/s = $10\sqrt{2}$ V
(d) The voltage across 50 Ω resistor/s = 10 V

Resonance and Quality Factor

21. In the circuit shown in the figure, the AC source gives a voltage $V = 20\cos(2000t)$. Neglecting source resistance, the voltmeter and ammeter reading will be



(a) 0 V, 0.47 A (b) 1.68 V, 0.47 A
(c) 0 V, 1.4 A (d) 5.6 V, 1.4 A

22. In a series L-C-R resonance circuit, if we change the resistance only, from a lower to higher value **[JEE Main 2021]**

(a) the bandwidth of resonance circuit will increase
(b) the resonance frequency will increase
(c) the quality factor will increase
(d) the quality factor and the resonance frequency will remain constant

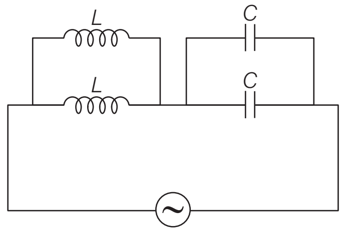
23. In an L-C-R circuit, capacitance is changed from C to $2C$. For the resonant frequency to remains unchanged, the inductance should be changed from L to

(a) $4L$ (b) $2L$ (c) $L/2$ (d) $L/4$

24. The self-inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of

(a) 4 μF (b) 8 μF
(c) 1 μF (d) 2 μF

25. Find resonance frequency in the given circuit.



(a) $\frac{1}{\sqrt{LC}}$ (b) $\frac{2}{\sqrt{LC}}$ (c) $\frac{1}{2\sqrt{LC}}$ (d) $\frac{4}{\sqrt{LC}}$

26. For an R-L-C circuit driven with voltage of amplitude V_m and frequency $\omega_0 = \frac{1}{\sqrt{L}}$, the current exhibits resonance. The quality factor Q is given by **[JEE Main 2018]**

(a) $\frac{\omega_0 L}{R}$ (b) $\frac{\omega_0 R}{L}$

(c) $\frac{R}{\omega_0 C}$ (d) $\frac{CR}{\omega_0}$

27. Which of the following combinations should be selected for better tuning of an L-C-R circuit used for communication? **[NCERT Exemplar]**

(a) $R = 20\ \Omega$, $L = 1.5\text{H}$, $C = 35\ \mu\text{F}$
(b) $R = 25\ \Omega$, $L = 2.5\text{ H}$, $C = 45\ \mu\text{F}$
(c) $R = 15\ \Omega$, $L = 3.5\text{H}$, $C = 30\ \mu\text{F}$
(d) $R = 25\ \Omega$, $L = 1.5\text{ H}$, $C = 45\ \mu\text{F}$

Power in AC Circuit

28. A coil of self-inductance L is connected in series with a bulb B and AC source. Brightness of the bulb decreases when

(a) frequency of the AC source is decreased
(b) number of turns in the coil is reduced
(c) a capacitance of reactance $X_C = X_L$ is included in the same circuit
(d) an iron rod is inserted in the coil

29. In an L-R circuit, the inductive reactance is equal to the resistance R of the circuit. An emf $E = E_0 \cos(\omega t)$ applied to the circuit. The power consumed in the circuit is

(a) $\frac{E_0^2}{R}$ (b) $\frac{E_0^2}{2R}$

(c) $\frac{E_0^2}{4R}$ (d) $\frac{E_0^2}{8R}$

30. The self-inductance of a choke coil is 10 mH. When it is connected with a 10 V DC source, then the loss of power is 20 W. When it is connected with 10 V AC source loss of power is 10 W, the frequency of AC source will be

(a) 50 Hz (b) 60 Hz
(c) 80 Hz (d) 100 Hz

31. $\frac{2.5}{\pi}\ \mu\text{F}$ capacitor and 3000 Ω resistance are joined in series to an AC source of 200 V and 50 s^{-1} frequency. The power factor of the circuit and the power dissipated in it will respectively

(a) 0.6, 0.06 W (b) 0.06, 0.6 W
(c) 0.6, 4.8 W (d) 4.8, 0.6 W

32. A resistance R draws power P when connected to an AC source. If an inductance is now placed in series with the resistance, such that the impedance of the circuit becomes Z, the power drawn will be

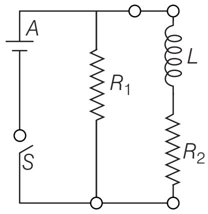
(a) $P\left(\frac{R}{Z}\right)$ (b) $P\sqrt{\frac{R}{Z}}$ (c) $P\,(R/Z)^2$ (d) P

33. In a series L-R circuit, power of 400W is dissipated from a source of 250 V, 50 Hz. The power factor of the circuit is 0.8. In order to bring the power factor to unity, a capacitor of value C is added in series to the L and R. Taking the value of C as $\left(\frac{n}{3\pi}\right)\mu\text{F}$, then value of n is **[JEE Main 2020]**

(a) 100 (b) 200
(c) 300 (d) 400

Growth and Decay of Current

34. An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 4\ \Omega$ and $R_2 = 2\ \Omega$ are connected to battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is



(a) $6e^{-5t}$ V (b) $\frac{12}{t}e^{-3t}$ V
(c) $6(1 - e^{-t/0.2})$ V (d) $12e^{-5t}$ V

35. An emf of 15 V is applied in a circuit coil containing 5 H inductance and 10 Ω resistance, the ratio of currents at time $t = \infty$ and $t = 1$ s is

(a) $\frac{e^{1/2}}{e^{1/2} - 1}$ (b) $\frac{e^2}{e^2 - 1}$
(c) $1 - e^{-1}$ (d) e^{-1}

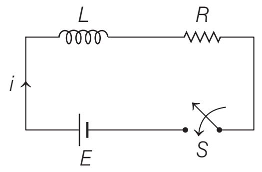
36. A coil of inductance 300 mH and resistance 2 Ω is connected to a source of voltage 2 V. The current reaches half of its steady state value in

(a) 0.05 s (b) 0.1 s
(c) 0.15 s (d) 0.3 s

37. A coil of self-inductance 10 mH and resistance 0.1 Ω is connected through a switch to a battery of internal resistance 0.9 Ω. After the switch is closed, the time taken for the current to attain 80% of the saturation value is (Take, ln 5 = 1.6) **[JEE Main 2019]**

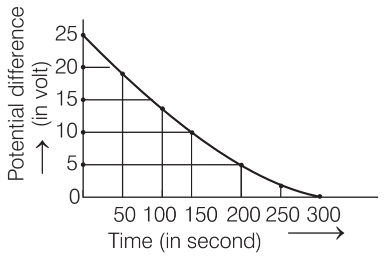
(a) 0.002 s (b) 0.324 s
(c) 0.103 s (d) 0.016 s

38. Consider the L-R circuit shown in the figure. If the switch S is closed at $t = 0$, then the amount of charge that passes through the battery between $t = 0$ and $t = \dfrac{L}{R}$ is **[JEE Main 2019]**



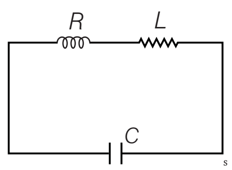
(a) $\dfrac{2.7\,EL}{R^2}$ (b) $\dfrac{EL}{2.7\,R^2}$
(c) $\dfrac{7.3\,EL}{R^2}$ (d) $\dfrac{EL}{7.3\,R^2}$

39. The figure shows an experimental plot discharging of a capacitor in an R-C circuit. The time constant τ of this circuit lies between

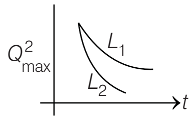


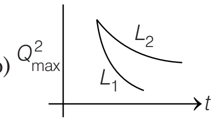
(a) 150 s and 200 s (b) 0 and 50 s
(c) 50 s and 100 s (d) 100 s and 150 s

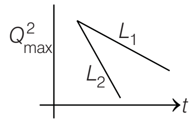
40. An L-C-R circuit is equivalent to a damped pendulum. In an L-C-R circuit, the capacitor is charged to Q_0 and then connected to the L and R as shown below

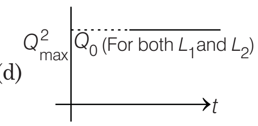


If a student plots graphs of the square of maximum charge (Q_{max}^2) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L, then which of the following represents this graph correctly? (Plots are schematic and not drawn to scale) **[JEE Main 2015]**

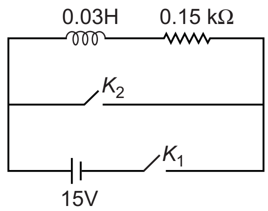
(a) 

(b) 

(c) 

(d) 

41. An inductor ($L = 0.03$ H) and a resistor ($R = 0.15$ kΩ) are connected in series to a battery of 15V emf in a circuit shown below. The key K_1 has been kept closed for a long time, then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1$ms, the current in the circuit will be ($e^5 \cong 150$) **[JEE Main 2015]**



(a) 100 mA (b) 67 mA (c) 6.7 mA (d) 0.67mA

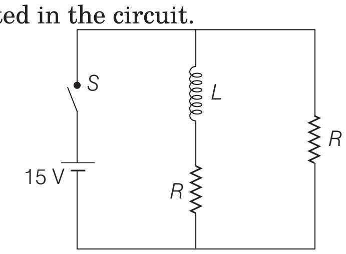
42. A series L-R circuit is connected to a battery of emf V. If the circuit is switched ON at $t = 0$, then the time at which the energy stored in the inductor reaches $\left(\dfrac{1}{n}\right)$ times of its maximum value, is **[JEE Main 2020]**

(a) $\dfrac{L}{R}\ln\left(\dfrac{\sqrt{n}+1}{\sqrt{n}-1}\right)$ (b) $\dfrac{L}{R}\ln\left(\dfrac{\sqrt{n}}{\sqrt{n}+1}\right)$
(c) $\dfrac{L}{R}\ln\left(\dfrac{\sqrt{n}-1}{\sqrt{n}}\right)$ (d) $\dfrac{L}{R}\ln\left(\dfrac{\sqrt{n}}{\sqrt{n}-1}\right)$

43. An emf of 20 V is applied at time $t = 0$ to a circuit containing in series 10 mH inductor and 5Ω resistor. The ratio of the currents at time $t = \infty$ and at $t = 40$ s is close to (Take, $e^2 = 7.389$) **[JEE Main 2020]**

(a) 1.06 (b) 1.15 (c) 1.46 (d) 0.84

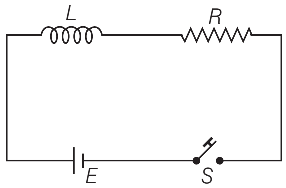
44. In the figure shown, a circuit contains two identical resistors with resistance $R = 5\,\Omega$ and an inductance with $L = 2$ mH. An ideal battery of 15 V is connected in the circuit. **[JEE Main 2019]**



What will be the current through the battery long after the switch is closed?

(a) 6 A (b) 3 A (c) 5.5 A (d) 7.5 A

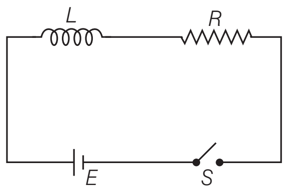
45. In the circuit shown in figure, switch S is closed at time $t = 0$. The charge which passes through the battery in one time constant is



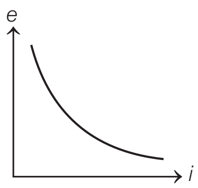
(a) $\dfrac{EL}{eR^2}$ (b) $\dfrac{eL}{ER}$

(c) $\dfrac{eR^2E}{L}$ (d) $E\left(\dfrac{L}{R}\right)$

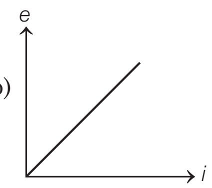
46. In an L-R circuit shown in figure, switch S is closed at time $t = 0$. If e denotes the induced emf across inductor and i, the current in the circuit at any time t, then which of the following graphs, shows the variation of e with i?



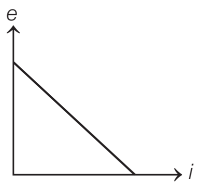
(a)



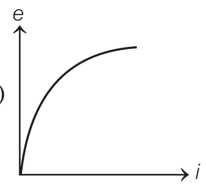
(b)



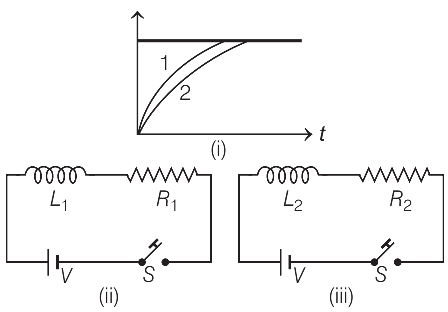
(c)



(d)



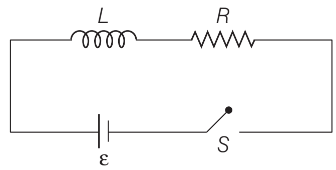
47. Current growth in two L-R circuits (ii) and (iii) is as shown in Fig (i). Let L_1, L_2, R_1 and R_2 be the corresponding values in two circuits, then



(a) $L_1 > L_2$ (b) $L_1 < L_2$

(c) $R_1 > R_2$ (d) $R_1 = R_2$

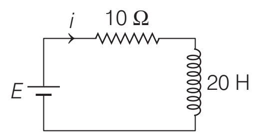
48. As shown in the figure, a battery of emf ε is connected to an inductor L and resistance R in series. The switch is closed at $t = 0$. The total charge that flows from the battery, between $t = 0$ and $t = t_c$ (t_c is the time constant of the circuit) is **[JEE Main 2020]**



(a) $\dfrac{\varepsilon L}{R^2}\left(1-\dfrac{1}{e}\right)$ (b) $\dfrac{\varepsilon L}{R^2}$

(c) $\dfrac{\varepsilon R}{eL^2}$ (d) $\dfrac{\varepsilon L}{eR^2}$

49. A 20 H inductor coil is connected to a 10 Ω resistance in series as shown in figure. The time at which rate of dissipation of energy (Joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is **[JEE Main 2019]**



(a) $\dfrac{2}{\ln 2}$ (b) $\dfrac{1}{2}\ln 2$ (c) $2\ln 2$ (d) $\ln 2$

L-C Oscillation, Choke Coil and Transformers

50. Choke coil is a device having
(a) low inductance and low resistance
(b) high inductance high resistance
(c) low inductance and high resistance
(d) high inductance and low resistance

51. A charged 40 μF capacitor is connected to a 16 mH inductor. What is the angular frequency of free oscillations of the circuit?

(a) 1.1 s (b) $1.25 \times 10^3\ \text{s}^{-1}$

(c) $2 \times 10^3\ \text{s}^{-1}$ (d) $2.5 \times 10^3\ \text{s}^{-1}$

52. In L-C circuit, the inductance $L = 40$ mH and capacitance $C = 100\,\mu\text{F}$. If a voltage $V(t) = 10\sin(314t)$ is applied to the circuit, the current in the circuit is given as **[JEE Main 2020]**

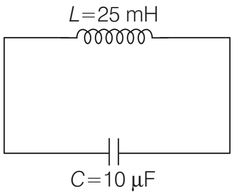
(a) $0.52\cos 314\,t$ (b) $0.52\sin 314\,t$

(c) $10\cos 314\,t$ (d) $5.2\cos 314\,t$

53. A 1 μF capacitor is charged to 25 V of potential. The battery is then disconnected and a pure 100 mH coil is connected across the capacitor, so that L-C oscillations are set up. The maximum current in the coil is

(a) 0.25 A (b) 0.01 A (c) 2.5 A (d) 1.6 A

54. If maximum energy is stored in a capacitor at $t = 0$, then find the time after which, current in the circuit will be maximum?



(a) $\frac{\pi}{2}$ ms (b) $\frac{\pi}{4}$ ms (c) π ms (d) 2 ms

55. The core of any transformer is laminated so as to
(a) reduce the energy loss due to eddy currents
(b) make it light weight
(c) make it robust and strong
(d) increase the secondary voltage

56. The turns ratio of transformer is given as 2 : 3. If the current passing through the primary coil is 3 A. Find the current through the load resistance.
(a) 4.5 A (b) 1.5 A (c) 2 A (d) 1 A

57. The number of turns in the primary coil of a transformer is 200 and the number of turns in secondary coil is 10. If 240 V AC is applied to the primary, the output from secondary will be
(a) 48 V (b) 24 V (c) 12 V (d) 6 V

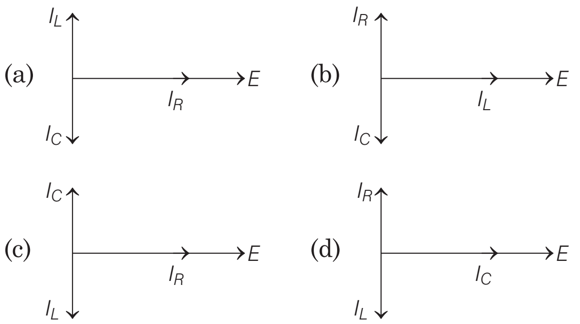
58. In an induction coil, the coefficient of mutual inductance is 4H. If current of 5A in the primary coil is cut-off for 1/1500 s, the emf at the terminals of the secondary coil will be
(a) 15 kV (b) 60 kV
(c) 10 kV (d) 30 kV

59. A transformer consisting of 300 turns in the primary and 150 turns in the secondary gives output power of 2.2 kW. If the current in the secondary coil is 10 A, then the input voltage and current in the primary coil are **[JEE Main 2019]**
(a) 440 V and 5 A
(b) 220 V and 20 A
(c) 220 V and 10 A
(d) 440 V and 20 A

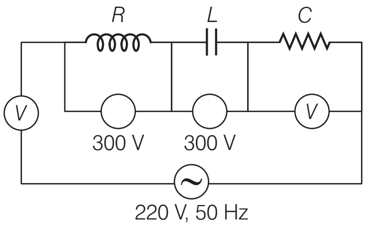
ROUND II Mixed Bag

Only One Correct Option

1. An alternating emf is applied across a parallel combination of a resistance R, capacitance C and an inductance L. If I_R, I_L and I_C are the currents through R, L and C respectively, then the diagram which correctly represents, the phase relationship among I_R, I_L, I_C and source emf E, is given by



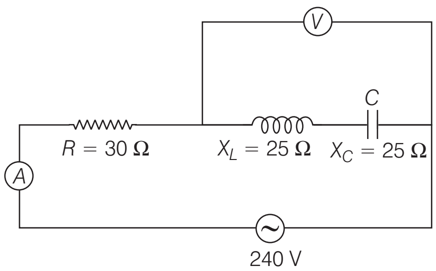
2. In the circuit shown below, what will be the readings of the voltmeter and ammeter?



(a) 800 V, 2A (b) 300 V, 2A
(c) 220 V, 2.2 A (d) 100 V, 2A

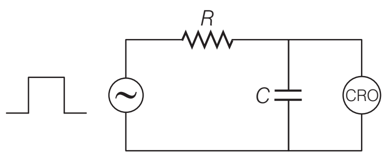
3. An AC source rated 220 V, 50 Hz is connected to a resistor. The time taken by the current to change from its maximum to the rms value is **[JEE Main 2021]**
(a) 2.5 ms (b) 25 ms
(c) 2.5 s (d) 0.25 ms

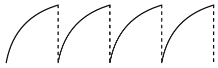
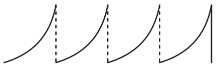
4. In the circuit shown in figure neglecting source resistance, the voltmeter and ammeter readings will be respectively,

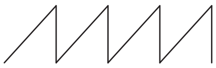


(a) 0 V, 3 A (b) 150 V, 3 A
(c) 150 V, 6 A (d) 0 V, 8 A

5. An R-C circuit as shown in the figure is driven by a AC source generating a square wave. The output wave pattern monitored by CRO would look close to



(a)  (b) 

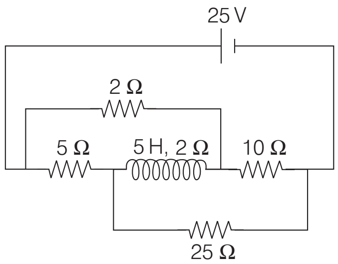
(c)  (d) 

6. If a circuit made up of a resistance 1 Ω and inductance 0.01 H; and alternating emf 200 V at 50 Hz is connected, then the phase difference between the current and the emf in the circuit is

(a) $\tan^{-1}(\pi)$ (b) $\tan^{-1}\left(\frac{\pi}{2}\right)$

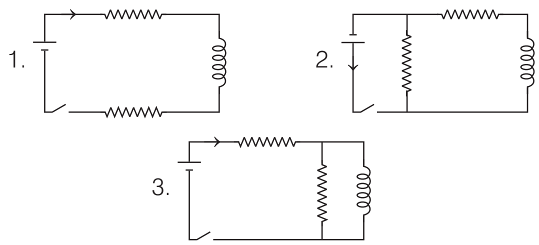
(c) $\tan^{-1}\left(\frac{\pi}{4}\right)$ (d) $\tan^{-1}\left(\frac{\pi}{3}\right)$

7. In the circuit shown, what is the energy stored in the coil at steady state?



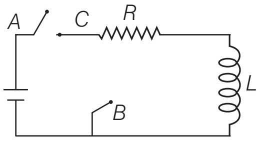
(a) 21.3 J (b) 42.6 J (c) Zero (d) 213 J

8. The figure shows three circuits with identical batteries, inductors and resistance. Rank the circuits according to the currents through the battery just after the switch is closed, greatest first.



(a) $i_2 > i_3 > i_1$ (b) $i_2 > i_1 > i_3$
(c) $i_1 > i_2 > i_3$ (d) $i_1 > i_3 > i_2$

9. In the circuit shown here, the point C is kept connected to point A till the current flowing through the circuit becomes constant. Afterward, suddenly point C is disconnected from point A and connected to point B at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L / R$ will be equal to **[JEE Main 2014]**



(a) $\frac{e}{1-e}$ (b) 1 (c) -1 (d) $\frac{1-e}{e}$

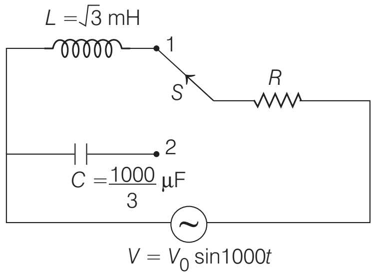
10. A circuit connected to an AC source of emf $e = e_0 \sin(100t)$ with t in seconds, gives a phase difference of $\frac{\pi}{4}$ between the emf e and current i. Which of the following circuits will exhibit this? **[JEE Main 2019]**

(a) R-C circuit with $R = 1\,\text{k}\Omega$ and $C = 1\,\mu\text{F}$
(b) R-L circuit with $R = 1\,\text{k}\Omega$ and $L = 1$ mH
(c) R-C circuit with $R = 1\,\text{k}\Omega$ and $C = 10\,\mu\text{F}$
(d) R-L circuit with $R = 1\,\text{k}\Omega$ and $L = 10$ mH

11. A 750 Hz, 20 V (rms) source is connected to a resistance of 100 Ω, an inductance of 0.1803 H and a capacitance of 10 μF all in series combination. The time in which the resistance (heat capacity 2 J/°C) will get heated by 10°C is close to. (Assume no loss of heat to the surroundings) **[JEE Main 2020]**

(a) 418 s (b) 245 s
(c) 365 s (d) 348 s

12. In the given AC circuit, when switch S is at position 1, the source emf leads current by $\pi/6$. Now, if the switch is at position 2, then



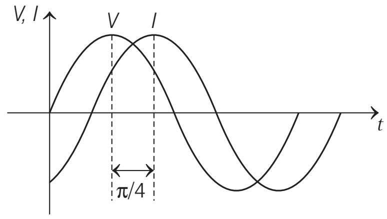
(a) current leads source emf by $\frac{\pi}{4}$

(b) current leads source emf by $\frac{\pi}{3}$

(c) source emf leads current by $\frac{\pi}{4}$

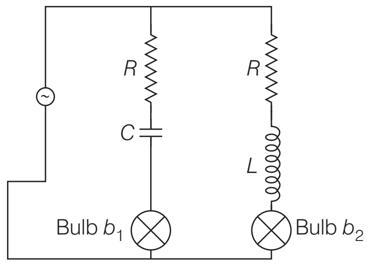
(d) source emf leads current by $\frac{\pi}{3}$

13. An AC voltage $V = V_0 \sin 100\,t$ is applied to the circuit, the phase difference between current and voltage is found to be $\frac{\pi}{4}$, then



(a) $R = 100\,\Omega, C = 1\,\mu\text{F}$ (b) $R = 1\,\text{k}\Omega, C = 10\,\mu\text{F}$
(c) $R = 10\,\text{k}\Omega, L = 1$ H (d) $R = 1\,\text{k}\Omega, L = 10$ H

14. Two identical incandescent light bulbs are connected as shown in figure. When the circuit is an AC voltage source of frequency f, which of the following observations will be correct?



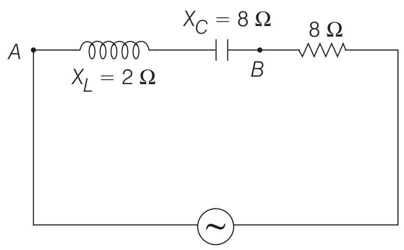
(a) Both bulbs will glow alternatively

(b) Both bulbs will glow with same brightness provided $f = \frac{1}{2\pi}\sqrt{(1/LC)}$

(c) Bulb b_1 will light up initially and goes OFF, bulb b_2 will be ON constantly

(d) Bulb b_1 will blink and bulb b_2 will be ON constantly

15. An inductor ($X_L = 2\,\Omega$), a capacitor ($X_C = 8\,\Omega$) and a resistance ($R = 8\,\Omega$) are connected in series with an AC source. The voltage output of AC source is given by $V = 10\cos(100\pi t)$.



The instantaneous potential difference between points A and B, when the applied voltage is (3/5)th of the maximum value of applied voltage is

(a) 0 V

(b) 6 V

(c) 8 V

(d) None of the above

16. A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60 Ω) is driven by an AC source of 24 V/50 Hz. The energy dissipated in the circuit in 60 s is **[JEE Main 2019]**

(a) 3.39×10^3 J

(b) 5.65×10^2 J

(c) 2.26×10^3 J

(d) 5.17×10^2 J

17. An L-C-R circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant b, the correct equivalence would be **[JEE Main 2020]**

(a) $L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k}$

(b) $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$

(c) $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$

(d) $L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$

Numerical Based Questions

18. A resistance R, an inductance $L = 0.01$ H and a capacitance C are connected in series. When a voltage $V = 400\cos(300t - 10^\circ)$ V is applied to the series combination, the current flowing is $10\sqrt{2}\cos(3000t - 55^\circ)$ A. Find the capacitance of capacitor in microfarads.

19. When 100 V DC is supplied across a solenoid, a current of 1.0 A flows in it. When 100 V AC is applied across the same coil, the current drops to 0.5 A. If the frequency of AC source is 50 Hz, then inductance (in H) of the solenoid is ....

20. An L-C-R series circuit with a resistance of 100 Ω is connected to an AC source of 200 V (rms) and angular frequency 300 rad/s. When only the capacitor is removed, the current lags behind the voltage by 60°. When only the inductor is removed, the current leads the voltage by 60°. The average power dissipated is .... W.

21. In a series circuit, $C = 2\,\mu F$, $L = 1$ mH and $R = 10\,\Omega$. When the current in the circuit is maximum, at that time, the ratio of the energies stored in the capacitor and the inductor will be $x : 5$, then the value of x is ....

22. A sinusoidal voltage of peak value 250 V is applied to a series L-C-R circuit, in which $R = 8\,\Omega$, $L = 24$ mH and $C = 60\,\mu F$. The value of power dissipated at resonant condition is x kW. The value of x to the nearest integer is .... **[JEE Main 2021]**

23. Seawater at a frequency $f = 9 \times 10^2$ Hz, has permittivity $\varepsilon = 80\varepsilon_0$ and resistivity $\rho = 0.25\,\Omega$-m. Imagine a parallel plate capacitor is immersed in seawater and is driven by an alternating voltage source $V(t) = V_0 \sin(2\pi f t)$, then the conduction current density becomes 10^x times the displacement current density after time $t = \frac{1}{800}$ s. The value of x is .... $\left(\text{Given, } \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}\right)$

[JEE Main 2021]

Answers

Round I

1. (d)	**2.** (d)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (a)	**7.** (a)	**8.** (b)	**9.** (a)	**10.** (a)
11. (b)	**12.** (b)	**13.** (a)	**14.** (c)	**15.** (a)	**16.** (b)	**17.** (c)	**18.** (a)	**19.** (b)	**20.** (a)
21. (d)	**22.** (a)	**23.** (c)	**24.** (c)	**25.** (a)	**26.** (a)	**27.** (c)	**28.** (d)	**29.** (c)	**30.** (c)
31. (c)	**32.** (a)	**33.** (d)	**34.** (d)	**35.** (b)	**36.** (b)	**37.** (d)	**38.** (b)	**39.** (a)	**40.** (a)
41. (d)	**42.** (d)	**43.** (a)	**44.** (a)	**45.** (a)	**46.** (c)	**47.** (b)	**48.** (d)	**49.** (c)	**50.** (d)
51. (b)	**52.** (a)	**53.** (a)	**54.** (b)	**55.** (a)	**56.** (a)	**57.** (c)	**58.** (d)	**59.** (a)	

Round II

1. (c)	**2.** (c)	**3.** (a)	**4.** (d)	**5.** (c)	**6.** (a)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (c)
11. (d)	**12.** (a)	**13.** (b)	**14.** (a)	**15.** (b)	**16.** (d)	**17.** (b)	**18.** 33	**19.** 0.55	
20. 400	**21.** 1	**22.** 4	**23.** 6						

Solutions

Round I

1. Here, $V = \left(\frac{120}{2}\right) 2 \sin 100\pi t \cos 100\pi t$

$$V = 60 \sin(200\,\pi t)$$

Maximum voltage, $V_0 = 60$ V and $\omega = 200\,\pi$ rad s^{-1}

or frequency, $\nu = \frac{\omega}{2\pi} = \frac{200\pi}{2\pi} = 100$ Hz

2. $\therefore\ I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{14}{\sqrt{2}} \approx 10$ A

3. Here, $\nu = 50$ Hz, $I_v = 5$ A, $I = ?$, $t = \frac{1}{300}$ s

$\therefore\quad I_0 = \sqrt{2}\, I_v = \sqrt{2} \times 5$ A

From $I = I_0 \sin \omega t$

$= 5\sqrt{2} \sin 100\,\pi \times \frac{1}{300} = 5\sqrt{2} \sin \frac{\pi}{3} = 5\sqrt{2} \times \frac{\sqrt{3}}{2} = 5\sqrt{\frac{3}{2}}$ A

4. Average value of output current is given by

$$I_{\text{av}} = \frac{\int_0^{T/2} I\,dt}{\int_0^{T/2} dt} = \frac{\int_0^{T/2} I_0 \sin \omega t\; dt}{T/2}$$

$$= \frac{2I_0}{T}\left[-\frac{\cos \omega t}{\omega}\right]_0^{T/2}$$

$$= \frac{2I_0}{T}\left[-\frac{\cos(\omega T/2)}{\omega} + \frac{\cos 0^\circ}{\omega}\right]$$

$$= \frac{2I_0}{\omega T}(-\cos \pi + \cos 0^\circ)$$

$$= \frac{2I_0}{2\pi}(1 + 1) = \frac{2I_0}{\pi}$$

5. The equation of AC is $I = I_1 \cos \omega t + I_2 \sin \omega t$

The resultant current is given by

$$I_0 = \sqrt{I_1^2 + I_2^2} \qquad \ldots\text{(i)}$$

Hence, the rms current from relation is

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{\sqrt{I_1^2 + I_2^2}}{\sqrt{2}} \qquad \text{[from Eq. (i)]}$$

$$= \sqrt{\frac{I_1^2 + I_2^2}{2}}$$

6. $I_0 = \frac{V_0}{R} = \frac{200}{100} = 2$ A

rms current, $I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{2}{\sqrt{2}} = 1.41$ A

7. Given, capacitance of the capacitor,

$$C = 60\,\mu\text{F} = 60 \times 10^{-6}\ \text{F},$$

$$V_{\text{rms}} = 110\ \text{V}$$

Frequency of AC supply, $f = 60$ Hz

Capacitive reactance,

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 60 \times 60 \times 10^{-6}} = 44.23\ \Omega$$

The rms value of the current in the circuit,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \frac{110}{44.23} = 2.49\ \text{A} \simeq 2.5\ \text{A}$$

8. $$P = \frac{1}{2} V_0 i_0 \cos \phi$$

$\Rightarrow\quad 1000 = \frac{1}{2} \times 200 \times i_0 \cos 60^\circ$

$\Rightarrow\quad i_0 = 20$ A

$\Rightarrow\quad i_{\text{rms}} = \frac{i_0}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 10\sqrt{2}$ A

9. When the circuit is connected to an AC source,

Voltage, $V = 12$ V

Current, $I = 0.2$ A

$\therefore$ Impedance, $Z = \frac{V}{I} = \frac{12}{0.2} = 60\ \Omega$

When it is connected to DC source,

Voltage, $V = 12\text{ V}$

Current, $I = 0.4\text{ A}$

$\Rightarrow$ Resistance, $R = \dfrac{V}{I} = \dfrac{12}{0.4} = 30\,\Omega$

As in case of DC supply, the capacitor act as an open circuit and no current flows through the circuit. So, the given circuit will not have capacitor in series combination. Therefore, the circuit should be a series L-R circuit.

10. Given, resistance, $R = 3\,\Omega$

Inductive reactance, $X_L = 10\,\Omega$

Capacitive reactance, $X_C = 14\,\Omega$

The impedance of the series L-C-R circuit,

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{(3)^2 + (14-10)^2}$$

$$Z = 5\,\Omega$$

11. When C is removed circuit becomes R-L circuit, hence

$$\tan\frac{\pi}{3} = \frac{X_L}{R} \quad \text{... (i)}$$

When L is removed circuit becomes R-C circuit, hence

$$\tan\frac{\pi}{3} = \frac{X_C}{R} \quad \text{... (ii)}$$

From Eqs. (i) and (ii), we obtain $X_L = X_C$.

This is condition of resonance and in resonance, $Z = R = 100\,\Omega$.

12. Here, $X_L = R, X_C = R/2$

$$\therefore \quad \tan\phi = \frac{X_L - X_C}{R} = \frac{R - \frac{R}{2}}{R} = \frac{1}{2}$$

$$\Rightarrow \quad \phi = \tan^{-1}(1/2)$$

Also, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \dfrac{R^2}{4}} = \dfrac{\sqrt{5}}{2}R$

13. Given, reactance, $X_L = 100\,\Omega$

Frequency of AC, $f = 1000\text{ Hz}$

As voltage leads current by 45°, so there must be some resistance in the coil otherwise $\Delta\phi = 90°$.

Using $\tan\phi = \dfrac{X_L}{R}$, we have

$$\tan 45° = \frac{X_L}{R} \text{ or } X_L = R$$

$\therefore$ Reactance of circuit containing resistance and inductance is

$$\sqrt{X_L^2 + R^2} = 100\,\Omega$$

$$\Rightarrow \sqrt{X_L^2 + X_L^2} = 100\,\Omega$$

or $\quad X_L = 50\sqrt{2}\,\Omega$

But $\quad X_L = L\omega$

$$\Rightarrow \quad L = \frac{X_L}{\omega} = \frac{X_L}{2\pi f} = \frac{50\sqrt{2}}{2\pi \times 1000}$$

$$\Rightarrow \quad L = 1.1 \times 10^{-2}\text{ H}$$

14. Here, $C = 100\,\mu\text{F} = 10^{-4}\text{ F}$, $R = 40\,\Omega$, $V_{\text{rms}} = 110\text{ V}$, $f = 60\text{ Hz}$

(i) As, $i_{\text{rms}} = \dfrac{V_{\text{rms}}}{\sqrt{R^2 + \dfrac{1}{\omega^2 C^2}}} = \dfrac{V_{\text{rms}}}{\sqrt{R^2 + \dfrac{1}{(2\pi f C)^2}}}$

$$= \frac{110}{\sqrt{40^2 + \dfrac{1}{(2\pi \times 60 \times 10^{-4})^2}}}$$

$$= \frac{100}{\sqrt{1600 + 703.62}}$$

$$= \frac{100}{48} = 2.292\text{ A}$$

Now, $i_{\text{rms}} = \sqrt{2}i_0 = \sqrt{2} \times 2.292 = 3.24\text{ A}$

(ii) For C-R circuit,

$$\tan\theta = \frac{1/\omega C}{R} = \frac{1}{2\pi f C R}$$

$$= \frac{1}{2\pi \times 60 \times 10^{-4} \times 40}$$

$$= 0.6631$$

$\theta = 33°33'$ (emf lags behind the current)

15. At angular frequency ω, the current in R-C circuit is given by

$$i_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}} \quad \text{... (i)}$$

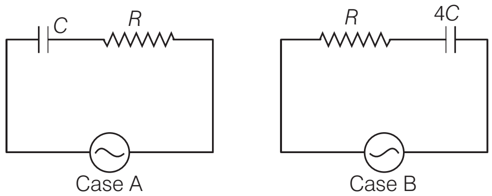
Also, $\dfrac{i_{\text{rms}}}{2} = \dfrac{V_{\text{rms}}}{\sqrt{R^2 + \left(\dfrac{1}{\frac{\omega}{3}C}\right)^2}} = \dfrac{V_{\text{rms}}}{\sqrt{R^2 + \dfrac{9}{\omega^2 C^2}}}$ ... (ii)

From Eqs. (i) and (ii), we get

$$3R^2 = \frac{5}{\omega^2 C^2}$$

$$\Rightarrow \quad \frac{\frac{1}{\omega C}}{R} = \sqrt{\frac{3}{5}} \Rightarrow \frac{X_C}{R} = \sqrt{\frac{3}{5}}$$

16. As, $Z = \sqrt{R^2 + \dfrac{1}{(\omega C)^2}}$ and $Z' = \sqrt{R^2 + \dfrac{1}{(\omega 4C)^2}}$



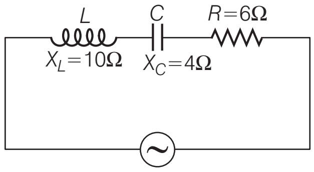
$$I_R^A = \frac{V}{Z}$$

$$I_R^B = \frac{V}{Z'}$$

As, $\quad Z' < Z$

$\Rightarrow \quad I_R^A < I_R^B$

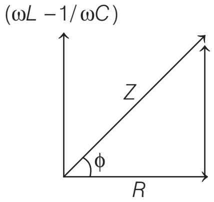
17.



We know that, power factor is $\cos\phi$.

i.e. $$\cos\phi = \frac{R}{Z} \quad \ldots\text{(i)}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \ldots\text{(ii)}$$



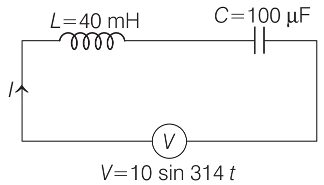
$$\Rightarrow \quad Z = \sqrt{6^2 + (10-4)^2}$$

$$\Rightarrow \quad Z = 6\sqrt{2} \mid \cos\phi = \frac{6}{6\sqrt{2}}$$

$$\cos\phi = \frac{1}{\sqrt{2}}$$

18. Impedance of given circuit,

$$Z = \sqrt{(X_C - X_L)^2} = X_C - X_L$$
$$= \frac{1}{\omega C} - \omega L$$
$$= \frac{1}{314 \times 100 \times 10^{-6}} - 314 \times 40 \times 10^{-3}$$
$$= 19.28\,\Omega$$



As $X_C > X_L$, circuit is capacitive, hence current in circuit leads emf by $\frac{\pi}{2}$ rad.

Current in circuit is given by

$$I = I_{\max}\sin\left(\omega t + \frac{\pi}{2}\right)$$
$$= \frac{V_{\max}}{Z}\cos\omega t$$
$$= \frac{10}{19.28} \times \cos 314\,t$$

or $$I = 0.52\cos 314\,t$$

19. (a) In a circuit having C alone, the voltage lags the current by $\frac{\pi}{2}$.

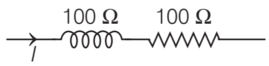
(b) In circuit containing R and L, the phase difference between current and voltage can have any value between 0 to $\pi/2$.

(c) In L-C circuit, the phase difference between current and voltage has a value of 0 or $\frac{\pi}{2}$ depending on the values of L and C.

(d) In a circuit containing L alone, the voltage leads the current by $\pi/2$.

20. We have, $X_L = \omega L = 100 \times 0.5 = 50\ \Omega$

$$X_1' = \frac{1}{\omega C} = \frac{1}{100 \times 100 \times 10^{-6}} = 100\ \Omega$$



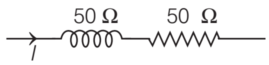
$$Z_1 = 100\sqrt{2},$$
$$I_1 = \frac{20}{100\sqrt{2}} = \frac{1}{5\sqrt{2}}$$

$$V \text{ across } 100\ \Omega = \frac{1}{5\sqrt{2}} \times 100$$
$$= \frac{20}{\sqrt{2}} \times \frac{\sqrt{3}}{\sqrt{2}} = 10\sqrt{2}$$

Phase difference between I_1 and V,

$$\cos\phi_1 = \frac{R_1}{Z_1} = \frac{100}{100\sqrt{2}},$$
$$\phi = \frac{\pi}{4}$$

I_1 and V,



$$Z_2 = 50\sqrt{2},\ I_2 = \frac{20}{50\sqrt{2}} = \frac{2}{5\sqrt{2}}$$

$$V \text{ across } 50\ \Omega = \frac{2}{50\sqrt{2}} \times 50 = \frac{20}{\sqrt{2}} = 10\sqrt{2}$$

$$\phi_2 = \frac{\pi}{4}\ I_2 \text{lag} V \text{ by } \frac{\pi}{4}$$

$$I = I_1 = I_2$$
$$I_{\text{net}} = \sqrt{I_1^2 + I_2^2}$$
$$I = \sqrt{\frac{4}{25 \times 2} + \frac{1}{25 \times 2}}$$
$$= \sqrt{\frac{5}{50}} = \frac{1}{\sqrt{10}} = 0.316\text{ A}$$

21. $Z = \sqrt{(R)^2 + (X_L - X_C)^2}$

$$R = 10\Omega,\ X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10\,\Omega$$

$$\therefore \quad X_C = \frac{1}{\omega C} = \frac{1}{2000 \times 50 \times 10^{-6}} = 10\Omega,\ i.e.\ Z = 10\Omega$$

Maximum current, $i_0 = \frac{V_0}{Z} = \frac{20}{10} = 2\text{ A}$

Hence, $i_{\text{rms}} = \frac{2}{\sqrt{2}} = 1.41\text{ A}$

and $V_{\text{rms}} = 4 \times 1.41 = 5.64\text{ V}$

22. Bandwidth $= R/L$

Bandwidth $\propto R$

So, bandwidth of resonance circuit will increase.

23. In the condition of resonance,

$$X_L = X_C \text{ or } \omega L = \frac{1}{\omega C} \quad \ldots\text{(i)}$$

Since, resonance frequency remains unchanged, so

$$\sqrt{LC} = \text{constant or } LC = \text{constant}$$

$\therefore \quad L_1 C_1 = L_2 C_2$

or $\quad L \times C = L_2 \times 2C \quad$ or $\quad L_2 = L/2$

24. Given, $\quad L = 10$ H, $f = 50$ Hz

For maximum power,

$$X_C = X_L \quad (\because \text{resonance condition})$$

or $\quad \dfrac{1}{\omega C} = \omega L \quad$ or $\quad C = \dfrac{1}{\omega^2 L}$

$\therefore \quad C = \dfrac{1}{4\pi^2 \times 50 \times 50 \times 10}$

or $\quad C = 0.1 \times 10^{-5}$ F $= 1\,\mu$F

25. In the given circuit diagram, both inductors of inductance L are parallel to each other, hence

$$L_{eg} = \frac{L_1 L_2}{L_1 + L_2} = \frac{L \times L}{L + L} = \frac{L}{2}$$

Similarly,

$$C_{eg} = C_1 + C_2 = C + C = 2C$$

$\therefore \quad$ Resonance frequency

$$\omega_r = \frac{1}{\sqrt{L_{eg} \cdot C_{eg}}} = \sqrt{\frac{1}{\frac{L}{2} \cdot 2C}} = \sqrt{\frac{1}{LC}} = \frac{1}{\sqrt{LC}}$$

26. Sharpness of resonance of a resonant L-C-R circuit is determined by the ratio of resonant frequency with the selectivity of circuit. This ratio is also called "quality factor" or Q-factor.

$$Q\text{-factor} = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R}$$

27. The L-C-R circuit used for communication should possess high quality factor (Q-factor) of resonance, which is given by

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Putting values in above equation,

For option (a), $Q = 10.52$

For option (b), $Q = 9.42$

For option (c), $Q = 0.77$

For option (d), $Q = 7.30$

Hence, option (c) is the best representation.

28. As, the iron rod is inserted, the magnetic field inside the coil magnetises, the iron increasing the magnetic field inside it. Hence, the inductance of the coil increases. Consequently, the inductive reactance of the coil increases. As a result, a larger fraction of the applied AC voltage appears across the inductor, leaving less voltage across the bulb. Therefore, the brightness of the light bulb decreases.

29. $P = E_{\text{rms}} i_{\text{rms}} \cos\phi = \dfrac{E_0}{\sqrt{2}} \times \dfrac{i_0}{\sqrt{2}} \times \dfrac{R}{Z}$

$$\Rightarrow P = \frac{E_0}{\sqrt{2}} \times \frac{E_0}{Z\sqrt{2}} \times \frac{R}{Z} \Rightarrow P = \frac{E_0^2 R}{2Z^2}$$

Given $X_L = R$, so $Z = \sqrt{2}R \Rightarrow P = \dfrac{E_0^2}{4R}$

30. With DC, $P = \dfrac{V^2}{R} \Rightarrow R = \dfrac{(10)^2}{20} = 5\ \Omega$

With AC, $P = \dfrac{V_{\text{rms}}^2 R}{Z^2} \Rightarrow Z^2 = \dfrac{(10)^2 \times 5}{10} = 50\ \Omega^2$

Also, $\quad Z^2 = R^2 + 4\pi^2\nu^2 L^2$

$\Rightarrow \quad 50 = (5)^2 + 4(3.14)^2 \nu^2 (10 \times 10^{-3})^2 \Rightarrow \nu = 80$ Hz

31. $Z = \sqrt{R^2 + \left(\dfrac{1}{2\pi\nu C}\right)^2} = \sqrt{(3000)^2 + \dfrac{1}{\left(2\pi \times 50 \times \dfrac{2.5}{\pi} \times 10^{-6}\right)^2}}$

$$\Rightarrow Z = \sqrt{(3000)^2 + (4000)^2} = 5 \times 10^3\ \Omega$$

So, power factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{3000}{5 \times 10^3} = 0.6$

and power, $\quad P = V_{\text{rms}}\, i_{\text{rms}} \cos\phi = \dfrac{V_{\text{rms}}^2 \cos\phi}{Z}$

$$\Rightarrow \quad P = \frac{(200)^2 \times 0.6}{5 \times 10^3} = 4.8 \text{ W}$$

32. When a resistor is connected to an AC source, the power drawn will be of

$$P = V_{\text{rms}} \cdot I_{\text{rms}} = V_{\text{rms}} \cdot \frac{V_{\text{rms}}}{R}$$

$\Rightarrow \quad V_{\text{rms}}^2 = PR$

When an inductor is connected in series with the resistor, then the power drawn will be

$$P' = V_{\text{rms}} \cdot I_{\text{rms}} \cos\phi$$

where, ϕ = phase difference.

$\therefore \quad P' = \dfrac{V_{\text{rms}}^2}{R} \cdot \dfrac{R}{Z} = \dfrac{P \cdot R}{R} \cdot \dfrac{R}{Z}$

$\Rightarrow \quad P' = \dfrac{P \cdot R}{Z} = P\left(\dfrac{R}{Z}\right)$

33. Given, for L-R circuit,

$$V_{\text{rms}} = 250 \text{ V},\ f = 50 \text{ Hz},\ P = 400 \text{ W}$$

and $\cos\phi = 0.8$

As we know, power is given by $P = V_{\text{rms}} i_{\text{rms}} \cos\phi$

$\Rightarrow \quad P = V_{\text{rms}} \dfrac{V_{\text{rms}}}{Z} \cos\phi \quad \Rightarrow \quad Z = \dfrac{V_{\text{rms}}^2}{P} \cos\phi$

$\Rightarrow \quad Z = \dfrac{(250)^2}{400} \times 0.8 = 125\ \Omega$

Also, $\cos\phi = \frac{R}{Z} \Rightarrow 0.8 = \frac{R}{125} \Rightarrow R = 100\ \Omega$

Impedance, $Z = \sqrt{R^2 + X_L^2} \Rightarrow 125 = \sqrt{(100)^2 + X_L^2}$

$X_L = 75\ \Omega$

Now, to obtain power factor unity, X_C must be equal to X_L.

i.e. $X_C = X_L = 75\ \Omega$

$\Rightarrow \quad \frac{1}{\omega C} = 75$

$\Rightarrow \quad C = \frac{1}{\omega \times 75} = \frac{1}{2\pi \times 50 \times 75}$

$\Rightarrow \quad C = \frac{1}{7500\pi}\ \text{F} \quad \text{or} \quad C = \frac{400}{3\pi}\ \mu\text{F}$

Given, $C = \frac{n}{3\pi}\ \mu\text{F}$

On comparing the both, we get

$n = 400$

34. In the R-L circuit, growth of current is given by

$$I = \frac{E}{R_2}\left[1 - e^{\frac{-R_2 t}{L}}\right]$$

Potential drop across L,

$$V_L = L\frac{dI}{dt} = L\left(\frac{-E}{R_2}\right)e^{\frac{R_2 t}{L}}\left(-\frac{R_2}{L}\right)$$

$$V_L = Ee^{-\frac{R_2 T}{L}} = 12e^{-2t/0.4} = 12e^{-5t}\ \text{V}$$

35. Here, $i = i_0$ at $t = \infty$. Let i be the current at $t = 1$ s

From $i = i_0(1 - e^{-\frac{R}{L}t}) = i_0\left(1 - e^{\frac{-10}{5}\times 1}\right) = i_0\left(1 - \frac{1}{e^2}\right)$

$\therefore \quad \frac{i_0}{i} = \frac{e^2}{e^2 - 1}$

36. The current at any instant is given by

$I = I_0(1 - e^{-Rt/L})$

$\Rightarrow \quad \frac{I_0}{2} = I_0(1 - e^{-Rt/L})$

or $\quad \frac{1}{2} = (1 - e^{-Rt/L})$

or $\quad e^{-Rt/L} = \frac{1}{2} \quad \text{or} \quad \frac{Rt}{L} = \ln 2$

$\therefore \quad t = \frac{L}{R}\ln 2 = \frac{300 \times 10^{-3}}{2} \times 0.693$

$= 150 \times 0.693 \times 10^{-3}$

$= 0.10395$ s

$= 0.1$ s

37. In an L-R circuit, current during charging of inductor is given by

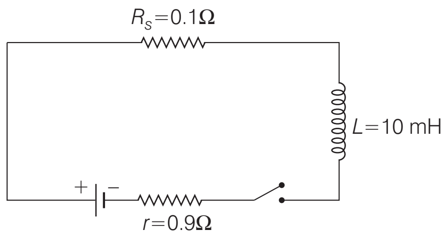
$$i = i_0(1 - e^{-\frac{R}{L}t})$$

where, i_0 = saturation current.

In given circuit,

inductance of circuit, $L = 10\ \text{mH} = 10 \times 10^{-3}$ H

Resistance of circuit, $R = (R_s + r) = 0.1 + 0.9 = 1\ \Omega$



Now, from $\quad i = i_0(1 - e^{-\frac{R}{L}\cdot t}) \qquad$ ...(i)

Given, $\quad i = 80\%$ of i_0

$\Rightarrow \quad i = \frac{80\, i_0}{100} = 0.8\, i_0$

Substituting the value of i in Eq. (i), we get

$0.8 = 1 - e^{-\frac{R}{L}\cdot t}$

$\Rightarrow \quad e^{-\frac{R}{L}t} = 0.2 \Rightarrow e^{\frac{R}{L}t} = 5$

$\Rightarrow \quad \ln(e)^{\frac{R}{L}t} = \ln 5$

$\Rightarrow \quad \frac{R}{L}t = \ln 5$

$\Rightarrow \quad t = \frac{L}{R}\cdot \ln(5) = \frac{10 \times 10^{-3}}{1} \times \ln(5)$

$= 10 \times 10^{-3} \times 1.6$

$= 1.6 \times 10^{-2}\ \text{s} = 0.016$ s

38. In an L-R circuit, current during charging is given by

$$I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$$

where, $I_0 = \frac{E}{R}$ = saturation current.

So, we have $\frac{dq}{dt} = I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$

$\Rightarrow \quad dq = I_0\left(1 - e^{-\frac{R}{L}t}\right)dt$

So, charge q that passes through battery from time $t = 0$ to $t = \frac{L}{R}$ is obtained by integrating the above equation within the specified limits, *i.e.*

$$q = \int_0^Q dq = \int_{t=0}^{t=\frac{L}{R}} I_0\left(1 - e^{-\frac{R}{L}t}\right)dt$$

$$= I_0\left[\left(t - \frac{1}{\left(-\frac{R}{L}\right)}\cdot e^{-\frac{R}{L}t}\right)\right]_0^{\frac{L}{R}}$$

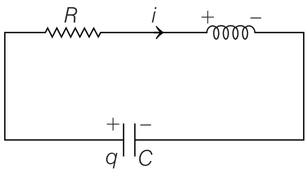
$$= \frac{E}{R}\left[\left\{\frac{L}{R} + \frac{L}{Re'}\right\} - \left\{0 + \frac{L}{R}\right\}\right]$$

$$= \frac{E}{R} \times \frac{L}{Re} = \frac{EL}{R^2 e}$$

$$\Rightarrow \quad Q = \frac{EL}{2.7\, R^2} \qquad (\because e \approx 2.72)$$

39. Time constant τ is the duration when the value of potential drops by 63% of its initial maximum value (*i.e.* V_0/e). Here, 37% of 25 V = 9.25 V which lies between 150 s to 200 s in the graph.

40. Consider the L-C-R circuit at any time t,



Now, applying KVL, we have

$$\frac{q}{C} - iR - \frac{Ldi}{dt} = 0$$

As current is decreasing with time, we can write

$$i = -\frac{dq}{dt}$$

$$\Rightarrow \quad \frac{q}{C} + \frac{dq}{dt}R + \frac{Ld^2q}{dt^2} = 0$$

or $$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = 0$$

This equation is equivalent to that of a damped oscillator.

Thus, we can write the solution as

$$Q_{\max}(t) = Q_0 \cdot e^{-Rt/2L} \quad \text{or} \quad Q^2_{\max} = Q_0^2\, e^{-\frac{Rt}{L}}$$

As $L_1 > L_2$ damping is faster for L_2

Hence, the correct graph is shown in option (a).

41. After long time, inductor behaves as short-circuit.

At $t = 0$, the inductor behaves as short-circuited. The current,

$$I_0 = \frac{E_0}{R} = \frac{15\text{ V}}{0.15\text{ k}\Omega} = 100\text{ mA}$$

As K_2 is closed, current through the inductor starts decay, which is given at any time t as

$$I = I_0\, e^{\frac{-tR}{L}} = (100\text{ mA})\, e^{\frac{-t \times 15000}{3}}$$

At $\quad t = 1$ ms,

$$I = (100\text{ mA})\, e^{-\frac{1\times10^{-3}\times15\times10^{3}}{3}}$$

$$I = (100\text{ mA})\, e^{-5} = 0.6737\text{ mA}$$

42. Let t be the required time at which the energy stored in inductor becomes $\left(\frac{1}{n}\right)$ times of its maximum value.

According to question, for given L-R circuit,

$$U = \frac{1}{n}U_{\max}$$

$$\frac{1}{2}LI^2 = \frac{1}{n} \times \frac{1}{2}LI^2_{\max}$$

$$\frac{1}{2}L\left[I_{\max}\left(1 - e^{\frac{-Rt}{L}}\right)\right]^2 = \frac{1}{n} \times \frac{1}{2}LI^2_{\max}$$

$$\frac{1}{2}LI^2_{\max}\left(1 - e^{\frac{-Rt}{L}}\right)^2 = \frac{1}{n} \times \frac{1}{2}L\,I^2_{\max}$$

$$\left(1 - e^{\frac{-Rt}{L}}\right)^2 = \frac{1}{n}$$

$$\Rightarrow \quad 1 - e^{\frac{-Rt}{L}} = \left(\frac{1}{n}\right)^{\frac{1}{2}}$$

$$e^{\frac{-Rt}{L}} = 1 - \frac{1}{\sqrt{n}}$$

$$\Rightarrow \quad e^{-\frac{Rt}{L}} = \frac{\sqrt{n} - 1}{\sqrt{n}}$$

Taking natural log on both sides, we get

$$\ln\left(e^{\frac{-Rt}{L}}\right) = \ln\left(\frac{\sqrt{n} - 1}{\sqrt{n}}\right)$$

or $$-\frac{Rt}{L}\ln(e) = \ln\left(\frac{\sqrt{n} - 1}{\sqrt{n}}\right)$$

$$= -\frac{Rt}{L} \times 1 = \ln\left(\frac{\sqrt{n} - 1}{\sqrt{n}}\right)$$

$$\Rightarrow \quad t = \frac{-L}{R}\ln\left(\frac{\sqrt{n} - 1}{\sqrt{n}}\right)$$

$$t = \frac{L}{R}\ln\left(\frac{\sqrt{n} - 1}{\sqrt{n}}\right)^{-1}$$

$$\Rightarrow \quad t = \frac{L}{R}\ln\left(\frac{\sqrt{n}}{\sqrt{n} - 1}\right)$$

43. In an L-R circuit, current growth occurs as

$$I = I_0\left(1 - e^{-\frac{R}{L}t}\right) \qquad \ldots\text{(i)}$$

where, I = instantaneous current,

I_0 = maximum current = current at $t = \infty = \dfrac{E}{R}$,

R = resistance of circuit,

L = inductance of circuit

and $\quad t$ = instantaneous time.

Here, $R = 5\,\Omega$, $L = 10$ mH $= 10 \times 10^{-3}$ H,

$E = 20$ V, $t = 40$ s

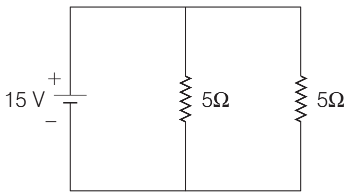
So, substituting these values in Eq. (i), we get

$$\frac{I}{I_0} = \left(1 - e^{-\frac{5}{10\times10^{-3}}\times 40}\right) = 1 - e^{-2\times10^4}$$

$$\Rightarrow \quad \frac{I_0}{I} = \frac{1}{1 - e^{-2\times 10^4}} \approx 1$$

$$\therefore \quad \frac{I_0}{I} \approx 1.06 \text{ (nearest option)}$$

44. After a sufficiently long time, in steady state, resistance offered by inductor is zero. So, circuit is reduced to



$$\therefore \text{ Current in circuit, } I = \frac{E}{R_{eq}} = \frac{15}{\left(\frac{5\times 5}{5+5}\right)} = \frac{15\times 2}{5} = 6 \text{ A}$$

45. In L-R circuit, the growing current at time t is given by

$$i = i_0[1 - e^{-t/\tau}], \text{ where } i_0 = \frac{E}{R} \text{ and } \tau = \frac{L}{R}$$

$\therefore$ Charge passed through the battery in one time constant is

$$q = \int_0^\tau i dt = \int_0^\tau i_0(1 - e^{-t/\tau})dt$$

$$q = i_0\tau - \left[\frac{i_0 e^{-t/\tau}}{-2/\tau}\right]_0^\tau = i_0\tau + i_0\tau[e^{-1} - 1]$$

$$= i_0\tau - i_0\tau + \frac{i_0\tau}{e}$$

$$q = \frac{i_0\tau}{e} = \frac{(E/R)(L/R)}{e} = \frac{EL}{eR^2}$$

46. In L-R circuit, current at any time t is given by

$$i = \frac{E}{R}\left(1 - e^{\frac{R}{L}t}\right) = \frac{E}{R} - \frac{E}{R}e^{-\frac{R}{L}t} \quad \ldots\text{(i)}$$

$$\frac{di}{dt} = \frac{E}{R}e^{-\frac{R}{L}t}\left(\frac{R}{L}\right) = \frac{E}{L}e^{-\frac{R}{L}t}$$

$$\text{Induced emf} = L\frac{di}{dt} = Ee^{-\frac{R}{L}t} \quad \ldots\text{(ii)}$$

From Eq. (i), $iR = E - Ee^{-\frac{R}{L}t}$

Using Eq. (ii), $iR = E - e$

or $e = E - iR$

Therefore, graph between e and i is a straight line with negative slope and positive intercept.

47. As is clear for Fig. (i), steady state current for $t =$ both the circuits is same. Therefore,

$$\frac{V}{R_1} = \frac{V}{R_2} \text{ or } R_1 = R_2$$

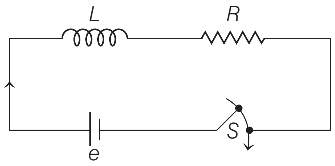
Again, from the same figure, we observe that

$$\tau_1 < \tau_2$$

$$\therefore \quad \frac{L_1}{R_1} < \frac{L_2}{R_2}$$

As $R_1 = R_2$, therefore $L_1 < L_2$

48. In an L-R circuit as shown in the figure, instantaneous current at time t is given by



$$i = I(1 - e^{-\frac{R.t}{L}})$$

where, $\frac{L}{R} = t_c =$ time constant of the circuit.

Now, charge that flows from $t = 0$ to $t = t_c$ from battery,

$$q = \int_0^{t_c} idt = I\int_0^{t_c}(1 - e^{-\frac{R}{L}.t})dt$$

$$= I\left[t - \frac{1}{-\frac{R}{L}}.e^{\frac{-R}{L}.t}\right]_0^{t_c} = I\left[t + \frac{L}{R}.e^{\frac{-R}{L}.t}\right]_0^{t_c}$$

$$= I\left[\left(t_c + \frac{L}{R}e^{-\frac{R}{L}t_c}\right) - \left(\frac{L}{R}e^0\right)\right]$$

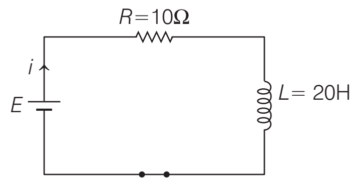
$$= I\left[\frac{L}{R} + \frac{L}{R}e^{-1} - \frac{L}{R}\right] = I.\frac{L}{R}.e^{-1}$$

Here, I is maximum current which occurs at $t = \infty$ and its value,

$$I = \frac{\varepsilon}{R}$$

$$\text{So,} \quad q = \frac{\varepsilon}{R}.\frac{L}{R}.\frac{1}{e} = \frac{\varepsilon L}{eR^2}$$

49. Given circuit is a series L-R circuit



In an L - R circuit, current increases is $i = \frac{E}{R}(1 - e^{-\frac{R}{L}.t})$

Now, energy stored in inductor is $U_L = \frac{1}{2}Li^2$

where, $L =$ self-inductance of the coil

and energy dissipated by resistor is $U_R = i^2R$.

Given, rate of energy stored in inductor is equal to the rate of energy dissipated in resistor. So, after differentiating, we get

$$iL\frac{di}{dt} = i^2R \Rightarrow \frac{di}{dt} = \frac{R}{L}i$$

$$\Rightarrow \quad \frac{E}{R}\cdot\frac{R}{L}e^{-\frac{R}{L}t} = \frac{R}{L}\cdot\frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\Rightarrow \quad 2e^{-\frac{R}{L}t} = 1 \Rightarrow e^{-\frac{R}{L}t} = \frac{1}{2}$$

Taking log on both sides, we have

$$\Rightarrow \quad \frac{-R}{L}t = \ln\left(\frac{1}{2}\right) \Rightarrow \frac{R}{L}t = \ln 2$$

$$\Rightarrow \quad t = \frac{L}{R}\ln 2 = \frac{20}{10}\ln 2$$

$$\Rightarrow \quad t = 2\ln 2$$

50. Choke coil is a device having high value of inductance and low resistance which is used to control AC.

51. Given, $C = 40\ \mu\text{F} = 40 \times 10^{-6}\text{F}$,

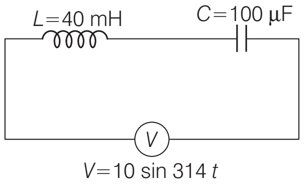
and $L = 16\ \text{mH} = 16 \times 10^{-3}\ \text{H}$

Angular frequency of oscillating circuit,

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(16 \times 10^{-3})(40 \times 10^{-6})}}$$

$$= \frac{10^4}{8} = 1.25 \times 10^3\ \text{s}^{-1}$$

52.



Impedance of given ciruit,

$$Z = \sqrt{(X_C - X_L)^2} = X_C - X_L$$

$$= \frac{1}{\omega C} - \omega L = \frac{1}{314 \times 100 \times 10^{-6}} - 314 \times 40 \times 10^{-3}$$

$$= 19.28\ \Omega$$

As $X_C > X_L$, circuit is capacitive, hence current in circuit leads emf by $\frac{\pi}{2}$ rad.

Current in circuit is given by

$$I = I_{\max}\sin\left(\omega t + \frac{\pi}{2}\right) = \frac{V_{\max}}{Z}\cos\omega t = \frac{10}{19.28} \times \cos 314t$$

or $\quad I = 0.52\cos 314t$

53. For L-C oscillations,

Energy stored in inductor = Energy stored in capacitor

$$\frac{1}{2}Li_m^2 = \frac{1}{2}CV_m^2$$

Given, $V_m = 25\ \text{V}, C = 10\ \mu\text{F} = 10^{-5}\ \text{F}$

and $L = 100\ \text{mH} = 10^{-1}\ \text{H}$

or $\quad i_m = V_m\sqrt{\frac{C}{L}} = 25\sqrt{\frac{10^{-5}}{10^{-1}}}$

$$= 25 \times 10^{-2}\ \text{A} = 0.25\ \text{A}$$

54. Given, $\quad L = 25\ \text{mH} = 25 \times 10^{-3}\ \text{H}$

$C = 10\ \mu\text{F} = 10^{-5}\ \text{F}$

If T be the time period in L-C oscillation, then

$$T = 2\pi\sqrt{LC}$$

$$= 2\pi\sqrt{25 \times 10^{-3} \times 10^{-5}}$$

$$= \pi \times 10^{-3}\ \text{s} = \pi\ \text{m-s}$$

Current in the circuit will be maximum, when

$$t = \frac{T}{4} = \frac{\pi}{4}\ \text{ms}$$

55. The core of transformer is laminated to reduce energy loss due to eddy currents because induction is reduced by laminating.

56. We have, $N_sI_s = N_pI_p$

$$\Rightarrow \quad \frac{N_S}{N_P} = \frac{I_P}{I_S}$$

$$\Rightarrow \quad \frac{2}{3} = \frac{3}{I_s}$$

$$\Rightarrow \quad I_s = \frac{3 \times 3}{2} = \frac{9}{2} = 4.5\ \text{A}$$

57. $E_s = \frac{n_s}{n_p}E_p = \frac{10}{200} \times 240 = 12\ \text{V}$

58. $e = \frac{Ldi}{dt} = 4 \times \frac{5}{1/1500}$

$= 30000\ \text{V} = 30\ \text{kV}$

59. Given, number of turns in primary, $N_1 = 300$

Number of turns in secondary, $N_2 = 150$

Output power, $P_2 = 2.2\ \text{kW} = 2.2 \times 10^3\ \text{W}$

Current in secondary coil, $I_2 = 10\ \text{A}$

Output power, $P_2 = I_2V_2$

$$\Rightarrow \quad V_2 = \frac{P_2}{I_2} = \frac{2.2 \times 10^3}{10} = 220\ \text{V} \qquad \text{... (i)}$$

We know that,

$$\frac{N_1}{N_2} = \frac{\text{Input voltage}}{\text{Output voltage}} = \frac{V_1}{V_2}$$

$$\Rightarrow \quad V_1 = \left(\frac{N_1}{N_2}\right)V_2$$

$$\Rightarrow \quad V_1 = \left(\frac{300}{150}\right) \times (220\ \text{V}) \qquad \text{[using Eq. (i)]}$$

$$V_1 = 440\ \text{V} \qquad \text{... (ii)}$$

Again, $\quad \frac{V_1}{V_2} = \frac{I_2}{I_1}$

$$\Rightarrow \quad I_1 = \left(\frac{V_2}{V_1}\right)I_2 = \frac{220}{440} \times 10$$

[using Eqs. (i) and (ii)]

$$\Rightarrow \quad I_1 = 5\ \text{A}$$

Round II

1. Current in inductance I_L lags behind the source voltage, current in resistance I_R is in phase with voltage, while current in capacitance I_C leads by a phase of $\pi/2$ with voltage. Hence, correct option is (c).

2. $V^2 = V_R^2 + (V_L - V_C)^2 \Rightarrow V_R = V = 220\text{ V}$

Also, $i = \dfrac{220}{100} = 2.2\text{ A}$

3.
$$i = i_0 \cos(\omega t)$$
$$i = i_0 \text{ at } t = 0$$
$$i = \frac{i_0}{\sqrt{2}} \text{ at } \omega t = \frac{\pi}{4}$$
$$t = \frac{\pi}{4\omega} = \frac{\pi}{4(2\pi f)} = \frac{1}{8f}$$
$$t = \frac{1}{400} = 2.5\text{ ms}$$

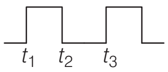
4. The voltages V_L and V_C are equal and opposite, so voltmeter reading will be zero.

Also, $R = 30\ \Omega$,

$X_L = X_C = 25\ \Omega$

So, $i = \dfrac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$

$= \dfrac{V}{R} = \dfrac{240}{30} = 8\text{ A}$

5. 

For t_1-t_2 Charging graph

t_2-t_3 Discharging graph

6. The phase difference, $\tan\phi = \dfrac{X_L}{R}$

and $X_L = \omega L = 2\pi f L$

$= 2\pi \times 50 \times 0.01 = \pi\ \Omega$

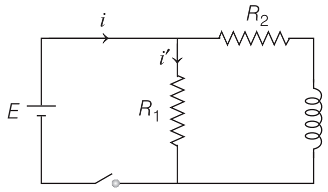
Also, $R = 1\ \Omega \Rightarrow \phi = \tan^{-1}(\pi)$

7. Four resistances form a balanced Wheatstone bridge.

So, current flowing in the coil will be zero and hence energy stored in the coil is zero.

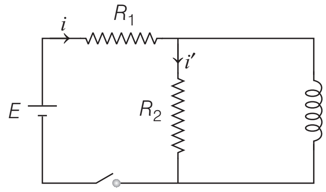
8. In circuit (1), on closing the switch, the current in the inductor is zero due to self-induction, i.e. $i_1 = 0$.

In circuit (2), on closing the switch, the current in the inductor is zero due to self-induction.



Therefore, $i_2 = i' = \dfrac{E}{R_1}$

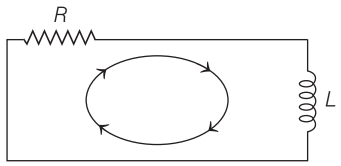
In circuit (3), on closing the switch, the current in the inductor is again zero due to the same reason.



Therefore, $i_3 = i' = \dfrac{E}{R_1 + R_2}$

Thus, it is obvious that, $i_2 > i_3 > i_1 (= 0)$.

9. After connecting C to B hanging the switch, the circuit will act like L-R discharging circuit.



Applying Kirchhoff's loop equation,

$$V_R + V_L = 0 \Rightarrow V_R = -V_L$$

$$\therefore \quad \frac{V_R}{V_L} = -1$$

10. Given, phase difference, $\phi = \dfrac{\pi}{4}$

As we know, for R-L or R-C circuit,

$$\tan\phi = \frac{\text{Capacitive reactance } (X_C) \text{ or inductive reactance } (X_L)}{\text{Resistance } (R)}$$

$$\tan\frac{\pi}{4} = \frac{X_C \text{ or } X_L}{R}$$

$$1 = \frac{X_C \text{ or } X_L}{R}$$

$$\Rightarrow \quad R = X_C \text{ or } X_L$$

Also, given $e = e_0 \sin(100\, t)$

Comparing the above equation with general equation of emf, *i.e.* $e = e_0 \sin \omega t$, we get

$\omega = 100\text{ rad/s} = 10^2\text{ rad/s}$

Now, checking option wise,

(a) For R-C circuit, with $R = 1\text{ k}\Omega = 10^3\ \Omega$ and $C = 1\,\mu\text{F} = 10^{-6}\text{F}$

So, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{10^2 \times 10^{-6}} = 10^4\ \Omega \Rightarrow R \neq X_C$

(b) For R-L circuit, with

$R = 1\text{ k}\Omega = 10^3\,\Omega$

and $L = 1\text{ mH} = 10^{-3}\text{ H}$

So, $X_L = \omega L = 10^2 \times 10^{-3} = 10^{-1}\Omega \Rightarrow R \neq X_L$

(c) For R-C circuit, with $R = 1\text{ k}\Omega = 10^3\,\Omega$

and $C = 10\,\mu\text{F} = 10 \times 10^{-6}\text{F} = 10^{-5}\text{ F}$

So, $X_C = \dfrac{1}{10^2 \times 10^{-5}} = 10^3\,\Omega \Rightarrow R = X_C$

(d) For R-L circuit, with $R = 1\,k\Omega = 10^3\,\Omega$

and $L = 10$ mH $= 10 \times 10^{-3}$ H $= 10^{-2}$H

$$X_L = 10^2 \times 10^{-2} = 1\,\Omega \Rightarrow R \neq X_L$$

11. Given, frequency, $f = 750$ Hz,

$$V_{\text{rms}} = 20\text{V}, R = 100\,\Omega, L = 0.1803\text{ H}$$

and $C = 10\,\mu\text{F}$

So, impedance of L-C-R circuit is

$$Z = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

$$= \sqrt{(100)^2 + \left(2\pi \times 750 \times 0.1803 - \frac{1}{2\pi \times 750 \times 10 \times 10^{-6}}\right)}$$

$$\approx 834\,\Omega$$

Now, power lost by L-C-R circuit, which occurs in resistance is given by

$$P = V_{\text{rms}} I_{\text{rms}} \cos\phi$$

As, $\cos\phi = \dfrac{R}{Z}$ and $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z}$

$$\therefore \quad P = \frac{V_{\text{rms}}^2 \cdot R}{Z^2} = \frac{(20)^2 \times 100}{(834)^2} = 0.0575\text{ Js}^{-1}$$

Heat developed in resistor, $H = Pt = S(\Delta\theta)$

where, t = time, S = heat capacity = 2 J/° C and

$\Delta\theta$ = temperature change = 10 ° C

$$\Rightarrow \quad t = \frac{S\Delta\theta}{P} = \frac{2 \times 10}{0.0575} = 348\text{ s}$$

12. In position-1

$$\tan\frac{\pi}{6} = \frac{\omega L}{R} = \frac{(1000)(\sqrt{3} \times 10^{-3})}{R}$$

or $\quad \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{R}$

$\therefore \quad R = 3\,\Omega$

In position-2

$$\tan\phi = \frac{X_C}{R} = \frac{(1/\omega C)}{R}$$

$$= \frac{1\Big/\left(1000 \times \frac{1000}{3} \times 10^{-6}\right)}{3} = 1$$

$\therefore \quad \phi = \dfrac{\pi}{4}$

Since, the circuit is C-R, hence current leads emf by $\pi/4$.

13. In the given graph current is leading the voltage by 45°. Therefore circuit is C-R.

$$\tan\phi = \frac{X_C}{R} \text{ and } \phi = \frac{\pi}{4}$$

$\therefore \quad X_C = R$

$\Rightarrow \quad \omega CR = 1$

$\Rightarrow \quad CR = \dfrac{1}{\omega} = \dfrac{1}{100}$

When $R = 1\,k\Omega = 10^3\,\Omega$, then

$$C = \frac{1}{10^5} = 10^{-5}\text{ F} = 10\,\mu\text{F}$$

14. This is a parallel circuit, for oscillation, the energy in L and C will be alternatively maximum.

15. $Z = \sqrt{R^2 + (X_C - X_L)^2} = 10\,\Omega$

$$\cos\phi = \frac{R}{Z} = \frac{4}{5}$$

$\therefore \quad \phi = 37°$

$$I_0 = \frac{V_0}{Z} = \frac{10}{10} = 1\text{ A}$$

$\therefore \quad I = 1\cos(100\pi t + 37°)$

$$V = 10\cos(100\pi t)$$

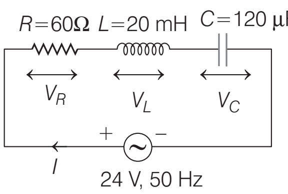
The applied voltage is $\dfrac{3}{5}$ th of the maximum applied voltage when, or $100\,\pi t = 53°$

$\therefore \quad I = 1\cos(53° + 37°) = 0$

$\therefore \quad V_R = 0$

$\therefore \quad V_{\text{AB}} = V_{\text{applied}} = \dfrac{3}{5} \times 10 = 6\text{ V}$

16. The given series R-L-C circuit is shown in the figure below



Here, V_R = potential across resistance (R),

V_L = potential across inductor (L)

and V_C = potential across capacitor (C).

Impedance of this series circuit is,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \text{...(i)}$$

$\because \quad X_L = \omega L = (2\pi f)(L) = 2\pi \times 50 \times 20 \times 10^{-3}\,\Omega$

$$X_L = 6.28\,\Omega \quad \text{...(ii)}$$

and $\quad X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi \times 50 \times 120 \times 10^{-6}} = \dfrac{250}{3\pi}\,\Omega \quad$ ...(iii)

and $X_L - X_C = \left(6.28 - \dfrac{250}{3\pi}\right) = -20.23\,\Omega \quad$ ...(iv)

RMS value of current in circuit,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{24}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$I_{\text{rms}} = \frac{24}{\sqrt{60^2 + (-20.23)^2}} = \frac{24}{63.18}$$

$$I_{\text{rms}} = 0.379\text{ A}$$

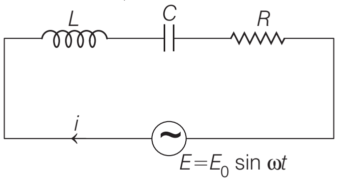
Therefore, energy dissipated is

$$= I_{rms}^2 \times R \times t$$

$$E = (0.379)^2 \times 60 \times 60$$

or $$= 517.10 = 5.17 \times 10^2 \text{ J}$$

17. For an L-C-R circuit,



By KVL, we have

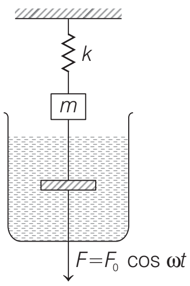
$$-L\frac{di}{dt} - \frac{q}{C} - iR + E = 0$$

Above can be rearranged as

$$L\frac{d^2q}{dt^2} + R\cdot\frac{dq}{dt} + \frac{1}{C}q = E_0 \sin \omega t \quad \text{...(i)}$$

Now, for a damped harmonic oscillator, we have

$$ma = -kx - bv + F_0 \cos \omega t$$



Rearranging above equation, we have

$$m\frac{d^2x}{dt^2} + b\cdot\frac{dx}{dt} + kx = F_0 \cos \omega t \quad \text{...(ii)}$$

On comparing Eqs. (i) and (ii), we get the following analogy

$$L \equiv m,\ R \equiv b \text{ and } \frac{1}{C} \equiv k$$

or $$L \leftrightarrow m,\ C \leftrightarrow \frac{1}{k} \text{ and } R \leftrightarrow b$$

18. The phase difference between the applied voltage and circuit current is $(55° - 10°) = 45°$ with current lagging. The angular frequency is $\omega = 3000$ rad/s. Since, current lags, $X_L > X_C$.

Net reactance, $X = (X_L - X_C)$.

Also, $X_L = \omega L = 3000 \times 0.01 = 30\ \Omega$

$$\tan\phi = X/R \text{ or } \tan 45° = X/R$$

$\therefore \quad X = R$

Now, $$Z = \frac{V_m}{I_m} = \frac{400}{10\sqrt{2}} = 28.3\ \Omega$$

$$Z^2 = R^2 + X^2 = 2R^2$$

$\therefore \quad R = Z/\sqrt{2} = 28.3/\sqrt{2} = 20\ \Omega;$

$$X = X_L - X_C = 30 - X_C = 20$$

$$X_C = 10\ \Omega \text{ or } \frac{1}{\omega C} = 10$$

$\Rightarrow \quad C = \dfrac{1}{3000} \qquad (\because \omega = 300\ \text{s}^{-1})$

$\Rightarrow \quad C = 33\mu\text{F}$

19. For DC, $R = \dfrac{V}{i} = \dfrac{100}{1} = 100\ \Omega$

For AC, $Z = \dfrac{V}{i} = \dfrac{100}{0.5} = 200\ \Omega$

$$Z = \sqrt{R^2 + (\omega L)^2}$$

$\Rightarrow \quad 200 = \sqrt{(100)^2 + 4\pi^2(50)^2L^2}$

$\therefore \quad L = 0.55$ H

20. $\tan\phi = \dfrac{X_L}{R} = \dfrac{X_C}{R}$

$\Rightarrow \quad \tan 60° = \dfrac{X_L}{R} = \dfrac{X_C}{R}$

$\Rightarrow \quad X_L = X_C = \sqrt{3}R \quad (\because$ the circuit is in resonance)

i.e. $\quad Z = \sqrt{R^2 + (X_L - X_C)^2} = R$

So, average power, $P = \dfrac{V^2}{R} = \dfrac{200 \times 200}{100} = 400$ W

21. Current will be maximum in the condition of resonance, so $i_{max} = \dfrac{V}{R} = \dfrac{V}{10}$ A

Energy stored in the coil,

$$W_L = \frac{1}{2}Li_{max}^2 = \frac{1}{2}L\left(\frac{E}{10}\right)^2$$

$$= \frac{1}{2} \times 10^{-3}\left(\frac{E^2}{100}\right) = \frac{1}{2} \times 10^{-5}E^2 \text{ J}$$

$\therefore$ Energy stored in the capacitor,

$$W_C = \frac{1}{2}CE^2 = \frac{1}{2} \times 2 \times 10^{-6}\ E^2 = 10^{-6}E^2 \text{ J}$$

$\therefore \quad \dfrac{W_C}{W_L} = \dfrac{1}{5}$

22. At resonance, power,

$$P = \frac{(V_{rms})^2}{R}$$

$$P = \frac{(250/\sqrt{2})^2}{8} = 3906.25 \text{ W} \approx 4 \text{ kW}$$

$\therefore \quad x = 4$

23. $$J_c = \frac{E}{\rho} = \frac{V}{\rho d}$$

$$J_d = \frac{1}{A}\frac{dq}{dt} = \frac{C}{A}\frac{dV_c}{dt} = \frac{\varepsilon}{d}\frac{dV_c}{dt}$$

$$\frac{J_c}{J_d} = \frac{\varepsilon_0 \tan\left(2\pi \times \dfrac{900}{800}\right)}{80\varepsilon_0\rho 2\pi f}$$

$$= \frac{2 \times 9 \times 10^9 \tan\left(\dfrac{9\pi}{4}\right)}{80 \times 0.25 \times 900} = 10^6$$

$\therefore \quad x = 6$