VERY SIMILAR
PRACTICE TEST 10

Hints and Explanations

1. **(a) :** Given, $\tau = a \times L + b \times I/\omega$

$$\therefore \quad [a] = \frac{[\tau]}{[L]} = \frac{[I\alpha]}{[I\omega]} = \frac{[T^{-2}]}{[T^{-1}]} = [T^{-1}]$$

$$[b] = \frac{[\tau]}{[I/\omega]} = \frac{[I\alpha]}{[I/\omega]} = [\alpha\omega] = [T^{-2}][T^{-1}] = [T^{-3}]$$

$\therefore \quad [a \times b] = [T^{-1}]\,[T^{-3}] = [M^0L^0T^{-4}]$

2. **(a) :** The electric field at the surface of the sphere is Aa and being radial it is along the outward normal. The flux of the electric field is, therefore,

$$\Phi = \oint E dS \cos 0° = Aa(4\pi a^2).$$

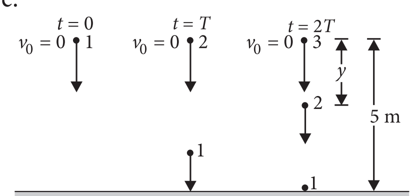
From Gauss's law, the charge contained in the sphere is

$$Q_{\text{inside}} = \varepsilon_0 \Phi = 4\pi\varepsilon_0 A a^3$$

$$= \frac{1}{9\times 10^9} \times 100 \text{ V m}^{-2} \times (0.20)^3$$

$$= 8.89 \times 10^{-11} \text{ C}$$

3. **(c) :** Let T be the time interval between the drops (1, 2, 3) falling from the tap as shown in the figure.



Since distance covered by the first drop in time $2T$ is 5 m,

$$5 = \frac{1}{2}g(2T)^2 = 2gT^2 \qquad \text{...(i)}$$

Further, distance covered by the second drop in time T (from $t = T$ to $t = 2T$),

$$y = \frac{1}{2}gT \qquad \text{...(ii)}$$

From eqns. (i) and (ii), $y = 1.25$ m

Distance of the second drop from the ground

$= 5 - y = 5 - 1.25 = 3.75$ m

4. **(c) :** The 30 cm length of the scale reads upto 60 kg.

$\therefore$ Maximum force, $F = 60$ kg wt $= 60 \times 9.8$ N $= 588$ N

and maximum extension, $x = 30 - 0 = 30$ cm $= 30 \times 10^{-2}$ m

Spring constant of the spring balance is

$$k = \frac{F}{x} = \frac{588\text{ N}}{30\times 10^{-2}\text{ m}} = 1960 \text{ N/m}$$

Let a body of mass m is suspended from this balance.

Then, time period of oscillation,

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{or} \quad T^2 = \frac{4\pi^2 m}{k}$$

$$\therefore \quad m = \frac{T^2 k}{4\pi^2} = \frac{(0.8)^2 \times (1960)}{4 \times (3.14)^2} = 31.8 \text{ kg}$$

Weight of the body $= mg = (31.8 \text{ kg})\,(9.8 \text{ m/s}^2) = 311.64$ N

5. **(d) :** R_{BC} (right hand side) $= \frac{8\times 8}{8+8}\Omega$

$= 4\ \Omega$ (as $2\ \Omega + 4\ \Omega + 2\ \Omega = 8\ \Omega$)

R_{AD} (right hand side) is again $4\ \Omega$.

Equivalent resistance of the circuit,

$R = 3\ \Omega + 4\ \Omega + 2\ \Omega = 9\ \Omega$

Current drawn from battery $I = \frac{V}{R} = \frac{9}{9} = 1$ A

At A, I is equally divided ($I/2$) between 8 Ω resistance and the remaining circuit of 8 Ω. At B, ($I/2$) is equally divided ($I/4$) between the 8 Ω resistor and the remaining circuit of resistance 8 Ω.

Thus, current through 4 Ω resistor is $I/4$, *i.e.*, 0.25 A.

6. **(a) :** Here, $m = 1$ kg, $v_i = 2$ m s^{-1}, $k = 0.5$ J

Initial kinetic energy,

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}\times(1\text{ kg})(2\text{ m s}^{-1})^2 = 2\text{ J}$$

Work done by retarding force

$$W = \int F_r dx = \int_{0.1}^{2.01} -\frac{k}{x}dx = -k\left[\ln x\right]_{0.1}^{2.01}$$

$$= -k\ln\left(\frac{2.01}{0.1}\right) = -0.5\ \ln(20.1) = -1.5\ \text{J}$$

According to work-energy theorem

$W = K_f - K_i$

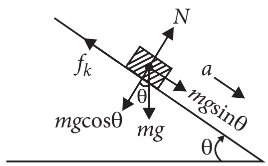
or $K_f = W + K_i = -1.5\ \text{J} + 2\ \text{J} = 0.5\ \text{J}$

7. (d) : Let μ_s and μ_k be the coefficients of static and kinetic friction between the box and the plank respectively.

When the angle of inclination θ reaches 30°, the block just slides,

$$\therefore\quad \mu_s = \tan\theta = \tan 30° = \frac{1}{\sqrt{3}} = 0.6$$

If a is the acceleration produced in the block, then



$ma = mg\sin\theta - f_k$

$\quad = mg\sin\theta - \mu_k N$

(where f_k is force of kinetic friction as $f_k = \mu_k N$)

$a = g(\sin\theta - \mu_k\cos\theta)$ (as $N = mg\cos\theta$)

As $g = 10\ \text{m s}^{-2}$ and $\theta = 30°$

$\therefore\ a = (10\ \text{m s}^{-2})(\sin 30° - \mu_k \cos 30°)$ ...(i)

If s is the distance travelled by the block in time t, then

$$s = \frac{1}{2}at^2 \quad (\text{as } u = 0) \text{ or } a = \frac{2s}{t^2}$$

But $s = 4.0$ m and $t = 4.0$ s (given)

$$\therefore\quad a = \frac{2(4.0\ \text{m})}{(4.0\ \text{s})^2} = \frac{1}{2}\ \text{m s}^{-2}$$

Substituting this value of a in equation (i), we get

$$\frac{1}{2}\ \text{m s}^{-2} = (10\ \text{m s}^{-2})\left(\frac{1}{2} - \mu_k\frac{\sqrt{3}}{2}\right)$$

$$\frac{1}{10} = 1 - \sqrt{3}\mu_k \quad \text{or} \quad \sqrt{3}\mu_k = 1 - \frac{1}{10} = \frac{9}{10} = 0.9$$

$$\mu_k = \frac{0.9}{\sqrt{3}} = 0.5$$

8. (d) : The time period T of oscillation of a magnet is given by

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

As I and M remain the same,

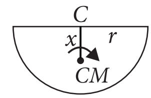
$$\therefore\quad T \propto \frac{1}{\sqrt{B}} \text{ or } \frac{T_2}{T_1} = \sqrt{\frac{B_1}{B_2}}$$

According to given problem,

$B_1 = 24\ \mu\text{T}$, $B_2 = 24\ \mu\text{T} - 18\ \mu\text{T} = 6\ \mu\text{T}$, $T_1 = 2$ s

$$\therefore\quad T_2 = (2\ \text{s})\sqrt{\frac{24\ \mu\text{T}}{6\ \mu\text{T}}} = 4\ \text{s}$$

9. (d) : We know, $I_C = mr^2/2$



Using parallel axes theorem,

$I_C = I_{CM} + mx^2$

$\therefore\ I_{CM} = I_C - mx^2 = mr^2/2 - mx^2$

10. (d) : If I_1 is the current through R_1 and I_2 is the current through L and R_2, then $I_1 = \frac{\varepsilon}{R_1}$ and $I_2 = I_0(1 - e^{-t/\tau})$,

Where $\tau = \frac{L}{R_2} = \frac{400\times10^{-3}}{2} = 0.2$ s

and $I_0 = \frac{\varepsilon}{R_2} = \frac{12}{2} = 6$ A

Thus, $I_2 = 6(1 - e^{-t/0.2})$

Potential drop across L, *i.e.*,

$\varepsilon - R_2 I_2 = 12\ \text{V} - 2 \times 6(1 - e^{-t/0.2})\ \text{V} = (12e^{-5t})\ \text{V}$

11. (d) : As $\varepsilon_{rms} = \frac{\varepsilon_0}{\sqrt{2}} = \frac{(1/\sqrt{2})}{\sqrt{2}} = \frac{1}{2}$ V,

$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{(1/\sqrt{2})}{\sqrt{2}} = \frac{1}{2}$ A,

and $\cos\phi = \cos \pi/3 = \frac{1}{2}$

$$P_{av} = \varepsilon_{rms} I_{rms} \cos\phi = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{8}\ \text{W}$$

12. (d) : Frequency remains unchanged with change of medium.

Refractive index, $n = \frac{c}{v} = \frac{1/\sqrt{\varepsilon_0\mu_0}}{1/\sqrt{\varepsilon\mu}} = \sqrt{\varepsilon_r\mu_r}$

Since μ_r is very close to 1, $n = \sqrt{\varepsilon_r} = \sqrt{4} = 2$

Thus, $\lambda_{medium} = \frac{\lambda}{n} = \frac{\lambda}{2}$

13. (a) : Gravitational potential due to the shell of radius a at any point inside it $= -\frac{GM}{a}$

Gravitational potential due to the particle at the centre at a point P distant $\frac{a}{2}$ from the centre

$$= -\frac{GM}{a/2} = -\frac{2GM}{a}$$

$\therefore$ Net gravitational potential at P

$$= -\frac{GM}{a} - \frac{2GM}{a} = -\frac{3GM}{a}$$

14. (c) : If W_1 and W_2 are widths of two slits, then

$$\frac{I_1}{I_2} = \frac{W_1}{W_2} = 4$$

Also, $\frac{I_1}{I_2} = \frac{A_1^2}{A_2^2}$ $\therefore$ $\frac{A_1^2}{A_2^2} = 4$ or $\frac{A_1}{A_2} = 2$

$$\frac{I_{max}}{I_{min}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2$$

$$= \frac{\left(\frac{A_1}{A_2} + 1\right)^2}{\left(\frac{A_1}{A_2} - 1\right)^2} = \frac{(2+1)^2}{(2-1)^2} = \frac{9}{1}$$

15. (d) : Here, $f_o = 1.5$ cm, $f_e = 6.25$ cm, $u_o = -2$ cm, $v_e = -25$ cm

For objective,

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o} \quad \therefore \quad \frac{1}{v_o} - \frac{1}{-2} = \frac{1}{1.5}$$

$$\frac{1}{v_o} = \frac{1}{1.5} - \frac{1}{2} \text{ or } v_o = 6 \text{ cm}$$

For eye piece,

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e} \Rightarrow \frac{1}{-25} - \frac{1}{u_e} = \frac{1}{6.25}$$

$$-\frac{1}{u_e} = \frac{1}{6.25} + \frac{1}{25} \text{ or } u_e = -5 \text{ cm}$$

Distance between two lenses = $|v_o| + |u_e|$
= 6 cm + 5 cm = 11 cm

16. (d) : According to Bohr's theory of hydrogen atom

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\upsilon = \frac{c}{\lambda} = RcZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\therefore \quad \upsilon \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \frac{n_2^2 - n_1^2}{n_1^2 n_2^2}$$

$$\upsilon \propto \left(\frac{n^2 - (n-1)^2}{n^2(n-1)^2}\right) = \frac{2n-1}{n^2(n-1)^2}$$

When $n >> 1$,

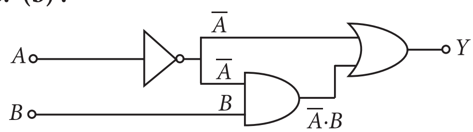
$$\upsilon \propto \frac{2n}{n^4}, \textit{i.e.}, \upsilon \propto \frac{1}{n^3} \text{ or } \upsilon \propto n^{-3}$$

17. (a) : Modulation index, $\mu = \frac{A_m}{A_c}$

$$\mu = \frac{0.5 \text{ V}}{10 \text{ V}} = 0.05$$

The side bands frequencies are $\upsilon_{SB} = \upsilon_c \pm \upsilon_m$
$= (1 \pm 0.010)$ MHz

18. (b) :



The output Y of the logic circuit is

$$Y = \bar{A} + \bar{A} \cdot B = \bar{A} \cdot (1 + B) = \bar{A} \cdot 1 = \bar{A}$$

19. (c) : Length of the wire at temperature T_2 is

$$L_t = L\left(1 + \frac{1}{100}\right) \therefore \quad 2L_t^2 = 2L^2\left(1 + \frac{1}{100}\right)^2$$

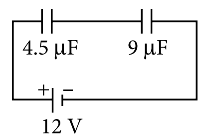
Now $2L_t^2$ = area of the plate at temperature T_2 (A_t) and $2L^2$ = area of the plate at temperature T_1 (A).

$$\therefore \quad A_t = A\left(1 + \frac{1}{100}\right)^2 = A\left(1 + \frac{2}{100}\right) = \frac{102A}{100}$$

Thus, the area increases by 2%.

20. (d)

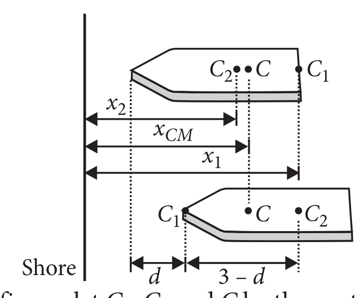
21. (8.0) : The given arrangement can be redrawn as



$\therefore$ Potential difference across 4.5 μF capacitor

$$= \frac{9\,\mu\text{F}}{(9\,\mu\text{F} + 4.5\,\mu\text{F})} \times 12 \text{ V} = 8 \text{ V}$$

22. (0.75) :



As shown in figure, let C_1, C_2 and C be the centres of mass of the boy, boat and the system (boy and boat) respectively. Let x_1 and x_2 be the distances of C_1 and C_2 from the shore. Then the centre of mass will be at a distance,

$$x_{CM} = \frac{30x_1 + 90x_2}{30 + 90}$$

As the boy moves from the stern to the bow, the boat moves backward through a distance d so that position of the centre of mass of the system remains unchanged.

$$x'_{CM} = \frac{30[x_1-(3-d)]+90(x_2+d)}{30+90}$$

As $x'_{CM} = x_{CM}$

$$\frac{30(x_1-3+d)+90(x_2+d)}{120} = \frac{30x_1+90x_2}{120}$$

or $-90 + 30\,d + 90\,d = 0$

or $d = 0.75$ m

23. (6.0): Pressure for soap bubble A

$$P_A = P_0 + \frac{4S}{r_A}$$

$$P_A = 8\text{ N m}^{-2} + \frac{4\times 0.04\text{ N m}^{-1}}{2\times 10^{-2}\text{ m}}$$

$P_A = 16\text{ Nm}^{-1}$

$$P_B = 8\text{ N m}^{-2} + \frac{4\times 0.04\text{ N m}^{-1}}{4\times 10^{-2}\text{ m}}$$

$P_B = 12\text{ N m}^{-2}$

$$V_A = \frac{4}{3}\pi r_B^3 \text{ and } V_B = \frac{4}{3}\pi r_B^3$$

$$\frac{V_B}{V_A} = \frac{r_B^3}{r_A^3} = \left(\frac{4}{2}\right)^3 \quad \text{...(i)}$$

Using ideal gas equation

$PV = nRT$

For bubble A, $P_AV_A = n_ART$ ...(ii)

For soap bubble B, $P_BV_B = n_BRT$ ...(iii)

From equation (i), (ii) and (iii)

$$\frac{n_B}{n_A} = \frac{P_B}{P_A}\left(\frac{V_B}{V_A}\right)$$

$$= \frac{12}{16}\times\left(\frac{4}{2}\right)^3 \Rightarrow \frac{n_B}{n_A} = 6$$

24. (66.7) : Power of the drill,

$P = 0.2\text{ hp} = (0.2)\,(750\text{ W}) = 150\text{ W}$

Work done (W) by the drill in 20 second

$= P \times 20\text{ s}$ (as P = work/time)

or $W = (150\text{ W})\,(20\text{ s}) = 3000\text{ J}$ ...(i)

Mass of iron, $m = 100\text{ g} = 0.1\text{ kg}$

Specific heat of iron, $c = 450\text{ J kg}^{-1}\,°\text{C}^{-1}$

If ΔT is the rise in temperature of iron,

$Q = mc\,\Delta T$

$= 0.1 \times 450 \times \Delta T$

$= (45\,\Delta T)\text{ J}$ ...(ii)

From equations (i) and (ii),

or $45\,\Delta T = 3000$

$$\text{or}\quad \Delta T = \frac{3000}{45}\,°\text{C} = 66.7\,°\text{C}$$

25. (1.5) : $\frac{1}{2}mv_{max}^2 = eV_0 = \frac{hc}{\lambda} - W_0$

$$\text{or } W_0 = \frac{hc}{\lambda} - eV_0$$

$$\therefore\ \frac{hc}{\lambda_1} - eV_1 = \frac{hc}{\lambda_2} - eV_2$$

$$\text{or}\quad V_2 = \frac{hc}{e}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right) + V_1$$

$$\text{or}\quad V_2 = \frac{6.6\times10^{-34}\times3\times10^8}{1.6\times10^{-19}\times10^{-9}}\left[\frac{1}{427.2} - \frac{1}{640.2}\right] + 0.54$$

$$\text{or}\quad V_2 = 12.375\times10^2\left[\frac{640.2-427.2}{427.2\times640.2}\right] + 0.54$$

$$\text{or}\quad V_2 = 12.375\times10^2\times\frac{213}{427.2\times640.2} + 0.54$$

or $V_2 = 1.5$ V

26. (a) : If $AlCl_3$ is present in ionic state in aqueous solution (Al^{3+} and three Cl^- ions), then standard heat of hydration of Al^{3+} and three Cl^- ions $= -4665 + (3 \times -381) = -5808\text{ kJ mol}^{-1}$

This hydration energy is greater than ionisation energy of Al, hence $AlCl_3$ would be ionic in aqueous solution.

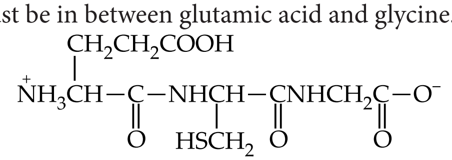
27. (d) : $r_n = a_0 \times n^2$, $\quad r_4 = a_0 \times 4^2 = 16a_0$

$$mv = 4\times\frac{h}{2\pi r_4} = \frac{4h}{2\pi\times16a_0} = \frac{h}{8\pi a_0}$$

$$\therefore\ \lambda = \frac{h}{mv} = 8\pi a_0$$

28. (a) : Urea (NH_2CONH_2) acts as stabiliser.

29. (a) : Since the tripeptide on hydrolysis gave two dipeptides *Glu-Cys* and *Cys-Gly*, hence cystine must be in between glutamic acid and glycine.



Glu-Cys-Gly

30. (c) : When positive and negative sols are mixed, they coagulate each other.

31. (c) : Oxidation state of S in $(NH_4)_2S_2O_8 = +6$

(Since $S_2O_8^{2-}$ has one peroxide bond)

Oxidation state of Os in $OsO_4 = +8$
Oxidation state of S in $H_2SO_5 = +6$
(Since it has one peroxide bond)
Oxidation state of O in $KO_2 = -1/2$

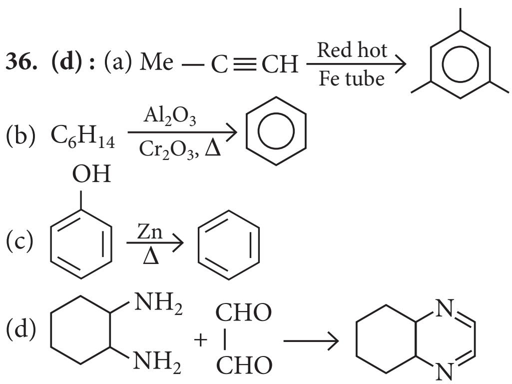
32. (b) : In the rate law expression, rate $= k[X][Y]^2$, if Y is reduced to one-fourth, the rate of reaction will be 1/16 times of the original rate.

33. (c) : + 3 and + 4 states are shown by Ce in aqueous solution.

34. (a) : The conversion of metal sulphide to metal oxide involves the process of roasting (*i.e.*, *x* is roasting). The metal oxides can then be converted to impure metal by reduction. *i.e.*, '*y*' is smelting.
The conversion of impure metal to pure metal involves a process of purification. Thus, it is electrolysis.

35. (d) : Be^{2+} being smaller in size has maximum hydration enthalpy which exceeds its lattice enthalpy.

36. (d) :



(not aromatic)

37. (b) : Thermal stability decreases as the size of atom increases (down the group). Thus, H_2O is most stable and H_2Te is least stable.

38. (c) : $2Al_{(s)} + 6HCl_{(aq)} \longrightarrow$

$$2Al^{3+}_{(aq)} + 6Cl^-_{(aq)} + 3H_{2(g)}$$

At STP, 6 moles of HCl produces 3 moles of H_2
or 3×22.4 lit of H_2

$\therefore$ 1 mole of HCl produces

$$\frac{3 \times 22.4}{6} = 11.2 \text{ lit of } H_2$$

Again at STP, 2 moles of Al produces 3 moles of H_2
or 3×22.4 lit of H_2

or 1 mole of Al produces $\frac{3 \times 22.4}{2} = 33.6$ lit of H_2

39. (d) :

Diatomic species	Bond order
NO	2.5
O_2^-	1.5
C_2^{2-}	3.0
He_2^+	0.5

Thus increasing order : $He_2^+ < O_2^- < NO < C_2^{2-}$

40. (d) : $CH_3-C(CH_3)_2-ONa + CH_3Cl \longrightarrow CH_3-C(CH_3)_2-O-CH_3 + NaCl$

Methyl *t*-butyl ether

whereas,

$CH_3-C(CH_3)_2-Cl + CH_3ONa \longrightarrow CH_3-C(CH_3)=CH_2 +$ (Alkene)

$+ CH_3OH + NaCl$

Secondary and tertiary alkyl halides readily undergo elimination reaction rather than ether formation in the presence of alkoxide.

41. (c) : Number of O-atoms per unit cell

$$= \frac{1}{8} \times 8 + \frac{1}{2} \times 6 = 4$$

Number of octahedral holes per unit cell

$$= 1 \times 4 = 4$$

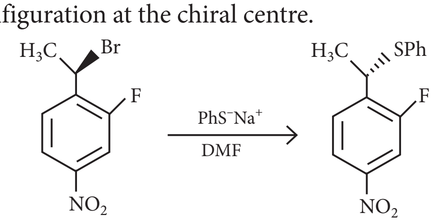
Number of Fe^{3+} ions per unit cell $= \frac{50 \times 4}{100} = 2$

Number of tetrahedral voids per unit cell $= 2 \times 4 = 8$

Number of Zn^{2+} ions per unit cell $= \frac{1}{8} \times 8 = 1$

Hence, formula is $ZnFe_2O_4$.

42. (a) : This reaction will not proceed via S_N1 mechanism as the carbocation formed will be destabilised by the $-I$ effect of flourine and $-R$ effect of —NO_2 group. Benzyl halides are more reactive than aryl halides in S_N2 reactions. Therefore, the reaction occurs in the side chain with inversion of configuration at the chiral centre.



43. (a) : Classical smog is formed during winter season in early morning hours.

44. (d) : $E = E^\circ + \frac{0.591}{n}\log[M^{n+}]$

Lower the concentration of M^{n+}, lower is the E.

45. (c) : $C_6H_5CH\,\boxed{O + H_2}\,CHCOCH_3 \xrightarrow{\text{Aq. NaOH}} C_6H_5CH{=}CHCOCH_3 + H_2O$

46. (7)

47. (5) : $-CH_3$ is an electron donating group, therefore, shows $+I$ effect.

48. (7.4): $C = \frac{1}{10}$ M

$K_4[Fe(CN)_6] \rightarrow 4K^+ + [Fe(CN)_6]^{4-}$

$\therefore\ n = 5$

Degree of dissociation, $\alpha = \frac{50}{100} = 0.5$

$\alpha = \frac{i-1}{n-1}, 0.5 = \frac{i-1}{5-1}, 0.5 = \frac{i-1}{4}$

$\Rightarrow\ i - 1 = 2\ \ \therefore\ \ i = 3$

So, osmotic pressure, $\pi = iCRT$

$= 3 \times \frac{1}{10} \times 0.0821 \times 300 = 90 \times 0.0821 = 7.389$ atm $\simeq 7.4$ atm

49. (2.59) : We know that, $\chi_C - \chi_H = 0.208\sqrt{\Delta}$

$\Delta = E_{C-H} - \sqrt{E_{C-C} \times E_{H-H}}$

$= 98.8 - \sqrt{83.1 \times 104.2} = 98.8 - 93.05 = 5.75$

$\therefore\ \chi_C - \chi_H = 0.208\sqrt{5.75} = 0.498$

$\chi_C = 0.498 + \chi_H = 0.498 + 2.1$

$= 2.59$

50. (8) : $CH_3NH_2 + HCl \longrightarrow CH_3NH_3^+Cl^-$

0.1	0.08	0
0.02	0	0.08

As it is a basic buffer solution,

$pOH = pK_b + \log\frac{0.08}{0.02} = -\log(5 \times 10^{-4}) + \log 4$

$= 3.30 + 0.602 = 3.902$

$pH = 14 - 3.902 = 10.09;$

$[H^+] = 8 \times 10^{-11}$ M

51. (c) : $A - (A - B) = A \cap (A - B)^c = A \cap (A \cap B^c)^c$

$= A \cap (A^c \cup B) = (A \cap A^c) \cup (A \cap B)$ [distributive law]

$= \phi \cup (A \cap B) = A \cap B$

52. (a) : Let $z = x + iy$. Then, $|z - 3 - i| = |z - 9 - i|$

$\Rightarrow\ \sqrt{(x-3)^2 + (y-1)^2} = \sqrt{(x-9)^2 + (y-1)^2}$

$\Rightarrow\ x = 6$

Also, $|z - 3 + 3i| = 3$

$\Rightarrow\ \sqrt{(x-3)^2 + (y+3)^2} = 3$

For $x = 6, y = -3.\ \therefore\ z = 6 - 3i$

$\therefore$ There is only one complex number.

53. (d) : For infinitely many solutions, $|A| = 0$

i.e., $\begin{vmatrix} (\alpha+1)^3 & (\alpha+2)^3 & -(\alpha+3)^3 \\ \alpha+1 & \alpha+2 & -(\alpha+3) \\ 1 & 1 & -1 \end{vmatrix} = 0$

$\Rightarrow\ 6\alpha + 12 = 0 \Rightarrow \alpha = -2$.

54. (c) : From the given information, we may write a relation $y = x + 20$, between the two sets of data.

[where x denotes the old values and y denotes the new values]

So, standard deviation of $x = \sqrt{10}$

Let $y_i = x_i + 20$ where $i = 1, 2, ..., 11$

$\therefore\ \bar{y} = \bar{x} + 20$

$\Rightarrow\ \frac{1}{n}\sum_{i=1}^{11}(y_i - \bar{y})^2 = \frac{1}{n}\sum_{i=1}^{11}(x_i - \bar{x})^2$

$\Rightarrow\ \sqrt{\frac{1}{n}\sum_{i=1}^{11}(y_i - \bar{y})^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{11}(x_i - \bar{x})^2} = \sqrt{10}$

Thus the standard deviation of y is $\sqrt{10}$.

55. (c) : $\exp\lim_{x\to 0}\frac{\tan\left(\frac{\pi}{4} + x\right) - 1}{x}$

$= \exp\lim_{x\to 0}\left(\frac{1 + \tan x}{1 - \tan x} - 1\right)\frac{1}{x}$

$= \exp\lim_{x\to 0}\frac{2\tan x}{x}\frac{1}{(1 - \tan x)} = e^2$

56. (d) : We have, $x = \frac{1 - \sin\phi}{\cos\phi},\ y = \frac{1 + \cos\phi}{\sin\phi}$

$xy + 1 = \frac{1 - \sin\phi + \cos\phi}{\cos\phi \cdot \sin\phi} = -(x - y).$

Hence, $xy + 1 + x - y = 0$

$\therefore\ x = \frac{y-1}{y+1},\ y = \frac{1+x}{1-x}.$

57. (b) : Let $I = \int_0^{\pi/2} \frac{2\sqrt{\cos\theta}}{3\left(\sqrt{\sin\theta}+\sqrt{\cos\theta}\right)} d\theta$

$\therefore \frac{3}{2}I = \int_0^{\pi/2} \frac{\sqrt{\cos\theta}}{\sqrt{\sin\theta}+\sqrt{\cos\theta}} d\theta$ ...(i)

$\frac{3}{2}I = \int_0^{\pi/2} \frac{\sqrt{\sin\theta}}{\sqrt{\cos\theta}+\sqrt{\sin\theta}} d\theta$ ...(ii)

Adding (i) and (ii), we get $3I = \int_0^{\pi/2} d\theta = \frac{\pi}{2}$

$\Rightarrow I = \pi/6$

58. (c) : The pair is

$$\left(\frac{\cos^2\theta}{4} + \sin^2\theta - \frac{1}{3}\right)x^2 + \frac{2\sin\theta}{\sqrt{3}}\cdot xy + \frac{\cos^2\theta}{4}\cdot y^2 = 0$$

$$a + b = \frac{\cos^2\theta}{4} + \sin^2\theta - \frac{1}{3} + \frac{\cos^2\theta}{4}$$

$$= \frac{\cos^2\theta}{2} + \sin^2\theta - \frac{1}{3} = \frac{1 + 3\sin^2\theta}{6}$$

$$h^2 - ab = \frac{\sin^2\theta}{3} - \left(\frac{\cos^2\theta}{4} + \sin^2\theta - \frac{1}{3}\right)\frac{\cos^2\theta}{4}$$

$$= \frac{(1+3\sin^2\theta)^2}{48}, \text{ on simplication}$$

$$\therefore \theta = \tan^{-1}\left(\frac{2\sqrt{h^2-ab}}{a+b}\right) = \tan^{-1}\left(\frac{\frac{2(1+3\sin^2\theta)}{4\sqrt{3}}}{\frac{1+3\sin^2\theta}{6}}\right)$$

$$= \tan^{-1}\sqrt{3} = \frac{\pi}{3}.$$

59. (a) : We know, $\frac{T_{r+1}}{T_r} = \frac{N-r+}{r}\cdot x$

Given $N = 2n+1 \Rightarrow \frac{T_{r+1}}{T_r} = \frac{2n+2-r}{r}\cdot x$

$\therefore T_{r+1} \geq T_r$

$\Rightarrow 2n+2-r \geq r \Rightarrow 2n+2 \geq 2r \Rightarrow r \leq n+1$

$\therefore r = n$

$$T_{r+1} = T_{n+1} = {}^{2n+1}C_{n+1} = \frac{(2n+1)!}{(n+1)!n!}.$$

60. (a) : The equation of the line passing through $P(1, -2, 3)$ and parallel to the given line is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6}$$

Suppose it meets the plane $x - y + z = 5$ at the point Q given by

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$$

i.e., $(2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$

This lies on $x - y + z = 5$. Therefore,

$2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5 \Rightarrow -7\lambda = -1 \Rightarrow \lambda = \frac{1}{7}$

So, the co-ordinates of Q are $(9/7, -11/7, 15/7)$.

Hence, required distance

$$= PQ = \sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}} = 1.$$

61. (d) : $x + y = z \Rightarrow 1 + \frac{dy}{dx} = \frac{dz}{dx}, \frac{dz}{dx} = 1 + \cos z$,

$$dx = \frac{dz}{1+\cos z} = \frac{1}{2}\sec^2\frac{z}{2}dz$$

$$\Rightarrow x = \tan\frac{z}{2} + c \text{ or } x = \tan\left(\frac{x+y}{2}\right) + c$$

$$y\left(\frac{\pi}{2}\right) = 0 \Rightarrow \frac{\pi}{2} = 1 + c \Rightarrow c = \frac{\pi}{2} - 1$$

$$x = \tan\left(\frac{x+y}{2}\right) + \frac{\pi}{2} - 1,$$

$$y = -x + 2\tan^{-1}\left(x + 1 - \frac{\pi}{2}\right)$$

$$y(0) = 2\tan^{-1}\left(1 - \frac{\pi}{2}\right) = -2\tan^{-1}\left(\frac{\pi}{2} - 1\right).$$

62. (a) : Since x lies in 3rd quadrant.

So, $\cos x$ and $\cos\frac{x}{2}$ are negative

Now, $\tan x = \frac{3}{4} \Rightarrow \cos x = -\frac{4}{5}$

Also, $\cos^2\frac{x}{2} = \frac{1}{2}\{\cos x + 1\} = \frac{1}{2}\left\{-\frac{4}{5}+1\right\} = \frac{1}{10}$

$\Rightarrow \cos\frac{x}{2} = \frac{-1}{\sqrt{10}}$

63. (d) : $\lim_{x\to 0} \frac{x - e^x + 1 - (1-\cos 2x)}{x^2} = -\frac{1}{2} - 2$

$$= -\frac{5}{2}$$

hence for continuity $f(0) = -\frac{5}{2}$

$\therefore [f(0)] = \left[-\frac{5}{2}\right] = -3; \{f(0)\} = \left\{-\frac{5}{2}\right\} = \frac{1}{2}$

Hence $[f(0)].\{f(0)\} = -\frac{3}{2} = -1.5$

64. (d) : The equation of the circle passing through the intersection of the circle

$x^2 + y^2 = 4$ and $x^2 + y^2 - 2x - 4y + 4 = 0$ is

$(x^2 + y^2 - 2x - 4y + 4) + \lambda(x^2 + y^2 - 4) = 0$

$\Rightarrow (1+\lambda)x^2 + (1+\lambda)y^2 - 2x - 4y + 4(1-\lambda) = 0$

$\Rightarrow \quad x^2 + y^2 - \frac{2x}{1+\lambda} - \frac{4y}{1+\lambda} + \frac{4(1-\lambda)}{1+\lambda} = 0$ ...(i)

Since the line $x + 2y = 0$ touches the circle (i)

$$\therefore \frac{\frac{1}{1+\lambda} + \frac{4}{1+\lambda}}{\sqrt{1^2 + 2^2}} = \sqrt{\left(\frac{1}{1+\lambda}\right)^2 + \left(\frac{2}{1+\lambda}\right)^2 - \frac{4(1-\lambda)}{1+\lambda}}$$

$$\Rightarrow \frac{\frac{5}{1+\lambda}}{\sqrt{5}} = \sqrt{\frac{5-(4)(1-\lambda^2)}{(1+\lambda^2}}$$

$$\Rightarrow \frac{\sqrt{5}}{1+\lambda} = \sqrt{\frac{1+4\lambda^2}{(1+\lambda)^2}} \Rightarrow \frac{5}{(1+\lambda)^2} = \frac{1+4\lambda^2}{(1+\lambda)^2}$$

$\Rightarrow 1 + 4\lambda^2 = 5 \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = 1$

Now put the value of $\lambda = 1$ in equation (i) we get

$x^2 + y^2 - x - 2y = 0$

65. (d) : Matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & \lambda & 5 \end{bmatrix}$ be non singular

only if $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & \lambda & 5 \end{vmatrix} \neq 0$

$\Rightarrow 1(25 - 6\lambda) - 2(20 - 18) + 3(4\lambda - 15) \neq 0$

$\Rightarrow 25 - 6\lambda - 4 + 12\lambda - 45 \neq 0$

$\Rightarrow 6\lambda - 24 \Rightarrow 0 \Rightarrow \lambda \neq 4.$

66. (d) : $P(n) = n(n + 1)$

$\therefore \quad P(3) = 3 \times 4 = 12$ (even)

$P(100) = 100 \times 101 = 10100$ (even)

$P(50) = 50 \times 51 = 2550$ (even)

As $P(3)$, $P(100)$, $P(50)$ are even numbers.

Hence, option (d) is correct.

67. (c) : We have first term $= a$ ...(i)

Second term $= b$ ...(ii)

and last term $= 2a$ ...(iii)

Let d be the common difference.

From (i), (ii) and (iii), $d = (b - a)$ and $n = \frac{b}{b-a}$

Then, sum $(S) = \frac{n}{2}[a + l]$

$$= \frac{b}{2(b-a)}[a + 2a] = \frac{3ab}{2(b-a)}$$

68. (a) : Let $L = \lim\limits_{x \to \frac{\pi}{4}} \dfrac{\int_2^{\sec^2 x} f(t)dt}{x^2 - \frac{\pi^2}{16}}$

Applying L' Hospital rule

$$L = \lim_{x \to \frac{\pi}{4}} \frac{f(\sec^2 x) \cdot 2\sec^2 x \tan x - 0}{2x}$$

$$L = \frac{f(2) \cdot 4}{\pi/2} = \frac{8f(2)}{\pi}.$$

69. (b) : Since α, β are the roots of the equation $6x^2 - 6x + 1 = 0$

$\therefore \quad \alpha + \beta = 1, \alpha\beta = 1/6$

Now, $\frac{1}{2}[a + b\alpha + c\alpha^2 + d\alpha^3] + \frac{1}{2}[a + b\beta + c\beta^2 + d\beta^3]$

$$= a + \frac{1}{2}b(\alpha + \beta) + \frac{1}{2}c(\alpha^2 + \beta^2) + \frac{1}{2}d(\alpha^3 + \beta^3)$$

$$= a + \frac{1}{2}b + \frac{1}{2}c[(\alpha+\beta)^2 - 2\alpha\beta] + \frac{1}{2}d[(\alpha+\beta)^3 - 3\alpha\beta(\alpha + \beta)]$$

$$= a + \frac{b}{2} + \frac{1}{2}c\left[(1)^2 - 2 \cdot \frac{1}{6}\right] + \frac{1}{2}d\left[(1)^3 - 3 \cdot \frac{1}{6}\right]$$

$$= a + \frac{b}{2} + \frac{c}{3} + \frac{d}{4}$$

70. (c) : $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$

$f'(x) = 6x^2 - 18ax + 12a^2$

For maxima or minima

$6(x^2 - 3ax + 2a^2) = 0$

$\Rightarrow x^2 - 3ax + 2a^2 = 0 \Rightarrow x = a, 2a$

$f''(x) = 12x - 18a$

$f''(a) = 12a - 18a = -6a < 0$

$f''(2a) = 12 \cdot 2a - 18a = 6a > 0$

$\therefore$ At $x = a$, $f(x)$ is maximum and at $x = 2a$, $f(x)$ is minimum

$\therefore \quad p = a, \ q = 2a$

Given $p^2 = q$

$\Rightarrow a^2 = 2a \Rightarrow a = 2$ $[\because a > 0]$

71. (840) : $\because {}^{16}C_r = {}^{16}C_{r+2}$

$\Rightarrow r + r + 2 = 16 \Rightarrow r = 7$

$\therefore \quad {}^rP_{r-3} = {}^7P_4 = 840$

72. **(6.92) :** The line through $(x_1, y_1, z_1), (x_2, y_2, z_2)$ is

$$\frac{x-x_1}{x_1-x_2}=\frac{y-y_1}{y_1-y_2}=\frac{z-z_1}{z_1-z_2}$$

$$\therefore \quad \frac{x+2}{-3}=\frac{y-6}{3}=\frac{z-4}{-3}$$

or $\quad \dfrac{x+2}{1}=\dfrac{y-6}{-1}=\dfrac{z-4}{1}$

$x = 0 \Rightarrow y = 4, z = 6, A \equiv (0, 4, 6)$

$y = 0 \Rightarrow x = 4, z = 10, B \equiv (4, 0, 10)$

$\therefore \quad AB=\sqrt{16+16+16}=4\sqrt{3}=6.92$

73. **(0.10) :** Let E_1 and E_2 are two events that Ram and Shyam will alive 30 years respectively.

Given that $P(E_1)=\dfrac{7}{11} \Rightarrow P(E_1^c)=\dfrac{4}{11}$

and $P(E_2)=\dfrac{7}{10} \Rightarrow P(E_2^c)=\dfrac{3}{10}$

$\therefore P(E_1^c \cap E_2^c)=P(E_1^c)\,P(E_2^c)=\dfrac{4}{11}\times\dfrac{3}{10}=\dfrac{12}{110}=0.10$

74. **(79) :** $T_9=a+8d=35$ and $T_{19}=a+18d=75$

On solving we get $d = 4$ and $a = 3$

Hence 20th term of an A.P. is

$a+19d=3+19\times 4=79$.

75. **(2) :** The values of λ are given by solution of

$$\begin{vmatrix} -\lambda^2 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0$$

$\Rightarrow \; -\lambda^2(\lambda^4-1)+1+\lambda^2+1+\lambda^2=0$

$\Rightarrow \; -\lambda^6+3\lambda^2+2=0 \Rightarrow \lambda^6-3\lambda^2-2=0$

Put $\lambda^2 = t, \;\; t^3-3t-2=0$

$\Rightarrow \; (t+1)^2(t-2)=0$

For $\lambda \in R, t > 0, (t+1)^2 \neq 0.$

$\therefore \quad t=2 \;\Rightarrow\; \lambda=\pm\sqrt{2}$