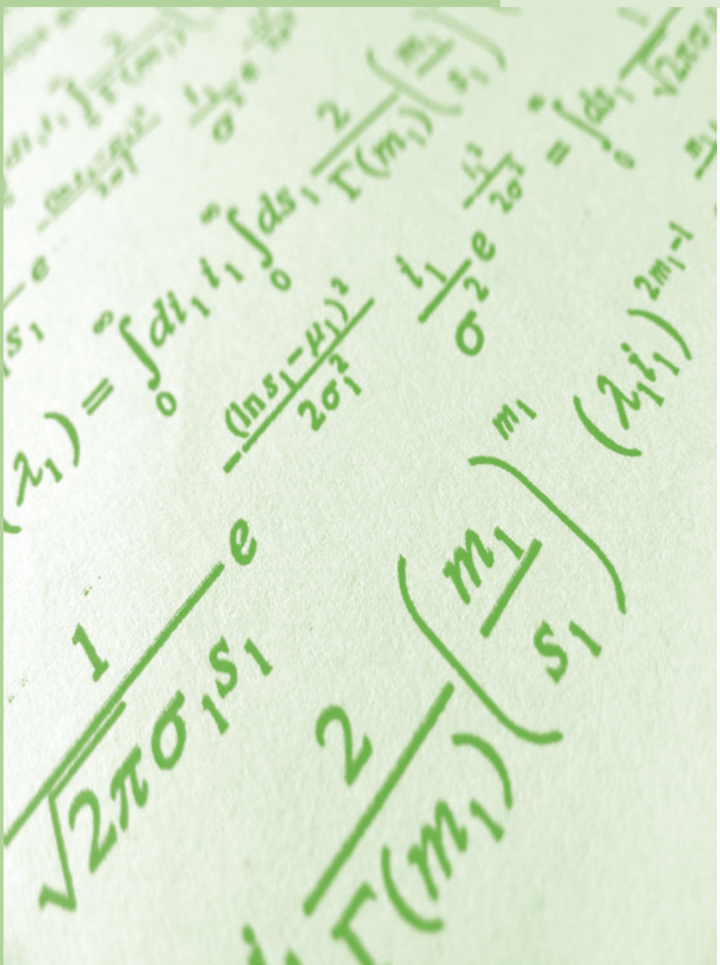
Chapter

20

Permutations and Combinations



REMEMBER

Before beginning this chapter, you should be able to:

- Have a knowledge of sets and terms related to it
- Apply operators on sets

KEY IDEAS

After completing this chapter, you would be able to:

- Study sum rule of disjoint counting and its general form
- Understand multiplication/product rule and its generalization
- Learn about permutation, its factorial notation and general formula
- Study combination and its general formula

INTRODUCTION

This chapter offers some techniques of counting without direct listing of the number of elements in a particular set or the number of outcomes of a particular experiment. We now present the two fundamental rules of counting, namely:

1. **The Sum Rule**
2. **The Multiplication Rule** or **Product rule**.

Sum Rule of Disjoint Counting

If there are two sets say A and B with A having m elements and B having n elements with no element in A appearing in B, then the number of elements in A or B is $(m + n)$.

Symbolically,

$$n(A \cup B) = n(A) + n(B), \text{ when } A \text{ and } B \text{ are disjoint.}$$

The symbol '$\cup$' stands for Union.

EXAMPLE 20.1

$A = \{1, 2, 3, 4\}$ and $B = \{a, e, i, o, u\}$ are two sets. In how many ways can a number from A or a letter from B be chosen?

SOLUTION

As no element of A is in B, we can apply the sum rule of disjoint counting.

$n(A \cup B) = n(A) + n(B) = 4 + 5 = 9.$

General Form of Sum Rule

If A and B are two sets, then the number of elements in A or B (not necessarily disjoint) is given by

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

The symbol '$\cap$' stands for intersection. It means 'common to'.

EXAMPLE 20.2

In how many ways can a prime or an odd number be chosen from $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$?

SOLUTION

We form two sets P and O as follows.

$P = \{2, 3, 5, 7\}$ (primes) and $O = \{1, 3, 5, 7, 9\}$ (odd numbers).

On applying the general form of Sum Rule we get,

$n(P \cup O) = n(P) + n(O) - n(P \cap O) = 4 + 5 - 3 = 6.$

We note that the numbers 3, 5 and 7 are counted among primes and also among odd numbers. So we discount 3 (common numbers) from the sum $n(P) + n(O)$.

Note The usage of the word OR prompts you to add.

Product Rule or Multiplication Rule

If two operations must be performed, and if the first operation can be performed in p_1 ways and the second in p_2 ways, then there are $p_1 \times p_2$ different ways in which the two operations can be performed one after the other.

EXAMPLE 20.3

A caterer's menu is to include 4 different sandwiches and 3 different desserts. In how many ways can one order for a sandwich and a dessert?

SOLUTION

Let S_1, S_2, S_3 and S_4 denote 4 sandwiches and D_1, D_2, D_3 denote 3 desserts.

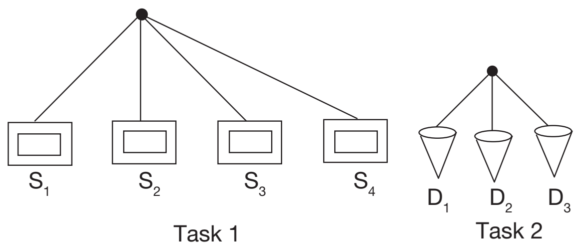


Figure 20.1

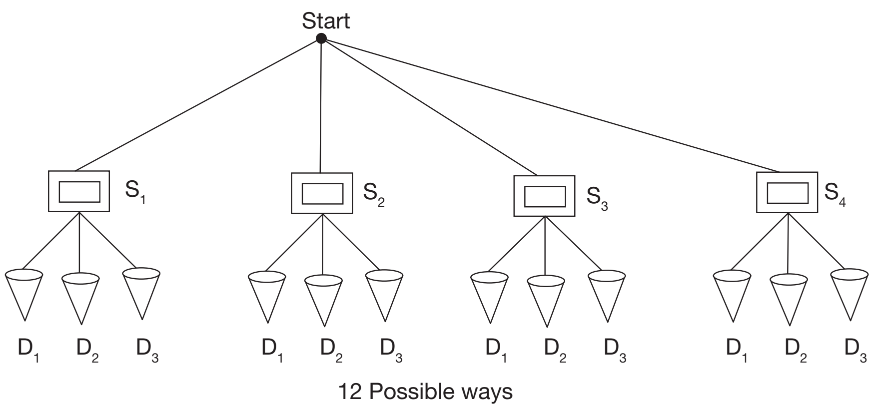


Figure 20.2

The tree diagram clearly suggests that there are 4 ways to choose a sandwich and for each of these 4 ways there are 3 ways to choose a dessert. There are $4 \times 3 = 12$ ways of choosing a sandwich and a dessert.

Generalization of Product Rule

Suppose that tasks T_1, T_2, T_3, …, T_r are to be performed in a sequence. If T_1 can be performed in p_1 ways, and for each of these ways, T_2 can be performed in p_2 ways, and for each of these $p_1 \times p_2$ ways of performing T_1, T_2 in sequence, T_3 can be performed in p_3 ways and so on, then the sequence T_1, T_2, T_3 , ..., T_r can be performed in $p_1 \times p_2 \times p_3 \times \cdots \times p_r$ ways.

EXAMPLE 20.4

A man has 7 trousers and 10 shirts. How many different outfits can he wear?

SOLUTION

Task 1: He may choose the trouser in 7 ways.

Task 2: He may choose the shirt in 10 ways.

According to the Product Rule, the total number of different outfits is 7×10, i.e., 70.

EXAMPLE 20.5

A class has 20 boys and 15 girls. If one representative from each gender has to be chosen, in how many ways can this be done?

SOLUTION

Task 1: Choosing a representative from boys.
This can be done in 20 ways.

Task 2: Choosing a representative from girls.
This can be done in 15 ways.

By the product rule, the number of ways of performing the two tasks is 20×15, i.e., 300 ways.

EXAMPLE 20.6

How many different outcomes arise from first tossing a coin and then rolling a die?

SOLUTION

There are 2 possibilities (i.e., head or tail) for the first task (tossing a coin) and after each of these outcomes there are 6 possibilities (i.e., any number from 1 to 6) for the second task (rolling a dice). Thus, by the product rule, there are 2×6, i.e., 12 possible outcomes, for the given compound task.

EXAMPLE 20.7

A password of 4 letters is to be formed with vowels alone. How many such passwords are possible if

(a) repetition of letters is allowed,

(b) repetition of letters is not allowed?

SOLUTION

The tasks T_1, T_2, T_3 and T_4 are about filling the 1st, 2nd, 3rd and 4th slots in the password.

(a) The first slot can be filled in 5 ways (*a*, *e*, *i*, *o* or *u*).

The second can also be filled in 5 ways (with repetition being allowed).

The third and fourth can also be filled in 5 ways each.

Using the generalisation, we get $5 \times 5 \times 5 \times 5 = 625$ passwords.

(b) The first slot can be filled in 5 ways (*a*, *e*, *i*, *o* or *u*). The second slot can be filled in 4 ways (as repetition is not allowed). The third and fourth in 3 and 2 ways respectively. Thus the total number of possible passwords are $5 \times 4 \times 3 \times 2 = 120$.

Note The usage of AND prompts you to multiply.

PERMUTATIONS

Each of the arrangements which can be made by taking some or all of a number of things is called a Permutation. Permutation implies 'arrangement', i.e., order of things is important.

EXAMPLE 20.8

Consider 4 elements *a*, *b*, *c* and *d*. List all permutations taken two at a time.

SOLUTION

We have two cases to deal

(a) with repetition allowed,

(b) with repetition not allowed.

Now list for

Case 1: *aa*, *ab*, *ba*, *ac*, *ca*, *ad*, *da*, *bb*, *bc*, *cb*, *bd*, *db*, *cc*, *cd*, *dc*, *dd*, i.e., there are 16 possibilities.

Case 2: *ab*, *ba*, *ac*, *ca*, *ad*, *da*, *bc*, *cb*, *bd*, *db*, *cd*, *dc*, i.e., there are 12 possibilities.

We have seen all the possibilities in both cases.

To count the number of permutations without actual listing of the arrangements, we use the product rule as a technique.

We have two tasks to perform–namely filling up the first slot in the arrangement and filling up the second slot in the arrangement.

Case 1: Repetition allowed.

$$\underset{\text{Task 1}}{\boxed{4}} \times \underset{\text{Task 2}}{\boxed{4}} = 16$$

Case 2: Repetition not allowed.

$$\underset{\text{Task 1}}{\boxed{4}} \times \underset{\text{Task 2}}{\boxed{3}} = 12$$

Note In Case (2), *aa*, *bb*, *cc*, *dd* are not allowed.

EXAMPLE 20.9

There are 10 railway stations between a station X and another station Y. Find the number of different tickets that must be printed so as to enable a passenger to travel from one station to any other.

SOLUTION

Including X and Y there are 12 stations. From any one station to any other, we need 11 different types of tickets. Since there are 12 stations, the different tickets possible are (12)(11) = 132.

Factorial Notation

If n is a positive integer, then the product of the first n positive integers is denoted by $n!$ or $\lfloor\underline{n}$ (read as n factorial). We define zero factorial as 1.

Accordingly,

$0\,! = 1.$

$1\,! = 1.$

$2\,! = 1 \times 2 = 2.$

$3\,! = 1 \times 2 \times 3 = 6.$

$4\,! = 1 \times 2 \times 3 \times 4 = 24.$

$5\,! = 120,\ 6\,! = 720,\ 7\,! = 5040.$

Note In some problems the answers will be left as factorials. We need not simplify numbers like 10!, 12!, 25!, etc. However a problem like $\dfrac{30!}{28!}$ can be simplified as $\dfrac{30 \times 29 \times 28!}{28!} = 30 \times 29 = 870$.

General Formula for Permutations (Repetitions not Allowed)

The number of permutations of n distinct objects, taken r at a time without repetition is ${}^nP_r = \dfrac{n!}{(n-r)!}$, for $r = 0, 1, 2, 3, \ldots, n$. ${}^nP_r$ is read as nPr.

Explanation

Consider r boxes, each of which can hold one item. When all the r boxes are filled, what we have is an arrangement of r items taken from the given n items. Hence the number of ways in which we can fill up the r boxes by taking r things from the given n things is equal to the number of permutations of n things taken r at a time.

$$\boxed{n} \quad \boxed{n-1} \quad \boxed{n-2} \cdots \boxed{n-r+1}$$

First box can be filled in n ways (because any one of the n items can be used to fill this box); having filled the first box, to fill the second box we now have only $(n-1)$ items; any one of these items can be used to fill the second box and hence the second box can be filled in $(n-1)$ ways; similarly, the third box in $(n-2)$ ways and so on. Thus the rth box in $\{n-r-1)\}$ ways, i.e., $(n-r+1)$ ways. Hence all the r boxes together can be filled up in $n(n-1)(n-2) \ldots (n-r+1)$ ways.

So, ${}^nP_r = n(n-1)(n-2) \ldots (n-r+1)$. This can be simplified by multiplying and dividing the right hand side by $(n-r)(n-r-1) \ldots 3.2.1$, giving us

$${}^nP_r = n(n-1)(n-2) \ldots [n-(r-1)] = \frac{n!}{(n-r)!}.$$

The number of permutations of n distinct items taken r at a time is

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

If we take n things at a time, then we get ${}^nP_n$. From the discussion we had for filling the boxes, we can find that the number of permutations of n things taken all at a time is $n!$.

$$\boxed{{}^nP_n = n!}$$

The value of ${}^nP_r$ without factorials is ${}^nP_r = n(n-1)(n-2) \ldots (n-r+1)$, for $r \neq 0$.

EXAMPLE 20.10

In how many ways can 8 athletes finish a race for Gold, Silver and Bronze medals?

SOLUTION

This is the number of permutations of 8 distinct objects taken three at a time without repetitions (here it means same person cannot get both silver and bronze).

Thus ${}^8P_3 = 8 \times 7 \times 6 = 336$ ways.

General Formula for Permutations with Repetitions Allowed

The number of permutations of n distinct objects taken r at a time with repetition allowed is n^r, for any integer $r \geq 0$.

Explanation We have r boxes with each box ready to accept one or more of the n distinct objects. Using product rule, the total ways of filling up these r boxes is $n \times n \times n \times n \times \ldots$ for r times $= n^r$.

$$\underbrace{\boxed{n}\quad\boxed{n}\quad\boxed{n}\cdots\boxed{n}}_{(r\ \text{times})} = n^r$$

EXAMPLE 20.11

In how many ways can 3 letters be put into 5 letter boxes when each box can take any number of letters?

SOLUTION

As each box can taken any number of letters, we can post each letter in 5 ways.

$$\underset{\text{Letter 1}}{\boxed{5}} \times \underset{\text{Letter 2}}{\boxed{5}} \times \underset{\text{Letter 3}}{\boxed{5}} = 5^3 = 125 \text{ ways.}$$

COMBINATIONS

Each of the groups or selections which can be made by taking some or all of a number of things is called a Combination.

In combinations, the order in which the things are taken is not considered as long as the specific things are included.

EXAMPLE 20.12

Consider a, b, c, d. List all combinations taken 3 at a time.

SOLUTION

The list includes abc, abd, acd, bcd.

Here abc, bca, cab are regarded the same as order is not important.

The number of combinations of n things taken r at a time is denoted by ${}^nC_r$.

General Formula for Combinations

We first look at the permutations of n items taken r at a time from a different perspective. We look at two tasks T_1 and T_2 as:

T_1: Select r objects.

T_2: Arrange all the r objects that got selected in T_1.

We understand that T_1 can be done in ${}^nC_r$ ways by definition, and its value yet to be determined and T_2 can be done in $r!$ ways. But then to get the permutations, we need to perform T_1 followed by T_2.

Thus by Fundamental Principle of Counting, both tasks can be done in ${}^nC_r \times r!$ ways.

Thus, ${}^nC_r \times r! = {}^nP_r$, i.e., ${}^nC_r = \dfrac{{}^nP_r}{r!} = \dfrac{n!}{(n-r)!r!}$.

Notes

1. ${}^nC_0 = {}^nC_n = 1$
2. ${}^nC_1 = {}^nC_n - 1 = n$
3. If ${}^nC_r = {}^nC_s$, then $r = s$ or $n = r + s$.

EXAMPLE 20.13

In a library there are 10 research scholars. In how many ways can we select 4 of them?

SOLUTION

Out of 10 scholars, we can select 4 of them in ${}^{10}C_4$ ways.

$${}^{10}C_4 = \frac{10\times 9\times 8\times 7}{1\times 2\times 3\times 4} = 210 \text{ ways.}$$

EXAMPLE 20.14

In how many ways can we select two vertices in a hexagon?

SOLUTION

A hexagon has 6 vertices. Any 2 vertices can be selected in ${}^6C_2$ ways $= \dfrac{6\times 5}{1\times 2} = 15$ ways.

EXAMPLE 20.15

From 8 gentlemen and 5 ladies, a committee of 4 is to be formed. In how many ways can this be done,

(a) when the committee consists of exactly three gentlemen?

(b) when the committee consists of at most three gentlemen?

SOLUTION

(a) We have to select three out of 8 gentlemen and one out of 5 ladies. Hence the number of ways in which this can be done $= {}^8C_3 \times {}^5C_1 = 280$.

(v) The committee is to contain at most three gentlemen, i.e., it may contain either 1, 2 or 3 gentlemen.

Hence, the total number of ways $= {}^8C_1 \times {}^5C_3 + {}^8C_2 \times {}^5C_2 + {}^8C_3 \times {}^5C_1 = 80 + 280 + 280 = 640$.

EXAMPLE 20.16

Find ${}^nC_3$, if ${}^nC_7 = {}^nC_4$.

SOLUTION

$${}^nC_7 = {}^nC_4 \;\Rightarrow\; n = 7 + 4 = 11$$

So ${}^nC_3 = {}^{11}C_3 = \dfrac{11\times 10\times 9}{1\times 2\times 3} = 165$.

EXAMPLE 20.17

How many distinct positive integers are possible with the digits 1, 3, 5, 7 without repetition?

SOLUTION

Possible number of,

single-digit numbers = 4

two-digit numbers = $4 \times 3 = 12$

three-digit numbers = $4 \times 3 \times 2 = 24$

four-digit numbers = $4 \times 3 \times 2 \times 1 = 24$

Thus, total number of distinct positive integers without repetition = $4 + 12 + 24 + 24 = 64$.

EXAMPLE 20.18

If ${}^nP_r = 990$ and ${}^nC_r$ 165, then find the value of r.

SOLUTION

${}^nP_r = r!\ {}^nC_r$

$$\Rightarrow \quad \frac{990}{165} = r!$$

$$\Rightarrow \quad r! = 6 \quad \Rightarrow \quad r = 3.$$

Alternative method:

${}^nP_r = 990 = 11 \times 10 \times 9 = {}^{11}P_3 \quad \Rightarrow \quad n = 11,\ r = 3$ also ${}^{11}C_3 = 165$.

$\therefore\ r = 3$.

EXAMPLE 20.19

In a plane there are 12 points, then answer the following questions:

(a) Find the number of different straight lines that can be formed by joining these points, when no combination of 3 points are collinear.

(b) Find the number of different straight lines that can be formed by joining these points, when 4 of these given points are collinear and no other combination of three points are collinear.

(c) Find the number of different triangles that can be formed by joining these points, when no combination of 3 points are collinear.

(d) Find the number of different triangles that can be formed by joining these points, when 5 of these given points are collinear and no other combination of three points are collinear.

SOLUTION

(a) We know passing through two points in a plane we can draw only one line, i.e., we require to select any two points from the given 12 points which is possible in ${}^{12}C_2$ ways.

$\therefore$ The number of different straight lines that can be formed by joining the given 12 points

$$= {}^{12}C_2 \frac{12 \times 11}{1 \times 2} = 66.$$

(b) Given, out of the 12 points, 4 points are collinear.

We know that collinear points form only one line.

$\therefore$ These four points when they are not collinear will actually form ${}^4C_2$ lines, which are not forming here.

$\therefore$ The number of the required lines $= {}^{12}C_2 - {}^4C_2 + 1 = 66 - 6 + 1 = 61$.

(c) We know, by joining three non collinear points a triangle forms.

$\therefore$ Three points can be selected from 12 points in ${}^{12}C_3$ ways.

$\therefore$ The required number of triangles $= {}^{12}C_3 = \dfrac{12 \times 11 \times 10}{1 \times 2 \times 3} = 220$.

(d) Given, 5 points are collinear.

$\Rightarrow$ ${}^5C_3$ triangles will not form.

$\therefore$ The required number of triangles $= {}^{12}C_3 - {}^5C_3 = 220 - 10 = 210$.

EXAMPLE 20.20

Find the number of diagonals of a polygon of 10 sides.

SOLUTION

Assume that there are 10 points in a plane where no 3 of them are collinear which are the vertices of the given polygon.

The number of different lines that can be formed by joining these 10 points is ${}^{10}C_2$.

We know in any polygon the lines joining non-adjacent vertices are called diagonals.

Hence, the required number of diagonals = Number of lines formed – number of sides of the polygon $= {}^{10}C_2 - 10 = 35$.

Note The number of diagonals of a polygon of n sides is given by

$$ {}^nC_2 - n = \frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}. $$

Using this formula the number of diagonals in the above problem $= \dfrac{10(10-3)}{2} = 35$.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. Find the value of 6!.
2. What is the value 0!?
3. Factorial is defined for ______ numbers.
4. The number of arrangements that can be made by taking r objects at a time from a group of n dissimilar objects, is denoted as ______.
5. What is the formula for ^nP_r?
6. In ^6C_r, what are the possible values of r?
7. What the value of ^nC_r?
8. What is the relation between ^nP_r and ^nC_r?
9. The number of straight lines that can be formed by n points in a plane, where no three points are collinear is ______ and in case p of these points are collinear is ______.
10. The number of triangles that can be formed by n points in a plane where no three points are collinear is ______ and when p of the given points are collinear is ______.
11. Find the number of 3-digit numbers, formed with the digits {2, 5, 4, 6} when repetition of the digits is allowed.
12. If $^nP_{100} = {^nP_{99}}$, then find the value of n.
13. If $^{100}C_3 = 161700$, then $^{100}C_{97}$ is equal to ______.
14. If $^nP_3 = 720$, then find the value of $^{11}P_n$.
15. Find the number of four-digit numbers that can be formed using the digits 1, 2, 5, 7, 4 and 6, if every digit can occur at most once in any number.
16. Find the number of integers greater than 4000 that can be formed by using the digits 3, 4, 5 and 2, if every digit can occur at most once in any number.
17. How many 6-letter words with distinct letters in each can be formed using the letters of the word EDUCATION? How many of these begin with I?
18. How many words with distinct letters can be formed by using all the letters of the word PLAYER which begin with P and end with R?
19. In a class, there are 45 students. On a new year eve, every student sends one greeting card to each of the other students. How many greeting cards were exchanged in all?
20. In how many ways can 6 prizes be distributed among 4 students, if each student can receive more than one prize?
21. If $^nP_r = 360$ and $^nC_r = 15$, then find the value of r.
22. A bag contains 3 yellow balls and 4 pink balls. In how many ways can 2 pink balls and 1 yellow ball be drawn from the bag?
23. In how many ways can 11 players be chosen from a group of 15 players?
24. A committee of 5 members is to be formed from 8 men and 6 women. Find the number of ways of forming the committee, if it has to contain 3 men and 2 women.
25. In how many ways can 3 diamond cards be drawn simultaneously from a pack of cards?
26. In a party there are 20 persons. If every person shook hand with every other person in the party exactly once, find the total number of handshakes exchanged in the party.
27. A regular polygon has 20 sides. Find the number of diagonals of the polygon.
28. How many different straight lines can be formed from 30 points in a plane? (no three points are collinear)
29. If the number of diagonals of a regular polygon is three times the number of its sides, find the number of sides of the polygon.
30. There are 20 points in a plane. How many different triangles can be formed with these points? (no three points are collinear)

Short Answer Type Questions

31. If $^nP_r = 1716$ and $r = 3$, then $^nC_r =$ ______.
32. A boy has 9 trousers and 12 shirts. In how many different ways can he select a trouser and a shirt?
33. How many three letter words are formed using the letters of the word FAILURE?
34. The number of selections that can be made to select 5 members from a group of 15 members is ______.

35. There are 8 points in a plane, how many different triangles can be formed using these points (no three points are collinear)?

36. A bag contains 9 yellow balls, 3 white balls and 4 red balls. In how many ways can two balls be drawn from the bag?

37. A question paper contains 20 questions. In how many ways can 4 questions be attempted?

38. If a polygon has 8 sides, then the number of diagonals of the polygon is ______.

39. In a class there are 15 boys and 10 girls. How many ways can a pair of one boy and one girl be selected from the class?

40. How many five-digit numbers can be formed using the digits {5, 6, 3, 9, 2}? (no digit can occur more than once in any number)?

41. In how many ways can 3 consonants be selected from the letters of the word EDUCATION?

42. Using all the letters of the word NOKIA, how many words can be formed, which begin with N and end with A?

43. Given 1 and 2 are two parallel lines. How many triangles can be formed with 12 points taking on 1 and 6 points on 2?

44. A question paper contain 15 questions. In how many ways can 7 questions be attempted?

45. A bag contains 5 white balls and 2 yellow balls. The number of ways of drawing 3 white balls is ______.

Essay Type Questions

46. A four-digit number is formed using the digits {0, 6, 7, 8, 9}. How many of these numbers are divisible by 3? (Each digit is occurred at most once in every number).

47. There are 25 points in a plane. Six of these are collinear and no other combination of 3 points are collinear. How many different straight lines can be formed by joining these points?

48. There are 20 points in a plane, of which 5 points are collinear and no other combination of 3 points are collinear. How many different triangles can be formed by joining these points?

49. Using the letters of the word 'TABLE'. How many words can be formed so that the middle place is always occupied by a vowel?

50. Find the value of ${}^6C_2 + {}^6C_3 + {}^7C_4 + {}^0C_5 + {}^9C_6$.

CONCEPT APPLICATION

Level 1

1. ${}^nC_n =$ ________.

(a) $n!$ (b) 1
(c) nn (d) n

2. If a polygon has 6 sides, then the number of diagonals of the polygon is ________.

(a) 18 (b) 12
(c) 9 (d) 15

3. How many two digit numbers can be formed using the digits {1, 2, 3, 4, 5}, if no digit occurs more than once in each number?

(a) 10 (b) 20
(c) 9 (d) 16

4. If ${}^nC_4 = 35$, then ${}^nP_4 =$ _________.

(a) 120 (b) 140
(c) 840 (d) 420

5. Using all the letters of the word 'QUESTION', how many different words can be formed?

(a) 8! (b) 7!
(c) $7 \times 7!$ (d) 9!

6. If ${}^nP_r = 24\ {}^nC_r$, then $r =$ _________.

(a) 24
(b) 6
(c) 4
(d) Cannot be determined

7. In how many ways can 5 prizes be distributed to 3 students, if each student is eligible for any number of prizes?

(a) 3^5 (b) 5^3
(c) ${}^5P_3$ (d) ${}^5C_3$

8. Using the letters of the word PUBLIC, how many four letter words can be formed which begin with B and end with P? (Repetition of letters is not allowed)

(a) 360 (b) 12
(c) 24 (d) 30

9. In a class there are 20 boys and 15 girls. In how many ways can 2 boys and 2 girls be selected?

(a) ${}^{35}C_4$ (b) ${}^{35}C_2$
(c) ${}^{20}C_2 \times {}^{15}C_2$ (d) 20×15

10. Using all the letters of the word 'OBJECTS', how many words can be formed which begin with B but do not end with S?

(a) 120 (b) 480
(c) 600 (d) 720

11. The number of diagonals of a regular polygon is 14. Find the number of the sides of the polygon.

(a) 7 (b) 8
(c) 6 (d) 9

12. In how many ways can 5 letters be posted into 7 letter boxes?

(a) ${}^{7}C_5$ (b) 5^7
(c) 7^5 (d) ${}^{7}P_5$

13. Sunil has 6 friends. In how many ways can he invite two or more of his friends for dinner?

(a) 58 (b) 57
(c) 63 (d) 49

14. Find the value of ${}^{7}C_4 - {}^{6}C_4 - {}^{5}C_3 - {}^{4}C_2$.

(a) 3 (b) 8
(c) 4 (d) 15

15. How many different words can be formed using all the letters of the word 'SPECIAL', so that the consonants always in the odd positions?

(a) 112 (b) 72
(c) 24 (d) 144

16. In how many ways can 3 consonants be selected from the English alphabet?

(a) ${}^{21}C_3$ (b) ${}^{26}C_3$
(c) ${}^{21}C_5$ (d) ${}^{26}C_5$

17. From 8 boys and 5 girls, a delegation of 5 students is to be formed. Find the number of ways this can be done such that delegation must contain exactly 3 girls.

(a) 140 (b) 820
(c) 280 (d) 410

18. There are 18 stations between Hyderabad and Bangalore. How many second class tickets have to be printed, so that a passenger can travel from one station to any other station?

(a) 380 (b) 190
(c) 95 (d) 100

19. How many numbers greater than 3000 can be formed using the digits 0, 1, 2, 3, 4 and 5, so that each digit occurs at most once in each number?

(a) 1000 (b) 300
(c) 1200 (d) 1380

20. Using all the letters of the word EDUCATION, how many words can be formed which begin with DU? (Repetition is not allowed).

(a) 8! (b) 7!
(c) 6! (d) 9!

21. Anil has 8 friends. In how many ways can he invite one or more of his friends to a dinner?

(a) 127 (b) 128
(c) 256 (d) 255

22. In how many ways can 4 letters be posted in 3 letter boxes?

(a) 4^3 (b) 3^4
(c) 6! (d) 4

23. Using the letters of the word PRIVATE, how many 6-letter words can be formed which begin with P and end with E?

(a) 3! (b) 4!
(c) 7! (d) 5!

24. Find the number of 4 digit odd numbers that can be formed using the digits 4, 6, 7, 9, 3, so that each digit occurs at most once in each number.

(a) 120 (b) 24
(c) 48 (d) 72

25. How many 5-digit numbers that are divisible by 5 can be formed using the digits {0, 1, 3, 5, 7, 6}? (each digit can be repeated any number of times)

(a) 1080 (b) 2160
(c) 6480 (d) 3175

26. How many four-digit even numbers can be formed using the digits {3, 5, 7, 9, 1, 0}? (Repetition of digits is not allowed)
(a) 120 (b) 60
(c) 360 (d) 100

27. There is a three-digit password and it is known that each digit can have four values 5, 6, 7 or 8. If there is exactly one correct password, how many distinct wrong passwords are there?
(a) 63 (b) 80
(c) 81 (d) 64

28. In how many ways can a committee consisting of 3 men and 4 women be formed from a group of 6 men and 7 women?
(a) ${}^6C_4{}^7C_3$ (b) ${}^6C_3{}^7C_5$
(c) ${}^6C_3{}^7C_4$ (d) ${}^7C_5{}^6C_4$

29. Thirty members attended a party. If each person shakes hands with every other person exactly once, then find the number of handshakes made in the party.
(a) ${}^{30}P_2$ (b) ${}^{30}C_2$
(c) ${}^{29}C_2$ (d) ${}^{60}C_2$

30. In how many ways can 6 members be selected from a group of 10 members?
(a) ${}^6C_4$ (b) ${}^{10}C_4$
(c) ${}^{10}C_5$ (d) ${}^{10}P_4$

Level 2

31. In a class there are 20 boys and 25 girls. In how many ways can a pair of a boy and a girl be selected?
(a) 400 (b) 500
(c) 600 (d) 20

32. How many different odd numbers are formed using the digits {2, 4, 0, 6}? (Repetition of digits is not allowed)
(a) 16 (b) 0
(c) 24 (d) 108

33. There are 15 stations from New Delhi to Mumbai. How many first class tickets can be printed to travel from one station to any other station?
(a) 210 (b) 105
(c) 240 (d) 135

34. In how many ways can 3 vowels be selected from the letters of the word EQUATION?
(a) 56 (b) 10
(c) 28 (d) 40

35. In how many ways can 3 consonants and 2 vowels be selected from the letters of the word TRIANGLE?
(a) 25 (b) 13
(c) 30 (d) 20

36. A plane contains 12 points of which 4 are collinear. How many different straight lines can be formed with these points?
(a) 50 (b) 66
(c) 60 (d) 61

37. A plane contains 20 points of which 6 are collinear. How many different triangles can be formed with these points?
(a) 1120 (b) 1140
(c) 1121 (d) 1139

38. Using the letters of the word 'ENGLISH', how many five letters words can begin with G?
(a) 2520 (b) 360
(c) 180 (d) 1260

39. Twelve teams are participating in a cricket tournament. If every team plays exactly one match with every other team, then the total number of matches played in the tournament is ________.
(a) 132 (b) 44
(c) 66 (d) 88

40. In how many ways can 4 consonants be chosen from the letters of the word SOMETHING?
(a) ${}^9C_4$ (b) ${}^6C_4$
(c) ${}^4C_4$ (d) ${}^4C_3$

41. How many three letter words can be formed using the letters of the word NARESH? (Repetition of letters is not allowed)
(a) 3! (b) ${}^5P_3$
(c) ${}^6P_3$ (d) ${}^6C_3$

PRACTICE QUESTIONS

42. A four digit number is to be formed using the digits 0, 1, 3, 5 and 7. How many of them are even numbers? (Each digit can occur for only one time).

(a) 48 (b) 60
(c) 24 (d) 120

43. How many numbers less than 1000 can be formed using the digits 0, 1, 3, 4 and 5 so that each digit occurs atmost once in each number?

(a) 53 (b) 69
(c) 68 (d) 60

44. There are 15 points in a plane. No three points are collinear except 5 points. How many different straight lines can be formed?

(a) 105 (b) 95
(c) 96 (d) 106

45. There are 12 points in a plane, no three points are collinear except 6 points. How many different triangles can be formed?

(a) 200 (b) 201
(c) 220 (d) 219

46. Twelve points are marked on a plane so that no three points are collinear. How many different triangles can be formed joining the points?

(a) 180 (b) 190
(c) 220 (d) 230

47. How many words can be formed from the letters of the word EQUATION using any four letters in each word?

(a) 840 (b) 1680
(c) 2080 (d) 3050

48. Seventeen points are marked on plane so that no three points are collinear. How many straight lines can be formed by joining these points so that

(a) 114 (b) 136
(c) 152 (d) 160

49. The following are the steps involved in solving ${}^{n}c_2 = 36$ for n. Arrange then in sequential order.

(A) $n^2 - n - 72 = 0$

(B) As $n > 0$, $n = 9$

(C) $n = 9$, $n = -8$

(D) $(n - 9)(n + 8) = 0$

(E) $\dfrac{n(n-1)}{1 \times 2} = 36$

(a) EACBD (b) EADCB
(c) EADBC (d) EDCDB

50. The following are the steps involved in find the value $\dfrac{n}{r}$ from ${}^{n}p_r = 1320$. Arrange them in sequential order.

(A) ${}^{n}p_r = \dfrac{12!}{9!} = \dfrac{12!}{(12-3)!}$

(B) $\Rightarrow \quad \dfrac{n}{r} = \dfrac{12}{3} = 4$

(C) $\Rightarrow \quad {}^{n}p_r = {}^{12}p_3$

(D) ${}^{n}p_r = 1320 = 12 \times 11 \times 10$

(a) DACB (b) DABC
(c) DBCA (d) DBAC

Level 3

51. How many 4-digit even numbers can be formed using the digits {1, 3, 0, 4, 7, 5}? (Each digit can occur only once)

(a) 48 (b) 60
(c) 108 (d) 300

52. Using the letters of the word CHEMISTRY, how many six letter words can be formed, which end with *Y*?

(a) ${}^{8}P_6$ (b) ${}^{9}P_6$
(c) ${}^{9}P_5$ (d) ${}^{8}P_5$

53. A telephone number has seven digits, no number starts with 0. In a city, how many different telephone numbers be formed using the digits 0 to 6? (Each digit can occur only once)

(a) 6! (b) $6 \cdot 6!$
(c) 7! (d) $2 \cdot 7!$

54. Using all the letters of the word PROBLEM, how many words can be formed such that the consonants occupy the middle place?

(a) 3000 (b) 4200
(c) 720 (d) 3600

55. Using the digits 0, 1, 2, 5 and 7 how many 4-digit numbers that are divisible by 5 can be formed if repetition of the digits is not allowed?

(a) 38 (b) 46

(c) 32 (d) 42

56. If ${}^{2n}C_4 : {}^{n}C_3 = 21 : 1$, then find the value of n.

(a) 4 (b) 5

(c) 6 (d) 7

57. How many three-digit numbers that are divisible by 5, can be formed, using the digits 0, 2, 3, 5, 7, if no digit occurs more than once in each number?

(a) 10 (b) 15

(c) 21 (d) 25

58. In how many ways can we select two vowels and three consonants from the letters of the word ARTICLE?

(a) 12 (b) 14

(c) 18 (d) 22

59. How many three-digit numbers can be formed using the digits {2, 4, 5, 7, 8, 9}, if no digit occurs more than once in each number?

(a) 80 (b) 90

(c) 120 (d) 140

60. The number of ways of selecting five members to form a committee from 7 men and 10 women is _____.

(a) 5266 (b) 6123

(c) 6188 (d) 8123

61. Twenty points are marked on a plane so that no three points are collinear except 7 points. How many triangles can be formed by joining the points?

(a) 995 (b) 1105

(c) 1200 (d) 1250

62. There are four different white balls and four different black balls. The number of ways that balls can be arranged in a row so that white and black balls are placed alternately is _____.

(a) $(4!)^2$ (b) $2(4!)^2$

(c) $4!$ (d) $(4!)^3$

63. In a party, there are 10 married couples. Each person shakes hands with every person other than her or his spouse. The total number of handshakes exchanged in that party is _____.

(a) 160 (b) 190

(c) 180 (d) 170

64. How many four-digit odd numbers can be formed using the digits 0, 2, 3, 5, 6, 8 (each digit occurs only once)?

(a) 64 (b) 72

(c) 86 (d) 96

65. The number of the words that can be formed using all the letters of the word BRAIN such that it starts with R and but does not end with A.

(a) 18 (b) 14

(c) 16 (d) 20

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. 720
2. 1
3. whole numbers
4. ${}^nP_r$
5. $\dfrac{n!}{(n-r)!}$
6. 0, 1, 2, 3, 4, 5, 6
7. $\dfrac{n!}{(n-r)!r!}$
8. ${}^nP_r = {}^nC_r r!$
9. ${}^nC_2$; ${}^nC_2 - {}^pC_2 + 1$
10. ${}^nC_3$; ${}^nC_3 - {}^pC_3$
11. $4^3 = 64$.
12. 100
13. 161700
14. 11!
15. ${}^6P_4$.
16. 12
17. (i) ${}^9P_6$ (ii) ${}^8P_5$.
18. 24
19. 1980
20. 4^6
21. 4
22. 18
23. ${}^{15}C_{11}$
24. 840
25. 286
26. 190
27. 170
28. 435
29. 9
30. ${}^{20}C_3$

Short Answer Type Questions

31. 286
32. 108
33. 210
34. ${}^{15}C_5$
35. 56
36. ${}^{16}C_2$
37. ${}^{20}C_4$
38. 20
39. 150
40. 120
41. 4
42. 6
43. 576
44. ${}^{15}C_7$
45. 10

Essay Type Questions

46. 60
47. 286
48. 1130
49. 48
50. 210

CONCEPT APPLICATION

Level 1

1. (b)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (a)	**6.** (c)	**7.** (a)	**8.** (b)	**9.** (c)	**10.** (c)
11. (a)	**12.** (c)	**13.** (b)	**14.** (c)	**15.** (d)	**16.** (a)	**17.** (c)	**18.** (a)	**19.** (d)	**20.** (b)
21. (d)	**22.** (b)	**23.** (d)	**24.** (d)	**25.** (b)	**26.** (b)	**27.** (a)	**28.** (c)	**29.** (b)	**30.** (b)

Level 2

31. (b)	**32.** (b)	**33.** (a)	**34.** (b)	**35.** (c)	**36.** (d)	**37.** (a)	**38.** (b)	**39.** (c)	**40.** (b)
41. (c)	**42.** (c)	**43.** (b)	**44.** (c)	**45.** (a)	**46.** (c)	**47.** (b)	**48.** (b)	**49.** (b)	**50.** (a)

Level 3

51. (c)	**52.** (d)	**53.** (b)	**54.** (d)	**55.** (d)	**56.** (b)	**57.** (c)	**58.** (a)	**59.** (c)	**60.** (c)
61. (b)	**62.** (b)	**63.** (c)	**64.** (d)	**65.** (a)					

CONCEPT APPLICATION

Level 1

1. Use, $^nC_r = \dfrac{n!}{(n-r)!}$.
2. The number of diagonals of a n-sided polygon is $\dfrac{n(n-3)}{2}$.
3. r objects can be arranged out of n objects is ^nP_r ways.
4. $^nP_r = {}^nC_r \times r!$
5. $^nP_n = n!$.
6. $r! = \dfrac{^nC_r}{^nC_r}$.
7. First prize can be distributed in 3 ways similarly 2nd, 3rd, 4th and 5th prizes can be distributed each in 3 ways. Now use the fundamental principle of counting.
8. Take 6 blanks, first blank is filled with B, last blank is filled with P and the remaining blanks can be filled with the remaining letters.
9. r objects can be selected from n objects in ^nC_r ways. Now use the fundamental principle, i.e., task T_1 can be done in m ways and task T_2 can be done in n ways, then the two tasks can be done simultaneously in mn ways.
15. First arrange the consonants in odd places is in 1, 3, 5 and 7 places. Now arrange the vowels in the remaining places and then use the fundamental principle.
16. Select 3 consonants from 21 consonants.
17. Select 3 girls from 5 girls and 2 boys from 8 boys then apply fundamental principle.
18. Total number of stations = 20. Select 2 stations from 20 stations.
20. Find the number of 7 letter words using the 7 letters.
22. (i) Each letter be posted in 3 ways.
    (ii) Now calculate the number of ways in which four letters can be posted by using fundamental theorem of counting.
23. As the letters begin with P and ends with E, four more letters are to be selected from the remaining 5 letters.
24. (i) The units digit of the number must be odd, i.e., it can be done in 3 ways.
    (ii) Now find the number of ways in which the three digits can be filled using four digits.
    (iii) Use the fundamental theorem of counting.
25. (i) If unit digit is 0 or 5, then the number is divisible by 5.
    (ii) If the units digit is 5, then ten thousands digit cannot be zero, now find the number of ways the other four digits can be arranged.
    (iii) Similarly when the units digit is 0, the other 4 digits can be arranged in $^5P_4$ ways. Use the fundamental theorem of counting.
26. (i) The units digit of the required number is 0.
    (ii) Find the number of ways in which the remaining 5 digits can be arranged in three places by using ^nP_r.
27. (i) Each digit of the password can have 4 values, hence first digit can be filled in 4 ways.
    (ii) Find the number of ways in which remaining digits can be filled.
    (iii) Use the fundamental principal of counting and find total number of passwords that are formed.
28. Select 3 men from 6 men and select 4 women from 7 women, then apply the fundamental principle.
29. Select two persons from 30 persons.
30. From a group of n members selecting r members at a time is denoted by ^nC_r.

Level 2

31. (i) The number of ways a boy and a girl can be selected individually is $^{20}C_1$ and $^{25}C_1$.
    (ii) Use the fundamental theorem of counting.
32. All the given digits are even, so no odd number can be formed with the given digits.
33. (i) The total number of stations are 15 (say n).

(ii) On a ticket, two stations should be printed.
(iii) The required number of ways = ${}^nC_2 \times 2$.

34. There are 5 vowels and 3 are to be chosen.

35. (i) Find the number of ways in which 3 consonants and 2 vowels can be selected from 5 consonants and 3 vowels.
(ii) Then use fundamental theorem of counting.

36. The number of lines that can be formed from n points in which m points are collinear is ${}^nC_2 - {}^mC_2 + 1$.

37. The number of triangles that can be formed from n points in which m points are collinear is ${}^nC_3 - {}^mC_3$.

38. As Y is filled in last blank, five more letters are to be selected from the remaining 8 letters.

39. Select any two teams from 15 teams.

40. There are 6 consonants and 4 are to be chosen.

41. Arrange 3-letter words out of 6 letters.

44. The number of lines that can be formed from n points in which m points are collinear is ${}^nC_2 - {}^mC_2 + 1$.

45. The number of triangles that can be formed from n points in which m points are collinear is ${}^nC_3 - {}^mC_2$.

46. From the 'n' points (no three points are collinear) in a plane, the number of triangles formed is ${}^nC_3$.

$\therefore$ The required number of triangles

$$= {}^{12}C_3 = \frac{12 \times 11 \times 10}{1 \times 2 \times 3} = 220.$$

47. There are 8 letters in the word EQUATION. The number of words with 4 letters formed with the 8 letters is ${}^8P_4 = 8 \times 7 \times 6 \times 5 = 1680$.

48. From the 'n' points of a plane, the number of straight lines formed is ${}^nC_2$.

$\therefore$ The required number of straight lines = ${}^{17}C_2$ = 136.

49. EADCB is the required sequential order.

50. DACB is the required sequential order.

Level 3

51. (i) If the digit in the units place is an even number, then the number is called even number.
(ii) The units digit of the four digit number must be 0 or 4.

			0

(or)

			4

(iii) If units digit is 4, then thousands digit can be filled in 4 ways and other two digits can be filled in ${}^4P_2$ ways.
(iv) If units digit is 0, then find the number of ways in which the remaining 3 digits can be filled by using 5 digits.

52. (i) Take 6 blanks.
(ii) As Y is filled in last blank, five more letters are to be selected from the remaining 8 letters.

53. (i) The first digit of the number cannot be zero, so it can be filled in 6 ways.
(ii) Now the second digit can be any of the 6 digits., i.e., it can be filled in 6 ways.
(iii) As the digits cannot be repeated, the number of ways the third digit can be filled is 5 ways and so on.
(iv) Now apply the fundamental theorem.

54. (i) There are 5 consonants and middle place can be filled in 5 ways.
(ii) Remaining places can be filled with remaining letters.

55. Unit place is 0 or 5 then the number is divisible by 5.

56. Given, $\dfrac{{}^{2n}C_4}{{}^nC_3} = \dfrac{21}{1}$

$${}^{2n}C_4 = 21\,{}^nC_3$$

$$\frac{(2n)!}{(2n-4)!4!} = 21\frac{n!}{(n-3)!3!}$$

$$(2n)(2n-1)(2n-2)(2n-3) = 84(n)(n-1)(n-2)$$

$$4(2n-1)(2n-3) = 84(n-2).$$

From the options, $n = 5$ satisfies the above equation.

57. If a number is divisible by 5, then the units digit must be either 0 or 5.

Case 1: If the units digit is 0, then the remaining two places can be filled by remaining 4 digits.

It can be done in ${}^4P_2 = 12$ ways.

Case 2: If the units digit is 5, then the remaining two places can be filled by the remaining 4 digits.

It can be done in $3 \times 3 = 9$ ways.

($\therefore$ The hundreds place can not be filled with 0)

$\therefore$ The required number of 3-digit numbers = $12 + 9 = 21$.

58. There are three vowels and four consonants in the word ARTICLE.

The number of ways of selecting 2 vowels and 3 consonants from 3 vowels and 4 consonants ${}^3C_2 \times {}^4C_3$ $= 3 \times 4 = 12$.

59. Here, we have 6 digits.

Since we have to find 3 digit numbers, the first digit can be filled in 6 ways, second digit can be filled in 5 and third digit can be filled in 4 ways.

($\because$ No digit is repeated)

$\therefore$ The required number of digits

$= 6 \times 5 \times 4 = 120$.

60. Here, the total number of persons is 17.

The number of ways of selecting 5 members from 17 members is ${}^{17}C_5 = 6188$.

61. We know that number of triangles formed by 'n' points in which 'm' are collinear is ${}^nC_3 - {}^mC_3$

The required number of triangles

$= {}^{20}C_3 - {}^7C_3$

$= 1140 - 35 = 1105$.

62. We have to arrange 4 different white balls and 4 different black balls as shown below

(i) WB WB WB WB

(ii) BW BW BW BW

This can be done in $4! \times 4! + 4! \times 4!$ ways, i.e., $2(4!)^2$.

63. There are 20 persons in the party.

Total handshakes = ${}^{20}C_2 = 190$

This includes 10 handshakes in which a person shakes hands with her or his spouse.

$\therefore$ The required number of handshakes

$= 190 - 10 = 180$.

64. If a number is odd, then the units digit of the number must contain any one of the numbers 1, 3, 5, 7 or 9.

Given digits are 0, 2, 3, 5, 6, 8.

Here units digit must contain either 3 or 5, i.e., 2 ways.

First place can be filled in 4 ways (since ten thousand pace can not be filled with 0).

Second place and third digit can be filled in 4 and 3 ways respectively.

The required number of odd numbers = $4 \times 4 \times 3 \times 2 = 96$.

65. Since the first place of the word always starts with R and last place is not A. The last place can be filled by any one of the remaining three letters. The remaining 3 places can be filled with 3 letters in 3! ways.

$\therefore$ The total number of words = $3 \times 3! = 18$.