Shortcuts and Important Results to Remember

1 If n letters corresponding to n envelopes are placed in the envelopes at random, then

(i) probability that all letters are in right envelopes $= \frac{1}{n!}$.

(ii) probability that all letters are not in right envelopes $= 1 - \frac{1}{n!}$.

(iii) probability that no letter is in right envelopes $= \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots + (-1)^n \frac{1}{n!}$.

(iv) probability that exactly r letters are in right envelopes $= \frac{1}{r!}\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots + (-1)^{n-r} \frac{1}{(n-r)!}\right]$.

2 When two dice are thrown, the probability of getting a total r (sum of numbers on upper faces), is

(i) $\frac{(r-1)}{36}$, if $2 \le r \le 7$ (ii) $\frac{(13-r)}{36}$, if $8 \le r \le 12$

3 When three dice are thrown, the probability of getting a total r (sum of numbers on upper faces), is

(i) $\frac{{}^{r-1}C_2}{216}$, if $3 \le r \le 8$ (ii) $\frac{25}{216}$, if $r = 9$

(iii) $\frac{27}{216}$, if $r = 10, 11$ (iv) $\frac{25}{216}$, if $r = 12$

(v) $\frac{{}^{(20-r)}C_2}{216}$, if $13 \le r \le 18$

4 If A and B are two finite sets (Let $n(A) = n$ and $n(B) = m$) and if a mapping is selected at random from the set of all mappings from A to B, the probability that the mapping is

(i) a one-one function is $\frac{{}^mP_n}{m^n}$.

(ii) a one-one onto function is $\frac{n!}{m^n}$.

(iii) a many one function is $1 - \frac{{}^mP_n}{m^n}$.

5 If r squares are selected from a chess board of size 8×8, then the probability that they lie on a diagonal line, is $\frac{4({}^7C_r + {}^6C_r + {}^5C_r + \ldots + {}^rC_r) + 2({}^8C_r)}{{}^{64}C_r}$ for $1 \le r \le 7$.

6 If n objects are distributed among n persons, then the probability that atleast one of them will not get anything, is $\frac{n^n - n!}{n^n}$.

7 Points about coin, dice and playing cards:

(a) **Coin** If 'one' coin is tossed n times 'n' coins are tossed once, then number of simple events (or simple points) in the space of the experiment is 2^n. All these events are equally likely.

(b) **Dice** If 'one' die is thrown 'n' times or 'n' dice are thrown once, then number of simple events (or simple points) in the space of the experiment is 6^n (here dice is cubical). All events are equally likely.

(c) **Playing Cards** A pack of playing cards has 52 cards. There are four suits Spade (♠ black face), Heart (♥ red face), Diamond (♦ red face) and Club (♣ black face) each having 13 cards. In 13 cards of each suit, there are 3 face (or court) cards namely King, Queen and Jack (or knave), so there are in all 12 face cards 4 King, 4 Queen and 4 Jacks (or knaves). 4 of each suit namely Ace (or Ekka), King, Queen and Jack (or knave).

(i) **Game of bridge** It is played by 4 players, each player is given 13 cards.

(ii) **Game of whist** It is played by two pairs of persons.

(iii) If two cards (one after the other) can be drawn out of a well-shuffled pack of 52 cards, then number of ways; (×) With replacement is $52 \times 52 = (52)^2 = 2704$ (β) Without replacement is $52 \times 51 = 2652$.

(iv) Two cards (simultaneously) can be drawn out of a well-shuffled pack of 52 cards, then number of ways is ${}^{52}C_2 = \frac{52 \times 51}{2} = 1326$.

8 Out of $(2n + 1)$ tickets consecutively numbered, three are drawn at random, then the probability that the numbers on them are in AP, is $\frac{3n}{4n^2 - 1}$.

9 Out of $3n$ consecutive integers, three are selected at random, then the probability that their sum is divided by 3, is $\frac{(3n^2 - 3n + 2)}{(3n-1)(3n-2)}$.

10 Two numbers a and b are chosen at random from the set $\{1, 2, 3, \ldots, 5n\}$, the probability that $a^4 - b^4$ is divisible by 5, is $\frac{17n - 5}{5(5n-1)}$.

11 Two numbers a and b are chosen at random from the set $\{1, 2, 3, \ldots, 3n\}$ the probability that $a^2 - b^2$ is divisible by 3, is $\frac{(5n-3)}{3(3n-1)}$.

12 Two numbers a and b are chosen at random from the set $\{1, 2, 3, \ldots, 3n\}$, the probability that $a^3 + b^3$ is divisible by 3, is $\frac{1}{3}$.

13 There are n stations between two cities A and B. A train is to stop at three of these n stations. The probability that no two of these three stations are consecutive, is $\frac{(n-3)(n-4)}{n(n-1)}$.