13 Probability

"In Cricket, people speculate the chances of a team winning a match on the basis of toss won or lost and which team decides to bat or field first! This is how conditional probability is used based on previous wins and losses, to calculate the chances of one event occurring with some relationship to other.

Topic Notes

- *Basic Concepts and Conditional Probability*
- *Bayes' Theorem, Random Variables and its Probability Distribution*

BASIC CONCEPTS AND CONDITIONAL PROBABILITY 1

TOPIC 1

SOME ELEMENTARY TERMS AND CONCEPTS

Probability means study of possibility or chances of occurrence of an event. It conveys the sense that it is not certain whether the event will take place. However, in the theory of probability, we assign a numerical value to the degree of uncertainty. In this chapter, we are going to study about the occurrence of an event under a certain condition of another event already occured. Such probability is also known as conditional probability. Further, conditional probability will help in understanding the concepts of independent events, multiplication rule of probability and Bayes' theorem. We shall also discuss random variable and its probability distribution, and mean.

In this section, we shall state some basic terms of probability along with their definition.

Experiment

It is an operation which produces some well-defined outcomes (or results).

Random Experiment

If an experiment is repeated under identical conditions and the outcomes of that experiment are not the same every time but each outcome is one of the several possible outcomes, then it is called a random experiment.

Important

➥ *A random experiment satisfies two conditions:*
(1) It has more than one possible outcome.
(2) It is not possible to predict the outcome in advance.

Illustration: (1) If a coin is tossed, we get two outcomes, namely head and tail; and it is not possible to predict in advance whether head or tail will appear.

(2) If a 6-faced die is thrown, we get six outcomes, namely 1, 2, 3, 4, 5 and 6; and it is not possible to predict in advance what number will appear.

Sample Space of a Random Experiment

The set of all possible outcomes of a random experiment is called sample space of that random experiment. It is denoted by S.

Illustration: (1) Throw of a die: $S = \{1, 2, 3, 4, 5, 6\}$

(2) Toss of a coin: $S = \{H, T\}$

(3) Toss of two coins: $S = \{HH, HT, TH, TT\}$

Event

Every subset of a sample space is an event.

Important

➥ *Events are generally subsets of sample space and are denoted by A, B, C etc.*

Illustration: In tossing a coin, {H} and {T} are events.

Impossible (or null) Event and Sure Event

Since null set is the subset of every set, it is a subset of the sample space also and hence it is an event. This event is called the null event (or impossible event).

Again, since every set is a subset of itself, so sample space is also an event. This event is called the sure event.

Illustration: (1) The appearance of number 8 in throwing a 6-faced die is an impossible event.

(2) The appearance of a natural number less than 7 in throwing a 6-faced die is a sure event.

Equally Likely Events

Two outcomes are said to be equally likely if one of them cannot be expected to occur in preference to the other. Or we can say that if the probability of occurrence of one event is same as that of the other event, then the two events are said to be equally likely events.

Illustration: In tossing of a coin, the appearance of head and tail are equally likely events.

Exhaustive Events

Events $E_1, E_2, E_3, \ldots, E_n$ are said to be exhaustive events if $E_1 \cup E_2 \cup E_3 \ldots \cup E_n = S$.

Illustration: Event E_1 as getting an odd number and E_2 as getting an even number on tossing a die are exhaustive events. $E_1 = \{1, 3, 5\}$ and $E_2 = \{2, 4, 6\}$, here $E_1 \cup E_2 = S$.

Mutually Exclusive Events

Two events A and B are called mutually exclusive if $A \cap B = \phi$.

Illustration: Let $S = \{1, 2, 3, 4, 5, 6\}$ and events A and B be two events such that $A = \{1, 3, 5\}$ and $B = \{2, 4, 6\}$.

Since $A \cap B = \phi$, events A and B are mutually exclusive.

| TOPIC 2 |

PROBABILITY

Let S be a finite sample space of a random experiment consisting of n outcomes. Also, let these outcomes be all equally likely. If m of these are favorable to the occurrence of an event A, then we define probability of A as

$$P(A) = \frac{n(A)}{n(S)} = \frac{m}{n}$$

Illustration: (1) Let S be the sample space of throwing a 6-faced die. Then, S = {1, 2, 3, 4, 5, 6}. Let A be the event that the number appearing on the face of the die is a prime number. Then, A = {2, 3, 5}.

So, $n(S) = 6$ and $n(A) = 3$.

$$\therefore \quad P(A) = \frac{3}{6}, \text{ or } \frac{1}{2}$$

(2) Let S be the sample space of tossing a coin two times. Then, S = {HH, HT, TH, TT}

Let A be the event 'getting at least one head'. Then, A = {HH, HT, TH}.

So, $n(S) = 4$ and $n(A) = 3$.

$$\therefore \quad P(A) = \frac{3}{4}$$

Important

- *The above definition of probability is not applicable if the sample space is an infinite set or its members are not equally likely.*
- *The probability of an event A lies between 0 and 1, i.e., $0 \le P(A) \le 1$.*
- *The probability of sure event is 1; and of the impossible event is zero.*
- *If $P(A) = \frac{m}{n}$, then $P(\text{not } A) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$*

$\Rightarrow P(A) + P(\text{not } A) = 1$

- *The odds in favour of occurrence of an event A are defined by $m : n - m$, i.e., $P(A) : P(A')$; and odds against of an event A are defined by $n - m : m$, i.e., $P(A') : P(A)$.*

⚠ Caution

- *P(not A) is also written as P(A'); and is read as 'complement of A'.*

Some Important Details of Playing Cards

A complete set of playing cards is called a deck or pack. It consists of a total of 52 cards.

These cards are equally distributed in four suits – Spades (♠), Hearts (♥), Clubs (♣) and Diamonds (♦). Each suit has 13 cards: an ace, a king, a queen, a jack and nine cards numbered 2 to 10.

Of the 52 cards of a pack, there are (1) 26 red cards – 13 heart cards + 13 diamond cards; and 26 black cards – 13 spade cards + 13 club cards; (2) 12 face cards – 4 kings, 4 queens and 4 jacks.

| TOPIC 3 |

ADDITION THEOREM ON PROBABILITY

Addition Theorem for Two Events

Addition theorem on probability states that if A and B are any two events associated with a random experiment, then the probability of occurrence of event 'A or B' is given as:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

In other words,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Corollary: If A and B are two mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$.

Example 1.1: If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$, and $P(A \cap B) = \frac{1}{8}$, find P (not A and not B). **[NCERT]**

Ans. We know that

$$P(\text{not A and not B}) = P(A' \cap B') = P[(A \cup B)'] = 1 - P(A \cup B) \quad \text{...(i)}$$

By addition theorem, we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8} = \frac{5}{8}$$

Using the value of $P(A \cup B)$ in (i), we have

$$P(\text{not A and not B}) = 1 - \frac{5}{8} = \frac{3}{8}$$

Addition Theorem for Three Events

Let E, F and G be any three events associated with a random experiment. Then,

$P(E \text{ or } F \text{ or } G) = P(E) + P(F) + P(G) - P(E \text{ and } F) - P(F \text{ and } G) - P(G \text{ and } E) + P(E \text{ and } F \text{ and } G)$

In other words,

$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(F \cap G) - P(G \cap E) + P(E \cap F \cap G)$

Corollary: If E, F and G are three mutually exclusive events, then $P(E \cup F \cup G) = P(E) + P(F) + P(G)$.

Set Notations for Different Types of Events

Event	Set Notation
Not A	A'
A and B	$A \cap B$
A or B	$A \cup B$
A but not B	$A \cap B'$
B but not A	$A' \cap B$
Neither A nor B	$A' \cap B'$
All three of A, B, C	$A \cap B \cap C$
Exactly one of A, B	$(A' \cap B) \cup (A \cap B')$
Exactly two of A, B, C	$(A' \cap B \cap C) \cup (A \cap B' \cap C) \cup (A \cap B \cap C')$

Important

➥ *Let A and B be any two events associated with a random experiment. Then,*

(1) $P(A \cap B') = P(A) - P(A \cap B)$

(2) $P(A' \cap B) = P(B) - P(A \cap B)$

(3) $P(A' \cap B') = P[(A \cup B)'] = 1 - P(A \cup B)$

(4) $P(A' \cup B') = P[(A \cap B)'] = 1 - P(A \cap B)$

TOPIC 4

CONDITIONAL PROBABILITY

Let A and B be two events associated with the same sample space S of the random experiment. Then, the probability of occurrence of A under the condition that B has already occurred and $P(B) \neq 0$ is called conditional probability, to be denoted by $P(A \mid B)$. Similarly, $P(B \mid A)$ is defined.

Meaning of P(A|B)

$$P(A \mid B) = \frac{\text{Number of outcomes of B which are favourable to A}}{\text{Number of outcomes favourable to B}}$$

$$= \frac{n(A \cap B)}{n(B)}$$

$$= \frac{n(A \cap B)/n(S)}{n(B)/n(S)}$$ [On dividing both the numerator and denominator by $n(S)$]

$$= \frac{P(A \cap B)}{P(B)}$$

Properties of Conditional Probability

Let A and B be any two arbitrary events associated with the same sample space of the random experiment.

Property 1

$$P(S \mid A) = P(A \mid A) = 1$$

Property 2

If $A \subset B$, then $A \cap B = A$

$$\Rightarrow \quad P(B \mid A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A)}{P(A)} = 1$$

Property 3

The conditional probability of not happening of B when A has already occurred is denoted by $P(B' \mid A)$ and is given by $P(B' \mid A) = 1 - P(B \mid A)$.

Property 4

If A and B are any two events associated with the same sample space S of the random experiment and F is an event of S such that $P(F) \neq 0$, then,

$$P[(A \cup B) \mid F] = P(A \mid F) + P(B \mid F) - P[(A \cap B) \mid F]$$

In particular, if A and B are disjoint events, then

$$P[(A \cup B) \mid F] = P(A \mid F) + P(B \mid F)$$

Example 1.2: A black and a red dice are rolled.

(A) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(B) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4. [NCERT]

Ans. When two dice are rolled,

Total number of outcomes, $n(S) = 36$

(A) Let A be the event of getting a sum greater than 9 and B be the event of getting a 5 on the black die. Then,

$A = \{(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)\}$

$B = \{(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)\}$

$\Rightarrow \quad A \cap B = \{(5, 5), (5, 6)\}$

$\therefore \quad P(A) = \frac{6}{36} = \frac{1}{6}; \; P(B) = \frac{6}{36} = \frac{1}{6};$

$P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

Now, required probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/18}{1/6} = \frac{1}{3}$$

(B) Let A be the event of getting a sum of 8 and B be the event of getting a number less than 4 on the red die. Then,

A = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

B = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)}

$\Rightarrow$ $A \cap B$ = {(6, 2), (5, 3)}

$\therefore$ $P(A) = \frac{5}{36}$; $P(B) = \frac{18}{36} = \frac{1}{2}$;

$$P(A \cap B) = \frac{2}{36} = \frac{1}{18}$$

Now, required probability = $P(A \mid B)$

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/18}{1/2} = \frac{1}{9}$$

Example 1.3: An instructor has a question bank consisting of 300 easy True/False questions, 200 difficult True / False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question? [NCERT]

Ans. Here, total questions = 300 + 200 + 500 + 400 = 1400

Let A be the event that selected question is an easy question. Then,

$$P(A) = \frac{300 + 500}{1400} = \frac{4}{7}$$

Let B be the event that selected question is a multiple choice question. Then,

$$P(B) = \frac{500 + 400}{1400} = \frac{9}{14}$$

Now, $A \cap B$ is the event so that the selected question is an easy multiple choice question. Then,

$$P(A \cap B) = \frac{500}{1400} = \frac{5}{14}$$

$\therefore$ Required probability = $P(A \mid B)$

$$= \frac{P(A \cap B)}{P(B)} = \frac{5/14}{9/14} = \frac{5}{9}$$

Example 1.4: Consider the experiment of throwing a die. If a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3.' [NCERT]

Ans. Here, S = {(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6), (1, H), (1, T), (2, H), (2, T), (4, H), (4, T), (5, H), (5, T)}

Let A be the event of getting a tail on the coin. Then,

A = {(1, T), (2, T), (4, T), (5, T)}

$$\Rightarrow \quad P(A) = \frac{4}{20} = \frac{1}{5}$$

Let B be the event that at least one die shows a 3. Then,

B = {(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (6, 3)}

$$\Rightarrow \quad P(B) = \frac{7}{20}$$

Now, $A \cap B = \phi$.

So, $P(A \cap B) = 0$

$\therefore$ Required probability = $P(A \mid B)$

$$= \frac{P(A \cap B)}{P(B)} = \frac{0}{7/20} = 0$$

Example 1.5: In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.

(A) Find the probability that she reads neither Hindi nor English newspapers.

(B) If she reads Hindi newspaper, find the probability that she reads English newspaper.

(C) If she reads English newspaper, find the probability that she reads Hindi newspaper.

[NCERT]

Ans. Let A be the event "student reads Hindi Newspaper"; B be the event "student reads English Newspaper". Then,

$$P(A) = \frac{60}{100} = 0.6; \quad P(B) = \frac{40}{100} = 0.4;$$

$$P(A \cap B) = \frac{20}{100} = 0.2 \qquad ...(i)$$

(A) Here, we need to find $P(A' \cap B')$

From (i), we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.6 + 0.4 - 0.2$$
$$= 0.8$$

Further,

$$P(A' \cap B') = P[(A \cup B)']$$
$$= 1 - P(A \cup B)$$
$$= 1 - 0.8 = 0.2$$

(B) Here, we need to find P(B | A).

We know that,

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.6} = \frac{1}{3}$$

(C) Here, we need to find P(A | B).

We know that,

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.4} = \frac{1}{2}$$

TOPIC 5

MULTIPLICATION THEOREM

If A and B be two arbitrary events associated with the same sample space of the random experiment, then

$P(A \cap B) = P(B).P(A \mid B)$ and $P(B \cap A) = P(A).\ P(B \mid A)$, provided $P(A) \neq 0$, $P(B) \neq 0$.

These further yields

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)};$$

and $$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

Example 1.6: Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black. [NCERT]

Ans. Let A and B be the events of drawing a black card in the first draw and in the second draw, respectively.

In the first draw, there are 26 black cards out of 52 cards.

So, $$P(A) = \frac{26}{52} = \frac{1}{2}$$

After first draw, there are 51 cards left.

So, in the second draw, there are 25 black cards out of 51 cards.

So, $$P(B \mid A) = \frac{25}{51}$$

Hence, P(both the cards are black)

$$= P(A \cap B)$$

$$= P(A).P(B \mid A) = \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$$

TOPIC 6

INDEPENDENT EVENTS

Two events are said to be independent events if the occurrence of one event is not affected by the occurrence of the other event.

Illustration: Let two cards be drawn from a pack of 52 cards, one after the other. Consider the events A and B as follows:

A : drawing an ace in the first draw;

B : drawing a queen in the second draw.

If the card drawn in the first draw is not replaced back, then the events A and B are dependent events.

If the card drawn in the first draw is replaced back, then the events A and B are independent events.

Important

➥ *For independent events A and B, $P(A) \neq 0$, $P(B) \neq 0$, we have*

(1) $P(A/B) = P(A)$; $P(B/A) = P(B)$ and $P(A \cap B) = P(A).P(B)$

(2) $P(A \cap B) = 1 - P(A')P(B')$

(3) A' and B are independent events.

(4) A' and B' are independent events.

Example 1.7: A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'the number is even,' and B be the event, 'the number is red'. Are A and B independent? [NCERT]

Ans. Let A be the event that the number on the die is even.

So, $P(A) = \frac{3}{6} = \frac{1}{2}$

Let B be the event that the number is red.

So, $P(B) = \frac{3}{6} = \frac{1}{2}$

Now, $A \cap B$ is the event when number is even and marked red.

$\Rightarrow \quad A \cap B = \{2\}$

So, $\quad P(A \cap B) = \frac{1}{6}$

Now, $\quad P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\Rightarrow \quad P(A \cap B) \neq P(A) \times P(B)$.

Hence, events A and B are not independent.

Example 1.8: Probability of solving a specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that:

(A) the problem is solved.

(B) exactly one of them solves the problem. [NCERT]

Ans. Here, $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$

$\Rightarrow P(A') = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2};$

and $P(B') = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$

(A) P(problem is solved) = P(A solves the problem or B solves the problem)

$= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$ [Using addition theorem]

$= P(A) + P(B) - P(A).P(B)$ [$\because$ A and B are independent events]

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{2} \times \frac{1}{3}$

$= \frac{5}{6} - \frac{1}{6}$

$= \frac{4}{6}$, or $\frac{2}{3}$

(B) P(exactly one of them solves the problem)

$= P(A \cap B') + P(A' \cap B)$

$= P(A)P(B') + P(A')P(B)$

$= \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3}$

$= \frac{1}{3} + \frac{1}{6}$

$= \frac{3}{6}$, or $\frac{1}{2}$

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. The probability of obtaining an even prime number on each die, when a pair of dice is rolled, is:

(a) 0 (b) $\frac{1}{3}$

(c) $\frac{1}{12}$ (d) $\frac{1}{36}$

Ans. (d) $\frac{1}{36}$

Explanation: When a pair of dice is rolled, then $n(S) = 36$. There is only one pair *i.e.,* (2, 2) having even prime number on each die. Thus, probability of obtaining an even prime number on each die is $\frac{1}{36}$.

2. If A and B are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \mid B) = \frac{1}{4}$, then $P(A' \cap B')$ equals:

(a) $\frac{1}{12}$ (b) $\frac{3}{4}$

(c) $\frac{1}{4}$ (d) $\frac{3}{16}$

Ans. (c) $\frac{1}{4}$

Explanation:

We know, $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow \quad \frac{1}{4} = \frac{P(A \cap B)}{1/3}$

$\Rightarrow \quad P(A \cap B) = \frac{1}{12}$

Now, $P(A' \cap B') = P[(A \cup B)']$

$= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - \left[\frac{1}{2} + \frac{1}{3} - \frac{1}{12}\right]$

$= 1 - \left(\frac{6+4-1}{12}\right)$

$= 1 - \frac{9}{12} = \frac{3}{12} = \frac{1}{4}$

Concept Applied

- *First find the value of $P(A \cap B)$, then find the value of $P(A' \cap B')$ by applying the desired formula.*

3. If A and B are events such that P(A) = 0.4, P(B) = 0.3 and P(A ∪ B) = 0.5, then P(B' ∩ A) equals:

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$

(c) $\frac{3}{10}$ (d) $\frac{1}{5}$

[NCERT Exemplar]

4. If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then P(A' ∩ B') equals:

(a) $\frac{4}{15}$ (b) $\frac{8}{45}$

(c) $\frac{1}{3}$ (d) $\frac{2}{9}$

[NCERT Exemplar]

Ans. (d) $\frac{2}{9}$

Explanation:

$$P(A' \cap B') = 1 - P(A \cup B)$$
$$= 1 - [P(A) + P(B) - P(A \cap B)]$$
$$= 1 - [P(A) + P(B) - P(A).\,P(B)]$$

[∵ A and B are independent events]

$$= 1 - \left(\frac{3}{5} + \frac{4}{9} - \frac{3}{5} \times \frac{4}{9}\right)$$
$$= 1 - \left(\frac{27+20-12}{45}\right)$$
$$= 1 - \frac{35}{45} = 1 - \frac{7}{9} = \frac{2}{9}$$

5. If A and B are two events such that P(A) + P(B) − P(A and B) = P(A), then:

(a) P(B|A) = 1 (b) P(A|B) = 1

(c) P(B|A) = 0 (d) P(A|B) = 0 [NCERT]

6. A die is thrown once. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then P(A ∪ B) is:

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$

(c) 0 (d) 1 [CBSE 2020]

7. If A and B are events such that P(A|B) = P(B|A), then:

(a) A ⊂ B but A ≠ B (b) A = B

(c) A ∩ B = ϕ (d) P(A) = P(B)

Ans. (d) P(A) = P(B)

Explanation: Here, P(A|B) = P(B|A)

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)} \Rightarrow P(A) = P(B)$$

$$\Rightarrow \quad P(A) = P(B)$$

8. A card is picked at random from a pack of 52 playing cards. Given that the picked card is a queen, the probability of this card to be a card of spade is:

(a) $\frac{1}{3}$ (b) $\frac{4}{13}$

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$ [CBSE 2020]

Ans. (c) $\frac{1}{4}$

Explanation: Let A be the event that the picked card is spade and B be the event that picked card is a queen.

There are 13 spade cards and 4 queens and only one queen is spade.

So, $n(A) = 13$, $n(B) = 4$

And $n(A \cap B) = 1$

Now, Required probability = P(A|B)

$$= \frac{P(A \cap B)}{P(B)}$$
$$= \frac{1/52}{4/52} = \frac{1}{4}$$

9. Two events A and B are independent, if:

(a) A and B are mutually exclusive

(b) P(A' ∩ B') = [1 − P(A)][1 − P(B)]

(c) P(A) = P(B)

(d) P(A) + P(B) = 1

Ans. (b) P(A' ∩ B') = [1 − P(A)][1 − P(B)]

Explanation: We know that if A and B are independent events, then

$$P(A \cap B) = P(A) \times P(B)$$

So, $$P(A' \cap B') = P(A') \times P(B')$$
$$= [1 - P(A)][1 - P(B)]$$

10. If A and B are two events such that A ⊂ B and P(B) ≠ 0, then which of the following is correct?

(a) $P(A|B) = \frac{P(B)}{P(A)}$ (b) P(A|B) < P(A)

(c) P(A|B) ≥ P(A) (d) None of these

11. Two events E and F are independent. If P(E) = 0.3 and P(E ∪ F) = 0.5, then P(E | F) − P(F | E) equals:

(a) $\frac{2}{7}$ (b) $\frac{3}{35}$

(c) $\frac{1}{70}$ (d) $\frac{1}{7}$

Ans. (c) $\frac{1}{70}$

Explanation: Since, E and F are independent events

$\therefore \quad P(E \cap F) = P(E).\,P(F)$

Now, $\quad P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$\therefore \quad 0.5 = 0.3 + P(F) - 0.3 \times P(F)$

$\Rightarrow \quad 0.2 = P(F)\,[1 - 0.3]$

$\Rightarrow \quad P(F) = \frac{2}{7}$

$$\therefore\ P(E\,|\,F) - P(F\,|\,E) = \frac{P(E \cap F)}{P(F)} - \frac{P(E \cap F)}{P(E)}$$

$$= \frac{P(E).P(F)}{P(F)} - \frac{P(E).P(F)}{P(E)}$$

$$= P(E) - P(F)$$

$$= 0.3 - \frac{2}{7}$$

$$= \frac{3}{10} - \frac{2}{7}$$

$$= \frac{21-20}{70}$$

$$= \frac{1}{70}$$

⚠ Caution

➥ *Use the concept of independent events to find P(F) and $P(E \cap F)$.*

12. If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(B'|A)$ is:

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$

(c) $\frac{3}{4}$ (d) 1 **[CBSE 2020]**

Ans. *(c)* $\frac{3}{4}$

Explanation: Here, A and B are independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$.

$$\text{Now, } P(B'|A) = \frac{P(B' \cap A)}{P(A)}$$

$$= \frac{P(B')P(A)}{P(A)}$$

[$\because$ A and B are independent events]

$$= P(B') = 1 - P(B)$$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

13. $P(A \cup B) = P(A \cap B)$, if and only if the relation between P(A) and P(B) is:

(a) $P(A) = P(A')$ (b) $P(A \cup B) = P(A' \cap B')$

(c) $P(A) = P(B)$ (d) None of these

[DIKSHA]

14. The covid pandemic has claimed several lives so far and therefore the government has launched a big vaccination drive to vaccinate all its citizens by giving them two doses of the covid vaccine. Let A be the event of a person being vaccinated and B be the event that a person has received three doses of the covid vaccine.

If $P(A) = \frac{1}{2}$, $P(B) = 0$, then $P(A|B)$ is:

(a) 0 (b) $\frac{1}{2}$

(c) not defined (d) 1

Ans. *(c) not defined*

Explanation: We know that

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{0} = \text{not defined}$$

15. Delhi Metro is the life line of Delhi these days and most people prefer using the metro for commuting to their offices. Let A be the event that during the office hour, passengers are travelling by metro and B the event that metro is running, subject to no technical faults or pandemic situation.

If A and B are two events such that $P(A) \neq 0$ and $P(B|A) = 1$, then:

(a) $A \subset B$ (b) $B \subset A$

(c) $B = \phi$ (d) $A = \phi$

Ans. *(a)* $A \subset B$

Explanation: It is given that P(B|A) = 1.

$\because \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$

$\Rightarrow \quad \frac{P(A \cap B)}{P(A)} = 1$

$\Rightarrow \quad P(A \cap B) = P(A)$

$\Rightarrow \quad A \subset B$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

16. Two cards are drawn at random and one-by-one without replacement from a well shuffled pack of 52 playing cards. Find the probability that one card is red and other is black. [CBSE 2020]

Ans. There are 26 red cards and 26 black cards in a deck of 52 playing cards.

Let R and B denote the events of drawing red card and black card, respectively.

Now, Required probability

= P(first card is red and second one is black)

+ P(first card is black and second one is red)

$= P(R) \times P(B|R) + P(B) \times P(R|B)$

$= \frac{26}{52} \times \frac{26}{51} + \frac{26}{52} \times \frac{26}{51}$

$= 2 \times \frac{26}{52} \times \frac{26}{51}$

$= \frac{26}{51}$

17. The probability that at least one of the two events A and B occur is 0.6. If A and B occur simultaneously with probability 0.3, evaluate $P(\overline{A}) + P(\overline{B})$. [NCERT Exemplar]

18. Three dice are thrown at the same time. Find the probability of getting three two's, if it is known that the sum of the numbers on the dice was six. [NCERT Exemplar]

Ans. On a throw of three dice, the sample space

$n(S) = 6^3 = 216$

Let, E_1 be the event that the sum of the numbers on the three dice was six and, E_2 be the event that three two's occurs.

$\Rightarrow \quad E_1 = \{(1, 1, 4), (1, 2, 3), (1, 3, 2), (1, 4, 1), (2, 1, 3), (2, 2, 2), (2, 3, 1), (3, 1, 2), (3, 2, 1), (4, 1, 1)\}$

$\Rightarrow n(E_1) = 10$ and $E_2 = \{(2, 2, 2)\}$ *i.e.* $n(E_2) = 1$

Also, $\quad E_1 \cap E_2 = \{(2, 2, 2)\}$

i.e. $\quad n(E_1 \cap E_2) = 1$

$\therefore \quad P(E_2/E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$

$= \frac{1/216}{10/216} = \frac{1}{10}$

19. A problem is given to three students whose probabilities of solving it are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{6}$ respectively. If the events of solving the problem are independent, find the probability that at least one of them solves it.

Ans. Let A, B, C be the events of solving the problem by the three students, respectively.

$\therefore P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$ and $P(C) = \frac{1}{6}$

Since, the events of solving the problem are independent.

$\therefore$ P(at least one of them solves it)

= 1 – P(none of them solves it)

$= 1 - P(\overline{A} \cap \overline{B} \cap \overline{C})$

$= 1 - P(\overline{A})P(\overline{B})P(\overline{C})$

$= 1 - [1 - P(A)][1 - P(B)][1 - P(C)]$

$= 1 - \left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{6}\right)$

$= 1 - \frac{2}{3} \times \frac{3}{4} \times \frac{5}{6}$

$= 1 - \frac{5}{12}$

$= \frac{7}{12}$

20. A and B are two events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$, and $P(A') = \frac{2}{3}$, then find $P(A' \cap B)$.

21. If P(A) = 0.5, P(B) = 0.4 and $P(A \cap B) = 0.3$, then what is the value of $P\left(\frac{A'}{B'}\right)$?

Ans. $P\left(\frac{A'}{B'}\right) = \frac{P(A' \cap B')}{P(B')} = \frac{P(A \cup B)'}{P(B')}$

$= \frac{1 - P(A \cup B)}{1 - P(B)}$

$$= \frac{1-[P(A)+P(B)-P(A\cap B)]}{1-P(B)}$$

$$= \frac{1-[0.5+0.4-0.3]}{1-0.4} = \frac{0.4}{0.6}$$

$$= \frac{2}{3}$$

22. **If A and B are two independent events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$, then find the value of P(neither A nor B).**

23. **Events E and F are such that P(not E or not F) = 0.25. State whether E and F are mutually exclusive.**

Ans. Given P(not E or not F) = 0.25

$\Rightarrow \quad P(E' \cup F') = P(E \cap F)' = 0.25$

We know, $P(E \cap F)' + P(E \cap F) = 1$

$\Rightarrow \quad P(E \cap F) = 1 - P(E \cap F)'$

$\Rightarrow \quad P(E \cap F) = 1 - 0.25 = 0.75 \neq 0$

Hence, E and F are not mutually exclusive.

24. **Four cards are successively drawn without replacement from a deck of 52 playing cards. What is the probability that all four cards are kings?**

Ans. Let, A denote the event that the first card is king,

B denote the event that the second card is king,

C denote the event that the third card is king, and D denote the event that the fourth card is king.

Now, $P(A \cap B \cap C \cap D)$

$= P(A) \times P(B|A) \times P(C|A \cap B) \times P(D|A \cap B \cap C)$

$$= \frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49} = \frac{1}{270725}$$

25. **A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. What is probability of drawing 2 green balls and one blue ball?**

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

26. **A and B are two events, such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$. Find:**

(A) P(A | B) **(B) P(B | A)**

(C) P(A' | B) **(D) P(A' | B')**

[NCERT Exemplar]

Ans. Here, $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$

and $P(A \cap B) = \frac{1}{4}$

(A) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/3} = \frac{3}{4}$

(B) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1/4}{1/2} = \frac{1}{2}$

(C) $P(A'|B) = 1 - P(A|B)$

$$= 1 - \frac{3}{4} = \frac{1}{4}$$

(D) $P(A'|B') = \frac{P(A' \cap B')}{P(B')}$

$$= \frac{1 - P(A \cup B)}{1 - P(B)}$$

$$= \frac{1 - [P(A) + P(B) - P(A \cap B)]}{1 - P(B)}$$

$$= \frac{1-\left[\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\right]}{1-\frac{1}{3}}$$

$$= \frac{1-\left(\frac{5}{6}-\frac{1}{4}\right)}{\frac{2}{3}}$$

$$= \frac{1-\frac{7}{12}}{\frac{2}{3}} = \frac{\frac{5}{12}}{\frac{2}{3}}$$

$$= \frac{5}{8}$$

Concept Applied

➥ $P(A'|B) = \frac{P(A' \cap B)}{P(B)}$

$= \frac{P(B) - P(A \cap B)}{P(B)}$

$= 1 - P(A/B)$

➥ $P(A' \cap B') = P(A \cup B)'$

$= 1 - P(A \cup B)$

27. Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected. [CBSE 2019]

Ans. Total number of students = 8

$\therefore$ Total number of ways to select 4 students out of 8 = $^8C_4 = \frac{8!}{4!4!} = 70.$

Now, number of ways to select 2 boys and 2 girls

$$= {}^3C_2 \times {}^5C_2 = \frac{3!}{2! \times 1!} \times \frac{5!}{2! \times 3!}$$

$$= 3 \times 10 = 30$$

$\therefore$ Required probability = $\frac{30}{70} = \frac{3}{7}$

28. Prove that if E and F are independent events, then the events E′ and F′ are also independent. [CBSE 2019]

Ans. Given, E and F are independent events.

$\therefore \quad P(E \cap F) = P(E)\,P(F)$

Now, $\quad P(E' \cap F') = 1 - P(E \cup F)$

$[\because P(E' \cap F') = P(E \cup F)']$

$= 1 - [P(E) + P(F) - P(E \cap F)]$

$= 1 - P(E) - P(F) + P(E \cap F)$

$= 1 - P(E) - P(F) + P(E)\,P(F)$

$= 1 - P(E) - P(F)\,[1 - P(E)]$

$= [1 - P(E)]\,[1 - P(F)]$

$= P(E')P(F')$

Hence, E′ and F′ are independent events.

29. The probability of finding a green signal on a busy crossing X is 30%. What is the probability of finding a green signal on X on two consecutive days out of three? [CBSE 2020]

Ans. Let G be the event of finding a green signal.

$\therefore$ Required probability = P(GGG′) + P(G′GG)

$$= \left(\frac{3}{10}\right)^2 \cdot \frac{7}{10} + \frac{7}{10}\left(\frac{3}{10}\right)^2$$

$$= \frac{9}{100} \cdot \frac{7}{10} + \frac{7}{10} \cdot \frac{9}{100}$$

$$= \frac{63}{1000} + \frac{63}{1000} = \frac{126}{1000}$$

$$= \frac{63}{500}$$

30. In the game of archery, the probability of Likith and Harish hitting the target are $\frac{2}{3}$ and $\frac{3}{4}$ respectively.

If both of them shoot an arrow, find the probability that the target is NOT hit by either of them. Show your steps.

Ans. Let events L and H be defined as follows:

L = Likith hit the target

H = Harish hit the target

$\therefore \quad P(L) = \frac{2}{3}, P(H) = \frac{3}{4}$

Now, required probability = P(Target is not hit by either of them)

$= P(\bar{L}\bar{H})$

$= P(\bar{L}) \times P(\bar{H})$

[$\because$ L and H are independent events]

$= [1 - P(L)]\,[1 - P(H)]$

$$= \left(1-\frac{2}{3}\right)\left(1-\frac{3}{4}\right)$$

$$= \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

31. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are black?

32. Two cards are drawn at random from a pack of 52 cards one-by-one without replacement. What is the probability of getting first card red and second card Jack? [CBSE Term-2 SQP 2022]

Ans. The required probability = P((The first is a red jack card and The second is a jack card) or (The first is a red non-jack card and The second is a jack card))

$$= \frac{2}{52} \times \frac{3}{51} + \frac{24}{52} \times \frac{4}{51} = \frac{1}{26}$$

[CBSE Marking Scheme Term-2 SQP 2022]

Explanation: There are 52 cards in a pack of cards.

No. of red cards = 26

No. of jacks = 4

$\therefore$Required probability

= P(First card non-jack red, second card jack)

+ P(first card red jack, second card jack)

$$= \frac{(26-2)}{52}\times\frac{4}{51}+\frac{2}{52}\times\frac{3}{51}$$

$$= \frac{24}{52}\times\frac{4}{51}+\frac{2}{52}\times\frac{3}{51}$$

$$= \frac{96+6}{52\times51}=\frac{102}{52\times51}$$

$$= \frac{34}{52\times17}$$

$$= \frac{2}{52}=\frac{1}{26}$$

33. **A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, then what is the probability of getting exactly one red ball?**

34. **There are 25 tickets bearing numbers from 1 to 25. One ticket is drawn at random. Find the probability that the number on it is a multiple of 5 or 6.**

35. **A speaks truth in 80% cases and B speak truth in 90% cases. In what percentage of cases are they likely to agree with each other in stating the same fact?**

[CBSE SQP 2020]

Ans. We have events A and B as,

A : A speaks truth.

B : B speaks truth.

$\therefore \quad P(A) = 80\% = \frac{80}{100} = \frac{8}{10}$

and $\quad P(B) = 90\% = \frac{90}{100} = \frac{9}{10}$

Now, Required probability

= P(A and B agree with each other)

= P(Both A and B speaks truth or both A and B lies)

$$= P(AB \text{ or } \bar{A}\bar{B})$$

$$= P(AB) + P(\bar{A}\bar{B})$$

$$= P(A)P(B) + P(\bar{A})P(\bar{B})$$

$$= P(A)P(B) + [1 - P(A)][1 - P(B)]$$

$$= \frac{8}{10}\times\frac{9}{10}+\left(1-\frac{8}{10}\right)\left(1-\frac{9}{10}\right)$$

$$= \frac{72}{100}+\frac{2}{100}=\frac{74}{100}=74\%$$

Hence, percentage of cases in which they are likely to agree with each other in stating the same fact, is 74%.

SHORT ANSWER Type-II Questions (SA-II)

[**3** marks]

36. **Two dice are tossed. Find whether the following two events A and B are independent.**

$A = \{(x, y); x + y = 11\}$ and $B = \{(x, y); x \neq 5\}$ where (x, y) denotes a typical sample point.

[NCERT Exemplar]

Ans. We have, $A = \{(x, y); x + y = 11\}$ and $B = \{(x, y); x \neq 5\}$.

$\therefore \quad A = \{(5, 6)\ (6, 5)\}$.

B = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Also, $A \cap B = \{(6, 5)\}$

$\Rightarrow n(A) = 2, n(B) = 30$ and $n(A \cap B) = 1$

$\therefore \quad P(A) = \frac{2}{36} = \frac{1}{18}$

$P(B) = \frac{30}{36} = \frac{5}{6}$

$\Rightarrow \quad P(A).\ P(B) = \frac{1}{18}\times\frac{5}{6} = \frac{5}{108}$

And $\quad P(A \cap B) = \frac{1}{36}$

$\Rightarrow \quad P(A).\ P(B) \neq P(A \cap B)$

So, A and B are not independent events.

⚠ Caution

- *Form the sample space carefully by using the given condition.*

37. **Two dice are thrown together and the total score is noted. The events E, F and G, are 'a total of 4', 'a total of 9 or more', and 'a total divisible by 5' respectively.**

Calculate P(E), P(F) and P(G) and decide which pairs of events, if any, are independent. **[NCERT Exemplar]**

Ans. Two dice are thrown together.

$\therefore \quad n(S) = 36$

Now,

Event E = A total of 4

$= \{(2, 2), (3, 1), (3, 3)\}$

$\Rightarrow \quad n(E) = 3$

Event F = Total of 9 or more.

$= \{(3, 6), (6, 3), (4, 5), (4, 6), (5, 4), (6, 4), (5, 5), (5, 6), (6, 5), (6, 6)\}$

$\Rightarrow \quad n(F) = 10$

Event G = A total divisible by 5

$= \{(1, 4), (4, 1), (2, 3), (3, 2), (4, 6), (6, 4), (5, 5)\}$

$\Rightarrow \quad n(G) = 7$

Here, $E \cap F = \phi$ and $E \cap G = \phi$

Also, $F \cap G = \{(4, 6), (6, 4), (5, 5)\}$

$\Rightarrow \quad n(F \cap G) = 3$ and $E \cap F \cap G = \phi$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

$$P(F) = \frac{n(F)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$

$$P(G) = \frac{n(G)}{n(S)} = \frac{7}{36}$$

$$P(F \cap G) = \frac{3}{36} = \frac{1}{12}$$

$$\text{And} \quad P(F).\,P(G) = \frac{5}{18} \cdot \frac{7}{36} = \frac{35}{648}$$

Here, we see that $P(F \cap G) \neq P(F).\,P(G)$

Hence, there is no pair which is independent.

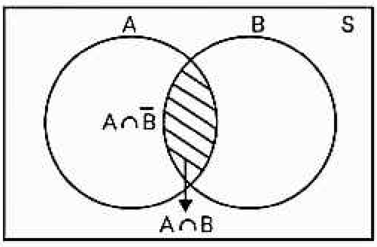
38. Prove that:

(A) $P(A) = P(A \cap B) + P(A \cap \overline{B})$

(B) $P(A \cup B) = P(A \cap B) + P(A \cap \overline{B}) + P(\overline{A} \cap B)$

[NCERT Exemplar]

Ans. (A)



We have, $A = A \cap S$.

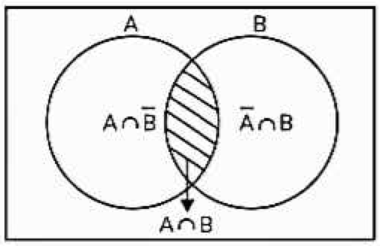
$= A \cap (B \cup \overline{B})$

$= (A \cap B) \cup (A \cap \overline{B})$

Also, $A \cap B$ and $A \cap \overline{B}$ are mutually exclusive.

$\therefore \quad P(A) = P(A \cap B) + P(A \cap \overline{B})$

(B)



By the diagram, we can see that

$$A \cup B = (A \cap B) \cup (A \cap \overline{B}) \cup (\overline{A} \cap B)$$

Also, $(A \cap B)$, $(A \cap \overline{B})$ and $(\overline{A} \cap B)$ are mutually exclusive.

$\therefore \; P(A \cup B) = P(A \cap B) + P(A \cap \overline{B}) + P(\overline{A} \cap B)$

⚠ Caution

➥ *Use venn diagram to solve the given situation. Then, it may be easy to get the solution.*

39. Assume that each born child is equally likely to be a boy on a girl. If a family has two children, what is the conditional probability that both are girls? Given that

(A) the youngest is a girl

(B) at least one is a girl [CBSE 2014]

Ans. Let G_i $(i = 1, 2)$ and B_i $(i = 1, 2)$ denote the i^{th} child is a girl and a boy, respectively.

Then, sample space is,

$S = \{G_1G_2, G_1B_2, B_1G_2, B_1B_2\}$

Let A be the event that both children are girls, B be the event that that youngest child is a girl and C be the event that at least one of the children is a girl.

Then, $A = \{G_1G_2\}$ *i.e.,* $n(A) = 1$

$B = \{G_1G_2, B_1G_2\}$ *i.e.,* $n(B) = 2$

$C = \{G_1G_2, G_1B_2, B_1G_2\}$ *i.e.,* $n(C) = 3$

$\therefore \; A \cap B = \{G_1G_2\}$ *i.e.* $n(A \cap B) = 1$

and $A \cap C = \{G_1G_2\}$ *i.e.* $n(A \cap C) = 1$

(A) Required probability $= P(A|B)$

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/4}{2/4} = \frac{1}{2}$$

(B) Required probability $= P(A|C)$

$$= \frac{P(A \cap C)}{P(C)}$$

$$= \frac{1/4}{3/4} = \frac{1}{3}$$

40. The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events 'A

coming in time' and 'B coming in time' are independent, find the probability of only one of them coming to the school in time. Give one advantage of coming to school on time. [CBSE 2013]

Ans. Let E be the event that A is coming in time.

$$\therefore \quad P(E) = \frac{3}{7}$$

And F be the event that B is coming in time.

$$\therefore \quad P(F) = \frac{5}{7}$$

Also, E and F are given to be independent events.

$\therefore$ Probability of only one of them coming to school in time

$$= P(E)P(\bar{F}) + P(\bar{E})P(F)$$

$$= \frac{3}{7}\left(1-\frac{5}{7}\right)+\left(1-\frac{3}{7}\right)\times\frac{5}{7}$$

$$= \frac{3}{7}\times\frac{2}{7}+\frac{4}{7}\times\frac{5}{7}$$

$$= \frac{6}{49}+\frac{20}{49}$$

$$= \frac{26}{49}$$

As education is very important for the overall growth and development of a child, so punctuality is the first concept need to be developed for getting the education.

41. **A fair die is rolled. Consider the following events:**

$A = \{2, 4, 6\}$, $B = \{4, 5\}$ and $C = \{3, 4, 5, 6\}$

Find (A) $P(A \cup B|C)$ (B) $P(A \cap B|C)$

42. **A bag contains 5 white, 7 red and 8 black balls. If four balls are drawn one by one without replacement, then find the probability of getting all white balls.**

43. **In a college, 70% students pass in physics, 75% students pass in mathematics and 10% students fail in both. One student is chosen at random. What is the probability that**

(A) he passes in physics and mathematics,

(B) he passes in mathematics, given that he passes in physics,

(C) he passes in physics, given that he passes in mathematics?

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

44. **In a hockey match, both teams A and B scored same number of goals upto the end of the game, so as to decide the winner, the refree asked both the captains to throw a die alternately and decided that the team, whose captain gets a six first, will be declared the winner. If the captain of team A was asked to start, find their respective probabilities of winning the match and state whether the decision of the refree was fair or not. [CBSE 2013]**

Ans. Respective probabilities of getting a six by the captains of both the teams A and B is

$$P(A) = P(B) = \frac{1}{6}$$

$$\therefore \quad P(\bar{A}) = P(\bar{B}) = 1-\frac{1}{6} = \frac{5}{6}$$

Since, A start the game, then if A wins then following events occurs in mutually exclusive ways:

$(A), (\bar{A}\bar{B}A), (\bar{A}\bar{B}\bar{A}\bar{B}A), \ldots$

$\therefore$ Probability that A wins

$$= P(A) + P(\bar{A}\bar{B}A) + P(\bar{A}\bar{B}\bar{A}\bar{B}A) + \ldots$$

$$= P(A)+P(\bar{A})P(\bar{B})P(A)+P(\bar{A})P(\bar{B})P(\bar{A})P(\bar{B})P(A)+\ldots$$

$$= \frac{1}{6}+\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^4\times\frac{1}{6}+\ldots$$

$$= \frac{1}{6}\left[1+\left(\frac{5}{6}\right)^2+\left(\frac{5}{6}\right)^4+\ldots\right]$$

Here, $\left[1+\left(\frac{5}{6}\right)^2+\left(\frac{5}{6}\right)^4+\ldots\right]$ is an infinite G.P., with first term 1 and common difference $\left(\frac{5}{6}\right)^2$

$$\therefore \quad \text{Its sum} = \left[\frac{1}{1-\frac{25}{36}}\right]$$

$$= \left(\frac{36}{36-25}\right) = \frac{36}{11}$$

$\therefore$ Probability that A wins

$$= \frac{1}{6}\times\frac{36}{11} = \frac{6}{11}$$

Hence, the probability of team A wining the match is $\frac{6}{11}$.

Since, the total probability of winning of A and B is 1.

$\therefore$ Probability of winning of B,

$$P(B) = 1 - P(A)$$
$$= 1 - \frac{6}{11}$$
$$= \frac{5}{11}$$

The decision is not fair, as the one who starts the game, has more chances of winning.

45. Consider the experiment of tossing a coin. If the coin shows head, toss it again, but if it shows tail, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4' given that 'there is atleast one tail'. [CBSE 2014]

Ans. The sample space for given experiment is

S = {(H, H), (H, T), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)}, and

Let A be the event that the die shows a number greater than 4 and B be the event that there is atleast one tail.

$\therefore$ A = {(T, 5), (T, 6)}

B = {(T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6), (H, T)}

So, A $\cap$ B = {(T, 5); (T, 6)}

Now, P(B) = P{(T, 1)} + P{(T, 2)} + P{(T, 3)} + P{(T, 4)} + P{(T, 5)} + P{(T, 6)} + P{(H, T)}

$$= \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{4}$$
$$= \frac{3}{4}$$

$$P(A \cap B) = P\{(T, 5)\} + P\{(T, 6)\} = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$$

$$\therefore \text{Required probability} = P(A|B) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{\frac{1}{6}}{\frac{3}{4}} = \frac{2}{9}$$

46. For a loaded die, the probabilities of outcomes are given as under:

P(1) = P(2) = 0.2, P(3) = P(5) = P(6) = 0.1 and P(4) = 0.3.

The die is thrown two times. Let A and B be the events 'same number each time', and a 'total score is 10 or more', respectively. Determine whether or not A and B are independent. [NCERT Exemplar]

47. If A and B are two independent events such that $P(\bar{A} \cap B) = \frac{2}{15}$ and $P(A \cap \bar{B}) = \frac{1}{6}$, then find P(A) and P(B). [CBSE 2015]

Ans. Here, A and B are independent events.

Also, $P(\bar{A} \cap B) = \frac{2}{15}$

$\Rightarrow$ $P(\bar{A}).P(B) = \frac{2}{15}$ ...(i)

[Since, A and B are independent, therefore, $\bar{A}$ and B are also independent]

And, $P(A \cap \bar{B}) = \frac{1}{6}$

$\Rightarrow$ $P(A).P(\bar{B}) = \frac{1}{6}$ ...(ii)

Let $P(A) = p$

$\Rightarrow$ $P(\bar{A}) = 1 - P(A) = 1 - p$

and $P(B) = q$

$\Rightarrow$ $P(\bar{B}) = 1 - P(B) = 1 - q$

Now from (i) and (ii), we get

$$(1 - p)q = \frac{2}{15} \quad \text{...(iii)}$$

and

$$p(1 - q) = \frac{1}{6} \quad \text{...(iv)}$$

Subtracting (iii) from (iv), we get

$$p - q = \frac{1}{6} - \frac{2}{15} = \frac{1}{30}$$

$$\Rightarrow \quad p = q + \frac{1}{30}$$

Putting this value of p in (iii), we get

$$\left(1 - q - \frac{1}{30}\right)q = \frac{2}{15}$$

$$\Rightarrow \quad \left(\frac{29}{30} - q\right)q = \frac{2}{15}$$

$$\Rightarrow \quad \frac{29}{30}q - q^2 = \frac{2}{15}$$

$$\Rightarrow \quad 30q^2 - 29q + 4 = 0$$

$$\Rightarrow \quad 30q^2 - 24q - 5q + 4 = 0$$

$$\Rightarrow 6q(5q - 4) - 1(5q - 4) = 0$$

$$\Rightarrow \quad (6q - 1)(5q - 4) = 0$$

$$\Rightarrow \quad q = \frac{1}{6}, q = \frac{4}{5}$$

For $q = \frac{4}{5}$, using (iv), we have

$$p\left(1 - \frac{4}{5}\right) = \frac{1}{6}$$

$\Rightarrow \quad p\left(\frac{1}{5}\right) = \frac{1}{6}$

$\Rightarrow \quad p = \frac{5}{6}$

For $q = \frac{1}{6}$, using (iv), we have

$$p\left(1 - \frac{1}{6}\right) = \frac{1}{6}$$

$\Rightarrow \quad p\left(\frac{5}{6}\right) = \frac{1}{6}$

$\Rightarrow \quad p = \frac{1}{5}$

$\therefore \quad P(A) = \frac{5}{6}, P(B) = \frac{4}{5}$

or $\quad P(A) = \frac{1}{5}, P(B) = \frac{1}{6}$

BAYES' THEOREM, RANDOM VARIABLE AND ITS PROBABILITY DISTRIBUTION

2

TOPIC 1

PARTITION OF A SAMPLE SPACE

A set of events $E_1, E_2, \ldots, E_n$ is said to represent a partition of the sample space S if

(1) $E_1 \cup E_2 \cup \ldots E_n = S$

(2) $E_i \cap E_j = \phi, i \neq j, i, j = 1, 2, 3, \ldots, n$

(3) $P(E_i) > 0$, for all $i = 1, 2, 3, \ldots, n$

In other words, the events $E_1, E_2, \ldots, E_n$ represent a partition of the sample space S if they are pair wise disjoint, exhaustive and have non-zero probabilities.

Illustration: Any non-empty event E and its complement E′ form a partition of the sample space S since they satisfy $E \cup E' = S$ and $E \cap E' = \phi$.

Caution

The partition of a sample space is not unique. There can be several partitions of the same sample space.

TOPIC 2

THEOREM OF TOTAL PROBABILITY

Let $E_1, E_2, \ldots, E_n$ be n non-empty events which constitutes a partition of sample space S. Let A be any event associated with S. Then,

$$P(A) = P(E_1).P(A \mid E_1) + P(E_2).P(A \mid E_2) + \ldots + P(E_n).P(A \mid E_n)$$

Example 2.1: Two-thirds of the students of a class are boys and the rest are girls. It is known that the probability of a girl getting A grade in Board's examination is 0.4 and of a boy getting A grade is 0.35. Find the probability that a student chosen at random will get A grade in the examination.

Ans. Let events E_1, E_2, A be defined as

E_1 : A boy is chosen;

E_2 : A girl is chosen;

A : The student gets A grade.

Clearly, E_1 and E_2 constitute a partition of sample space S.

$$\therefore \; P(E_1) = \frac{2}{3}, P(E_2) = \frac{1}{3}$$

Also, $P(A \mid E_1) = 0.35$, $P(A \mid E_2) = 0.4$

We need to determine P(A).

Using theorem of total probability, we have

$$P(A) = P(E_1).P(A \mid E_1) + P(E_2).P(A \mid E_2)$$

$$= \frac{2}{3} \times 0.35 + \frac{1}{3} \times 0.4$$

$$= \frac{7}{30} + \frac{4}{30}, \text{ or } \frac{11}{30}$$

Example 2.2: An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn, are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red? [NCERT]

Ans. Let events E_1, E_2, A be defined as

E_1 : The red ball is drawn in the first draw;

E_2 : The black ball is drawn in the first draw;

A : The ball drawn in the second draw is red.

Clearly, E_1 and E_2 constitute a partition of sample space S.

$$\text{Also, } P(E_1) = \frac{5}{10} = \frac{1}{2}, \; P(E_2) = \frac{5}{10} = \frac{1}{2}$$

When 2 additional balls of red color are put in the urn, there are 7 red and 5 black balls in the urn.

$$\therefore \quad P(A \mid E_1) = \frac{7}{12}$$

When 2 additional balls of black colour are put in the urn, there are 5 red and 7 black balls in the urn.

$$\therefore \quad P(A \mid E_2) = \frac{5}{12}$$

We need to determine P(A).

Using theorem of total probability, we have

$$P(A) = P(E_1).P(A|E_1) + P(E_2).P(A|E_2)$$

$$= \frac{1}{2} \times \frac{7}{12} + \frac{1}{2} \times \frac{5}{12}$$

$$= \frac{12}{24}, \text{ or } \frac{1}{2}$$

Example 2.3: A bag contains $(2n + 1)$ coins. It is known that $(n - 1)$ of these coins have a head on both sides, whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, determine the value of n. [NCERT]

Ans. Let events E_1, E_2, A be defined as

E_1 : Picking a coin with head on both sides;

E_2 : Picking a fair coin;

A : Getting head on tossing the coin.

Clearly, E_1 and E_2 constitute a partition of sample space S.

Also, $P(E_1) = \frac{n-1}{2n+1}$, $P(E_2) = \frac{n+2}{2n+1}$,

$P(A \mid E_1) = 1$, $P(A \mid E_2) = \frac{1}{2}$

We need to determine P(A).

Using theorem of total probability, we have

$$P(A) = P(E_1).P(A \mid E_1) + P(E_2).P(A \mid E_2)$$

$$= \frac{n-1}{2n+1} \times 1 + \frac{n+2}{2n+1} \times \frac{1}{2}$$

$$= \frac{3n}{2(2n+1)}$$

Equating $\frac{3n}{2(2n+1)}$ to $\frac{31}{42}$, we get $n = 31$

TOPIC 3

BAYES' THEOREM

Let $E_1, E_2, \ldots, E_n$ be n non-empty events which constitutes a partition of sample space S. Let A be any event of non-zero probability. Then,

$$P(E_i \mid A) = \frac{P(E_i).P(A|E_i)}{\sum_i^n P(E_i).P(A|E_i)} \text{ for any } i = 1, 2, 3, \ldots, n$$

Example 2.4: A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag. [NCERT]

Ans. Let events E_1, E_2, A be defined as

E_1 : Ball is drawn from the first bag;

E_2 : Ball is drawn from the second bag;

A : Ball drawn is red.

$\therefore$ $P(E_1) = P(E_2) = \frac{1}{2}$

Also, $P(A \mid E_1) = \frac{4}{8} = \frac{1}{2}$, $P(A \mid E_2) = \frac{2}{8} = \frac{1}{4}$

We need to determine $P(E_1 \mid A)$.

Using Baye's Theorem, we have

$$P(E_1 \mid A) = \frac{P(E_1).P(A \mid E_1)}{P(E_1).P(A \mid E_1) + P(E_2).P(A \mid E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{4}} = \frac{2}{3}$$

Example 2.5: An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of their accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver? [NCERT]

Ans. Let events E_1, E_2, E_3, A be defined as

E_1 : Company insured scooter driver;

E_2 : Company insured car driver;

E_3 : Company insured truck driver;

A : Person meets with an accident.

$\therefore$ $P(E_1) = \frac{2000}{12000} = \frac{1}{6}$, $P(E_2) = \frac{4000}{12000} = \frac{1}{3}$,

$P(E_3) = \frac{6000}{12000} = \frac{1}{2}$.

Also, $P(A \mid E_1) = 0.01$, $P(A \mid E_2) = 0.03$, $P(A \mid E_3) = 0.15$

We need to determine $P(E_1 \mid A)$.

Using Bayes' Theorem, we have

$P(E_1 \mid A)$

$$= \frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1)+P(E_2).P(A|E_2)+P(E_3).P(A|E_3)}$$

$$= \frac{\frac{1}{6} \times 0.01}{\frac{1}{6} \times 0.01 + \frac{1}{3} \times 0.03 + \frac{1}{2} \times 0.15} = \frac{1}{52}$$

Example 2.6: Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?

Ans. Let events E_1, E_2, A be defined as

E_1 : Girl gets 5 or 6 on throwing a die;

E_2 : Girl gets 1, 2, 3 or 4 on throwing a die;

A : Exactly one head appears on the coin.

$\therefore P(E_1) = \frac{2}{6} = \frac{1}{3}, P(E_2) = \frac{4}{6} = \frac{2}{3}$

Now, probability of getting exactly one head on tossing a coin three times, *i.e.*,

$P(A \mid E_1) = P(HTT) + P(THT) + P(TTH)$

$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{3}{8}$$

Probability of getting exactly one head on tossing a coin once, *i.e.*,

$P(A \mid E_2) = \frac{1}{2}$

We need to determine $P(E_2|A)$

Using Bayes' Theorem, we have

$$P(E_2 \mid A) = \frac{P(E_2).P(A \mid E_2)}{P(E_1).P(A \mid E_1) + P(E_2).P(A \mid E_2)}$$

$$= \frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}} = \frac{8}{11}$$

TOPIC 4

RANDOM VARIABLE

A real valued function X defined on a sample space of a random experiment is called a random variable.

Illustration: In the toss of three fair coins, the number of heads obtained is a random variable. It can take the values 0, 1, 2 or 3.

Discrete/Continuous Random Variable

If a random variable takes finite number of values or it takes values of an infinite sequence of real numbers, then it is called discrete random variable.

If the range of a random variable is an interval, then it is called continuous random variable.

Illustration: Number of print mistakes on a randomly selected page of a book is a discrete random variable; whereas person's blood pressure is a continuous random variable.

⚠ Caution

➥ *From now onwards, we shall use the term "random variable" for "discrete random variable".*

TOPIC 5

PROBABILITY DISTRIBUTION OF A DICRETE RANDOM VARIABLE

Let X be a discrete random variable taking values x_1, x_2, x_3 ,......, x_n.

Let p_1, p_2, p_3 ,......, p_n denote the corresponding probabilities, *i.e.*, $p_i = P(X = x_i)$, for every i = 1, 2, 3, ..., n satisfying $p_i > 0$ and $\sum_{i=1}^{n} p_i = 1$.

Then the tabular representation

X	x_1	x_2	x_3	......	x_n
$P(X = x_i)$	p_1	p_2	p_3	......	p_n

is known as probability distribution of discrete random variable X.

Example 2.7: Two cards are drawn at random from a well-shuffled pack of 52 cards. Find the probability distribution of number of spade cards.

Ans. The number of ways in which two cards can be drawn from a well-shuffled pack of 52 cards $= {}^{52}C_2 = 1326$

Let X be the number of spade cards. Then, X can take values 0, 1, 2.

Now,

$P(X = 0)$ = P(drawing no spade card)

$$= \frac{{}^{39}C_2}{{}^{52}C_2} = \frac{741}{1326} = \frac{19}{34}$$

$P(X = 1)$ = P (drawing only 1 spade card)

$$= \frac{{}^{13}C_1 \times {}^{39}C_1}{{}^{52}C_2} = \frac{13}{34}$$

$P(X = 2)$ = P(drawing 2 spade cards)

$$= \frac{{}^{13}C_2}{{}^{52}C_2} = \frac{2}{34} = \frac{1}{17}$$

So, the probability distribution of X is:

X	0	1	2
$P(X = x_i)$	$\frac{19}{34}$	$\frac{13}{34}$	$\frac{1}{17}$

Example 2.8: State which of the following are not the probability distributions of a random variable. Give reasons for your answer.

(A)

X	0	1	2
P(X)	0.4	0.4	0.2

(B)

X	0	1	2	3	4
P(X)	0.4	0.4	0.2	− 0.1	0.3

(C)

Y	−1	0	1
P(Y)	0.6	0.1	0.2

(D)

Z	3	2	1	0	−1
P(Z)	0.3	0.2	0.4	0.1	0.05

[NCERT]

Ans. For the probability distribution we need to check the following two things:

(1) All probabilities must be positive; and

(2) The sum of the probabilities should be 1.

(A) Here, all probabilities are positive and their sum (0.4 + 0.4 + 0.2) is equal to 1.

So, it is a probability distribution.

(B) Here, all probabilities are not positive [P(X = 3) = − 0.1]

So, it is not a probability distribution.

(C) Here, the sum of all the probabilities is not equal to 1. [0.6 + 0.1 + 0.2 = 0.9 < 1]

So, it is not a probability distribution.

(D) Here, the sum of all the probabilities is not equal to 1.

[0.3 + 0.2 + 0.4 + 0.1 + 0.05 = 1.05 > 1]

So, it is not a probability distribution.

Example 2.9: Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as

(A) number greater than 4;

(B) six appears on at least one die. [NCERT]

Ans. (A) Let X be the random variable which denotes the number of numbers greater than 4 in two tosses of a die.

So, X may take values 0, 1 or 2.

Now, P(X = 0) = Probability of getting numbers less than or equal to 4 on both the tosses

$$= \frac{4}{6} \times \frac{4}{6} = \frac{4}{9}$$

P(X = 1) = Probability of getting numbers less than or equal to 4 on one toss and greater than 4 on another toss.

$$= \frac{4}{6} \times \frac{2}{6} + \frac{2}{6} \times \frac{4}{6} = \frac{4}{9}$$

P(X = 2) = Probability of getting numbers greater than 4 on both the tosses.

$$= \frac{2}{6} \times \frac{2}{6} = \frac{1}{9}$$

Thus, required probability distribution is:

X	0	1	2
P(X = x)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(B) Let X be the random variable which denotes the number of sixes in two tosses of a die.

So, X may take values 0, 1 or 2.

Now,

P(X = 0) = Probability of no six on two tosses

$$= \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$$

P(X = 1) = Probability of six on one toss and on six on another toss

$$= \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} = \frac{10}{36}$$

P(X = 2)

Thus, required probability distribution is:

X	0	1	2
P(X = x)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

Example 2.10: The random variable X has a probability distribution P(X) of the following form, where k is some number:

$$P(X) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

(A) Determine the value of k.

(B) Find P(X < 2), P(X ≤ 2), P(X ≥ 2). [NCERT]

Ans. (A) Here, $k + 2k + 3k = 1$

$$\Rightarrow k = \frac{1}{6}$$

(B) P(X < 2) = P(0) + P(1) =

$$k + 2k = 3k = 3 \times \frac{1}{6} = \frac{1}{2};$$

P(X ≤ 2) = P(0) + P(1) + P(2)

$$= k + 2k + 3k = 6k = 6 \times \frac{1}{6} = 1;$$

P(X ≥ 2) = P(2) + P(otherwise)

$$= 3k = 3 \times \frac{1}{6} = \frac{1}{2}$$

TOPIC 6

MEAN OF A RANDOM VARIABLE

Let X be a discrete random variable taking values x_1, x_2, x_3,......, x_n.

Let p_1, p_2, p_3,......, p_n denote the corresponding probabilities, *i.e.*, $p_i = P(X = x_i)$, for every i = 1, 2, 3, ..., n satisfying $p_i > 0$ and $\sum_{i=1}^{n} p_i = 1$.

Then, the mean of X, denoted by μ, is the number $\sum_{i=1}^{n} x_i p_i$

⚠ Caution

➥ *The mean of a random variable X is also called the expectation of X, denoted by E(X).*

In other words, the mean or expectation of a random variable X is the sum of the products of all possible values of X and their corresponding probabilities.

Example 2.11: Find the mean number of heads in three tosses of a coin. [NCERT]

Ans. Let X be the random variable of "number of heads".

Then, X = 0, 1, 2, 3.

$$P(X = 0) = P(\text{no head}) = P(\text{all tails}) = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$$

$$P(X = 1) = P(\text{1 head and 2 tails}) = P(HTT) + P(TTH) + P(THT) = 3 \times \left\{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right\} = \frac{3}{8}$$

$$P(X = 2) = P(\text{2 heads and 1 tail}) = P(HHT) + P(HTH) + P(THH) = 3\times\left\{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right\} = \frac{3}{8}$$

$$P(X = 3) = P(\text{all heads}) = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$$

Thus, the mean of X

$$= \sum_{i=1}^{n} x_i p_i = \left(0\times\frac{1}{8}\right) + \left(1\times\frac{3}{8}\right) + \left(2\times\frac{3}{8}\right) + \left(3\times\frac{1}{8}\right)$$

$$= \frac{12}{8} = 1.5$$

⚠ Caution

➥ *No need to make the probability distribution of X, unless specified.*

OBJECTIVE Type Questions

[**1** mark]

Multiple Choice Questions

1. Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was a head, is:

(a) $\frac{4}{5}$ (b) $\frac{1}{2}$

(c) $\frac{1}{5}$ (d) $\frac{2}{5}$

Ans. *(a)* $\frac{4}{5}$

Explanation: Let events E_1, E_2, A respectively be defined as

E_1 : A speaks truth; E_2 : A does not speak truth;

A : Head appears on tossing a coin.

$$\therefore\ P(E_1) = \frac{4}{5},\quad P(E_2) = 1 - \frac{4}{5} = \frac{1}{5},$$

$$P(A \mid E_1) = \frac{1}{2},\quad P(A \mid E_2) = \frac{1}{2}$$

We need to determine $P(E_1 \mid A)$.

Using Bayes' theorem, we have

$$P(E_1 \mid A) = \frac{P(E_1).P(A \mid E_1)}{P(E_1).P(A \mid E_1) + P(E_2).P(A \mid E_2)}$$

$$= \frac{\frac{4}{5}\times\frac{1}{2}}{\frac{4}{5}\times\frac{1}{2} + \frac{1}{5}\times\frac{1}{2}} = \frac{4}{5}$$

2. Bag I contains 2 white and 3 red balls and Bag II contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Then, the probability that it was drawn from bag B, is:

(a) $\frac{25}{52}$ (b) $\frac{25}{51}$

(c) $\frac{15}{22}$ (d) $\frac{5}{52}$

3. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face, is:

(a) 1 (b) 2

(c) 5 (d) $\frac{8}{3}$

Ans. *(b) 2*

Explanation: Let X be the random variable denoting the number obtained on throwing a die.

Then, X takes the values 1, 2 and 5.

The corresponding probabilities are:

$P(X = 1) = \frac{3}{6}$, or $\frac{1}{2}$; $P(X = 2) = \frac{2}{6}$ or $\frac{1}{3}$;

$P(X = 5) = \frac{1}{6}$

So, the mean of X is

$$\mu = 1\times\frac{1}{2}+2\times\frac{1}{3}+5\times\frac{1}{6} = \frac{12}{6} = 2$$

4. The probability distribution of a discrete random variable X is given below:

X	2	3	4	5
P(X)	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

The value of *k* is:

(a) 8 **(b) 16**

(c) 32 **(d) 48**

[NCERT Exemplar]

5. Urn I contains 6 red and 4 black balls and urn II contains 4 red and 6 black balls. One ball is drawn at random from urn I and placed in urn II. If one ball is drawn at random from urn II, then the probability that it is a red ball, is:

(a) $\frac{3}{5}$ (b) $\frac{4}{11}$

(c) $\frac{2}{5}$ (d) $\frac{23}{55}$

Ans. *(d)* $\frac{23}{55}$

Explanation: Let the events B_1, B_2 and E respectively be defined as:

B_1 : Red ball is transferred from Urn I to Urn II;

B_2 : Black ball is transferred from Urn I red to Urn II;

E : Red ball is drawn from Urn II.

Then, $P(B_1) = \frac{6}{10}$, $P(B_2) = \frac{4}{10}$, $P(E|B_1) = \frac{5}{11}$,

$P(E|B_2) = \frac{4}{11}$

We need to determine P(E).

Using Total Probability Theorem, we have

$$P(E) = P(B_1).P(E|B_1) + P(B_2).P(E|B_2)$$

$$= \frac{6}{10}\times\frac{5}{11}+\frac{4}{10}\times\frac{4}{11}$$

$$= \frac{46}{110} \text{ or } \frac{23}{55}$$

6. The chances that doctor A will diagnose a disease X correctly is 60%. The chances that a patient will die by his treatment after correct diagnose is 40% and the chance of death by wrong diagnosis is 70%. A patient of doctor A, who had disease X, died. What is the chance that his disease was diagnosed correctly?

(a) $\frac{7}{13}$ (b) $\frac{6}{13}$

(c) $\frac{3}{10}$ (d) $\frac{7}{10}$

7. A box B_1 contains 1 white, 3 red and 2 black balls. Another box B_2 contains 2 white, 3 red and 4 black balls. A third box B_3 contains 3 white, 4 red and 5 black balls. If two balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other is red, then the probability of these two balls being drawn from box B_2, is:

(a) $\frac{116}{181}$ (b) $\frac{126}{181}$

(c) $\frac{65}{181}$ (d) $\frac{55}{181}$

Ans. *(d)* $\frac{55}{181}$

Explanation: Let the events E_1, E_2, E_3 and A respectively be defined as:

E_1 : Both balls are from box B_1;

E_2 : Both balls are from box B_2;

E_3 : Both balls are from box B_3;

A : One ball is white and the other is red.

$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

Also, $P(A|E_1) = \frac{1\times3}{^6C_2} = \frac{1}{5}$,

$P(A|E_2) = \frac{2\times3}{^9C_2} = \frac{1}{6}$;

$P(A|E_3) = \frac{3\times4}{^{12}C_2} = \frac{2}{11}$

We need to determine $P(E_2|A)$.

Using Bayes' theorem, we have

$P(E_2|A)$

$$= \frac{P(E_2).P(A|E_2)}{P(E_1).P(A|E_1) + P(E_2).P(A|E_2) + P(E_3).P(A|E_3)}$$

$$= \frac{\frac{1}{3}\times\frac{1}{6}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{6}+\frac{1}{3}\times\frac{2}{11}}$$

$$= \frac{\frac{1}{6}}{\frac{1}{5}+\frac{1}{6}+\frac{2}{11}}$$

$$= \frac{5\times11}{6\times11+5\times11+5\times6\times2}$$

$$= \frac{55}{181}$$

8. **Suppose that two cards are drawn at random from a deck of 52 playing cards. Let X be the number of aces obtained. Then the value of E(X) is:**

(a) $\frac{37}{22}$ **(b)** $\frac{5}{13}$

(c) $\frac{1}{13}$ **(d)** $\frac{2}{13}$

Ans. *(d)* $\frac{2}{13}$

Explanation: Let X be the random variable denoting the number of aces obtained.

Then, X can takes the values 0, 1, 2.

The corresponding probabilities are:

$P(X = 0) = \frac{^{48}C_2}{^{52}C_2}$;

$P(X = 1) = \frac{^4C_1 \times {^{48}C_1}}{^{52}C_2}$;

$P(X = 2) = \frac{^4C_2}{^{52}C_2}$

So, $E(X) = \sum_{i=1}^{n} X_i P(X_i)$

$$= 0\times\frac{^{48}C_2}{^{52}C_2} + 1\times\frac{^4C_1\times{^{48}C_1}}{^{52}C_2} + 2\times\frac{^4C_2}{^{52}C_2}$$

$$= 0 + \frac{192}{1326} + \frac{12}{1326}$$

$$= \frac{204}{1326}, \text{ or } \frac{2}{13}$$

9. **Ⓐ Two dice are thrown simultaneously. If X denotes the number of sixes, then the expected value of X is:**

(a) $E(X) = \frac{1}{3}$ **(b)** $E(X) = \frac{2}{3}$

(c) $E(X) = \frac{1}{6}$ **(d)** $E(X) = \frac{5}{6}$

[NCERT Exemplar]

10. **A random variable X has the following distribution:**

X = x	0	1	2	3
P(X = x)	$\frac{1}{3}$	$\frac{1}{2}$	0	$\frac{1}{6}$

The mean of X is:

(a) $\frac{1}{4}$ **(b)** 4

(c) 1 **(d)** 2

Ans. *(c)* *1*

Explanation: Here,

$E(X) = \Sigma XP(X)$

$$= 0\times\frac{1}{3}+1\times\frac{1}{2}+2\times0+3\times\frac{1}{6}$$

$$= \frac{1}{2}+\frac{1}{2} = 1$$

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

11. *For an audition of a reality singing competition, interested candidates were asked to apply* *under one of the two musical genres - folk or classical and under one of the two age categories - below 18 or 18 and above.*

The following information is known about the 2000 applications received:

- *960 of the total applications were for the folk genre.*
- *192 of the folk applications were for the below 18 category.*
- *104 of the classical applications were for the 18 and above category.*

(A) What is the probability that an application selected at random is for the 18 and above category provided it is under the classical genre? Show your work.

(B) An application selected at random is found to be under the below 18 category.

Find the probability that it is under the folk genre. Show your work. [Delhi Gov. 2022]

Ans. Let the events A, B, C and D be defined as follows:

A = Application is for folk genre.

B = Application is for classical genre.

C = Application is for age category below 18.

D = Application is for age category 18 or above.

$$\therefore \quad P(A) = \frac{960}{2000} = \frac{12}{25}$$

$$P(B) = 1 - P(A)$$

$$= 1 - \frac{12}{25} = \frac{13}{25}$$

Also, $n(A \cap C) = 192$

$\Rightarrow$ $n(A \cap B) = 960 - 192 = 768$

And $n(B \cap D) = 104$

$\Rightarrow$ $n(B \cap C) = 1040 - 104 = 936$

$$\left[\because \text{Number of applications of classical genre} = 2000 - 960 = 1040\right]$$

(A) Required probability

$$= P(D|B)$$

$$= \frac{P(D \cap B)}{P(B)}$$

$$= \frac{\frac{104}{2000}}{\frac{13}{25}} = \frac{\frac{13}{250}}{\frac{13}{25}} = \frac{1}{10}$$

(B) Required probability

$$= P(A|C)$$

$$= \frac{P(A)P(C|A)}{P(A)P(C|A) + P(B)P(C|B)}$$

[Using Bayes' theorem]

$$= \frac{P(A) \times \frac{n(C \cap A)}{n(A)}}{P(A) \times \frac{n(C \cap A)}{n(A)} + P(B) \times \frac{n(C \cap B)}{n(B)}}$$

[Using conditional probability]

$$= \frac{\frac{12}{25} \times \frac{192}{960}}{\frac{12}{25} \times \frac{192}{960} + \frac{13}{25} \times \frac{936}{1040}}$$

$$= \frac{\frac{12}{125}}{\frac{12}{125} + \frac{117}{250}}$$

$$= \frac{\frac{12}{125}}{\frac{24+117}{250}} = \frac{12}{125} \times \frac{250}{141}$$

$$= \frac{8}{47}$$

12. *A factory has two machines P and Q for manufacturing an item. Past record shows that machine P produces 60% of the items of the output and machine Q produces 40% of the items of the output. Further, 2% of the items produced by machine P and 1% produced by machine Q are defective. All the items produced in a day are put in one stockpile.*

One item is picked at random from the stockpile.

(A) What is the probability that this item is defective?

(B) If the item is defective one, then what is the probability that this item is produced by machine P?

Ans. (A) Let the events E_1, E_2 and A respectively be defined as:

E_1 : Item is produced by machine P;

E_2 : Item is produced by machine Q;

A : Picked up item is defective.

Then, $P(E_1) = 0.6$; $P(E_2) = 0.4$; $P(A|E_1) = 0.02$; $P(A|E_2) = 0.01$

By theorem of Total Probability, we have

$$P(A) = P(E_1).P(A|E_1) + P(E_2).P(A|E_2)$$

$$= 0.6 \times 0.02 + 0.4 \times 0.01$$

$$= 0.012 + 0.004$$

$$= 0.016$$

(B) Required probability is $P(E_1|A)$.

$\therefore$ By Bayes' Theorem, we have

$$P(E_1|A) = \frac{P(E_1).P(A \mid E_1)}{P(E_1).P(A \mid E_1) + P(E_2).P(A \mid E_2)}$$

$$= \frac{0.6 \times 0.02}{0.6 \times 0.02 + 0.4 + 0.01}$$

$$= \frac{0.012}{0.016} = \frac{3}{4} = 0.75$$

13. *A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots.*

(A) Let the target is hit by A, B and C one by one. The probability that A, B and C all will hit, is:

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$

(c) $\frac{2}{5}$ (d) $\frac{1}{5}$

(B) Referring to (A), what is the probability that B, C will hit and A will lose?

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$

(c) $\frac{7}{10}$ (d) $\frac{4}{10}$

(C) With references to the events mentioned in (A), what is the probability that any two of A, B and C will hit?

(a) $\frac{1}{30}$ (b) $\frac{11}{30}$

(c) $\frac{17}{30}$ (d) $\frac{13}{30}$

(D) What is the probability that none of them will hit the target?

(a) $\frac{1}{30}$ (b) $\frac{1}{60}$

(c) $\frac{1}{15}$ (d) $\frac{2}{15}$

(E) What is the probability that at least one of A, B or C will hit the target?

(a) $\frac{59}{60}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) $\frac{1}{60}$

[CBSE Question Bank 2021]

Ans. (A) *(c)* $\frac{2}{5}$

Explanation: Probability of A hitting target, $P(A) = \frac{4}{5}$

Probability of B hitting target, $P(B) = \frac{3}{4}$

Probability of C hitting target, $P(C) = \frac{2}{3}$

Probability of A, B, C hitting the target

$$= P(A) \times P(B) \times P(C)$$

$$= \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3}$$

$$= \frac{2}{5}$$

(B) *(a)* $\frac{1}{10}$

Explanation: Probability of hitting the target by B and C, missing by A

$$= P(\bar{A}) \times P(B) \times P(C)$$

$$= \left(1 - \frac{4}{5}\right) \times \frac{3}{4} \times \frac{2}{3}$$

$$= \frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$$

(E) *(a)* $\frac{59}{60}$

Explanation: Required probability

$= 1 -$ P(none of them hit the target)

$= 1 - P(\bar{A})P(\bar{B})P(\bar{C})$

$= 1 - \frac{1}{60}$ [Using part (D)]

$= \frac{59}{60}$

14. *In a residential school of Haryana, a section of Class XI class has chosen various subjects. School reported that 40% students chose mathematics, 25% students chose Biology, 60% students chose Physics and 55% studens chose Chemistry. They further gave* *information that 50% students has chosen both Physics and Chemistry, 30% students has chosen both Physics and Mathematics, whereas 15% students has chosen both Mathematics and Biology.*

(A) What is the probability that a student has chosen Mathematics, if it is known that he has chosen Biology ?

(B) What is the probability that a student has chosen Physics, if it is known that he has chosen Chemistry?

Ans. (A) Let A and B be the events defined as:

A : Student chose Mathematics;

B : Student chose Biology;

$A \cap B$: Student chose both mathematics and biology.

Then, $P(A) = \frac{40}{100}$; $P(B) = \frac{25}{100}$;

$P(A \cap B) = \frac{15}{100}$.

Now,

Required probability = P(A|B)

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{15/100}{25/100} = \frac{3}{5}$$

15. *The reliability of a COVID PCR test is specified as follows:*

Of people having COVID, 90% of the test detects the disease but 10% goes undetected. Of people free of COVID, 99% of the test is judged COVID negative but 1% are diagnosed as showing COVID positive. From a large population of which only 0.1% have COVID, one person is selected at random, given the COVID PCR test, and the pathologist reports him/her as COVID positive.

(A) What is the probability of the 'person to be tested as COVID positive' given that 'he is actually having COVID'?

(a) 0.001 (b) 0.1

(c) 0.8 (d) 0.9

(B) What is the probability of the 'person to be tested as COVID positive' given that 'he is actually not having COVID'?

(a) 0.01 (b) 0.99

(c) 0.1 (d) 0.001

(C) What is the probability that the 'person is actually not having COVID'?

(a) 0.998 (b) 0.999

(c) 0.001 (d) 0.111

(D) What is the probability that the 'person is actually having COVID' given that 'he is tested as COVID positive'?

(a) 0.83 (b) 0.0803

(c) 0.083 (d) 0.089

(E) What is the probability that the 'person selected will be diagnosed as COVID positive'?

(a) 0.1089 (b) 0.01089

(c) 0.0189 (d) 0.189

[CBSE Question Bank 2021]

Ans. (A) *(d) 0.9*

Explanation: P(E) = Probability that person selected has COVID = $0.1\% = \frac{0.1}{100} = 0.001$

P(G) = Probability that the person is tested as COVID +ve

$\therefore$ P(G|E) = Probability that the test judges COVID +ve, if person actually has COVID

$$= 90\% = \frac{90}{100} = 0.9$$

(B) *(a) 0.01*

Explanation: P(F) = Probability that person selected does not have COVID

$$= 1 - P(E)$$

$$= 1 - 0.001 = 0.999$$

$\therefore$ P(G|F) = Probability that the person judges COVID +ve, if the person does not have COVID

$$= 1\% = \frac{1}{100} = 0.01$$

(C) *(b) 0.999*

Explanation: P(F) = Probability that person selected does not have COVID

$$= 1 - P(E)$$

$$= 1 - 0.001$$

$$= 0.999$$

(E) *(b) 0.01089*

Explanation: P(patient diagnosed as COVID positive)

$= P(G)$

$= P(E)\,P(G|E) + P(F)\,P(G|F)$

$= 0.001 \times 0.9 + 0.999 \times 0.01$

$= 0.0009 + 0.00999$

$= 0.01089$

16. *There is a chamber of lawyers in a session's court. They were discussing a case related to the number of bullets a person had in the pistol. The case deals with the arms act with an offense of possession of unauthorized bullets. There is a bit complexity in the problem due to involvement of probability. Reading about the case, a lawyer said, "the pistol contains 4 gold and 3 silver bullets. Event E is the drawing of 3 bullets in a random manner. Being the lawyers, they do not know much about the probability of conviction. As a student of class XII, help them to understand the following questions:*

(A) Probability of getting no silver bullet is:

(a) $\frac{1}{35}$ (b) $\frac{2}{35}$

(c) $\frac{3}{35}$ (d) $\frac{4}{35}$

(B) Probability of getting 1 silver and 2 gold bullets is:

(a) $\frac{17}{39}$ (b) $\frac{14}{33}$

(c) $\frac{18}{35}$ (d) $\frac{17}{35}$

(C) Probability of getting 2 silver and 1 gold bullets is:

(a) $\frac{5}{17}$ (b) $\frac{12}{35}$

(c) $\frac{7}{34}$ (d) $\frac{2}{27}$

(D) Probability of getting 3 silver bullets is:

(a) $\frac{1}{35}$ (b) $\frac{2}{37}$

(c) $\frac{9}{62}$ (d) $\frac{4}{43}$

(E) Mean of event of drawing silver bullets is:

(a) $\frac{9}{7}$ (b) $\frac{7}{9}$

(c) $\frac{8}{15}$ (d) $\frac{9}{15}$

Ans. (A) *(d)* $\frac{4}{35}$

Explanation: P(getting no silver bullet)

= P(getting all gold bullets)

$$= \frac{^4C_3}{^7C_3} = \frac{4}{35}$$

(B) *(c)* $\frac{18}{35}$

Explanation: P(getting 1 silver and 2 gold bullets)

$$= \frac{^4C_2 \times {^3C_1}}{^7C_3} = \frac{18}{35}$$

(E) *(a)* $\frac{9}{7}$

Explanation: Let X be a random variable "denoting the number of silver bullets drawn". Then, X takes values 0, 1, 2, 3.

Using parts (A) to (D), we have

$P(X = 0) = \frac{4}{35}$; $P(X = 1) = \frac{18}{35}$; $P(X = 2) = \frac{12}{35}$; $P(X = 3) = \frac{1}{35}$

So, mean of $X = 0\times\frac{4}{35} + 1\times\frac{18}{35} + 2\times\frac{12}{35} + 3\times\frac{1}{35}$

$$= (0 + 18 + 24 + 3) \times \frac{1}{35}$$

$$= \frac{45}{35} = \frac{9}{7}$$

17. *In answering a question on a multiple choice test for class XII, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. Let E_1, E_2, E be the events that the students knows the answer, guesses the answer and answers correctly, respectively.*

(A) What is the value of $P(E_1)$?

(a) $\frac{2}{5}$ (b) $\frac{1}{3}$

(c) 1 (d) $\frac{3}{5}$

(B) Value of $P(E|E_1)$ is:

(a) $\frac{1}{3}$ (b) 1

(c) $\frac{2}{3}$ (d) $\frac{4}{15}$

(C) $\sum_{k=1}^{k=2} P(E|E_k)\,P(E_k)$ equals:

(a) $\frac{11}{15}$ (b) $\frac{4}{15}$

(c) $\frac{1}{5}$ (d) 1

(D) Value of $\sum_{k=1}^{k=2} P(E_k)$

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$

(c) 1 (d) $\frac{3}{5}$

(E) What is the probability that the student knows the answer given that he answered it correctly?

(a) $\frac{2}{11}$ (b) $\frac{5}{3}$

(c) $\frac{9}{11}$ (d) $\frac{13}{3}$

[CBSE Question Bank 2021]

Ans. (A) *(d)* $\frac{3}{5}$

Explanation: Given E_1 is the event that student knows the answer.

$\therefore \quad P(E_1) = \frac{3}{5}$

(B) *(b)* 1

Explanation: Probability of giving the answer correct on knowing the answer is 1. Therefore, required probability is 1.

(C) *(a)* $\frac{11}{15}$

Explanation: $\sum_{k=1}^{2} P\left(\frac{E}{E_k}\right)P(E_k)$

$$= P\left(\frac{E}{E_1}\right)P(E_1) + P\left(\frac{E}{E_2}\right)P(E_2)$$

$$= 1\times\frac{3}{5} + \frac{1}{3}\times\frac{2}{5}$$

$$= \frac{3}{5} + \frac{2}{15}$$

$$= \frac{9+2}{15} = \frac{11}{15}$$

(E) *(c)* $\frac{9}{11}$

Explanation:

Required probability $= P\left(\frac{E_1}{E}\right)$

$$= \frac{P\left(\frac{E}{E_1}\right)\times P(E_1)}{P\left(\frac{E}{E_1}\right)\times P(E_1) + P\left(\frac{E}{E_2}\right)\times P(E_2)}$$

$$= \frac{1\times\frac{3}{5}}{1\times\frac{3}{5}+\frac{2}{5}\times\frac{1}{3}}$$

$$= \frac{\frac{3}{5}}{\frac{3}{5}+\frac{2}{15}}$$

$$= \frac{\frac{3}{5}}{\frac{9+2}{15}}$$

$$= \frac{9}{11}$$

18. *An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20 percent of the population is accident prone.*

(A) **What is the probability that a new policyholder will have an accident within a year of purchasing a policy?**

(B) **Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?**

[CBSE Term-2 SQP 2022]

Ans. Let E_1 = The policy holder is accident prone.

E_2 = The policy holder is not accident prone.

E = The new policy holder has an accident within a year of purchasing a policy.

(A) $P(E) = P(E_1) \times P\left(\frac{E_1}{E}\right) + P(E_2) \times P\left(\frac{E}{E_2}\right)$

$$= \frac{20}{100} \times \frac{6}{10} + \frac{80}{100} \times \frac{2}{10}$$

$$= \frac{7}{25}$$

(B) By Bayes' Theorem,

$$P\left(\frac{E_1}{E}\right) = \frac{P(E_1) \times P\left(\frac{E}{E_1}\right)}{P(E)}$$

$$= \frac{\frac{20}{100} \times \frac{6}{10}}{\frac{280}{1000}}$$

$$= \frac{3}{7}$$

[CBSE Marking Scheme Term-2 SQP 2022]

Explanation: Consider E_1 = Policy holder is prove to

E_2 = Policy holder is not prove to accident

E_3= Policy holder met with an accident in year of purchase.

(A) Here, $P(E_1) = \frac{20}{100} = \frac{2}{10}$

$$P(E_2) = 1 - \frac{20}{100} = \frac{80}{100} = \frac{8}{10}$$

$$P\left(\frac{E}{E_1}\right) = 0.6 = \frac{6}{10}$$

$$P\left(\frac{E}{E_2}\right) = 0.2 = \frac{2}{10}$$

$\therefore$ $P(E) = P(E_1) \times P\left(\frac{E}{E_1}\right) + P(E_2) \times P\left(\frac{E}{E_2}\right)$

$$= \frac{2}{10} \times \frac{6}{10} + \frac{8}{10} \times \frac{2}{10}$$

$$= \frac{12+16}{100} = \frac{28}{100} = \frac{7}{25}$$

(B) Here, we need to find the probability of $P\left(\frac{E_1}{E}\right)$

So, by Baye's theorem,

$$P\left(\frac{E_1}{E}\right) = \frac{P\left(\frac{E}{E_1}\right) \times P(E_1)}{P(E)}$$

$$= \frac{\frac{6}{10} \times \frac{2}{10}}{\frac{7}{25}}$$

$$= \frac{12 \times 25}{100 \times 7} = \frac{3}{7}$$

⚠ **Caution**

➥ *Read the case carefully, to get an idea about the probabilites given and what is needed to be calculated to get the desired results.*

19. *On a Diwali's night, four members of a family plan to play a cards' game. Head of the family is to act as an initiator of the game. He draws a card at random from a well-shuffled deck of 52 cards and noted the outcome and thereafter deck is reshuffled without replacing the card. Another card is then drawn from the deck by a member.*

(A) What is the probability that 'both the cards are of the same suit'?

(B) Ⓐ What is the probability that 'one card is an ace and the other is a black queen'?

Ans. (A) P(both the cards are of the same suit)

= P(both are spades) + P(both are diamonds) + P(both are hearts) + P(both are clubs)

$$= \frac{^{13}C_2}{^{52}C_2} + \frac{^{13}C_2}{^{52}C_2} + \frac{^{13}C_2}{^{52}C_2} + \frac{^{13}C_2}{^{52}C_2}$$

$$= 4 \times \frac{1}{17} = \frac{4}{17}$$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

20. **Ⓐ A box has 5 blue and 4 red balls. One ball is drawn at random and not replaced. Its colour is also not noted. Then, another ball is drawn at random. What is the probability of second ball being blue? [NCERT Exemplar]**

21. Two dice are thrown 'n' times in succession. What is the probability of obtaining a double six atleast once?

Ans. Probability of getting double six in two dice

$$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

So, probability of not getting double six

$$= 1 - \frac{1}{36} = \frac{35}{36}$$

∴ Required probability

= 1 – (Probability of not getting double six)^n

$$= 1 - \left(\frac{35}{36}\right)^n$$

22. The probability distribution of a random variable X is given below:

X	–5	–4	–3	–2	–1	0	1	2	3	4	5
P(X)	p	$2p$	$3p$	$4p$	$5p$	$7p$	$8p$	$9p$	$10p$	$11p$	$12p$

Then, find the value of 'p'.

Ans. ∵ Sum of probability distribution = 1

$$\Rightarrow p + 2p + 3p + 4p + 5p + 7p + 8p + 9p + 10p + 11p + 12p = 1$$

$$\Rightarrow \quad 72p = 1$$

$$\Rightarrow \quad p = \frac{1}{72}$$

Hence, the value of p is $\frac{1}{72}$.

23. An urn contains 6 red and 3 black balls. Two balls are randomly drawn. Let X represents the number of black balls. What are the possible values of X?

Ans. Here, from the given urn, we are drawing 2 balls.

So, the possible values of X are 0, 1, 2.

24. If E_1, E_2 ..... E_n constitute a partition of sample space S and A is any event of non-zero probability, then what is the value of $P(E_i|A)$?

SHORT ANSWER Type-I Questions (SA-I)

[**2** marks]

25. Bhavani is going to play a game of chess against one of four opponents in an intercollege sports competition. Each opponent is equally likely to be paired against her. The table below shows the chances of Bhavani losing, when paired against each opponent.

Opponent	Opponent 1	Opponent 2	Opponent 3	Opponent 4
Bhavani's chances of losing	12%	60%	x %	84%

If the probability that Bhavani loses the game that day is $\frac{1}{2}$, find the probability for Bhavani to be losing the game when paired against Opponent 3. Show your steps.

Ans. Let E_1, E_2, E_3, E_4 and A be the events defined as follows:

E_1 = Bhavani is paired against opponent 1.

E_2 = Bhavani is paired against opponent 2.

E_3 = Bhavani is paired against opponent 3.

E_4 = Bhavani is paired against opponent 4.

A = Bhavani losses the game.

Then, $P(E_1) = P(E_2)$

$$= P(E_3) = P(E_4) = \frac{1}{4}$$

Also, $P(A|E_1) = 12\% = \frac{12}{100}$

$$P(A|E_2) = 60\% = \frac{60}{100}$$

$$P(A|E_3) = x\% = \frac{x}{100}$$

$$P(A|E_4) = 84\% = \frac{84}{100}$$

Now, using theorem of total probability, we have,

$$P(A) = P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2) + P(E_3)\,P(A|E_3) + P(E_4)\,P(A|E_4)$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{4}\times\frac{12}{100} + \frac{1}{4}\times\frac{60}{100} + \frac{1}{4}\times\frac{x}{100} + \frac{1}{4}\times\frac{84}{100}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{400}(12 + 60 + x + 84)$$

$\Rightarrow \quad 200 = 156 + x$

$\Rightarrow \quad x = 44$

Hence, the probability for Bhavani losing the game against opponent 3 is 44%.

26. Find the probability distribution of X, the number of heads in a simultaneous toss of two coins. [CBSE 2019]

Ans. When we toss two coins simultaneously, we may get 0 head, 1 head or 2 heads.

Then, possible values of X are 0, 1, 2.

Now, $P(X = 0) = P(\text{getting no head}) = P(TT) = \frac{1}{4}$

$P(X = 1) = P(\text{getting one head}) = P(HT \text{ or } TH) = \frac{2}{4} = \frac{1}{2}$

$P(X = 2) = P(\text{getting two heads}) = P(HH) = \frac{1}{4}$

∴ Probability distribution is

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

27. The random variable X has a probability distribution P(X) of the following form, where 'k' is some number.

$$P(X = x) = \begin{cases} k, & \text{if } X = 0 \\ 2k, & \text{if } X = 1 \\ 3k, & \text{if } X = 2 \\ 0, & \text{otherwise} \end{cases}$$

Find the value of k. [CBSE 2019]

Ans. Make the given information in tabular form

X	0	1	2	Otherwise
P(X)	k	$2k$	$3k$	0

Since, sum of all probabilities is 1.

∴ $\Sigma P(X = x) = 1$

$\Rightarrow P(X = 0) + P(X = 1) + P(X = 2) = 1$

$\Rightarrow k + 2k + 3k = 1$

$\Rightarrow 6k = 1$

$\Rightarrow k = \frac{1}{6}$

28. For the following probability distribution, determine mean of the random variable X.

X	2	3	4
P(X)	0.2	0.5	0.3

29. From a lot of 15 bulbs which include 5 defectives, a sample of 2 bulbs is drawn at random (without replacement). Find the probability distribution of the number of defective bulbs.

30. A bag contains 4 white and 5 black balls. Another bag contains 9 white and 7 black balls. A ball is transferred from the first bag to the second bag and then a ball is drawn from the second bag. Find the probability that the ball drawn is white.

[NCERT Exemplar]

Ans. Let, E_1 be the event that ball transferred from the first bag is white and E_2 be the event that the ball transferred from the first bag is black.

And E be the event that ball drawn from the second bag is white.

∴ $P(E|E_1) = \frac{10}{17}$ and $P(E|E_2) = \frac{9}{17}$

And $P(E_1) = \frac{4}{9}$ and $P(E_2) = \frac{5}{9}$

∴ $P(E) = P(E_1)\,P(E|E_1) + P(E_2)P(E|E_2)$

$= \frac{4}{9}.\frac{10}{17} + \frac{5}{9}.\frac{9}{17}$

$= \frac{40+45}{153} = \frac{85}{153} = \frac{5}{9}$

31. A bag contains 1 red and 3 white balls. Find the probability distribution of the number of red balls if 2 balls are drawn at random from the bag one-by-one without replacement.

[CBSE Term-2 SQP 2022]

Ans. Let X be the random variable defined as the number of red balls.

Then X = 0, 1

$P(X = 0) = \frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$

$P(X = 1) = \frac{1}{4} \times \frac{3}{3} + \frac{3}{4} \times \frac{1}{3} = \frac{6}{12} = \frac{1}{2}$

Probability Distribution Table:

X	0	1
P(X)	$\frac{1}{2}$	$\frac{1}{2}$

[CBSE Marking Scheme Term-2 SQP 2022]

Explanation: Total number of balls in the bag = 4

Now, 2 balls are drawn from the bag one by one without replacement.

Let, X be the random variables defined as the number of red balls.

Then, $X = 0, 1$

$$P(X=0) = \frac{3}{4}\times\frac{2}{3}=\frac{1}{2}$$

$$P(X=1) = \frac{1}{4}\times\frac{3}{3}+\frac{3}{4}\times\frac{1}{3}$$

$$= \frac{6}{12}=\frac{1}{2}$$

∴ Probability distribution is:

X	0	1
P(X)	$\frac{1}{2}$	$\frac{1}{2}$

⚠ Caution

➥ *As there is only 1 red ball in the bag, so in 2 draws we can get only 1 red ball at maximum.*

32. Suppose you have two coins which appear identical in your pocket. You know that, one is fair and one is two-headed. If you take one out, toss it and get a head, what is the probability that it was a fair coin?

[NCERT Exemplar]

33. Suppose 10,000 tickets are sold in a lottery each for ₹1. First prize is of ₹3000 and second prize is of ₹2000. There are three third prizes of ₹500 each. If you buy one ticket, then what is your expectation? [NCERT Exemplar]

Ans. Let X be a random variable for the prize.

X	0	500	2000	3000
P(X)	$\frac{9995}{10000}$	$\frac{3}{10000}$	$\frac{1}{10000}$	$\frac{1}{10000}$

We know, $E(X) = \Sigma XP(X)$

$$\Rightarrow \quad E(X) = 0\times\frac{9995}{10000} + 500\times\frac{3}{10000} + 2000\times\frac{1}{10000}+3000\times\frac{1}{10000}$$

$$= \frac{1500}{10000}+\frac{2000}{10000}+\frac{3000}{10000}$$

$$= \frac{1500+2000+3000}{10000}$$

$$= \frac{6500}{10000} = ₹0.65$$

⚠ Caution

➥ *Here, we will use the formula for expectation $E(X) = \Sigma XP(X)$ to get the desired result.*

34. The probability distribution of a discrete random variable X is given as under:

X	1	2	4	2A	3A	5A
P(X)	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{1}{25}$	$\frac{1}{25}$

Calculate the value of A, if E(X) = 2.94

[NCERT Exemplar]

Ans. We have, $\Sigma XP(X)$

$$= 1\times\frac{1}{2} + 2\times\frac{1}{5} + 4\times\frac{3}{25} + 2A\times\frac{1}{10} + 3A\times\frac{1}{25} + 5A\times\frac{1}{25}$$

$$= \frac{1}{2}+\frac{2}{5}+\frac{12}{25}+\frac{2A}{10}+\frac{3A}{25}+\frac{5A}{25}$$

$$= \frac{69+26A}{50}$$

Since, $E(X) = \Sigma XP(X)$

$$\Rightarrow \quad 2.94 = \frac{69+26A}{50}$$

$$\Rightarrow \quad 26A = 50\times 2.94 - 69$$

$$A = \frac{147-69}{26}$$

$$= \frac{78}{26} = 3$$

Hence, the value of A is 3.

SHORT ANSWER Type-II Questions (SA-II)

[**3** marks]

35. A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is choosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, find the probability of one of them being red and another black.

[CBSE 2015]

Ans. Probability of choosing bag A = $P(A) = \frac{2}{6} = \frac{1}{3}$

Probability of choosing bag B = $P(B) = \frac{4}{6} = \frac{2}{3}$

Let E be the event of drawing a red and a black balls.

$\therefore \quad P\left(\frac{E}{A}\right) = \frac{6\times 4}{^{10}C_2}$

and $\quad P\left(\frac{E}{B}\right) = \frac{7\times 3}{^{10}C_2}$

$\therefore$ Required probability

$= P(E)$

$= P(A) \times P\left(\frac{E}{A}\right) + P(B) \times P\left(\frac{E}{B}\right)$

$= \frac{1}{3}\times\frac{6\times 4}{^{10}C_2} + \frac{2}{3}\times\frac{7\times 3}{^{10}C_2}$

$= \frac{8}{\frac{10!}{2!\times 8!}} + \frac{14}{\frac{10!}{2!\times 8!}}$

$= \frac{8}{45} + \frac{14}{45} = \frac{22}{45}$

36. There are two bags, one of which contains 3 black and 4 white balls, while the other contains 4 black and 3 white balls. A die is thrown. If it shows up 1 or 3, a ball is taken from the first bag, but if it shows up any other number, a ball is chosen from the second bag. Find the probability of choosing a black ball.

Ans. Consider, E_1 as an event of choosing first bag, or die showing up 1 or 3.

$\therefore \quad P(E_1) = \frac{2}{6} = \frac{1}{3}$

Consider E_2 as an event of chosing second bag, or die showing up 2, 4, 5 or 6.

$\therefore \quad P(E_2) = \frac{4}{6} = \frac{2}{3}$

and A = Event that a black ball is drawn.

Then, $P(A|E_1) = \frac{3}{7}$ and $P(A|E_2) = \frac{4}{7}$

$\therefore$ Required probability

$= P(A)$

$= P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right)$

$= \frac{1}{3}\times\frac{3}{7} + \frac{2}{3}\times\frac{4}{7}$

$= \frac{1}{7} + \frac{8}{21}$

$= \frac{3+8}{21} = \frac{11}{21}$

37. Three defective bulbs are mixed up with 7 good ones. 3 bulbs are drawn at random. Find the probability distribution of defective bulbs.

Ans. Given, number of defective bulbs = 3 and number of good bulbs = 7

$\therefore$ Total number of bulbs = 3 + 7 = 10

Let, X denotes the number of defective bulbs.

Then, X takes the value 0, 1, 2, 3.

So, $\quad P(X = 0) = P(\text{getting 0 defective bulbs})$

$= \frac{^3C_0 \times {^7C_3}}{^{10}C_3}$

$= \frac{\left(1\times\frac{7\times 6\times 5}{3\times 2\times 1}\right)}{\frac{10\times 9\times 8}{3\times 2\times 1}}$

$= \frac{7\times 6\times 5}{10\times 9\times 8} = \frac{7}{24}$

$P(X = 1) = P(\text{getting 1 defective bulb})$

$= \frac{^3C_1 \times {^7C_2}}{^{10}C_3}$

$= \frac{3\times\frac{7\times 6}{2\times 1}}{\frac{10\times 9\times 8}{3\times 2\times 1}}$

$= \frac{3\times 7\times 6\times 3}{10\times 9\times 8} = \frac{21}{40}$

$P(X = 2) = P(\text{getting 2 defective bulbs})$

$= \frac{^3C_2 \times {^7C_1}}{^{10}C_3}$

$= \frac{\frac{3\times 2}{2\times 1}\times 7}{\frac{10\times 9\times 8}{3\times 2\times 1}}$

$= \frac{3\times 2\times 7\times 3}{10\times 9\times 8} = \frac{7}{40}$

$P(X = 3) = P(\text{getting 3 defective bulbs})$

$= \frac{^3C_3 \times {^7C_0}}{^{10}C_3}$

$= \frac{1\times 1}{\left(\frac{10\times 9\times 8}{3\times 2\times 1}\right)}$

$= \frac{3\times 2\times 1}{10\times 9\times 8} = \frac{1}{120}$

$\therefore$ The probability distribution table is as follows:

X	0	1	2	3
P(X)	$\frac{7}{24}$	$\frac{21}{40}$	$\frac{7}{40}$	$\frac{1}{120}$

38. Three persons A, B and C apply for a job of manager in a private company. Chances of their selection (A, B, C) are in the ratio 1 : 2 : 4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3 respectively.

If the changes does not take place, find the probability that it is due to the appointment of C. [CBSE 2016]

Ans. Let E be the event to improve profits of the company.

Probability of selection of A, $P(A) = \frac{1}{1+2+4}$

$= \frac{1}{7}$

Probability of selection of B, $P(B) = \frac{2}{7}$

Probability of selection of C, $P(C) = \frac{4}{7}$

Probability that A does not introduce changes,

$$P(\bar{E}|A) = 1 - 0.8 = 0.2$$

Probability that B does not introduce changes,

$$P(\bar{E}|B) = 1 - 0.5 = 0.5$$

Probability that C does not introduce changes,

$$P(\bar{E}|C) = 1 - 0.3 = 0.7$$

So, required probability = $P(C|\bar{E})$

$$= \frac{P(C)P(\bar{E}|C)}{P(A)P(\bar{E}|A) + P(B)P(\bar{E}|B) + P(C)P(\bar{E}|C)}$$

$$= \frac{\frac{4}{7}\times 0.7}{\frac{1}{7}\times 0.2 + \frac{2}{7}\times 0.5 + \frac{4}{7}\times 0.7} = 0.7$$

39. **In a shop X, 30 tins of ghee of type A and 40 tins of ghee of type B which look alike, are kept for sale. While in shop Y, similar 50 tins of ghee of type A and 60 tins of ghee of type B are there. One tin of ghee is purchased from one of the randomly selected shop and is found to be of type B. Find the probability that it is purchased from shop Y. [CBSE 2020]**

40. **Let X denote the number of colleges where you will apply after your result and $P(X = x)$ denotes your probability of getting admission in x number of colleges. It is given that**

$$P(X = x) = \begin{cases} kx, & \text{if } x = 0 \text{ or } 1 \\ 2kx, & \text{if } x = 2 \\ k(5-x), & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{if } x > 4 \end{cases}$$

where k is a positive constant. Find the value of k. Also, find the probability that you will get admission in:

(A) exactly one college

(B) atmost 2 colleges

(C) atleast 2 colleges [CBSE 2016]

Ans. The probability distribution of X is

X	0	1	2	3	4
P(X)	0	k	$4k$	$2k$	k

The given distribution is a probability distribution.

$$\therefore \quad \sum_{i=1}^{5} P_i = 1$$

$\Rightarrow 0 + k + 4k + 2k + k = 1$

$\Rightarrow \quad 8k = 1$

$\Rightarrow \quad k = \frac{1}{8} = 0.125$

(A) P(getting admission in exactly one college)
$= P(X = 1) = k = 0.125$

(B) P(getting admission in atmost 2 colleges)
$= P(X \le 2)$
$= P(X = 0) + P(X = 1) + P(X = 2)$
$= 0 + k + 4k = 5k = 5 \times 0.125$
$= 0.625$

(C) P(getting admission in atleast 2 colleges)
$= P(X \ge 2)$
$= P(X = 2) + P(X = 3) + P(X = 4)$
$= 4k + 2k + k = 7k = 7 \times 0.125$
$= 0.875$

41. **A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find the mean of X. [CBSE 2014]**

Ans. Here, the ages of the given 15 students are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years.

$\therefore$ Required probability distribution of X is:

X	14	15	16	17	18	19	20	21
P(X)	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Now, Mean, $\bar{X} = \Sigma XP(X)$

$$= 14\times\frac{2}{15} + 15\times\frac{1}{15} + 16\times\frac{2}{15} + 17\times\frac{3}{15}$$

$$+ 18\times\frac{1}{15} + 19\times\frac{2}{15} + 20\times\frac{3}{15} + 21\times\frac{1}{15}$$

$= \frac{1}{15}(28 + 15 + 32 + 51 + 18 + 38 + 60 + 21)$

$= \frac{263}{15}$

42. **There are four cards numbered 1, 3, 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean of X.**

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

43. Three bags contain a number of red and white balls as follows:
bag 1: 3 red balls,
bag 2: 2 red balls and 1 white ball
and bag 3: 3 white balls
The probability that bag i will be chosen and a ball is selected from it is $\frac{i}{6}$, where i = 1, 2, 3. What is the probability that
(A) a red ball will be selected?
(B) a white ball is selected?
[NCERT Exemplar]

Ans. Let E_1, E_2 and E_3 be the events that bag 1, 2 and 3, respectively are selected and a ball is choosen from it.

$\therefore P(E_1) = \frac{1}{6}, P(E_2) = \frac{2}{6} = \frac{1}{3}, P(E_3) = \frac{3}{6} = \frac{1}{2}$

(A) Consider, E as an event that red ball is selected. Then, probability of selection of red ball is:

$$P(E) = P(E_1).P(E/E_1) + P(E_2).P(E/E_2) + P(E_3).P(E/E_3)$$
$$= \frac{1}{6}.\frac{3}{3} + \frac{1}{3}.\frac{2}{3} + \frac{1}{2}.0$$
$$= \frac{1}{6} + \frac{2}{9} = \frac{3+4}{18} = \frac{7}{18}$$

(B) Let G be the event that white ball is selected.

$$\therefore P(G) = P(E_1)P(G/E_1) + P(E_2)P(G/E_2) + P(E_3)P(G/E_3)$$
$$= \frac{1}{6}.0 + \frac{2}{6}.\frac{1}{3} + \frac{1}{2}.\frac{3}{3}$$
$$= \frac{1}{9} + \frac{1}{2} = \frac{2+9}{18} = \frac{11}{18}$$

Caution

- *Here, we could directly get the probability of getting white ball by applying formula $P(G) = 1 - P(E)$.*

44. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of the random variable X and hence find the mean of the distribution. [CBSE 2014]

Ans. First six positive integers are 1, 2, 3, 4, 5, 6.

If two numbers are selected at random from above six numbers then sample space is given by

S = {(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5)}

$\Rightarrow n(S) = 30$

Now, X is a random variable which denotes larger of the two numbers, so it can take values 2, 3, 4, 5 or 6.

$\therefore$ Required probability distribution is given as

P(X = 2) = Probability of getting (1, 2) or (2, 1)

$$= \frac{2}{30}$$

P(X = 3) = Probability of getting (1, 3) or (2, 3) (3, 1) or (3, 2)

$$= \frac{4}{30}$$

P(X = 4) = Probability of getting (1, 4) or (2, 4) or (3, 4) or (4, 1) or (4, 2) or (4, 3)

$$= \frac{6}{30}$$

P(X = 5) = Probability of getting (1, 5) or (2, 5) or (3, 5) or (4, 5) or (5, 1) or (5, 2) or (5, 3) or (5, 4)

$$= \frac{8}{30}$$

P(X = 0) = Probability of getting (1, 6) or (2, 6) or (3, 6) or (4, 6) or (5, 6) or (6, 1) or (6, 2) or (6, 3) or (6, 4) or (6, 5)

$$= \frac{10}{30}$$

∴ The probability distribution table is:

X	2	3	4	5	6
P(X)	$\frac{2}{30}$	$\frac{4}{30}$	$\frac{6}{30}$	$\frac{8}{30}$	$\frac{10}{30}$

∴ Required mean

$= E(X) = \Sigma XP(X)$

$$= 2\times\frac{2}{30} + 3\times\frac{4}{30} + 4\times\frac{6}{30} + 5\times\frac{8}{30} + 6\times\frac{10}{30}$$

$$= \frac{1}{30}(4 + 12 + 24 + 40 + 60)$$

$$= \frac{140}{30} = \frac{14}{3}$$

45. **Find the probability distribution of the maximum of the two scores obtained when a die is thrown twice. Determine also the mean of the distribution. [NCERT Exemplar]**

46. A letter is known to have come either from 'TATA NAGAR' or from 'CALCUTTA'. On the envelope, just two consecutive letters TA are visible. What is the probability that the letter came from TATA NAGAR?

Ans. Let E_1 be the event that letter is from 'TATA NAGAR' and E_2 be the event that letter is from 'CALCUTTA'. Also, let E be the event that on the envelope, two consecutive letters are TA.

Now, $P(E_1) = \frac{1}{2}$ and $P(E_2) = \frac{1}{2}$

Now, two consecutive letters on TATA NAGAR

= {TA, AT, TA, AN, NA, AG, GA, AR}

∴ Probability of getting TA from TATA NAGAR

$= P(E|E_1)$

$= \frac{2}{8}$

Similarly, two consecutive letters on CALCUTTA

= {CA, AL, LC, CU, UT, TT, TA}

∴ Probability of getting TA from CALCUTTA

$= P(E|E_2)$

$= \frac{1}{7}$

Required probability $= P(E_1/E)$

∴ By baye's theorem

$$P(E_1 | E) = \frac{P(E_1)P(E|E_1)}{P(E_1)(E|E_1)+P(E_2)P(E|E_2)}$$

$$= \frac{\frac{1}{2}\times\frac{2}{8}}{\frac{1}{2}\times\frac{2}{8}+\frac{1}{2}\times\frac{1}{7}}$$

$$= \frac{\frac{1}{8}}{\frac{1}{8}+\frac{1}{14}} = \frac{\frac{1}{8}}{\frac{7+4}{56}}$$

$$= \frac{7}{11}$$

47. **A shopkeeper sells three types of flower seeds A_1, A_2 and A_3. They are sold as a mixture, where the proportions are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35%. Calculate the probability**

(A) of a randomly chosen seed to germinate;

(B) that it will not germinate given that the seed is of type A_3;

(C) that it is type A_2 given that a randomly chosen seed does not germinate.

[NCERT Exemplar]

48. A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job 30% of time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by A?

[CBSE 2019]

Ans. Consider,

E_1 : The item is manufactured by operator A.

E_2 : The item is manufactured by operator B.

E_3 : The item is manufactured by operator C.

E : The item is defective.

∴ $P(E_1) = \frac{50}{100} = \frac{5}{10}$, $P(E_2) = \frac{30}{100} = \frac{3}{10}$

$P(E_3) = \frac{20}{100} = \frac{2}{10}$

Also, $P(E|E_1) = \frac{1}{100}$, $P(E|E_2) = \frac{5}{100}$,

$P(E|E_3) = \frac{7}{100}$

Now, probability that item is produced by operator A given that it is defective = $P(E_1|E)$.

$\therefore$ By Bayes' theorem

$P(E_1|E)$

$$= \frac{P(E_1)P(E \mid E_1)}{P(E_1).P(E \mid E_1) + P(E_2).P(E \mid E_2) + P(E_3).(E \mid E_3)}$$

$$= \frac{\frac{5}{10} \times \frac{1}{100}}{\frac{5}{10} \times \frac{1}{100} + \frac{3}{10} \times \frac{5}{100} + \frac{2}{10} \times \frac{7}{100}}$$

$$= \frac{5}{5+15+14}$$

$$= \frac{5}{34}$$

49. ④**Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A-grade. What is the probability that the student is a hostlier?**

50. A man is known to speak truth 3 out of 7 times. He throws a die and reports that it is 5. Find the probability that it is actually 5.

Ans. Let A, B and C be the following events.

A : Man speaks truth.

B : Man does not speak truth.

C : Man reports that it is 5.

$\therefore$ $P(A) = \frac{3}{7}$, $P(B) = 1 - \frac{3}{7} = \frac{4}{7}$

Also, $P(C|A) = \frac{1}{6}$

[$\because$ probability of getting 5 on throwing a die is $\frac{1}{6}$.]

and, $P(C|B) = 1 - \frac{1}{6} = \frac{5}{6}$

Now, required probability = $P(A|C)$

$\therefore$ By Bayes' theorem,

$$P(A|C) = \frac{P(A)P(C|A)}{P(A)P(C|A) + P(B)P(C|B)}$$

$$= \frac{\frac{3}{7} \times \frac{1}{6}}{\frac{3}{7} \times \frac{1}{6} + \frac{4}{7} \times \frac{5}{6}}$$

$$= \frac{3}{3+20} = \frac{3}{23}$$

Now, probability that item is produced by operator A given that it is defective = $P(E_1|E)$.

$\therefore$ By Bayes' theorem

$P(E_1|E)$

$$= \frac{P(E_1)P(E \mid E_1)}{P(E_1).P(E \mid E_1) + P(E_2).P(E \mid E_2) + P(E_3).(E \mid E_3)}$$

$$= \frac{\frac{5}{10} \times \frac{1}{100}}{\frac{5}{10} \times \frac{1}{100} + \frac{3}{10} \times \frac{5}{100} + \frac{2}{10} \times \frac{7}{100}}$$

$$= \frac{5}{5 + 15 + 14}$$

$$= \frac{5}{34}$$

49. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A-grade. What is the probability that the student is a hostlier?

50. A man is known to speak truth 3 out of 7 times. He throws a die and reports that it is 5. Find the probability that it is actually 5.

Ans. Let A, B and C be the following events.

A : Man speaks truth.

B : Man does not speak truth.

C : Man reports that it is 5.

$\therefore$ $P(A) = \frac{3}{7}$, $P(B) = 1 - \frac{3}{7} = \frac{4}{7}$

Also, $P(C|A) = \frac{1}{6}$

$\left[\because \text{probability of getting 5 on throwing a die is } \frac{1}{6}.\right]$

and, $P(C|B) = 1 - \frac{1}{6} = \frac{5}{6}$

Now, required probability = $P(A|C)$

$\therefore$ By Bayes' theorem,

$$P(A|C) = \frac{P(A)P(C|A)}{P(A)P(C|A) + P(B)P(C|B)}$$

$$= \frac{\frac{3}{7} \times \frac{1}{6}}{\frac{3}{7} \times \frac{1}{6} + \frac{4}{7} \times \frac{5}{6}}$$

$$= \frac{3}{3 + 20} = \frac{3}{23}$$

TOPPER'S CORNER

SHORT ANSWER Type-I Questions (SA-I)

[**2** marks]

1. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find if A and B are independent events.

Ans.

A : number obtained is even $= \{2, 4, 6\}$

B : number obtained is red $= \{1, 2, 3\}$

$P(A) = \frac{3}{6} = \frac{1}{2}$

$P(B) = \frac{3}{6} = \frac{1}{2}$

$A \cap B$ = number obtained is red and even $= \{2\}$

$P(A \cap B) = \frac{1}{6}$

$P(A)\,P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(A \cap B) \neq P(A)\,P(B)$ Hence events A and B are not independent events

[CBSE Topper 2017]

2. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Ans.

Let E_1 = "Event of obtaining the sum 8"

$E_1 = \{(2,6)(6,2)(3,5)(5,3)(4,4)\}$

and F = "Event that red die result in a number less than 4."

$$F = \begin{Bmatrix} (1,1)\ (2,1)\ (3,1)\ (4,1)\ (5,1)\ (6,1) \\ (1,2)\ (2,2)\ (3,2)\ (4,2)\ (5,2)\ (6,2) \\ (1,3)\ (2,3)\ (3,3)\ (4,3)\ (5,3)\ (6,3) \end{Bmatrix}$$

$P(E/F) = \frac{P(E \cap F)}{P(F)}$ or $\frac{n(E \cap F)}{n(F)}$

$E \cap F = \{(6,2), (5,3)\}$

$n(E \cap F) = \frac{2}{36}$, $n(F) = \frac{18}{36}$

$P(E_1/F) = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{2}{18} = \boxed{\frac{1}{9}}$ Ans

[CBSE Topper 2018]

3. The random variable X has a probability distribution P(X) of the following form, where 'k' is some number.

$$P(X = x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of 'k'.

Ans.

Let E_1 = "Event that the girl threw 3, 4, 5 or 6"

E_2 = "Event that the girl threw 1, 2."

and A = "Event that the girl gets exactly one tail"

$P(E_1) = \frac{4}{6} = \frac{2}{3}$, $P(E_2) = \frac{2}{6} = \frac{1}{3}$

$P(A/E_1) = \frac{1}{2}$, $P(A/E_2) = 3/8$

Now, $P(E_1/A) = \dfrac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$

$P(E_1/A) = \dfrac{\frac{2}{3}\times\frac{1}{2}}{\frac{2}{3}\times\frac{1}{2} + \frac{1}{3}\times\frac{3}{8}}$

$P(E_1/A) = \dfrac{\frac{2}{6}}{\frac{2}{6}+\frac{1}{8}} = \dfrac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{8}} = \dfrac{\frac{1}{3}}{\frac{8+3}{24}}$

$P(E_1/A) = \frac{1}{3}\times\frac{24}{11} = \boxed{\frac{8}{11}}$ ans.

[CBSE Topper 2018]

6. Two numbers are selected at random (without replacement) from the first five positive integers. Let X denote the larger of the two numbers obtained. Find the mean of X.

Ans.

Let X denote the larger of the two numbers

$X = 2, 3, 4, 5$

$P(X=2) = \frac{2}{20} = \frac{1}{10}$

$P(X=3) = \frac{4}{20} = \frac{2}{10}$

$P(X=4) = \frac{6}{20} = \frac{3}{10}$

$P(X=5) = \frac{8}{20} = \frac{4}{10}$

Probability Distribution :-

X	2	3	4	5
P(X)	1/10	2/10	3/10	4/10

Mean

$E(X) = \Sigma p_i x_i$

$\Sigma p_i x_i = 2\times\frac{1}{10} + 3\times\frac{2}{10} + 4\times\frac{3}{10} + 5\times\frac{4}{10}$

$\Sigma p_i x_i = \frac{2}{10} + \frac{6}{10} + \frac{12}{10} + \frac{20}{10} = \frac{40}{10} = \boxed{4}$ ans.

[CBSE Topper 2018]

Ans.

$\sum_{i=0}^{2} P(X_i) = 1$ $\quad$ [∵ $\sum_{i=0}^{n} P(x_i) = 1$] [∵ sum of all probabilities = 1, exhaustive & exclusive]

⇒ $k + 2k + 3k = 1$

⇒ $6k = 1$

$k = \frac{1}{6}$

[CBSE Topper 2019]

SHORT ANSWER Type-II Questions (SA-II)

[**3** marks]

4. There are 4 cards numbered 1, 3 , 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean of X.

Ans.

X	P(X)	X P(X)	P(X) X^2
4	P((1,3),(3,1)) $= \frac{1}{4}\times\frac{1}{3} + \frac{1}{4}\times\frac{1}{3} = \frac{2}{12}$	$4\times\frac{2}{12} = \frac{2}{3}$	$\frac{2}{3}\times 4 = \frac{8}{3}$
6	P((1,5),(5,1)) $= \frac{1}{4}\times\frac{1}{3} + \frac{1}{4}\times\frac{1}{3} = \frac{2}{12}$	$\frac{2}{12}\times 6 = 1$	$1\times 6 = 6$
8	P((1,7),(7,1),(3,5),(5,3)) $= \frac{1}{4}\times\frac{1}{3} + \frac{1}{4}\times\frac{1}{3} + \frac{1}{4}\times\frac{1}{3} + \frac{1}{4}\times\frac{1}{3} = \frac{4}{12}$	$\frac{4\times 8}{12} = \frac{8}{3}$	$\frac{8}{3}\times 8 = \frac{64}{3}$
10	P((3,7),(7,3)) $= \frac{1}{4}\times\frac{1}{3} + \frac{1}{4}\times\frac{1}{3} = \frac{2}{12}$	$10\times\frac{2}{12} = \frac{5}{3}$	$\frac{5}{3}\times 10 = \frac{50}{3}$
12	P((5,7),(7,5)) $= \frac{1}{4}\times\frac{1}{3} + \frac{1}{4}\times\frac{1}{3} = \frac{2}{12}$	$12\times\frac{2}{12} = 2$	$2\times 12 = 24$

$\bar{X} = \sum_{i=4}^{12} P(x_i)\, x_i$

$= \frac{2}{3} + 1 + \frac{8}{3} + \frac{5}{3} + 2$

$= 3 + \frac{15}{3} = 8$

Mean = 8

[CBSE Topper 2017]

5. Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained . If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die?