Shortcuts and Important Results to Remember

1 If $T_n = An + B$, i.e. nth term of an AP is a linear expression in n, where A, B are constants, then coefficient of n i.e., A is the common difference.

2 If $S_n = Cn^2 + Dn$ is the sum of n terms of an AP, where C and D are constants, then common difference of AP is $2C$ i.e., 2 times the coefficient of n^2.

3 (i) $d = T_n - T_{n-1}$ $[n \geq 2]$ (ii) $T_n = S_n - S_{n-1}$ $[n \geq 2]$
(iii) $d = S_n - 2S_{n-1} + S_{n-2}$ $[n \geq 3]$

4 If for two different AP's

$$\frac{S_n}{S_n'} = \frac{An^2 + Bn}{Cn^2 + Cn} \quad \text{or} \quad \frac{An + B}{Cn + D}$$

Then, $\dfrac{T_n}{T_n'} = \dfrac{A(2n-1) + B}{C(2n-1) + D}$

5 If for two different AP's

$$\frac{T_n}{T_n'} = \frac{An + B}{Cn + D}, \text{ then } \frac{S_n}{S_n'} = \frac{A\left(\frac{n+1}{2}\right) + B}{C\left(\frac{n+1}{2}\right) + D}$$

6 If $T_p = q$ and $T_q = p$, then $T_{p+q} = 0$, $T_r = p + q - r$

7 If $pT_p = qT_q$ of an AP, then $T_{p+q} = 0$

8 If $S_p = S_q$ for an AP, then $S_{p+q} = 0$

9 If $S_p = q$ and $S_q = p$ of an AP, then $S_{p+q} = -(p+q)$

10 If $T_p = P$ and $T_q = Q$ for a GP, then $T_n = \left[\dfrac{P^{n-q}}{Q^{n-p}}\right]^{1/(p-q)}$

11 If $T_{m+n} = p$, $T_{m-n} = q$ for a GP, then

$$T_m = \sqrt{pq}, \quad T_n = p\left(\frac{q}{p}\right)^{m/2n}$$

12 If $T_m = n$, $T_n = m$ for a HP, then

$$T_{m+n} = \frac{mn}{(m+n)}, T_{mn} = 1, T_p = \frac{mn}{p}$$

13 If $T_p = qr$, $T_q = pr$ for a HP, then $T_r = pq$

14 No term of HP can be zero and there is no formula to find S_n for HP.

15 a, b, c are in AP, GP or HP as $\dfrac{a-b}{b-c} = \dfrac{a}{a}$ or $\dfrac{a}{b}$ or $\dfrac{a}{c}$.

16 If A, G, H be AM, GM and HM between a and b, then

$$\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \begin{cases} A, & \text{when } n = 0 \\ G, & \text{when } n = -\frac{1}{2} \\ H, & \text{when } n = -1 \end{cases}$$

17 If A and G are the AM and GM between two numbers a, b, then a, b are given by $A \pm \sqrt{(A+G)(A-G)}$

18 If a, b, c are in GP, then $a + b, 2b, b + c$ are in HP.

19 If a, b, c are in AP, then $\lambda^a, \lambda^b, \lambda^c$ are in GP, where $\lambda > 0, \lambda \neq 1$.

20 If $-1 < r < 1$, then GP is said to be convergent, if $r < -1$ or $r > 1$, then GP is said to be divergent and if $r = -1$, then series is oscillating.

21 If a, b, c, d are in GP, then
$(a \pm b)^n, (b \pm c)^n, (c \pm d)^n$ are in GP, $\forall\, n \in I$

22 If a, b, c are in AP as well as in GP, then $a = b = c$.

23 The equations $a_1x + a_2y = a_3$, $a_4x + a_5y = a_6$ has a unique solution, if $a_1, a_2, a_3, a_4, a_5, a_6$ are in AP and common difference $\neq 0$.

24 For n positive quantities $a_1, a_2, a_3, \dots, a_n$

$$\text{AM} \geq \text{GM} \geq \text{HM}$$

sign of equality (AM = GM = HM) holds when quantities are equal

i.e. $a_1 = a_2 = a_3 = \dots = a_n$.

25 For two positive numbers a and b (AM) (HM) $= (\text{GM})^2$, the result will be true for n numbers, if they are in GP.

26 If odd numbers of (say $2n + 1$) AM's, GM's and HM's be inserted between two numbers, then their middle means [i.e., $(n+1)$th mean] are in GP.

27 If a^2, b^2, c^2 are in AP.

$$\Rightarrow \quad \frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b} \text{ are in AP.}$$

28 Coefficient of x^{n-1} and x^{n-2} in
$(x - a_1)(x - a_2)(x - a_3) \dots (x - a_n)$
are $-(a_1 + a_2 + a_3 + \dots + a_n)$ and Σa_1a_2, respectively
where, $\Sigma a_1a_2 = \dfrac{(\Sigma a_1)^2 - \Sigma a_1^2}{2}$.

29 $1 + 3 + 5 + \dots$ upto n terms $= n^2$

30 $2 + 6 + 12 + 20 + \dots$ upto n terms $= \dfrac{n(n+1)(n+2)}{3}$

31 $1 + 3 + 7 + 13 + \dots$ upto n terms $= \dfrac{n(n^2+2)}{3}$

32 $1 + 5 + 14 + 30 + \dots$ upto n terms $= \dfrac{n(n+1)^2(n+2)}{12}$

33 If $a_1, a_2, a_3, \dots, a_n$ are the non-zero terms of a non-constant AP, then

$$\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + \frac{1}{a_3a_4} + \dots + \frac{1}{a_{n-1}a_n} = \frac{(n-1)}{a_1a_n}$$