

CHAPTER – 5

ARITHMETIC PROGRESSION

***n*th Term of an ARITHMETIC PROGRESSION (AP)**

*n*th term a_n of the AP with first term a and common difference d is given by

$$a_n = a + (n - 1) d.$$

IMPORTANT QUESTIONS

Find the 15th term of the 21, 24, 27, . . .

Solution: Here, $a = 21$, $d = 24 - 21 = 3$

We know that $a_n = a + (n - 1)d$

So, $a_{15} = a + 14d = 21 + 14(3) = 21 + 42 = 63$

Which term of the AP : 3, 9, 15, 21, . . . , is 99?

Solution: Here, $a = 3$, $d = 9 - 3 = 6$

We know that $a_n = a + (n - 1)d$

Let $a_n = 99 \Rightarrow a + (n - 1)d = 99$

$\Rightarrow 3 + (n - 1)6 = 99 \Rightarrow (n - 1)6 = 99 - 3 = 96$

$\Rightarrow n - 1 = \frac{96}{6} = 16 \Rightarrow n = 16 + 1 = 17$

Hence, 17th term of the given AP is 99

Determine the AP whose 3rd term is 5 and the 7th term is 9.

Solution: We have $a_3 = a + (3 - 1) d = a + 2d = 5$ (1)

and $a_7 = a + (7 - 1) d = a + 6d = 9$ (2)

Solving the pair of linear equations (1) and (2), we get $a = 3$, $d = 1$

Hence, the required AP is 3, 4, 5, 6, 7, . . .

Questions for practice

1. Find the 10th term of the AP : 2, 7, 12, . . .
2. Which term of the AP : 21, 18, 15, . . . is -81 ?
3. Which term of the AP : 3, 8, 13, 18, . . . , is 78?
4. How many two-digit numbers are divisible by 3?
5. How many three-digit numbers are divisible by 7?
6. How many multiples of 4 lie between 10 and 250?
7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.
8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.
9. If the 3rd and the 9th terms of an AP are 4 and -8 respectively, which term of this AP is zero?
10. Which term of the AP : 3, 15, 27, 39, . . . will be 132 more than its 54th term?
11. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.
12. The sum of 4th term and 8th term of an AP is 24 and the sum of 6th and 10th terms is 44. Find the AP.
13. The sum of 5th term and 9th term of an AP is 72 and the sum of 7th and 12th terms is 97. Find the AP.
14. If the numbers $n - 2$, $4n - 1$ and $5n + 2$ are in AP, find the value of n .
15. Find the value of the middle most term (s) of the AP : $-11, -7, -3, \dots, 49$.
16. The sum of the first three terms of an AP is 33. If the product of the first and the third term exceeds the second term by 29, find the AP.
17. The sum of the 5th and the 7th terms of an AP is 52 and the 10th term is 46. Find the AP.

18. Find the 20th term of the AP whose 7th term is 24 less than the 11th term, first term being 12.
19. If the 9th term of an AP is zero, prove that its 29th term is twice its 19th term.
20. Which term of the AP: 53, 48, 43,... is the first negative term?
21. A sum of Rs 1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interests form an AP? If so, find the interest at the end of 30 years making use of this fact.
22. In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

***n*th Term from the end of an ARITHMETIC PROGRESSION (AP)**

Let the last term of an AP be ' l ' and the common difference of an AP is ' d ' then the n th term from the end of an AP is given by

$$l_n = l - (n - 1) d.$$

IMPORTANT QUESTIONS

Find the 11th term from the last term (towards the first term) of the AP : 10, 7, 4, . . . , - 62.

Solution : Here, $a = 10$, $d = 7 - 10 = -3$, $l = -62$,

We know that n th term from the last is given by $l_n = l - (n - 1) d$.

$$\therefore l_{11} = l - 10d = -62 - 10(-3) = -62 + 30 = -32$$

Questions for practice

- Find the 20th term from the last term of the AP : 3, 8, 13, . . . , 253.
- Find the 10th term from the last term of the AP : 4, 9, 14, . . . , 254.
- Find the 6th term from the end of the AP 17, 14, 11, (-40).
- Find the 8th term from the end of the AP 7, 10, 13, 184.
- Find the 10th term from the last term of the AP : 8, 10, 12, . . . , 126.

Sum of First n Terms of an ARITHMETIC PROGRESSION (AP)

The sum of the first n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where a = first term, d = common difference and n = number of terms.

or

$$S_n = \frac{n}{2}[a + l]$$

where l = last term

IMPORTANT QUESTIONS

Find the sum of the first 22 terms of the AP : 8, 3, -2, . . .

Solution : Here, $a = 8$, $d = 3 - 8 = -5$, $n = 22$.

We know that $S = \frac{n}{2}[2a + (n-1)d]$

$$\therefore S = \frac{22}{2}[16 + (22-1)(-5)] = 11(16 - 105) = 11(-89) = -979$$

So, the sum of the first 22 terms of the AP is -979.

Questions for practice

- If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term.
- How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?

3. How many terms of the AP : 9, 17, 25, . . . must be taken to give a sum of 636?
4. Find the sum of first 24 terms of the list of numbers whose n th term is given by $a_n = 3 + 2n$
5. Find the sum of the first 40 positive integers divisible by 6.
6. Find the sum of the first 15 multiples of 8.
7. Find the sum of the odd numbers between 0 and 50.
8. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.
9. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?
10. Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.
11. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.
12. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.
13. If $a_n = 3 - 4n$, show that a_1, a_2, a_3, \dots form an AP. Also find S_{20} .
14. In an AP, if $S_n = n(4n + 1)$, find the AP.
15. In an AP, if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of k .
16. Show that $a_1, a_2, \dots, a_n, \dots$ form an AP where a_n is defined as : (i) $a_n = 3 + 4n$ (ii) $a_n = 9 - 5n$
Also find the sum of the first 15 terms in each case.
17. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the n th terms.
18. Find the sum of first 17 terms of an AP whose 4th and 9th terms are -15 and -30 respectively.
19. If sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.
20. Find the sum of all the 11 terms of an AP whose middle most term is 30.
21. Find the sum of last ten terms of the AP: 8, 10, 12,---, 126.
22. How many terms of the AP: $-15, -13, -11, \dots$ are needed to make the sum -55 ? Explain the reason for double answer.

MCQ QUESTIONS (1 mark)

1. The 10th term of the AP: 5, 8, 11, 14, ... is
(a) 32 (b) 35 (c) 38 (d) 185
2. In an AP if $a = -7.2$, $d = 3.6$, $a_n = 7.2$, then n is
(a) 1 (b) 3 (c) 4 (d) 5
3. In an AP, if $d = -4$, $n = 7$, $a_n = 4$, then a is
(a) 6 (b) 7 (c) 20 (d) 28
4. In an AP, if $a = 3.5$, $d = 0$, $n = 101$, then a_n will be
(a) 0 (b) 3.5 (c) 103.5 (d) 104.5
5. The list of numbers $-10, -6, -2, 2, \dots$ is
(a) an AP with $d = -16$ (b) an AP with $d = 4$
(c) an AP with $d = -4$ (d) not an AP
6. The 11th term of the AP: $-5, -5/2, 0, 5/2, \dots$ is
(a) -20 (b) 20 (c) -30 (d) 30
7. The first four terms of an AP, whose first term is -2 and the common difference is -2 , are

(a) $-2, 0, 2, 4$ (b) $-2, 4, -8, 16$ (c) $-2, -4, -6, -8$ (d) $-2, -4, -8, -16$

8. The 21st term of the AP whose first two terms are -3 and 4 is
(a) 17 (b) 137 (c) 143 (d) -143
9. If the 2nd term of an AP is 13 and the 5th term is 25, what is its 7th term?
(a) 30 (b) 33 (c) 37 (d) 38
10. Which term of the AP: 21, 42, 63, 84,... is 210?
(a) 9th (b) 10th (c) 11th (d) 12th
11. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?
(a) 5 (b) 20 (c) 25 (d) 30
12. What is the common difference of an AP in which $a_{18} - a_{14} = 32$?
(a) 8 (b) -8 (c) -4 (d) 4
13. Two APs have the same common difference. The first term of one of these is -1 and that of the other is -8 . Then the difference between their 4th terms is
(a) -1 (b) -8 (c) 7 (d) -9
14. If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its 18th term will be
(a) 7 (b) 11 (c) 18 (d) 0
15. The 4th term from the end of the AP: $-11, -8, -5, \dots, 49$ is
(a) 37 (b) 40 (c) 43 (d) 58
16. If the first term of an AP is -5 and the common difference is 2, then the sum of the first 6 terms is (a) 0 (b) 5 (c) 6 (d) 15
17. The sum of first 16 terms of the AP: 10, 6, 2,... is
(a) -320 (b) 320 (c) -352 (d) -400
18. In an AP if $a = 1$, $a_n = 20$ and $S_n = 399$, then n is
(a) 19 (b) 21 (c) 38 (d) 42
19. The sum of first 100 multiples of 3 is
(a) 30300 (b) 15150 (c) 300 (d) none of these
20. The sum of first five multiples of 3 is
(a) 45 (b) 55 (c) 65 (d) 75