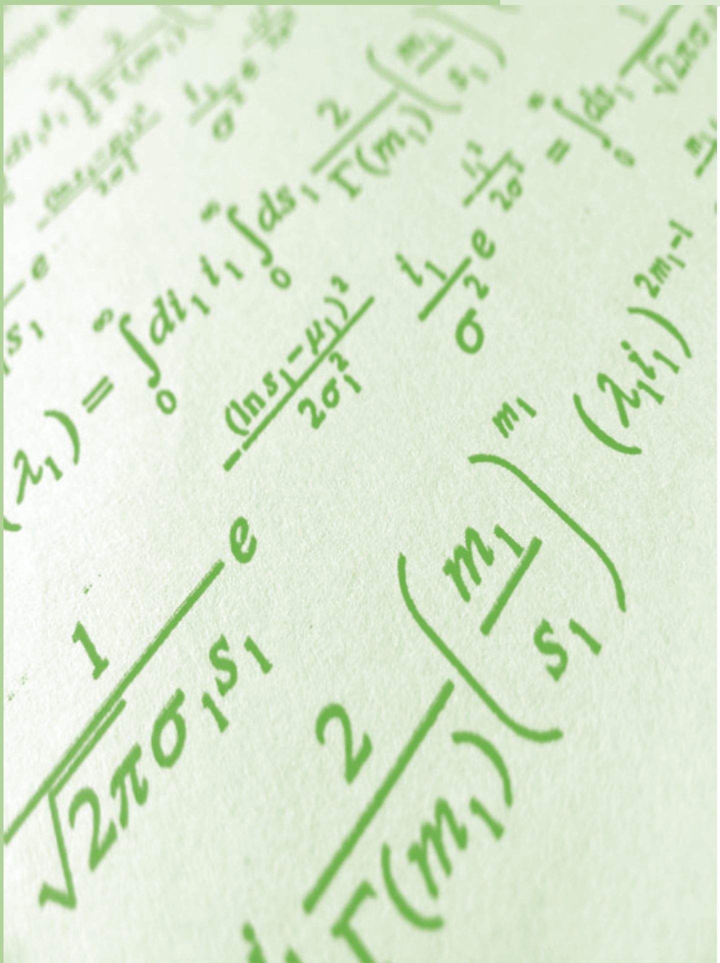
Chapter

8 Trigonometry



REMEMBER

Before beginning this chapter, you should be able to:

- Basic trigonometric identities
- Know systems of measurement of angle

KEY IDEAS

After completing this chapter, you would be able to:

- Convert angles using three systems of measurements of angles
- Study trigonometric identities and solve problems using them
- Understand trigonometric ratios of specific and compound angles
- Learn signs of trigonometric ratios in different quadrants and how to use trigonometric table
- Calculate heights of poles; longer distances, etc., using trigonometric identities

INTRODUCTION

The word trigonometry is originated from the Greek word 'tri' means three, 'gonia' means angle and metron means measure. Hence, the word trigonometry means three angle measure, i.e., it is the study of geometrical figures which have three angles, i.e., triangles.

The great Greek mathematician Hipapachus of 140 BC gave relation between the angles and sides of a triangle. Further trigonometry is developed by Indian (Hindu) mathematicians. This was migrated to Europe via Arabs.

Trigonometry plays an important role in the study of Astronomy, Surveying, Navigation and Engineering. Now a days it is used to predict stock market trends.

ANGLE

A measure formed between two rays having a common initial point is called an angle. The two rays are called the arms or sides of the angle and the common initial point is called the vertex of the angle.

In the Fig. 8.1, OA is said to be the initial side and the other ray OB is said to be the terminal side of the angle.

The angle is taken positive when measured in anti-clockwise direction and is taken negative when measured in clockwise direction (see Fig. 8.2).

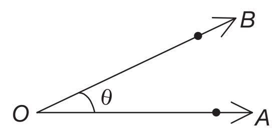


Figure 8.1

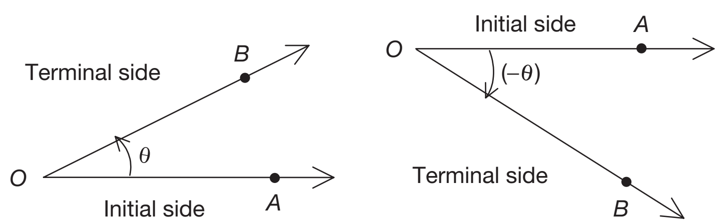


Figure 8.2

Systems of Measurement of Angle

We have the following systems of the measurement of angle.

Sexagesimal System

In this system, the angle is measured in degrees(°).

Degree When the initial ray is rotated through $\left(\frac{1}{360}\right)$ of one revolution, we say that an angle of one degree (1°) is formed at the initial point. A degree is divided into 60 equal parts and each part is called one minute (1′).

Further, a minute is divided into 60 equal parts called seconds (″).

So, 1 right angle = 90°

$1° = 60'$ (minutes) and

$1' = 60''$ (seconds).

Note This system is also called as the British system.

Centesimal System

In this system, the angle is measured in grades.

Grade When the initial ray is rotated through $\left(\frac{1}{400}\right)$ of one revolution, an angle of one grade is said to be formed at the initial point. It is written as 1^g.

Further, one grade is divided into 100 equal parts called minutes and one minute is further divided into 100 equal parts called seconds.

So, 1 right angle = 100^g

$1^g = 100'$ (minutes) and

$1' = 100''$ (seconds).

Note This system is also called as the French system.

Circular System

In this system, the angle is measured in radians.

Radian The angle subtended by an arc of length equal to the radius of a circle at its centre is said to have a measure of one radian. It is written as 1^c (see Fig. 8.3).

Note This measure is also known as radian measure.

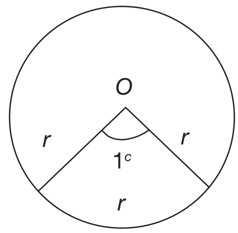


Figure 8.3

Relation Between the Units of the Three Systems

When a rotating ray completes one revolution, the measure of angle formed about the vertex is $360°$ or 400^g or $2\pi^c$.

So,

$$360° = 400^g = 2\pi^c$$

(or)

$$90° = 100^g = \frac{\pi^c}{2}.$$

For convenience, the above relation can be written as, $\frac{D}{90} = \frac{G}{100} = \frac{R}{\frac{\pi}{2}}$.

Where D denotes degrees, G grades and R radians.

Remember

1. $1° = \frac{\pi}{180}$ radians = 0.0175 radians (approximately)
2. $1^c = \frac{180}{\pi}$ degrees = $57°17'\,44''$ (approximately)

Notes

1. The measure of an angle is a real number.
2. If no unit of measurement is indicated for any angle, it is considered as radian measure.

EXAMPLE 8.1

Convert 45° into circular measure.

SOLUTION

Given, $D = 45°$

We have, $\dfrac{D}{90} = \dfrac{R}{\frac{\pi}{2}}$

So, $\dfrac{45}{90} = \dfrac{R}{\frac{\pi}{2}}$

$$\text{or}\quad \frac{1}{2} \times \frac{\pi}{2} = R$$

$$\text{or,}\quad R = \frac{\pi}{4}.$$

Hence, circular measure of 45° is $\dfrac{\pi^c}{4}$.

EXAMPLE 8.2

Convert 150^g into sexagesimal measure.

SOLUTION

Given, $G = 150^g$

We have, $\dfrac{D}{90} = \dfrac{G}{100}$

So, $\dfrac{D}{90} = \dfrac{150}{100}$

$$\text{or}\quad D = \frac{3}{2} \times 90 = 135.$$

Hence, sexagesimal measure of 150^g is 135°.

EXAMPLE 8.3

What is the sexagesimal measure of angle measuring $\dfrac{\pi^c}{3}$?

SOLUTION

Given, $R = \dfrac{\pi^c}{3}$

We have, $\dfrac{D}{90} = \dfrac{R}{\frac{\pi}{2}}$

So, $\dfrac{D}{90} = \dfrac{\frac{\pi}{3}}{\frac{\pi}{2}}$

$$\Rightarrow \quad D = \frac{2}{3} \times 90 = 60°.$$

Hence, the sexagesimal measure of $\dfrac{\pi^c}{3}$ is 60°.

TRIGONOMETRIC RATIOS

Let AOB be a right triangle with $\angle AOB$ as 90°. Let $\angle OAB$ be θ. Notice that $0° < \theta < 90°$, i.e., θ is an acute angle (see Fig. 8.4).

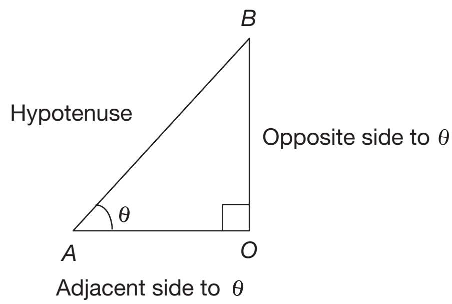


Figure 8.4

We can define six possible ratios among the three sides of the triangle AOB, known as trigonometric ratios. They are defined as follows.

1. Sine of the angle θ (or) simply $\sin\theta$:

$$\sin\theta = \frac{\text{Side opposite to angle } \theta}{\text{Hypotenuse}} = \frac{OB}{AB}.$$

2. Cosine of the angle θ or simply $\cos\theta$:

$$\cos\theta = \frac{\text{Side adjacent to angle } \theta}{\text{Hypotenuse}} = \frac{OA}{AB}.$$

3. Tangent of the angle θ or simply $\tan\theta$:

$$\tan\theta = \frac{\text{Side opposite to } \theta}{\text{Side adjacent to } \theta} = \frac{OB}{OB}.$$

4. Cotangent of the angle θ or simply $\cot\theta$:

$$\cot\theta = \frac{\text{Side adjacent to } \theta}{\text{Side opposite to } \theta} = \frac{OA}{OB}.$$

5. Cosecant of the angle θ or simply $\operatorname{cosec}\theta$:

$$\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Side opposite to } \theta} = \frac{AB}{OB}.$$

6. Secant of the angle θ or simply sec θ:

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \theta} = \frac{AB}{OA}.$$

Observe that,

1. $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$ and $\cot\theta = \dfrac{1}{\tan\theta}$.

2. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$.

EXAMPLE 8.4

If $\sin\theta = \dfrac{3}{5}$, then find the values of tan θ and sec θ.

SOLUTION

Given, $\sin\theta = \dfrac{3}{5}$.

Let AOB be the right triangle such that $\angle OAB = \theta$.
Assume that $OB = 3$ and $AB = 5$ (see Fig. 8.5).

Then, $OA = \sqrt{AB^2 - OB^2} = \sqrt{25-9} = 4$.

So $\tan\theta = \dfrac{\text{Side opposite to } \theta}{\text{Side adjacent to } \theta} = \dfrac{OB}{OA} = \dfrac{3}{4}$ and

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \theta} = \frac{AB}{OA} = \frac{5}{4}.$$

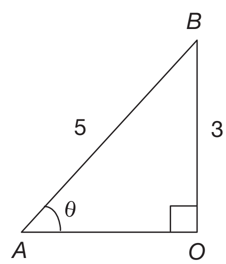


Figure 8.5

Some Pythagorean Triplets

1. 3, 4, 5
2. 5, 12, 13
3. 8, 15, 17
4. 7, 24, 25
5. 9, 40, 41

Trigonometric Identities

1. $\sin^2\theta + \cos^2\theta = 1$
2. $\sec^2\theta - \tan^2\theta = 1$
3. $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

Table of Values of Trigonometric Ratios for Specific Angles

Trigonometric Ratios	Angles				
	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\mathrm{cosec}\,\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

From the above table, we observe that

1. $\sin\theta = \cos\theta$, $\tan\theta = \cot\theta$ and $\sec\theta = \mathrm{cosec}\,\theta$, if $\theta = 45°$.
2. $\sin\theta$ and $\tan\theta$ are increasing functions, when $0° \leq \theta \leq 90°$.
3. $\cos\theta$ is a decreasing function, when $0° \leq \theta \leq 90°$.

EXAMPLE 8.5

Find the value of $\tan 45° + 2\cos 60° - \sec 60°$.

SOLUTION

$$\tan 45° + 2\cos 60° - \sec 60° = 1 + 2\left(\frac{1}{2}\right) - 2 = 1 + 1 - 2 = 0.$$

$\therefore$ $\tan 45° + 2\cos 60° - \sec 60° = 0$.

EXAMPLE 8.6

Using the trigonometric table, evaluate

(a) $\sin^2 30° + \cos^2 30°$.

(b) $\sec^2 60° - \tan^2 60°$.

SOLUTION

(a) $\sin^2 30° + \cos^2 30° = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1.$

Hence, $\sin^2 30° + \cos^2 30° = 1$.

(b) $\sec^2 60^\circ - \tan^2 60^\circ = (2)^2 - \left(\sqrt{3}\right)^2 = 4 - 3 = 1.$

Hence, $\sec^2 60^\circ - \tan^2 60^\circ = 1$.

EXAMPLE 8.7

Find the values of $\dfrac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ}$ and $\tan 30^\circ$. What do you observe?

SOLUTION

$$\frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ} = \frac{\sqrt{3} - \dfrac{1}{\sqrt{3}}}{1 + \sqrt{3}.\dfrac{1}{\sqrt{3}}} = \frac{\dfrac{(3-1)}{\sqrt{3}}}{2} = \frac{1}{\sqrt{3}} \text{ and } \tan 30^\circ = \frac{1}{\sqrt{3}}.$$

Hence, $\dfrac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ} = \tan 30^\circ$.

Trigonometric Ratios of Compound Angles

1. $\sin(A + B) = \sin A \cos B + \cos A \sin B$ and $\sin(A - B) = \sin A \cos B - \cos A \sin B$
2. $\cos(A + B) = \cos A \cos B - \sin A \sin B$ and $\cos(A - B) = \cos A \cos B + \sin A \sin B$
3. $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$ and $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

Also, by taking $A = B$ in the above relations, we get,

1. $\sin 2A = 2\sin A\cos A$
2. $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$
3. $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

EXAMPLE 8.8

Find the value of $\sin 75^\circ$.

SOLUTION

We have, $\sin 75^\circ = \sin(45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ = \dfrac{1}{\sqrt{2}}\dfrac{\sqrt{3}}{2} + \dfrac{1}{\sqrt{2}} \cdot \dfrac{1}{2} = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$.

$\therefore\ \sin 75^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$.

EXAMPLE 8.9

Find the value of tan15°.

SOLUTION

We have, $\tan 15° = \tan(45° - 30°) = \dfrac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} = \dfrac{1 - \dfrac{1}{\sqrt{3}}}{1 + \dfrac{1}{\sqrt{3}}} = \dfrac{\sqrt{3} - 1}{\sqrt{3} + 1}$

$$= \frac{(\sqrt{3} - 1)}{(\sqrt{3} + 1)} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)}$$

$$= \frac{(\sqrt{3} - 1)^2}{3 - 1} = \frac{3 + 1 - 2\sqrt{3}}{2} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}.$$

$\therefore\ \tan 15° = \dfrac{\sqrt{3} - 1}{\sqrt{3} + 1}$ or $2 - \sqrt{3}$.

EXAMPLE 8.10

Eliminate θ from the equations $x = p \sin\theta$ and $y = q \cos\theta$.

SOLUTION

We know that trigonometric ratios are meaningful when they are associated with some θ, i.e., we cannot imagine any trigonometric ratio with out θ. Eliminate θ means, eliminating the trigonometric ratio itself by suitable identities.

Given, $x = p \sin\theta$ and $y = q \cos\theta$

$$\Rightarrow \quad \frac{x}{p} = \sin\theta \text{ and } \frac{y}{q} = \cos\theta$$

We know that, $\sin^2\theta + \cos^2\theta = 1$.

So,
$$\left(\frac{x}{p}\right)^2 + \left(\frac{y}{q}\right)^2 = 1$$

Hence, the required equation is $\dfrac{x^2}{p^2} + \dfrac{y^2}{q^2} = 1$.

EXAMPLE 8.11

Find the relation obtained by eliminating θ from the equations $x = r \cos\theta + s \sin\theta$ and $y = r \sin\theta - s \cos\theta$.

SOLUTION

Given,
$$x = r \cos\theta + s \sin\theta$$

$$\Rightarrow \quad x^2 = (r \cos\theta + s \sin\theta)^2$$

$$= r^2 \cos^2\theta + 2rs \cos\theta \cdot \sin\theta + s^2 \sin^2\theta.$$

Also, $y = r \sin\theta - s \cos\theta$.

$$\text{or } y^2 = r^2 \sin^2\theta + s^2 \cos^2\theta - 2rs \sin\theta \cdot \cos\theta$$
$$\text{or, } x^2 + y^2 = r^2 (\cos^2\theta + \sin^2\theta) + s^2(\sin^2\theta + \cos^2\theta)$$
$$\text{or, } r^2 (1) + s^2 (1) \; [\because \sin^2\theta + \cos^2\theta = 1] = r^2 + s^2.$$

Hence, the required relation is $x^2 + y^2 = r^2 + s^2$.

EXAMPLE 8.12

Eliminate θ from the equations

$x = \operatorname{cosec}\theta + \cot\theta$

$y = \operatorname{cosec}\theta - \cot\theta$

SOLUTION

Given, $x = \operatorname{cosec}\theta + \cot\theta$ and $y = \operatorname{cosec}\theta - \cot\theta$.

Multiplying these equations, we get

$$xy = (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta)$$
$$= \operatorname{cosec}^2\theta - \cot^2\theta = 1.$$

Hence, the required relation is $xy = 1$.

EXAMPLE 8.13

Eliminate θ from the equations $m = \tan\theta + \cot\theta$ and $n = \tan\theta - \cot\theta$.

SOLUTION

Given,

$m = \tan\theta + \cot\theta$ (1)

$n = \tan\theta - \cot\theta$ (2)

Adding Eqs. (1) and (2), we get

$$m + n = 2\tan\theta$$
$$\Rightarrow \quad \tan\theta = \frac{m+n}{2}$$

Subtracting Eq. (2) from Eq. (1), we get

$$m - n = 2\cot\theta$$
$$\Rightarrow \quad \cos\theta = \frac{m-n}{2}$$
$$\therefore \tan\theta \cdot \cot\theta = \left(\frac{m+n}{2}\right) \cdot \left(\frac{m-n}{2}\right)$$
$$\Rightarrow \quad \tan\theta \cdot \frac{1}{\tan\theta} = \frac{m^2 - n^2}{4}$$
$$\Rightarrow \quad 1 = \frac{m^2 - n^2}{4} \text{ (or) } m^2 - n^2 = 4.$$

Hence, by eliminating 'θ', we obtain the relation $m^2 - n^2 = 4$.

EXAMPLE 8.14

If $\cos(A+B)=\dfrac{1}{2}$ and $B=\sqrt{2}$, then find A and B.

SOLUTION

Given,

$\cos(A+B)=\dfrac{1}{2}$

$\cos(A+B)=\cos 60°$

$A+B=60°$ (1)

$\sec B=\sqrt{2}=\sec 45°$

$B=45°$ (2)

From Eqs. (1) and (2), we have

$A=15°$ and $B=45°$.

EXAMPLE 8.15

Find the length of the chord which subtends an angle of 120° at the centre 'O' and which is at a distance of 5 cm from the centre.

SOLUTION

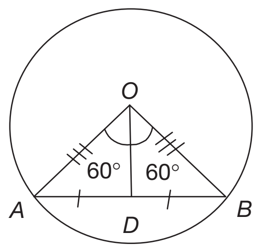


Figure 8.6

Let the chord be AB and OD be the distance of chord from the centre of circle.

Given $\angle AOB = 120°$ and $OD = 5$ cm, clearly, $\Delta OAD \cong \Delta OBD$, by SSS axiom

$$\angle AOD=\angle BOD=\frac{1}{2}(\angle AOB)=60° \text{ in } \Delta AOD$$

$$\tan 60°=\frac{AD}{OD}$$

$$\Rightarrow \quad \sqrt{3}=\frac{AD}{5} \quad \Rightarrow \quad AD=5\sqrt{3}.$$

The length of the chord $AB=2AD=10\sqrt{3}$ cm.

EXAMPLE 8.16

Evaluate: $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}$.

SOLUTION

Given, $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}$.

Rationalize the denominator, i.e.,

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}\cdot\frac{1-\sin\theta}{1-\sin\theta}}$$

$$=\sqrt{\frac{(1-\sin\theta)^2}{(1)^2-\sin^2\theta}}=\sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}=\sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}(\because\ \sin^2\theta+\cos^2\theta=1)$$

$$=\sqrt{\left(\frac{1-\sin\theta}{\cos\theta}\right)^2}=\frac{1-\sin\theta}{\cos\theta}$$

$$=\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}=\sec\theta-\tan\theta\left(\because\ \sec\theta=\frac{1}{\cos\theta},\ \tan\theta=\frac{\sin\theta}{\cos\theta}\right)$$

$$\therefore\ \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=\sec\theta-\tan\theta.$$

Standard Position of the Angle

The angle is said to be in its standard position if its initial side coincides with the positive X-axis (see Fig. 8.7).

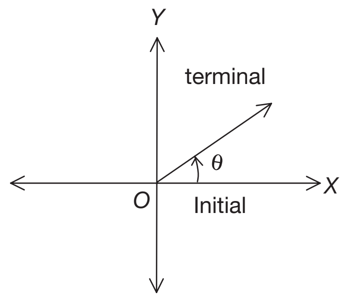


Figure 8.7

Notes

1. Rotation of terminal side in anti-clockwise direction we consider the angle formed is positive and rotation in clockwise direction the angle formed is negative.
2. Depending upon the position of terminal side we decide the angle in different quadrants.

Co-terminal Angles

The angles that differ by either 360° or the integral multiples of 360° are called co-terminal angles (see Fig. 8.8).

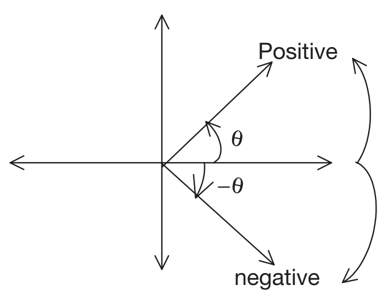


Figure 8.8

Example: $60°$, $360° + 60° = 420°$, $2 \cdot 360° + 60° = 780°$ are co-terminal angles.

Notes

1. If θ is an angle then its co-terminal angle is in the form of $(n \cdot 360° + \theta)$.
2. The terminal side of co-terminal angles in their standard position coincides (see Fig. 8.9).

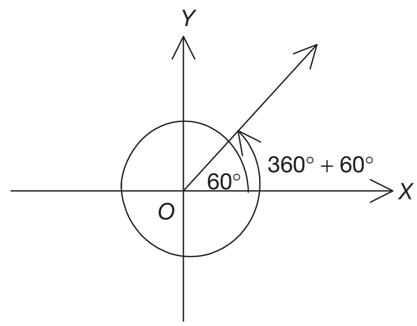


Figure 8.9

Signs of Trigonometric Ratios

1. If θ lies in the first quadrant, i.e., $0 < \theta < \dfrac{\pi}{2}$, then all the trigonometric ratios are taken positive.
2. If θ lies in the second quadrant, i.e., $\dfrac{\pi}{2} < \theta < \pi$, then only $\sin\theta$ and $\operatorname{cosec}\theta$ are taken positive and all the other trigonometric ratios are taken negative.
3. If θ lies in the third quadrant, i.e., $\pi < \theta < \dfrac{3\pi}{2}$, then only $\tan\theta$ and $\cot\theta$ are taken positive and all the other trigonometric ratios are taken negative.
4. If θ lies in the fourth quadrant, i.e., $\dfrac{3\pi}{2} < \theta < 2\pi$, then only $\cos\theta$ and $\sec\theta$ are taken positive and all the other trigonometric ratios are taken negative.

Trigonometric Ratios of $(90° - \theta)$

$\sin(90° - \theta) = \cos\theta$ $\qquad$ $\cos(90° - \theta) = \sin\theta$
$\tan(90° - \theta) = \cot\theta$ $\qquad$ $\cot(90° - \theta) = \tan\theta$
$\operatorname{cosec}(90° - \theta) = \sec\theta$ $\qquad$ $\sec(90° - \theta) = \operatorname{cosec}\theta$

Trigonometric Ratios of $(90° + \theta)$

$\sin(90° + \theta) = \cos\theta$ $\qquad$ $\cos(90° + \theta) = -\sin\theta$
$\tan(90° + \theta) = -\cot\theta$ $\qquad$ $\cot(90° + \theta) = -\tan\theta$
$\operatorname{cosec}(90° + \theta) = \sec\theta$ $\qquad$ $\sec(90° + \theta) = -\operatorname{cosec}\theta$

Trigonometric Ratios of $(180° - \theta)$

$\sin(180° - \theta) = \sin\theta$ $\qquad$ $\cos(180° - \theta) = -\cos\theta$
$\tan(180° - \theta) = -\tan\theta$ $\qquad$ $\cot(180° - \theta) = -\cot\theta$
$\operatorname{cosec}(180° - \theta) = \operatorname{cosec}\theta$ $\qquad$ $\sec(180° - \theta) = -\sec\theta$

Trigonometric Ratios of (180° + θ)

$\sin(180° + \theta) = -\sin\theta$ $\cos(180° + \theta) = -\cos\theta$

$\tan(180° + \theta) = \tan\theta$ $\cot(180° + \theta) = \cot\theta$

$\operatorname{cosec}(180° + \theta) = -\operatorname{cosec}\theta$ $\sec(180° + \theta) = -\sec\theta$

Similarly, the trigonometric ratios of $270° \pm \theta$ and $360° \pm \theta$ can be written.

Note The trigonometric ratios of $(-\theta)$ are the same as the trigonometric ratios of $(360° - \theta)$. So, $\sin(-\theta) = \sin(360° - \theta) = -\sin\theta$ and so on.

EXAMPLE 8.17

What is the value of tan 315°?

SOLUTION

$\tan 315° = \tan(360° - 45°) = -\tan 45° = -1$

$\therefore\ \tan 315° = -1.$

EXAMPLE 8.18

Find the value of $\sin^2 135° + \sec^2 135°$.

SOLUTION

$$\sin 135° = \sin(180° - 45°) = \sin 45° = \frac{1}{\sqrt{2}}.$$

$$\sec 135° = \sec(180° - 45°) = -\sec 45° = -\sqrt{2}.$$

$$\therefore\ \sin^2 135° + \sec^2 135° = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(-\sqrt{2}\right)^2 = \frac{1}{2} + 2 = \frac{5}{2}.$$

EXAMPLE 8.19

If $\cos A = \frac{5}{13}$ and A is not in first quadrant, then find the value of $\frac{\sin A - \cos A}{\tan A + 1}$.

SOLUTION

Given that $\cos A = \frac{5}{13}$ and A is not in first quadrant

$\Rightarrow$ A is in fourth quadrant.

$\sin A = \frac{-12}{13}$ and $\tan A = \frac{-12}{5}$

Now, $$\frac{\sin A - \cos A}{\tan A + 1} = \frac{\frac{-12}{13} - \frac{5}{13}}{\frac{-12}{5} + 1} = \frac{-17}{13} \times \frac{-5}{7} = \frac{85}{91}.$$

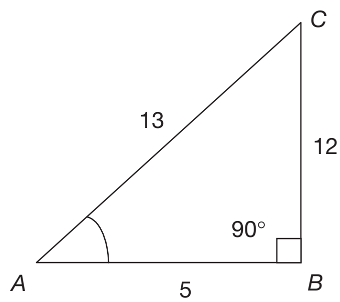


Figure 8.10

EXAMPLE 8.20

If $ABCD$ is a cyclic quadrilateral, then find the value of $\cos A \cos B - \cos C \cos D$.

SOLUTION

Given, $ABCD$ is a cyclic quadrilateral

$A + C = 180°$ and $B + D = 180°$

Now, $\cos A \cos B - \cos C \cos D$

$= \cos A \cos B - \cos(180° - A) \cos(180° - B) = \cos A \cos B - (-\cos A)(-\cos B)$

$= \cos A \cos B - \cos A \cos B = 0.$

EXAMPLE 8.21

If $\cot 15° = m$, then find $\dfrac{\cot 195° + \cot 345°}{\tan 15° - \cot 105°}$.

SOLUTION

Given, $\cot 15° = m \quad \Rightarrow \quad \tan 15° = \dfrac{1}{m}$ and $\tan 75° = m$ ($\because \tan(90° - \theta) = \cot\theta$).

$$\frac{\cot 195° + \cot 345°}{\tan 15° - \cot 105°} = \frac{\cot(180° + 15°) + \cot(360° - 15°)}{\tan 15° - \cot(90° + 15°)} = \frac{\cot 15° - \cot 15°}{\tan 15° - (-\tan 15°)} = 0.$$

EXAMPLE 8.22

If $\sin\theta$ and $\cos\theta$ are the roots of the equation $mx^2 + nx + 1 = 0$, then find the relation between m and n.

SOLUTION

The given equation is $mx^2 + nx + 1 = 0$

Here, $a = m$, $b = n$ and $c = 1$

$$\sin\theta + \cos\theta = \frac{-b}{a} = \frac{-n}{m}$$

$$\sin\theta + \cos\theta = \frac{c}{a} = \frac{1}{m}$$

Consider,

$$\sin\theta + \cos\theta = \frac{-n}{m}$$

$$\text{or} \quad (\sin\theta + \cos\theta)^2 = \left(\frac{-n}{m}\right)^2$$

$$\text{or,} \quad \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{n^2}{m^2}$$

$$\Rightarrow \quad 1 + 2\sin\theta\cos\theta = \frac{n^2}{m^2} \; (\because \; \sin^2\theta + \cos^2\theta = 1)$$

$$1 + 2\left(\frac{1}{m}\right) = \frac{n^2}{m^2}\left(\because \; \sin\theta\cos\theta = \frac{1}{m}\right)$$

$$1 + \frac{2}{m} = \frac{n^2}{m^2}$$

$$\Rightarrow \quad n^2 - m^2 = 2m.$$

EXAMPLE 8.23

If $\sin\alpha = \frac{1}{3}$ and $\cos\beta = \frac{4}{5}$, then find $\sin(\alpha + \beta)$.

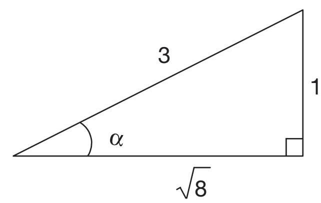


Figure 8.11

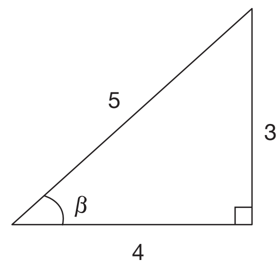


Figure 8.12

SOLUTION

Given, $\sin\alpha = \frac{1}{3}$

$$\cos\alpha = \frac{\sqrt{8}}{3}$$

$$\cos\beta = \frac{4}{5}$$

$$\sin\beta = \frac{3}{5}$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= \frac{1}{3}\cdot\frac{4}{5} + \frac{\sqrt{8}}{3}\cdot\frac{3}{5} = \frac{4 + 3\sqrt{8}}{15}.$$

$$\sin(\alpha + \beta) = \frac{4 + 3\sqrt{8}}{15}.$$

EXAMPLE 8.24

Express the following as a single trigonometric ratio

(a) $\sqrt{3}\cos\theta - \sin\theta$

(b) $\sin\theta - \cos\theta$

SOLUTION

(a) Given, $\sqrt{3}\cos\theta - \sin\theta = 2\left(\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta\right)$

$$= 2(\cos\theta\cdot\cos 30^\circ - \sin\theta\cdot\sin 30^\circ) = 2(\cos(\theta + 30^\circ))$$

$$\Rightarrow \quad \sqrt{3}\cos\theta - \sin\theta = 2\cos(\theta + 30^\circ).$$

(b) $\sin\theta - \cos\theta = \sqrt{2}\left(\dfrac{\sin\theta - \cos\theta}{\sqrt{2}}\right)$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\theta - \frac{1}{\sqrt{2}}\cos\theta\right) = \sqrt{2}\left[\sin\theta\cos\left(\frac{\pi}{4}\right) - \cos\theta\sin\left(\frac{\pi}{4}\right)\right]$$

$$= \sqrt{2}\sin\left(\theta - \frac{\pi}{4}\right) \because\ [\sin(A-B) = \sin A\cos B - \cos A\sin B].$$

EXAMPLE 8.25

If $A + B = 90°$, then prove that

(a) $\sin^2 A + \sin^2 B = 1$

(b) $\tan^2 A - \cot^2 B = 0$

SOLUTION

Given, $A + B = 90°$

$$\Rightarrow \quad B = 90° - A.$$

(a) $\sin^2 A + \sin^2 B = \sin^2 A + \sin^2 (90° - A) = \sin^2 A + \cos^2 A$ ($\therefore$ $\sin(90° - \theta) = \cos\theta$) $= 1$ $\sin^2 A + \sin^2 B = 1$.

(b) $\tan^2 A - \cot^2 B = \tan^2 A - \cot^2 (90° - A) = \tan^2 A - \tan^2 A$ ($\because$ $\tan\theta = \cot(90° - \theta)$) $= 0$ $\tan^2 A - \cot^2 B = 0$.

EXAMPLE 8.26

Simplify the followings:

(a) $\begin{vmatrix} \cos A & \sin A \\ \sin A & -\cos A \end{vmatrix}$

(b) $\log(\cot 1°) + \log \cot 2° + \log(\cot 3°) + \cdots + \log(\cot 89°)$

(c) $\sin 1° \cdot \sin 2° \ldots \sin 181°$

SOLUTION

(a) $\begin{vmatrix} \cos A & \sin A \\ \sin A & -\cos A \end{vmatrix} = \cos A(-\cos A) - \sin A \sin A = -\cos^2 A - \sin^2 A$

$= -(\sin^2 A + \cos^2 A) = -1$

$\therefore \quad \begin{vmatrix} \cos A & \sin A \\ \sin A & -\cos A \end{vmatrix} = -1.$

(b) $\log(\cot 1°) + \log \cot 2° + \cdots + \log \cot 89°$

$= \log(\cot 1° \cdot \cot 2° \ldots \cot 89°)$. [$\because$ $\log a + \log b + \cdots + \log n = \log(abc \ldots n)$]

We know that,

$\cot 1° = \tan 89°$, $\cot 2° = \tan 88°$ and so on

$\cot 1° \cdot \cot 89° = \cot 1° \cdot \tan 1° = 1$ and so on

$\cot 1° \cdot \cot 2° \ldots \cot 89° = 1$

$\Rightarrow \quad \log(\cot 1° \cot 2° \ldots \cot 89°) = \log(1) = 0.$

(c) $\sin 1° \cdot \sin 2° \cdot \sin 3° \ldots \sin 181°.$

We know that, $\sin 180° = 0$

$\sin 1° \cdot \sin 2° \ldots \sin 180° \cdot \sin 181° = 0.$

Trigonometric Tables

The values of trigonometric ratios of different angles can be found by using natural sines, natural cosines, natural tangents, etc. These tabular values are approximate and tally upto three decimal places.

The following example gives an idea how to find any value of trigonometric ratios.

EXAMPLE 8.27

Find the value of $\sin 65° 28'$.

SOLUTION

Step 1: We have to refer the table of natural sines in order to find the value of $\sin 65°28'$.

Step 2: We have to slide our view from left to right in the row containing 65° until we reach the intersection with 28′ and directly minutes table, so as to locate the nearest approximation in the minutes, i.e., in this problem 24′ and note down the value.

Step 3: We locate the difference of minutes (i.e., here 4′) in the mean difference and add it to the value in step 2.

To get the value of $\sin 65° 28'$, consider the following procedure.

0′	6′	12′	18′	24′	…	54′	Mean Differences				
							1	2	3	4	5
0.9063	9070	9078	9085	9092	…	9128	1	2	4	5	6

Now,

$$\sin 65° 28' = \sin 65° \cdot 24' + 4'$$
$$= 0.9092 + 0.0005$$
$$= 0.9097.$$

Hence, $\sin 65° 28' = 0.9097$.

EXAMPLE 8.28

Find the area of the right angle triangle with one of the acute angles being 65° and hypotenuse 6 cm.

SOLUTION

Let the right triangle be ABC, $\angle B = 90°$, $\angle A = 65°$ and $AC = 6$ cm.

From ΔABC,

$$\cos 65° = \frac{AB}{AC}$$

$$0.4226 = \frac{AB}{6}$$

$$AB = 2.5356.$$

$$\sin 65° = \frac{BC}{AC} \Rightarrow 0.9063 = \frac{BC}{6}$$

$$BC = 5.4378.$$

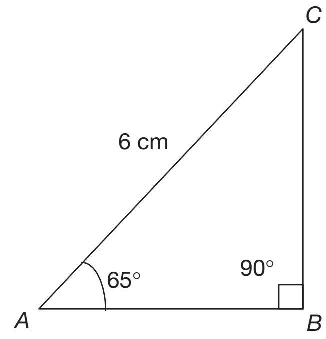


Figure 8.13

Area of $\Delta ABC = \frac{1}{2} AB \times BC = \frac{1}{2}(2.5356)\ (5.4378)$

$= 6.8940$ cm^2. (approximately)

Hence, the area of the ΔABC is 6.8940 cm^2.

EXAMPLE 8.29

Find the length of the chord which subtends an angle of 110° at the centre of the circle of radius 7 cm.

SOLUTION

Let the chord be AB. O be the centre of the circle and OD is shortest distance of the chord from the centre of circle.

Given $OA = OB = 7$ cm, $\angle AOB = 110°$.

Clearly, $\Delta AOD \cong \Delta BOD$ by SSS axiom

$$\angle AOD = \angle BOD = \frac{1}{2}\angle AOB = 55°.$$

In ΔAOD,

$$\sin 55° = \frac{AD}{AO}$$

$$0.8192 = \frac{AD}{7}$$

$$AD = 5.7344.$$

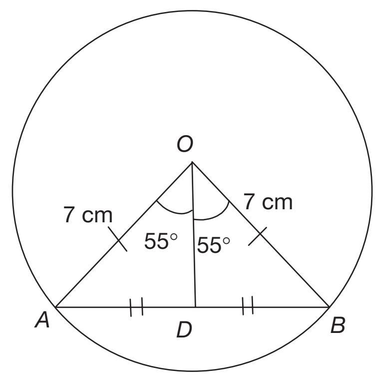


Figure 8.14

The length of the chord $= 2AD$

$= 2(5.7344)$

$= 11.4688$ cm.

HEIGHTS AND DISTANCES

1. Let AB be a vertical line and PA and P^1B be two horizontal lines as shown in the Fig. 8.15.
2. Let $\angle APB = \alpha$ and $\angle PBP^1 = \beta$. Then,
   - **(i)** α is called the angle of elevation of the point B as seen from the point P and
   - **(ii)** β is called the angle of depression of the point P as seen from the point B.

Note Angle of elevation is always equal to the angle of depression.

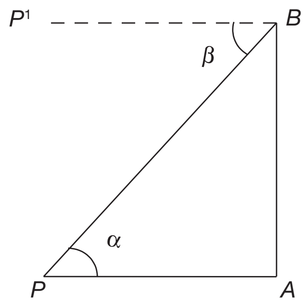


Figure 8.15

EXAMPLE 8.30

From a point on the ground which is at a distance of 50 m from the foot of the tower, the angle of elevation of the top of the tower is observed to be 30°. Find the height of the tower.

SOLUTION

Let the height of the tower be h m.

From ΔPAB, $\tan 30° = \dfrac{AB}{PA}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{50} \quad \Rightarrow \quad h = \frac{50}{\sqrt{3}} \text{ (or) } \frac{50\sqrt{3}}{3}.$$

Hence, the height of the tower is $\dfrac{50\sqrt{3}}{3}$ m.

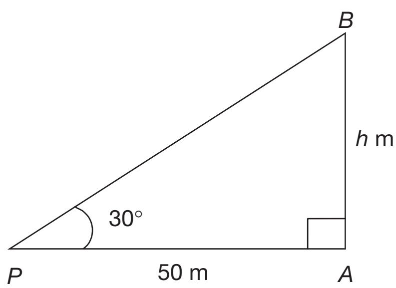


Figure 8.16

EXAMPLE 8.31

The angle of elevation of the top of a tower is 45°. On walking 20 m towards the tower along the line joining the foot of the observer and foot of the tower, the angle of elevation is found to be 60°. Find the height of the tower.

SOLUTION

Let the height of the tower be h m.

Let $QA = x$ m.

In ΔPAB,

$$\tan 45° = \frac{AB}{PA} \Rightarrow 1 = \frac{h}{20+x} \Rightarrow 20 + x = h$$

$$\Rightarrow x = h - 20 \quad (1)$$

From $\Delta\ QAB$,

$$\tan 60° = \frac{AB}{QA}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}\,x \Rightarrow h = \sqrt{3}\,(h-20), \text{ (using Eq. (1))}$$

$$\Rightarrow (\sqrt{3}-1)\,h = 20\sqrt{3} \Rightarrow h = \frac{20\sqrt{3}}{\sqrt{3}-1} = \frac{20\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)\,(\sqrt{3}+1)} = \frac{20(3+\sqrt{3})}{3-1} = 10(3+\sqrt{3}).$$

Hence, the height of the tower is $10(3+\sqrt{3})$ m.

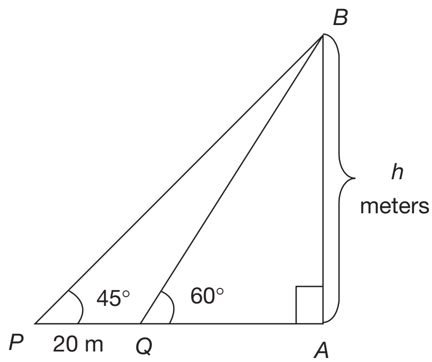


Figure 8.17

EXAMPLE 8.32

From the top of a building 100 m high, the angles of depression of the bottom and the top of an another building just opposite to it are observed to be 60° and 45° respectively. Find the height of the building.

SOLUTION

Let the height of the building be h m.

Let $AC = BD = d$ m.

From ΔBDE,

$$\tan 45° = \frac{ED}{BD} \Rightarrow 1 = \frac{100-h}{d}$$

$$\Rightarrow d = 100 - h \quad (1)$$

From $\Delta\ ACE$,

$$\tan 60° = \frac{CE}{AC}$$

$$\Rightarrow \sqrt{3} = \frac{100}{d} \Rightarrow \sqrt{3}d = 100 \Rightarrow d = \frac{100}{\sqrt{3}}$$

$$\Rightarrow \quad 100 - h = \frac{100}{\sqrt{3}} \text{ (using Eq. (1))} \quad \Rightarrow \quad h = 100 - \frac{100}{\sqrt{3}}$$

$$= 100\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = \frac{100(3-\sqrt{3})}{3}.$$

Hence, the height of the tower is $\dfrac{100(3-\sqrt{3})}{3}$ m.

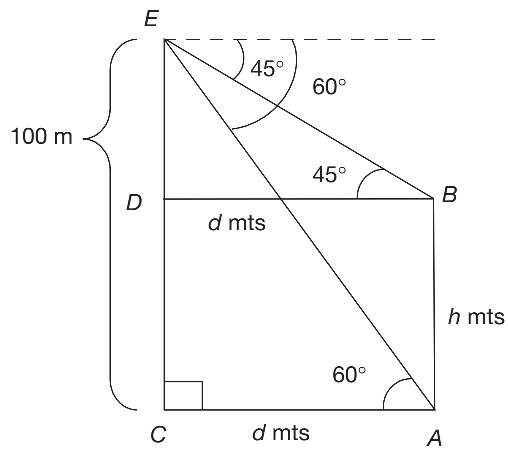


Figure 8.18

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. If $\sin\theta = \frac{1}{2}$ where $0° \leq \theta \leq 180°$, then the possible values of θ are _______.
2. $\cot\theta$ interms of $\sin\theta =$ _______ $(0 \leq \theta \leq 90°)$.
3. If A and B are two complementary angles, then $\sin A \cdot \cos B + \cos A \cdot \sin B =$ _______.
4. If the angle of a sector is 45° and the radius of the sector is 28 cm then the length of the arc is _______.
5. If $ABCD$ is a cyclic quadrilateral, then $\tan A + \tan C =$ _______.
6. $\frac{1-\cos 2\theta}{2} =$ ______ (in terms of $\sin\theta$).
7. $\cos 1° \cdot \cos 2° \cdot \cos 3° \cdots \cos 120° =$ _______.
8. The $\frac{3\pi}{2}$ is equivalent to _______ in centesimal system.
9. If $A + B = 360°$, then $\frac{\tan A + \tan B}{1 - \tan A \tan B} =$ _________.
10. If $\tan\theta + \cot\theta = 2$, then $\tan^{10}\theta + \cot^{10}\theta =$ _______ (where $0 < \theta < 90°$).
11. Write an equation eliminating θ from the equations $a = d\sin\theta$ and $c = d\cos\theta$.
12. Convert 250^g into other two measures.
13. $\sin(180 + \theta) + \cos(270 + \theta) + \cos(90 + \theta) + \sin(360 + \theta) =$ _______.
14. If $\sin\theta + \cos\theta = 1$ and $0° \leq \theta \leq 90°$, then the possible values of θ are _______.
15. Evaluate $\sin^2 45° + \cos^2 60° + \text{cosec}^2 30°$.
16. If $ABCD$ is a cyclic quadrilateral, then find the value of $\cos A + \cos B + \cos C + \cos D$.
17. $\text{cosec}(7\pi + \theta) \cdot \sin(8\pi + \theta) =$ _______.
18. If $\theta_1 = \frac{7}{25}$ and $\theta_2 = \frac{24}{25}$, then find the relation between θ_1 and θ_2.
19. Find the value of tan 1140°.
20. If $\sin(A+B) = \cos(A-B) = \frac{\sqrt{3}}{2}$ then $\cot 2A =$ _______.
21. If ΔABC is an isosceles triangle and right angled at B, then $\frac{\tan A + \tan C}{\cot A + \cot C} =$ ________.
22. $[\sin(x - \pi) + \cos(x - \pi/2)] \cdot \cos(x - 2\pi) =$ _______.
23. $\tan(A + B)\tan(A - B) =$ _______.
24. The angles of a quadrilateral are in the ratio 1 : 2 : 3 : 4. Then the smallest angle in the centesimal system is _______.
25. If $\alpha + \beta = 90°$ and $\alpha = \frac{\beta}{2}$ then $\tan\alpha \cdot \tan\beta =$ _______.
26. $[\sin\alpha + \sin(180 - \alpha) + \sin(180 + \alpha)]\ \text{cosec}\,\alpha =$ _______.
27. Express $\frac{\tan\theta + 1}{\tan\theta - 1}$ as a single trigonometric ratio.
28. If $\text{cosec}\,\theta + \cot\theta = 3$, then find $\cos\theta$.
29. The top of a building from a fixed point is observed at an angle of elevation 60° and the distance from the foot of the building to the point is 100 m. then the height of the building is _______.
30. If $\cot\theta = \frac{4}{3}$ where $180 < \theta < 270$, then $\sin\theta + \cos\theta =$ _______.

Short Answer Type Questions

31. If the tip of the pendulum of a clock travels 13.2 cm in one oscillation and the length of the pendulum is 6.3 cm, then the angle made by the pendulum in half oscillation in radian system is _______.
32. If $\text{cosec}\,\theta$, $\sec\theta$ and $\cot\theta$ are in HP, then $\frac{\sin\theta + \tan\theta}{\cos\theta} =$ _______.
33. $\cot\frac{\pi}{18} \cdot \cot\frac{\pi}{9} \cdot \cot\frac{\pi}{4} \cdot \cot\frac{4\pi}{18} \cdot \cot\frac{7\pi}{18} =$ _______.
34. If $\cot\theta = \frac{3}{4}$ and θ is acute, then find the value of $\frac{\tan\theta + \cot\theta}{\sec\theta + \text{cosec}\,\theta}$.

35. Simplify $\sin(A + 45°) \sin(A - 45°)$.

36. Eliminate θ from the following equations: $x = a \sin \theta$, $y = b \cos \theta$ and $z = a \sin^2 \theta + b \cos^2 \theta$.

37. If $\sin A = \frac{3}{5}$ and A is not in the first quadrant, then find $\frac{\cos A + \sin 2A}{\tan A + \sec A}$.

38. If $\cos(A - B) = \frac{5}{13}$ and $\sin(A + B) = \frac{4}{5}$, then find $\sin 2B$.

39. If $\operatorname{cosec} \theta - \cot \theta = 2$, then find the value of $\operatorname{cosec}^2 \theta + \cot^2 \theta$.

40. Prove that $\frac{1 + \cos A}{1 - \cos A} = (\operatorname{cosec} A + \cot A)^2$.

41. If $\tan 28° = n$, then find the value of $\frac{\tan 152° + \tan 62°}{\tan 242° + \tan 28°}$.

42. If $3\sin A + 4\cos A = 4$, then find $4\sin A - 3\cos A$.

43. A ladder of length 50 m rests against a vertical wall, at height of 30 m from the grand. Find the inclination of the ladder with the horizontal. Also find the distance between the foot of the ladder and the wall.

44. Eliminate θ from the following equations: $x \sin \alpha + y \cos \alpha = p$ and $x \cos \alpha - y \sin \alpha = q$.

45. Prove that $\left(\frac{1 - \sin \alpha}{\cos \alpha} + \frac{\cos \alpha}{1 + \sin \alpha}\right)\left(\sec \alpha + \frac{1}{\cot \alpha}\right) = 2$.

Essay Type Questions

46. Show that $6(\sin x + \cos x)^4 + 12(\sin x - \cos x)^2 + 8(\sin^6 x + \cos^6 x) = 26$.

47. Prove that $\frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} = \frac{1 + \cos A}{\sin A}$.

48. The angle of depression of the top of the tower from the top of a building is 30° and angle of elevation of the top of the tower from the bottom of the building is 45° and if the height of the tower is 20 m then find the height of the building.

49. A vertical pole is 60 m high. The angles of depression of two points P and Q on the ground are 30° and 45° respectively. If the points P and Q lie on either side of the pole, then find the distance PQ.

50. Prove that $\sin^8 \theta - \cos^8 \theta = \cos 2\theta\,(2\sin^2 \theta \cos^2 \theta - 1)$.

CONCEPT APPLICATION

Level 1

1. If $\sin x° = \sin \alpha x$, then α is

(a) $\frac{180}{\pi}$ (b) $\frac{\pi}{270}$

(c) $\frac{270}{\pi}$ (d) $\frac{\pi}{180}$

2. If in a triangle ABC, A and B are complementary, then $\tan C$ is

(a) ∞ (b) 0

(c) 1 (d) $\sqrt{3}$

3. If $\alpha = \frac{4}{5}$ and $\beta = \frac{4}{5}$, then which of the following is true?

(a) $\alpha < \beta$ (b) $\alpha > \beta$

(c) $\alpha = \beta$ (d) None of these

4. $\sin^2 20 + \sin^2 70$ is equal to ______.

(a) 1 (b) −1

(c) 0 (d) 2

5. $\cos 50° 50^1 \cos 9° 10^1 - \sin 50° 50^1 \sin 9° 10^1 =$

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) $\frac{\sqrt{3}}{2}$

6. $\sin \theta \cos (90° - \theta) + \cos \theta \sin (90° - \theta)$ ______.

(a) −1 (b) 2

(c) 0 (d) 1

7. A wheel makes 20 revolutions per hour. The radians it turns through 25 minutes is

PRACTICE QUESTIONS

(a) $\frac{50\pi^c}{7}$ (b) $\frac{250\pi^c}{3}$

(c) $\frac{150\pi^c}{7}$ (d) $\frac{50\pi^c}{3}$

8. $\frac{\sin^4\theta - \cos^4\theta}{\sin^2\theta - \cos^2\theta} =$

(a) -1 (b) 2

(c) 0 (d) 1

9. Simplified expression of $(\sec\theta + \tan\theta)(1 - \sin\theta)$ is

(a) $\sin^2\theta$ (b) $\cos^2\theta$

(c) $\tan^2\theta$ (d) $\cos\theta$

10. If $a = \sec\theta - \tan\theta$ and $b = \sec\theta + \tan\theta$, then

(a) $a = b$ (b) $\frac{1}{a} = \frac{-1}{b}$

(c) $a = \frac{1}{b}$ (d) $a - b = 1$

11. If $\sec\alpha + \tan\alpha = m$, then $\sec^4\alpha - \tan^4\alpha - 2\sec\alpha\tan\alpha$ is

(a) m^2 (b) $-m^2$

(c) $\frac{1}{m^2}$ (d) $\frac{-1}{m^2}$

12. If $\sin^4 A - \cos^4 A = 1$, then $(A/2)$ is $(0 < A \leq 90°)$

(a) 45° (b) 60°

(c) 30° (d) 40°

13. The value of tan 15° tan 20° tan 70° tan 75° is

(a) -1 (b) 2

(c) 0 (d) 1

14. In a ΔABC, $\tan\left(\frac{A+C}{2}\right) =$

(a) $\tan\frac{B}{2}$ (b) $\cot\frac{B}{2}$

(c) $-\tan B$ (d) $\cot B$

15. If $\tan(A - 30°) = 2 - \sqrt{3}$, then find A.

(a) $\frac{\pi^c}{2}$ (b) $\frac{\pi^c}{4}$

(c) $\frac{\pi^c}{6}$ (d) $\frac{\pi^c}{3}$

16. If $\sin^4\theta - \cos^4\theta = K^4$ then $\sin^2\theta - \cos^2\theta$ is

(a) K^4 (b) K^3

(c) K^2 (d) K

17. $\frac{\tan^3\theta - 1}{\tan\theta - 1} =$

(a) $\sec^2\theta + \tan\theta$ (b) $\sec^2\theta - \tan\theta$

(c) 0 (d) $\tan\theta - \sec^2\theta$

18. For all values of θ, $1 + \cos\theta$ can be _______.

(a) positive (b) negative

(c) non-positive (d) non-negative

19. If $\sin 3\theta = \cos(\theta - 6°)$, where 3θ and $(\theta - 6°)$ are acute angles then the value of θ is

(a) 42° (b) 24°

(c) 12° (d) 26°

20. $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A) =$

(a) -1 (b) 2

(c) 0 (d) 1

21. If $x = a(\operatorname{cosec}\theta + \cot\theta)$ and $y = b(\cot\theta - \operatorname{cosec}\theta)$, then

(a) $xy - ab = 0$ (b) $xy + ab = 0$

(c) $\frac{x}{a} + \frac{y}{b} = 1$ (d) $x^2y^2 = ab$

22. The value of $\frac{\cos^4 x + \cos^2 x\sin^2 x + \sin^2 x}{\cos^2 x + \sin^2 x\cos^2 x + \sin^4 x}$ is

(a) 2 (b) 1

(c) 3 (d) 0

23. $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$ is equal to

(a) $2\sec^2\theta$ (b) $2\cos^2\theta$

(c) 0 (d) 1

24. If $\tan(\alpha + \beta) = \frac{1}{2}$ and $\tan\alpha = \frac{1}{3}$, then $\tan\beta =$

(a) $\frac{1}{6}$ (b) $\frac{1}{7}$

(c) 1 (d) $\frac{7}{6}$

25. The value of log sin 0° + log sin 1° + log sin 2° + ⋯ + log sin 90° is

(a) 0 (b) 1

(c) -1 (d) undefined

26. Which of the following is not possible?

(a) $\sin\theta = \frac{3}{5}$ (b) $\sec\theta = 100$

(c) $\operatorname{cosec}\theta = 0.14$ (d) None of these

27. $\sin^2 20° + \cos^2 160° - \tan^2 45° =$

(a) 2 (b) 0

(c) 1 (d) −2

28. $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}=$

(a) $\frac{2}{1-2\cos^2\theta}$

(b) $\frac{2}{2\sin^2\theta-1}$

(c) Both (a) and (b)

(d) None of these

29. The length of the side (in cm) of an equilateral triangle inscribed in a circle of radius 8 cm is

(a) $16\sqrt{3}$ (b) $12\sqrt{3}$

(c) $8\sqrt{3}$ (d) $10\sqrt{3}$

30. Which among the following is true?

(a) $\sin 1° > \sin 1^c$

(b) $\sin 1° < \sin 1^c$

(c) $\sin 1° = \sin 1^c$

(d) None of these

Level 2

31. If $2\sin\alpha + 3\cos\alpha = 2$, then $3\sin\alpha - 2\cos\alpha =$

(a) ±3 (b) ±1

(c) 0 (d) ±2

32. If $\frac{\sin^2\alpha - 3\sin\alpha + 2}{\cos^2\alpha} = 1$, then α can be

(a) 60° (b) 45°

(c) 0° (d) 30°

33. If $\cot A = \frac{5}{12}$ and A is not in the first quadrant, then $\frac{\sin A - \cos A}{1+\cot A}$ is

(a) $\frac{-74}{25}$ (b) $\frac{-84}{221}$

(c) $\frac{-87}{223}$ (d) None of these

34. If $\frac{1+\sin\alpha}{1-\sin\alpha} = \frac{m^2}{n^2}$, then $\sin\alpha$ is

(a) $\frac{m^2+n^2}{m^2-n^2}$ (b) $\frac{m^2-n^2}{m^2+n^2}$

(c) $\frac{m^2+n^2}{n^2-m^2}$ (d) $\frac{n^2-m^2}{m^2+n^2}$

35. If $\sin\theta - \cos\theta = \frac{3}{5}$, then $\sin\theta\cos\theta =$

(a) $\frac{16}{25}$ (b) $\frac{9}{16}$ (c) $\frac{9}{25}$ (d) $\frac{8}{25}$

36. If $ABCD$ is a cyclic quadrilateral, then the value of $\cos^2 A - \cos^2 B - \cos^2 C + \cos^2 D$ is

(a) 0 (b) 1

(c) −1 (d) 2

37. The length of minute hand of a wall clock is 12 cm. Find the distance covered by the tip of the minute hand in 25 minutes.

(a) $\frac{220}{7}$ cm (b) $\frac{110}{7}$ cm

(c) $\frac{120}{7}$ cm (d) $\frac{240}{7}$ cm

38. $\sin^2 2° + \sin^2 4° + \sin^2 6° + \cdots + \sin^2 90° =$ ________

(a) 22 (b) 23

(c) 44 (d) 45

39. A straight highway leads to the foot of a tower of height 50 m. From the top of the tower, the angles of depression of two cars standing on the highway are 30° and 60°. What is the distance between the two cars? (in metres)

(a) $\frac{100}{\sqrt{3}}$ (b) $50\sqrt{3}$

(c) $\frac{50}{\sqrt{3}}$ (d) $100\sqrt{3}$

40. The angle of elevation of the top of a hill from the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°.

If the tower is 50 m high, then what is the height of the hill?

(a) 180 m (b) 150 m

(c) 100 m (d) 120 m

41. $\tan 38° - \cot 22° =$

(a) $\frac{1}{2}\text{cosec}\, 38° \sec 22°$

(b) $2 \sin 22° \cos 38°$

(c) $-\frac{1}{2}\text{cosec}\, 22° \sec 38°$

(d) None of these

42. $\frac{1-\cos\theta}{\sin\theta} + \frac{\sin\theta}{1-\cos\theta} =$

(a) $2 \sin\theta$ (b) $2 \cos\theta$

(c) $2\,\text{cosec}\,\theta$ (d) $2 \sec\theta$

43. $\sqrt{-4+\sqrt{8+16\,\text{cosec}^4\alpha \sin^4\alpha}} =$

(a) $\text{cosec}\,\alpha - \sin\alpha$

(b) $2\text{cosec}\,\alpha + \sin\alpha$

(c) $2\text{cosec}\,\alpha - \sin\alpha$

(d) $\text{cosec}\,\alpha - 2\sin\alpha$

44. The angles of depression of the top and the bottom of a 7 m tall building from the top of a tower are 45° and 60° respectively. Find the height of the tower in metres.

(a) $7(3+\sqrt{3})$ (b) $\frac{7}{2}(3-\sqrt{3})$

(c) $\frac{7}{2}(3+\sqrt{3})$ (d) $7(3-\sqrt{3})$

45. If $\tan 86° = m$, then $\frac{\tan 176° + \cot 4°}{m + \tan 4°}$ is

(a) $\frac{m^2-1}{m^2+1}$. (b) $\frac{m^2+1}{1-m^2}$.

(c) $\frac{1-m^2}{1+m^2}$. (d) $\frac{m^2+1}{m^2-1}$.

46. The following sentences are the steps involved in proving the result $\frac{\cos x}{1-\tan x} + \frac{\sin x}{1-\cot x} = \cos x + \sin x$. Arrange them in sequential order from first to last.

(A) $\frac{\cos^2 x}{\cos x - \sin x} + \frac{\sin^2 x}{\sin x - \cos x}$

(B) $\frac{\cos^2 x - \sin^2 x}{\cos x - \sin x}$

(C) $\frac{\cos x}{1-\frac{\sin x}{\cos x}} + \frac{\sin x}{1-\frac{\cos x}{\sin x}}$

(a) (A), (B) and (C)

(b) (C), (A) and (B)

(c) (C), (B) and (A)

(d) None of these

47. The following sentences are the steps involved in eliminating θ from the equation $x = y \tan\theta$ and $a = b \sec\theta$. Arrange them in sequential order from first to last.

(A) Subtract $\left(\frac{x}{y}\right)^2$ from $\left(\frac{a}{b}\right)^2$

(B) $\left(\frac{x}{y}\right)^2 - \left(\frac{a}{b}\right)^2 = 1$

(C) Taking squares on both the sides.

(D) Find $\frac{x}{y}$ and $\frac{a}{b}$

(a) (D), (A), (C) and (B)

(b) (D), (C), (B) and (A)

(c) (D), (B), (A) and (C)

(d) (D), (C), (A) and (B)

Level 3

48. There is a small island in the river which is 100 m wide and a tall tree stands on the island. P and Q are points directly opposite each other on the two banks and in line with the tree. If the angles of elevation of the top of the tree from P and Q are respectively are 30° and 45°, find the height of the tree (in metres).

(a) $50(\sqrt{3}-1)$ (b) $50(\sqrt{3}+1)$

(c) $100(\sqrt{3}+1)$ (d) $100(\sqrt{3}-1)$

49. A balloon is connected to a meteorological ground station by a cable of length 215 m inclined at 60° to the horizontal. Determine the height of the balloon from the ground. Assume that there is no slack in the cable.

(a) $107.5\sqrt{3}$ m (b) $100\sqrt{3}$ m

(c) $215\sqrt{3}$ m (d) $215/\sqrt{3}$ m

50. If $\sin^2 A = 2\sin A\cos A$ and $\sin 20° = K$, then the value of $\cos 20° \cos 40° \cos 80° \cos 160° =$

(a) K (b) $-\sqrt{1-K^2}$

(c) $\frac{\sqrt{1-K^2}}{8}$ (d) $-\frac{\sqrt{1-K^2}}{8}$

51. If $\sqrt{2}\cos\theta - \sqrt{6}\sin\theta = 2\sqrt{2}$, then the value of θ can be

(a) 0° (b) −45°

(c) 30° (d) −60°

52. A circus artist climbs from the ground along a rope which is stretched from the top of a vertical pole and tied at the ground at a certain distance from the foot of the pole. The height of the pole is 12 m and the angle made by the rope with the ground is 30°. Calculate the distance covered by the artist in reaching the top of the pole.

(a) 24 m (b) 6 m

(c) 12 m (d) None of these

53. Find the value of $\sin^2 5° + \sin^2 10° + \sin^2 15° + \cdots + \sin^2 90°$.

(a) 8 (b) 9

(c) $\frac{17}{2}$ (d) $\frac{19}{2}$

54. If $\sec\theta + \tan\theta = 2$, then find the value of $\sin\theta$.

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$

(c) $-\frac{3}{5}$ (d) $-\frac{2}{5}$

55. If $\cos\theta + \left(\frac{1}{\sqrt{3}}\right)\sin\theta = \frac{2}{\sqrt{3}}$, then find θ in circular measure.

(a) $\frac{\pi^c}{10}$ (b) $\frac{\pi^c}{9}$

(c) $\frac{\pi^c}{6}$ (d) $\frac{\pi^c}{3}$

56. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} =$ ____

(a) $\sec\theta + \tan\theta$

(b) $\sec\theta - \cot\theta$

(c) $\operatorname{cosec}\theta + \tan\theta$

(d) $\operatorname{cosec}\theta - \tan\theta$

57. If $\frac{\sin^2\theta - 5\sin\theta + 3}{\cos^2\theta} = 1$, then θ can be

(a) 30° (b) 45°

(c) 60° (d) 0°

58. If $\cot\theta = \frac{24}{7}$ and θ is not in the first quadrant, then find the value of $\tan\theta - \sec\theta$.

(a) 1 (b) $\frac{4}{3}$

(c) $\frac{3}{2}$ (d) $\frac{5}{4}$

59. If $\sin 20° = p$, then find the value of $\left(\frac{\sin 380° - \sin 340°}{\cos 380° + \cos 340°}\right)$.

(a) $\sqrt{1-p^2}$ (b) $\sqrt{\frac{1-p^2}{p}}$

(c) $\frac{p}{\sqrt{1-p^2}}$ (d) None of these

60. Find the value $\tan\left(22\frac{1°}{2}\right)$.

(a) $\sqrt{2}-1$ (b) $1+\sqrt{2}$

(c) $2+\sqrt{3}$ (d) $2-\sqrt{3}$

61. If the sun ray's inclination increases from 45° to 60°, the length of the shadow of a tower decreases by 50 m. Find the height of the tower (in m).

(a) $50(\sqrt{3}-1)$ (b) $75(3-\sqrt{3})$

(c) $100(\sqrt{3}+1)$ (d) $25(3+\sqrt{3})$

62. The angles of depression of two points from the top of the tower are 30° and 60°. If the height of the tower is 30 m, then find the maximum possible distance between the two points.

(a) $40\sqrt{3}$ m (b) $30\sqrt{3}$ m

(c) $20\sqrt{3}$ m (d) $10\sqrt{3}$ m

63. From a point on the ground, the angle of elevation of an aeroplane flying at an altitude of 500 m changes from 45° to 30° in 5 seconds. Find the speed of the aeroplane (in kmph).

(a) $720\left(\sqrt{3}-1\right)$ (b) $720\left(\sqrt{3}+1\right)$

(c) $360\left(\sqrt{3}-1\right)$ (d) $360\left(\sqrt{3}+1\right)$

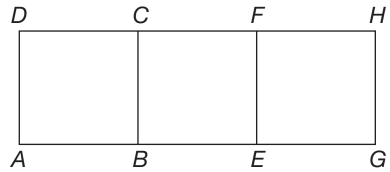
64. From the top of a building, the angles of elevation and depression of top and bottom of a tower are 60° and 30° respectively. If the height of the building is 5 m, then find the height of the tower.

(a) $10\sqrt{3}$ m (b) 15 m

(c) $15\sqrt{3}$ m (d) 20 m

65. In the figure given below (not to scale), *ABCD*, *BEFC* and *EGHF* are three squares.

Find $\angle FAE + \angle HAG$.



(a) 30°

(b) 45°

(c) 60°

(d) None of these

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. 30° or 150°
2. $\dfrac{\sqrt{1-\sin^2\theta}}{\sin\theta}$
3. 1
4. 22 cm
5. 0°.
6. $\sin^2\theta$
7. 0
8. 300^g
9. 0
10. 2
11. $a^2 + c^2 = d^2$
12. $\dfrac{5\pi^c}{4}$
13. 0
14. 0° and 90°
15. $\dfrac{19}{4}$
16. 0
17. −1
18. $\theta_1 = \theta_2$
19. $\sqrt{3}$
20. 0
21. 1
22. 0
23. $\dfrac{\tan^2 A - \tan^2 B}{1-\tan^2 A\tan^2 B}$.
24. 40^g
25. 1
26. 1
27. $-\tan\left(\theta + \dfrac{\pi}{4}\right)$
28. $\dfrac{4}{5}$
29. $100\sqrt{3}$ m
30. $\dfrac{-7}{5}$

Short Answer Type Questions

31. $\dfrac{\pi^c}{3}$
32. 2
33. 1
34. $\dfrac{5}{7}$
35. $-\dfrac{1}{2}\cos 2A$
36. $bx^2 + ay^2 = abz$
37. $\dfrac{22}{25}$
38. $\dfrac{-16}{65}$
39. $\dfrac{17}{8}$
41. $\dfrac{1-n^2}{1+n^2}$
42. ±3
43. 40 m.
44. $x^2 + y^2 = p^2 + q^2$

Essay Type Questions

48. $\dfrac{20(1+\sqrt{3})}{\sqrt{3}}$ m
49. $60(\sqrt{3}+1)$ m

ANSWER KEYS

CONCEPT APPLICATION

Level 1

1. (d)	**2.** (a)	**3.** (b)	**4.** (a)	**5.** (b)	**6.** (d)	**7.** (d)	**8.** (d)	**9.** (d)	**10.** (c)
11. (c)	**12.** (a)	**13.** (d)	**14.** (b)	**15.** (b)	**16.** (a)	**17.** (a)	**18.** (d)	**19.** (b)	**20.** (d)
21. (b)	**22.** (b)	**23.** (a)	**24.** (b)	**25.** (d)	**26.** (b)	**27.** (d)	**28.** (b)	**29.** (b)	**30.** (b)

Level 2

31. (b)	**32.** (d)	**33.** (c)	**34.** (c)	**35.** (c)	**36.** (b)	**37.** (b)	**38.** (b)	**39.** (d)	**40.** (d)
41. (d)	**42.** (c)	**43.** (c)	**44.** (a)	**45.** (d)	**46.** (b)	**47.** (d)			

Level 3

48. (d)	**49.** (c)	**50.** (b)	**51.** (b)	**52.** (c)	**53.** (d)	**54.** (a)	**55.** (c)	**56.** (a)	**57.** (a)
58. (b)	**59.** (c)	**60.** (a)	**61.** (d)	**62.** (a)	**63.** (c)	**64.** (d)	**65.** (b)		

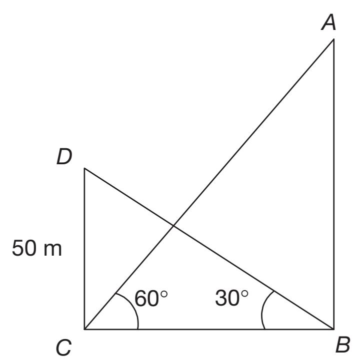
CONCEPT APPLICATION

Level 1

1. Use the relation between degrees and radians.
2. $A + B = 90°$ and $A + B + C = 180°$.
3. Find $\sin\beta$ and compare $\sin\alpha$ and $\sin\beta$.
4. $\sin\theta = \cos(90 - \theta)$
5. Use $\cos A \cos B - \sin A \sin B = \cos(A + B)$.
6. $\cos(90 - \theta) = \sin\theta$ and $\sin(90 - \theta) = \cos\theta$.
7. Find the angle covered in one hour.
8. $a^4 - b^4 = (a^2 - b^2)(a^2 + b^2)$.
9. Write $\sec\theta$ and $\tan\theta$ in terms of $\sin\theta$ and $\cos\theta$.
10. Use the identity $\sec^2\theta - \tan^2\theta = 1$.
11. Simplify the given expression and find $\sec\alpha - \tan\alpha$.
12. $a^4 - b^4 = (a^2 - b^2)(a^2 + b^2)$.
13. $\tan\theta = \cot(90 - \theta)$ and $\tan\theta \cdot \cot\theta = 1$.
14. $A + B + C = 180°$.
15. Use the identity $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$.
16. $a^4 - b^4 = (a^2 - b^2)(a^2 + b^2)$.
17. Use $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.
18. Recall the range of $\cos\theta$.
19. Use complementary angles.
20. Replace $\operatorname{cosec} A$ by $\dfrac{1}{\sin A}$ and $\sec A$ by $\dfrac{1}{\cos A}$.
21. Use $\operatorname{cosec}^2\theta - \cot^2\theta = 1$.
22. Simplify the numerator and denominator by taking common terms appropriately.
23. Take LCM and simplify.
24. $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$.
25. $\log a + \log b + \log c + \ldots = \log(a.b.c\ldots)$.
26. Recall the ranges of $\sin\theta$ and $\cos\theta$.
27. $\sin(180 - \theta) = \sin\theta$.
28. Simplify the expression.
29. Equal chords subtend equal angles at the centre. $(2r \sin 60°)$.
30. (i) $1°$ is always less than 1^c.
    (ii) The value $\sin\theta$ increases from $0°$ to $90°$.

Level 2

31. Put $3\sin\alpha - 2\cos\alpha = k$, square the two equations and add.
32. Use the identity $\cos^2\alpha + \sin^2\alpha = 1$ and convert the equation into quadratic form in terms of $\sin\alpha$ and then solve.
33. (i) $\cot A$ is positive in the first and the third quadrants. As $\cot A$ not in first quadrant, $\cot A$ in third quadrant.
    (ii) Using the right angle triangle find the values of $\sin A$, $\cos A$ and $\cot A$ in third quadrant.
34. Apply componendo – dividendo rule, i.e., if $\dfrac{a}{b} = \dfrac{c}{d}$, then $\dfrac{a+b}{a-b} = \dfrac{c+d}{c-d}$.
35. Square the given equation, then use the identity $\cos^2\theta + \sin^2\theta = 1$ to evaluate the value of $\sin\theta \cos\theta$.
36. (i) In a cyclic quadrilateral, sum of the opposite angles is $180°$.
    (ii) Use the result, $\cos(180 - \theta) = -\cos\theta$ and simplify.
37. $l = r\theta$, where l = length, r = radians and θ = angle in radians.
38. Use the results $\sin^2 88 = \cos^2 2$ and $\sin^2\theta + \cos^2\theta = 1$ to simplify the given expression.
39. Use $\tan\theta$ = opposite side/adjacent side in the triangles so formed.
40.



Use the figure, by taking the values of $\tan 30°$ from ΔBCD and $\tan 60°$ from ΔABC, find CD.

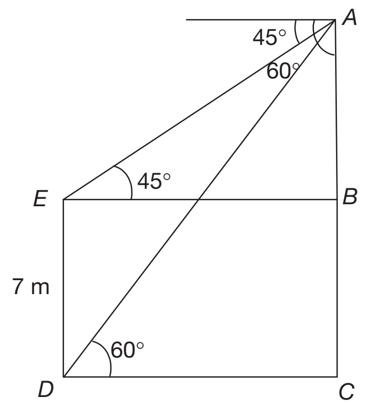
41. (i) Convert the given trigonometric values in terms of $\sin\theta$, $\cos\theta$, then simplify to obtain the required value.

(ii) Use the formula, cost $(A + B) = \cos A \cos B - \sin A \cdot \sin B$.

42. Take LCM and use the identity $\sin^2\theta + \cos^2\theta = 1$ to simplify the given equation.

43. Express the square root function as $(a + b)^2$ and remove the first square root. Again express the obtained function as $(a - b)^2$ and then simplify.

44. The following figure is drawn as per the description in the problem.



Use the figure, to find DC from ΔADC and AB from ΔAEB as $DC = BE$.

45. Express $\tan 176° = -\cot 86°$, $\cot 4° = \tan 86°$ and $\tan 4° = \cot 86°$. Then substitute these values to simplify the given expression.

46. (C), (A) and (D) are in sequential order from first to last.

47. (D), (C), (A) and (B) are in sequential order from first to last.

Level 3

48. Draw the figure as per the description given in the problem. Two right triangles are formed. Use appropriate trigonometric ratios by considering each of the triangle and find the height of the tree.

49. Draw a right triangle according to the data and take the ratio of $\sin\theta$ to evaluate the height of the balloon from the ground.

50. (i) Multiply and divide the given expression with $\sin 20°$.

(ii) Use $\sin^2 A = 2\sin A \cos A$.

(iii) $\sin(180 - A) = \sin A$.

51. (i) Divide the equation with $2\sqrt{2}$.

Then substitute $\frac{1}{2} = \cos 60°$ and $\frac{\sqrt{3}}{2} = \sin 60°$.

(ii) Now apply the formula,

$\cos(A + B) = \cos A \cos B - \sin A \sin B$ then obtain the value of θ.

52. (i) Draw a figure as per the situation described then use the concept, $\sin\theta = \dfrac{\text{Opposite side to } \theta}{\text{Hypotenuse}}$.

(ii) Given θ and opposite side find hypotenuse which is the required length of the rope.

53. $\sin^2 5° + \sin^2 10° + \sin^2 15° + \cdots + \sin^2 90°$

$= (\sin^2 5° + \sin^2 85°) + (\sin^2 10° + \sin^2 80°) + \cdots +$

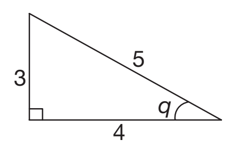
$\sin^2 45 + \sin^2 90°(\sin 85° = \cos 5°)$

$$= 8 + \left(\frac{1}{\sqrt{2}}\right)^2 + 1 = 9 + \frac{1}{2} = \frac{19}{2}.$$

54. Given,

$$\sec\theta + \tan\theta = 2 \qquad (1)$$

$$\sec\theta - \tan\theta = \frac{1}{2} \qquad (2)$$



Adding Eqs. (1) and (2), we get

$$2\sec\theta = \frac{5}{2}$$

$$\Rightarrow \quad \sec\theta = \frac{5}{4}$$

$$\Rightarrow \quad \cos\theta = \frac{4}{5}$$

$$\sin\theta = \frac{3}{5}.$$

55. Given,

$$\cos\theta + \left(\frac{1}{\sqrt{3}}\right)\sin\theta = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \quad \left(\frac{\sqrt{3}}{2}\right)\cos\theta + \left(\frac{1}{2}\right)\sin\theta = 1$$

$$\Rightarrow \quad \sin 60^\circ\ \cos\theta + \sin\theta\ \cos 60^\circ = 1$$

$$\Rightarrow \quad \sin(60^\circ + \theta) = \sin 90^\circ$$

$$\Rightarrow \quad 60^\circ + \theta = 90^\circ$$

$$\Rightarrow \quad \theta = 30^\circ = \frac{\pi^c}{6}.$$

56. $$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \frac{1+\sin\theta}{\cos\theta} = \sec\theta + \tan\theta.$$

57. $\sin^2\theta - 5\sin\theta + 3 = \cos^2\theta$

$$\Rightarrow \quad \sin^2\theta - 5\sin\theta + 3 = 1 - \sin^2\theta$$

$$\Rightarrow \quad 2\sin^2\theta - 5\sin\theta + 2 = 0$$

$$\Rightarrow \quad (2\sin\theta - 1)\ (\sin\theta - 2) = 0$$

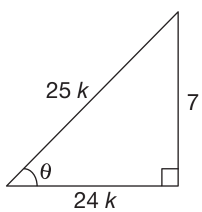
$$\sin\theta = 2 \text{ or } \frac{1}{2}.$$

But $\sin\theta = 2$ is not possible

$$\therefore \quad \sin\theta = \frac{1}{2} \quad \Rightarrow \quad \theta = 30^\circ.$$

58. Given, $\cot\theta = \dfrac{24}{7}$

As $\tan\theta$ is positive and $\theta \notin Q_1$,



$\theta \in Q_3$.

$$\therefore \quad \sec\theta = -\frac{25}{24}$$

$$\tan\theta = \frac{7}{24}$$

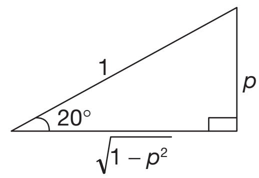
$$\text{Now } \tan\theta - \sec\theta = \frac{7}{24} + \frac{25}{24}$$

$$= \frac{32}{24} = \frac{4}{3}.$$

59. Given, $\sin 20^\circ = p$

$$\sin 380^\circ = \sin(360^\circ + 20^\circ)$$

$$= \sin 20^\circ.$$



$$\sin 340^\circ = \sin(360^\circ - 20^\circ)$$

$$= -\sin 20^\circ$$

$$\cos 380^\circ = \cos(360^\circ + 20^\circ)$$

$$= \cos 20^\circ$$

$$\cos 340^\circ = \cos(360^\circ - 20^\circ)$$

$$= \cos 20^\circ$$

$$\therefore \quad \left(\frac{\sin 380 - \sin 340}{\cos 380 + \cos 340}\right) = \frac{\sin 20 - (-\sin 20)}{\cos 20 + \cos 20}$$

$$= \frac{2\sin 20}{2\cos 20}$$

$$= \tan 20^\circ = \frac{p}{\sqrt{1-p^2}}.$$

60. We have, $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$$\text{Let } \theta = \left(22\frac{1}{2}\right)^\circ \quad \Rightarrow \quad 2\theta = 45^\circ$$

$$\therefore \quad \tan 45^\circ = \frac{2\tan\theta}{1-\tan^2\theta}.$$

$$1 = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\Rightarrow \quad 1 - \tan^2\theta = 2\tan\theta$$

$$\Rightarrow \quad \tan^2\theta + 2\tan\theta - 1 = 0$$

$$\tan\theta = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{8}}{2} = -1 \pm \sqrt{2}.$$

As θ is $\left(22\frac{1}{2}\right)$, $\tan\left(22\frac{1}{2}\right)^\circ = \sqrt{2} - 1$.

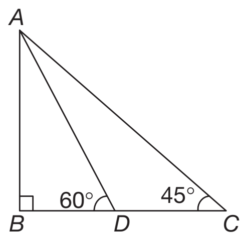
61. Let AB be the height of the tower.

Let $CD = 50$ m (given)

In ΔABC, $\tan 45^\circ = \frac{AB}{BC}$

$$\Rightarrow \quad 1 = \frac{AB}{BD + CD} \qquad (1)$$

$$\Rightarrow \quad AB = BD + 50.$$



In $\Delta\ ABD$,

$$\tan 60^\circ = \frac{AB}{BD} \quad \Rightarrow \quad \sqrt{3} = \frac{AB}{BD}$$

$\Rightarrow \quad BD = \frac{AB}{\sqrt{3}}$ substitute BD in Eq. (1)

$$AB = \frac{AB}{\sqrt{3}} + 50$$

$$AB\left(1 - \frac{1}{\sqrt{3}}\right) = 50$$

$$AB\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = 50$$

$$AB = \frac{50\sqrt{3}}{\sqrt{3}-1} = \frac{50\sqrt{3}\left(\sqrt{3}+1\right)}{3-1}$$

$$= 25\left(3 + \sqrt{3}\right) \text{ m.}$$

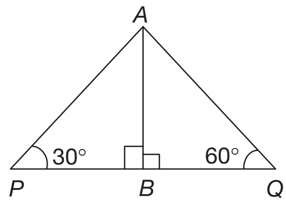
62. For the maximum possible distance, two points lie on either side of the tower.

Let AB be the height of the tower.

$AB = 30$ m (given)

Let P and Q be the given points.

In ΔABP,



$$\tan 30^\circ = \frac{AB}{PB} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{30}{PB} \quad \Rightarrow \quad PB = 30\sqrt{3}.$$

In ΔABQ, $\tan 60^\circ = \frac{AB}{BQ}$

$$\Rightarrow \quad \sqrt{3} = \frac{30}{BQ} \quad \Rightarrow \quad BQ = \frac{30}{\sqrt{3}} \text{ m.}$$

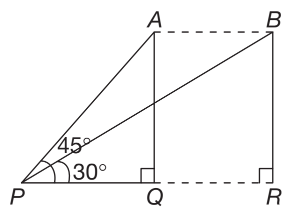
Now, $PQ = PB + BQ$

$$= 30\sqrt{3} + \frac{30}{\sqrt{3}} = 30\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)$$

$$= 30\left(\frac{4}{\sqrt{3}}\right) = 40\sqrt{3} \text{ m.}$$

63. Let A and B be the initial and final positions of the plane.

Given, $AQ = 500$ m $\therefore$ $BR = 500$ m $(AQ = BR)$



In ΔAPQ,

$$\tan 45^\circ = \frac{AQ}{PQ}$$

$$1 = \frac{500}{PQ} \quad \therefore PQ = 500 \text{ m.}$$

In ΔBPR,

$$\tan 30^\circ = \frac{BR}{PR}$$

$$\frac{1}{\sqrt{3}} = \frac{500}{PQ + QR} \quad \Rightarrow \quad 500 + QR = 500\sqrt{3}$$

$$QR = 500\left(\sqrt{3} - 1\right) \text{ m} \quad \therefore AB = 500\left(\sqrt{3} - 1\right) \text{ m.}$$

$$\text{Speed of the plane} = \frac{AB}{\text{time}}$$

$$= \left(\frac{500\left(\sqrt{3}-1\right)}{5} \right) \text{m/s}$$

$$= 100\left(\sqrt{3}-1\right) \text{m/s}$$

$$= 100\left(\sqrt{3}-1\right)\left(\frac{18}{5}\right) \text{kmph}$$

$$= 360\left(\sqrt{3}-1\right) \text{kmph.}$$

64. AB be the height of the building.

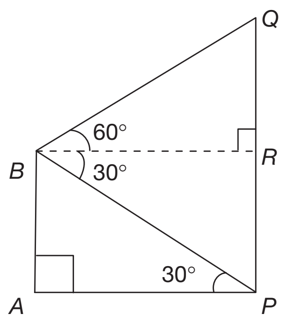
$\therefore$ $AB = 5$ m (given)

Let PQ be the height of the tower.

In ΔABP,

$$\tan 30° = \frac{AB}{AP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{5}{AP}.$$

$$\therefore\ AP = 5\sqrt{3} \Rightarrow \therefore BR = 5\sqrt{3} \text{ m}$$



In ΔBQR $\tan 60° = \dfrac{QR}{BR}$

$$\sqrt{3} = \frac{QR}{5\sqrt{3}} \Rightarrow QR = 15 \text{ m.}$$

From the figure, $PQ = PR + QR$

$= 5 + 15 = 20$ m.

65. Let $\angle FAE = \theta_1$, and $\angle HAG = \theta_2$

$$\tan\theta_1 = \frac{FE}{AE} = \frac{1}{2}$$

$$\tan\theta_2 = \frac{HG}{AG} = \frac{1}{3}$$

$$\tan(\theta_1 + \theta_2) = \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2}$$

$$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}$$

$$= \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

$\tan(\theta_1 + \theta_2) = \tan 45°$

$\Rightarrow$ $\theta_1 + \theta_2 = 45°$

$\angle FAE + \angle HAG = 45°$.