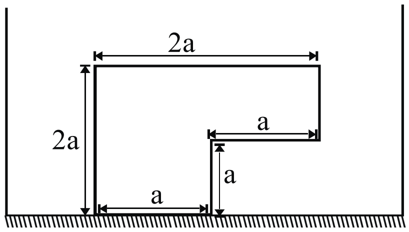
GUIDED REVISION

SECTION-I

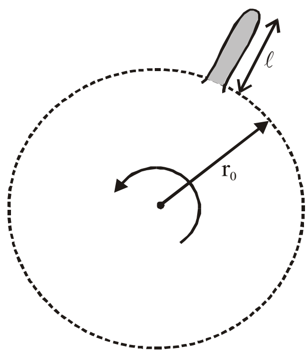
Single Correct Answer Type **8 Q. [3 M (–1)]**

1. A piece of material whose specific gravity is 'S' is placed in a container as shown in figure and water is poured slowly, then piece will



(A) topple clockwise if $S < \frac{1}{2}$.
(B) topple anticlockwise if $S < \frac{1}{2}$.
(C) topple clockwise if $S > 2$.
(D) topple anticlockwise if $S > 2$.

2. A test tube filled with water is being spun around in an ultracentrifuge with angular velocity ω. The test tube is lying along a radius and the free surface of the water is at radius r_0 (Figure). The length of the test tube is ℓ and ρ is the density of the water. Ignore gravity and ignore atmospheric pressure. Choose the correct statement.



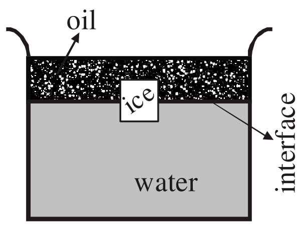
(A) The pressure at the mid point of test tube is given by $\frac{1}{8}\rho\ell\omega^2(\ell + 4r_0)$

(B) The pressure at the mid point of test tube is given by $\frac{1}{2}\rho\ell\omega^2\left(\frac{\ell}{2} + r_0\right)$

(C) The pressure at the mid point of test tube is given by $\frac{1}{4}\rho\ell\omega^2\left(\frac{\ell}{2} + r_0\right)$

(D) The pressure remains constant through out the tube.

3. An ice cube is floating in water above which a layer of a lighter oil is poured. As the ice melts completely, the level of interface and the upper most level of oil will respectively :-

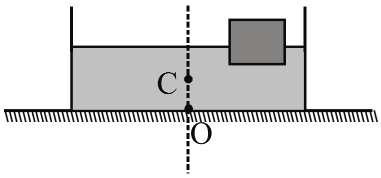


(A) rise and fall (B) fall and rise (C) rise and not change (D) not change and fall

4. Water is flowing on a horizontal fixed surface, such that its flow velocity varies with y (vertical direction) as $v = k\left(\frac{2y^2}{a^2} - \frac{y^3}{a^3}\right)$. If coefficient of viscosity for water is η, what will be shear stress between layers of water at $y = a$.

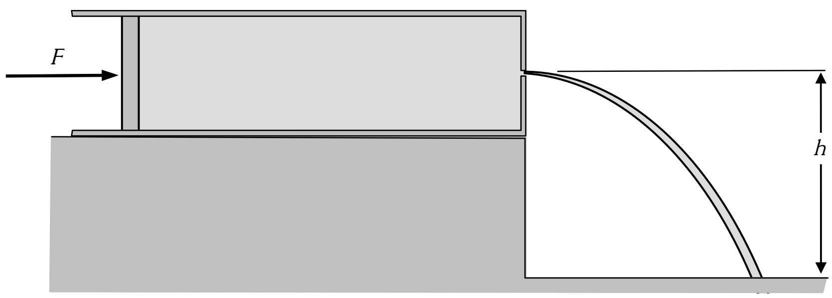
(A) $\frac{\eta k}{a}$ (B) $\frac{\eta}{ka}$ (C) $\frac{\eta a}{k}$ (D) None of these

5. There is water in container with center of mass at C. Now a small wooden piece is place towards right as shown in the figure. After putting the wooden piece.



(A) Pressure at base remains same and centre of mass of water and wooden piece will be right of line OC.
(B) Pressure at base remains same and centre of mass of water and wooden piece will be on line OC.
(C) Pressure at base changes and centre of mass of water and wooden piece will be right of line OC.
(D) Pressure at base changes and centre of mass of water and wooden piece will be on line OC.

6. An ideal liquid of density ρ is filled in a horizontally fixed syringe fitted with piston. There is no friction between the piston and the inner surface of the syringe. Cross-section area of the syringe is A. At one end of the syringe, an orifice of negligible cross-section area is made. When the piston is pushed into the syringe, the liquid comes out of the orifice following parabolic path and falls on the ground. With what speed the liquid strikes the ground? Neglect the air drag :-



(A) $\sqrt{\dfrac{F+\rho ghA}{\rho A}}$ (B) $\sqrt{\dfrac{F+2\rho ghA}{\rho A}}$ (C) $\sqrt{\dfrac{2F+\rho ghA}{\rho A}}$ (D) $\sqrt{\dfrac{2(F+\rho ghA)}{\rho A}}$

7. A vertical tank, open at the top, is filled with a liquid and rests on a smooth horizontal surface. A small hole is opened at the centre of one side of the tank. The area of cross-section of the tank is N times the area of the hole, where N is a large number. Neglect mass of the tank itself. The initial acceleration of the tank is :-

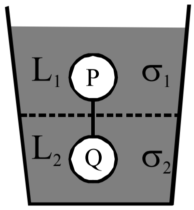
(A) $\dfrac{g}{2N}$ (B) $\dfrac{g}{\sqrt{2}N}$ (C) $\dfrac{g}{N}$ (D) $\dfrac{g}{2\sqrt{N}}$

8. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_C is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is **[IIT-JEE-2012]**
(A) more than half-filled if ρ_C is less than 0.5
(B) more than half-filled if ρ_C is more than 1.0
(C) half-filled if ρ_C is more than 0.5
(D) less than half-filled if ρ_c is less than 0.5

Multiple Correct Answer Type 2 Q. [4 M (–1)]

9. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity $\vec{V}_P$ and Q alone in L_1 has terminal velocity $\vec{V}_Q$, then

[JEE Advanced-2015]



(A) $\dfrac{|\vec{V}_P|}{|\vec{V}_Q|} = \dfrac{\eta_1}{\eta_2}$ (B) $\dfrac{|\vec{V}_P|}{|\vec{V}_Q|} = \dfrac{\eta_2}{\eta_1}$ (C) $\vec{V}_P . \vec{V}_Q > 0$ (D) $\vec{V}_P . \vec{V}_Q < 0$

10. A soap film (surface tension = T) is formed on a rectangular wire frame as shown in figure. A small, light, inextensible loop of thread of length "ℓ" is gently placed over the soap film. The loop stays on the film in the irregular fashion in which it was placed. Now, a hole is pricked in the film inside the loop with a needle and the thread finally comes to equilibrium position. Choose the correct option(s)

(A) The thread will take a square shape

(B) The thread will take a circular shape

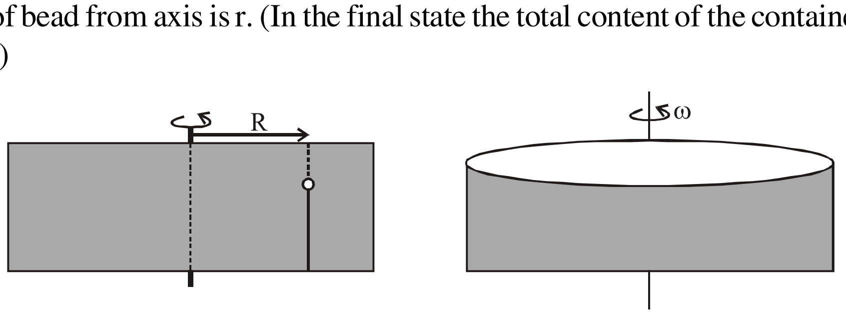
(C) Tension in the thread will be $2T\ell$

(D) Tension in the thread will be $\dfrac{T\ell}{\pi}$

Linked Comprehension Type (Single Correct Answer Type) (2 Para × 2Q & 2 Para × 3Q.) [3 M (-1)]

Paragraph for Question No. 11 and 12

A closed cylindrical container with a vertical axis is completely filled with liquid (ρ_ω). A plastic bead of density ρ_{body} and volume V is placed at distance R from the axis and is anchored to the bottom of the container by a thin thread of length ℓ. If as a result of the containers revolutions, the beads sink by h and final distance of bead from axis is r. (In the final state the total content of the container rotates at the same angular speed.)



11. What is force exerted by liquid on the bead ?

(A) ρ_ω V g

(B) $(\rho_\omega - \rho_{body})$ V g

(C) $(\rho_\omega - \rho_{body})$ V $\sqrt{g^2 + (\omega^2 r)^2}$

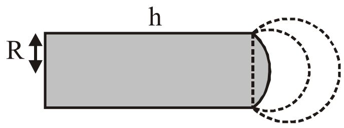
(D) ρ_ω V $\sqrt{g^2 + (\omega^2 r)^2}$

12. What is ω :-

(A) $\omega = \sqrt{\dfrac{(R+r)g}{(\ell+h)r}}$ (B) $\omega = \sqrt{\dfrac{(R-r)g}{(\ell-h)r}}$ (C) $\omega = \sqrt{\dfrac{(R+r)g}{(\ell-h)r}}$ (D) $\omega = \sqrt{\dfrac{(R-r)g}{(\ell+h)r}}$

Paragraph for Questions 13 and 14

A glass tube with thin walls is placed in a chamber of rarefied air. One end of the tube is closed and the other is covered by a stretched liquid film. The pressure of the air is p_0 and its temperature is T_0 both inside and outside the tube. The length of the tube is h, its radius is R. The surface tension of the liquid is α. The temperature in the tube starts to rise slowly.



13. At what temperature T will the enclosed air have a maximum pressure?

(A) $\dfrac{p_{max}\left[h+\left(\frac{2}{3}\right)R\right]T_0}{p_0 h}$ (B) $\dfrac{p_{max}\left[h+\left(\frac{1}{3}\right)R\right]T_0}{p_0 h}$

(C) $\dfrac{p_{max}\left[\left(\frac{2}{3}\right)h+R\right]T_0}{p_0 h}$ (D) $\dfrac{p_{max}\left[\frac{h}{2}+R\right]T_0}{p_0 h}$

14. How much work is performed by the enclosed air until the state of maximum pressure is reached? Assume that, in the pressure and temperature ranges investigated, the liquid is far away from its boiling point.

(A) $3\alpha R^2\pi + p_0\frac{2}{3}R^3\pi$ (B) $2\alpha R^2\pi + p_0\frac{2}{3}R^3\pi$

(C) $\alpha R^2\pi + p_0\frac{4}{3}R^3\pi$ (D) $4\alpha R^2\pi + p_0\frac{2}{3}R^3\pi$

Paragraph for Questions no. 15 to 17

When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper. **[IIT-JEE-2010]**

15. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming r <<R) is

(A) $2\pi rT$ (B) $2\pi RT$ (C) $\dfrac{2\pi r^2T}{R}$ (D) $\dfrac{2\pi R^2T}{r}$

16. If $r = 5\times10^{-4}$ m, $\rho = 10^3$ kgm^{-3}, $g = 10$ ms^{-2}, $T = 0.11$ Nm^{-1}, the radius of the drop when it detaches from the dropper is approximately

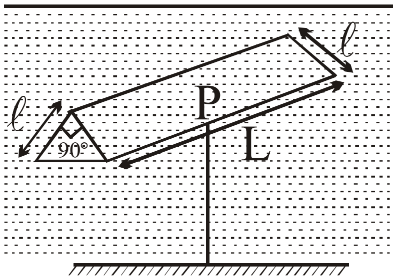
(A) 1.4×10^{-3} m (B) 3.3×10^{-3} m (C) 2.0×10^{-3} m (D) 4.1×10^{-3} m

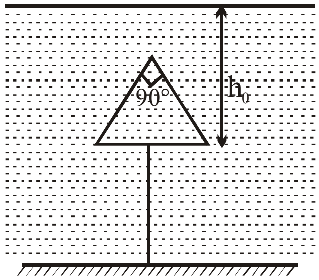
17. After the drop detaches, its surface energy is

(A) 1.4×10^{-6} J (B) 2.7×10^{-6} J (C) 5.4×10^{-6} J (D) 8.1×10^{-6} J

Paragraph for Question No. 18 to 20

A prism shaped styrofoam of density $\rho_{styrofoam} < \rho_{water}$ is held completely submerged in water. It lies with its base horizontal. The base of foam is at a depth h_0 below water surface and atmospheric pressure is P_0. Surface is open to atmosphere. Styrofoam prism is held in equilibrium by the string attached symmetrically. (Take : $\rho_{styrofoam} = \rho_f$; $\rho_{water} = \rho_w$).





18. Net force exerted by water on the styrofoam is

(A) $\sqrt{2}\rho_w g\ell^2 L$ (B) $2\rho_w g\ell^2 L$ (C) $\rho_w g\frac{\ell^2 L}{2}$ (D) $\rho_w g\ell^2 L$

19. Magnitude of force on any one of the slant face of styrofoam is

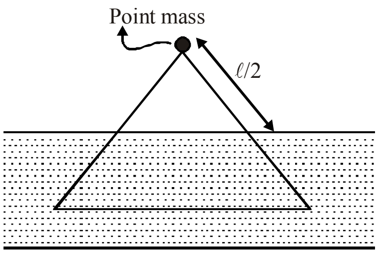
(A) $(P_0 + \rho_w g(h_0 - \sqrt{2}\ell))L\ell$

(B) $(P_0 + \rho_w g(h_0 - \frac{\ell}{\sqrt{2}}))L\ell$

(C) $(P_0 - \rho_w g(h_0 - \frac{\ell}{\sqrt{2}}))L\ell$

(D) $(P_0 + \rho_w g(h_0 - \frac{\ell}{2\sqrt{2}}))L\ell$

20. Now string is cut and styrofoam is allowed to come to surface. A point mass is to be placed symmetrically on the upper surface of styrofoam such that it is in equilibrium with its base in horizontal plane. In equilibrium position styrofoam has half of its slant length submerged. Surface tension of water is T, contact angle is 0°. Determine mg to achieve equilibrium.



(A) $\frac{\rho_f gL\ell^2}{2} + \frac{3}{4}\rho_w gL\ell^2 - 2T[L+\ell]$

(B) $\frac{-\rho_f gL\ell^2}{2} + \frac{3}{8}\rho_w gL\ell^2 - \sqrt{2}T[L+\ell]$

(C) $\frac{-\rho_f gL\ell^2}{2} + \frac{3}{8}\rho_w gL\ell^2 - \frac{T[L+\ell]}{\sqrt{2}}$

(D) $\frac{-\rho_f gL\ell^2}{2} + \frac{3}{8}\rho_w gL\ell^2 + \sqrt{2}T[L+\ell]$

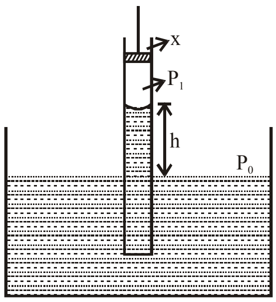
SECTION-II

Numerical Answer Type Question **3 Q. [3(0)]**
(upto second decimal place)

1. A U-tube filled with a liquid of volumetric coefficient of 10^{-5}/°C lies in a vertical plane. The height of liquid column in the left vertical limb is 100 cm. The liquid in the left vertical limb is maintained at a temperature = 0°C while the liquid in the right limb is maintained at a temperature = 100°C. Find the difference in levels in the two limbs.

2. An expansible balloon filled with air floats on the surface of a lake with 2/3 of its volume submerged. How deep must it be sunk in the water so that it is just inequilibrium neither sinking further nor rising? It is assumed that the temperature of the water is constant & that the height of the water barometer is 9 meters.

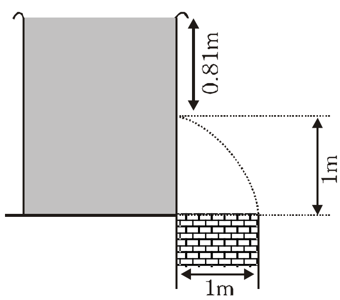
3. A capillary of radius r = 0.2 mm is dipped vertically in a liquid of density, ($\rho = 1 gm/cc$). A small piston (x) is inserted inside the capillary which maintains the constant pressure of air ($P_1 = 2 \times 10^4$ dynes/cm^2) above the hemispherical meniscus. If ambient pressure is $P_0 = 10^5$ dynes/cm^2 and surface tension of liquid is T = 72 dynes/cm, then find the height h of liquid in cm. (weight of curved part of liquid is ignored)



SECTION-III

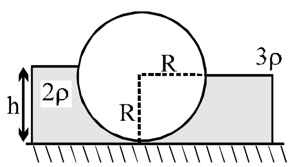
Numerical Grid Type (Ranging from 0 to 9) 1 Q. [4 M (0)]

1. For the arrangement shown in the figure. The time interval in seconds after which the water jet ceases to cross the wall is found to be $(\alpha)^3$. Area of the cross section of the tank $A = \sqrt{5}\ m^2$ and area of the orifice a = 32 cm^2. [Assume that the container remaining fixed]. Find the value of α.
(Take g = 10 m/s^2)

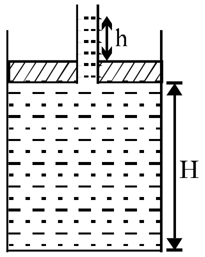


Subjective Type 6 Q. [4 M (0)]

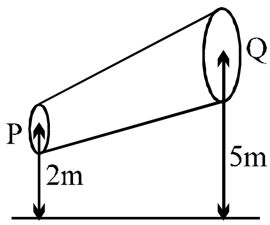
1. In the figure shown, the heavy cylinder (radius R) resting on a smooth surface separates two liquids of densities 2ρ and 3ρ. Find the height 'h' for the equilibrium of cylinder.



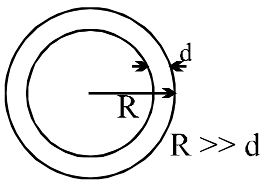
2. A piston of mass M = 3kg and radius R = 4cm has a hole into which a thin pipe of radius r = 1cm is inserted. The piston can enter a cylinder tightly and without friction, and initially it is at the bottom of the cylinder. 750gm of water is now poured into the pipe so that the piston & pipe are lifted up as shown. Find the height H of water in the cylinder and height h of water in the pipe.



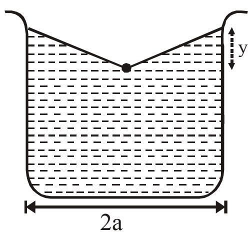
3. A non-viscous liquid of constant density 1000 kg/m^3 flows in a streamline motion along a tube of variable cross section. The tube is kept inclined in the vertical plane as shown in the figure. The area of cross section of the tube at two points P and Q at heights of 2 meters and 5 meters are respectively 4 × 10^{-3}m^2 and 8 × 10^{-3} m^3. The velocity of the liquid at point P is 1 m/s. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q.



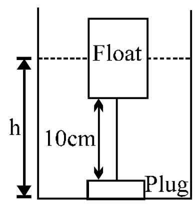
4. A soap bubble has radius R and thickness d (<< R) as shown. It collapses into a spherical drop. Find the ratio of excess pressure in the drop to the excess pressure inside the bubble.



5. A container of width 2a is filled with a liquid. A thin wire of weight per unit length λ is gently placed over the liquid surface in the middle of the surface as shown in the figure. As a result, the liquid surface is depressed by a distance y (y << a). Determine the surface tension of the liquid. **[2004, 2M]**



6. A level controller is shown in the figure. It consists of a thin circular plug of diameter 10cm and a cylindrical float of diameter 20cm tied together with a light rigid rod of length 10cm. The plug fits in snugly in a drain hole at the bottom of the tank which opens into atmosphere. As water fills up and the level reaches height h, the plug opens. Find h. Determine the level of water in the tank when the plug closes again. The float has a mass 3kg and the plug may be assumed as massless.



SECTION-I

Single Correct Answer Type **8 Q. [3 M (–1)]**

1. Ans. (A) **2. Ans.** (A) **3. Ans.** (A) **4. Ans.** (A)

5. Ans. (D) 6. **Ans.** (D) **7. Ans.** (C) **8. Ans.** (A)

Multiple Correct Answer Type **2 Q. [4 M (–1)]**

9. Ans. (A, D) **10. Ans.** (B, D)

Linked Comprehension Type **(2 Para × 2Q & 2 Para × 3Q.) [3 M (-1)]**

(Single Correct Answer Type)

11. Ans. (D) **12. Ans.** (B) **13. Ans.** (A) **14. Ans.** (B)

15. Ans. (C) **16. Ans.** (A) **17. Ans.** (B) **18. Ans.** (C)

19. Ans. (D) **20. Ans.** (B)

SECTION-II

Numerical Answer Type Question **3 Q. [3(0)]**

(upto second decimal place)

1. Ans. 0.1 cm **2. Ans.** 4.5m **3. Ans.** 80.72

SECTION-III

Numerical Grid Type (Ranging from 0 to 9) **1 Q. [4 M (0)]**

1. Ans. 5

Subjective Type **6 Q. [4 M (0)]**

1. Ans. $R\sqrt{\frac{3}{2}}$ **2. Ans.** $h = \frac{2m}{\pi}$, $H = \frac{11}{32\pi}$ m

3. Ans. + 29625 J/m^3, – 30000 J/m^3 **4. Ans.** $\left(\frac{R}{24d}\right)^{\frac{1}{3}}$ **5. Ans.** $\frac{\lambda a}{2y}g$

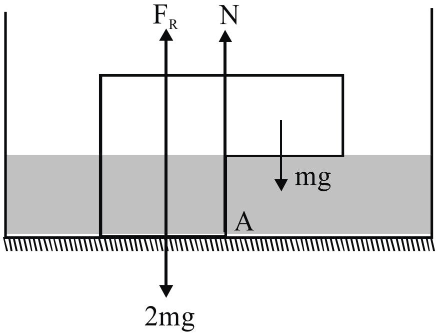
6. Ans. $h_1 = \frac{2(3+\pi)}{15\pi} = 0.26$; $h_2 = \frac{3+\pi}{10\pi} = 0.195$

SOLUTIONS
SECTION-I

Single Correct Answer Type **8 Q. [3 M (–1)]**

1. Ans. (A)

Sol.



FBD when the material just topples,

Balancing torque, about A : $\left(-F_B + 2mg\right)\frac{a}{2} = mg.\frac{a}{2} \Rightarrow F_B = mg \Rightarrow \rho_S = \rho_W$

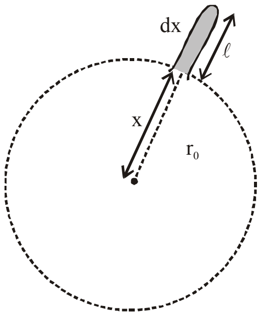
If $\rho_S < \rho_W$, substance will topple clockwise.

If $\rho_S > \rho_W$, substance will not topple & normal will shift to balance the torque by F_B

2. Ans. (A)

Sol. For the section of length dx,

$dT = \rho A\, dx\, \omega^2 x$



$$\therefore \text{ T at centre} = \int_{r_0}^{\left(\frac{\ell}{2}+r_0\right)} \rho A\omega^2 x dx = \frac{1}{2}\rho A\omega^2\left[\left(\frac{\ell}{2}+r_0\right)^2 - r_0^2\right]$$

$$= \frac{1}{2}\rho A\omega^2\left[\frac{\ell^2}{4} + r_0^2 + \ell r_0 - r_0^2\right] = \frac{1}{2}\rho A\omega^2\left[\frac{\ell^2 + 4\ell r_0}{4}\right] = \frac{1}{8}\rho A\omega^2 \ell\left[\ell + 4r_0\right]$$

$$\therefore \text{ Pressure} = \frac{T}{A} = \frac{1}{8}\rho\omega^2\ell\left[\ell + 4r_0\right]$$

3. Ans. (A)

Sol. (i) $V_{ice} > V_{water}$ so upper layer falls

(ii) Displaced volume of water = $\left[M_{ice} - \frac{F_B}{g}\right]\frac{1}{\rho_{water}}$

ice melts then B decreases then level of water rises.

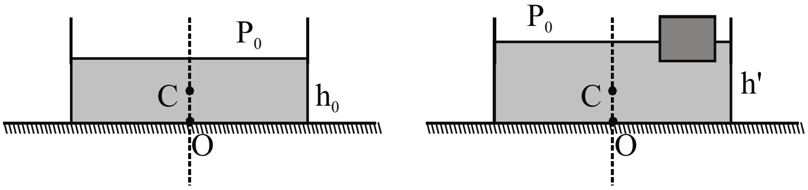
4. Ans. (A)

Sol. $F = -\eta A \frac{dv}{dy}$

$$\text{shear stress} = \frac{F}{A} = -\eta\frac{dv}{dy}$$

$$= -\eta k\left[\frac{4y}{a^2} - \frac{3y^2}{a^3}\right] = -\eta k\left[\frac{4}{a} - \frac{3}{a}\right] = = \frac{-\eta k}{a}$$

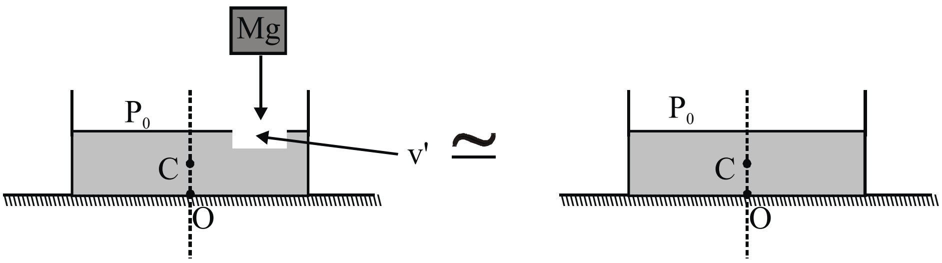
5. Ans. (D)



$P_{base} = \rho g h_0 + P_0$ $\qquad$ $P'_{base} = \rho g h' + P_0$

$h' > h_0 \Rightarrow P'_{base} > P_{base}$

Sol.



$mg = \rho g v'$
so centre of mass
will be on line OC

6. Ans. (D)

Sol. $V_y^2 = 2gh$

Horizontal

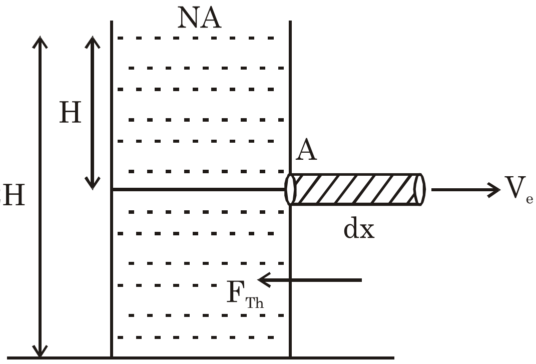
$$Fdx = \frac{1}{2}(\rho A dx)V_x^2$$

$$\Rightarrow V_x^2 = \frac{2F}{\rho A}$$

$$V = \sqrt{V_x^2 + V_y^2}$$

7. Ans. (C)

Sol.



By torricelli's Law

$V_e = \sqrt{2gH}$

Thrust force on the tank

$$\Rightarrow F_{Th} = V_r \cdot \frac{dm}{dt}$$

$$= V_e \cdot \rho A \frac{dx}{dt}$$

$$= V_e . \rho A V_e$$

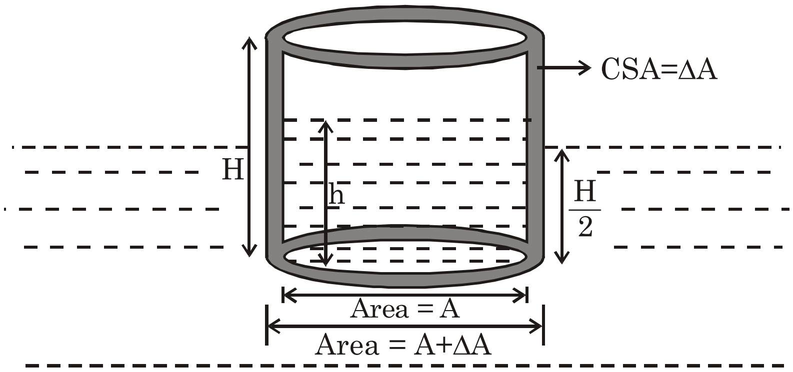
$$= \rho A V_e^2$$

$$\text{So, } a_{Tank} = \frac{F_{Th}}{m} = \frac{\rho A V_e^2}{m}$$

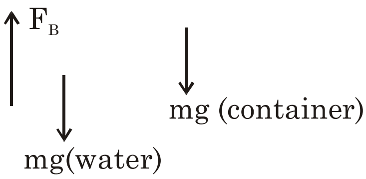
$$= \frac{\rho A 2gH}{\rho(NA2H)} = \frac{g}{N}$$

8. Ans. (A)

Sol.



For equilibrium $\Rightarrow$ FBD $\Rightarrow$



$$F_B = mg_{(Water)} + mg_{(container)}$$

$$\rho_W (A + \Delta A)\frac{H}{2} g = \rho_W hAg + \rho_C \rho_W H\Delta Ag$$

$$hA = \left(\frac{AH}{2}\right) + \left(\frac{\Delta AH}{2}\right) - (\rho_C \Delta AH)$$

$$h = \frac{H}{2} + \frac{H\Delta A}{A}(0.5 - \rho_C)$$

This shows that $h > \frac{H}{2}$ if $\rho_C < 0.5$

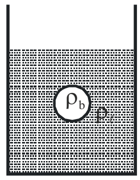
So Ans. (A)

Multiple Correct Answer Type 2 Q. [4 M (–1)]

9. Ans. (A, D)

Sol. Consider a body of density ρ_b kept in density ρ_ℓ whose viscosity is η and terminal velocity V. Then

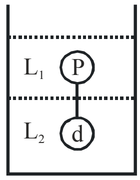
$$\vec{F}_{viscous} + \vec{F}_{mg} + \vec{F}_{Buoyancy} = 0$$



$$\vec{F}_{viscous} + \rho_b \frac{4}{3}\pi R^3(-\hat{j}) + \rho\ell \frac{4}{3}\pi R^3(\hat{j}) = 0$$

$$\therefore\ \vec{F}_{viscous} = (\rho_b - \rho_\ell)\frac{4}{3}\pi R^3(\hat{j}) \Rightarrow 6\pi\eta RV = (\rho_b - \rho_\ell)\frac{4}{3}\pi R^3$$

$\therefore$ if $\rho_b > \rho_1$ then $\quad \vec{F}_{viscous} \uparrow\ V \propto \frac{1}{\eta}$



& if $\rho_b < \rho_\ell$

$\vec{F}_{viscous} \downarrow$

as per given diagram we can say

$\sigma_2 > \sigma_1$; $\rho_1 < \sigma_1$ & $\rho_2 > \sigma_2$

$\Rightarrow \rho_2 > \sigma_2 > \sigma_1 > \rho_1$

$\therefore$ if we put P in L_2 where $|\vec{V}_P| \propto \frac{1}{\eta_2}$ when $\rho_1 < \sigma_2$ $\therefore\ \vec{F}_{viscous} \downarrow$ $\quad \therefore\ \vec{V}_P \uparrow$

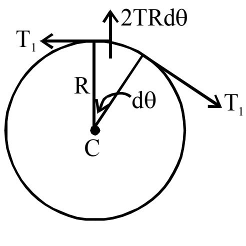
$\therefore$ if we put Q in L_1 where $|\vec{V}_Q| \propto \frac{1}{\eta_1}$ when $\rho_2 > \sigma_1$ $\therefore\ \vec{F}_{viscous} \uparrow$ $\quad \therefore\ \vec{V}_P \downarrow$

$$\Rightarrow \frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2}\ \&\ \vec{V}_P.\vec{V}_Q < 0$$

10. Ans. (B, D)

Sol. (i) due to equal magnitude of surface tension force from all sides, the thread will take circular shape.

(ii)



$$2T_1\frac{d\theta}{2} = 2TRd\theta$$

$$\Rightarrow T_1 = 2TR = \frac{2T\ell}{2\pi} = \frac{T\ell}{\pi}$$

Linked Comprehension Type (2 Para × 2Q & 2 Para × 3Q.) [3 M (-1)]
(Single Correct Answer Type)

11. Ans. (D)

Sol. Force exerted by liquid = (mass of liquid displaced) × $\sqrt{g^2 + (\omega^2 r)^2}$

$$= \rho_w v \times \sqrt{g^2 + (\omega^2 r)^2}$$

12. Ans. (B)

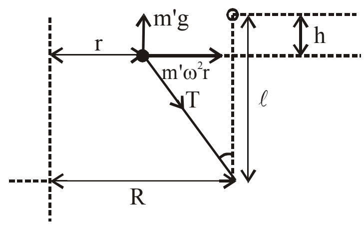
Sol. $R - r = \ell \sin\theta$ ..... (i)

$\ell - h = \ell \cos\theta$ ...... (ii)

$m' = (\rho_w - \rho_{body})v$ & $m'g = T\cos\theta$ ..... (iii)

$m'\omega^2 r = T\sin\theta$ ..... (iv)

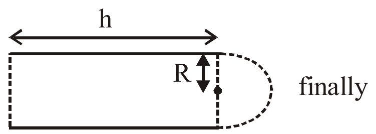
from (i), (ii), (iii) & (iv)



$$\frac{\omega^2 r}{g} = \tan\theta = \frac{R - r}{\ell - h}$$

13. Ans. (A)

Sol. Pressure is maximum when liquid film bulges into a hemisphere



Initially : $P_0 (\pi R^2 h) = nRT_0$ ... (i)

Finally : $P_{max} (pR^2 h + \frac{2}{3}\pi R^3) = nRT$ ... (ii)

(ii)/(i) gives : $\dfrac{P_{max}\left(h + \frac{2}{3}R\right)}{P_0 h} = \dfrac{T}{T_0}$

$$T = \frac{P_{max}\left(h + \frac{2R}{3}\right)T_0}{P_0 h}$$

14. Ans. (B)

Sol. By work enrgy theorem

$W_{by\ gas} + W_{by\ atm} + W_{by\ S.T} = 0$

$W_{by\ gas} + (-P_0\ \Delta V) + (-\alpha\ \Delta A) = 0$

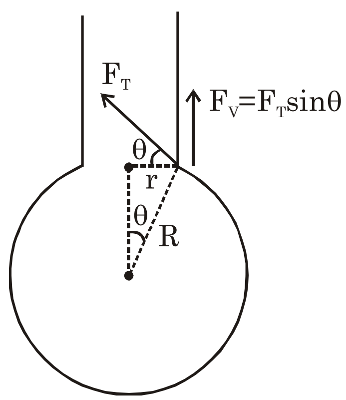
$W_{by\ gas} = P_0 \Delta V + \alpha \Delta A$

$$= P_0\left[\frac{2}{3}\pi R^3\right] + \alpha \cdot \left[2\pi R^2\right]$$

$$= P_0 \frac{2}{3}\pi R^3 + \alpha \cdot 2\pi R^2$$

15. Ans. (C)

Sol.



$F_T = T \cdot 2\pi r$

$F_V = F_T \sin\theta$

$= T \cdot 2\pi r \cdot \dfrac{r}{R}$

$= \dfrac{2\pi r^2 T}{R}$

16. Ans. (A)

Sol. Drop detaches from dropper

When :

$F_V = mg$

$$\frac{2\pi r^2 T}{R} = \rho \cdot \frac{4}{3}\pi R^3 g$$

$$R = \left(\frac{3r^2T}{2g\rho}\right)^{1/4}$$

$$= \left[\frac{3 \times \left(5 \times 10^{-4}\right)^2 \times (0.11)}{2 \times 10 \times 1000}\right]^{1/4}$$

$\approx 1.4 \times 10^{-3}$ m

17. Ans. (B)

Sol. Surface energy of drop after detachment = T × surface area

$= T \times 4\pi R^2 = 0.11 \times 4 \times \pi \times (1.4 \times 10^{-3})^2$

$\approx 2.7 \times 10^{-6}$ Joule

18. Ans. (C)

Sol. Net force exerted by liquid on styrofoam is buoyant force $= \rho_W Vg = \rho_W (A)\, Lg = \rho_W g \dfrac{\ell^2 L}{2}$

19. Ans. (D)

Sol. Average pressure on slant surface $P_{avg} = \dfrac{(P_0 + \rho_W g h_0) + \left(P_0 + \rho_W g h_0 - \dfrac{l}{\sqrt{2}}\right)}{2} = \left(P_0 + \rho_W g h_0 - \dfrac{l}{2\sqrt{2}}\right)$

Force on any one of the slant face $= \left[P_0 + \rho_w g(h_0 - \dfrac{\ell}{2\sqrt{2}})\right] L\ell$

20. Ans. (B)

Sol. Balancing force in vertical

$(M_{styrofoam} + m)g + F_{surface\ Tension} = F_{bouyancy}$

$$\rho_f\left(\frac{1}{2}\ell^2 L\right)g + mg + T\left(2L\sin 45^\circ + \frac{2\ell}{\sqrt{2}}\right) = \rho_W \frac{1}{2}\left(\frac{\ell}{\sqrt{2}} + \sqrt{2}\ell\right)\left(\frac{\ell}{\sqrt{2}} - \frac{\ell}{2\sqrt{2}}\right)Lg$$

$$mg = \frac{-\rho_f g L \ell^2}{2} + \frac{3}{8}\rho_W g L \ell^2 - \sqrt{2}T[L + \ell]$$

SECTION-II

Numerical Answer Type Question **3 Q. [3(0)]**
(upto second decimal place)

1. **Ans.** 0.1 cm

Sol. $\rho_1 g h_1 = \rho_2 g h_2$

$$\frac{h_2}{h_1} = \frac{\rho_1}{\rho_2} = \frac{\rho_1}{\rho_1(1-\gamma\Delta T)}$$

Y = 10^{-5}/°C, ΔT = 100°C, h_1 = 300 cm

$h_2 - h_1$ = 0.1 cm ans.

2. **Ans.** 4.5m

Sol. $P_\ell V_\ell = P_f V_f$

$$P_0(V) = P_f\left(\frac{2V}{3}\right)$$

$$P_f = \frac{3}{2}P_0 = P_0 + \rho_w g h$$

$$\frac{P_0}{2} = \rho_w g h$$

$$\frac{\rho_w g \times 9}{2} = \rho_w g h$$

[h = 4.5 m]

3. **Ans. 80.72**

Sol. $P_1 - \frac{2T}{r} + \rho g h = P_0$

$$h = \frac{P_0 - P_1 + \frac{2T}{r}}{\rho g}$$

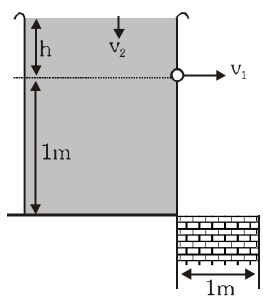
SECTION-III

Numerical Grid Type (Ranging from 0 to 9) **1 Q. [4 M (0)]**

1. **Ans. 5**

Sol For minimum velocity at orifice

$$\Rightarrow (v_{min}) \times \sqrt{\frac{2h}{g}} = 1 \Rightarrow v_{min} = \sqrt{5}\ m/s$$



This will give us minimum height for this velocity

$$\Rightarrow \sqrt{2gh_{min}} = \sqrt{5} \Rightarrow h_{min} = \frac{1}{4}m = 0.25m$$

$v_1 = \sqrt{2gh}$ & $v_2 = -dh/dt$

By equation of continuity $av_1 = Av_2$

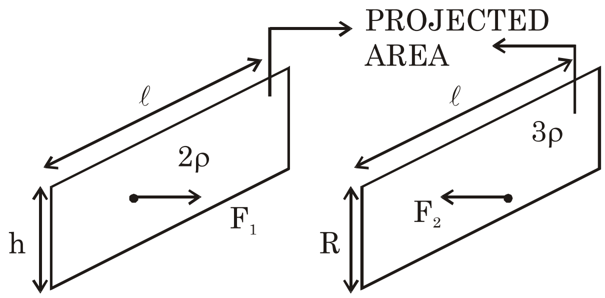
$$\Rightarrow a\sqrt{2gh} = A\left(-\frac{dh}{dt}\right) \Rightarrow \int_{0.81}^{0.25}\frac{dh}{\sqrt{h}} = \int_0^t \frac{\left(a\sqrt{2g}\right)}{A}dt$$

$\Rightarrow$ t = 125 sec = $(5)^3 = (\alpha)^3 \Rightarrow \alpha = 5$

Subjective Type **6 Q. [4 M (0)]**

1. Ans. $R\sqrt{\frac{3}{2}}$

Sol.



Equating force horizontally from both sides

$$F_{av_1} = F_{av_2}$$

$$P_{av_1} \cdot A_1 = P_{av_2} \cdot A_2$$

$$\left(\frac{0+2\rho gh}{2}\right)(h\ell) = \left(\frac{0+3\rho gR}{2}\right)(R\ell)$$

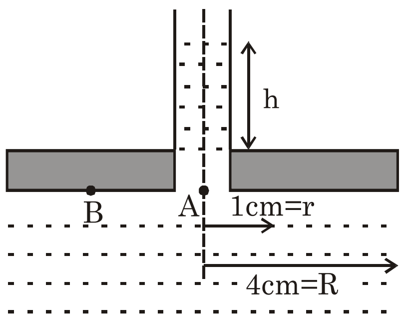
$$\rho gh^2 = \frac{3}{2}\rho gR^2$$

$$h^2 = \frac{3}{2}R^2$$

$$h = \sqrt{\frac{3}{2}}R$$

2. **Ans.** $h = \frac{2m}{\pi}$, $H = \frac{11}{32\pi}$ m

Sol. (i)



Equating pressure at points A and B

$$P_A = P_B$$

$$P_{atm} + \rho_W gh = P_{atm} + \frac{mg}{\pi(R^2 - r^2)}$$

$$1000 \times 10 \times h = \frac{3 \times 10}{\pi\left[\left(\frac{4}{100}\right)^2 - \left(\frac{1}{100}\right)^2\right]}$$

$$h = \frac{30}{\pi[15]} = \frac{2}{\pi} m$$

(ii) Total mass of water = 0.75kg = $\rho_W \times V_{Total}$

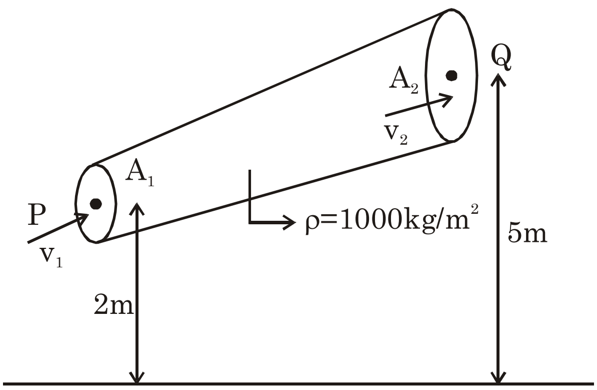
$0.75 = 1000\ [\pi r^2 h + \pi R^2 H]$

$$0.75 = 1000\pi\left[\frac{1}{10000} \times \frac{2}{\pi} + \frac{16}{10000} H\right]$$

$$H = \left(\frac{11}{32\pi}\right) m$$

3. **Ans.** + 29625 J/m^3, – 30000 J/m^3

Sol.



$A_1 = 4 \times 10^{-3}\ m^2$

$A_2 = 8 \times 10^{-3}\ m^2$

$v_1 = 1$ m/sec

$\rho_W = 10^3\ kg/m^3$

Continuity equation at P & Q

$A_1 v_1 = A_2 v_2$

$(4 \times 10^{-3})\ (1) = (8 \times 10^{-3})\ (v_2)$

$$v_2 = \frac{1}{2} m/sec$$

Work done per unit volume by pressure = ΔP = pressure difference

Applying Bernoulli's theorem between P & Q

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$P_1 - P_2 = \rho g(h_2 - h_1) + \frac{\rho}{2}(v_2^2 - v_1^2) = 10^4(5-2) + 500\left(\frac{1}{4} - 1\right)$$

$$= 30000 - \frac{1500}{4} = 29625\ J/m^3$$

Work done per unit volume by gravity force = $\frac{mgh}{v} = (\rho g h)$

So, work done/volume = $\rho g\ (h_1 - h_2)$

= 1000 × 10 [2 – 5]

= – 30000 J/m^3

4. **Ans.** $\left(\frac{R}{24d}\right)^{\frac{1}{3}}$

Sol. P_{excess}/bubble = $\frac{4T}{R}$

$\rho(4\pi R^2)d = \rho\left(\frac{4\pi}{3}r^3\right)$

$r = [3R^2d]^{1/3}$

P_{excess}/drop $= \frac{2T}{r}$

$$\frac{P_{excess}/drop}{P_{excess}/bubble} = \frac{\frac{2T}{r}}{\frac{4T}{R}} = \frac{R}{2r} = \frac{R}{2\left[3R^2d\right]^{1/3}} = \left[\frac{R}{24d}\right]^{1/3}$$

5. **Ans.** $\frac{\lambda a}{2y}g$

Sol. By force balance on the wire,

$2T\cos\theta\ell = \lambda\ell g$

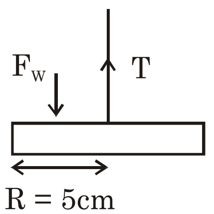
$$\frac{2Ty}{\sqrt{a^2+y^2}}\ell = \lambda\ell g$$

$$\left[T = \frac{\lambda ag}{2y}\right]$$

6. **Ans.** $h_1 = \frac{2(3+\pi)}{15\pi} = 0.26$; $h_2 = \frac{3+\pi}{10\pi} = 0.195$

Sol. Plug opens when level of water reaches 'h'.

- FBD of plug



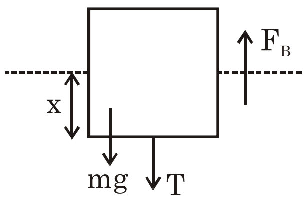
$F_W = T$

$P.\pi R^2 = T$

$$T = \rho_W ghA = 1000\times10\times h\times\pi\left(\frac{5}{100}\right)^2$$

$T = 25\pi h$

- FBD of float



$F_B = mg + T$

$\rho_W gxA = 3 \times 10 + T$

$$10000x\pi \times \left(\frac{10}{100}\right)^2 = 30 + 25\,\pi h$$

$100\,\pi x = 30 + 25\,\pi h$

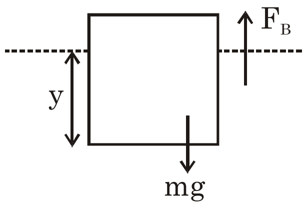
$100\pi\,(h - 0.1) = 30 + 25\,\pi h$

$75\,\pi h = 30 + 10\,\pi$

$$h = \frac{(3+\pi)10}{75\pi} = \left(\frac{2(3+\pi)}{15\pi}\right)\text{m}$$

- To again close the plug

  $mg = F_B$ (for the float) $\rightarrow$ [When water has risen to height y for the float]



$F_B = mg$

$\rho_W gyA = 3g$

$$1000 \times 10 \times y \times \pi \times \left(\frac{10}{100}\right)^2 = 3g$$

$100\pi y = 3g$

$$y = \frac{30}{100\pi} = \frac{3}{10\pi}\text{m}$$

$$\text{So } h = 0.1\text{ m} + \frac{3}{10\pi}\text{m}$$

$$= \left(\frac{1}{10} + \frac{3}{10\pi}\right)\text{m}$$

$$h = \left(\frac{\pi+3}{10\pi}\right)\text{m}$$