VERY SIMILAR
PRACTICE TEST 2

Hints and Explanations

1. **(d) :** $x = 1.2t^2$,

$$\text{Velocity, } v = \frac{dx}{dt} = \frac{d}{dt}(1.2t^2) = 2.4t$$

$$\text{Acceleration, } a = \frac{dv}{dt} = \frac{d}{dt}(2.4t) = 2.4 = \text{a constant}$$

Thus the given motion is uniformly accelerated.

2. **(c) :** According to radioactive decay,

$N = N_0 e^{-\lambda t}$

where,

N_0 = Number of radioactive nuclei present in the sample at $t = 0$

N = Number of radioactive nuclei left undecayed after time t

λ = decay constant

For 20% decay

$$\frac{80N_0}{100} = N_0 e^{-\lambda t_1} \quad \text{...(i)}$$

For 80% decay

$$\frac{20N_0}{100} = N_0 e^{-\lambda t_2} \quad \text{...(ii)}$$

Dividing equation (i) by (ii), we get

$4 = e^{-\lambda(t_1 - t_2)} \Rightarrow 4 = e^{\lambda(t_2 - t_1)}$

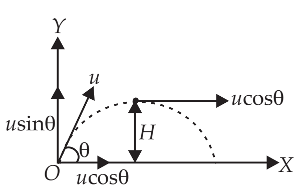
Taking natural logarithms of both sides of above equation, we get

$\ln 4 = \lambda(t_2 - t_1)$

$$2\ln 2 = \frac{\ln 2}{T_{1/2}}(t_2 - t_1)$$

$t_2 - t_1 = 2 \times T_{1/2} = 2 \times 20 \text{ min} = 40 \text{ min}$

3. **(c) :**



Here, angle of projection, $\theta = 60°$

Let u be the velocity of projection of the particle.

Kinetic energy of a particle at the point of projection O is

$$K = \frac{1}{2}mu^2 \quad \text{...(i)}$$

where m is the mass of a particle.

Velocity of the particle at the highest point (*i.e.* at maximum height) is $u\cos\theta$.

$\therefore$ Kinetic energy of the particle at the highest point is

$$K' = \frac{1}{2}m(u\cos\theta)^2 = \frac{1}{2}mu^2\cos^2\theta = \frac{1}{2}mu^2\cos^2 60°$$

$$= \frac{1}{2}mu^2\left(\frac{1}{2}\right)^2 = \frac{K}{4} \quad \text{(Using (i))}$$

4. **(c) :** $\dfrac{I_{max}}{I_{min}} = \dfrac{(a+b)^2}{(a-b)^2} = \dfrac{25}{1}$

$$\frac{a+b}{a-b} = \frac{5}{1} \quad \text{or} \quad 5a - 5b = a + b$$

$$\text{or} \quad 4a = 6b \quad \text{or} \quad \frac{a}{b} = \frac{3}{2} \quad \therefore \quad \frac{I_1}{I_2} = \frac{a^2}{b^2} = \frac{9}{4}$$

5. **(a) :** $E = G^p h^q c^r$ ...(i)

Equating dimensions on both sides of equation (i), we get

$[M^1L^2T^{-2}] = [M^{-1}L^3T^{-2}]^p[ML^2T^{-1}]^q[LT^{-1}]^r$

$= [M^{-p+q} L^{3p+2q+r} T^{-2p-q-r}]$

Applying principle of homogeneity of dimensions, we get

$-p + q = 1$ ...(ii)

$3p + 2q + r = 2$ ...(iii)

$-2p - q - r = -2$ ...(iv)

Adding (iii) and (iv), we get

$p + q = 0$ ...(v)

Adding (ii) and (v), we get

$$2q = 1 \quad \text{or} \quad q = \frac{1}{2}$$

From (ii)

$$p = q - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$$

Substituting the values of p and q in equation (iii), we get $\dfrac{3}{2} + 1 + r = 2$ or $r = \dfrac{5}{2}$

$$\text{Hence, } p = -\frac{1}{2}, q = \frac{1}{2}, r = \frac{5}{2}$$

6. (b) : When the body is floating in a liquid, then

Weight of the body = Weight of liquid displaced

i.e. $V_{body}\rho_{body}g = V_{inside}\rho_{liquid}g$

Let V be the volume of the body.

In water, $V\rho_{body}g = \frac{2}{3}V\rho_{water}g$

$$\rho_{body} = \frac{2}{3}\rho_{water} \quad ...(i)$$

In oil, $V\rho_{body}g = \frac{1}{2}V\rho_{oil}g$

$$\rho_{body} = \frac{1}{2}\rho_{oil} \quad ...(ii)$$

From (i) and (ii) we get

$$\frac{\rho_{oil}}{\rho_{water}} = \frac{4}{3}$$

Specific gravity of oil

$$\frac{\rho_{oil}}{\rho_{water}} = \frac{4}{3}$$

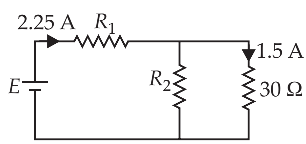
7. (d) : Current through R_2 is (2.25 – 1.5) A = 0.75 A.

Voltage across 30 Ω = 1.5 × 30 = 45 V

As R_2 and 30 Ω are in parallel

$\therefore$ Voltage across, R_2 = 45 V

$$\therefore \quad R_2 = \frac{45\text{ V}}{0.75\text{ A}}$$



$R_2 = 60\ \Omega$

Also, $R_1 = 60\ \Omega$ $\quad$ ($\because$ $R_1 = R_2$ (Given))

Voltage across, R_1 = 2.25 × 60 Ω = 135 V

$\therefore$ E = (135 + 45)V = 180 V

8. (d) : For good demodulation,

$$\frac{1}{\upsilon} << RC \quad \text{or} \quad RC >> \frac{1}{\upsilon}$$

9. (d) : In an open pipe, the fundamental frequency is $\frac{v}{2L_O}$ and all harmonics are present.

Third overtone means fourth harmonic.

$\therefore$ Frequency of third overtone $\upsilon_4 = \frac{4v}{2L_O}$

where v is the velocity of sound and L_O is the length of the open pipe.

In a closed pipe, the fundamental frequency is $\frac{v}{4L_C}$ and only odd harmonics are present. Second overtone mean fifth harmonic

$\therefore$ Frequency of second overtone $\upsilon_5 = \frac{5v}{4L_C}$

where L_C is a length of a closed pipe.

As $\upsilon_4 = \upsilon_5$

$$\frac{4v}{2L_O} = \frac{5v}{4L_C} \quad \text{or} \quad L_C = \frac{10}{16}L_O = \frac{10}{16}\times 8\text{ cm} = 5\text{ cm}$$

10. (b) : de Broglie wavelength associated with charged particle is given by,

$$\text{or} \quad \lambda = \frac{h}{\sqrt{2mqV}}, \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{2m_\alpha q_\alpha V}{2m_p q_p V}}$$

where subscripts, p and α represent proton and α particle respectively.

$$\text{As} \quad \frac{m_p}{m_\alpha} = \frac{1}{4} \text{ and } \frac{q_\alpha}{q_p} = \frac{2}{1} \Rightarrow \frac{\lambda_p}{\lambda_\alpha} = \sqrt{4\times 2} = 2\sqrt{2}$$

11. (c) : Here, the springs are connected in series, their resultant force constant K is given by

$$\frac{1}{K} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{k_1+k_2}{k_1k_2} \quad \text{or} \quad K = \frac{k_1k_2}{k_1+k_2}$$

12. (a) : Given α = 0.98 and ΔI_E = 5.0 mA

From the definition of,

$$\alpha = \frac{\Delta I_C}{\Delta I_E}$$

Change in collector current

$$\Delta I_C = (\alpha)(\Delta I_E) = 0.98 \times 5.0 = 4.9\text{ mA}$$

Change in base current,

$$\Delta I_B = \Delta I_E - \Delta I_C = (5.0 - 4.9)\text{ mA} = 0.1\text{ mA}$$

13. (d) : In SHM,

Potential energy of a particle, $U = \frac{1}{2}m\omega^2x^2$

Total energy of a particle, $E = \frac{1}{2}m\omega^2A^2$

where symbols have their usual meaning

At $x = \frac{A}{2}$, the fraction of total energy which is potential,

$$\frac{U}{E} = \frac{\frac{1}{2}m\omega^2x^2}{\frac{1}{2}m\omega^2A^2} = \frac{x^2}{A^2} = \left[\frac{A/2}{A}\right]^2 = \frac{1}{4}$$

14. (c) : When the lens is in air

$$\frac{1}{f_a} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{20} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

When lens is in water,

$$\frac{1}{f_w} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or $\dfrac{1}{f_w}=\left(\dfrac{1.5-1.33}{1.33}\right)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)$

or $\dfrac{f_w}{20}=(1.5-1)\left(\dfrac{1.33}{1.5-1.33}\right)$

or $f_w=20\times0.5\times\dfrac{1.33}{0.17}=78.2$ cm

$= 78.2$ cm

The change in focal length = 78.2 – 20 = 58.2 cm

15. (b) : According to Wien's law, $\lambda_m\propto\dfrac{1}{T}$

$\therefore\quad \dfrac{\lambda_m}{\lambda'_m}=\dfrac{T'}{T}=\dfrac{3000}{2000}=\dfrac{3}{2}\quad$ or $\quad\lambda'_m=\dfrac{2}{3}\lambda_m$

16. (a) : Here, $t=2$ mm, $x=1.6$ mm, $K=?$

As potential difference remains the same, capacity must remain the same

$$\therefore\quad \frac{\varepsilon_0 A}{d-t+\frac{t}{k}}=\frac{\varepsilon_0 A}{d-1.6}$$

where d is the separation between the plates initially.

$x=t\left(1-\dfrac{1}{K}\right)\Rightarrow 1.6=2\left(1-\dfrac{1}{K}\right)$, which gives $K=5$.

17. (a) : Distance covered in 5th second is

$$D_5=0+\frac{a}{2}(2\times5-1)=\frac{9a}{2}\qquad(\because\ u=0)$$

Distance covered in 5 seconds is

$$S_5=0+\frac{1}{2}a\times5^2=\frac{25a}{2}\qquad(\because\ u=0)$$

$$\therefore\quad \frac{D_5}{S_5}=\frac{9}{25}$$

18. (c) : As $\Delta Q=mc\Delta T$

$$\therefore\quad Q=\int\Delta Q=\int_{T_1}^{T_2}mc\Delta T$$

Here, $c=DT^3$, $T_1=20$ K and $T_2=30$ K

$$\therefore\ Q=\int_{20}^{30}mDT^3dT=mD\int_{20}^{30}T^3dT=mD\left[\frac{T^4}{4}\right]_{20}^{30}$$

$$=\frac{mD}{4}[(30)^4-(20)^4]=\frac{mD}{4}\times10^4[81-16]$$

$$=\left(\frac{65}{4}\right)\times10^4 Dm$$

19. (a) : Here, $\phi=4t^2-4t+1$

Induced emf, $|\varepsilon|=\dfrac{d\phi}{dt}$

$$=\frac{d}{dt}(4t^2-4t+1)=8t-4$$

Induced current, $I=\dfrac{|\varepsilon|}{R}=\dfrac{8t-4}{10}$

At $t=\dfrac{1}{2}$ s, $I=\dfrac{8\times\frac{1}{2}-4}{10}=0$ A

20. (b) : Total average energy density of electromagnetic wave is

$$<u>=\frac{1}{2}\varepsilon_0E_{\text{rms}}^2+\frac{1}{2\mu_0}B_{\text{rms}}^2$$

$$=\frac{1}{2}\varepsilon_0E_{\text{rms}}^2+\frac{1}{2\mu_0}\left(\frac{E_{\text{rms}}^2}{c^2}\right)\quad\left(\because B_{\text{rms}}=\frac{E_{\text{rms}}}{c}\right)$$

$$=\frac{1}{2}\varepsilon_0E_{\text{rms}}^2+\frac{1}{2\mu_0}E_{\text{rms}}^2\varepsilon_0\mu_0$$

$$=\frac{1}{2}\varepsilon_0E_{\text{rms}}^2+\frac{1}{2}\varepsilon_0E_{\text{rms}}^2=\varepsilon_0E_{\text{rms}}^2$$

$= (8.85\times10^{-12})\times(720)^2=4.58\times10^{-6}$ J m^{-3}.

21. (6.0) : Here $r_1=6R+R=7R$; $T_1=24$ hours

$r_2=2.5R+R=3.5R$

According to Kepler's third law of planetary motion,

$$\frac{T_2^2}{T_1^2}=\frac{r_2^3}{r_1^3}\ \Rightarrow\ T_2=T_1\left(\frac{r_2}{r_1}\right)^{3/2}=24\left(\frac{3\ 5R}{7R}\right)^{3/2}$$

$$=\frac{24}{(2)^{3/2}}=\frac{24}{2\sqrt{2}}=6\sqrt{2}\text{ hours}$$

The value of n is 6.

22. (2.0) : By Gauss theorem, $\oint\vec{E}\cdot d\vec{S}=\dfrac{q_{\text{enclosed}}}{\varepsilon_0}$

$$E\,4\pi r^2=\frac{1}{\varepsilon_0}\int_0^r\kappa r^a\,4\pi r^2dr$$

$$\Rightarrow\quad E=\frac{\kappa}{\varepsilon_0r^2}\left[\frac{r^{a+3}}{a+3}\right]\ \Rightarrow\ E=\frac{\kappa}{\varepsilon_0}\frac{[r^{a+1}]}{a+3}$$

$$E\left(r=\frac{R}{2}\right)=\frac{1}{8}E(r=R)\qquad\text{(Given)}$$

$$\therefore\ \frac{\kappa}{\varepsilon_0(a+3)}\left[\frac{R}{2}\right]^{a+1}=\frac{1}{8}\frac{\kappa}{\varepsilon_0(a+3)}[R]^{a+1}$$

$$\Rightarrow\ \left[\frac{R}{2}\right]^{a+1}=\frac{1}{8}\left[R^{a+1}\right]\ \Rightarrow\ 2^{a+1}=8\ \therefore\ a=2$$

23. (4.0) : For an adiabatic process,

$TV^{\gamma-1}=$ constant; $T_iV_i^{\gamma-1}=T_fV_f^{\gamma-1}$

Substituting the given values, we get

$$T_iV_i^{\gamma-1}=aT_i\left(\frac{V_i}{8}\right)^{\gamma-1}\ \Rightarrow\ a=8^{\gamma-1}$$

For a monoatomic gas, $\gamma = \frac{5}{3}$

$\therefore\ a = 8^{\frac{5}{3}-1} = 8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^2 = 4.$

24. (3.0) : Here m_1 = 10 kg, m_2 = 20 kg, m_3 = 30 kg, F = 60 N

The common acceleration of the system is

$$a = \frac{F}{m_1+m_2+m_3} = \frac{60}{10+20+30} = 1\ \mathrm{m\ s^{-2}}$$

$\therefore\ T_1 = m_1 a = 10a$

$T_2 = (m_1 + m_2)a = (10 + 20)a = 30a$

$$\frac{T_2}{T_1} = \frac{30a}{10a} = 3$$

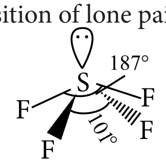
25. (0.5) : The maximum current is obtained at resonance where the net impedance is only resistive which is the resistance of the coil only. This gives the resistance of the coil as 10 Ω. Now, this coil along with the internal resistance of the cell gives a current of 0.5 A.

26. (a) : FeO (being basic) combines with silica (SiO_2) an acidic flux to give $FeSiO_3$ slag.

$$FeO + SiO_2 \longrightarrow \underset{\text{(slag)}}{FeSiO_3}$$

27. (c) : (a) θ = less than 109°28′

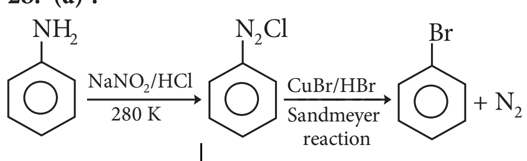
(b) Incorrect position of lone pair



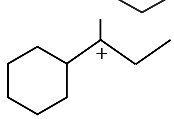
(c) Square planar structure with θ = 90°

(d) θ ≠ 90° in SiF_4 because it is tetrahedral.

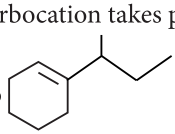
28. (a) :



29. (d) :



Intermediate 3° carbocation is formed which is more stable. Thus, no rearrangement in carbocation takes place.

So  product is not possible.

30. (b) : Degree of hydrolysis, $h = \sqrt{\frac{K_w}{K_a.c}}$

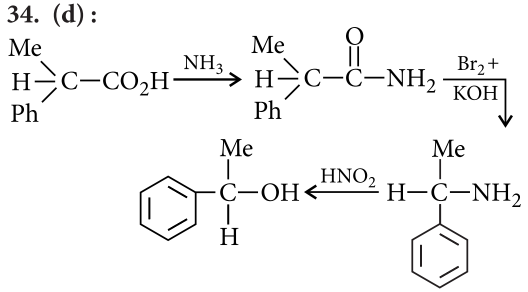
$$= \sqrt{\frac{1\times10^{-14}}{1\times10^{-5}\times0.001}} = \sqrt{\frac{1\times10^{-14}}{1\times10^{-5}\times1\times10^{-3}}}$$

$$= \sqrt{10^{-6}} = 10^{-3}$$

31. (c) : It is poly-β-hydroxybutyrate-co-β-hydroxy valerate (PHBV) which is biodegradable polymer. All others are non-biodegradable polymers.

32. (d) **33. (b)**

34. (d) :



35. (c) : Ionic radii of isoelectronic ions decrease with increase of nuclear charge thus, it shows a decrease from O^{2-} to Al^{3+}.

36. (a) : $\Delta T_f = K_f \times i \times m$

$0.0558 = 1.86 \times i \times 0.01 \ \Rightarrow\ i = 3$

Given complex is a strong electrolyte.

For strong electrolyte, $i = n$

n = No. of ions = 3 ions

Therefore, formula of the complex is $[Co(NH_3)_5Cl]Cl_2$.

37. (b) : In the equation; $\overset{+4}{SO_2} \longrightarrow \overset{0}{S}$

SO_2 acts as an oxidising agent,

$$\text{Eq. mass of } SO_2 = \frac{\text{Molar mass}}{\text{Change in oxidation number}}$$

$$= \frac{64}{4} = 16$$

Twice of 16 = 32

38. (d) : Energy absorbed by each molecule $= 4.4 \times 10^{-19}$ J

Energy required to break the bond $= 4.0 \times 10^{-19}$ J

Remaining energy gets converted to kinetic energy

$= (4.4 \times 10^{-19} - 4.0 \times 10^{-19})$ J

$= 0.4 \times 10^{-19}$ J per molecule

$\therefore$ Kinetic energy per atom $= 0.2 \times 10^{-19}$ J $= 2 \times 10^{-20}$ J

39. (a) : The physical adsorption isobar shows a decrease in x/m throughout with rise in temperature.

40. (a) : For endothermic reaction, ΔH = +ve. For reaction to be spontaneous, ΔS must be positive and also $T\Delta S$ must be greater than ΔH in magnitude. The reaction is then said to be entropy driven.

41. (d) : Compound with N, S and carbon will form NaSCN to give red ppt.

$$Na + C + N + S \longrightarrow NaSCN$$
$$Fe^{3+} + SCN^- \longrightarrow \underset{\text{(Blood red)}}{[Fe(SCN)]^{2+}}$$

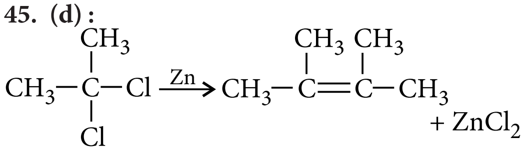
42. (d) : Due to lesser amount of D.O. in warm water, the growth of fish in it is not as healthy as in cold water.

43. (b) : $8P + 3Ca(OH)_2 + 6H_2O \rightarrow 3Ca(H_2PO_2)_2 + \underset{(X)}{2PH_3}$

(A) is H_3PO_2 (hypophosphorous acid), a monobasic acid. PH_3 is less basic than NH_3. The bond angle in (X) is less than that present in NH_3. H_3PO_2 on heating gives orthophosphoric acid and phosphine (X).

44. (b) : The colour arise by charge transfer. In MnO_4^-, an electron is momentarily transferred from oxygen to the metal and thus oxygen changes from O^{2-} to O^- and Mn from (+7) to (+6).

45. (d) :



(A)

46. (217) : Density of the crystal d (g cm^{-3})

$$= \frac{ZM}{N_A \times a^3}$$

$$Z = \frac{d \times N_A \times a^3}{M} = \frac{2 \times 6 \times 10^{23} \times (5 \times 10^{-8})^3}{75} = 2$$

Thus, the unit cell of cubic lattice will be body centred. For *bcc* lattice,

$4r$ (radius of atom) = Diagonal of cube = $\sqrt{3}a$

$$r = \frac{\sqrt{3}}{4} \times a \times 10^2 = \frac{1.732 \times 5 \times 10^2}{4} = 216.5 \approx 217 \text{ pm}$$

47. (3) : Among the given species, I_3^-, N_2O and N_3^- are linear species.

$$^-:\ddot{N} = \overset{+}{N} = \ddot{N}:^-,\ ^-N = \overset{+}{N} = O,\ [I - I - I]^-$$

48. (11.08) : For first order reaction,

$$t_{1/2} = \frac{0.693}{k}$$

$$t_{1/2} = 200 \text{ min} = 200 \times 60 = 12000 \text{ s}$$

$$\Rightarrow k = \frac{0.693}{12000 \text{ s}} = 5.8 \times 10^{-5} \text{ s}^{-1}$$

Also, for first order decomposition of H_2O_2, rate constant is

$$\log k = 14.2 - \frac{1.0 \times 10^4}{T} \text{K}$$

Comparing the above equation with the Arrhenius equation,

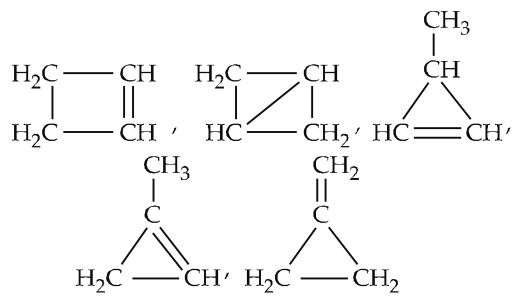
$$\log k = \log A - \frac{E_a}{2.303RT}$$

$$\Rightarrow E_a = 2.303 \times 8.314 \times 1.0 \times 10^4$$
$$= 1.91 \times 10^5 \text{ J mol}^{-1}$$

The multiplication of E_a and k

$$= 1.91 \times 10^5 \times 5.8 \times 10^{-5} = 11.078$$

49. (5) : The possible cyclic isomers of the compound with molecular formula C_4H_6 are



50. (6) : $[Pt(NH_3)_2(SCN)_2]$- *cis* and *trans*-isomers = 2
$[Pt(NH_3)_2(NCS)(SCN)]$- *cis* and *trans*-isomers = 2
$[Pt(NH_3)_2(NCS)_2]$ - *cis* and *trans*-isomers = 2
The complex is square planar so, it does not have a pair of enantiomers on account of the presence of plane of symmetry.

51. (d) : The given function is $f(x) = \sin[x]$

Clearly $-\frac{\pi}{4} \le x < 0 \Rightarrow [x] = -1$

and $0 \le x < \frac{\pi}{4} \Rightarrow [x] = 0$

$\therefore$ Range = $\{0, -\sin 1\}$

52. (a) : ${}^nC_5 + {}^nC_6 > {}^{n+1}C_5 \Rightarrow {}^{n+1}C_6 > {}^{n+1}C_5$

$$\Rightarrow \frac{(n+1)!}{6!(n-5)!} \cdot \frac{5!(n-4)!}{(n+1)!} > 1 \Rightarrow \frac{(n-4)}{6} > 1$$

$\Rightarrow n - 4 > 6 \Rightarrow n > 10$

Hence, least value of n = 11.

53. (a) : We have $y^2 = x^3 \Rightarrow 2y \cdot \frac{dy}{dx} = 3x^2$

$$\Rightarrow \frac{dy}{dx}=\frac{3x^2}{2y}$$

Given that, slope of tangent at (m^2, m^3)

= slope of normal at (M^2, M^3)

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(m^2,m^3)}=\frac{-1}{\left(\frac{dy}{dx}\right)_{(M^2,M^3)}}$$

$$\Rightarrow \frac{3m^4}{2m^3}=-\frac{2M^3}{3M^4}\Rightarrow mM=-\frac{4}{9}$$

54. (a) : We have

$$T_{r+1}={}^{21}C_r\left(\sqrt[3]{\frac{a}{\sqrt{b}}}\right)^{21-r}\left(\sqrt{\frac{b}{\sqrt[3]{a}}}\right)^{r}$$

$$={}^{21}C_r\,a^{7-(r/2)}b^{(2/3)r-(7/2)}$$

Since the powers of a and b are same,

$$\therefore\quad 7-\frac{r}{2}=\frac{2}{3}r-\frac{7}{2}\Rightarrow r=9$$

55. (a) : Let $I=\int_0^{\pi/2}\frac{dx}{1+\cot x}=\int_0^{\pi/2}\frac{\cos x\,dx}{\cos x+\sin x}$

Replacing x by $\frac{\pi}{2}-x$, we get

$$I=\int_0^{\pi/2}\frac{\cos\left(\frac{\pi}{2}-x\right)dx}{\cos\left(\frac{\pi}{2}-x\right)+\sin\left(\frac{\pi}{2}-x\right)}$$

$$=\int_0^{\pi/2}\frac{\sin x\,dx}{\sin x+\cos x}$$

$$\therefore\quad I+I=\int_0^{\pi/2}\frac{\cos x\,dx}{\cos x+\sin x}+\int_0^{\pi/2}\frac{\sin x\,dx}{\sin x+\cos x}$$

$$=\int_0^{\pi/2}dx=[x]_0^{\pi/2}=\frac{\pi}{2}$$

$$\text{So, }2I=\frac{\pi}{2}\ \Rightarrow\ I=\frac{\pi}{4}\ \Rightarrow\ \int_0^{\pi/2}\frac{dx}{1+\cot x}=\frac{\pi}{4}$$

56. (d) : $f'(0^-)=\lim\limits_{h\to 0}\dfrac{a\sin h+be^h+ch^3-b}{-h}$

$$=-(a+b)$$

$$f'(0^+)=\lim_{h\to 0}\frac{a\sin h+be^h+ch^3-b}{h}=a+b$$

$\because$ $f(x)$ is differentiable at $x=0$

$\therefore$ $a+b=-(a+b)\ \Rightarrow\ (a+b)=0.$

57. (c) : The ellipse is $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

$S_1\equiv(-2,0),\ S\equiv(2,0)$

$$\Rightarrow\ S_1S=4=2ae=2a\times\frac{1}{3}\Rightarrow a=6$$

$$\therefore\quad b^2=a^2(1-e^2)=36\left(1-\frac{1}{9}\right)=32$$

$$\Rightarrow\quad b=4\sqrt{2}\ \therefore \text{Minor axis}=2b=8\sqrt{2}.$$

58. (c) : From given relation, we have

$x=\cos\theta\pm i\sin\theta$

Take $x=\cos\theta+i\sin\theta$

$\therefore$ $x^n=\cos n\theta+i\sin n\theta$ [By De-Moivre's theorem]

and $1/x^n=\cos n\theta-i\sin n\theta$

$\therefore$ $x^n+(1/x^n)=2\cos n\theta$

$\therefore$ $x^{2n}-2x^n\cos n\theta+1=0$

59. (a) : $n=15,\ \Sigma x^2=2830,\ \Sigma x=170$

(New)$\Sigma x^2=2830-400+900=3330$

(New)$\Sigma x=170-20+30=180$

Corrected variance, *i.e.*, σ^2

$$=\frac{(\text{New})\sum x^2}{n}-\left(\frac{(\text{New})\sum x}{n}\right)^2$$

$$=\frac{3330}{15}-\left(\frac{180}{15}\right)^2=78.0$$

60. (c) : We have, $\sin x+\sin^2x=1$

or $\sin x=1-\sin^2x$ or $\sin x=\cos^2x$

$\therefore$ $\cos^{12}x+3\cos^{10}x+3\cos^8x+\cos^6x-2$

$=\sin^6x+3\sin^5x+3\sin^4x+\sin^3x-2$

$=(\sin^2x)^3+3(\sin^2x)^2\sin x+3(\sin^2x)(\sin x)^2+(\sin x)^3-2$

$=(\sin^2x+\sin x)^3-2=(1)^3-2=-1.$

61. (d) : $\lim\limits_{x\to\infty}\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}$

$$=\lim_{x\to\infty}\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}$$

(Using Rationalisation)

$$=\lim_{x\to\infty}\frac{\sqrt{1+\frac{1}{\sqrt{x}}}}{\sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x\sqrt{x}}}}+1}=\frac{1}{1+1}=\frac{1}{2}$$

62. (b) : Since $\dfrac{1}{p+q},\dfrac{1}{r+p}$ and $\dfrac{1}{r+q}$ are in A.P.

$$\therefore\quad \frac{1}{r+p}-\frac{1}{p+q}=\frac{1}{q+r}-\frac{1}{r+p}$$

$$\Rightarrow\ \frac{p+q-r-p}{(r+p)(p+q)}=\frac{r+p-q-r}{(q+r)(r+p)}$$

$\Rightarrow \dfrac{q-r}{p+q} = \dfrac{p-q}{q+r}$ or $q^2 - r^2 = p^2 - q^2$

Hence, p^2, q^2, r^2 are in A.P.

63. (b) : $\Delta = (2+i)\begin{vmatrix} 1 & 1 & i \\ 1 & 1+2i & 1+i \\ 1 & 2 & 1-i \end{vmatrix}$

$= (2+i)\begin{vmatrix} 0 & -2i & -1 \\ 0 & -1+2i & 2i \\ 1 & 2 & 1-i \end{vmatrix}$

(By $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$)

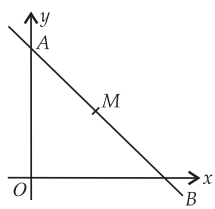
$= (2+i)\{-4i^2 + (-1+2i)\} = (2+i)(4-1+2i)$

$= (2+i)(3+2i) = 4 + 7i$.

64. (a) : Coordinates of $A \equiv \left(0, \dfrac{p}{\sin\alpha}\right)$

Coordinates of $B \equiv \left(\dfrac{p}{\cos\alpha}, 0\right)$

Coordinates of $M \equiv \left(\dfrac{p}{2\cos\alpha}, \dfrac{p}{2\sin\alpha}\right)$



Now, let $x = \dfrac{p}{2\cos\alpha}$ ...(i) ,

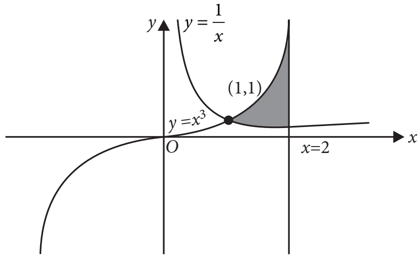
$y = \dfrac{p}{2\sin\alpha}$ ...(ii)

Since, $\cos^2\alpha + \sin^2\alpha = 1$

$\Rightarrow \left(\dfrac{p}{2x}\right)^2 + \left(\dfrac{p}{2y}\right)^2 = 1$ [From (i) and (ii)]

$\Rightarrow \dfrac{p^2}{4}\left[\dfrac{x^2+y^2}{x^2y^2}\right] = 1 \Rightarrow x^{-2} + y^{-2} = 4p^{-2}$

65. (d) : The point of intersection of the curves $y = x^3$ and $y = 1/x$ is (1, 1)



$\therefore$ The required area $= \displaystyle\int_1^2 \left(x^3 - \dfrac{1}{x}\right)dx$

$= \left[\dfrac{x^4}{4} - \log x\right]_1^2 = 4 - \log_e 2 - \dfrac{1}{4}$

$= \left(\dfrac{15}{4} - \log_e 2\right)$ sq. units

66. (d) : Given that $\tan\theta - \cot\theta = a$

and $\sin\theta + \cos\theta = b$

Now, $(b^2 - 1)^2 (a^2 + 4)$

$= \{(\sin\theta + \cos\theta)^2 - 1\}^2 \{(\tan\theta - \cot\theta)^2 + 4\}$

$= [1 + \sin 2\theta - 1]^2 [\tan^2\theta + \cot^2\theta - 2 + 4]$

$= \sin^2 2\theta(\operatorname{cosec}^2\theta + \sec^2\theta)$

$= 4\sin^2\theta\cos^2\theta\left[\dfrac{1}{\sin^2\theta} + \dfrac{1}{\cos^2\theta}\right] = 4$

67. (a) : For binomial distribution mean $= np = 20$ ...(i)

variance $= npq = 16$ ...(ii)

$20q = 16$ (from (i) and (ii))

$\Rightarrow q = \dfrac{4}{5} \Rightarrow 1 - p = \dfrac{4}{5} \Rightarrow p = \dfrac{1}{5}$

Now, $n \times \dfrac{1}{5} = 20 \Rightarrow n = 100$

$\therefore (n, p) = \left(100, \dfrac{1}{5}\right)$

68. (d) : $1^2 + 2^2 + ... + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

$= \dfrac{n \times (n+1) \times (n+1/2)}{3} > \dfrac{n \cdot n \cdot n}{3} = \dfrac{n^3}{3}$

$[\because n + 1 > n$ and $n + 1/2 > n]$

69. (a) : The given lines are perpendicular so, vectors $\vec{b}_1 = -3\hat{i} + 2\alpha\hat{j} + 2\hat{k}$ and $\vec{b}_2 = 3\alpha\hat{i} + \hat{j} - 5\hat{k}$ are perpendicular. So $\vec{b}_1 \cdot \vec{b}_2 = 0$

$\Rightarrow -9\alpha + 2\alpha - 10 = 0 \Rightarrow -7\alpha = 10$

$\Rightarrow \alpha = -10/7$

70. (b) : We have, $\dfrac{dy}{dx} + \sqrt{\dfrac{1-y^2}{1-x^2}} = 0$

$\Rightarrow \dfrac{dy}{dx} = -\sqrt{\dfrac{1-y^2}{1-x^2}} \Rightarrow \dfrac{dy}{\sqrt{1-y^2}} = -\dfrac{dx}{\sqrt{1-x^2}}$

$\Rightarrow \displaystyle\int \dfrac{dy}{\sqrt{1-y^2}} = -\int \dfrac{dx}{\sqrt{1-x^2}}$

$\Rightarrow \sin^{-1} y + \sin^{-1} x = \sin^{-1} a$

$\Rightarrow x\sqrt{1-y^2} + y\sqrt{1-x^2} = a$

71. (45) : If $2 - i$ is the root then $2 + i$ is also the root

Sum of roots $= 2 - i + 2 + i = 4$

$\Rightarrow \dfrac{-12}{a} = 4 \Rightarrow a = -3$

Product of roots $= \dfrac{b}{a} = (2-i)(2+i)$

$$\Rightarrow \frac{b}{a} = 4-(i)^2 = 4+1=5$$

$\Rightarrow b = 5 \times (-3) \Rightarrow b = -15$

$\therefore ab = -3 \times (-15) = 45$

72. (3) : Let $A = \begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix}$,

where $t = \frac{2\pi}{3} \Rightarrow 3t = 2\pi$

$$\therefore A^2 = \begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix}\begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} \cos 2t & -\sin 2t \\ \sin 2t & \cos 2t \end{bmatrix}$$

Now, $A^2 \times A = A^3$

$$= \begin{bmatrix} \cos 2t & -\sin 2t \\ \sin 2t & \cos 2t \end{bmatrix}\begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix}$$

$$= \begin{bmatrix} \cos 3t & -\sin 3t \\ \sin 3t & \cos 3t \end{bmatrix} = \begin{bmatrix} \cos 2\pi & -\sin 2\pi \\ \sin 2\pi & \cos 2\pi \end{bmatrix}$$

$\Rightarrow A^3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\therefore$ Least value of $k = 3$.

73. (2) : The equation of any plane containing the given line is $(x + y + 2z - 3) + \lambda (2x + 3y + 4z - 4) = 0$

$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (2 + 4\lambda)z - (3 + 4\lambda) = 0$ ...(i)

If the plane is parallel to z-axis whose direction cosines are 0, 0, 1 ; then the normal to the plane will be perpendicular to z-axis.

$\therefore (1+2\lambda)(0)+(1+3\lambda)(0) + (2 + 4\lambda)(1)=0$

$\Rightarrow \lambda = -\frac{1}{2}$

Putting value of λ in (i), we get required plane

$$\left(x+y+2z-3\right)-\frac{1}{2}\left(2x+3y+4z-4\right)=0$$

$\Rightarrow y + 2 = 0$ ...(ii)

$\therefore$ S.D. = distance of any point say (0, 0, 0) on z-axis from plane (ii) $= \frac{2}{\sqrt{(1)^2}} = 2$

74. (3) : $(\vec{a}\times\vec{b})^2 + (\vec{a}\cdot\vec{b})^2 = 144$

$\Rightarrow |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2 + (\vec{a}\cdot\vec{b})^2 = 144$

$\Rightarrow 16\cdot|\vec{b}|^2 = 144 \Rightarrow |\vec{b}|^2 = 9 \Rightarrow |\vec{b}| = 3$

75. (2) : $f(x)$ is increasing.

So its greatest value is $f(3) = 27$.

Let the G.P. be $a, ar, ar^2, \ldots$ with, $-1 < r < 1$

$\frac{a}{1-r} = 27$ and $a - ar = 3$

On solving, we get $r = 2/3$