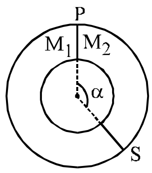
GUIDED REVISION

SECTION-I

Single Correct Answer Type **8 Q. [3 M (–1)]**

1. A ring shaped tube contains two ideal gases with equal masses and molar masses $M_1 = 32$ and $M_2 = 28$. The gases are separated by one fixed partition P and another movable stopper S which can move freely without friction inside the ring. The angle α is



(A) 182° (B) 170° (C) 192° (D) 180°

2. What is fraction of molecule below an altitude h in atmosphere? Assume uniform gravitational field, isothermal conditions, mass of a molecule m, Boltzman constant k, temperature T.

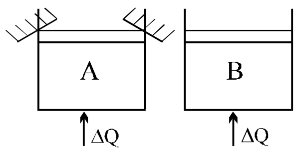
(A) $f = e^{(mgh/kT)}$ (B) $f = e^{-(mgh/kT)}$ (C) $f = 1 - e^{-(mgh/kT)}$ (D) $f = 1 - e^{(mgh/kT)}$

3. A student records ΔQ, ΔU & ΔW for a thermodynamic cycle $A \to B \to C \to A$. Certain entries are missing. Find correct entry in following options.

	AB	BC	CA
ΔW	40J		30J
ΔU		50J	
ΔQ	150J	10J	

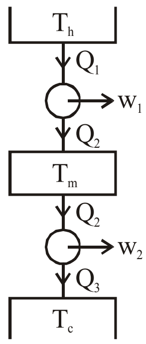
(A) $W_{BC} = -70$ J (B) $\Delta Q_{CA} = 130$ J (C) $\Delta U_{AB} = 190$ J (D) $\Delta U_{CA} = -160$ J

4. Two identical vessels A & B contain equal amount of ideal monoatomic gas. The piston of A is fixed but that of B is free. Same amount of heat is absorbed by A & B. If B's internal energy increases by 100 J the change in internal energy of A is :-



(A) 100 J (B) $\frac{500}{3}$ J (C) 250 J (D) none of these

5. Suppose that two heat engines are connected in series, such that the heat exhaust of the first engine is used as the heat input of the second engine as shown in figure. The efficiencies of the engines are η_1 and η_2, respectively. The net efficiency of the combination is given by



(A) $\eta_{net} = \eta_2 + (1 - \eta_1)\eta_2$ (B) $\eta_{net} = \frac{\eta_1}{(1-\eta_1)\eta_2}$

(C) $\eta_{net} = \eta_1 + (1 - \eta_1)\eta_2$ (D) $\eta_{net} = \frac{1-\eta_1}{(1-\eta_2)\eta_2}$

6. At temperature T, N molecules of gas A each having mass m and at the same temperature 2N molecules of gas B each having mass 2m are filled in a container. The mean square velocity of molecules of gas B is v^2 and mean square of x component of velocity of molecules of gas A is w^2. The ratio of w^2/v^2 is :

(A) 1 (B) 2 (C) 1/3 (D) 2/3

7. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively) :-

[JEE-Mains-2016]

(A) $n=\frac{C-C_V}{C-C_P}$ (B) $n=\frac{C_P}{C_V}$ (C) $n=\frac{C-C_P}{C-C_V}$ (D) $n=\frac{C_P-C}{C-C_V}$

8. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that

$C_p - C_v = a$ for hydrogen gas

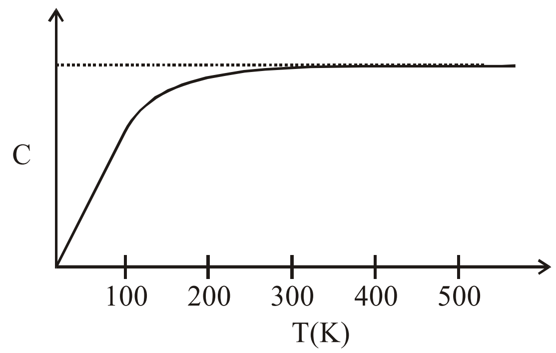
$C_p - C_v = b$ for nitrogen gas

The correct relation between a and b is : **[JEE-Main 2017]**

(A) a = 14 b (B) a = 28 b (C) $a = \frac{1}{14}b$ (D) a = b

Multiple Correct Answer Type 3 Q. [4 M (–1)]

9. The figure below shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation :- **[JEE-2013]**



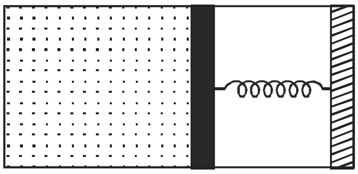
(A) the rate at which heat is absorbed in the range 0–100 K varies linearly with temperature T.

(B) heat absorbed in increasing the temperature from 0–100 K is less than the heat required for increasing the temperature from 400–500 K.

(C) there is no change in the rate of heat absorption in the range 400–500 K

(D) the rate of heat absorption increases in the range 200–300 K

10. An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder, the correct statement(s) is(are) :- **[JEE-Advance-2015]**



(A) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $\frac{1}{4}P_1V_1$

(B) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3P_1V_1$

(C) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\frac{7}{3}P_1V_1$

(D) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $\frac{17}{6}P_1V_1$

11. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is/are true ? **[JEE-Advance 2017]**

(A) The resistive force experienced by the plate is proportional to v

(B) The pressure difference between the leading and trailing faces of the plate is proportional to uv.

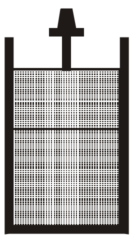
(C) The plate will continue to move with constant non-zero acceleration, at all times

(D) At a later time the external force F balances the resistive force.

Linked Comprehension Type (1 Para × 2Q. 1 Para × 3Q. 1 Para × 4Q.) [3 M (-1)] (Single Correct Answer Type)

Paragraph for Questions 12 and 13

In the figure a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monatomic gas are $C_V = \frac{3}{2}R$, $C_P = \frac{5}{2}R$, and those for an ideal diatomic gas are $C_V = \frac{5}{2}R, C_P = \frac{7}{2}R$. **[JEE-Advance-2014]**



12. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be

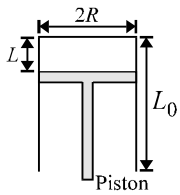
(A) 550 K (B) 525 K (C) 513 K (D) 490 K

13. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be

(A) 250 R (B) 200 R (C) 100 R (D) –100 R

Paragraph for Questions No. 14 to 16 (3 questions)

A fixed thermally conducting cylinder has a radius R and length L_0. The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface, as shown in the figure. The atmospheric pressure is P_0.



14. The piston is now pulled out slowly and held at a distance $2L$ from the top. The pressure in the cylinder between its top and the piston will then be **[JEE 2007]**

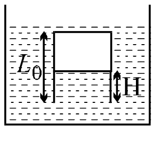
(A) P_0 (B) $P_0/2$ (C) $\frac{P_0}{2} + \frac{Mg}{\pi R^2}$ (D) $\frac{P_0}{2} - \frac{Mg}{\pi R^2}$

15. While the piston is at a distance $2L$ from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is :- **[JEE 2007]**

(A) $\left(\frac{2P_0\pi R^2}{\pi R^2 P_0 + Mg}\right)(2L)$ (B) $\left(\frac{P_0\pi R^2 - Mg}{\pi R^2 P_0}\right)(2L)$

(C) $\left(\frac{P_0\pi R^2 + Mg}{\pi R^2 P_0}\right)(2L)$ (D) $\left(\frac{P_0\pi R^2}{\pi R^2 P_0 - Mg}\right)(2L)$

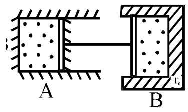
16. The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in the figure. The density of the water is ρ. In equilibrium, the height H of the water column in the cylinder satisfies **[JEE 2007]**



(A) $\rho g(L_0 - H)^2 + P_0(L_0 - H) + L_0P_0 = 0$
(B) $\rho g(L_0 - H)^2 - P_0(L_0 - H) - L_0P_0 = 0$
(C) $\rho g(L_0 - H)^2 + P_0(L_0 - H) - L_0P_0 = 0$
(D) $\rho g(L_0 - H)^2 - P_0(L_0 - H) + L_0P_0 = 0$

Paragraph for Questions No. 17 to 20 (4 questions)

Two cylinder A and B having piston connected by massless rod (as shown in figure). The cross-sectional area of two cylinders are same & equal to 'S'. The cylinder A contains m gm of an ideal gas at Pressure P & temperature T_0. The cylinder B contain identical gas at same temperature T_0 but has different mass. The piston is held at the state in the position so that volume of gas in cylinder A & cylinder B are same & is equal to V_0.



The walls & piston of cylinder A are thermally insulated, whereas cylinder B is maintained at temperature T_0 reservoir. The whole system is in vacuum. Now the piston is slowly released and it moves towards left & mechanical equilibrium is reached at the state when the volume of gas in cylinder A becomes $\frac{V_0}{2}$. Then (here γ for gas = 1.5)

17. The mass of gas in cylinder B

(A) $2\sqrt{2}$ m (B) $3\sqrt{2}$ m (C) $\sqrt{2}$ m (D) none

18. The change in internal energy of gas in cylinder A

(A) $(\sqrt{2}-1)\,PV_0$ (B) $2(\sqrt{2}-1)\,PV_0$ (C) $\frac{PV_0}{(\sqrt{2}-1)}$ (D) none

19. If work done by gas in cylinder B is W_B & work done by gas in cylinder A is W_A then

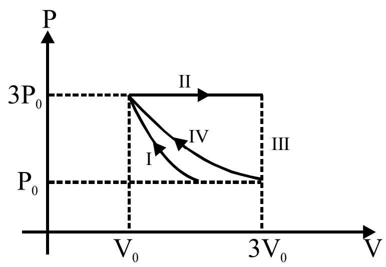
(A) $W_A = -W_B$ (B) $|W_A| > |W_B|$ (C) $|W_A| < |W_B|$ (D) we can't say anything

20. What will be the compressive force in connecting rod at equilibrium

(A) PS (B) $\sqrt{2}$ PS (C) $2^{3/2}$ PS (D) none

Matching List Type (4 × 4) **1 Q. [3 M (–1)]**

21. One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List–II. **[JEE-Advance 2018]**



List–I		List–II	
P.	In process I	1.	Work done by the gas is zero
Q.	In process II	2.	Temperature of the gas remains unchanged
R.	In process III	3.	No heat is exchanged between the gas and its surroundings
S.	In process IV	4.	Work done by the gas is $6\,P_0V_0$

(A) $P \rightarrow 4$; $Q \rightarrow 3$; $R \rightarrow 1$; $S \rightarrow 2$

(B) $P \rightarrow 1$; $Q \rightarrow 3$; $R \rightarrow 2$; $S \rightarrow 4$

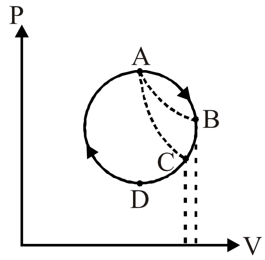
(C) $P \rightarrow 3$; $Q \rightarrow 4$; $R \rightarrow 1$; $S \rightarrow 2$

(D) $P \rightarrow 3$; $Q \rightarrow 4$; $R \rightarrow 2$; $S \rightarrow 1$

SECTION-IV

Matrix Match Type (4 × 5) **2 Q. [8 M (for each entry +2(0)]**

1. For an ideal gas a process PV diagram is a circle. An adiabatic from A passes through C. An isotherm from A passes through B. We take a part of the circular cyclic process. Comment on the sign of the quantity of column-I.

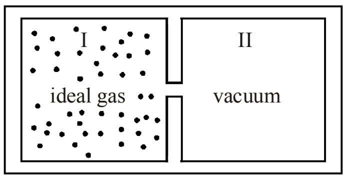


Column-I		**Column-II**	
(A)	Heat given to the gas in going from A to C along circle	(P)	Positive
(B)	Heat given to the gas in going from B to C along circle	(Q)	Negative
(C)	Heat given to the gas in going from C to D along circle	(R)	Zero
		(S)	can't be said

2. **Column I** contains a list of processes involving expansion of an ideal gas. Match this with **Column II**. describing the thermodynamic change during this process. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS. **[JEE 2008]**

Column I

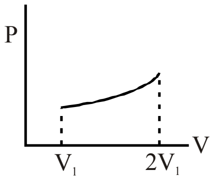
(A) An insulated container has two chambers separated by a valve. Chamber I contains an ideal gas and the chamber II has vacuum. The valve is opened..



(B) An ideal monoatomic gas expands to twice its original volume such that pressure $P \propto \frac{1}{V^2}$, where V is the volume of the gas.

(C) An ideal monoatomic gas expands to twice its original volume such that its pressure $P \propto \frac{1}{V^{4/3}}$, where V is its volume.

(D) An ideal monoatomic gas expands such that its pressure P and volume V follows the behaviour shown in the graph.



Column II

(P) The temperature of the gas decreases

(Q) The temperature of the gas increases or remains constant

(R) The gas loses heat

(S) The gas gains heat

Subjective Type 8 Q. [4 M (0)]

1. 0.12 mole of a monoatomic ideal gas at pressure $P_0 = 35\ Nm^{-2}$ and volume $V_0 = 8\ m^3$ is brought to pressure $P_1 = 10\ Nm^{-2}$ and volume $V_1 = 48\ m^3$ by a process following the equation $P = aV + b$; where $a = -0.625\ Nm^{-5}$ and $b = 40\ Nm^{-2}$. Find :

(i) the maximum temperature during the process.

(ii) the heat transferred Q from the volume V_0 to any other volume V along the line.

(iii) the value of V at which Q is maximum.

(iv) the heat transferred along the line from V_0 to V $(Q = Q_{max})$

(v) the heat transferred along the line from V $(Q = Q_{max})$ to V_1.

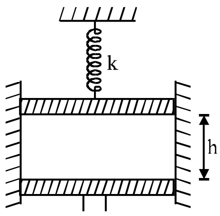
2. A barometer is faulty. When the true barometer reading are 73 and 75 cm of Hg, the faulty barometer reads 69 cm and 70 cm respectively.

(i) What is the total length of the barometer tube?

(ii) What is the true reading when the faulty barometer reads 69.5 cm ?

(iii) What is the faulty barometer reading when the true barometer reads 74 cm?

3. An ideal gas at NTP is enclosed in a adiabatic vertical cylinder having area of cross section $A = 27cm^2$, between two light movable pistons as shown in the figure. Spring with force constant $k = 3700$ N/m is in a relaxed state initially. Now the lower piston is moved upwards a height h/2, h being the initial length of gas column. It is observed that the upper piston moves up by a distance h/16. Find h taking γ for the gas to be 1.5. Also find the final temperature of the gas.



4. The figure shows an insulated cylinder divided into three parts A, B and C. Pistons I and II are connected by a rigid rod and can move without friction inside the cylinder. Piston I is perfectly conducting while piston II is perfectly insulating. The initial state of the gas $(\gamma = 1.5)$ present in each compartment A, B and C is as shown.

Now, compartment A is slowly given heat through a heater H such that the final volume of C becomes $\frac{4V_0}{9}$.

Assume the gas to be ideal and find.

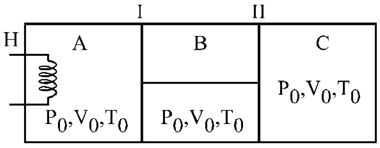
(a) Final pressures in each compartment A, B and C

(b) Final temperatures in each compartment A, B and C

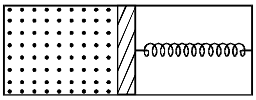
(c) Heat supplied by the heater

(d) Work done by gas in A and B.

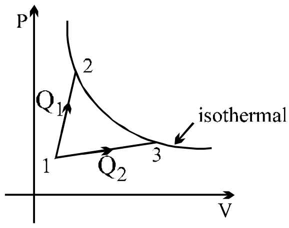
(e) Heat flowing across piston I.



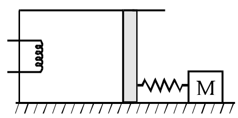
5. A thermally insulated vessel is divided into two equal parts by a heat-insulating piston which can move in the vessel without the friction. The left part of the vessel contains one mole of an ideal monatomic gas, & the right part is empty. The piston is connected to the right wall of the vessel through a spring whose length in free state is equal to the length of the vessel as shown in the figure. Determine the heat capacity C of the system, neglecting the heat capacities of the vessel, piston and spring.



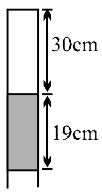
6. A gas takes part in two processes in which it is heated from the same initial state 1 to the same final temperature. The processes are shown on the P-V diagram by the straight line 1-2 and 1-3. 2 and 3 are the points on the same isothermal curve. Q_1 and Q_2 are the heat transfer along the two processes. Then in which case will the heat transfer be more.



7. An adiabatic cylinder has 8 gram of helium. A light smooth adiabatic piston is connected to a light spring of force constant 300 N/m. The other end of the spring is connected with a block of mass 1 kg kept on a rough horizontal surface of coefficient of friction $\mu = 0.3$. Area of cross-section of cylinder is $A = 25\ cm^2$. Initially the spring is in a relaxed position and the temperature of the gas is 400 K. The gas is heated slowly for some time by means of an electric heater so as to bring the block M on the verge of motion. Take $P_{atm} = 10^5\ N/m^2$. Find
(a) the work done by the gas
(b) the final temperature
(c) heat supplied by the heater



8. A certain amount of air at 300 K is trapped in a glass tube between its closed end and a 19 cm long mercury column as shown in figure.



(a) What will be the length of air column if the tube is inverted isothermally?
(b) To what temperature should the air column be heated so that it regains its original volume?
(c) How much heat needs to be supplied to the air column for this purpose? Area of cross-section of the tube = 1 cm^2, $\gamma_{air} = 7/5$. Atmospheric pressure =76cm of Hg $\cong 10^5$ Pa. Neglect surface tension.

SECTION-I

Single Correct Answer Type 8 Q. [3 M (–1)]

1. Ans. (C) **2. Ans. (C)** **3. Ans. (D)** **4. Ans. (B)**
5. Ans. (C) **6. Ans. (D)** **7. Ans. (C)** **8. Ans. (A)**

Multiple Correct Answer Type 3 Q. [4 M (–1)]

9. Ans. (A, B, C, D) **10. Ans. (A,B,C)** **11. Ans. (A,B,D)**

Linked Comprehension Type (1 Para × 2Q. 1 Para × 3Q. 1 Para × 4Q.) [3 M (-1)]
(Single Correct Answer Type)

12. Ans. (D) **13. Ans. (D)** **14. Ans. (A)** **15. Ans. (D)**
16. Ans. (C) **17. Ans. (B)** **18. Ans. (B)** **19. Ans. (C)**
20. Ans. (C)

Matching List Type (4 × 4) 1 Q. [3 M (–1)]

21. Ans. (C)

SECTION-IV

Matrix Match Type (4 × 5) 2 Q. [8 M (for each entry +2(0)]

1. Ans. (A) P (B) Q (C) Q **2. Ans. (A) Q; (B) P,R; (C) P,S; (D) Q,S**

Subjective Type 8 Q. [4 M (0)]

1. Ans. (i) 642 K, (ii) $100 (V - 8) - 1.25 (V^2 - 8^2)$, (iii) $V = 40\ m^3$, (iv) 1280 J, (v) – 80 J

2. Ans. (i) 74 cm, (ii) 73.94 cm, (iii) 69.52 cm **3. Ans.** 1.6 m, 364 K

4. Ans. (a) Final pressure in A = $\frac{27}{8} P_0$ = Final pressure in C, Final pressure in B = $\frac{21}{4} P_0$

(b) Final temperature in A (and B) = $\frac{21}{4} T_0$, Final temperature in C = $\frac{3}{2} T_0$,

(c) $18\ P_0V_0$

(d) work done by gas in A = $+ P_0V_0$, work done by gas in B = 0,

(e) $\frac{17}{2} P_0V_0$

5. Ans. C = 2R **6. Ans.** $Q_1 < Q_2$ **7. Ans.** (a) 2.515, (b) 404.8 K, (c) 122.179 J

8. Ans. (a) 18 cm, (b) 500 K, (c) $\frac{21}{4}$

SOLUTIONS
SECTION-I

Single Correct Answer Type 8 Q. [3 M (–1)]

1. Ans. (C)

Sol. For movable stopper to be in equilibrium,

$F_1 = F_2 \Rightarrow P_1A = P_2A$

$P_1 = P_2$

$$\frac{n_1RT}{V_1} = \frac{n_2RT}{V_2}$$

$$\frac{n_1}{V_1} = \frac{n_2}{V_2}$$

$$n_1 = \frac{m}{32}, n_2 = \frac{m}{28}$$

$V_1 = (2\pi - \alpha)A$

$V_2 = \alpha A$

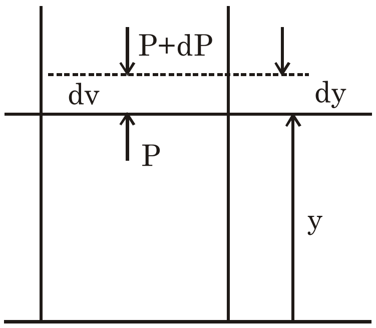
Where A is cross sectional area of tube

$$\frac{V_2}{V_1} = \frac{n_2}{n_1} \Rightarrow \frac{\alpha}{2\pi - \alpha} = \frac{32}{28}$$

$$\Rightarrow \alpha = \frac{16\pi}{15}$$

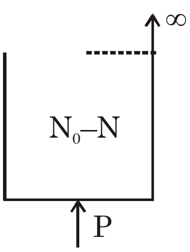
2. Ans. (C)

Sol.



Let total number of molecules are N_0 and molecules below height y are N.

for volume of gas, from height h to ∞



$P \times A = (N_0 - N)mg$ ... (i)

Where m is mass of 1 molecule

for volume dV,

PdV = dN kT (T is constant)

$P \times Ady = dN\, kT$

from (i)

$(N_0 - N)\, mg\, dy = dN\, kT$

$$\frac{dN}{N_0 - N} = \frac{mg}{kT} dy$$

for y = 0, N = 0

$$\int_0^N \frac{dN}{N_0 - N} = \frac{mg}{kT} \int_0^y dy$$

$$-\left[\ell n\left(N_0 - N\right)\right]_0^N = \frac{mg}{kT} y$$

$$\left(\frac{N_0 - N}{N_0}\right) = e^{-mgy/kT}$$

$$1 - \frac{N}{N_0} = e^{-mgy/kT}$$

$$\frac{N}{N_0} = 1 - e^{-mgy/kT}$$

3. Ans. (D)

Sol. For any process

$\Delta Q = \Delta U + \Delta W$

For AB $\Rightarrow \Delta U = 110$ J

For BC $\Rightarrow \Delta W = -40$ J

For cycle $\Rightarrow \Delta U = 0$ J

$\Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA} = 0$

$\Delta U_{CA} = -160$ J

4. Ans. (B)

Sol. For isochoric process, $\Delta U = \Delta Q$

$$\frac{3}{2} nR\Delta T = 100$$

$\Delta Q = \Delta U + \Delta W$

For isobaric $\Delta Q = n\frac{3}{2}R\Delta T + Rn\Delta T$

$$= \frac{5}{2}\left(nR\Delta T\right)$$

$$\Delta Q = \frac{5}{3}\left(\frac{3}{2}nR\Delta T\right) = \frac{500}{3}$$

$$\Delta Q = \Delta U = \frac{500}{3}$$

5. **Ans. (C)**

Sol. $\eta_1 = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$

$Q_1 = \frac{Q_2}{1-\eta_1}$

$\eta_2 = \frac{Q_2 - Q_3}{Q_2} = 1 - \frac{Q_3}{Q_2}$

$Q_3 = Q_2\,(1 - \eta_2)$

Now $\eta = \frac{Q_1 - Q_3}{Q_1} = 1 - \frac{Q_3}{Q_1}$

$= 1 - \frac{Q_2(1-\eta_2)}{Q_2}(1-\eta_1)$

6. **Ans. (D)**

Sol. For gas A

$v_{rms}^2 = v_x^2 + v_y^2 + v_z^2$

$v_x^2 = \frac{v_{rms}^2}{3} = \omega^2$

$\omega^2 = \frac{1}{3}\left(\frac{3RT}{M}\right)$

For gas B

$v^2 = \frac{3RT}{2M}$

$\frac{v^2}{\omega^2} = \frac{2}{3}$

7. **Ans. (C)**

Sol. Specific heat $C = \frac{R}{1-n} + C_V$ for polytropic process

$\therefore \ \frac{R}{1-n} + C_V = C$

$\frac{R}{1-n} = C - C_V \Rightarrow \frac{R}{C - C_V} = 1 - n$

(Where $R = C_P - C_V$)

$\Rightarrow \ n = \frac{C - C_P}{C - C_V}$

8. **Ans. (A)**

Sol. $C_P - C_V = R$

where C_P and C_V are molar specific heat of gas at constant pressure and at constant volume respectively (in J/mol k)

R is universal gas constant (J/mole k)

⇒ When C_P & C_V are gram specific heat in J/g-k

$$C_P - C_V = \frac{R}{M_W}$$

$$a = \frac{R}{2}, b = \frac{R}{28}$$

Multiple Correct Answer Type 3 Q. [4 M (–1)]

9. Ans. (A, B, C, D)

Sol. (A) $\Delta Q = mC\Delta T$

$$\frac{\Delta Q}{\Delta T} = mC$$

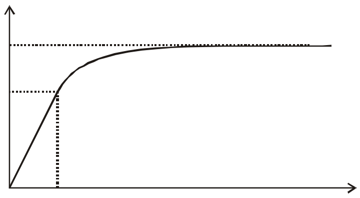
at temperature is increased at a constant rate

$\Delta T \propto \Delta t$

Rate of heat absorption,

$$\frac{\Delta Q}{\Delta t} \propto C$$

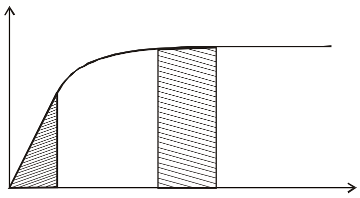
From 0 to 100 K the major part of graph lies in linear region and very small part in non-linear region, therefore to a reasonable approximation between 0 K – 100 K, graph of C vs T is linear.



$C \propto T$

$$\frac{\Delta Q}{\Delta t} \propto T$$

(B) by comparing area under curve



(C) from 400 K to 500 K, Graph of C vs T become asymptotic hence rate of heat absorption become constant

$$\frac{\Delta Q}{\Delta t} \propto C$$

C = constant

(D) The rate of heat absorption increases as C is increasing.

C is increasing between 200 – 300, hence $\frac{\Delta Q}{\Delta t}$ is also increasing.

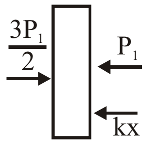
10. Ans. (A,B,C)

Sol. (A) As spring is initially relaxed, surrounding pressure is P_1.

$$\frac{kx}{A} = \frac{P_1}{2}$$

$\frac{1}{2}kx^2$

$= \frac{1}{2}\left(\frac{P_1}{2}\right)(xA) = \frac{P_1}{4}[V_2 - V_1] = \frac{P_1V_1}{4}$



(B) $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

if $V_2 = 2V_1$ & $T_2 = 3T_1$

$\Rightarrow P_2 = \frac{3}{2}P_1$

$\Delta U = \frac{f}{2}(nR\Delta T) = \frac{3}{2}(P_2V_2 - P_1V_1)$

$= \frac{3}{2} \times 2P_1V_1 = 3P_1V_1$

(C) $V_2 = 3V_1$, $T_2 = 4T_1$

$\Rightarrow P_2 = \frac{4}{3}P_1$

$W_{gas} + W_{sp} + W_{atm} = 0$

$W_{gas} = -W_{sp} - W_{atm}$

$= -\left(-\frac{1}{2}kx^2\right) - (-P_1\Delta V)$

$= \frac{1}{2}kx \times x + P_1(V_2 - V_1)$

$= \frac{1}{2}(P_2 - P_1) \times Ax + P_1 \times 2V_1$

$= \frac{1}{2} \times \frac{1}{3}P_1 \times 2V_1 + 2P_1V_1$

$= \left(\frac{1}{3} + 2\right)P_1V_1 = \frac{7}{3}P_1V_1$

(D) According to the first law of thermodynamics

$\Delta Q = \Delta U + W$

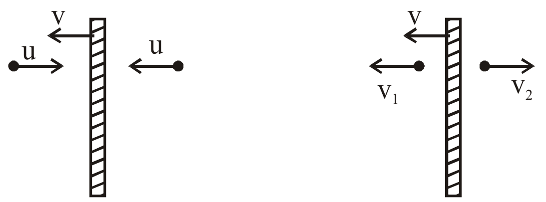
$= \frac{3}{2}(nR\Delta T) + \frac{7}{3}P_1V_1$

$= \frac{3}{2}(P_2V_2 - P_1V_1) + \frac{7}{3}P_1V_1$

$= \left(\frac{9}{2} + \frac{7}{3}\right)P_1V_1 = \frac{41}{6}P_1V_1$

11. Ans. (A,B,D)

Sol.



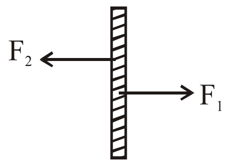
Just before the collision Just after the collision

$v_1 = u + 2v$ $v_2 = (u - 2v)$

$\Delta v_1 = (2u + 2v)$ $\Delta v_2 = (2u - 2v)$

$F_1 = \frac{dp_1}{dt} = \rho A(u + v)\,(2u + 2v)$ $F_2 = \frac{dp_2}{dt} = \rho A(u - v)\,(2u - 2v)$

$= 2\rho A\,(u + v)^2$ $= 2\rho A\,(u - v)^2$



$\Delta F = F_1 - F_2$ (ΔF net force due to the air molecules on the plate)

$= 2\rho A\,(4uv) = 8\rho Auv$

$P = \frac{\Delta F}{A} = 8\rho(uv)$

$F_{net} = (F - \Delta F) = ma$ (m is mass of the plate)

$F - (8\rho Au)v = ma$ a depends on v

as v is changing, acceleration isnot constant.

(D) After some time

ΔF will increase as v is increasing

$\Delta F = 8\rho Auv$

and finally, $\Delta F = F$

Linked Comprehension Type (1 Para × 2Q. 1 Para × 3Q. 1 Para × 4Q.) [3 M (-1)] (Single Correct Answer Type)

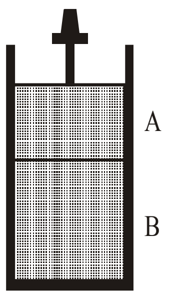
12. Ans. (D)

Sol. $\theta_A = \theta_B$

Process in A is isobaric

and

Process in B is isochoric



$nC_p\,(T - 400) = nC_V\,(700 - T)$

$2\frac{7}{2}\,R\,(T - 400) = 2 \times \frac{3}{2}\,R\,(700 - T)$

$7\,T - 2800 = 2100 - 3\,T$

$T = 490$

13. Ans. (D)

Sol. Now both processes are isobaric as piston is free to move

$\theta_A = \theta_B$

$n\frac{7}{2}$ R (T – 400) = n $\frac{5}{2}$ R (700 – T)

7 T – 2800 = 3500 – 5 T

12 T = 6300

T = 525

W = nRΔT + nRΔT_1

W = – 100 R

14. Ans. (A)

Sol. As the hole at the top is open

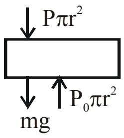
$P_{inside} = P_{outside}$

15. Ans. (D)

Sol. Since the process is isothermal, $P_1V_1 = P_2V_2$

$AP_0 2L = P(x)A$

$$x = \frac{P_0(2L)}{P_1}$$



$Mg + P\pi r^2 = P_0\pi r^2$

$P = P_0^{\,2} - Mg/\pi r^2$

$$P = \frac{P_0}{\left(\frac{P_0\pi r^2 - mg}{\pi r^2}\right)}(2L)$$

16. Ans. (C)

Sol. Process is isothermal ⇒ $P_1V_1 = P_2V_2$

$P_0L_0 = P(L_0 - H)$ ... (i)

$P = P_0 + \rho gh$

$P_0 + \rho g(L_0 - H) = P$ ... (ii)

On solving (i) & (ii)

$\rho g(L_0 - H)^2 + P_0(L_0 - H) - L_0P_0 = 0$

17. Ans. (B)

Sol. Initially $V_A = V_B = V_0$

$T_A = T_0$, $T_B = T_0$ (always)

$m_A = m$, $P_i = P$

Finally, $V_A = \frac{V_0}{2}$, $V_B = \frac{3}{2}V_0$

$P_A = P_B$ (at equilibrium)

For A

process is adiabatic

$TV^{\gamma-1}$ = constant

γ = 1.5 (given)

$TV^{1/2}$ = constant

$$T_0V_0^{1/2} = T\left(\frac{V_0}{2}\right)^{1/2}$$

$T_A = T_0\sqrt{2}$

Now, $P_A = P_B$

$$\frac{nRT_A}{V_A} = \frac{nRT_B}{V_B}$$

$$m\frac{T_0\sqrt{2}}{\frac{V_0}{2}} = m_B\frac{T_0}{\frac{3V_0}{2}}$$

$[m_B = 3\sqrt{2}\,m]$

18. Ans. (B)

Sol. $\Delta V = nC_V\Delta T$

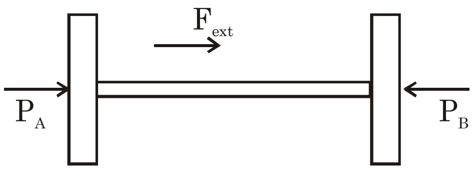
$$= n \times \frac{R}{\gamma - 1} \times (T_2 - T_1)$$

$$= \frac{nR(\sqrt{2}-1)T_0}{0.5}$$

$$= PV_0 \times 2(\sqrt{2}-1)$$

19. Ans. (C)

Sol. Since piston is slowly moving towards left



According to work energy theorem.

$W_B + W_{ext} + W_A = 0$

$W_B = -(W_A - W_{ext})$

Since $W_A < 0$, $W_{ext} < 0 \Rightarrow |W_B| = |W_A| + |W_{ext}|$

$|W_B| > |W_A|$

20. Ans. (C)

Sol. At equilibrium,

Compressive force, $F = p_f \times S$

For A $\Rightarrow \dfrac{PV_0}{T_0} = \dfrac{P_f \times \frac{V_0}{2}}{T_0\sqrt{2}} \Rightarrow P_f = P_x 2\sqrt{2} = 2^{3/2}P$

Matching List Type (4 × 4) 1 Q. [3 M (–1)]

21. Ans. (C)

Sol. Process – I is an adiabatic process

$\Delta Q = \Delta U + W$ $\quad\quad\quad$ $\Delta Q = 0$

$W = -\Delta U$

Volume of gas is decreasing $\Rightarrow W < 0$

$\Delta U > 0$

$\Rightarrow$ Temperatuer of gas increases.

$\Rightarrow$ No heat is exchanged between the gas and surrounding.

Process – II is an isobaric process

(Pressure remain constant)

$W = P\,\Delta V = 3P_0[3V_0 - V_0] = 6P_0V_0$

Process - III is an isochoric process
(Volume remain constant)
$\Delta Q = \Delta U + W$
$W = 0$
$\Delta Q = \Delta U$
Process – IV is an isothermal process
(Temperature remains constant)
$\Delta Q = \Delta U + W$
$\Delta U = 0$

SECTION-IV

Matrix Match Type (4 × 5) 2 Q. [8 M (for each entry +2(0)]

1. Ans. (A) P (B) Q (C) Q

Sol. For (A)
(A) For cyclic process ABCA
$Q_{ABC\ circle} + Q_{CA\ direct} > 0$
Since cycle is clockwise
Now $Q_{CA\ Direct} = 0$, process is adiabatic so
$\Delta Q_{ABC\ circle} > 0$
$A \to C$ from circle $\Delta Q > 0$
For (B)
$B \to C$
$W < 0$ (compresion)
$\Delta U < 0$ (PV product decreases) $\Rightarrow \Delta Q < 0$
For (C)
$C \to D$
$W < 0$ (compresion)
$\Delta U < 0$ (PV product decreases) $\Rightarrow \Delta Q < 0$

2. Ans. (A) Q; (B) P,R; (C) P,S; (D) Q,S

Sol. (A) $Q = 0$, $W = Pdv = 0$ so $\Delta U = 0 \Rightarrow T =$ constant

(B) $P \propto v^{-2}$ & $Pv = \mu RT \Rightarrow v \propto \frac{1}{T}$

Since volume increases so temperature decreases.
For polytropic process $Pv^2 =$ constant

$$C = C_v + \frac{R}{1-x} = C_v + \frac{R}{1-2} = \frac{3}{2}R - R = \frac{R}{2}$$

Since Vis increasing, T decreases ($\Delta T < 0$)

$$Q = \mu C\Delta T = \mu\left(\frac{R}{2}\right)\Delta T = \text{Negative}$$

(C) $C = C_v + \dfrac{R}{1-\frac{4}{3}} = -\dfrac{3}{2}R \Rightarrow Q = \mu C\Delta T = -\mu\left(\dfrac{3}{2}R\right)\Delta T \equiv$ Positive

$PV = nRT$

$$\frac{1}{V^{4/3}}V = nRT$$

$$T \propto \frac{1}{V^{1/3}}$$

if V increases, T decreases

$\Delta V > 0 \Rightarrow \Delta T < 0$

(D) W = positive, ΔT = positive ΔU = positive

$\Rightarrow Q = W + \Delta U$ = positive

Product of PV is increasing

Hence T is increasing

Subjective Type — 8 Q. [4 M (0)]

1. Ans. (i) 642 K, (ii) $100 (V - 8) - 1.25 (V^2 - 8^2)$, (iii) $V = 40\ m^3$, (iv) 1280 J, (v) – 80 J

Sol. (i) $P = aV + b$

$$\frac{nRT}{V} = aV + b$$

$$T = \frac{1}{nR}\left(aV^2 + Vb\right)$$

for maximum value of T,

$$\frac{dT}{dV} = 0$$

$$\frac{dT}{dV} = \frac{1}{nR}\left[2aV + b\right] = 0$$

$$V = \frac{-b}{2a} = 32m^3$$

$$P = -a \times \frac{b}{2a} + b = \frac{b}{2} = 20N/m^2$$

hence, $T = \dfrac{PV}{nR}$

T = 642 K

(ii) $\Delta Q = \Delta U + \Delta W$

$$= \frac{3}{2}nR\Delta T + \int_{V_0}^{V} Pdv$$

$$= \frac{3}{2}\left(PV - P_0V_0\right) + \int_{V_0}^{V}\left(aV + b\right)dV$$

$$= \frac{3}{2}\left[\left(aV - b\right)V - \left(aV_0 + b\right)V_0\right] + \frac{a}{2}\left(V^2 - V_0^2\right) + b\left(V - V_0\right)$$

$$Q = 2a\left(V^2 - V_0^2\right) + \frac{5}{2}b\left(V - V_0\right) \qquad \text{... (i)}$$

(iii) for Q_{max} $\dfrac{dQ}{dV} = 0$

$$\frac{dQ}{dV} = 2a\left(2V\right) + \frac{5}{2}b = 0$$

$$V = -\frac{5b}{8a} = 40m^3$$

(iv) Using equation (i)

$V = 40\ m^3,\ V_0 = 8\ m^3$

$A = -125\ (40^2 - 8^2) + 100\ (40 - 8)$

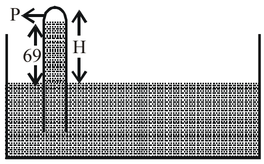
$= 1280\ J$

(v) Using equation (i) $V_1 = 48\ m^3,\ V = 40\ m^3$

$A = -1.25\ (48^2 - 40^2) + 100\ (48 - 40)$

$Q = -80\ J$

2. **Ans.** (i) 74 cm, (ii) 73.94 cm, (iii) 69.52 cm

Sol. (i) 

$73 = P + 69$

$P = 4$

Now $PV = nRT$

$4 \times (H - 69) = nRT$ ......(i)

$\because\ 75 = P_1 + 70$

$P_1 = 5$

Now $5(H - 70) = nRT$ ......(ii)

for (i) & (ii)

$4(H - 69) = 5(H - 70)$

$H = 74$

(ii) $69.5 + P = h$

$PV = nRT$

$(h - 69.5)\ (74 - 69.5) = nRT$ ......(iii)

from (i) & (iii)

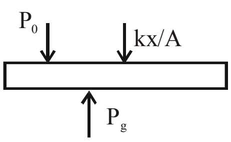
$(h - 69.5)\ (74 - 69.5) = 4 \times (5)$

$h - 69.5 = \dfrac{20}{4.5} \quad \Rightarrow h = 73.94$

(iii) $74 = h + p$

Now $p \times v = nRT \Rightarrow (74 - h)(74 - h) = 5 \times 4 \Rightarrow h = 69.52$

3. **Ans.** 1.6 m, 364 K

Sol. 

$P_g = P_0 + \dfrac{kx}{A} = P_0 + \dfrac{kh}{16A}$

For adiabatic process $P_1V_1^\gamma = P_2V_2^\gamma$

$$P_0\left(hA\right)^\gamma = P_g\left[\left(\frac{h}{2} + \frac{h}{16}\right)A\right]^\gamma$$

$$10^5\left(hA\right)^\gamma = \left[10^5 + \frac{3700 \times h}{16 \times 27 \times 10^{-4}}\right]\left[\frac{9h}{16}A\right]^\gamma$$

$$\frac{64}{27} = 1 + \frac{370 \times h}{16 \times 27}$$

$$\frac{37}{27} = \frac{370 \times h}{16 \times 27} \Rightarrow h = \frac{16}{10}$$

PV = nRT

$P_0hA = nR \times 273$ (initial)

$$\left\{P_0 + \frac{kh}{16A}\right\}\left\{\frac{Ph}{16}A\right\} = nRT \quad \text{(final)}$$

divide

$$\left\{1 + \frac{kh}{16A \times P_0}\right\}\frac{9}{16} = \frac{T}{273}$$

$$T = 273 \times \frac{9}{16}\left[1 + \frac{370}{270}\right]$$

$$T = 273 \times \frac{4}{3} = 364k$$

4. Ans. (a) Final pressure in A = $\frac{27}{8}P_0$ = Final pressure in C, Final pressure in B = $\frac{21}{4}P_0$

(b) Final temperature in A (and B) = $\frac{21}{4}T_0$, Final temperature in C = $\frac{3}{2}T_0$,

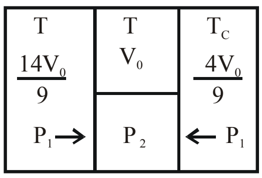
(c) 18 P_0V_0

(d) work done by gas in A = + P_0V_0, work done by gas in B = 0,

(e) $\frac{17}{2}P_0V_0$

Sol. Pressure of A & C will be same & Temperature of A & B will same

For "C"



$$P_0V_0^{1.5} = P_1\left(\frac{4V_0}{9}\right)^{1.5} \Rightarrow P_1 = \frac{27}{8}P_0$$

For "C"

$TV^{\gamma-1}$ = constant

$$T_0V_0^{0.5} = T_C\left(\frac{4V_0}{9}\right)^{0.5} \Rightarrow T_C = \frac{3T_0}{2}$$

For (A)

$$\frac{PV}{RT} = n$$

$n_i = n_f$

$$\frac{P_0V_0}{RT_0} = \frac{\left(\frac{27}{8}P_0\right)\left(\frac{14}{9}V_0\right)}{nT}$$

$T = \frac{21}{4}T_0$

For (B)

$\frac{P_0V_0}{PT_0} = \frac{P_2 \times V_0}{PT}$

$\frac{P_0V_0}{T_0} = \frac{4P_2V_0}{21T_0}$

$\frac{21P_0}{4} = P_2$

(C) Net work done will be zero.

$\Delta Q = \Delta U_A + \Delta U_B + \Delta U_C + (w_{net})$

$\Delta Q = \Delta U_A + \Delta U_B + \Delta U_C$

$= 2 \times 2nR(T_f - T_i) + 2Rn\left[\frac{3T_0}{2} - T_0\right]$

$= 4 \times \left(\frac{21}{4}P_0V_0 - P_0V_0\right) + P_0V_0$

$= 17P_0V_0 + P_0V_0 = 18P_0V_0$

$\Delta Q = \Delta U_B + \Delta W_B$

$\Delta Q = nC_V\Delta T \quad (\Delta W_B = 0)$

$= n2R(T_f - T_i)$

$= 2\left[\frac{21}{4}P_0V_0 - P_0V_0\right] = \frac{17}{2}P_0V_0$

(D) $\Delta w_B = 0$

$\Delta w_A = -\Delta w_C = -[-nC_V\Delta T]_C = nC_V\Delta T_C = 2nR[T_{fC} - T_{iC}]$

$= 2\left[\frac{27}{8}P_0 \times \frac{4V_0}{9} - P_0V_0\right] = P_0V_0$

5. **Ans.** C = 2R

Sol. $C = C_V + \frac{PdV}{dT}$

Now

$PdV + VdP = nRdT$

$PdV = \frac{k\ell}{A}Adx = k\ell dx$

$\left(\frac{k\ell}{A}\right)(Adx) + (A\ell)\frac{kdx}{A} = 1 \times RdT$

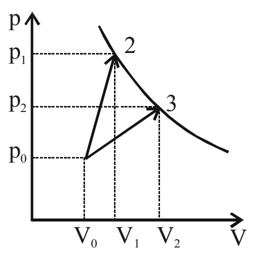
$\frac{2k\ell dx}{dT} = R$

$\frac{k\ell dx}{dT} = \frac{R}{2}$

$C = \frac{3}{2}R + \frac{R}{2} = 2R$

6. **Ans.** $Q_1 < Q_2$

Sol.



According to the first law of thermodynamics, the amount of heat ΔQ_1 received by a gas going over from state 1 (p_0, V_0) to state 2 (p_1, V_1) is

$\Delta Q_1 = \Delta U_1 + W_1$

where ΔU_1 is the change in its internal energy, and W_1 is the work done by the gas,

$$W_1 = \frac{(p_0 + p_1)(V_1 - V_0)}{2}$$

As the gas goes over from state 1 to state 3 (p_2, V_2) (points 2 and 3 lie on the same isotherm), the following relations are fulfilled :

$\Delta Q_2 = \Delta U_2 + W_2$,

$$W_2 = \frac{(p_0 + p_2)(V_2 - V_0)}{2}$$

Since the final temperature of the gas in states 2 and 3 is the same, $\Delta U_1 = \Delta U_2$. In order to find out in which process the gas receives a larger amount of heat, we must compare the works W_1 and W_2 :

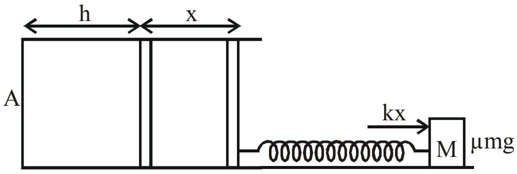
$$W_1 - W_2 = \frac{(p_0 + p_1)(V_1 - V_0)}{2} - \frac{(p_0 + p_2)(V_2 - V_0)}{2}$$

$$= \frac{(p_0V_1 - p_0V_2) + (p_2V_0 - p_1V_0)}{2} < 0$$

since $p_0V_1 < p_0V_2$ and $p_2V_0 < p_1V_0$. Consequently, $W_2 > W_1$ and $\Delta Q_2 > \Delta Q_1$, i.e. the amount of heat received by the gas in the process $1 \rightarrow 3$ is larger.

7. **Ans.** (a) 2.515, (b) 404.8 K, (c) 122.179 J

Sol.



(a) $\mu mg = 3 = kx \Rightarrow x = \frac{3}{k} = 10^{-2}$

$W = P_0A \times x + \frac{1}{2}kx^2 = 10^5 \times 25 \times 10^{-4} \times 10^{-2} + \frac{1}{2} \times 300 \times 10^{-4}$

$= 2.5 + 0.015 = 2.515$ J

(b) $P_0 = 10^5\ N/m^2$, $T_0 = 400\ K$, $V_0 = Ah$

$$P_2 = P_0 + \frac{kx}{A} = 1.012 \times 10^5 N/m^2$$

$V_2 = V_0 + Ax$

Initial $P_0 \times (Ah) = nR \times 400$

final $A\left(P_0 + \frac{kx}{A}\right)(h + x) = nR \times T_f$

$\Rightarrow T_f = 404.8$

(c) $\Delta Q = W + nC_V\Delta T$

$= W + \frac{2 \times 3}{2} R\left[T_f - 400\right]$

$= 2.515 + 3 \times 8.31 \times 4.8$

$= 2.515 + 119.664$

$= 122.179$

8. **Ans.** (a) 18 cm, (b) 500 K, (c) $\frac{21}{4}$

Sol. Initially,

(a) $P_1 = P_0 - \rho gh = 76 - 19 = 57$ cm of Hg

$V_1 = A \times 30$

Finally,

$P_2 = P_0 + \rho gh = 95$ cm of Hg

$V_2 = Ax$

Since process is isothermal,

$P_1V_1 = P_2V_2$

$57 \times A \times 30 = 95 \times Ax$

$x = 18$ cm

(b) Heating at constant pressure,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow T_2 = 300 \times \frac{30 \times A}{18A}$$

$T_2 = 500$ K

(c) For isobaric process,

$$\Delta Q = nC_P\Delta T = \frac{7}{2}(nR\Delta T) = \frac{7}{2}P(V_2 - V_1) = 5.25\ J$$