VERY SIMILAR PRACTICE TEST 6

Hints and Explanations

1. **(b) :** All the non-zero digits are significant. The trailing zero (s) in a number with decimal point are significant. Power of 10 is irrelevant for the determination of significant figures. Hence, 4.8000×10^4 has 5 significant figures.
All the non-zero digits are significant. All the zeros between two non-zero digits are significant, no matter where the decimal point is. The trailing zero(s) in a number with decimal point are significant. Hence, 48000.50 has 7 significant figures.

2. **(c) :** In first case, potential gradient, $K = \frac{\varepsilon_0}{l}$
where ε_0 is the emf of the battery in potentiometer circuit. As per question

$$\varepsilon = \frac{Kl}{5} = \frac{\varepsilon_0}{l} \times \frac{l}{5} = \frac{\varepsilon_0}{5}$$

In second case, length of potentiometer wire

$$= l + \frac{l}{2} = \frac{3l}{2}$$

Potential gradient, $K' = \frac{\varepsilon_0}{3l/2} = \frac{2\varepsilon_0}{3l}$

If l' is the new balancing length, then

$$\varepsilon = \frac{\varepsilon_0}{5} = \frac{2\varepsilon_0}{3l} \times l' \quad \text{or} \quad l' = \frac{3}{10} l$$

3. **(a) :** Time taken by body A, $t_1 = 5$ s
Acceleration of body $A = a_1$
Time taken by body B, $t_2 = 5 - 2 = 3$ s
Acceleration of body $B = a_2$
Distance covered by first body in 5^{th} second after its start,

$$S_5 = u + \frac{a_1}{2}(2t_1 - 1) = 0 + \frac{a_1}{2}(2 \times 5 - 1) = \frac{9}{2} a_1$$

Distance covered by the second body in the 3^{rd} second after its start,

$$S_3 = u + \frac{a_2}{2}(2t_2 - 1) = 0 + \frac{a_2}{2}(2 \times 3 - 1) = \frac{5}{2} a_2$$

Since $S_5 = S_3$

$$\therefore \quad \frac{9}{2} a_1 = \frac{5}{2} a_2$$

or $a_1 : a_2 = 5 : 9$

4. **(c) :** Magnetic moment, $M = IA$

$$M = \frac{qv}{2\pi r} \times \pi r^2 = \left(q \times \frac{\omega}{2\pi}\right)\pi r^2 = \frac{1}{2} q\omega r^2$$

Angular momentum,
$L = mvr = m(\omega r)r = m\omega r^2$

$$\therefore \quad \frac{M}{L} = \frac{\frac{1}{2} q\omega r^2}{m\omega r^2} = \frac{1}{2}\frac{q}{m}$$

5. **(a) :** Here, $M = 2000$ kg, $m = 10$ g $= 0.01$ kg
Force on car = rate of change of momentum of bullets

$$F = nmv = 10 \times 0.01 \times 500 = 50 \text{ N}$$

$$a = \frac{F}{M} = \frac{50}{2000} = 0.025 \text{ m s}^{-2}$$

6. **(c) :** In case of a solenoid as $B = \mu_0 nI$,
$\phi = B(nlS) = \mu_0 n^2 lSI$ and hence

$$L = \frac{\phi}{I} = \mu_0 n^2 lS = \mu_0 \frac{N^2}{l} S \qquad \left(\text{as } n = \frac{N}{l}\right)$$

When N and l are doubled, then

$$L' = \mu_0 \frac{(2N)^2}{2l} S = 2\mu_0 \frac{N^2}{l} S = 2L$$

i.e., inductance of the solenoid will be doubled.

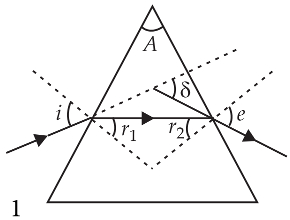
7. **(a) :** Let the mass of the unexploded bomb be $5m$.
It explodes into the two pieces of masses m and $4m$ respectively.
Initial momentum of the unexploded bomb
$= 5m(40\hat{i} + 50\hat{j} - 25\hat{k})$
After explosion, momentum of the smaller piece
$= m\vec{v}_1 = m(200\hat{i} + 70\hat{j} + 15\hat{k})$
and momentum of the larger piece $= 4m\vec{v}_2$
where $\vec{v}_1$ and $\vec{v}_2$ are the velocities of the two pieces respectively.
According to law of conservation of momentum, we get

$$5m(40\hat{i} + 50\hat{j} - 25\hat{k}) = m(200\hat{i} + 70\hat{j} + 15\hat{k}) + 4m\vec{v}_2$$

$$4m\vec{v}_2 = 5m(40\hat{i} + 50\hat{j} - 25\hat{k}) - m(200\hat{i} + 70\hat{j} + 15\hat{k})$$

$$\vec{v}_2 = \frac{1}{4}(180\hat{j} - 140\hat{k}) = 45\hat{j} - 35\hat{k}$$

8. **(c) :** For normal incidence,
$i = 0°, r_1 = 0°$
As $r_1 + r_2 = A$
$\therefore\ r_2 = A - r_1 = 30°$



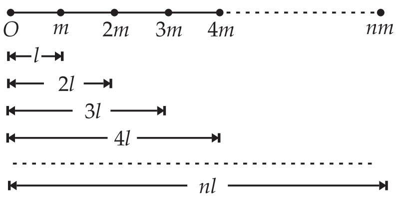
As $\mu = \dfrac{\sin e}{\sin r_2}$

$\therefore\ \sin e = \mu \sin r_2$

$= \sqrt{2} \sin 30° = \dfrac{1}{\sqrt{2}}$

$e = 45°$
$\delta = i + e - A = 0° + 45° - 30° = 15°.$

9. **(a) :**



The distance of centre of mass of given configuration of the particles from the fixed point O is

$$X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + ... + m_nx_n}{m_1 + m_2 + m_3 + ... + m_n}$$

$$X_{CM} = \frac{(m)(l) + (2m)(2l) + (3m)(3l) + ... + (nm)(nl)}{m + 2m + 3m + ... + nm}$$

$$= \frac{ml[1^2 + 2^2 + 3^2 + ... + n^2]}{m[1 + 2 + 3 + ... + n]}$$

$$= \frac{\frac{(l)(n)(n+1)(2n+1)}{6}}{\frac{(n)(n+1)}{2}} = \frac{(2n+1)l}{3} \text{ cm}$$

10. **(d) :** $\Delta x = \dfrac{D(\mu - 1)t}{d}$. Also $\Delta x = \dfrac{nD\lambda}{d}$.

$\therefore\ \dfrac{D(\mu-1)t}{d} = \dfrac{nD\lambda}{d}$ or $(\mu - 1)t = n\lambda$

or $\dfrac{t_1}{t_2} = \dfrac{n_1}{n_2} \Rightarrow t_2 = \dfrac{n_2 t_1}{n_1}$

or $t_2 = \dfrac{20 \times 4.8}{30} = 3.2$ mm.

11. **(c) :** $dW = PdV = \dfrac{RT}{V}dV$ ...(i)

As $V = KT^{2/3}$ $\therefore\ dV = K\dfrac{2}{3}T^{-1/3}\,dT$

$\therefore\ \dfrac{dV}{V} = \dfrac{K\frac{2}{3}T^{-1/3}dT}{KT^{2/3}} = \dfrac{2}{3}\dfrac{dT}{T}$

From (i), $W = \displaystyle\int_{T_1}^{T_2} RT\frac{dV}{V} = \int_{T_1}^{T_2} RT\,\frac{2}{3}\frac{dT}{T}$

$W = \dfrac{2}{3}R(T_2 - T_1) = \dfrac{2}{3}R \times 60 = 40R$

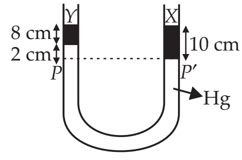
12. **(b) :** Energy contained in a cylinder
U = average energy density × volume

$= \dfrac{1}{2}\varepsilon_0 E_0^2 \times Al$

$= \dfrac{1}{2} \times (8.85 \times 10^{-12}) \times (50)^2 \times (10 \times 10^{-4}) \times 1$

$= 1.1 \times 10^{-11}$ J

13. **(a) :** As shown in adjacent figure, in the two arms of a tube pressure remains same on surface PP'.



Hence,

$8 \times \rho_Y \times g + 2 \times \rho_{Hg} \times g = 10 \times \rho_X \times g$

$\therefore\ 8\rho_Y + 2 \times 13.6 = 10 \times 3.36$

or $\rho_Y = \dfrac{33.6 - 27.2}{8} = 0.8 \text{ g cm}^{-3}$

14. **(c) :** AC power gain is ratio of change in output power to the change in input power. AC power gain

$= \dfrac{\text{Change in output power}}{\text{Change in input power}} = \dfrac{\Delta V_o \times \Delta I_c}{\Delta V_i \times \Delta I_b}$

$= \left(\dfrac{\Delta V_o}{\Delta V_i}\right) \times \left(\dfrac{\Delta I_c}{\Delta I_b}\right) = A_V \times \beta_{AC}$

where A_V is voltage gain and $(\beta)_{AC}$ is AC current gain.

Also, $A_V = \beta_{AC} \times \text{resistance gain}\left(\dfrac{R_o}{R_i}\right)$

Given, $A_V = 50$, $R_o = 200\ \Omega$, $R_i = 100\ \Omega$

Hence, $50 = \beta_{AC} \times \dfrac{200}{100}$

$\therefore\ \beta_{AC} = 25$

Now, AC power gain $= A_c \times \beta_{AC} = 50 \times 25 = 1250$

15. **(b) :** Poisson's ratio, $\sigma = \dfrac{\Delta d / d}{\Delta l / l}$

Area, $A = \pi r^2 = \pi\dfrac{d^2}{4}$ $(\because\ d = 2r)$

$\therefore\ \Delta A = \dfrac{2\pi d\Delta d}{4} = \dfrac{\pi}{2}d\Delta d$

$\therefore\ \dfrac{\Delta A}{A} = \dfrac{\pi\frac{d}{2}\Delta d}{\pi\frac{d^2}{4}} = 2\dfrac{\Delta d}{d}$

Given: $\dfrac{\Delta A}{A} \times 100 = 2\%$

$\therefore\ 2 = 2\dfrac{\Delta d}{d} \times 100$ or $\dfrac{\Delta d}{d} \times 100 = 1\%$ ...(i)

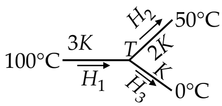
Given: $\sigma = \frac{\Delta d / d}{\Delta l / l} = 0.4$ or $\frac{\Delta d}{d} = 0.4\frac{\Delta l}{l}$

$\therefore \frac{\Delta l}{l}\times 100 = \frac{1}{0.4}\frac{\Delta d}{d}\times 100$

$= 2.5 \times 1\% = 2.5\%$ (Using (i))

16. (d) : Here, $\upsilon_s = 12$ kHz,
$\upsilon_c = 2.51$ MHz $= 2510$ kHz
The upper side band frequency
$= 2510 + 12 = 2522$ kHz
The lower side band frequency
$= 2510 - 12 = 2498$ kHz

17. (a) : Let L and A be length and area of cross-section of each rod.



At steady state,
$H_1 = H_2 + H_3$

$$\frac{(100-T)(3K)A}{L} = \frac{(T-50)2KA}{L} + \frac{(T-0)KA}{L}$$

$3(100 - T) = 2(T - 50) + T$
$300 - 3T = 2T - 100 + T$
$6T = 400$

or $T = \frac{400}{6} = \frac{200}{3}$°C

18. (b) : Here, $m_p = 1.007825$ u, $m_n = 1.008665$ u
mass of $_1H^2$ nucleus, $m_N(_1H^2) = 2.014102$ u
The deuteron nucleus contains one proton and one neutron.
Therefore, mass of nucleons constituting deuteron,
$m_p + m_n = 1.007825 + 1.008665 = 2.01649$ u
Mass defect, $\Delta M = (m_p + m_n) - m_N(_1H^2)$
$= 2.01649 - 2.014102 = 0.002388$ u
$= 0.002388 \times 931.5$ MeV/c^2
$= 2.224$ MeV/c^2
Binding energy, $E_b = \Delta M c^2 = 2.224$ MeV
Binding energy per nucleon

$= \frac{E_b}{A} = \frac{2.224}{2} = 1.112$ MeV

19. (d) : Here, $\tau = pE\sin\theta$

$10\sqrt{2} = p\times 10^4 \sin 30° = 10^4 \times \frac{p}{2}$

$p = \frac{20\sqrt{2}}{10^4} = 2\sqrt{2}\times 10^{-3}$

P.E. $= pE\cos\theta$

$= 2\sqrt{2}\times 10^{-3}\times 10^4 \cos 30° = 20\sqrt{2}\times\frac{\sqrt{3}}{2}$

$= 10\sqrt{6} = 10\times 2.45$ J $= 24.5$ J

20. (d) : The torque $= I\alpha$

The restoring torque $I\frac{d^2\theta}{dt^2} = -mg\frac{l\theta}{2}$

$$\frac{d^2\theta}{dt^2} = -\left(\frac{mgl/2}{ml^2/3}\right)\theta = -\left(\frac{3g}{2l}\right)\theta \Rightarrow \frac{d^2\theta}{dt^2} = -\omega^2\theta,$$

where, $\omega = \sqrt{3g/2l}$

$\therefore T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2}{3}\frac{l}{g}}$

21. (5.0): The capacitance of a parallel plate capacitor in air is given by

$C = \frac{\varepsilon_0 A}{d}$

By introducing a slab of thickness t, the new capacitance C' becomes

$$C' = \frac{\varepsilon_0 A}{d' - t\left(1 - \frac{1}{K}\right)}$$

The charge ($Q = CV$) remains the same in both the cases.
Hence,

$$\frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d' - t\left(1-\frac{1}{K}\right)} \text{ or } d = d' - t\left(1 - \frac{1}{K}\right)$$

Here, $d' = d + 2.4 \times 10^{-3}$ m, $t = 3$ mm $= 3 \times 10^{-3}$ m
Substituting these values, we get

$$d = d + (2.4\times 10^{-3}) - 3\times 10^{-3}\left(1 - \frac{1}{K}\right)$$

or $(2.4\times 10^{-3}) = 3\times 10^{-3}\left(1-\frac{1}{K}\right)$

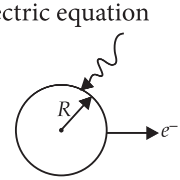
Solving it, we get $K = 5$

22. (7.0): Here, radius of sphere $R = 1$ cm $= 1 \times 10^{-2}$ m
Work function, $W = 4.7$ eV
Energy of incident radiation

$= \frac{hc}{\lambda} = \frac{1240 \text{ eV nm}}{200 \text{ nm}}$ (Take $hc = 1240$ eV nm)

$= 6.2$ eV
According to Einstein's photoelectric equation

$\frac{hc}{\lambda} = W + eV_s$



6.2 eV = 4.7 eV + eV_s
$V_s = 1.5$ V
The sphere will stop emitting photoelectrons, when the potential on its surface becomes equal to 1.5 V.

$\therefore \quad \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R} = 1.5$

$\dfrac{1}{4\pi\varepsilon_0}\dfrac{Ne}{R} = 1.5$

where N = Number of photoelectrons emitted,
e = charge of each electron.

$$N = \frac{1.5 \times R}{\frac{1}{4\pi\varepsilon_0} \times e} = \frac{1.5 \times 1 \times 10^{-2}}{9 \times 10^9 \times 1.6 \times 10^{-19}}$$

$$N = \frac{15}{16} \times \frac{1}{9} \times 10^8 = \frac{5}{48} \times 10^8$$

$$N = \frac{50}{48} \times 10^7 = 1.04 \times 10^7 \qquad \therefore \; Z = 7$$

23. (1.0): Activity, $A = \lambda N$

$A = \dfrac{1}{\tau} N \qquad \left(\text{As } \lambda = \dfrac{1}{\tau}\right)$

where τ is the mean life time.
$N = A\tau = (10^{10}$ decay/s$)(10^9$ s$) = 10^{19}$ atoms
Mass of the sample, $m = N \times$ (mass of 1 atom)
$= 10^{19} \times 10^{-25}$ kg $= 10^{-6}$ kg $= 1$ mg

24. (4) : According to Kepler's third law
$T^2 \propto r^3$

$\therefore \quad \dfrac{T_A^2}{T_B^2} = \dfrac{r_A^3}{r_B^3}.$

or $\quad \dfrac{r_A}{r_B} = \left(\dfrac{T_A}{T_B}\right)^{2/3} = (8)^{2/3} = 4 \quad$ or $\quad r_A = 4r_B$

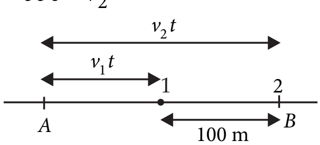
25. (4.0): Let the velocities of car 1 and car 2 be v_1 m s^{-1} and v_2 m s^{-1}.

$\therefore$ Apparent frequencies of sound emitted by car 1 and car 2 as detected at end point are

$\upsilon_1 = \dfrac{\upsilon_0 v}{v - v_1}$ and $\upsilon_2 = \dfrac{\upsilon_0 v}{v - v_2}$

$\therefore \quad 330 = \dfrac{300 \times 330}{330 - v_1}$ or $v_1 = 30$ m s^{-1}

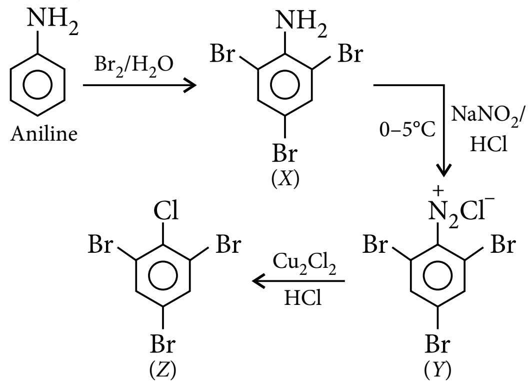
and $360 = \dfrac{300 \times 330}{330 - v_2}$ or $v_2 = 55$ m s^{-1}



The distance between both the cars just when the 2nd car reaches point B(as shown in figure) is
100 m $= v_2 t - v_1 t$

$$t = \frac{100}{v_2 - v_1} = \frac{100}{55 - 30} = 4 \text{ s}$$

26. (d) :



2, 4, 6-Tribromo chlorobenzene

27. (b) : Temporary hardness is due to dissolved HCO_3^- of Ca^{2+} and Mg^{2+}.
$CaO + H_2O \longrightarrow Ca(OH)_2$
$Ca(HCO_3)_2 + Ca(OH)_2 \longrightarrow 2CaCO_3\downarrow + 2H_2O$
Permanent hardness is due to dissolved Cl^- and SO_4^{2-} of Ca^{2+} and Mg^{2+}.
$CaCl_2 + Na_2CO_3 \longrightarrow CaCO_3\downarrow + 2NaCl$
$CaSO_4 + Na_2CO_3 \longrightarrow CaCO_3\downarrow + Na_2SO_4$

28. (a) : The increased concentration of the reactants on the surface influences the rate of reaction.

29. (d) : $\Delta H_{solution}$ is the summation of all the heats involved in the formation of solution.

30. (b) : (b) is correct as very high energy is required for the conversion of CuS to CuO. It follows (a) is wrong.
(c) is wrong as FeS gets converted to FeO even at low temperature below 800°C.
(d) is wrong as $FeSiO_3$ is formed at 1400°C during smelting and not during roasting.

31. (a) : 1 atomic mass unit on the scale of 1/6 of C–12 = 2 amu on the scale of 1/12 of C–12.
Now, atomic mass of an element

$$= \frac{\text{Mass of one atom of the element}}{1 \text{ amu (Here on the scale of } \frac{1}{6} \text{ of C-12)}}$$

$$= \frac{\text{Mass of one atom of the element}}{2 \text{ amu (Here on the scale of } \frac{1}{12} \text{ of C-12)}}$$

$\therefore$ Numerically the mass of a substance will become half of the normal scale.

32. (d)

33. (d) : K_p is constant at constant temperature. As volume is halved, pressure will be doubled. Hence, equilibrium will shift in the backward direction, *i.e.*, degree of dissociation decreases.

34. (b)

35. (c) : Planar conjugated cyclic compounds containing $4n\pi$ electrons are anti-aromatic, *e.g.*, cyclooctatetraene ($8\pi\ e^-$s).

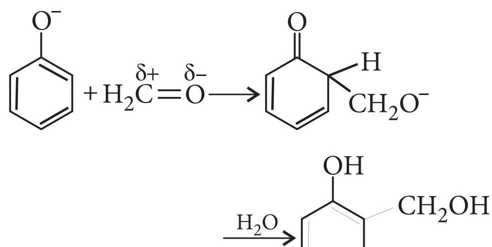
36. (c) : The cations will be deposited as metals in the sequence of decreasing reduction potentials. Cations having $E°$ value < – 0.83 V (reduction potential of water) will not be deposited from aqueous solutions.

37. (b) **38. (a)**

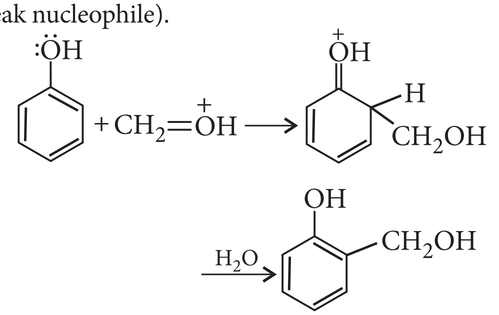
39. (b) : With rise of temperature, the most probable speed increases.

40. (c) : As time increases slope will decrease.

41. (c) : Condensation of phenol with formaldehyde is an electrophilic substitution reaction. Base converts phenol into phenoxide ion which being more reactive, reacts easily with $CH_2=O$ (a weak electrophile).



In presence of an acid, $CH_2=O$ (a weak electrophile) is protonated to $CH_2=\overset{+}{O}H$ (a strong electrophile) which easily reacts with phenol (a weak nucleophile).



42. (c) : (iii) NO(15) : $KK\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_z^2\ \pi 2p_x^2 = \pi 2p_y^2, \pi^* 2p_x^1$

Bond order $= \frac{1}{2}(8-3) = 2.5$

(iv) NO^{2+} :

Bond order $= \frac{1}{2}(7-2) = 2.5$

(ii) NO^+ :

Bond order $= \frac{1}{2}(8-2) = 3.0$

(i) NO^- :

Bond order $= \frac{1}{2}(8-4) = 2.0$

(v) NO^{2-} :

Bond order $= \frac{1}{2}(8-5) = 1.5$

Hence, the correct order is

$$\underset{(v)}{NO^{2-}} < \underset{(i)}{NO^-} < \underset{(iv)}{NO^{2+}} = \underset{(iii)}{NO} < \underset{(ii)}{NO^+}$$

43. (a) : $\underset{\text{'X'}}{CH_3CH_2OH} \xrightarrow[CH_2Cl_2]{PCC} \underset{\text{'Y'}}{CH_3CHO} \xrightarrow{I_2/NaOH} CHI_3\downarrow$

44. (d) : Element *X* belongs to fourth period and fifteenth group.

Period	1	2	3	4
Group 15	Nil	N(7)	P(15)	As(33)

Let us configure As(33). Now,

As(33) → $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^3$

So, *s* and *d*-orbitals are fully-filled and *p*-orbital is half-filled.

45. (a) : More negative or lower the reduction potential, more is the reducing property. Thus, the order of reducing power is $Y > Z > X$.

46. (6) : Alkyl halides that can form 3°, allylic or benzylic carbocations react by S_N1 mechanism. These are $(CH_3)_3CBr$, $BrCH_2CH=CH_2$, $C_6H_5CH_2Br$, $(CH_3)_3CCH_2Br$, $C_6H_5CHBrCH_3$ and $CH_3CH=CHCH_2Cl$.

47. (2) : For reversible process

$$W_{rev} = -2.303\ nRT \log_{10}\frac{P_1}{P_2}$$

$$= -2.303 \times 2 \times 8.314 \times 300 \log_{10}\frac{1.01\times 10^5}{5.05\times 10^6}$$

$= +\ 1.9518 \times 10^4$ joule

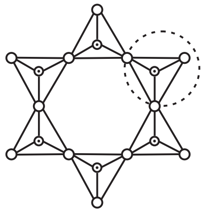
Since, W_{rev} is a measure of free energy change

∴ $-\Delta G = -W_{rev} = -W_{max}$

or $\Delta G = 1.9518 \times 10^4$ joule $\cong 2 \times 10^4$ joule

48. (3) : μ_{eff} value of 1.73 B.M. corresponds to one unpaired electron. $Ti^{3+} = 3d^1$ (Ti = [Ar] $3d^2\ 4s^2$).

49. (2) : Beryl is a cyclic silicate.
The formula of Beryl is $Be_3Al_2(SiO_3)_6$.
Every SiO_4^{4-} unit shares two O atoms. The structure is



SiO_4^{4-} is sharing two O atoms.

50. (2) :

	C	H	N
Weight ratio :	9 :	1 :	3.5
Molar ratio :	$\frac{9}{12}$:	$\frac{1}{1}$:	$\frac{3.5}{14}$
	$=\frac{3}{4}$:	$\frac{1}{1}$:	$\frac{1}{4}$

Simplest ratio : 3 : 4 : 1
Empirical formula = C_3H_4N
$(C_3H_4N)_n = 108$
$(12 \times 3 + 1 \times 4 + 14)n = 108$
$54\, n = 108 \Rightarrow n = 108/54 = 2$
Molecular formula = $C_6H_8N_2$

51. (b) : The given functions are

$$g(x)=1+x-[x] \text{ and } f(x)=\begin{cases}-1, & x<0\\ 0, & x=0\\ 1, & x>0\end{cases}$$

We see that $g(x)=1+x-[x]=1+(x-[x])$
$=1+\{x\}\geq 1\ (\because\ 0\leq\{x\}<1)$
$\therefore\ f(g(x)) = 1$ for all x

52. (c) : Let $I=\int\frac{2^x}{\sqrt{1-4^x}}dx$

Put $2^x = t \Rightarrow 2^x \log 2\, dx = dt$

$$\therefore\ I=\frac{1}{\log 2}\int\frac{dt}{\sqrt{1-t^2}} = \frac{1}{\log 2}\sin^{-1}\frac{t}{1}+C$$

$$=\frac{1}{\log 2}\sin^{-1}2^x+C$$

53. (c) : For $|x| < 1$, $x^{2n} \to 0$ as $n \to \infty$

$|x|>1, \frac{1}{x^{2n}}\to 0$ as $n\to\infty$

$$f(x)=\begin{cases}\log_e(2+x), & |x|<1\\ \lim\limits_{n\to\infty}\dfrac{x^{-2n}\log_e(2+x)-\sin x}{x^{-2n}+1}=-\sin x, & \text{if } |x|>1\\ \frac{1}{2}(\log_e(2+x)-\sin x), & |x|=1\end{cases}$$

$$\lim_{x\to1^+}f(x)=-\sin 1;\quad \lim_{x\to1^-}f(x)=\log 3.$$

54. (c) : $\cos\frac{\pi}{8}\cdot\cos\frac{3\pi}{8}\cdot\cos\frac{5\pi}{8}\cdot\cos\frac{7\pi}{8}$

$$=\cos\frac{\pi}{8}\cdot\sin\frac{\pi}{8}\cdot\left(-\sin\frac{\pi}{8}\right)\cdot\left(-\cos\frac{\pi}{8}\right)$$

$$=\frac{1}{4}\left(2\sin\frac{\pi}{8}\cos\frac{\pi}{8}\right)^2=\frac{1}{4}\sin^2\frac{\pi}{4}=\frac{1}{4}\cdot\frac{1}{2}=\frac{1}{8}.$$

55. (b) : Let $y=\frac{ax+b}{p}\ \therefore\ \bar{y}=\frac{a\bar{x}+b}{p}$

Now, $y-\bar{y}=\frac{1}{p}a(x-\bar{x}) \Rightarrow (y-\bar{y})^2=\frac{a^2}{p^2}(x-\bar{x})^2$

$$\Rightarrow\ \frac{1}{n}\sum(y-\bar{y})^2=\frac{a^2}{p^2}\frac{1}{n}\sum(x-\bar{x})^2$$

$\therefore$ S.D. of $y=\left|\frac{a}{p}\right|$S.D. of $x=\left|\frac{a}{p}\right|\sigma_x$

56. (b) : Total number of arrangements of n books = $n!$.
If two specified books always together then number of ways = $(n-1)! \times 2$
Hence required number of ways = $n! - (n-1)! \times 2$
$= n(n-1)! - (n-1)! \times 2 = (n-1)!\,(n-2)$.

57. (c) : Let first term = A and common difference = D
$\therefore\ a = A + (p-1)D,\ b = A + (q-1)D,$
$c = A + (r-1)D$

$$\begin{vmatrix}a & p & 1\\ b & q & 1\\ c & r & 1\end{vmatrix}=\begin{vmatrix}A+(p-1)D & p & 1\\ A+(q-1)D & q & 1\\ A+(r-1)D & r & 1\end{vmatrix}$$

Operate $C_1 \to C_1 - DC_2 + DC_3$

$$=\begin{vmatrix}A & p & 1\\ A & q & 1\\ A & r & 1\end{vmatrix}=A\begin{vmatrix}1 & p & 1\\ 1 & q & 1\\ 1 & r & 1\end{vmatrix}=0$$

58. (d) : The *d.r's* of the first line are $\frac{1}{3},\frac{1}{6},\frac{1}{2}$ ...(i)

The second line is along the vector

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & 1\\ 1 & 1 & -2\end{vmatrix}=-5\hat{i}+7\hat{j}+\hat{k}$$

The *d.r's* are $-5, 7, 1$ ...(ii)

From (i) and (ii) $\Rightarrow -\frac{5}{3}+\frac{7}{6}+\frac{1}{2}=0$.

$\therefore$ The lines are perpendicular.

59. (d) : Put $\cos x = t$

$$\Rightarrow \lim_{x\to 0}\frac{(\cos x)^{1/3}-(\cos x)^{1/2}}{\sin^2 x}=\lim_{t\to 1}\frac{t^{\frac{1}{3}}-t^{\frac{1}{2}}}{1-t^2}$$

$$=\lim_{t\to 1}\frac{\frac{1}{3}t^{\frac{-2}{3}}-\frac{1}{2}t^{\frac{-1}{2}}}{-2t}=\frac{\frac{1}{3}-\frac{1}{2}}{-2}=\frac{1}{12}\quad \text{(By L.H. Rule)}$$

60. (a) : $y = 4x^2 \Rightarrow x^2=\frac{1}{4}y$

$\therefore\ \frac{x^2}{a^2}-\frac{y^2}{16}=1$ becomes $\frac{y}{4a^2}-\frac{y^2}{16}=1$

$\Rightarrow\ 4y - a^2y^2 = 16a^2 \Rightarrow a^2y^2 - 4y + 16a^2 = 0$

$\Rightarrow\ D \ge 0$ for intersection of two curves

$\Rightarrow\ 16 - 4a^2 (16a^2) \ge 0 \Rightarrow 1 - 4a^4 \ge 0$

$\Rightarrow\ (2a^2)^2 \le 1 \Rightarrow |\sqrt{2}a| \le 1 \Rightarrow -\frac{1}{\sqrt{2}} \le a \le \frac{1}{\sqrt{2}}$

61. (b) : The given numbers are in A.P.

$\therefore\ 2\log_9(3^{1-x}+2)=\log_3(4\cdot 3^x-1)+1$

$\Rightarrow\ 2\log_{3^2}(3^{1-x}+2)=\log_3(4\cdot 3^x-1)+\log_3 3$

$\Rightarrow\ \frac{2}{2}\log_3(3^{1-x}+2)=\log_3[3(4\cdot 3^x-1)]$

$\Rightarrow\ 3^{1-x}+2=3(4\cdot 3^x-1)$

$\Rightarrow\ \frac{3}{y}+2=12y-3$, where $y=3^x$

$\Rightarrow\ 12y^2-5y-3=0$

$y=\frac{-1}{3}$ or $\frac{3}{4} \Rightarrow 3^x=\frac{-1}{3}$ or $3^x=\frac{3}{4}$

$x=\log_3(3/4) \Rightarrow x=1-\log_3 4$.

62. (a) : For $n = 1$, $P(1) : 65 + k$ is divisible by 64.
$\therefore$ k, should be -1 since, $65 - 1 = 64$ is divisible by 64.

63. (c) : $\frac{d}{dx}(\sin x.\ y) = x \sin x$

$\therefore\ y \sin x = \int x\sin x\,dx = -x \cos x + \sin x + c$

$\Rightarrow\ (y - 1) \sin x = c - x \cos x.$

64. (b) : $2x - y = 1 \Rightarrow x = \alpha, y = 2\alpha - 1$
The image of the point $(\alpha, 2\alpha - 1)$ in the line $x + y = 0$ is given by

$$\frac{x-\alpha}{1}=\frac{y-(2\alpha-1)}{1}=-2\frac{(\alpha+2\alpha-1)}{1+1}=1-3\alpha$$

$\therefore\ x = 1 - 2\alpha, y = 2\alpha - 1 + 1 - 3\alpha = -\alpha$
Elimination of α gives the image $x - 2y = 1$.

65. (c) : $\begin{vmatrix} x & \frac{x(x-1)}{2} & \frac{x(x-1)(x-2)}{6} \\ y & \frac{y(y-1)}{2} & \frac{y(y-1)(y-2)}{6} \\ z & \frac{z(z-1)}{2} & \frac{z(z-1)(z-2)}{6} \end{vmatrix}$

$$=\frac{xyz}{12}\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

Use $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$ and expanding along R_3, we have,

$\frac{xyz}{12}(x-y)(y-z)(z-x)$

66. (b) : $y' = \frac{a}{x} + 2bx + 1$, $y'(-1) = 0$, $y'(2) = 0$

$\Rightarrow\ -a - 2b + 1 = 0,\ \frac{a}{2} + 4b + 1 = 0$

$\Rightarrow\ a = 2,\ b = -\frac{1}{2} \Rightarrow a + b = \frac{3}{2}$.

67. (c) : Let us define the events as
E_1 : Jockey B rides horse A
E_2 : Jockey C rides horse A
E : The horse A wins

$\therefore\ P(E_1)=\frac{2}{3}$ [Since odds in favour of E_1 are 2 : 1]

and $P(E|E_1)=\frac{1}{6}$

Again $P(E_2)=1-P(E_1)=1-\frac{2}{3}=\frac{1}{3}$

and $P(E|E_2)=3P(E|E_1)=\frac{1}{2}$

Now, the required probability = $P(E)$
$= P(E_1 \cap E) + P(E_2 \cap E)$
$= P(E_1)\, P(E|E_1) + P(E_2)\, P(E|E_2)$
$=\frac{2}{3}\cdot\frac{1}{6}+\frac{1}{3}\cdot\frac{1}{2}=\frac{5}{18}$

Therefore the odds against winning of A are 13 : 5.

68. (d) : $(1+x+x^3+x^4)^{10}=(1+x)^{10}(1+x^3)^{10}$

$=(1+{}^{10}C_1\cdot x+{}^{10}C_2\cdot x^2+...)(1+{}^{10}C_1\cdot x^3+{}^{10}C_2\cdot x^6+...)$

$\therefore$ Coefficient of $x^4 = {}^{10}C_1\cdot{}^{10}C_1+{}^{10}C_4=310$

69. (c) : We have, $\int_{\sqrt{2}}^{x}\frac{dt}{t\sqrt{t^2-1}}=\frac{\pi}{2}$

$\Rightarrow\ \left[\sec^{-1}t\right]_{\sqrt{2}}^{x}=\frac{\pi}{2}$

$$\Rightarrow \sec^{-1}x - \sec^{-1}\sqrt{2} = \frac{\pi}{2} \Rightarrow \sec^{-1}x = \frac{\pi}{2}+\frac{\pi}{4} = \frac{3\pi}{4}$$

$$\Rightarrow x = -\sqrt{2}.$$

70. (a) : Given, $|z_1| = |z_2| = |z_3| = 1$

Now, $|z_1| = 1 \Rightarrow |z_1|^2 = 1 \Rightarrow z_1\overline{z}_1 = 1$

Similarly, $z_2\overline{z}_2 = 1, z_3\overline{z}_3 = 1$

Now, $\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1$

$$\Rightarrow \left|\frac{z_1\overline{z}_1}{z_1}+\frac{z_2\overline{z}_2}{z_2}+\frac{z_3\overline{z}_3}{z_3}\right| = |\overline{z}_1+\overline{z}_2+\overline{z}_3| = 1$$

$$\Rightarrow |z_1+z_2+z_3| = 1$$

71. (6) : $\sqrt{3x^2-7x-30}+\sqrt{2x^2-7x-5} = x+5$

$$\Rightarrow \sqrt{3x^2-7x-30} = (x+5)-\sqrt{2x^2-7x-5}$$

On squaring, we get, $\sqrt{2x^2-7x-5} = 5$

$\Rightarrow 2x^2 - 7x - 30 = 0 \Rightarrow x = 6.$

72. (2.138) : $\vec{a} = 3\hat{i}-\hat{j}+5\hat{k}, \vec{b} = 2\hat{i}+3\hat{j}+\hat{k}$

Projection of $\vec{a}$ on $\vec{b}$ is $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$$= \frac{(3\hat{i}-\hat{j}+5\hat{k})\cdot(2\hat{i}+3\hat{j}+\hat{k})}{\sqrt{2^2+3^2+1^2}}$$

$$= \frac{6-3+5}{\sqrt{14}} = \frac{8}{\sqrt{14}}. = 2.138$$

73. (14) : $\sum_{k=1}^{n}\tan^{-1}\frac{2k}{1+(k^2+k+1)(k^2-k+1)}$

$$\Rightarrow \sum_{k=1}^{n}\tan^{-1}\frac{(k^2+k+1)-(k^2-k+1)}{1+(k^2+k+1)(k^2-k+1)}$$

$$\Rightarrow \sum_{k=1}^{n}\{\tan^{-1}(k^2+k+1)-\tan^{-1}(k^2-k+1)\}$$

$= \tan^{-1}(n^2+n+1) - \tan^{-1}1$

When $n \to \infty$,

then $\sum_{k=1}^{\infty}\tan^{-1}\left(\frac{2k}{2+k^2+k^4}\right) = \frac{\pi}{2}-\frac{\pi}{4} = \frac{\pi}{4}$

Hence, $\frac{56}{\pi}\times\frac{\pi}{4}$ is 14

74. (11) : The foot of (1, –5, –10) to the plane is

$$\frac{x-1}{1} = \frac{y+5}{-1} = \frac{z+10}{1} = -\frac{(1+5-10-5)}{1+1+1} = 3$$

$\therefore$ $x = 4, y = -8, z = -7, (a, b, c) = (4, -8, -7),$

$|a+b+c| = |4-8-7| = |-11| = 11.$

75. (3) : Let $\tan x = 2t, \tan y = 3t, \tan z = 5t$

$$\sum \tan x = (\tan x \tan y \tan z) \Rightarrow t^2 = \frac{1}{3}$$

$\tan^2x + \tan^2y + \tan^2z = t^2(4+9+25) = 38t^2$

$\Rightarrow K = 3$