GUIDED REVISION

SECTION-I

Single Correct Answer Type **4 Q. [3 M (–1)]**

1. If a satellite orbits as close to the earth's surface as possible, then which of the following statement is incorrect

(A) its speed is maximum

(B) time period of its rotation is minimum

(C) the total energy of the earth plus satellite system minimum

(D) the total energy of the earth plus satellite system is maximum

2. Satellites A and B are orbiting around the earth in orbits of ratio R and 4R respectively. The ratio of their areal velocities is :

(A) 1 : 2 (B) 1 : 4 (C) 1 : 8 (D) 1 : 16

3. The Sun travels in approximately circular orbit of radius R around the center of the galaxy and completes one revolution in time T. The Earth also revolves around the Sun in time t. Assume orbit of the Earth to be a circle of radius r (r << R) and whole mass of the galaxy centered on its center. By using only these given informations, find an expression for the ratio of the mass of the galaxy to that of the Sun.

(A) $\left(\frac{R}{r}\right)^3\left(\frac{t}{T}\right)^2$ (B) $\left(\frac{R}{r}\right)^3\left(\frac{T}{t}\right)^2$ (C) $\left(\frac{R}{r}\right)^2\left(\frac{t}{T}\right)^3$ (D) $\left(\frac{R}{r}\right)^2\left(\frac{T}{t}\right)^3$

4. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is : **[JEE-Main 2014]**

(A) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (B) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (C) $\sqrt{\frac{GM}{R}}$ (D) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

Multiple Correct Answer Type **3 Q. [4 M (–1)]**

5. Two tunnels are dug from one side of the earth's surface to the other side, first along a diameter and the second along a chord. Now two particles are dropped from one end of each of the tunnels. Both the particles oscillate simple harmonically along the tunnels. Let T_1 and T_2 be the time period v_1 and v_2 be the maximum speed of the particle in the two tunnels. Then

(A) $T_1 = T_2$ (B) $T_1 > T_2$ (C) $v_1 = v_2$ (D) $v_1 > v_2$

6. Satellite A is in a circular orbit of radius r and satellite B in another circular orbit of radius 4 r, both revolving around the earth. The masses of the satellites A and B are in the ratio 3 : 1. If the symbols V, E, u and T represent the speed, total energy, escape velocity and period of revolution of satellite, then

(A) $V_A/V_B=2$ (B) $E_A/E_B=12$ (C) $u_A/u_B=2$ (D) $T_A/T_B=\frac{1}{8}$.

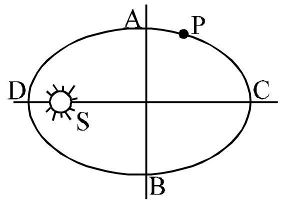
7. A partical of mass m is thrown in a parabolic path towards earth having mass M and radius R. It comes to a greatest approach distance R/2 from surface of earth. Magnitude of total energy change required to make the particle moving in an elliptical path having eccentricity 1/2, with the greatest approach point either aphelion or perihelion are :

(A) $\frac{1}{3}\frac{GMm}{R}$ (B) $\frac{1}{6}\frac{GMm}{R}$ (C) $\frac{1}{2}\frac{GMm}{R}$ (D) $\frac{GMm}{R}$

Linked Comprehension Type (1 Para × 2Q.) [3 M (-1)]
(Single Correct Answer Type)

Paragraph for Question No. 8 and 9

Figure shows the orbit of a planet P around the sun S. AB and CD are the minor and major axes of the ellipse.



8. If t_1 is the time taken by the planet to travel along ACB and t_2 the time along BDA, then

(A) $t_1 = t_2$ (B) $t_1 > t_2$
(C) $t_1 < t_2$ (D) nothing can be concluded

9. If U is the potential energy and K kinetic energy then $|U| > |K|$ at

(A) Only D (B) Only C (C) both D & C (D) neither D nor C

Matching List Type (4 × 4) 1 Q. [3 M (–1)]

10. A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2 respectively. Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1 ; and v_2, L_2, K_2 and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II. **[JEE-Advance 2018]**

List–I	List–II
P. $\frac{v_1}{v_2}$	1. $\frac{1}{8}$
Q. $\frac{L_1}{L_2}$	2. 1
R. $\frac{K_1}{K_2}$	3. 2
S. $\frac{T_1}{T_2}$	4. 8

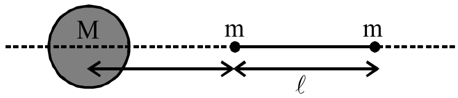
(A) P → 4 ; Q → 2 ; R → 1 ; S → 3 (B) P → 3 ; Q → 2 ; R → 4 ; S → 1
(C) P → 2 ; Q → 3 ; R → 1 ; S → 4 (D) P → 2 ; Q → 3 ; R → 4 ; S → 1

SECTION-III

Numerical Grid Type (Ranging from 0 to 9) 2 Q. [4 M (0)]

1. A binary star consists of two stars A (mass 2.2 M_S) and B (mass 11 M_S), where M_S is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is:- **[IIT-JEE 2010]**

2. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length ℓ and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3\ell$ from M, the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$. The value of k is : **[JEE-Advance 2015]**



SECTION-IV

Matrix Match Type (4 × 5) **1 Q. [8 M (for each entry +2(0)]**

1. In elliptical orbit of a planet, as the planet moves from apogee position to perigee position,

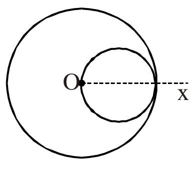
	Column–I		**Column–II**
(A)	Speed of planet	(P)	Remains same
(B)	Distance of planet from centre of Sun	(Q)	Decreases
(C)	Potential energy	(R)	Increases
(D)	Angular momentum about centre of Sun	(S)	Can not say

Subjective Type **7 Q. [4 M (0)]**

1. A particle of mass m was transferred from the centre of the base of a uniform hemisphere of mass M and radius R into infinity. What work was performed in the process by the gravitational force exerted on the particle by the hemisphere?

2. A pair of stars rotates about a common center of mass. One of the stars has a mass M which is twice as large as the mass m of the other. Their centres are at a distance d apart, d being large compared to the size of either star. (a) Derive an expression for the period of rotation of the stars about their common centre of mass in terms of d,m, G. (b) Compare the angular momentum of the two stars about their common centre of mass by calculating the ratio L_m / L_M. (c) Compare the kinetic energies of the two stars by calculating the ratio K_m/K_M.

3. A sphere of radius R has its centre at the origin. It has a uniform mass density ρ_o except that there is a spherical hole of radius r = R/2 whose centre is at x = R/2 as in fig. (a) Find gravitational field at points on the axis for $|x| > R$ (b) Show that the gravitational field inside the hole is uniform, find its magnitude and direction.



4. A small body of mass m is projected with a velocity just sufficient to make it reach from the surface of a planet (of radius 2R and mass 3M) to the surface of another planet (of radius R and mass M). The distance between the centers of the two spherical planets is 6R. The distance of the body from the center of bigger planet is 'x' at any moment. During the journey, find the distance x where the speed of the body is (a) maximum (b) minimum. Assume motion of body along the line joining centres of planets.

5. A hypothetical spherical planet of radius R and its density varies as $\rho = Kr$, where K is constant and r is the distance from the center. Determine the pressure caused by gravitational pull inside (r < R) the planet at a distance r measured from its center.

6. Two uniform spherical stars made of same material have radii R and 2R. Mass of the smaller planet is m. They start moving from rest towards each other from a large distance under mutual force of gravity. The collision between the stars is inelastic with coefficient of restitution 1/2.
(a) Find the kinetic energy of the system just after the collision.
(b) Find the maximum separation between their centres after their first collision.

7. A remote sensing satellite is revolving in an orbit of radius x over the equator of earth. Find the area on earth surface in which satellite can not send message.

ANSWER KEY	GR # GRAVITATION

SECTION-I

Single Correct Answer Type **4 Q. [3 M (–1)]**

1. Ans. (D) **2. Ans. (A)** **3. Ans. (A)** **4. Ans. (B)**

Multiple Correct Answer Type **3 Q. [4 M (–1)]**

5. Ans. (A, D) **6. Ans. (A,B,C,D)** **7. Ans. (B,C)**

Linked Comprehension Type **(1 Para × 2Q.) [3 M (-1)]**
(Single Correct Answer Type)

8. Ans. (B) **9. Ans. (C)**

Matching List Type (4 × 4) **1 Q. [3 M (–1)]**

10. Ans. (B)

SECTION-III

Numerical Grid Type (Ranging from 0 to 9) **2 Q. [4 M (0)]**

1. Ans. 6 **2. Ans. 7**

SECTION-IV

Matrix Match Type (4 × 5) **1 Q. [8 M (for each entry +2(0)]**

1. Ans. (A)-R, (B)-Q, (C)-Q (D)-P

Subjective Type **7 Q. [4 M (0)]**

1. Ans. $A = -\frac{3}{2}\frac{GmM}{R}$ **2. Ans.** (a) $T = \frac{2\pi d^{3/2}}{\sqrt{3Gm}}$, (b) 2, (c) 2

3. Ans. $\vec{g} = +\frac{\pi G\rho_0 R^3}{6}\left[\frac{1}{(x-(R/2))^2} - \frac{8}{x^2}\right]\hat{i}$, $\vec{g} = -\frac{2\pi G\rho_0 R}{3}\hat{i}$

4. Ans. (a) 2R, (b) $3\sqrt{3}\left(\sqrt{3}-1\right)R$ **5. Ans.** $\frac{1}{4}K^2G\pi(R^4 - r^4)$

6. Ans. (a) $\frac{2Gm^2}{3R}$ (b) 4R **7. Ans.** $\left(1 - \frac{\sqrt{x^2 - R^2}}{x}\right)4\pi R^2$

SOLUTIONS
SECTION-I

Single Correct Answer Type 4 Q. [3 M (–1)]

1. Ans. (D)

Sol. Speed = $\sqrt{\frac{GM}{r}}$, Time period = $\left(\frac{2\pi}{\sqrt{GM}}\right)r^{3/2}$, Total energy = $-\frac{GMm}{2r}$

2. Ans. (A)

Sol. Areal velocity = $\frac{dA}{dt} = \frac{v_0 r}{2}$

& $V_0 = \sqrt{\frac{GM}{r}}$, V_0 = orbital speed

r = radius of orbit

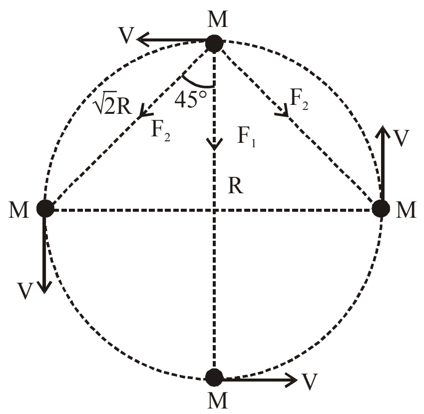
3. Ans. (A)

Sol. Let M_G & m_s represent masses of the galaxy and the Sun.

According to Kepler's law $t^2 = \frac{4\pi^2}{Gm_s}r^3$ and $T^2 = \frac{4\pi^2}{GM_G}R^3 \Rightarrow \frac{M_G}{m_s} = \left(\frac{R}{r}\right)^3\left(\frac{t}{T}\right)^2$

4. Ans. (B)

Sol.



Net force on one particle

$F_{net} = F_1 + 2F_2 \cos 45° =$ Centripetal force

$\Rightarrow \frac{GM^2}{(2R)^2} + \left[\frac{2GM^2}{(\sqrt{2}R)^2}\cos 45°\right] = \frac{MV^2}{R}$

$V = \frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

Multiple Correct Answer Type 3 Q. [4 M (–1)]

5. Ans. (A, D)

Sol. ω in both is same

so $T_1 = T_2$

and by $v_{max} = A\omega$ $\quad V_1 > V_2$

6. Ans. (A,B,C,D)

Sol. $V = \sqrt{\frac{GM}{r}}$

$$\frac{V_A}{V_B} = \sqrt{\frac{r_B}{r_A}} = 2$$

$$E = \frac{-GMm}{2r}, \qquad \frac{E_A}{E_B} = \frac{m_A}{m_B} \times \frac{r_B}{r_A} = 12$$

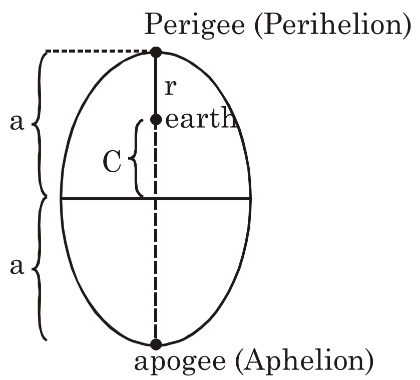
$$u_A = \sqrt{\frac{2GM}{r}}, \qquad \frac{u_A}{u_B} = 2$$

$$T = \frac{2\pi}{\sqrt{GM}}(r)^{3/2}$$

$$\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{3/2} = \frac{1}{8}$$

7. Ans. (B,C)

Sol. Initially parabolic path means escape velocity & hence (TE = 0)



$$r_{min} = (a - c) = \frac{a}{2}$$

$$r_{max} = (a + c) = \frac{3a}{2}$$

$$e = \frac{c}{a} = \frac{1}{2} \Rightarrow c = \frac{a}{2}$$

$$r = R + h = R + \frac{R}{2} = \frac{3R}{2}$$

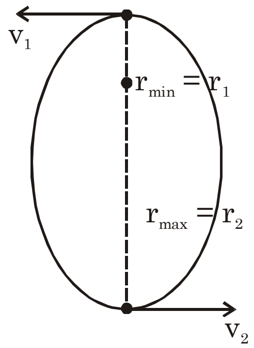
If $r_{min} = \frac{3R}{2} \Rightarrow a = 3R$

and if $r_{max} = \frac{3R}{2} \Rightarrow a = R$

If a = 3R, then, $C = \frac{a}{2} = \frac{3R}{2}$

$$r_{min} = (a - c) = \frac{3R}{2} = r_1$$

and $r_{max} = (a + c) = \frac{9R}{2} = r_2$



L = constant

$mv_1r_1 = mv_2r_2$

$$v_1\left(\frac{3R}{2}\right) = v_2\left(\frac{9R}{2}\right)$$

$v_1 = 3v_2$

TE = conserved

$$\frac{1}{2}mv_1^2 - \frac{GmM}{r_1} = \frac{1}{2}mv_2^2 - \frac{GmM}{r_2}$$

$$\frac{v_1^2}{2} - \frac{GM}{\left(\frac{3R}{2}\right)} = \frac{\left(\frac{v_1}{3}\right)^2}{2} - \frac{GM}{\left(\frac{9R}{2}\right)}$$

$$\frac{v_1^2}{2}\left(1 - \frac{1}{9}\right) = \frac{GM}{R}\left(\frac{2}{3} - \frac{2}{9}\right)$$

$$v_1^2\left(\frac{4}{9}\right) = \frac{GM}{R}\left(\frac{4}{9}\right)$$

$$v_1^2 = \frac{GM}{R}$$

$$TE = \frac{1}{2}mv_1^2 - \frac{GMm}{r_1}$$

$$= \frac{1}{2}m\frac{GM}{R} - \frac{GMm}{\left(\frac{3R}{2}\right)}$$

$$\Rightarrow \underset{(\text{finally})}{TE} = \frac{GmM}{R}\left(\frac{1}{2} - \frac{2}{3}\right) = \left[-\frac{GmM}{R}\left(\frac{1}{6}\right)\right]$$

So, change in TE required = $\frac{GmM}{6R}$

Similarly solving for $(a = R)$

then we will get

$$TE_{final} = -\frac{GmM}{2R}$$

So, change in $TE = \frac{GmM}{2R}$

Alternate Method

For perihilion

$a(1-e) = R + h = \frac{3R}{2}$

$\Rightarrow a = 3R$

Also, for elliptical path

$T.E. = -\frac{GMm}{2a} = \frac{-GMm}{6R}$

$\Rightarrow$ change in $TE = \frac{GMm}{6R}$

For aphelion :

$a(1+e) = R + h = \frac{3R}{2}$

$\Rightarrow a = R$

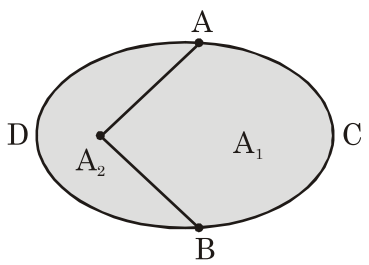
& $T.E. = -\frac{GMm}{2a} = -\frac{GMm}{2R}$

$\Rightarrow$ change in $TE = \frac{GMm}{2R}$

Linked Comprehension Type (1 Para × 2Q.) [3 M (-1)]
(Single Correct Answer Type)

8. Ans. (B)

Sol. According to Keplar's Law



$\frac{dA}{dt} = \text{constant}$

$\Rightarrow A \propto t \Rightarrow \frac{A_1}{A_2} = \frac{t_1}{t_2}$

$A_1 > A_2$

$\Rightarrow t_1 > t_2$

9. Ans. (C)

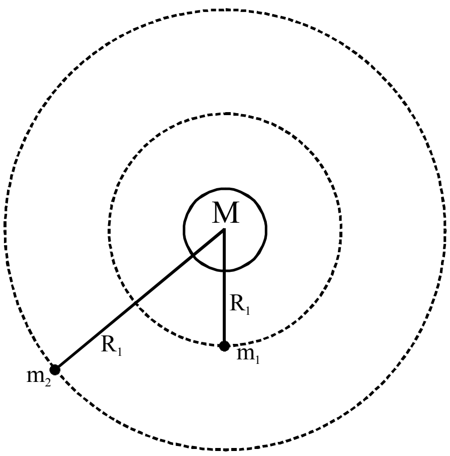
Sol. For a bounded system, TE < 0

$\Rightarrow |U| > |KE|$

Matching List Type (4 × 4) 1 Q. [3 M (–1)]

10. Ans. (B)

Sol.



$\left.\begin{array}{l}\frac{m_1}{m_2} = 2 \\ \frac{R_1}{R_2} = \frac{1}{4}\end{array}\right\}$ given

$$\frac{GMm_1}{R_1^2}=\frac{m_1v_1^2}{R_1}$$

$$v_1^2=\frac{GM}{R_1},\ v_2^2=\frac{GM}{R_2}$$

$$\frac{v_1^2}{v_2^2}=\frac{R_2}{R_1}=4$$

(P) $\frac{v_1}{v_2}=2$

(Q) L = mvR

$$\frac{L_1}{L_2}=\frac{m_1v_1R_1}{m_2v_2R_2}=2\times2\times\frac{1}{4}=1$$

(R) $K=\frac{1}{2}mv^2$

$$\frac{K_1}{K_2}=\frac{m_1v_1^2}{m_2v_2^2}=2\times(2)^2=8$$

(S) T = 2πR/V

$$\frac{T_1}{T_2}=\frac{R_1}{v_1}\times\frac{v_2}{R_2}=\frac{R_1}{R_2}\times\frac{v_2}{v_1}=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$$

SECTION-III

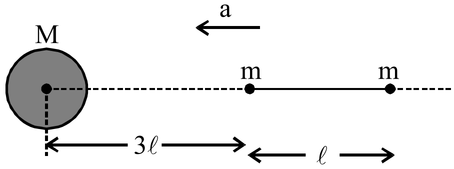
Numerical Grid Type (Ranging from 0 to 9) 2 Q. [4 M (0)]

1. **Ans. 6**

Sol. $\frac{L_{total}}{L_B}=\frac{m_1r_1^2}{m_2r_2^2}+1$

2. **Ans. 7**

Sol. Due to gravitational interaction connected masses have some acceleration.
Let both small masses are moving with acceleration 'a' towards larger mass M



Force eq. for mass nearer to larger mass

$$\frac{GMm}{(3\ell)^2}-\frac{Gm^2}{\ell^2}=ma \qquad ...(i)$$

Force eq. for mass away from larger mass

$$\frac{GMm}{(4\ell)^2}+\frac{Gm^2}{\ell^2}=ma \qquad ...(ii)$$

from equation (i) & (ii)

$$\frac{GM}{9\ell^2} - \frac{Gm}{\ell^2} = \frac{GM}{16\ell^2} + \frac{Gm}{\ell^2}$$

$$\Rightarrow \frac{M}{9} - \frac{M}{16} = m + m$$

$$\frac{7M}{144} = 2m$$

$$\Rightarrow m = \frac{7M}{288} = k\left(\frac{M}{288}\right)$$

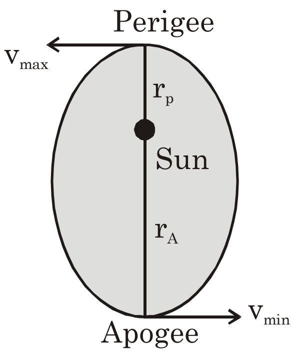
$\Rightarrow K = 7$

SECTION-IV

Matrix Match Type (4 × 5) **1 Q. [8 M (for each entry +2(0)]**

1. Ans. (A)-R, (B)-Q, (C)-Q (D)-P

Sol.



(A) $v \propto \frac{1}{r}$

(B) $r_p < r_A$

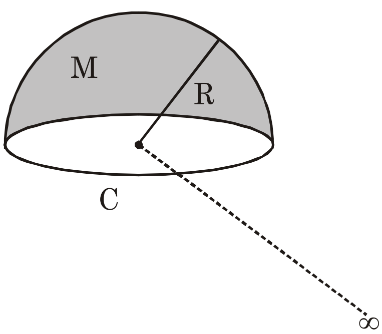
(C) $PE \propto -\frac{1}{r}$

(D) $L = \text{constant}$

Subjective Type **7 Q. [4 M (0)]**

1. Ans. $A = -\frac{3}{2}\frac{GmM}{R}$

Sol.



$W_g = \Delta U_g$

$= -m\left(\Delta V_g\right)$

$$= -m\left(V_{gf} - V_{gi}\right)$$

$$= -m\left(V_{\infty} - V_C\right)$$

$$W_g = -m\left(0 - \left(\frac{-3GM}{2R}\right)\right)$$

$$= -\frac{3GmM}{2R}$$

2. **Ans.** (a) $T=\dfrac{2\pi d^{3/2}}{\sqrt{3Gm}}$, (b) 2, (c) 2

Sol. (a) The COM of the system divides the distance between the stars in the inverse ratio of thin masses. If d_1 & d_2 are the distances of M & m from the COM. (COM = Centre of mass)

$$d_1 = \frac{d}{M+m} \times m \;\;\&\;\; d_2 = \frac{d}{M+m} \times M$$

The stars will rotate in circles of radii d_1 & d_2 about their COM. The same force of attraction provides the necessary contripetal force for their circular motion.

$$\therefore \;\; \frac{GmM}{d^2} = M\omega_1^2 d_1 = m\omega_2^2 d_2$$

$$\text{or} \;\; \omega_1^2 = \frac{Gm}{d^2 d_1} = \frac{Gm}{d^2} \times \frac{M+m}{d \times m} = \frac{G(M+m)}{d^3}$$

$$\text{and} \;\; \omega_2^2 = \frac{G(M)}{d^2 d_2} = \frac{GM}{d^2} \times \left(\frac{M+m}{d \times M}\right) = \frac{G(M+m)}{d^3}$$

$$\Rightarrow \omega_1 = \omega_2 = \sqrt{\frac{G(M+m)}{d^3}}$$

(b) From the fact that the moment of momentum is also angular momentum.

$$\frac{I_M}{I_m} = \frac{(Mv_1)d_1}{(Mv_2)d_2} = \frac{M}{m}\frac{d_1^2}{d_2^2} = \frac{M}{m} \times \frac{m^2}{M^2} = \frac{m}{M}$$

$$\text{(c)} \;\; \frac{K_M}{K_m} = \frac{\frac{1}{2}Mv_1^2}{\frac{1}{2}Mv_2^2} = \frac{M}{m} \times \frac{\omega_1^2 d_1^2}{\omega_2^2 d_2^2} = \frac{M}{m}\frac{d_1^2}{d_2^2}$$

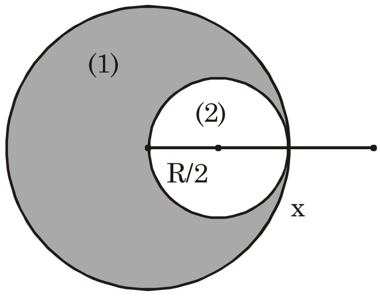
$$\Rightarrow \frac{K_M}{K_m} = \frac{M}{m} \times \frac{m^2}{M^2} = \frac{m}{M}$$

3. **Ans.** $\vec{g} = +\dfrac{\pi G\rho_0 R^3}{6}\left[\dfrac{1}{(x-(R/2))^2} - \dfrac{8}{x^2}\right]\hat{i}$, $\vec{g} = -\dfrac{2\pi G\rho_0 R}{3}\hat{i}$

Sol. (a) $\vec{g} = \vec{E}_g$

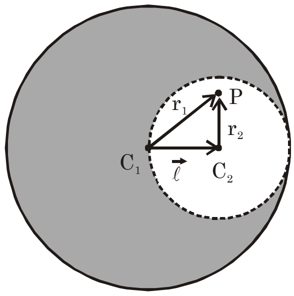
$$= \left(\vec{E}_g\right)_1 - \left(\vec{E}_g\right)_2$$

$$\vec{E}_{g_1} = -\left(\frac{GM_1}{r_1^2}\right)\hat{i} = -\left(\frac{G\rho_0 \frac{4\pi}{3} R^3}{x^2}\right)\hat{i}$$



$$\vec{E}_{g_2} = -\frac{GM_2}{r_2^2}\hat{i} = -\left(\frac{G\rho_0\left(\frac{4\pi}{3}\left(\frac{R}{2}\right)^3\right)}{\left(x-\frac{R}{2}\right)^2}\right)\hat{i}$$

(b) $\vec{E}_p = \left(\vec{E}_g\right)_1 - \left(\vec{E}_g\right)_2$



$$= \left(\frac{-\rho_0 \vec{r}_1}{3\left(\frac{1}{4\pi G}\right)}\right) - \left(\frac{-\rho_0 \vec{r}_2}{3\left(\frac{1}{4\pi G}\right)}\right)$$

$$= \frac{-4\pi G\rho_0}{3}\left(\vec{r}_1 - \vec{r}_2\right)$$

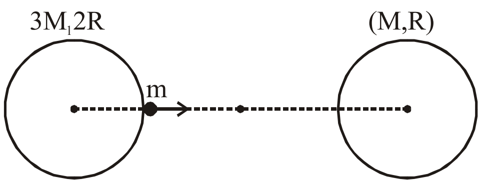
$$= \frac{-4\pi}{3} G\rho_0 \left(\vec{\ell}\right) = \text{constant with position.}$$

hence, $\vec{E}_p$ is uniform inside cavity

$$\vec{\ell} = \frac{R}{2}\hat{i}$$

4. **Ans.** (a) 2R, (b) $3\sqrt{3}\left(\sqrt{3}-1\right)R$

Sol.



(a) speed will maximum at projection point

(b) To just reach the other planet small body must be able to reach the point where gravitation field due to both planet is zero. At this point speed will be minimum and equal to zero.

$$\frac{G \times 3M}{x^2} = \frac{GM}{(6R-x)^2} \Rightarrow \sqrt{3}\left(6R - x\right) = x$$

$$\Rightarrow \left(1+\sqrt{3}\right)x = 6\sqrt{3}R$$

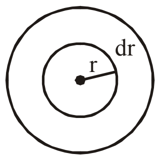
$$x = \frac{6\sqrt{3}\left(\sqrt{3}-1\right)R}{2}$$

5. **Ans.** $\frac{1}{4}K^2G\pi(R^4 - r^4)$

Sol. $dm = kr4\pi r^2 dr$

$m = k\pi r^4$

$E = \frac{Gm}{r^2}$



$-dP \times 4\pi r^2 = G\frac{m}{r^2}dm$

$$-dP = \frac{Gk\pi r^4 \times k4\pi r^3 dr}{4\pi r^4}$$

$-dP = Gk^2\pi r^3 dr$

$$-\int_{P=0}^{P} dP = Gk^2\pi\int_{r=R}^{r=r} r^3 dr$$

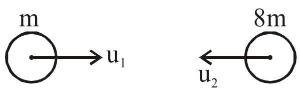
$$P = G\pi k^2 \frac{R^4 - r^4}{4}$$

6. **Ans.** (a) $\frac{2Gm^2}{3R}$ (b) 4R

Sol. Mass of bigger planet = 8m

Let u_1 & u_2 are speed just before collision

So, from momentum conservation; $u_1 = 8u_2$ ... (i)



From energy conservation, $\frac{1}{2}mu_1^2 + \frac{1}{2}(8m)\times u_2^2 - \frac{G(m)8m}{3R} = 0$ ... (ii)

Let v_1 & v_2 are speed just after collision

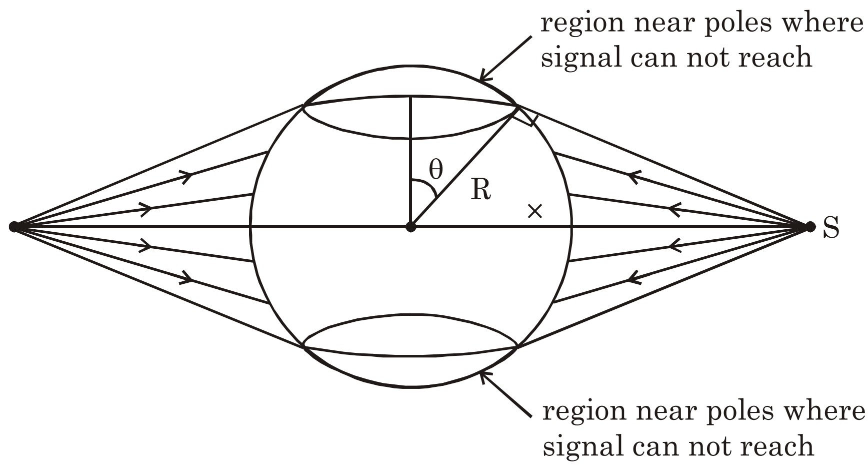
from $\Delta P = 0$,

$mu_1 - 8mu_2 = -mv_1 + 8mv_2$ ... (iii)

Apply energy conservation between just after collision and maximum sepration (when they come to rest, due to mutual actraction.

7. **Ans.** $\left(1-\frac{\sqrt{x^2-R^2}}{x}\right)4\pi R^2$

Sol.



$\sin\theta = \frac{R}{x}$

$\cos\theta = \sqrt{1-\frac{R^2}{x^2}}$

Area required $= 2(2\pi R^2(1-\cos\theta))$

$= 4\pi R^2\left(1-\frac{\sqrt{x^2-R^2}}{x}\right)$