09

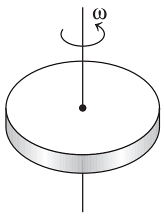
Rotational Motion

IN THIS CHAPTER ....

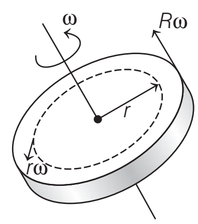
If all particles of a rigid body perform circular motion and the centres of these circles are steady on a definite straight line, then the motion of the rigid body is called *rotational motion*.

Basic Concepts of Rotational Motion

In rotation of a rigid body about a fixed axis, every particle of the body moves in a circle, which lies in a plane perpendicular to the axis and has its centre on the axis. For *e.g.* A disc (rigid body) of radius R and mass M rotating about a fixed axis passing through its centre as shown in figure.



Rotational motion is characterised by angular displacement $d\theta$ and angular velocity $\omega = \dfrac{d\theta}{dt}$.



If angular velocity is not uniform, then rate of change of angular velocity is known as *angular acceleration* (α).

Angular acceleration, $\alpha = \dfrac{\Delta\omega}{\Delta t}$ (unit of α = rad s^{-2})

If the angular velocity of rotation varies with the magnitude of tangential velocity as $v = r\omega$. The rate of change of tangential velocity is known as *tangential acceleration* (a_t) given by

$$a_t = \frac{\Delta v}{\Delta t} = r \cdot \frac{\Delta \omega}{\Delta t}$$

$$\Rightarrow \qquad a_t = r\alpha$$

Moment of Inertia

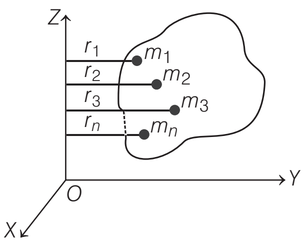
Moment of inertia of a rotating body is its property to oppose any change in its state of uniform rotation.

The moment of inertia of a rigid body about any axis of rotation is the sum of the product of masses of the particles and the square of their respective distances from axis of rotation.

$$I = MR^2$$

$$I = m_1r_1^2 + m_2r_2^2 + \ldots + m_nr_n^2$$

or
$$I = \sum_{i=1}^{n} m_ir_i^2$$



where, $m_1, m_2, \ldots, m_n$ are the masses of n particles and $r_1, r_2, \ldots, r_n$ be their distances from axis of rotation.

The unit of moment of inertia in SI system is kg-m². It is neither a scalar nor a vector, *i.e.* it is a tensor.

Radius of Gyration

The radius of gyration of a body about a given axis is the perpendicular distance of a point from the axis at which the whole mass of the body could be concentrated without any change in the moment of inertia of the body about that axis.

If a body has mass M and radius of gyration is K, then moment of inertia,

$$I = MK^2$$

$$K = \sqrt{\frac{I}{M}}$$

Radius of gyration is also defined as the root mean square distance of all the particles about the axis of rotation.

i.e.
$$K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \ldots + r_n^2}{n}}$$

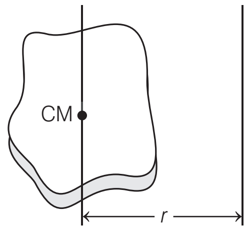
Note Radius of gyration depends upon shape and size of the body, position and configuration of the axis of rotation.

Theorems on Moment of Inertia

There are two theorems based on moment of inertia are given below

Theorem of Parallel Axes

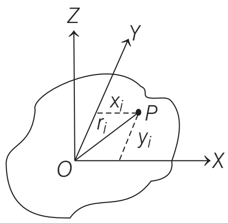
It states that the moment of inertia of a rigid body about any axis is equal to the sum of the moment of inertia about a parallel axis through its centre of mass (I_{CM}), the product of the mass of the body (M) and the square of the perpendicular distance between the two axes.



$$I = I_{CM} + Mr^2$$

Theorem of Perpendicular Axes

It states that the moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the lamina about any two mutually perpendicular axes in its plane and intersecting each other at the point, where the perpendicular axis passes through it.

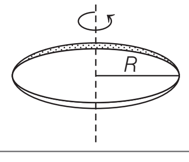


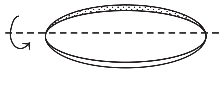
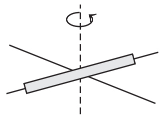
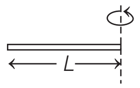
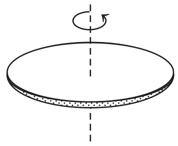
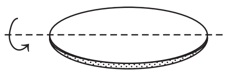
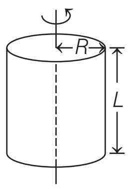
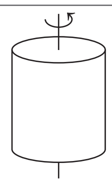
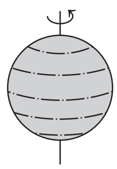
Let X and Y axes be chosen in the plane of the body and Z-axis perpendicular to this plane, three axes being mutually perpendicular, then according to the theorem

$$I_Z = I_X + I_Y$$

where, I_X, I_Y and I_Z are the moments of inertia about the X, Y, Z axes respectively.

Values of Moment of Inertia for Simple Geometrical Objects

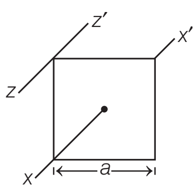
Body	Axis of Rotation	Figure	Moment of Inertia	K	K^2/R^2
Thin circular ring, radius R	About an axis passing through CG and perpendicular to its plane		MR^2	R	1

Body	Axis of Rotation	Figure	Moment of Inertia	K	K^2/R^2
Thin circular ring, radius R	About its diameter		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Thin rod length, L	Perpendicular to rod at mid-point		$\frac{1}{12}ML^2$	$\frac{L}{\sqrt{12}}$	
Thin rod length, L	About an axis passing through its edge and perpendicular to the rod		$\frac{ML^2}{3}$	$\frac{L}{\sqrt{3}}$	
Circular disc, radius R	Perpendicular to disc at centre		$\frac{1}{2}MR^2$	$\frac{R}{2}$	$\frac{1}{2}$
Circular disc, radius R	Diameter		$\frac{1}{4}MR^2$	$\frac{R}{4}$	$\frac{1}{4}$
Hollow cylinder, radius R	About its own axis		MR^2	R	1
Solid cylinder, radius R	About its own axis		$\frac{MR^2}{2}$	$\frac{R}{2}$	$\frac{1}{2}$
Solid sphere, radius R	About its diametric axis		$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}R$	$\frac{2}{5}$

Example 1. *Consider a uniform square plate of side a and mass m. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is* **[AIEEE 2008]**

(a) $\frac{5}{6}ma^2$ (b) $\frac{1}{12}ma^2$ (c) $\frac{7}{12}ma^2$ (d) $\frac{2}{3}ma^2$

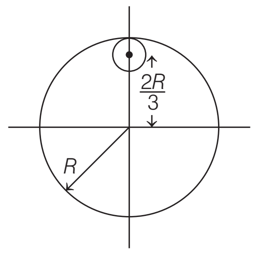
Sol. (*d*)



Moment of inertia of square plate about xx' is $\frac{ma^2}{6}$, therefore moment of inertia about zz' can be computed using parallel axes theorem,

$$I_{zz'} = I_{xx'} + m\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{ma^2}{6} + \frac{ma^2}{2} = \frac{2ma^2}{3}$$

Example 2. *From a uniform circular disc of radius R and mass 9 M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is* **[JEE Main 2018]**



(a) $4MR^2$ (b) $\frac{40}{9}MR^2$ (c) $10MR^2$ (d) $\frac{37}{9}MR^2$

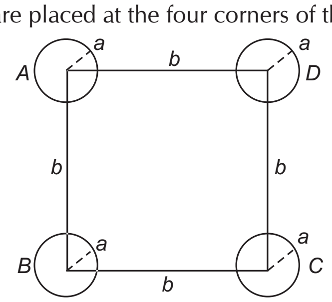
Sol. (*a*) Moment of inertia of remaining solid
= Moment of inertia of complete solid
− Moment of inertia of removed portion

$$\therefore \quad I = \frac{9MR^2}{2} - \left[\frac{M(R/3)^2}{2} + M\left(\frac{2R}{3}\right)^2\right] \Rightarrow I = 4MR^2$$

Example 3. *Four spheres, each of diameter 2a and mass M are placed with their centres on the four corners of a square of side b. Then, moment of inertia of the system about one side of the square taken as the axis is*

(a) $\frac{2}{5}M(4a^2 + 5b^2)$ (b) $\frac{2}{3}M(4a^2 + 5b^2)$
(c) $\frac{1}{3}M(2a^2 + 5b^2)$ (d) $\frac{1}{4}M(2a^2 + 5b^2)$

Sol. (*a*) $ABCD$ is a square of side b. Four spheres, each of mass M and radius a are placed at the four corners of the square.



Moment of inertia of the system about any side, say CD
= MI of sphere at A about CD
+ MI of sphere at B about CD
+ MI of sphere at C about CD
+ MI of sphere at D about CD

$$= \left(\frac{2}{5}Ma^2 + Mb^2\right) + \left(\frac{2}{5}Ma^2 + Mb^2\right) + \frac{2}{5}Ma^2 + \frac{2}{5}Ma^2$$

$$= \frac{8}{5}Ma^2 + 2Mb^2$$

$$= \frac{2}{5}M(4a^2 + 5b^2)$$

Example 4. *Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm) about its axis be I. The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is* ***[JEE Main 2019]***

(a) 16 *cm* (b) 14 *cm*
(c) 12 *cm* (d) 18 *cm*

Sol. (a) Moment of inertia of hollow cylinder about its axis is

$$I_1 = \frac{M}{2}(R_1^2 + R_2^2)$$

where, R_1 = inner radius and R_2 = outer radius.
Moment of inertia of thin hollow cylinder of radius R about its axis is

$$I_2 = MR^2$$

Given, $I_1 = I_2$ and both cylinders have same mass (M). So, we have

$$\frac{M}{2}(R_1^2 + R_2^2) = MR^2$$

$$(10^2 + 20^2)/2 = R^2$$

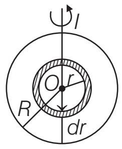
$$\Rightarrow \quad R^2 = 250$$

$$R \approx 16 \text{ cm}$$

Example 5. *A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its centre. The moment of inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = aMR^2$. The value of the coefficient a is* ***[JEE Main 2019]***

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) $\frac{8}{5}$ (d) $\frac{3}{2}$

Sol. (c) Consider an elementary ring of thickness dx and radius r as shown below



Varying density, $\rho(r) = \rho_0 r$, where ρ_0 is constant.
Area of circular elementary ring.

$$A = 2\pi r dr$$

$\therefore$ Mass of ring, M = density × area

$$= \rho \times 2\pi r dr$$

$\therefore$ Total mass of the plate,

$$M = \int_0^R \rho 2\pi r\, dr = \int_0^R \rho_0 r \times 2\pi r dr = \int_0^R \rho_0 \times 2\pi \times r^2 dr$$

$$\Rightarrow \quad M = \frac{2\pi\rho_0 R^3}{3}$$

$\therefore$ Moment of inertia about centre of mass,

$$I_{CM} = \int_0^R Mr^2 = \int_0^R \rho_0 r \times 2\pi r\, dr \times r^2$$

$$\Rightarrow \quad I_{CM} = \frac{2\pi\rho_0 R^5}{5}$$

Now, using parallel axes theorem, we know that

$$\Rightarrow \quad I = I_{CM} + MR^2$$

$$= \frac{2\pi\rho_0 R^5}{5} + \frac{2\pi\rho_0 R^3}{3} \times R^2$$

$$= \pi\rho_0 R^5\left(\frac{2}{5} + \frac{2}{3}\right)$$

$$\Rightarrow \quad I = \frac{16}{15}\pi\rho_0 R^5$$

$$= \frac{8}{5}\left(\frac{2\pi\rho_0 R^3}{3}\right) \times R^2$$

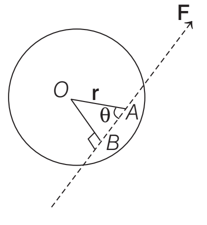
$$= \frac{8}{5} \times M \times R^2 = a(MR)^2 \quad \text{(given)}$$

Hence, $a = \frac{8}{5}$

Moment of a Force or Torque

Torque (or moment of a force) is the turning effect of a force applied at a point on a rigid body about the axis of rotation.
Mathematically, torque, $\tau = \mathbf{r} \times \mathbf{F} = |\mathbf{r} \times \mathbf{F}|\,\hat{\mathbf{n}} = rF\sin\theta\hat{\mathbf{n}}$
where, $\hat{\mathbf{n}}$ is a unit vector along the axis of rotation.
Torque is an axial vector and its SI unit is newton-metre (N-m).

- The torque about axis of rotation is independent of choice of origin O, so long as it is chosen on the axis of rotation AB.
- Only normal component of force contributes towards the torque. Radial component of force does not contribute towards the torque.



- A torque produces angular acceleration in a rotating body. Thus, torque, $\tau = I\alpha$.
- Moment of a couple (or torque) is given by product of position vector $\mathbf{r}$ between the two forces and either force $\mathbf{F}$. Thus, $\tau = \mathbf{r} \times \mathbf{F}$.
- If under the influence of an external torque, τ the given body rotates by $d\theta$, then work done, $dW = \tau \cdot d\theta$.
- In rotational motion, power may be defined as the scalar product of torque and angular velocity, *i.e.* Power $P = \tau \cdot \omega$.

Example 6. *Let* $\mathbf{F}$ *be the force acting on a particle having position vector* $\mathbf{r}$ *and* τ *be the torque of this force about the origin. Then,* **[AIEEE 2003]**

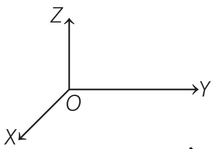
(a) $\mathbf{r}\cdot\tau = 0$ and $\mathbf{F}\cdot\tau \neq 0$ (b) $\mathbf{r}\cdot\tau \neq 0$ and $\mathbf{F}\cdot\tau = 0$

(c) $\mathbf{r}\cdot\tau \neq 0$ and $\mathbf{F}\cdot\tau \neq 0$ (d) $\mathbf{r}\cdot\tau = 0$ and $\mathbf{F}\cdot\tau = 0$

Sol. (d) As $\tau = \mathbf{r}\times\mathbf{F}$

τ is perpendicular to both $\mathbf{r}$ and $\mathbf{F}$, so $\mathbf{r}\cdot\tau$ as well as $\mathbf{F}\cdot\tau$ has to be zero.

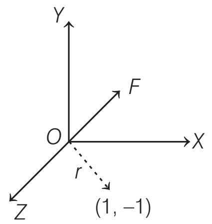
Example 7. *A force of* $-F\hat{\mathbf{k}}$ *acts on O, the origin of the coordinate system. The torque about the point* (1, – 1) *is* **[AIEEE 2006]**



(a) $F(\hat{\mathbf{i}} - \hat{\mathbf{j}})$ (b) $-F(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

(c) $F(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ (d) $-F(\hat{\mathbf{i}} - \hat{\mathbf{j}})$

Sol. (c) The torque about the given position, $\tau = \mathbf{r}\times\mathbf{F}$



Here, $\mathbf{r} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$ and $\mathbf{F} = -F\hat{\mathbf{k}}$

$$\therefore \quad \tau = (\hat{\mathbf{i}} - \hat{\mathbf{j}}) \times (-F\,\hat{\mathbf{k}})$$
$$= F\,[(-\hat{\mathbf{i}}\times\hat{\mathbf{k}}) + (\hat{\mathbf{j}}\times\hat{\mathbf{k}})]$$
$$= F\,(\hat{\mathbf{j}} + \hat{\mathbf{i}}) = F\,(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

Example 8. *The magnitude of torque on a particle of mass* 1 *kg is* 2.5 *N-m about the origin. If the force acting on it is* 1 *N and the distance of the particle from the origin is* 5 *m, then the angle between the force and the position vector is (in rad)* **[JEE Main 2019]**

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

Sol. (d) Given, $m = 1\,\text{kg}$

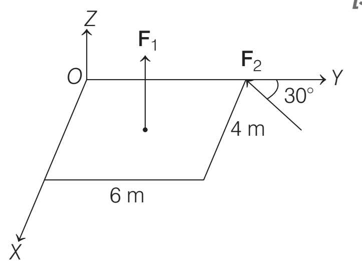
$$|\tau| = 2.5\ \text{N-m},\ F = 1\,\text{N and } r = 5\ \text{m}$$

We know that,

torque, $|\tau| = rF\sin\theta$

$$\Rightarrow \quad 2.5 = 5\times 1\times\sin\theta$$
$$\Rightarrow \quad \sin\theta = \frac{1}{2}$$
$$\text{or} \quad \theta = \frac{\pi}{6}\ \text{rad}$$

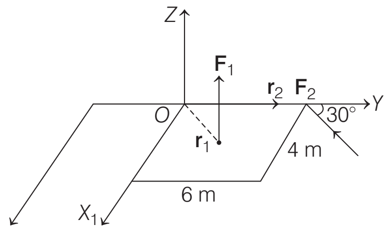
Example 9. *A slab is subjected to two forces* $\mathbf{F}_1$ *and* $\mathbf{F}_2$ *of same magnitude F as shown in the figure. Force* $\mathbf{F}_2$ *is in XY-plane while force* $\mathbf{F}_1$ *acts along Z-axis at the point* $(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$. *The moment of these forces about point O will be* **[JEE Main 2019]**



(a) $(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})F$ (b) $(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})F$

(c) $(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})F$ (d) $(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})F$

Sol. (b) According to the question, as shown in the figure below



$$\mathbf{r}_1 = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} \text{ and } \mathbf{r}_2 = 6\hat{\mathbf{j}}$$
$$\mathbf{F}_1 = F\,\hat{\mathbf{k}}$$

and $\mathbf{F}_2 = (-\sin 30^\circ\,\hat{\mathbf{i}} - \cos 30^\circ\,\hat{\mathbf{j}})\,F$

Moment of force is given as $\tau = \mathbf{r}\times\mathbf{F}$

where, r is the perpendicular distance and $\mathbf{F}$ is the force.

$\therefore$ Moment due to $\mathbf{F}_1$,

$$\tau_1 = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times (F\,\hat{\mathbf{k}})$$
$$= -2F\,\hat{\mathbf{j}} + 3F\,\hat{\mathbf{i}} \quad \ldots\text{(i)}$$

Moment due to $\mathbf{F}_2$,

$$\tau_2 = (6\hat{\mathbf{j}}) \times (-\sin 30^\circ\,\hat{\mathbf{i}} - \cos 30\,\hat{\mathbf{j}})\,F$$
$$= 6\sin 30^\circ F\,\hat{\mathbf{k}} = 3F\,\hat{\mathbf{k}} \quad \ldots\text{(ii)}$$

$\therefore$ Resultant torque,

$$\tau = \tau_1 + \tau_2 = 3F\,\hat{\mathbf{i}} - 2F\,\hat{\mathbf{j}} + 3F\,\hat{\mathbf{k}}$$
$$= (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})\,F$$

Angular Momentum

The moment of linear momentum of a given body about an axis of rotation is called its angular momentum.

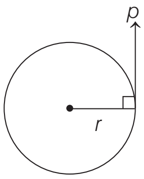
If $\mathbf{p} = m\mathbf{v}$ be the linear momentum of a particle and $\mathbf{r}$ is its position vector from the point of rotation, then

Angular momentum, $\mathrm{L} = \mathrm{r}\times\mathrm{p} = r\,p\sin\theta\,\hat{\mathbf{n}} = mvr\sin\theta\,\hat{\mathbf{n}}$,

where $\hat{\mathbf{n}}$ is a unit vector in the direction of rotation. Angular momentum is an axial vector and its SI unit is kg-m^{2}s^{-1} or J-s.

- For rotational motion of a rigid body, angular momentum is equal to the product of angular velocity and moment of inertia of the body about the axis of rotation.

Mathematically, $L = I\omega$.



- The rate of change of angular momentum of a body is equal to the external torque applied on it and takes place in the direction of torque. Thus,

$$\tau = \frac{dL}{dt} = \frac{d}{dt}(I\omega) = I\frac{d\omega}{dt} = I\alpha \qquad \left[\because \alpha = \frac{d\omega}{dt}\right]$$

- Total effect of a torque applied on a rotating body in a given time is called angular impulse. **Angular impulse** is equal to total change in angular momentum of the system in given time. Thus, angular impulse,

$$J = \int_0^{\Delta t} \tau\, dt = \Delta L = L_f - L_i$$

- The angular momentum of a system of particles about the origin is $\quad L = \sum^{n} r_i \times p_i$

Law of Conservation of Angular Momentum

According to the law of conservation of angular momentum, if no external torque is acting on a system, then total vector sum of angular momentum of different particles of the system remains constant.

We know that, $\quad \tau_{ext} = \dfrac{d\mathbf{L}}{dt}$

If $\tau_{ext} = 0$, then $\dfrac{d\mathbf{L}}{dt} = 0 \Rightarrow \quad L = \text{constant}$.

Therefore, in the absence of any external torque, the total angular momentum of a system must remain conserved.

As $\mathbf{L} = I\omega$, the law of conservation of momentum leads us to the conclusion.

For an isolated system, $I\omega = \text{constant}$ or $I_1\omega_1 = I_2\omega_2$

This principle is often made use by gymnast, swimmers, circus acrobats and ballet dancers etc.

Example 10. *The time dependence of the position of a particle of mass m = 2 kg is given by* $\mathbf{r}(t) = 2t\hat{\mathbf{i}} - 3t^2\hat{\mathbf{j}}$. *Its angular momentum, with respect to the origin, at time t = 2 s is* ***[JEE Main 2019]***

(a) $36\,\hat{\mathbf{k}}$ (b) $-34\,(\hat{\mathbf{k}} - \hat{\mathbf{i}})$

(c) $-48\,\hat{\mathbf{k}}$ (d) $48\,(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

Sol. (c) Position of particle, $\mathbf{r} = 2t\hat{\mathbf{i}} - 3t^2\hat{\mathbf{j}}$

where, t is instantaneous time.

Velocity of particle, $\mathbf{v} = \dfrac{d\mathbf{r}}{dt} = 2\hat{\mathbf{i}} - 6t\hat{\mathbf{j}}$

Now, angular momentum of particle with respect to origin is given by

$$\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$$
$$= m\{(2t\hat{\mathbf{i}} - 3t^2\hat{\mathbf{j}}) \times (2\hat{\mathbf{i}} - 6t\hat{\mathbf{j}})\}$$
$$= m(-12t^2(\hat{\mathbf{i}} \times \hat{\mathbf{j}}) - 6t^2(\hat{\mathbf{j}} \times \hat{\mathbf{i}}))$$

As, $\quad \hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = 0$

$\Rightarrow \quad \mathbf{L} = m(-12t^2\hat{\mathbf{k}} + 6t^2\hat{\mathbf{k}})$

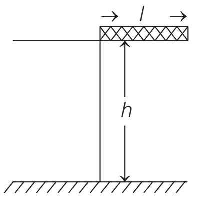
As, $\quad \hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$ and $\hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}$

$\Rightarrow \quad \mathbf{L} = -m(6t^2)\hat{\mathbf{k}}$

So, angular momentum of particle of mass 2 kg at time $t = 2$ s is

$$\mathbf{L} = (-2 \times 6 \times 2^2)\hat{\mathbf{k}} = -48\,\hat{\mathbf{k}}$$

Example 11. *A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5 m. When released, it slips off the table in a very short time* $\tau = 0.01\,s$, *remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in rad) close to* ***[JEE Main 2019]***



(a) 0.02 (b) 0.3 (c) 0.5 (d) 0.28

Sol. (c) Now, angular impulse of weight = change in angular momentum

$$\therefore \quad mg\frac{l}{2} \times \tau = \frac{ml^2}{3}\omega \Rightarrow \omega = \frac{3g \times \tau}{2 \times l}$$

Substituting the given values, we get

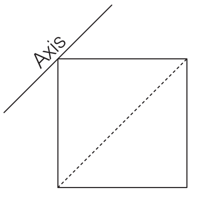
$$\omega = \frac{3 \times 10 \times 0.01}{2 \times 0.3} = 0.5 \text{ rad s}^{-1}$$

Time of fall of box, $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2 \times 5}{10}} \approx 1\,\text{s}$

So, angle turned by box in reaching ground,

$$\theta = \omega t = 0.5 \times 1 = 0.5 \text{ rad}$$

Example 12. *Four point masses, each of mass m are fixed at the corners of a square of side l. The square is rotating with angular frequency ω, about an axis passing through one of the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is* ***[JEE Main 2020]***



(a) $ml^2\omega$ (b) $4\,ml^2\omega$ (c) $3\,ml^2\omega$ (d) $2\,ml^2\omega$

Sol. (c) The angular momentum of the square about the given axis will be the sum of angular momentum due to each point masses.

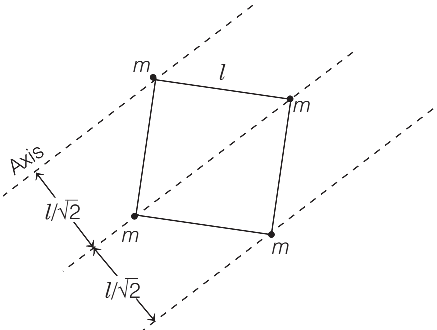
i.e. $L = L_1 + L_2 + L_3 + L_4$

$= I_1\omega_1 + I_2\omega_2 + I_3\omega_3 + I_4\omega_4$

Here, angular frequency will be same.

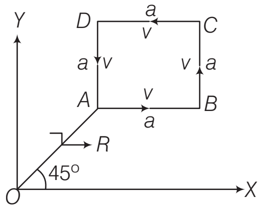
i.e., $\omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega$ (say)

$\therefore$ $L = (I_1 + I_2 + I_3 + I_4)\omega$



$$L = \left\{ m(0)^2 + 2 \times m\left(\frac{l}{\sqrt{2}}\right)^2 + m(\sqrt{2}l)^2 \right\} \omega = 3ml^2\omega$$

Example 13. *A particle of mass m is moving along the side of a square of side a, with a uniform speed v in the XY-plane as shown in the figure.*



*Which of the following statements is false, for the angular momentum **L** about the origin?* **[JEE Main 2016]**

(a) $L = -\frac{mv}{\sqrt{2}} R\,\hat{k}$, when the particle is moving from *A* to *B*.

(b) $L = mv\left(\frac{R}{\sqrt{2}} + a\right)\hat{k}$, when the particle is moving from *B* to *C*.

(c) $L = mv\left(\frac{R}{\sqrt{2}} - a\right)\hat{k}$, when the particle is moving from *C* to *D*.

(d) $L = \frac{mv}{\sqrt{2}} R\,\hat{k}$, when the particle is moving from *D* to *A*.

Sol. (c, d) For a particle of mass *m* is moving along the side of a square of side *a* such that

Angular momentum **L** about the origin, $L = r \times p = rp \sin\theta\, \hat{n}$

or $L = r(p)\,\hat{n}$

When a particle is moving from *D* to *A*, $L = \frac{R}{\sqrt{2}} mv(-\hat{k})$

A particle is moving from *A* to *B*, $L = \frac{R}{\sqrt{2}} mv(-\hat{k})$

and it moves from *C* to *D*, $L = \left(\frac{R}{\sqrt{2}} + a\right) mv\,(\hat{k})$

For *B* to *C*, we have

$$L = \left(\frac{R}{\sqrt{2}} + a\right) mv\,(\hat{k})$$

Example 14. *A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its centre. Two beads of mass m and negligible size are at the centre of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be* **[JEE Main 2019]**

(a) $\frac{M\omega_0}{M + 3m}$

(b) $\frac{M\omega_0}{M + m}$

(c) $\frac{M\omega_0}{M + 2m}$

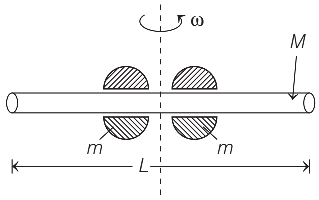
(d) $\frac{M\omega_0}{M + 6m}$

Sol. (d) As there is no external torque on system.

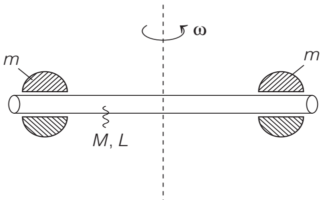
$\therefore$ Angular momentum of system is conserved.

$\Rightarrow$ $I_i\omega_i = I_f\omega_f$

Initially,



Finally,



$$\Rightarrow \quad \frac{ML^2}{12}\cdot\omega_0 + 0 = \left(\frac{ML^2}{12} + 2(m)\left(\frac{L}{2}\right)^2\right)\omega$$

So, final angular speed of system is

$$\Rightarrow \quad \omega = \frac{\frac{ML^2}{12}\cdot\omega_0}{\left(\frac{ML^2 + 6mL^2}{12}\right)} = \frac{M\omega_0}{M + 6m}$$

Angular Momentum of a Rigid Body

When a rigid body describes pure rotational motion, all its constituent particles describe circular motion about the axis of rotation. In such case, angular momentum of the rigid body,

$$L = \Sigma mvr = \Sigma mr^2\omega = (\Sigma mr^2)\cdot\omega = I\omega$$

Direction of **L** is same as ω. Hence, ω angular momentum is an axial vector.

When a rigid body is rotating about its centre of mass axis with angular velocity ω_{CM}, simultaneously moving translationally with a linear velocity v, the angular momentum of that body about a point P in the laboratory frame is given by

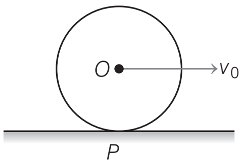
$$\mathbf{L}_p = I_{CM}\,\omega + r \times m\,\mathbf{v}_{CM}$$

Example 15. *A solid sphere rolls without slipping on a rough surface with centre of mass having a constant speed v_0. If mass of the sphere be m and r be its radius, then what is the value of angular momentum of the sphere about the point of contact with rough surface?*

(a) $\frac{7}{5}mv_0 r$ (b) $\frac{5}{7}mv_0 r$

(c) $\frac{3}{5}mv_0 r$ (d) $\frac{5}{3}mv_0 r$

Sol. (a) The angular momentum of the sphere about the point of contact P will be



$$\mathbf{L}_p = I_{CM}\omega + \mathbf{r} \times \mathbf{P}_{CM} = I_{CM}\omega + \mathbf{r} \times m\mathbf{v}_{CM}$$

As sphere is rolling without slipping, thus

$$\omega = \frac{v_0}{r}$$

$$\mathbf{L}_p = \left(\frac{2}{3}mr^2\right)\left(\frac{v_0}{r}\right) + r \times m \times v_0$$

$$= \frac{2}{5}mv_0 r + mv_0 r$$

$$= \frac{7}{5}mv_0 r$$

Pure Rotational Motion (Spinning)

When the body rotates in such a manner that its axis of rotation does not move, then its motion is called spinning motion.

In spinning, rotational kinetic energy is given by

$$K_R = \frac{1}{2}I\omega^2$$

$\because$ $$I = mK^2$$

and $$v = \omega R$$

$\therefore$ $$K_R = \frac{1}{2}mK^2\frac{v^2}{R^2}$$

$$= \frac{1}{2}mv^2\left(\frac{K^2}{R^2}\right)$$

Here, $\frac{K^2}{R^2}$ is a constant for different bodies. Value of $\frac{K^2}{R^2} = 1$ for ring and cylindrical shell and $\frac{K^2}{R^2} = \frac{1}{2}$ for disc and solid cylinder and $\frac{K^2}{R^2} = \frac{2}{5}$ for a solid sphere.

Equations of Rotational Motion

The kinematical quantities in rotational motion, angular displacement (θ), angular velocity (ω) and angular acceleration (α) respectively, then the kinematic equations for rotational motion with uniform angular acceleration are

$$\omega = \omega_0 t + \alpha t,$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

and $$\omega^2 = \omega_0^2 + 2 \times (\theta - \theta_0)$$

where, θ_0 = initial angular displacement of the rotating body and ω_0 = initial angular velocity of the body.

Dynamics of Rotational Motion about a Fixed Axis

From the given table, we compare translation motion and rotational motion about a fixed axis.

Equivalence between Translation and Rotational Motions

Translation Motion	Rotational Motion
Displacement, x	Angular displacement, θ
Linear velocity, $v = \frac{dx}{dt}$	Angular velocity, $\omega = \frac{d\theta}{dt}$
Linear momentum, $p = mv$	Angular momentum, $L = I\omega$
Acceleration, $a = \frac{dv}{dt}$	Angular acceleration, $\alpha = \frac{d\omega}{dt}$
Force, $F = ma$	Torque, $\tau = I\alpha$
Impulse, $I = F\Delta t = \Delta p$	Rotational impulse, $J = \int \tau dt = \Delta L$
Work, $W = \int F \cdot dS$	Work $= \int \tau \cdot d\theta$
Power, $P = F \cdot v$	Rotational power, $P = \tau \cdot \omega$

Example 16. *A disc of mass 5 kg and radius 50 cm rolls on the ground at the rate of 10 ms^{-1}. The kinetic energy of the disc is (Given, $I = \frac{1}{2}MR^2$)*

(a) 300 J (b) 325 J (c) 350 J (d) 375 J

Sol. (d) Here, mass of the disc, $M = 5$ kg

Radius of the disc, $R = 50$ cm $= \frac{1}{2}$ m

Linear velocity of the disc, $v = 10$ ms^{-1}

As, $$v = R\omega$$

$\therefore$ $$10 = \frac{1}{2}\omega$$

or $$\omega = 10 \times 2 = 20 \text{ rad s}^{-1}$$

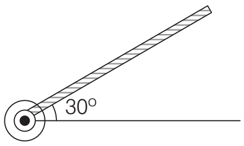
Also, moment of inertia of disc about an axis through its centre,

$$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 5 \times \left(\frac{1}{2}\right)^2 = \frac{5}{8}\text{kgm}^2$$

$\therefore$ KE of translation $= \frac{1}{2}Mv^2 = \frac{1}{2} \times 5 \times (10)^2 = 250$ J and KE of rotation $= \frac{1}{2} I\omega^2 = \frac{1}{2} \times \frac{5}{8} \times (20)^2 = 125$ J

Total KE $= 250 + 125 = 375$ J

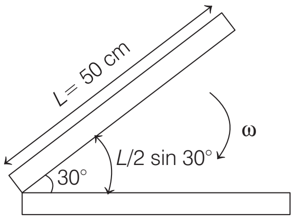
Example 17. *A rod of length 50 cm is pivoted at one end. It is raised such that, if makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be (Take, $g = 10\ ms^{-2}$)* **[JEE Main 2019]**



(a) $\frac{\sqrt{30}}{2}$ (b) $\sqrt{30}$

(c) $\frac{\sqrt{20}}{3}$ (d) $\sqrt{\frac{30}{2}}$

Sol. (*b*) Loss of potential energy of rod = Gain of kinetic energy



$$\therefore \qquad \Delta PE = \frac{1}{2} I\omega^2$$

(where, I = moment of inertia of rod and ω = angular frequency of rod)

$$\Rightarrow \quad Mg \times \frac{L}{2} \sin 30° = \frac{1}{2} \times I \times \omega^2$$

$$\Rightarrow \quad Mg \frac{L}{2} \times \frac{1}{2} = \frac{1}{2} \times I \times \omega^2$$

$$\Rightarrow \quad \frac{Mg\ L \times 2}{4 \times I} = \omega^2$$

$$\Rightarrow \quad \sqrt{\frac{MgL}{4 \times \frac{ML^2}{3}} \times \frac{2}{1}} = \omega \qquad \left[\because I = \frac{ML^2}{3} \text{ for rod}\right]$$

$$\omega = \sqrt{30} \text{ rad/s}$$

Example 18. *A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I, then the angular acceleration of the disc is* **[JEE Main 2019]**

(a) $\frac{k}{2I}\theta$ (b) $\frac{k}{I}\theta$ (c) $\frac{k}{4I}\theta$ (d) $\frac{2k}{I}\theta$

Sol. (*d*) Given, kinetic energy $= k\theta^2$

We know that, kinetic energy of a rotating body about its axis $= \frac{1}{2} I\omega^2$

where, I is moment of inertia and ω is angular velocity.

$$\text{So,} \qquad \frac{1}{2} I\omega^2 = k\theta^2$$

$$\text{or} \qquad \omega^2 = \frac{2k\theta^2}{I}$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{2k}{I}}\,\theta \qquad \dots \text{(i)}$$

Differentiating the above equation w.r.t. time on both sides, we get

$$\frac{d\omega}{dt} = \sqrt{\frac{2k}{I}} \cdot \frac{d\theta}{dt} = \sqrt{\frac{2k}{I}} \cdot \omega \qquad \left[\because \omega = \frac{d\theta}{dt}\right]$$

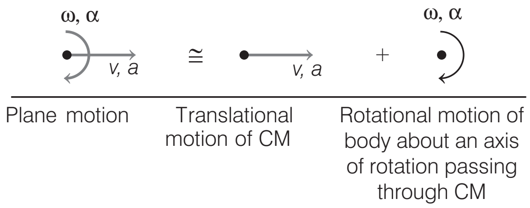
$\therefore$ Angular acceleration,

$$\alpha = \frac{d\omega}{dt} = \sqrt{\frac{2k}{I}} \cdot \omega = \sqrt{\frac{2k}{I}} \cdot \sqrt{\frac{2k}{I}}\,\theta \qquad \text{[using Eq. (i)]}$$

$$\text{or} \qquad \alpha = \frac{2k}{I}\theta$$

Rolling Motion

When a body performs translatory motion as well as rotational motion, then this type of motion is known as rolling motion or combined rotational and translational motion. *e.g.* Motion of football rolling on a surface.



The kinetic energy of rolling body is the sum of kinetic energy of a translation motion and kinetic energy of rotation motion.

Net kinetic energy,

$$K_N = K_T + K_R = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

$$= \frac{1}{2} mv^2 + \frac{1}{2} mv^2 \frac{K^2}{R^2}$$

$$\therefore \qquad K_N = \frac{1}{2} mv^2 \left(1 + \frac{K^2}{R^2}\right)$$

Classification of Rolling Motion

Depending on the fact that relative velocity of point of contact of the body undergoing rolling motion, with the platform (on which the body is performing plane motion) is zero or non-zero, rolling motion is classified into two categories, which are as follows

(i) Pure rolling or rolling without slipping/sliding or perfect rolling motion.

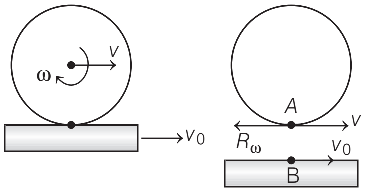
(ii) Impure rolling or rolling with slipping/sliding or imperfect rolling motion.

Pure Rolling Motion

If the relative velocity of the point of contact (between body and platform) is zero, then the rolling motion is said to be pure rolling motion.

For pure rolling motion, $v_{AB} = 0$

i.e. $\quad (v - R\omega) - v_0 = 0 \;\Rightarrow\; v - R\omega = v_0$



If the platform is stationary, *i.e.* $v_0 = 0$, then for pure rolling motion $v = R\omega$.

Note Friction is responsible for pure rolling motion. In rolling motion, friction is non-dissipative in nature, *i.e.* work done by friction force is zero because point of contact is relatively at rest.

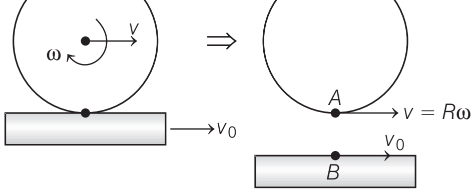
Impure Rolling Motion

In impure rolling motion, the point of contact of the body with the platform is not relatively at rest w.r.t. platform on which it is performing rolling motion, as a result sliding occurs at the point of contact.

For impure rolling motion,

$$v_{AB} \neq 0, \quad i.e. \quad v - R\omega \neq v_0$$

If platform is stationary, *i.e.* $v_0 = 0$, then condition for impure rolling motion is $v \neq R\omega$.



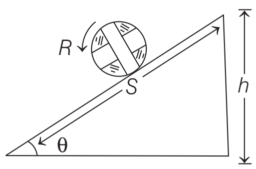
Here as, $\quad v \neq R\omega$,

So, $\quad a \neq R\alpha$

Note In this case, friction is opposing the relative motion, *i.e.* rolling motion and is dissipative in nature, *i.e.* work done by friction force is non-zero.

Rolling Motion on an Inclined Plane

When a body of mass m and radius R rolls down on inclined plane of height h and angle of inclination θ, it loses potential energy. However, it acquires both linear and angular speeds and hence gain kinetic energy of translation and that of rotation.



By conservation of mechanical energy,

$$mgh = \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$$

- **Velocity at the lowest point,** $v = \sqrt{\dfrac{2gh}{1 + \dfrac{K^2}{R^2}}}$
- **Acceleration in motion** From second equation of motion, $v^2 = u^2 + 2as$

  By substituting $u = 0, s = \dfrac{h}{\sin\theta}$ and $v = \sqrt{\dfrac{2gh}{1 + \dfrac{K^2}{R^2}}}$, we get

  $$a = \frac{g\sin\theta}{1 + \dfrac{K^2}{R^2}}$$

- **Time of descent** From first equation of motion,

  $$v = u + at$$

  By substituting $u = 0$ and value of v and a from above expressions $t = \dfrac{1}{\sin\theta}\sqrt{\dfrac{2h}{g}\left[1 + \dfrac{K^2}{R^2}\right]}$

  From the above expressions, it is clear that,

  $$v \propto \frac{1}{\sqrt{1 + \dfrac{K^2}{R^2}}}; a \propto \frac{1}{1 + \dfrac{K^2}{R^2}}; t \propto \sqrt{1 + \frac{K^2}{R^2}}$$

Note (i) Factor $\left(\dfrac{K^2}{R^2}\right)$ is a measure of moment of inertia of a body and its value is constant for given shape of the body and it does not depend on the mass and radius of a body.

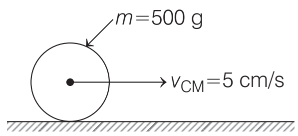
(ii) Velocity, acceleration and time of descent (for a given inclined plane) all depends on $\dfrac{K^2}{R^2}$. Lesser the moment of inertia of the rolling body, lesser will be the value of $\dfrac{K^2}{R^2}$. So, greater will be its velocity and acceleration and lesser will be the time of descent.

Example 19. *A uniform sphere of mass* 500 g *rolls without slipping on a plane horizontal surface with its centre moving at a speed of* 5.00 cm/s. *Its kinetic energy is* **[JEE Main 2020]**

(a) 6.25×10^{-4} J $\qquad$ (b) 1.13×10^{-3} J

(c) 8.75×10^{-4} J $\qquad$ (d) 8.75×10^{-3} J

Sol. (c) The given situation is as shown in the figure below.



In pure rolling, object possesses both translational kinetic energy and rotational kinetic energy.

So, $\quad KE_{\text{sphere}} = \frac{1}{2}mv_{CM}^2 + \frac{1}{2}I\omega^2$

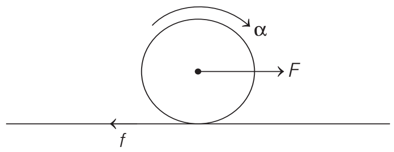
Here, $\quad I = \frac{2}{5}mr^2$ and $\omega = \frac{v}{r} \quad [\because v_{CM} = v]$

So,
$$KE_{\text{sphere}} = \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{2}{5}mr^2\times\frac{v^2}{r^2}$$
$$= \left(\frac{1}{2}+\frac{1}{5}\right)mv^2$$
$$= \frac{7}{10}\times 500\times 10^{-3}\times 25\times 10^{-4}$$
$$= 8.75\times 10^{-4}\text{ J}$$

Example 20. *A homogeneous solid cylindrical roller of radius R and mass m is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is* **[JEE Main 2019]**

(a) $\frac{F}{2\,m\,R}$ (b) $\frac{2F}{3\,m\,R}$ (c) $\frac{3F}{2\,m\,R}$ (d) $\frac{F}{3\,mR}$

Sol. (*b*) When force F is applied at the centre of roller of mass m as shown in the figure below



Its acceleration is given by
$$\frac{(F-f)}{m} = \alpha \quad \text{... (i)}$$
where, f = force of friction and m = mass of roller.
Torque on roller is provided by friction f and it is
$$\tau = fR = I\alpha \quad \text{...(ii)}$$
where, $\quad I$ = moment of inertia of solid cylindrical roller $= mR^2/2$

and $\quad \alpha$ = angular acceleration of cylinder $= a/R$.

Hence, $\quad \tau = \frac{mR^2}{2}\cdot\frac{a}{R} = \frac{maR}{2}$

From Eq. (ii), $(\tau = fR)$
$$f = \frac{ma}{2} \quad \text{...(iii)}$$
From Eqs. (i) and (iii), we get
$$F = \frac{3}{2}ma$$
$$\Rightarrow \quad a = \frac{2F}{3m}$$
So, $$\alpha = \frac{2F}{3mR} \quad \left[\because \alpha = \frac{a}{R}\right]$$

Example 21. *Three bodies, a ring, a solid cylinder and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of the bodies are identical. Which of the body reaches the ground with maximum velocity?*

(*a*) *Ring*

(*b*) *Cylinder*

(*c*) *Sphere*

(*d*) *All have the same velocity*

Sol. (*c*) We assume conservation of energy of the rolling body, *i.e.* there is no loss of energy due to friction etc. The potential energy lost by the body in rolling down the inclined plane $(= mgh)$ must, therefore be equal to kinetic energy gained.

Since, the bodies start from rest. Therefore, the kinetic energy gained is equal to the final kinetic energy of the bodies.
$$K = \frac{1}{2}mv_{CM}^2\left(1+\frac{k^2}{R^2}\right)$$
where, v_{CM} is the final velocity of (the centre of mass) of the body.

Equating K and mgh, we have
$$mgh = \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$$
or
$$v^2 = \left(\frac{2\,gh}{1+k^2/R^2}\right)$$

Note It is independent of the mass of the rolling body.

For a ring, $\quad k^2 = R^2$
$$v_{\text{ring}} = \sqrt{\frac{2\,gh}{1+1}} = \sqrt{gh}$$
For a solid cylinder, $\quad k^2 = \frac{R^2}{2}$
$$v_{\text{disc}} = \sqrt{\frac{2\,gh}{1+1/2}} = \sqrt{\frac{4}{3}gh}$$
For a solid sphere, $\quad k^2 = \frac{2R^2}{5}$
$$v_{\text{sphere}} = \sqrt{\frac{2\,gh}{1+\frac{2}{5}}} = \sqrt{\frac{10}{7}gh}$$
From the results obtained it is clear that among the three bodies, the sphere has the greatest and the ring has the least velocity of the centre of mass at the bottom of the inclined plane.

Practice Exercise

ROUND I Topically Divided Problems

Moment of Inertia

1. Moment of inertia of a thin rod of mass M and length L about an axis passing through its centre is $\frac{ML^2}{12}$. Its moment of inertia about a parallel axis at a distance of $\frac{L}{4}$ from this axis is given by

(a) $\frac{ML^2}{48}$ (b) $\frac{ML^3}{48}$
(c) $\frac{ML^2}{12}$ (d) $\frac{7\,ML^2}{48}$

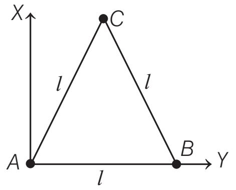
2. Moment of inertia of a solid cylinder of length L and diameter D about an axis passing through its centre of gravity and perpendicular to its geometric axis is

(a) $M\left(\frac{D^2}{4}+\frac{L^2}{12}\right)$ (b) $M\left(\frac{L^2}{16}+\frac{D^2}{8}\right)$
(c) $M\left(\frac{D^2}{4}+\frac{L^2}{6}\right)$ (d) $M\left(\frac{L^2}{12}+\frac{D^2}{16}\right)$

3. Three identical thin rods each of length l and mass M are joined together to form a ladder H. What is the moment of inertia of the system about one of the sides of H ?

(a) $\frac{Ml^2}{4}$ (b) $\frac{Ml^2}{3}$
(c) $\frac{2Ml^2}{3}$ (d) $\frac{4Ml^2}{3}$

4. Three particles each of mass m gram, are situated at the vertices of an equilateral triangle ABC of side l cm (as shown in figure). The moment of inertia of the system about a line AX perpendicular to AB and in the plane of ABC in g-cm^2 unit will be



(a) $\frac{3}{4}ml^2$ (b) $2\,ml^2$ (c) $\frac{5}{4}ml^2$ (d) $\frac{3}{2}ml^2$

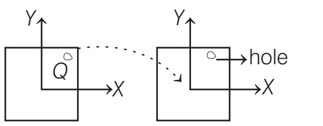
5. Two thin uniform circular rings each of radius 10 cm and mass 0.1 kg are arranged such that they have a common centre and their planes are perpendicular to each other. The moment of inertia of this system about an axis passing through their common centre and perpendicular to the plane of one of the rings in kg-m^2 is

(a) 15×10^{-3} (b) 5×10^{-3}
(c) 1.5×10^{-3} (d) 18×10^{-4}

6. Moment of inertia of a circular wire of mass M and radius R about its diameter is **[AIEEE 2002]**

(a) $\frac{MR^2}{2}$ (b) MR^2 (c) $2\,MR^2$ (d) $\frac{MR^2}{4}$

7. A uniform square plate has a small piece Q of an irregular shape removed and glued to the centre of the plate leaving a hole behind. The moment of inertia about the Z-axis is **[NCERT Exemplar]**

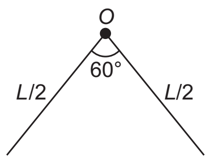


(a) increased
(b) decreased
(c) the same
(d) changed in unpredicted manner

8. Two discs have same mass and thickness and their materials have densities d_1 and d_2. The ratio of their moments of inertia about central axis will be

(a) $d_1 : d_2$ (b) $d_1d_2 : 1$
(c) $1 : d_1d_2$ (d) $d_2 : d_1$

9. A thin rod of length L and mass M is bent at the middle point O at an angle of 60°. The moment of inertia of the rod about an axis passing through O and perpendicular to the plane of the rod will be



(a) $\frac{ML^2}{6}$ (b) $\frac{ML^2}{12}$ (c) $\frac{ML^2}{24}$ (d) $\frac{ML^2}{3}$

10. Four point masses, each of value m, are placed at the corners of a square $ABCD$ of side l. The moment of inertia of this system about an axis passing through A and parallel to BD is **[AIEEE 2006]**

(a) $2ml^2$ (b) $\sqrt{3}\,ml^2$

(c) $3ml^2$ (d) ml^2

11. The moment of inertia of uniform semi-circular disc of mass M and radius r about a line perpendicular to the plane of the disc through the centre is **[AIEEE 2005]**

(a) $\frac{1}{4}Mr^2$ (b) $\frac{2}{5}Mr^2$

(c) Mr^2 (d) $\frac{1}{2}Mr^2$

12. If I_1 is the moment of inertia of a thin rod about an axis perpendicular to its length and passing through its centre of mass and I_2 is the moment of inertia of ring about an axis perpendicular to plane of ring and passing through its centre formed by bending the rod, then

(a) $\frac{I_1}{I_2} = \frac{3}{\pi^2}$ (b) $\frac{I_1}{I_2} = \frac{2}{\pi^2}$

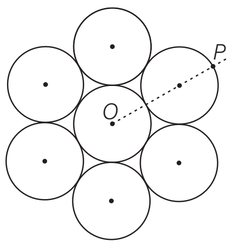
(c) $\frac{I_1}{I_2} = \frac{\pi^2}{2}$ (d) $\frac{I_1}{I_2} = \frac{\pi^2}{3}$

13. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B such that **[AIEEE 2004]**

(a) $I_A = I_B$ (b) $I_A > I_B$

(c) $I_A < I_B$ (d) $\frac{I_A}{I_B} = \frac{d_A}{d_B}$

14. Seven identical circular planar discs, each of mass M and radius R are welded symmetrically as shown in the figure. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is **[JEE Main 2018]**



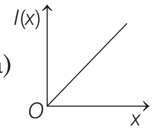
(a) $\frac{19}{2}MR^2$

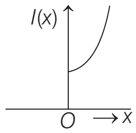
(b) $\frac{55}{2}MR^2$

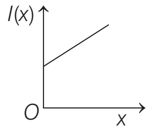
(c) $\frac{73}{2}MR^2$

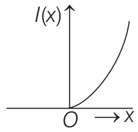
(d) $\frac{181}{2}MR^2$

15. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is $I(x)$. Which one of the graphs represents the variation of $I(x)$ with x correctly? **[JEE Main 2019]**

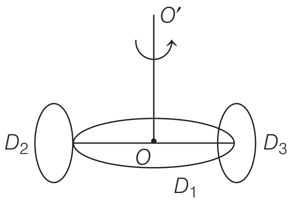
(a)


(b)


(c)


(d)


16. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO' passing through the centre of D_1, as shown in the figure will be **[JEE Main 2019]**



(a) $\frac{2}{3}MR^2$ (b) $\frac{4}{5}MR^2$ (c) $3MR^2$ (d) MR^2

17. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$, where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is **[JEE Main 2019]**

(a) $\frac{MR^2}{2}$ (b) $\frac{MR^2}{6}$ (c) $\frac{MR^2}{3}$ (d) $\frac{2MR^2}{3}$

18. The radius of gyration of a uniform rod of length l, about an axis passing through a point $\frac{l}{4}$ away from the centre of the rod and perpendicular to it, is **[JEE Main 2020]**

(a) $\frac{1}{8}l$ (b) $\sqrt{\frac{3}{8}}\,l$ (c) $\sqrt{\frac{7}{48}}\,l$ (d) $\frac{1}{4}l$

19. Mass per unit area of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre is **[JEE Main 2020]**

(a) $2\pi a^4\left(\frac{A}{4} + \frac{B}{5}\right)$ (b) $2\pi a^4\left(\frac{aA}{4} + \frac{B}{5}\right)$

(c) $\pi a^4\left(\frac{A}{4} + \frac{aB}{5}\right)$ (d) $2\pi a^4\left(\frac{A}{4} + \frac{aB}{5}\right)$

20. The linear mass density of a thin rod AB of length L varies from A to B as $\lambda(x) = \lambda_0\left(1 + \frac{x}{L}\right)$, where x is the distance from A. If M is mass of the rod, then its moment of inertia about an axis passing through A and perpendicular to the rod is **[JEE Main 2020]**

(a) $\frac{5}{12}ML^2$ (b) $\frac{7}{18}ML^2$

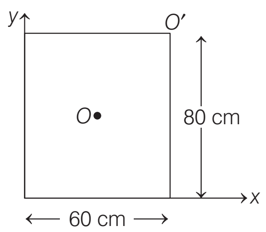
(c) $\frac{2}{5}ML^2$ (d) $\frac{3}{7}ML^2$

21. Consider two uniform discs of same thickness and different radii $R_1 = R$ and $R_2 = \alpha R$ made of the same material. If the ratio of their moments of inertia I_1 and I_2 respectively, about their axes is $I_1 : I_2 = 1 : 16$, then the value of α is **[JEE Main 2020]**

(a) 2 (b) $2\sqrt{2}$

(c) 4 (d) $\sqrt{2}$

22.



For a uniform rectangular sheet, shown in the above figure, the ratio of moments of inertia about the axes perpendicular to the sheet and passing through O (the centre of mass) and O' (corner point) is **[JEE Main 2020]**

(a) 1/8 (b) 2/3

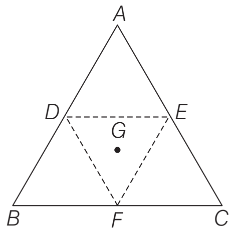
(c) 1/4 (d) 1/2

23. Moment of inertia of a cylinder of mass M, length L and radius R about an axis passing through its centre and perpendicular to the axis of the cylinder is $I = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right)$. If such a cylinder is to be made for a given mass of a material. To have minimum possible moment of inertia, the ratio L/R for cylinder is **[JEE Main 2020]**

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$

(c) $\sqrt{\frac{3}{2}}$ (d) $\sqrt{\frac{2}{3}}$

24. An equilateral triangle ABC is cut from a thin solid sheet of wood (see figure). D, E and F are the mid points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I, then **[JEE Main 2019]**



(a) $I' = \frac{3}{4}I_0$ (b) $I' = \frac{15}{16}I_0$

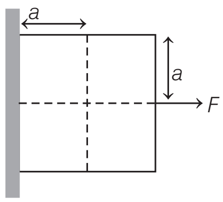
(c) $I' = \frac{I_0}{4}$ (d) $I' = \frac{9}{16}I_0$

Torque, Angular Momentum and Conservation of Angular Momentum

25. Four 2 kg masses are connected by $\frac{1}{4}$ m spokes to an axle. A force of 24 N acts on a lever (1/2) m long to produce angular acceleration α. The magnitude of α (in rad s^{-2}) is

(a) 24 (b) 12 (c) 6 (d) 3

26. A horizontal force F is applied such that the block remains stationary, then which of the following statement is false?



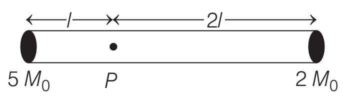
(a) $F = mg$ (where F is the frictional force).

(b) $F = N$ (where N is the normal force).

(c) F will not produce torque.

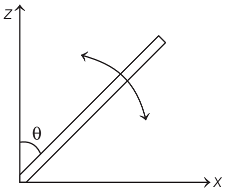
(d) N will not produce torque.

27. A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be **[JEE Main 2019]**



(a) $\frac{g}{13l}$ (b) $\frac{g}{2l}$ (c) $\frac{7g}{3l}$ (d) $\frac{g}{3l}$

28. A slender uniform rod of mass M and length l is pivoted at one end, so that it can rotate in a vertical plane (see the figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical, is **[JEE Main 2017]**



(a) $\frac{2g}{3l}\sin\theta$ (b) $\frac{3g}{2l}\cos\theta$

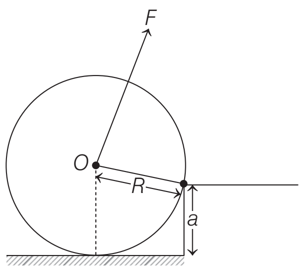
(c) $\frac{2g}{3l}\cos\theta$ (d) $\frac{3g}{2l}\sin\theta$

29. A particle of mass m is moving along a trajectory given by $x = x_0 + a\cos\omega_1 t$ and $y = y_0 + b\sin\omega_2 t$. The torque acting on the particle about the origin at $t = 0$ is **[JEE Main 2019]**

(a) zero (b) $m(-x_0 b + y_0 a)\,\omega_1^2\,\hat{\mathbf{k}}$

(c) $-m(x_0 b\omega_2^2 - y_0 a\omega_1^2)\,\hat{\mathbf{k}}$ (d) $+my_0 a\,\omega_1^2\,\hat{\mathbf{k}}$

30. A uniform cylinder of mass M and radius R is to be pulled over a step of height a $(a < R)$ by applying a force F at its centre O perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is **[JEE Main 2020]**



(a) $Mg\frac{a}{R}$ (b) $Mg\sqrt{1-\frac{a^2}{R^2}}$

(c) $Mg\sqrt{\left(\frac{R}{R-a}\right)^2-1}$ (d) $Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$

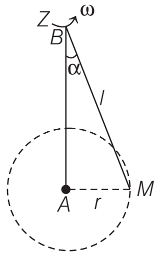
31. A ring of diameter 0.4 m and of mass 10 kg is rotating about its axis at the rate of 1200 rpm. The angular momentum of the ring is

(a) 60.28 kg-m^{2}s^{-1} (b) 55.26 kg-m^{2}s^{-1}

(c) 40.28 kg-m^{2}s^{-1} (d) 50.28 kg-m^{2}s^{-1}

32. A mass M hangs on a massless rod of length l which rotates at a constant angular frequency. The mass M moves with steady speed in a circular path of constant radius. Assume that, the system is in steady circular motion with constant angular velocity ω. The angular momentum of M about point A is L_A which lies in the position z-direction and the angular momentum of M about B is L_B.

The correct statement for this system is



[JEE Main 2021]

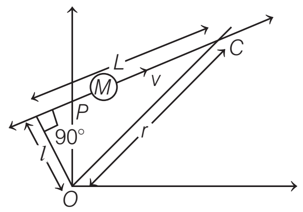
(a) L_A and L_B both are constant in magnitude and direction

(b) L_B is constant in direction with varying magnitude

(c) L_B is constant, both in magnitude and direction

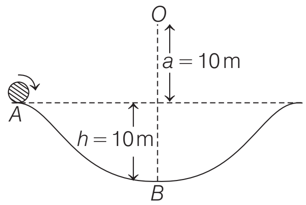
(d) L_A is constant, both in magnitude and direction

33. A particle of mass M moves along the line PC with velocity v as shown. What is the angular momentum of the particle about O?



(a) mvL (b) mvl (c) mvr (d) Zero

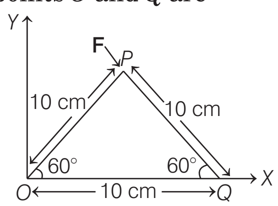
34. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be (Take, $g = 10\ \text{m/s}^2$) **[JEE Main 2019]**



(a) 8 kg-m^2/s (b) 3 kg-m^2/s

(c) 2 kg-m^2/s (d) 6 kg-m^2/s

35. A triangular plate is shown. A force $\mathbf{F} = 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$ is applied at point P. The torque at point P with respect to points O and Q are



[JEE Main 2021]

(a) $-15-20\sqrt{3}, 15-20\sqrt{3}$ (b) $15+20\sqrt{3}, 15-20\sqrt{3}$

(c) $15-20\sqrt{3}, 15+20\sqrt{3}$ (d) $-15+20\sqrt{3}, 15+20\sqrt{3}$

36. Initial angular velocity of a circular disc of mass M is ω_1, then two small spheres of mass m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc? **[AIEEE 2002]**

(a) $\left(\frac{M+m}{M}\right)\omega_1$ (b) $\left(\frac{M+m}{m}\right)\omega_1$

(c) $\left(\frac{M}{M+4m}\right)\omega_1$ (d) $\left(\frac{M}{M+2m}\right)\omega_1$

37. A cord is wound round the circumference of a wheel of radius r. The axis of the wheel of horizontal and moment of inertia about it is I. A weight mg is attached to the end of the cord and falls from rest. After falling through a distance h, the angular velocity of the wheel will be

(a) $\left(\frac{2gh}{1+mr}\right)^{1/2}$ (b) $\left(\frac{2mgh}{I+mr^2}\right)^{1/2}$

(c) $\left(\frac{2mgh}{1+2m}\right)^{1/2}$ (d) $(2gh)^{1/2}$

38. A 3 kg particle moves with constant speed of 2 ms^{-1} in the XY-plane in the y-direction along the line $x = 4$ m. The angular momentum (in kg-m^2s^{-1}) relative to the origin and the torque about the origin needed to maintain this motion are respectively

(a) 12, 0 (b) 24, 0

(c) 0, 24 (d) 0, 12

39. If the earth suddenly changes its radius x times the present value, the new period of rotation would be

(a) $6x^2h$ (b) $12x^2h$

(c) $24x^2h$ (d) $48x^2h$

40. A thin and circular disc of mass and radius R is rotating in a horizontal plane about axis passing through its centre and perpendicular of its plane with an angular velocity ω. If another disc of same dimensions but of mass $\frac{M}{4}$ is placed gently on the first disc co-axially, then the new angular velocity of the system is

(a) $\frac{5}{4}\omega$ (b) $\frac{2}{3}\omega$ (c) $\frac{4}{5}\omega$ (d) $\frac{3}{2}\omega$

41. A ballet dancer spins with 2.8 rev s^{-1} with her arms out stretched. When the moment of inertia about the same axis becomes 0.7 I, the new rate of spin is

(a) 3.2 rev s^{-1} (b) 4.0 rev s^{-1}

(c) 4.8 rev s^{-1} (d) 5.6 rev s^{-1}

42. A merry-go-round, made of a ring-like platform of radius R and mass M, is revolving with angular speed ω. A person of mass M is standing on it. At one instant, the person jumps off the round, radially away from the centre of the round (as seen from the round). The speed of the round afterward is **[NCERT Exemplar]**

(a) 2ω (b) ω (c) $\frac{\omega}{2}$ (d) 0

43. If earth where to shrink to half its present diameter without any change in its mass, the duration of the day will be

(a) 48 h (b) 6 h (c) 12 h (d) 24 h

44. A man of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis. The mass of the platform with the man on it rotates at 12.0 rpm. If the man now moves to centre of the platform, the rotational speed would become

(a) 16.5 rpm (b) 25.7 rpm

(c) 32.3 rpm (d) 31.2 rpm

45. Consider a uniform rod of mass $M = 4m$ and length l pivoted about its centre. A mass m moving with velocity v making angle $\theta = \frac{\pi}{4}$ to the rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod-mass system just after the collision is **[JEE Main 2020]**

(a) $\frac{3\sqrt{2}}{7}\frac{v}{l}$ (b) $\frac{4}{7}\frac{v}{l}$ (c) $\frac{3}{7}\frac{v}{l}$ (d) $\frac{3}{7\sqrt{2}}\frac{v}{l}$

Dynamics of Rotational Motion

46. The moment of inertia of a body about a given axis is 1.2 kg-m^2. Initially, the body is at rest. In order to produce a rotational kinetic energy of 1500 J and angular acceleration of 25 $rads^{-2}$ must be applied about that axis for a duration of

(a) 4 s (b) 2 s (c) 8 s (d) 10 s

47. A ring and a disc of different masses are rotating with the same kinetic energy. If we apply a retarding torque τ on the ring stops after making n revolutions, then in how many revolutions will the disc stop under the same retarding torque?

(a) n (b) $2n$ (c) $4n$ (d) $n/2$

48. A thin metal disc of radius 0.25 m and mass 2 kg starts from rest and rolls down an inclined plane. If its rotational kinetic energy is 4 J at the foot of the inclined plane, then its linear velocity at the same point is

(a) 1.2 ms^{-1} (b) $2\sqrt{2}$ ms^{-1}

(c) 20 ms^{-1} (d) 2 ms^{-1}

49. The motor of an engine is rotating about its axis with an angular velocity of 100 rev min^{-1}. It come to rest in 15s, after being switched off. Assuming constant angular deceleration. What are the number of revolutions made by it before coming to rest?

(a) 15.6 (b) 32.6 (c) 12.5 (d) 40

50. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of **[AIEEE 2009]**

(a) $\frac{1}{3}\frac{l^2\omega^2}{g}$ (b) $\frac{1}{6}\frac{l\omega}{g}$ (c) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (d) $\frac{1}{6}\frac{l^2\omega^2}{g}$

51. The oxygen molecule has a mass of 5.30×10^{-26} kg and a moment of inertia of 1.94×10^{-46} kg-m^{-2} about an axis through its centre perpendicular to the lines joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that is KE of rotation is $\frac{2}{3}$ of its KE translation.

Find the average angular velocity of the molecule.

(a) 3.75×10^{12} rad/s (b) 5.75×10^{12}rad/s
(c) 9.75×10^{12} rad/s (d) 6.75×10^{12} rad/s

52. A circular disc rolls down an inclined plane. The ratio of the rotational kinetic energy to total kinetic energy is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

53. Moment of inertia of a body about a given axis is 1.5 kg-m^2. Initially, the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular acceleration of 20 rad/s^2 must be applied about the axis for a duration of **[JEE Main 2019]**

(a) 5 s (b) 2 s (c) 3 s (d) 2.5 s

54. Two discs of moment of inertia I_1 and I_2 about their respective axes and rotating with angular speed ω_1 and ω_2 are brought into contact face-to-face with their axes of rotation coincident. Then the loss of kinetic energy of the system in the process is

(a) $\frac{I_1 I_2}{2(I_1+I_2)}(\omega_1-\omega_2)^2$ (b) $-\frac{I_1 I_2}{2(I_1+I_2)}(\omega_1-\omega_2)^2$

(c) $\frac{I_1 I_2}{(I_1+I_2)}(\omega_1-\omega_2)^2$ (d) zero

55. A thin uniform rod AB of mass m and length L is hinged at one end A to the level floor. Initially, it stands vertically and is allowed to fall freely to the floor in the vertical plane. The angular velocity of the rod, when its end B strikes the floor is (g is acceleration due to gravity)

(a) $\left(\frac{mg}{L}\right)$ (b) $\left(\frac{mg}{3L}\right)^{1/2}$ (c) $\left(\frac{g}{L}\right)$ (d) $\left(\frac{3g}{L}\right)^{1/2}$

56. A particle performs uniform circular motion with an angular momentum L. If the frequency of a particle's motion is doubled and its kinetic energy is halved, the angular momentum becomes

(a) $2L$ (b) $4L$ (c) $L/2$ (d) $L/4$

57. A wheel is rotating freely with an angular speed ω on a shaft. The moment of inertia of the wheel is I and the moment of inertia of the shaft is negligible. Another wheel of moment of inertia $3I$ initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is **[JEE Main 2020]**

(a) $\frac{3}{4}$ (b) $\frac{5}{6}$ (c) 0 (d) $\frac{1}{4}$

58. Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centres. The moment of inertia and angular velocity of the first disc are 0.1 kg-m^2 and 10 rad s^{-1} respectively, while those for the second one are 0.2 kg-m^2 and 5 rad s^{-1}, respectively. At some instant, they get stuck together and start rotating as a single system about their common axis with some angular speed. The kinetic energy of the combined system is **[JEE Main 2020]**

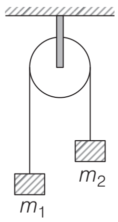
(a) $\frac{10}{3}$ J (b) $\frac{20}{3}$ J (c) $\frac{5}{3}$ J (d) $\frac{2}{3}$ J

59. Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$ are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$, about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is **[JEE Main 2019]**

(a) $-\frac{I_1\omega_1^2}{24}$ (b) $-\frac{I_1\omega_1^2}{12}$ (c) $\frac{3}{8}I_1\omega_1^2$ (d) $\frac{I_1\omega_1^2}{6}$

60. A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its centre of mass (see figure). A massless string is wrapped over its rim and two blocks of masses m_1 and $m_2 (m_1 > m_2)$ are attached to the ends of the string.

The system is released from rest. The angular speed of the wheel when m_1 descends by a distance h is **[JEE Main 2020]**



(a) $\left[\frac{2(m_1+m_2)gh}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}$ (b) $\left[\frac{m_1+m_2}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}} gh$

(c) $\left[\frac{m_1-m_2}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}} gh$ (d) $\left[\frac{2(m_1-m_2)gh}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}$

Rolling Motion

61. A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless), so that they slide down the plane. Then, maximum acceleration down the plane is for (no rolling) **[AIEEE 2002]**

(a) solid sphere (b) hollow sphere
(c) ring (d) All same

62. A sphere and a hollow cylinder roll without slipping down two separate inclined planes and travel the same distance in the same time. If the angle of the plane down which the sphere rolls is 30°, the angle of the other plane is

(a) 60° (b) 53°
(c) 37° (d) 45°

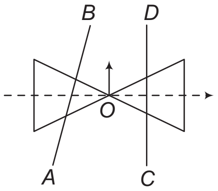
63. A sphere of mass 2 kg and radius 0.5 m is rolling with an initial speed of 1ms^{-1} goes up an inclined plane which makes an angle of 30° with the horizontal plane, without slipping. How long will the sphere take to return to the starting point A? **[JEE Main 2021]**

(a) 0.60 s (b) 0.52 s
(c) 0.57 s (d) 0.80 s

64. A solid sphere rolls down without slipping on an inclined plane at angle 60° over a distance of 10 m. The acceleration (in ms^{-2}) is

(a) 4 (b) 5
(c) 6.06 (d) 7

65. A roller is made by joining together two corners at their vertices O. It is kept on two rails AB and CD which are placed a symmetrically (see the figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see the figure). It is given a light path, so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to **[JEE Main 2016]**



(a) turn left
(b) turn right
(c) go straight
(d) turn left and right alternately

66. A solid cylinder on moving with constant speed v_0 reaches the bottom of an incline of 30°. A hollow cylinder of same mass and radius moving with the same constant speed v_0 reaches the bottom of a different incline of inclination θ. There is no slipping and both of them go through the same distance in the same time; θ is then equal to

(a) 37° (b) 30°
(c) 42° (d) 45°

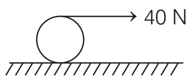
67. A hemispherical bowl or radius R is kept on a horizontal table. A small sphere of radius r ($r << R$) is placed at the highest point at the inside of the bowl and let go. The sphere rolls without slipping. Its velocity at the lowest point is

(a) $\sqrt{5\,gR/7}$ (b) $\sqrt{3\,gR/2}$
(c) $\sqrt{4\,gR/3}$ (d) $\sqrt{10\,gR/7}$

68. A marble and a cube have the same mass starting from rest, the marble rolls and the cube slides down a frictionless ramp. When they arrive at the bottom, the ratio of speed of the cube to the centre of mass and speed of the marble is

(a) 7 : 5 (b) $\sqrt{7}:\sqrt{5}$
(c) $\sqrt{2}:1$ (d) 5 : 2

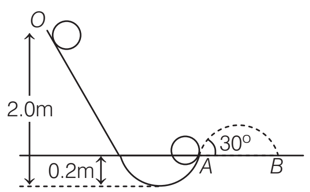
69. A string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string) **[JEE Main 2019]**



(a) 10 rad/s^2 (b) 16 rad/s^2
(c) 20 rad/s^2 (d) 12 rad/s^2

70. A tennis ball (treated as hollow spherical shell) starting from O rolls down a hill. At point A, the ball becomes air borne leaving at an angle of 30° with the horizontal. The ball strikes the ground at B, then what is the value of the distance AB?

(Moment of inertia of a spherical shell of mass m and radius R about its diameter $= \frac{2}{3}mR^2$) **[JEE Main 2013]**



(a) 1.87 m (b) 2.08 m
(c) 1.57 m (d) 1.77 m

ROUND II Mixed Bag

Only One Correct Option

1. If the moment of inertia of a disc about an axis tangential and parallel to its surface be I, then what will be the moment of inertia about the axis tangential but perpendicular to the surface?

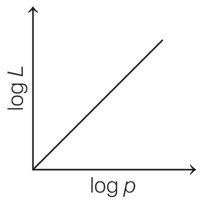
(a) $\frac{6}{5}I$ (b) $\frac{3}{4}I$ (c) $\frac{3}{2}I$ (d) $\frac{5}{4}I$

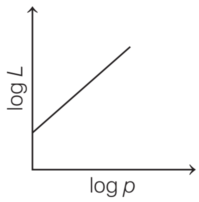
2. A ring of radius R is first rotated with an angular velocity ω_0 and then carefully placed on a rough horizontal surface. The coefficient of friction between the surface and the ring is μ. Time after which its angular speed is reduced to half is

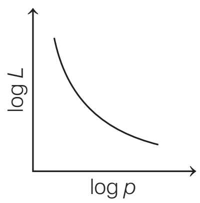
(a) $\frac{\omega_0 \mu R}{2g}$ (b) $\frac{2\omega_0 R}{\mu g}$

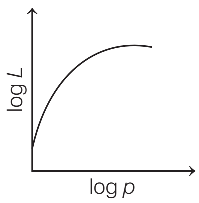
(c) $\frac{\omega_0 R}{2\mu g}$ (d) $\frac{\omega_0 g}{2\mu R}$

3. The curve between $\log_e L$ and $\log_e p$ is (L is angular momentum and p is linear momentum)

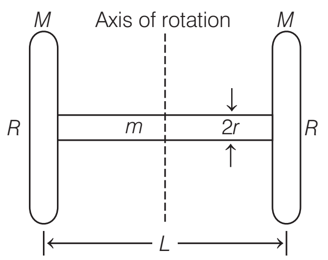
(a) 

(b) 

(c) 

(d) 

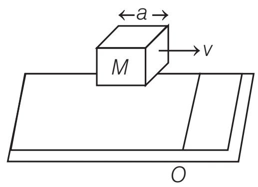
4. Two thin discs each of mass M and radius R are placed at either end of a rod of mass m, length l and radius r. Moment of inertia of the system about an axis passing through the centre of rod and perpendicular to its length is



(a) $\frac{mL^2}{12}+\frac{1}{4}MR^2+\frac{1}{4}ML^2$ (b) $\frac{ML^2}{12}+\frac{1}{2}mR^2+\frac{1}{2}mL^2$

(c) $\frac{1}{2}mL^2+\frac{mR^2}{2}+\frac{ML^2}{12}$ (d) $\frac{mL^2}{12}+MR^2+\frac{1}{2}ML^2$

5. A cubical block of side a is moving with velocity v on a horizontal smooth plane as shown. It hits a ridge at point O. The angular speed of the block after is hits O is



(a) $\frac{3v}{4a}$ (b) $\frac{3v}{2a}$

(c) $\frac{\sqrt{3}\,v}{\sqrt{2}\,a}$ (d) zero

6. Four particles each of mass m are lying symmetrically on the rim of a disc of mass M and radius R. Moment of inertia of this system about an axis passing through one of the particles and perpendicular to plane of disc is

(a) $16mR^2$ (b) $3(M+16m)\frac{R^2}{2}$

(c) $(3M+12m)\frac{R^2}{2}$ (d) zero

7. If the radius r of earth suddenly changes to x times the present values, the new period of rotation would be

(a) $dT/dt = (T/r)\,(dr/dt)$ (b) $dT/dt = (2T/r)\,(dr/dt)$

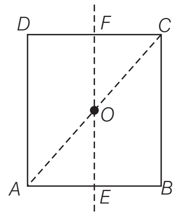
(c) $dT/dt = (r/T)\,(dr/dt)$ (d) $dT/dt = \left(\frac{1}{2}T/r\right)\left(\frac{dr}{dt}\right)$

8. The mass of the earth is increasing at the rate of 1 part in 5×10^{19} per day by the attraction of meteors falling normally on the earth's surface. Assuming that the density of earth is uniform, the rate of change of the period of rotation of the earth is

(a) 2.0×10^{-20} (b) 2.66×10^{-19}

(c) 4.33×10^{-18} (d) 5.66×10^{-17}

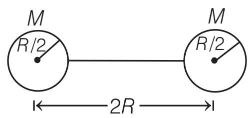
9. For the given uniform square lamina $ABCD$, whose centre is O as shown in figure. **[AIEEE 2007]**



(a) $I_{AC}=\sqrt{2}\,I_{EF}$ (b) $\sqrt{2}\,I_{AC}=I_{EF}$

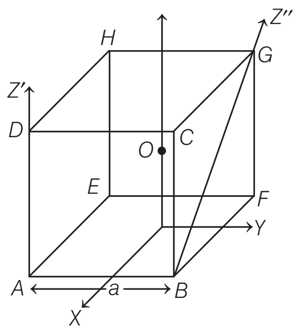
(c) $I_{AD}=3\,I_{EF}$ (d) $I_{AC}=I_{EF}$

10. Two spheres each of mass M and radius $R/2$ are connected with a massless rod of length $2R$ as shown in the figure. What will be the moment of inertia of the system about an axis passing through the centre of one of the sphere and perpendicular to the rod?



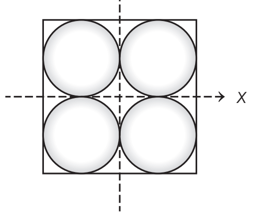
(a) $\frac{21}{5}MR^2$ (b) $\frac{2}{5}MR^2$
(c) $\frac{5}{2}MR^2$ (d) $\frac{5}{21}MR^2$

11. With referene to figure of a cube of edge a and mass m, state whether the following is true.
(O is the centre of the cube).



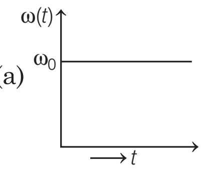
(a) The moment of inertia of cube about Z-axis is $I_Z = I_X + I_Y$
(b) The moment of inertia of cube about Z' is $I'_Z = I_Z + \frac{ma^2}{2}$
(c) The moment of inertia of cube about Z'' is $I''_Z = I_Z + \frac{mz^2}{2}$
(d) $I_X \neq I_Y$

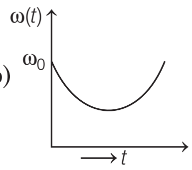
12. Four holes of radius R are cut from a thin square plate of side $4R$ and mass M. The moment of inertia of the remaining portion about z-axis is

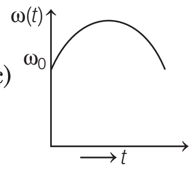


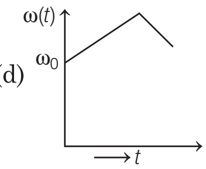
(a) $\frac{\pi}{12}MR^2$ (b) $\left(\frac{4}{3}-\frac{\pi}{4}\right)MR^2$
(c) $\left(\frac{4}{3}-\frac{\pi}{6}\right)MR^2$ (d) $\left(\frac{8}{3}-\frac{10\pi}{16}\right)MR^2$

13. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0. When the tortoise moves along a chord of the platform with a constant velocity (w.r.t. the platform), the angular velocity of the platform will vary with the time t as

(a) 

(b) 

(c) 

(d) 

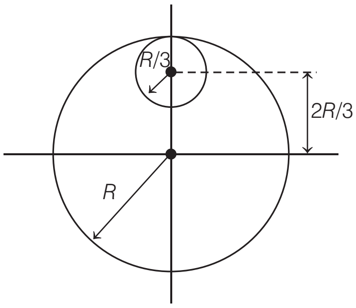
14. A pulley of radius 2m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in s) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg-m^2, the number of rotation made by the pulley before its direction of motion of reversed is **[AIEEE 2011]**
(a) less than 3
(b) more than 3 but less than 6
(c) more than 6 but less than 9
(d) more than 9

15. A solid cylinder is rolling down on an inclined plane of angle θ. The coefficient of static friction between the plane and cylinder is μ_s. The condition for the cylinder not to slip is
(a) $\tan\theta \geq 3\mu_s$ (b) $\tan\theta > 3\mu_s$
(c) $\tan\theta \leq 3\mu_s$ (d) $\tan\theta < 3\mu_s$

16. The moment of inertia of a rod about an axis through its centre and perpendicular to it is $\frac{1}{12}ML^2$ (where, M is the mass and L the length of the rod). The rod is bent in the middle to that the two halves make an angle of 60°. The moment of inertia of the bent rod about the same axis would be
(a) $\frac{1}{48}ML^2$ (b) $\frac{1}{12}ML^2$ (c) $\frac{1}{24}ML^2$ (d) $\frac{ML^2}{8\sqrt{3}}$

17. Four point masses, each of value m, are placed at the corners of a square $ABCD$ of side l. The moment of inertia of the system about an axis passing through A and parallel to BD is **[AIEEE 2006]**
(a) $\sqrt{3}\,ml^2$ (b) $3\,ml^2$
(c) ml^2 (d) $2\,ml^2$

18. From a circular disc of radius R and mass $9\,M$, a small disc or radius $R/3$ is removed. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is **[IIT JEE 2005]**

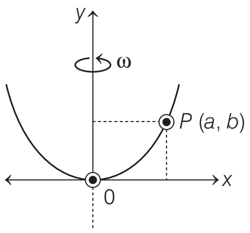


(a) $4\,MR^2$ (b) $\frac{40}{9}MR^2$ (c) $10\,MR^2$ (d) $\frac{37}{9}MR^2$

19. As shown in the figure, a bob of mass m is tied by a massless string whose other end portion is wound on a flywheel (disc) of radius r and mass m. When released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be **[JEE Main 2020]**

(a) $\frac{1}{r}\sqrt{\frac{2gh}{3}}$ (b) $r\sqrt{\frac{3}{4gh}}$ (c) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$ (d) $r\sqrt{\frac{3}{2gh}}$

20. A bead of mass m stays at point $P(a, b)$ on a wire bent in the shape of a parabola $y = 4Cx^2$ and rotating with angular speed ω (see figure). The value of ω is (neglect friction) **[JEE Main 2020]**



(a) $2\sqrt{2gC}$ (b) $\sqrt{\frac{2g}{C}}$

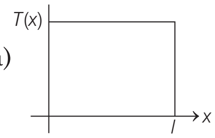
(c) $2\sqrt{gC}$ (d) $\sqrt{\frac{2gC}{ab}}$

21. From a solid sphere of mass M and radius R, a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is **[JEE Main 2015]**

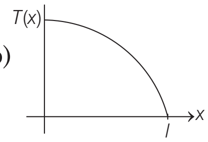
(a) $\frac{MR^2}{32\sqrt{2}\pi}$ (b) $\frac{MR^2}{16\sqrt{2}\pi}$ (c) $\frac{4MR^2}{9\sqrt{3}\pi}$ (d) $\frac{4MR^2}{3\sqrt{3}\pi}$

22. A uniform rod of length l is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is $T(x)$ at a distance x from the axis, then which of the following graphs depicts it most closely? **[JEE Main 2019]**

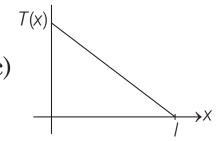
(a)



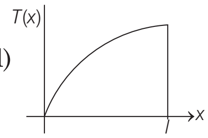
(b)



(c)



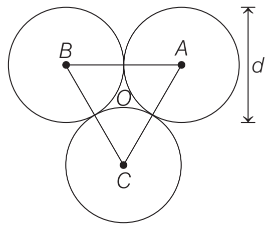
(d)



23. A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is k. If radius of the ball be R, then the fraction of total energy associated with its rotational energy will be

(a) $\frac{k^2}{k^2 + R^2}$ (b) $\frac{R^2}{k^2 + R^2}$ (c) $\frac{k^2 + R^2}{R^2}$ (d) $\frac{k^2}{R^2}$

24.



Three solid sphere each of mass m and diameter d are stuck together such that the lines connecting the centres form an equilateral triangle of side of length d. The ratio I_0 / I_A of moment of inertia I_0 of the system about an axis passing the centroid and about centre of any of the spheres I_A and perpendicular to the plane of the triangle is **[JEE Main 2020]**

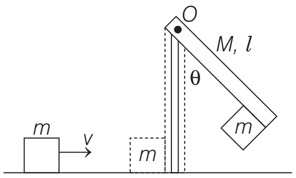
(a) $\frac{15}{13}$ (b) $\frac{13}{15}$ (c) $\frac{13}{23}$ (d) $\frac{23}{13}$

25. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius $2R$. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis. The ratio I_1 / I_2 is given by **[JEE Main 2019]**

(a) 285 (b) 185
(c) 65 (d) 140

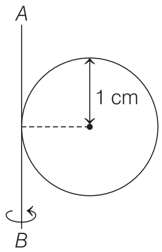
26. A block of mass $m = 1$ kg slides with velocity $v =$ 6 m/s on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision, making angle θ before

momentarily coming to rest. If the rod has mass $M = 2$ kg and length $l = 1$ m, then the value of θ is approximately
(Take, $g = 10\,\text{m/s}^2$) **[JEE Main 2020]**



(a) 63° (b) 55°
(c) 69° (d) 49°

27. A metal coin of mass 5g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5s, is close to **[JEE Main 2019]**



(a) 4.0×10^{-6} N-m
(b) 2.0×10^{-5} N-m
(c) 1.6×10^{-5} N-m
(d) 7.9×10^{-6} N-m

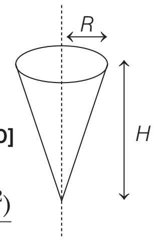
28. A person of mass M is sitting on a swing to length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance $l (l << L)$, is close to **[JEE Main 2019]**

(a) $Mgl\,(1 - \theta_0^2)$
(b) $Mgl\,(1 + \theta_0^2)$
(c) Mgl
(d) $Mgl\left(1 + \frac{\theta_0^2}{2}\right)$

29. A round uniform body of radius R, mass M and moment of inertia I, rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then, its acceleration is **[AIEEE 2007]**

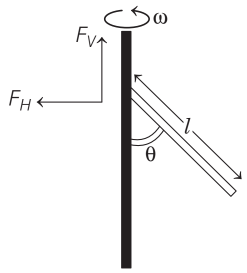
(a) $\frac{g \sin \theta}{1 + I/MR^2}$ (b) $\frac{g \sin \theta}{1 + MR^2/I}$
(c) $\frac{g \sin \theta}{1 - I/MR^2}$ (d) $\frac{g \sin \theta}{1 - MR^2/I}$

30. Shown in the figure is a hollow icecream cone (it is open at the top). If its mass is M, radius of its top R and height H, then its moment of inertia about its axis is **[JEE Main 2020]**



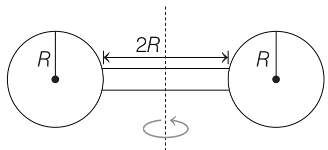
(a) $\frac{MR^2}{2}$ (b) $\frac{M(R^2 + H^2)}{4}$
(c) $\frac{MH^2}{3}$ (d) $\frac{MR^2}{3}$

31. A uniform rod of length l is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω, the rod makes an angle θ with it (see figure). To find θ, equate the rate of change of angular momentum (direction going into the paper) $\frac{ml^2}{12} \omega^2 \sin\theta \cos\theta$ about the centre of mass to the torque provided by the horizontal and vertical forces F_H and F_V about the centre of mass. The value of θ is then such that **[JEE Main 2020]**



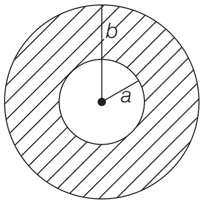
(a) $\cos\theta = \frac{g}{l\omega^2}$ (b) $\cos\theta = \frac{g}{2l\omega^2}$
(c) $\cos\theta = \frac{2g}{3l\omega^2}$ (d) $\cos\theta = \frac{3g}{2l\omega^2}$

32. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length $2R$ and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is **[JEE Main 2019]**



(a) $\frac{137}{15} MR^2$ (b) $\frac{209}{15} MR^2$ (c) $\frac{17}{15} MR^2$ (d) $\frac{152}{15} MR^2$

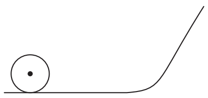
33. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is **[JEE Main 2019]**



(a) $\sqrt{\dfrac{a^2+b^2+ab}{2}}$ (b) $\dfrac{a+b}{2}$

(c) $\sqrt{\dfrac{a^2+b^2+ab}{3}}$ (d) $\dfrac{a+b}{3}$

34. A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\dfrac{h_{\text{sph}}}{h_{\text{cyl}}}$ is given by **[JEE Main 2019]**



(a) $\dfrac{2}{\sqrt{5}}$ (b) $\dfrac{14}{15}$ (c) 1 (d) $\dfrac{4}{5}$

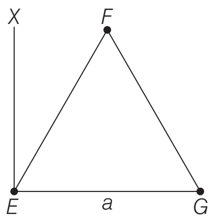
35. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane : (i) a ring of radius R, (ii) a solid cylinder of radius $R/2$ and (iii) a solid sphere of radius $R/4$. If in each case, the speed of the centre of mass at the bottom of the incline is same, the ratio of the maximum height they climb is

(a) $10:15:7$ (b) $4:3:2$

(c) $20:15:14$ (d) $2:3:4$

Numerical Value Questions

36. A massless equilateral triangle EFG of side a (as shown in figure) has three particles of mass m situated at its vertices. The moment of inertia of the system about the line EX perpendicular to EG in the plane of EFG is $\dfrac{N}{20}ma^2$, where N is an integer. The value of N is .... **[JEE Main 2020]**



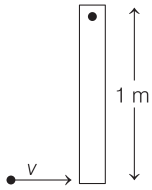
37. A force $\mathbf{F} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ N acts at a point $(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$ m. Then, the magnitude of torque about the point $(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}})$m will be $\sqrt{x}$ N-m. The value of x is .... **[JEE Main 2020]**

38. A person of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis at 5 rpm. The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform, when the person reaches its centre? **[JEE Main 2020]**

39. One end of a straight uniform 1 m long bar is pivoted on horizontal table. It is released from rest when it makes an angle 30° from the horizontal (see figure). Its angular speed when its hits the table is given as $\sqrt{n}\ \text{s}^{-1}$, where n is an integer. The value of n is .... **[JEE Main 2020]**

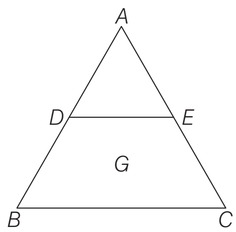
40. A thin rod of mass 0.9 kg and length 1 m is suspended at rest from one end, so that it can freely oscillate in the vertical plane.

A particle of mass 0.1 kg moving in a straight line with velocity 80 m/s hits the rod at its bottom-most point and sticks to it (see figure). The angular speed (in rad/s) of the rod immediately after the collision will be .... **[JEE Main 2020]**



41. A circular disc of mass M and radius R is rotating about its axis with angular speed ω_1. If another stationary disc having radius $\dfrac{R}{2}$ and same mass M is dropped co-axially on to the rotating disc. Gradually, both discs attain constant angular speed ω_2. The energy lost in the process is $p\%$ of the initial energy. Value of p is .... **[JEE Main 2020]**

42. ABC is a plane lamina of the shape of an equilateral triangle. D, E are mid-points of AB, AC and G is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through G and perpendicular to the plane ABC is I_0. If part ADE is removed, the moment of inertia of the remaining part about the same axis is $\dfrac{NI_0}{16}$, where N is an integer. Value of N is .... **[JEE Main 2020]**



Answers

Round I

1. (d)	**2.** (d)	**3.** (d)	**4.** (c)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (d)	**9.** (b)	**10.** (c)
11. (d)	**12.** (d)	**13.** (c)	**14.** (d)	**15.** (b)	**16.** (c)	**17.** (d)	**18.** (c)	**19.** (d)	**20.** (b)
21. (a)	**22.** (c)	**23.** (c)	**24.** (b)	**25.** (b)	**26.** (d)	**27.** (a)	**28.** (d)	**29.** (d)	**30.** (d)
31. (d)	**32.** (d)	**33.** (b)	**34.** (d)	**35.** (a)	**36.** (c)	**37.** (b)	**38.** (b)	**39.** (c)	**40.** (c)
41. (b)	**42.** (a)	**43.** (b)	**44.** (d)	**45.** (a)	**46.** (b)	**47.** (b)	**48.** (b)	**49.** (c)	**50.** (d)
51. (d)	**52.** (b)	**53.** (b)	**54.** (b)	**55.** (d)	**56.** (d)	**57.** (a)	**58.** (b)	**59.** (a)	**60.** (d)
61. (d)	**62.** (d)	**63.** (c)	**64.** (c)	**65.** (a)	**66.** (c)	**67.** (d)	**68.** (b)	**69.** (b)	**70.** (b)

Round II

1. (a)	**2.** (c)	**3.** (b)	**4.** (a)	**5.** (a)	**6.** (b)	**7.** (b)	**8.** (a)	**9.** (d)	**10.** (a)
11. (b)	**12.** (d)	**13.** (c)	**14.** (b)	**15.** (c)	**16.** (b)	**17.** (b)	**18.** (a)	**19.** (c)	**20.** (a)
21. (c)	**22.** (b)	**23.** (a)	**24.** (c)	**25.** (d)	**26.** (a)	**27.** (b)	**28.** (b)	**29.** (a)	**30.** (a)
31. (d)	**32.** (a)	**33.** (c)	**34.** (b)	**35.** (c)	**36.** 25	**37.** 195	**38.** 9	**39.** 15	**40.** 20
41. 20	**42.** 11								

Solutions

Round I

1. Apply parallel axes theorem,

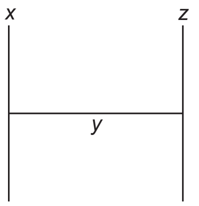
$$I = I_{CM} + Mh^2, \text{ we get}$$

$$\Rightarrow \quad = \frac{ML^2}{12} + M\left(\frac{L}{4}\right)^2 = \frac{7\,ML^2}{48}$$

2. Moment of inertia,

$$I = M\left(\frac{L^2}{12} + \frac{r^2}{4}\right) = M\left(\frac{L^2}{12} + \frac{D^2}{16}\right)$$

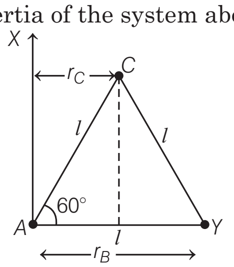
3. Moment of inertia of the system about rod x shown in the figure



$$I = I_x + I_y + I_z$$

$$= 0 + \left(\frac{Ml^2}{12} + \frac{Ml^2}{4}\right) + Ml^2 = \frac{4}{3}Ml^2$$

4. Moment of inertia of the system about axis AX



$$= I_A + I_B + I_C$$

$$= M_A(r_A)^2 + M_B(r_B)^2 + M_C(r_C)^2$$

$$= M(0)^2 + m(l)^2 + m(l\cos 60^\circ)^2$$

$$= ml^2 + \frac{ml^2}{4} = \frac{5\,ml^2}{4} \qquad \left(\because \cos 60^\circ = \frac{1}{2}\right)$$

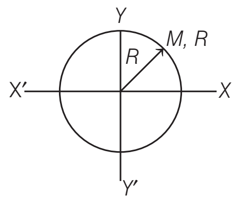
5. Here, $m_1 = m_2 = 0.1$ kg

$$r_1 = r_2 = 10 \text{ cm} = 0.1 \text{ m}$$

and $\quad I = I_1 + I_2 = m_1 r_1^2 + \frac{1}{2} m_2 r_2^2 = \frac{3}{2} m_1 r_1^2 \qquad (\because m_1 = m_2)$

$$= \frac{3}{2} \times 0.1\,(0.1)^2 = 1.5 \times 10^{-3} \text{ kg-m}^2$$

6. Moment of inertia of circular wire about its axis is MR^2. Consider two diameters XX' and YY'. Moment of inertia about any of these diameters is same, let us say I.



From perpendicular axes theorem,

$$I + I = MR^2 \text{ or } I = \frac{MR^2}{2}$$

7. According to the theorem of perpendicular axes, $I_Z = I_X + I_Y$.

With the hole, I_X and I_Y both decrease gluing the removed piece at the centre of square plate does not affect I_Z. Hence, I_Z decreases, overall.

8. As, $m_1 = m_2$

$$\Rightarrow \qquad \pi R_1^2 x d_1 = \pi R_2^2 x d_2$$

$$\frac{R_1^2}{R_2^2} = \frac{d_2}{d_1}$$

Now, $$\frac{I_1}{I_2} = \frac{\frac{1}{2}mR_1^2}{\frac{1}{2}mR_2^2} = \frac{R_1^2}{R_2^2} = \frac{d_2}{d_1}$$

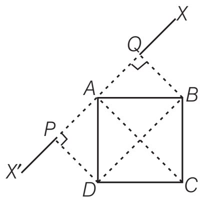
9. Moment of inertia of a uniform rod about one end

$$= \frac{mL^2}{3}$$

$\therefore$ Moment of inertia of the system, which rod is bent

$$= 2 \times \left(\frac{M}{2}\right)\frac{(L/2)^2}{3} = \frac{ML^2}{12}$$

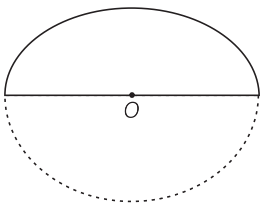
10. The situation is shown in figure



$$I_{XX'} = m \times DP^2 + m \times BQ^2 + m \times CA^2$$

$$= m \times 2 \times \left(\frac{\sqrt{2}\, l}{2}\right)^2 + m \times (\sqrt{2}\, l)^2 = 3ml^2$$

11. The mass of complete (circular) disc is



The moment of inertia of disc about the given axis is

$$I = \frac{2\,Mr^2}{2} = Mr^2$$

Let the moment of inertia of semi-circular disc be I_1.

The disc may be assumed as combination of two semi-circular parts.

Thus, $\quad I_1 = I - I_1 \quad$ or $\quad I_1 = \frac{I}{2} = \frac{Mr^2}{2}$

12. Here, as ring is made by bending the rod $l = 2\pi R$.

$$R = \frac{l}{2\pi}$$

$$I_1 = \frac{ml^2}{12},$$

$$I_2 = mR^2 = \frac{ml^2}{4\pi^2}$$

$$\frac{I_1}{I_2} = \frac{ml^2}{12} / \frac{ml^2}{4\pi^2} = \frac{\pi^2}{3}$$

13. Let same mass and same outer radii of solid sphere and hollow sphere be M and R, respectively.

The moment of inertia of solid sphere A about its diameter,

$$I_A = \frac{2}{5}MR^2 \qquad \text{...(i)}$$

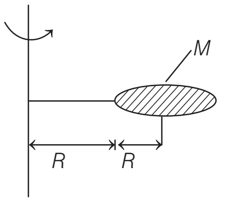
Similarly, the moment of inertia of hollow sphere (spherical shell) B about its diameter,

$$I_B = \frac{2}{3}MR^2 \qquad \text{...(ii)}$$

It is clear from Eqs. (i) and (ii), $I_A < I_B$.

14. First we found moment of inertia (MI) of system using parallel axes theorem about centre of mass, then we use it to find moment of inertia about given axis.

Moment of inertia of an outer disc about the axis through centre



$$= \frac{MR^2}{2} + M(2R)^2$$

$$= MR^2\left(\frac{1}{2} + 4\right) = \frac{9}{2}MR^2$$

For 6 such discs,

Moment of inertia $= 6 \times \frac{9}{2}MR^2 = 27MR^2$

So, moment of inertia of system

$$= \frac{MR^2}{2} + 27MR^2 = \frac{55}{2}MR^2$$

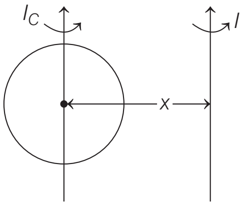
Hence, $$I_P = \frac{55}{2}MR^2 + (7M \times 9R^2)$$

$$\Rightarrow \qquad I_P = \frac{181}{2}MR^2$$

$$I_{\text{system}} = \frac{181}{2}MR^2$$

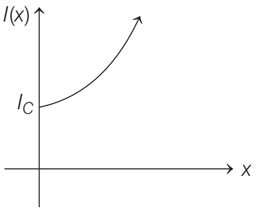
15. Moment of inertia of a solid sphere about an axis through its centre of mass is $I_C = \frac{2}{5}MR^2$

Moment of inertia about a parallel axis at a distance x from axis through its COM is



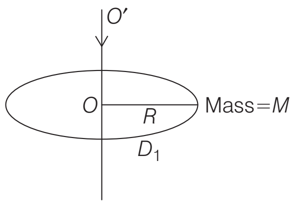
$$I = I_C + Mx^2 \qquad \text{(by parallel axes theorem)}$$

So graph of I *versus* x is parabolic as shown



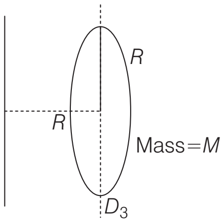
16. For disc D_1, moment of inertia across axis OO' will be

$$I_1 = \frac{1}{2} MR^2 \quad \ldots\text{(i)}$$



For discs D_2 and D_3, OO' is an axis parallel to the diameter of disc. Using parallel axes theorem,

$$I_2 = I_3 = I_{\text{diameter}} + Md^2 \quad \ldots\text{(ii)}$$



Here, $I_{\text{diameter}} = \frac{1}{4} MR^2$ and $d = R$

$$\therefore \quad I_2 = I_3 = \frac{1}{4} MR^2 + MR^2 = \frac{5}{4} MR^2$$

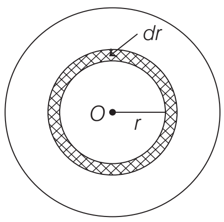
Now, total moment of inertia of the system,

$$I = I_1 + I_2 + I_3$$
$$= \frac{1}{2} MR^2 + 2 \times \frac{5}{4} MR^2$$
$$= 3MR^2$$

17. Given, surface mass density,

$$\sigma = kr^2$$

So, mass of the disc can be calculated by considering small element of area $2\pi rdr$ on it and then integrating it for complete disc, i.e.



$$dm = \sigma \, dA = \sigma \times 2\pi rdr$$

$$\int dm = M = \int_0^R (kr^2) \, 2\pi rdr$$

$$\Rightarrow \quad M = 2\pi k \frac{R^4}{4} = \frac{1}{2} \pi k R^4$$

$$\Rightarrow \quad k = \frac{2M}{\pi R^4} \quad \ldots\text{(i)}$$

Moment of inertia about the axis of the disc,

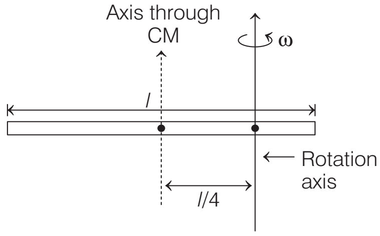
$$I = \int dI = \int dmr^2 = \int \sigma dAr^2$$
$$= \int_0^R kr^2 \, (2\pi rdr) \, r^2$$

$$\Rightarrow \quad I = 2\pi k \int_0^R r^5 \, dr = \frac{2\pi k R^6}{6} = \frac{\pi k R^6}{3} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$I = \frac{2}{3} MR^2$$

18. To calculate radius of gyration, first we calculate moment of inertia of rod about given axis.



Moment of inertia of rod about an axis through centre of mass, $I_c = \frac{ml^2}{12}$

where, m = mass and l = length of rod.

Moment of inertia of rod about given axis of rotation by parallel axes theorem will be

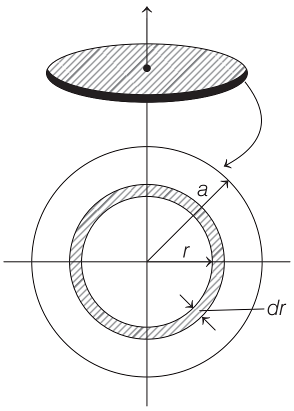
$$I = I_c + mh^2 = m\frac{l^2}{12} + m\left(\frac{l}{4}\right)^2 = \frac{7}{48} ml^2$$

If k = radius of gyration of rod, then moment of inertia of rod, $\quad I = mk^2$

$$\Rightarrow \quad mk^2 = \frac{7}{48} ml^2$$

$\therefore$ Radius of gyration of rod, $k = \sqrt{\frac{7}{48}} l$

19. Consider an elemental ring of thickness d at a distance from the centre.



Area of this elemental ring, $dA = 2\pi r dr$

Given, mass per unit area, $\sigma(r) = A + Br$

Mass of elemental ring,

$$dm = \sigma(r) \cdot dA = (A + Br) 2\pi r \, dr$$

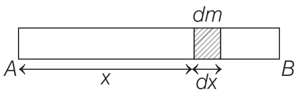
Moment of inertia of this elemental ring about axis,

$$dI = dm \cdot r^2$$
$$= 2\pi r^3 \, (A + Br) \cdot dr$$

Moment of inertia of complete disc of radius a,

$$I = \int_0^a dI = \int_0^a 2\pi r^3 (A + Br) \, dr = \left[2\pi A \frac{r^4}{4} + 2\pi B \frac{r^5}{5} \right]_0^a$$
$$= \frac{2\pi A a^4}{4} + \frac{2\pi B a^5}{5} = 2\pi a^4 \left(\frac{A}{4} + \frac{aB}{5} \right)$$

20. According to question,



Linear mass density, $\lambda(x) = \lambda_0 \left(1 + \frac{x}{L} \right)$

Moment of inertia of small element about given axis,

$$dI = (dm)x^2 = (\lambda dx)x^2$$
$$\Rightarrow \quad dI = \lambda_0 \left(1 + \frac{x}{L} \right) x^2 dx$$

Total moment of inertia about same axis due to the complete rod,

$$I = \int dI = \int_0^L \lambda_0 \left(1 + \frac{x}{L} \right) x^2 dx = \lambda_0 \left[\frac{x^3}{3} + \frac{x^4}{4L} \right]_0^L$$
$$= \frac{7L^3 \lambda_0}{12} \qquad \text{...(i)}$$

Now, mass of complete rod,

$$M = \int dm = \int \lambda dx$$
$$= \int_0^L \lambda_0 \left(1 + \frac{x}{L} \right) dx$$
$$= \lambda_0 \left[x + \frac{x^2}{2L} \right]_0^L$$
$$= \lambda_0 \left[L + \frac{L^2}{2L} - 0 \right] = \frac{3\lambda_0 L}{2}$$
$$\Rightarrow \quad \lambda_0 = \frac{2M}{3L}$$

Putting the value of λ_0 in Eq. (i), we get

$$I = \frac{7}{18} ML^2$$

21. Both the discs are made up of same material.

Let the density of material used for both discs be ρ and their thickness be t.

Now, moment of inertia of first disc,

$$I_1 = \frac{M_1 R_1^2}{2} = \frac{\rho A_1 t R_1^2}{2}$$
$$= \frac{\rho(\pi R_1^2) t R_1^2}{2} = \frac{\pi \rho R_1^4 t}{2}$$
$$\because \quad R_1 = R \qquad \text{(given)}$$
$$\therefore \quad I_1 = \frac{\pi \rho R^4 t}{2} \qquad \text{...(i)}$$

and moment of inertia of second disc,

$$I_2 = \frac{M_2 R_2^2}{2} = \frac{\rho A_2 t R_2^2}{2}$$
$$= \frac{\rho(\pi R_2^2) t R_2^2}{2} = \frac{\pi \rho R_2^4 t}{2}$$
$$\because \quad R_2 = \alpha R \qquad \text{(given)}$$
$$\therefore \quad I_2 = \frac{\pi \rho (\alpha R)^4 t}{2} = \frac{\alpha^4 \pi \rho R^4 t}{2} \qquad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{I_1}{I_2} = \frac{\left(\frac{\pi \rho R^4 t}{2} \right)}{\left(\frac{\alpha^4 \pi \rho R^4 t}{2} \right)}$$
$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{1}{\alpha^4} \qquad \text{...(iii)}$$

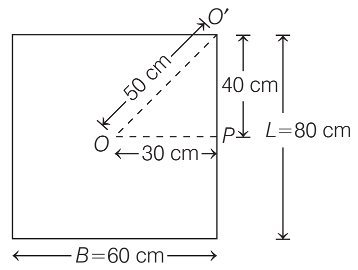
It is given that,

$$\frac{I_1}{I_2} = \frac{1}{16} \qquad \text{...(iv)}$$

On comparing the Eqs. (iii) and (iv), we get

$$\frac{1}{\alpha^4} = \frac{1}{16}$$
$$\Rightarrow \quad \alpha = (16)^{1/4}$$
$$= (2^4)^{1/4} = 2$$

22.



Let the mass of this rectangular sheet be M.

From figure,

$$O'P = \frac{L}{2} = \frac{80}{2} \text{ cm} = 40 \text{ cm}$$
$$OP = \frac{B}{2} = \frac{60}{2} \text{ cm} = 30 \text{ cm}$$
$$OO' = \sqrt{(OP)^2 + (O'P)^2}$$

(by Pythagoras' theorem)

$$= \sqrt{(30)^2 + (40)^2}$$
$$= \sqrt{900 + 1600}$$
$$= \sqrt{2500} = 50 \text{ cm}$$

Now, moment of inertia about the axis perpendicular to the plane of sheet and passing through point O is

$$I_O = \frac{M}{12}[L^2 + B^2] = \frac{M}{12}[(80)^2 + (60)^2]$$
$$= \frac{M}{12}[6400 + 3600]$$
$$= \frac{M}{12}[10000] \quad \text{...(i)}$$

and moment of inertia about the axis perpendicular to the plane of sheet and passing through point O' is

$I_{O'} = I_O + Md^2$ (using theorem of parallel axes)

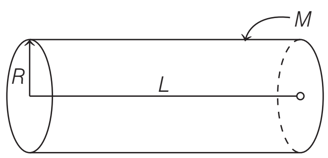
Here, $d = OO' = 50$ cm

So, $$I_{O'} = \frac{M}{12}[10000] + M[50]^2$$
$$= \frac{M}{12}[10000] + M[2500]$$
$$= \frac{M[10000] + M[30000]}{12}$$
$$= \frac{M}{12}[40000] \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{I_O}{I_{O'}} = \frac{\frac{M}{12}[10000]}{\frac{M}{12}[40000]} = \frac{1}{4}$$

23.



Mass of cylinder, $M = \rho\pi R^2 L$ ...(i)

(where, ρ = density of material)

$$\Rightarrow \quad L = \frac{M}{\rho\pi R^2} \quad \text{...(ii)}$$

Moment of inertia of a cylinder,

$$I = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right)$$
$$\Rightarrow \quad I = M\left(\frac{R^2}{4} + \frac{M^2}{12\rho^2\pi^2 R^4}\right) \quad \text{[from Eq. (ii)]}$$

For moment of inertia to be minimum,

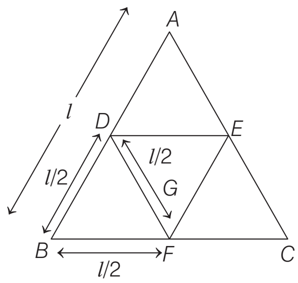
$$\frac{dI}{dR} = 0$$
$$\Rightarrow \quad \frac{d}{dR}\left[M\left(\frac{R^2}{4} + \frac{M^2}{12\rho^2\pi^2R^4}\right)\right] = 0$$
$$\Rightarrow \quad \frac{2R}{4} + \frac{M^2}{12\rho^2\pi^2}\cdot(-4R^{-5}) = 0$$

[$\because M$ = constant]

$$\Rightarrow \quad \frac{2R}{4} = \frac{4M^2}{12\rho^2\pi^2R^5}$$
$$\Rightarrow \quad R^6 = \frac{2}{3}\frac{M^2}{\rho^2\pi^2}$$
$$\Rightarrow \quad R^6 = \frac{2}{3}\cdot\frac{\rho^2\pi^2R^4L^2}{\rho^2\pi^2} \quad \text{[from Eq. (i)]}$$
$$\Rightarrow \quad \left(\frac{R}{L}\right)^2 = \frac{2}{3}$$
$$\Rightarrow \quad \frac{L}{R} = \sqrt{\frac{3}{2}}$$

24. Suppose the mass of the ΔABC be M and length of the side be l.

When the ΔDEF is being removed from it, then the mass of the removed Δ will be $M/4$ and length of its side will be $l/2$ as shown below



Since we know, moment of inertia of the triangle about the axis, passing through its centre of gravity is $I = kml^2$, where k is a constant.

Then for ΔDEF, moment of inertia of the triangle about the axis,

$$I = k\left(\frac{M}{4}\right)\left(\frac{l}{2}\right)^2 = \frac{kMl^2}{16} \quad \text{...(i)}$$

Moment of inertia of ΔABC is

$$I_0 = kMl^2 \quad \text{...(ii)}$$

The moment of inertia of the remaining part will be

$$I' = I_0 - I = kMl^2 - \frac{kMl^2}{16}$$

[$\because$ using Eqs. (i) and (ii)]

$$= \frac{15kMl^2}{16}$$

or $$I' = \frac{15}{16}I_0$$

25. As, force $\times$ distance $= \tau = I\alpha$

$$\Rightarrow \quad F\sin 30° \times \frac{1}{2} = 4\left[2\times\left(\frac{1}{4}\right)^2\right]\alpha$$

or $$24 \times \frac{1}{2} \times \frac{1}{2} = \frac{\alpha}{2}$$

$\therefore$ $$\alpha = 12 \text{ rad s}^{-2}$$

26. As the block remains stationary, therefore for translatory equilibrium,

$$\Sigma f_x = 0 \quad \therefore \quad F = N$$

and $$\Sigma f_y = 0 \quad \therefore \quad F = mg$$

For rotational equilibrium, $\Sigma\tau = 0$.

By taking the torque of different force about point O,

$$\tau_f + \tau_f + \tau_N + \tau_{mg} = 0$$

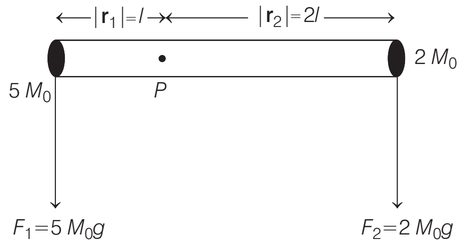
As f and mg passing through point O

$\therefore \qquad \tau_f + \tau_N = 0$

As $\tau_f \neq 0$

$\therefore \tau_N \neq 0$ and torque by friction and normal reaction will be in opposite direction or N will not produce torque.

27. The given condition can be drawn in the figure below



Torque (τ) about $P = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2$ ... (i)

$\Rightarrow \qquad \tau = l \times 5M_0 g$ (outwards)$- 2l \times 2M_0 g$ (inwards)

$\Rightarrow \qquad \tau = 5M_0 gl - 4M_0 gl$ (outwards)

$\Rightarrow \qquad \tau = M_0 gl$ (outwards) ...(ii)

Now we know that, torque is also given by

$$\tau = I\alpha \qquad \text{...(iii)}$$

Here, I = moment of inertia (w.r.t. point P) of rod and α = angular acceleration.

For point P, $\quad I = (5M_0) \times l^2 + (2M_0)(2l)^2 \quad [\because I = MR^2]$

$\Rightarrow \qquad I = 13M_0 l^2$ ...(iv)

Putting value of I from Eq. (iv) in Eq. (iii), we get

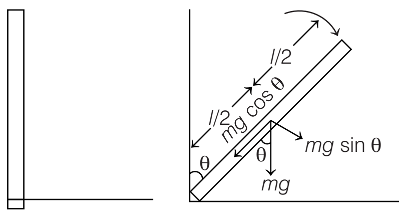
$$\tau = (13M_0 l^2)\,\alpha \qquad \text{...(v)}$$

From Eqs. (ii) and (v), we get

$$M_0 gl = 13M_0 l^2 \alpha$$

$\Rightarrow \qquad \alpha = \dfrac{g}{13l}$

28. As the rod rotates in vertical plane, so a torque is acting on it, which is due to the vertical component of weight of rod.



Initial condition $\qquad$ At any time t

Now, torque τ = force × perpendicular distance of line of action of force from axis of rotation

$$= mg \sin\theta \times \frac{l}{2}$$

Again, torque, $\tau = I\alpha$

where, I = moment of inertia $= \dfrac{ml^2}{3}$

[force and torque frequency along axis of rotation passing through in end]

α = angular acceleration

$\therefore \qquad mg \sin\theta \times \dfrac{l}{2} = \dfrac{ml^2}{3}\alpha$

$\therefore \qquad \alpha = \dfrac{3g\sin\theta}{2l}$

29. Given equations of trajectory are

$$x = x_0 + a\cos\omega_1 t$$

$$y = y_0 + b\sin\omega_2 t$$

Now, $v_x = dx/dt = -a\omega_1 \sin\omega_1 t$

and $\qquad v_y = \dfrac{dy}{dt} = b\omega_2 \cos\omega_2 t$

Similarly, $a_x = \dfrac{dv_x}{dt} = -a\omega_1^2 \cos(\omega_1 t)$

Similarly, $a_y = \dfrac{dv_y}{dt} = -b\omega_2^2 \sin(\omega_2 t)$

Now, at $t = 0$; $x = x_0 + a$; $y = y_0$ and
$a_x = -a\omega_1^2$; $a_y = 0$ ... (i)

Torque acting on the particle is given as

$$\tau = \mathbf{r} \times \mathbf{F} = m(\mathbf{r} \times \mathbf{a})$$

Here, $\mathbf{r} = (x_0 + a)\hat{\mathbf{i}} + y_0\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$

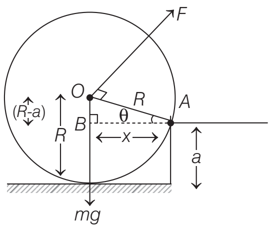
and $\qquad \mathbf{a} = -a\omega_1^2\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$

So, torque at $t = 0$,

$\tau = m(-a\omega_1^2) \times y_0(-\hat{\mathbf{k}})$ [using Eq. (i)]

$\Rightarrow \qquad \tau = +my_0 a\omega_1^2\hat{\mathbf{k}}$

30. Force F is applied to cylinder as shown



Force F must be large enough to create a torque about point A, so that it is able to counteract the torque due to weight about point A.

$\therefore$ Torque due to force about point A = Torque due to weight about point A

$\Rightarrow \qquad F \cdot R = Mgx$

From ΔOAB, $x = \sqrt{R^2 - (R-a)^2}$

$\Rightarrow \qquad F \cdot R = Mg\sqrt{R^2 - (R-a)^2}$

$\Rightarrow \qquad F = \dfrac{Mg}{R}\sqrt{R^2 - (R-a)^2}$

$\Rightarrow \qquad F = Mg\sqrt{1 - \left(\dfrac{R-a}{R}\right)^2}$

31. Here, $r = 0.2$ m, $M = 10$ kg,

$$n = 1200 \text{ rpm} = 20 \text{ rps}$$
$$L = I\omega = (Mr^2)(2\pi n)$$
$$= 10 \times (0.2)^2 \times 2 \times \frac{22}{7} \times 20$$
$$= 50.28 \text{ kg-m}^2\text{s}^{-1}$$

32. Angular momentum, $\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$

The direction of $\mathbf{L}$ with respect to A is along the Z-axis and magnitude is mvr. So, L_A is constant, both in magnitude and direction.

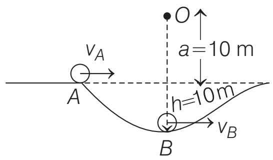
But the direction of $\mathbf{L}$ with respect to B is continuously changing with time with constant magnitude.

33. Angular momentum

= linear momentum × perpendicular distance of line of action of linear momentum from the axis of rotation

$= mv \times l$

34. The given figure is shown below as



As friction is absent, energy at A = energy at B

$$\Rightarrow \quad \frac{1}{2}mv_A^2 + mgh = \frac{1}{2}mv_B^2$$
$$\Rightarrow \quad v_A^2 + 2gh = v_B^2$$
$$\text{or} \quad v_B^2 = (5)^2 + 2 \times 10 \times 10 = 225$$
$$\Rightarrow \quad v_B = 15 \text{ ms}^{-1}$$

Angular momentum about point O,

$$= mv_B r_B$$
$$= 20 \times 10^{-3} \times 15 \times 20$$
$$= 6 \text{ kg-m}^2\text{ s}^{-1}$$

35. Torque at point P w.r.t. O,

$$\tau_O = \mathbf{r}_O \times \mathbf{F} = (5\hat{\mathbf{i}} + 5\sqrt{3}\hat{\mathbf{j}}) \times (4\hat{\mathbf{i}} - 3\hat{\mathbf{j}})$$
$$= (-15 - 20\sqrt{3})\hat{\mathbf{k}}$$

Torque at point P w.r.t. Q,

$$\tau_Q = \mathbf{r}_Q \times \mathbf{F} = (-5\hat{\mathbf{i}} + 5\sqrt{3}\hat{\mathbf{j}}) \times (4\hat{\mathbf{i}} - 3\hat{\mathbf{j}})$$
$$= (15 - 20\sqrt{3})\,\hat{\mathbf{k}}$$

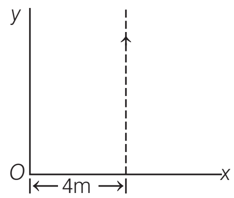
36. Conservation of angular momentum gives

$$\frac{1}{2}MR^2\omega_1 = \left(\frac{1}{2}MR^2 + 2mR^2\right)\omega_2$$
$$\Rightarrow \quad \frac{1}{2}MR^2\omega_1 = \frac{1}{2}R^2(M + 4m)\,\omega_2$$
$$\text{or} \quad \omega_2 = \left(\frac{M}{M + 4m}\right)\omega_1$$

37. By conservation of energy,

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$
$$= \frac{1}{2}I\omega^2 + \frac{1}{2}mr^2\omega^2 = \frac{\omega^2}{2}[I + mr^2]$$
$$\omega = \left(\frac{2mgh}{I + mr^2}\right)^{1/2}$$

38. Here, $m = 3$ kg, $v = 2$ ms^{-1}



$$r = 4 \text{ m}, L = ?, T = ?$$
$$L = mvr\sin 90° = 3 \times 2 \times 4 \times 1 = 24 \text{ kg m}^2\text{s}^{-1}$$
$$\text{and} \quad \tau = \frac{dL}{dt} = 0$$

39. As no torque is applied, angular momentum

$$L = I\omega = \text{constant} = \left(\frac{2}{5}MR^2\right)\left(\frac{2\pi}{T}\right) = \text{constant. } i.e.$$
$$\Rightarrow \quad \frac{R^2}{T} = \text{constant}$$
$$\Rightarrow \quad \frac{R_1^2}{T_1} = \frac{R_2^2}{T_2}$$
$$\therefore \quad T_2 = \left(\frac{R_2}{R_1}\right)^2 T_1 = \left(\frac{xR_1}{R_1}\right)^2 \times 24\,h = 24x^2h$$

40. According to conservation of angular momentum,

$$I_1\omega_1 = I_2\omega_2$$
$$\Rightarrow \quad \frac{1}{2}MR^2\omega = \left(\frac{1}{2}MR^2 + \frac{1}{2}\left\{\frac{M}{4}\right\}R^2\right)\omega_2$$
$$\therefore \quad \omega_2 = \frac{4}{5}\omega$$

41. Here, $n_1 = 2.8$ rps, $n_2 = ?$

$$l_2 = 0.7\, I_1$$
$$\text{As,} \quad \frac{\omega_2}{\omega_1} = \frac{I_1}{I_2} = \frac{1}{0.7}$$
$$\therefore \quad \frac{n_2}{n_1} = \frac{10}{7} \qquad (\because \omega = 2\pi n)$$
$$n_2 = \frac{10}{7}n_1 = \frac{10}{7} \times 2.8 = 4.0 \text{ rps}$$

42. When the person jumps off the round, radially away from the centre, no torque is exerted, *i.e.* $\tau = 0$. According to the principle of conservation of angular momentum, $I \times \omega$ = constant. As mass reduces to half (from $2M$ to M), moment of inertia I becomes half. Therefore, ω must become twices ($= 2\omega$).

43. As $L = I\omega$ = constant

$$\therefore \quad \frac{2}{5}MR^2 \times \left(\frac{2\pi}{7}\right) = \text{constant}$$
$$i.e. \quad \frac{R^2}{T} = \text{constant}$$

where R is halved, R^2 becomes (1/4)th. Therefore, T becomes 1/4th, *i.e.* 6 h.

44. Here, mass of man, $m = 80$ kg

Mass of platform, $M = 200$ kg

Let R be the radius of platform.

When man is standing on the rim,

$$I_1 = M(R/2)^2 + mR^2$$
$$= \left(\frac{R}{2}\right)^2 (M + 4m)$$

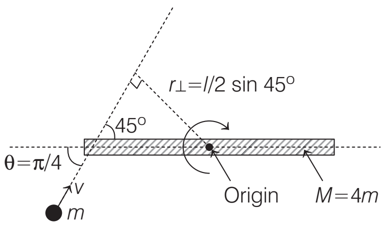
When man reaches the centre of platform,

$$I_2 = M(R/2)^2 + m \times 0 = M(R/2)^2$$

As angular momentum is conserved,

$$\frac{I_1}{I_2} = \frac{\omega_2}{\omega_1} = \frac{2\pi n_2}{2\pi n_1} = \frac{n_2}{n_1}$$
$$n_2 = \frac{I_1}{I_2} \times n_2$$
$$= \frac{(M+4m)(R/2)^2}{M(R/2)^2} \times 12$$
$$= \frac{(200 + 4 \times 80)}{200} \times 12$$
$$= \frac{520 \times 12}{200}$$
$$n_1 = 31.2 \text{ rpm}$$

45. Collision of particle and rod occurs as shown in the figure



Angular momentum of particle about origin (centre of rod) before collision,

$$L_i = m \cdot v \cdot r_\perp$$

where, $r_\perp$ = perpendicular distance of direction of velocity from centre of rod.

From geometry of figure,

$$r_\perp = \frac{l}{2} \sin 45° = \frac{l}{2\sqrt{2}}$$

So, initial angular momentum of system,

$$L_i = m \cdot v \cdot \frac{l}{2\sqrt{2}}$$

Now, when mass m stuck to one of the ends of rod, let system rotates about origin with initial angular speed ω, then final angular momentum of system just after collision,

$$L_f = I_{\text{system}} \cdot \omega$$
$$= (I_{\text{rod}} + I_{\text{particle}})\,\omega$$
$$= \left(\frac{4m \cdot l^2}{12} + m\left(\frac{l}{2}\right)^2\right)\omega = \frac{7}{12} ml^2\omega$$

By angular momentum conservation,

$$L_i = L_f$$
$$\Rightarrow \quad \frac{mvl}{2\sqrt{2}} = \frac{7}{12} ml^2\omega$$
$$\Rightarrow \quad \omega = \frac{3\sqrt{2}}{7}\left(\frac{v}{l}\right)$$

46. As, $I = 1.2$ kg-m^2, $E_r = 1500$ J, $\alpha = 25$ rad s^{-2} $\omega_1 = 0$, $t = ?$

Now, $\quad E_r = \frac{1}{2} I\omega^2$

$$\Rightarrow \quad \omega = \sqrt{\frac{2E_r}{I}}$$
$$= \sqrt{\frac{2 \times 1500}{1.2}} = 50 \text{ rad s}^{-1}$$

From, $\quad \omega_2 = \omega_1 + \alpha t$

$$50 = 0 + 25t$$
$$\Rightarrow \quad t = 2 \text{ s}$$

47. Work done in stopping = Change in KE = Final KE − Initial KE, *i.e.* $\tau\theta = K$ = constant. As τ is same in the two cases, θ must be same, *i.e.* number of revolution must be same.

Then, total number of revolutions made by disc when it stops = $n + n = 2n$.

48. Here, $r = 0.25$ m and $m = 2$ kg

Rotational KE $= \frac{1}{2} I\omega^2 = \frac{1}{2} \times \left(\frac{1}{2} mr^2\right)\omega^2$

$$\Rightarrow \quad 4 = \frac{1}{4} mv^2 = \frac{1}{4} \times 2v^2$$
$$\therefore \quad v = \sqrt{8} = 2\sqrt{2} \text{ ms}^{-1}$$

49. As $\quad \omega_f = \omega_i + \alpha t$

Here, $\quad 0 = \omega_0 - \alpha t$

$$\therefore \quad \alpha = \frac{\omega_0}{t} = \frac{(100 \times 2\pi)/60}{15} = 0.7 \text{ rad s}^{-2}$$

Angle rotated before coming to rest,

$$\theta = \frac{\omega_0^2}{2\alpha}$$

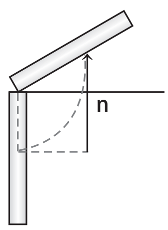
or

$$\theta = \frac{\left(\frac{100 \times 2\pi}{60}\right)^2}{2 \times 0.7} = 78.33 \text{ rad}$$

Hence, number of rotations, $n = \frac{\theta}{2\pi} = \frac{78.33}{2\pi} = 12.5$

50. By conservation of energy,

Initial total energy = Final total energy



$$(TE)_i = (TE)_f$$

$$\frac{1}{2} I\omega^2 = mgh$$

$$\frac{1}{2} \times \frac{1}{3} ml^2\omega^2 = mgh$$

$$\Rightarrow \quad h = \frac{1}{6} \frac{l^2\omega^2}{g}$$

51. Mass of oxygen molecule, $M = 5.30 \times 10^{-26}$ kg

Moment of inertia, $I = 1.94 \times 10^{-46}$ kg-m^2

Mean speed of the molecule, $v = 500$ m/s

Given, KE of rotation $= \frac{2}{3} \times$ KE of translation

$$\frac{1}{2} I\omega^2 = \frac{2}{3} \times \frac{1}{2} Mv^2$$

or $$\omega = \sqrt{\frac{2Mv^2}{I}}$$

$$= \sqrt{\frac{2 \times 5.30 \times 10^{-26} \times (500)^2}{1.94 \times 10^{-46}}}$$

$$= 1.35 \times 10^{10} \times 500$$

$$= 6.75 \times 10^{12} \text{ rad/s}$$

52. Rotational kinetic energy, $K_R = \frac{1}{2} I\omega^2$

$$K_R = \frac{1}{2} \times \frac{MR^2}{2} \times \omega^2 = \frac{1}{4} Mv^2 \qquad (\because v = R\omega)$$

Translational kinetic energy, $K_T = \frac{1}{2} Mv^2$

Total kinetic energy $= K_T + K_R = \frac{1}{2} Mv^2 + \frac{1}{4} Mv^2$

$$= \frac{3}{4} Mv^2$$

$$\therefore \frac{\text{Rotational kinetic energy}}{\text{Total kinetic energy}} = \frac{\frac{1}{4} Mv^2}{\frac{3}{4} Mv^2} = \frac{1}{3}$$

53. Rotational kinetic energy of a body is given by

$$KE_{\text{rotational}} = \frac{1}{2} I\omega^2$$

where, $\omega = \omega_0 + \alpha t$

So, $$KE_{\text{rotational}} = \frac{1}{2} I(\omega_0 + \alpha t)^2 \qquad \text{... (i)}$$

Here, $I = 1.5$ kgm^2,

KE $= 1200$ J

and $\alpha = 20$ rad/s^2 and $\omega_0 = 0$

Substituting these values in Eq.(i), we get

$$1200 = \frac{1}{2} (1.5) (20 \times t)^2$$

$$\Rightarrow \quad t^2 = \frac{2 \times 1200}{1.5 \times 400} = 4$$

$$\therefore \quad t = 2 \text{ s}$$

54. Here, $$K_f = \frac{1}{2} (I_1 + I_2) \left[\frac{(I_1\omega_1 + I_2\omega_2)^2}{(I_1 + I_2)^2}\right]$$

$$= \frac{1}{2} \frac{(I_1\omega_1 + I_2 \omega_2)^2}{I_1 + I_2}$$

and $$K_i = \frac{1}{2} (I_1\omega_1^2 + I_2 \omega_2^2)$$

$$\Rightarrow \quad \Delta K = K_f - K_i$$

$$= -\frac{I_1 I_2}{2 (I_1 + I_2)} (\omega_1 - \omega_2)^2$$

55. As, the rod is hinged at one end, its moment of inertia about this end is $I = \frac{ML^2}{3}$.

Total energy in upright position

= Total energy on striking the floor

$$0 + \frac{MgL}{2} = \frac{1}{2} I\omega^2 + 0 = \frac{1}{2} \frac{ML^2}{3} \omega^2$$

$$\Rightarrow \quad g = \frac{L\omega^2}{3}$$

or $$\omega = \sqrt{\frac{3g}{L}}$$

56. Given, angular momentum,

$$L = I\omega$$

$$\Rightarrow \quad L = I(2\pi n) \qquad [\because \omega = 2\pi n]$$

$$\Rightarrow \quad L = 2\pi n I \qquad \text{...(i)}$$

Rotational kinetic energy,

$$K = \frac{1}{2} I\omega^2$$

$$\Rightarrow \quad I = \frac{2K}{\omega^2} \qquad \text{...(ii)}$$

Now, from Eqs. (i) and (ii), we have

$$L = 2\pi n \times \frac{2K}{\omega^2}$$

$$= 2\pi n \times \frac{2K}{(2\pi n)^2}$$

$$L = \frac{K}{\pi n}$$

$$\Rightarrow \quad \frac{L_2}{L_1} = \frac{K_2}{K_1} \times \frac{n_1}{n_2}$$

$$= \frac{K_1/2}{K} \times \frac{n_1}{2n_1} \qquad \left[\because K_2 = \frac{K_1}{2} \text{ and } n_2 = 2n_1\right]$$

$$\Rightarrow \quad \frac{L_2}{L_1} = \frac{1}{4}$$

$$\Rightarrow \quad L_2 = \frac{L_1}{4} = \frac{L}{4} \qquad [\because L_1 = L]$$

57. Let ω_c be the final angular speed.

According to law of conservation of angular momentum,

Initial angular momentum = Final angular momentum

$$I\omega + 0 = I\omega_c + 3I\omega_c$$
$$I\omega = 4I\omega_c$$
$$\Rightarrow \quad \omega_c = \frac{\omega}{4}$$

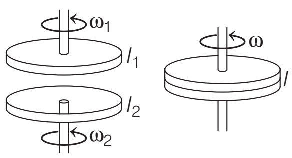
Loss of rotational kinetic energy,

$$(\Delta KE)_{loss} = \text{Initial KE} - \text{Final KE}$$
$$\Rightarrow \quad (\Delta KE)_{loss} = \frac{1}{2}I\omega^2 - \frac{1}{2}(I+3I)\omega_c^2$$
$$= \frac{1}{2}I\omega^2 - \frac{1}{2}(I+3I)\left(\frac{\omega}{4}\right)^2$$
$$= \frac{1}{2}I\omega^2 - \frac{1}{2}(4I)\left(\frac{\omega}{4}\right)^2$$
$$= \frac{1}{2}I\omega^2\left(1-\frac{1}{4}\right)$$
$$= \frac{3}{8}I\omega^2$$

The resultant fractional loss in KE of the system

$$= \frac{(\Delta KE)_{loss}}{(KE)_{initial}}$$
$$= \frac{(3/8)I\omega^2}{(1/2)I\omega^2} \qquad \left[\because (KE)_{initial} = \frac{1}{2}I\omega^2\right]$$
$$= \frac{3}{4}$$

58. The given situation is shown in figure



As there is no external torque, angular momentum of system is conserved.

i.e. $L_1 + L_2 = L_{system}$

$$\Rightarrow \quad I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$$
$$\Rightarrow \quad \omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$
$$\omega = \frac{0.1\times 10 + 0.2\times 5}{(0.1+0.2)} \quad \text{or } \omega = \frac{20}{3}\ \text{rad}/s$$

Kinetic energy of combined system is

$$K = \frac{1}{2}I_{system}\omega^2$$
$$= \frac{1}{2}(0.1+0.2)\times\left(\frac{20}{3}\right)^2 = \frac{20}{3}\ \text{J}$$

59. Initial kinetic energy of the given system,

$$(KE)_i = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}\left(\frac{I_1}{2}\right)\left(\frac{\omega_1}{2}\right)^2$$
$$= \left(\frac{1}{2}+\frac{1}{16}\right)I_1\omega_1^2 = \frac{9}{16}I_1\omega_1^2 \qquad \dots\text{(i)}$$

Now, using angular momentum conservation law (assuming angular velocity after contact is ω)

Initial angular momentum = Final angular momentum

$$I_1\omega_1 + \left(\frac{I_1}{2}\right)\left(\frac{\omega_1}{2}\right) = I_1\omega_2' + \frac{I_1}{2}\omega_2'$$
$$\Rightarrow \quad \frac{5}{4}\omega_1 = \frac{3}{2}\omega_2'$$
$$\text{or} \quad \omega_2' = \frac{5}{6}\omega_1 \qquad \dots\text{(ii)}$$

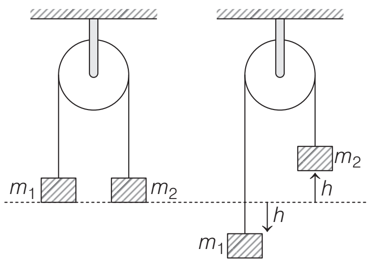
Now, final kinetic energy (after contact) is

$$(KE)_f = \frac{1}{2}I_1\omega_2'^2 + \frac{1}{2}\left(\frac{I_1}{2}\right)\omega_2'^2$$
$$= \frac{1}{2}I_1\left(\frac{5}{6}\omega_1\right)^2 + \frac{1}{4}I_1\left(\frac{5}{6}\omega_1\right)^2 \quad \text{[using Eq. (ii)]}$$
$$= \left(\frac{25}{72}+\frac{25}{144}\right)I_1\omega_1^2$$
$$= \frac{25}{48}I_1\omega_1^2 \qquad \dots\text{(iii)}$$

Hence, change in KE,

$$\Delta KE = (KE)_f - (KE)_i$$
$$= \frac{25}{48}I_1\omega_1^2 - \frac{9}{16}I_1\omega_1^2 \qquad \text{[using Eq. (i)]}$$
$$\Delta KE = -\frac{1}{24}I_1\omega_1^2$$

60. Loss of gravitational potential energy of system appears in the form of kinetic energy of system.



Loss of potential energy of masses

= Kinetic energy of masses + Kinetic energy of rotation of wheel

$$\Rightarrow \quad m_1gh - m_2gh = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2$$

Assuming no slip, we have

$$v = \omega R$$
$$\therefore \quad (m_1 - m_2)gh = \frac{1}{2}(m_1+m_2)(\omega^2R^2) + \frac{1}{2}I\omega^2$$
$$\Rightarrow \quad \omega = \sqrt{\frac{2(m_1-m_2)gh}{\left(m_1+m_2+\frac{I}{R^2}\right)R^2}}$$
$$\text{or} \quad \omega = \sqrt{\frac{2(m_1-m_2)gh}{(m_1+m_2)R^2+I}}$$
$$\text{or} \quad \omega = \left[\frac{2(m_1-m_2)gh}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}$$

61. Since, the inclined plane is frictionless, then there will be no rolling and the mass will only slide down.
Hence, acceleration is same for all the given bodies.

62. For rolling, $t = \sqrt{\dfrac{2l\,(1 + K^2/R^2)}{g \sin \theta}}$ = same

(given in question)

$$\therefore \quad \frac{2l\,(1+K_1^2/R^2)}{g \sin \theta_1} = \frac{2l\,(1+K_2^2/R^2)}{g \sin \theta_2}$$

$$\Rightarrow \quad \frac{1 + K_1^2/R^2}{\sin \theta_1} = \frac{1 + K_2^2/R^2}{\sin \theta_2} \quad \ldots\text{(i)}$$

For sphere, $K_1^2 = \dfrac{2}{5} R^2, \quad \theta_1 = 30°$

For hollow cylinder, $\dfrac{K_2^2}{R^2} = 1, \theta_2 = ?$

$\therefore$ From Eq. (i), we get

$$\Rightarrow \quad \frac{1 + \frac{2}{5}}{\sin 30°} = \frac{1+1}{\sin \theta_2}$$

$$\text{or} \quad \sin \theta_2 = \frac{5}{7} = 0.7143$$

$$\text{or} \quad \theta = 45.58° \simeq 45°$$

63. For rolling, $a = \dfrac{g \sin \theta}{\left(1 + \dfrac{I}{MR^2}\right)}$

$$= \frac{10 \times 1/2}{1 + 2/5}$$

$$= \frac{25}{7}\,\text{ms}^{-2} \qquad \left(\because \text{for sphere, } I = \frac{2}{5} MR^2\right)$$

$$\Rightarrow \quad t = \frac{2v}{a} = \frac{2 \times 1 \times 7}{25} = 0.56\text{ s}$$

$$\simeq 0.57\text{ s}$$

64. Here, $\theta = 60°$, $l = 10$ m, $a = ?$

For solid sphere, $K^2 = \dfrac{2}{5} R^2$

$$\therefore \quad a = \frac{9.8 \sin 60°}{1 + \frac{2}{5}} \qquad \left(\because a = \frac{g \sin \theta}{1 + \frac{K^2}{R^2}}\right)$$

$$\text{or} \quad a = \frac{5}{7} \times 9.8 \times \frac{\sqrt{3}}{2} = 6.06\text{ ms}^{-2}$$

65. As, the wheel rolls forward the radius of the wheel decreases along AB, hence for the same number of rotations, it moves less distance along AB. So, it turns left.

66. For solid cylinder, $\theta = 30°$, $\dfrac{K_1^2}{R_1^2} = \dfrac{1}{2}$

For hollow cylinder, $\theta = ?, \dfrac{K_2^2}{R_2^2} = 1$

For rolling on inclined plane,

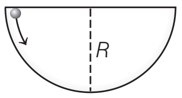
$$\frac{1 + \frac{K_1^2}{R_1^2}}{\sin \theta_1} = \frac{1 + \frac{K_2^2}{R_2^2}}{\sin \theta_2}$$

$$\text{Hence,} \quad \frac{\left(1 + \frac{1}{2}\right)}{\sin 30°} = \frac{1+1}{\sin \theta}$$

$$\therefore \quad \sin \theta = \frac{2}{3} = 0.6667$$

$$\theta = 41.8° \simeq 42°$$

67. As, it is clear from figure,



On reaching the bottom of the bowl, loss in PE = mgR, and gain in

$$\text{KE} = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

$$\Rightarrow \quad |\Delta K| = \frac{1}{2} mv^2 + \frac{1}{2} \times \left(\frac{2}{5} mr^2\right) \omega^2$$

$$= \frac{1}{2} mv^2 + \frac{1}{5} mv^2$$

$$= \frac{7}{10} mv^2$$

As, gain in KE = loss in PE

$$\therefore \quad \frac{7}{10} mv^2 = mgR$$

$$v = \sqrt{\frac{10\,gR}{7}}$$

68. If h is height of the ramp, then in rolling of marble, speed

$$v = \sqrt{\frac{2\,gh}{1 + K^2/R^2}}$$

The speed of the cube to the centre of mass,

$$v' = \sqrt{2\,gh}$$

$$\therefore \quad \frac{v'}{v} = \sqrt{1 + \frac{K^2}{R^2}}$$

For marble sphere, $K^2 = \dfrac{2}{5} R^2$

$$\therefore \quad \frac{v'}{v} = \sqrt{1 + \frac{2}{5}}$$

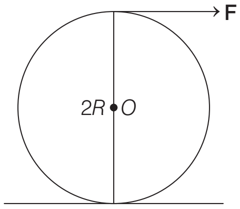
$$= \sqrt{\frac{7}{5}} = \sqrt{7} : \sqrt{5}$$

69. Given, $m = 5$ kg, $R = 0.5$ m.

Horizontal force, $F = 40$ N

As, cylinder is rolling without slipping.

Hence, torque is producing rotation about centre O.



So, $\tau = \mathbf{r} \times \mathbf{F}$ (Here, $r = R$)

$\theta = 90°$

So, $\tau = \mathbf{r} \times \mathbf{F} = rF$

or $\tau = 0.5 \times 40 = 20$ N-m ...(i)

If α is acceleration of centre of mass 'O' then torque is,

$$\tau = I\alpha$$

where, $I = MR^2$

$\therefore$ $\tau = MR^2\alpha$ ...(ii)

Comparing Eq. (i) with Eq. (ii),

$$MR^2\alpha = 20$$

$$\Rightarrow \quad \alpha = \frac{20}{5 \times (0.5)^2}$$

or $\alpha = 16 \text{ rad/s}^2$

70. Total mechanical energy at O

$$= mgh = 2\,mg$$

Now, total mechanical energy at A

$$= \frac{1}{2} mv^2 \left(1 + \frac{K^2}{R^2}\right) + 0.2\,mgh$$

Since, total mechanical energy is conserved

$$\Rightarrow \quad \frac{1}{2} mv^2 \left(1 + \frac{K^2}{R^2}\right) + 0.2\,mgh = 2\,mg$$

$$\Rightarrow \quad \frac{1}{2} v^2 \left(1 + \frac{\left(R\sqrt{\frac{2}{5}}\right)^2}{R^2}\right) + 0.2\text{ g} = 2\text{ g}$$

$$\left[\because K = \sqrt{\frac{2}{5}}\,R \text{ for spherical sphere}\right]$$

$$\Rightarrow \quad \frac{1}{2} v^2 \left[1 + \frac{2}{5}\right] + 2 = 20$$

$$\Rightarrow \quad v^2 \left[\frac{7}{5}\right] = 36$$

$$\Rightarrow \quad v = \sqrt{\frac{36 \times 5}{7}} = \sqrt{\frac{180}{7}} = 5.1 \text{ m/s}$$

Now, range of projectile

$$= \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{(5.1)^2 \sin 60°}{10} \approx 2.08 \text{ m}$$

Round II

1. MI of disc about tangent in a plane $= \frac{5}{4} MR^2 = I$

$$\therefore \quad MR^2 = \frac{4}{5} I$$

MI of disc about tangent I to plane, $I' = \frac{3}{2} MR^2$

$$\therefore \quad I' = \frac{3}{2}\left(\frac{4}{5} I\right) = \frac{6}{5} I$$

2. Angular retardation, $\alpha = \frac{\tau}{I} = \frac{\mu\, mgR}{mR^2} = \frac{\mu g}{R}$

As, $\omega = \omega_0 - \alpha t$

$$\therefore \quad t = \frac{\omega_0 - \omega}{\alpha} = \frac{\omega_0 - \omega_0/2}{\mu g / R} = \frac{\omega_0 R}{2\mu g}$$

3. As, $L = rP$

$$\Rightarrow \quad \log_e L = \log_e P + \log_e r$$

If graph is drawn between $\log_e L$ and $\log_e P$, then it will be straight line which will not pass through the origin because of presence of constant term ($\log_e r$) in the equation.

4. Moment of inertia of rod about the given axis $= \frac{mL^2}{12}$

Moment of inertia of each disc about disc about its diameter

$$= \frac{MR^2}{4}$$

Using theorem of parallel axes, moment of inertia of each disc about the given axis $= \frac{MR^2}{4} + M\left(\frac{L}{2}\right)^2$

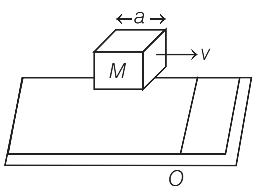
$$= \frac{MR^2}{4} + \frac{ML^2}{4}$$

$\therefore$ For theorem of parallel axes, moment of inertia about the given axis is

$$I = \frac{mL^2}{12} + \left(\frac{MR^2}{4} + \frac{ML^2}{4}\right)$$

$$I = \frac{mL^2}{12} + \frac{MR^2}{4} + \frac{ML^2}{4}$$

5. Angular momentum of block w.r.t. O before collision at $O = Mv\frac{a}{2}$. On collision, the block will rotate about the side passing through O. Now its angular momentum $= I\omega$



By law of conservation of angular momentum,

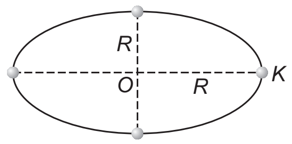
$$Mv\frac{a}{2} = I\omega$$

$$\Rightarrow \quad Mv\frac{a}{2} = \left(\frac{Ma^2}{6} + \frac{Ma^2}{2}\right)\omega$$

$$\Rightarrow \quad \omega = \frac{3v}{4a}$$

where I is moment of inertia of the block about the axis perpendicular to the plane passing through O.

6. According to the theorem of parallel axes, moment of inertia of disc about an axis passing through K and perpendicular to plane of disc,



$$= \frac{1}{2}MR^2 + MR^2$$

$$= \frac{3}{2}MR^2$$

Total moment of inertia of the system

$$= \frac{3}{2}MR^2 + m(2R)^2 + m(\sqrt{2}R)^2 + m(\sqrt{2}R)^2$$

$$= 3(M + 16m)\frac{R^2}{2}$$

7. As no torque is being applied, angular momentum,

$$L = I\omega = \text{constant}$$

$$\Rightarrow \quad \left(\frac{2}{5}Mr^2\right)\frac{2\pi}{T} = \text{constant}$$

$$\text{or} \quad \frac{r^2}{T} = \text{constant}$$

Differentiating w.r.t. time t, we get

$$\frac{T \cdot 2r\dfrac{dr}{dt} - r^2\dfrac{dT}{dt}}{T^2} = 0$$

$$\text{or} \quad 2Tr\frac{dr}{dt} = r^2\frac{dT}{dt}$$

$$\text{or} \quad \frac{dT}{dt} = \frac{2T}{r}\frac{dr}{dt}$$

8. As angular momentum is conserved in the absence of a torque, therefore

$$I_0\omega_0 = I\omega$$

$$\left(\frac{2}{3}MR^2\right)\left(\frac{2\pi}{T_0}\right) = \left[\frac{2}{5}MR^2 + \frac{2}{5}\frac{MR^2}{5\times10^{19}}\right]\frac{2\pi}{T}$$

$$\frac{T}{T_0} = 1 + \frac{1}{5\times10^{19}}$$

$$\frac{T}{T_0} - 1 = \frac{1}{5\times10^{19}} = 2\times10^{-20}$$

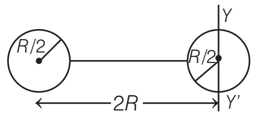
9. From symmetry considerations,

$I_{AC} = \frac{1}{2}$ moment of inertia of lamina about an axis through O and perpendicular to the plane $ABCD$.

and $I_{EF} = \frac{1}{2}$ moment of inertia of lamina about an axis through O and perpendicular to the plane $ABCD$

$$\therefore \quad I_{AC} = I_{EF}$$

10.



Moment of inertia of the system about YY'

$I_{yy'}$ = Moment of inertia of sphere P about YY' + Moment of inertia of sphere Q about YY'

Moment of inertia of sphere P about YY'

$$= \frac{2}{5}M\left(\frac{R}{2}\right)^2 + M(x)^2$$

$$= \frac{2}{5}M\left(\frac{R}{2}\right)^2 + M(2R)^2$$

$$= \frac{MR^2}{10} + 4MR^2$$

Moment of inertia of sphere Q about YY' is $\frac{2}{5}M\left(\frac{R}{2}\right)^2$

Now, $I_{YY'} = \frac{MR^2}{10} + 4MR^2 + \frac{2}{5}M\left(\frac{R}{2}\right)^2 = \frac{21}{5}MR^2$

11. Choice (a) is false, as theorem of perpendicular axes applies only to a plane lamina.

Now, Z axis parallel to Z' axis and distance between them $= \frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}}$. Therefore, according to the theorem of parallel axes,

$$I_Z' = I_Z + m\left(\frac{a}{\sqrt{2}}\right)^2 = I_Z + \frac{ma^2}{2}$$

Choice (b) is true.

Again, choice (c) is false as Z'' axis is not parallel to Z-axis.

Choice (d) is also false as from symmetry, we find that $I_X = I_Y$.

12. If M is mass of the square plate before cutting the holes, then mass of portion of each hole,

$$m = \frac{M}{16R^2}\times\pi R^2 = \frac{\pi}{16}M$$

$\therefore$ Moment of inertia of remaining portion,

$$I = I_{\text{square}} - 4I_{\text{hole}}$$

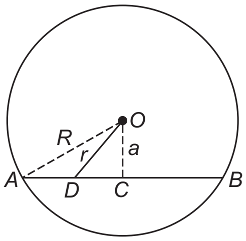
$$= \frac{M}{12}(16R^2 + 16R^2) - 4\left[\frac{mR^2}{2} + m(\sqrt{2}R)^2\right]$$

$$= \frac{M}{12} \times 32 R^2 - 10 mR^2$$

$$= \frac{8}{3} MR^2 - \frac{10\pi}{16} MR^2$$

$$= \left(\frac{8}{3} - \frac{10\pi}{16}\right) MR^2$$

13. As there is no external torque, angular momentum will remain constant. When the tortoise moves from A to C, moment of inertia of the platform and tortoise decreases. Therefore, angular velocity of the system increases. When the tortoise moves from C to B, moment of inertia increases. Therefore, angular velocity decreases.



If M = mass of platform,

R = radius of platform,

m = mass of tortoise moving along the chord AB

and a = perpendicular distance of O from AB.

Initial angular momentum, $I_1 = mR^2 + \frac{MR^2}{2}$

At any time t, let the tortoise reach D moving with velocity v.

$\therefore \quad AD = vt$

$AC = \sqrt{R^2 - a^2}$

As $\quad DC = AC - AD = (\sqrt{R^2 - a^2} - vt)$

$\therefore \quad OD = r = a^2 + [\sqrt{R^2 - a^2} - vt]^2$

Angular momentum at time t

$$I_2 = mr^2 + \frac{MR^2}{2}$$

As angular momentum is conserved

$\therefore \quad I_1\omega_0 = I_2\omega(t)$

This shows that variation of $\omega(t)$ with time is non-linear.

14. To reverse the direction,

$$\int \tau \, d\theta = 0 \qquad \text{(work done is zero)}$$

As, $\quad \tau = F \times r = (20t - 5t^2) \times 2 = 40t - 10t^2$

$$\alpha = \frac{\tau}{I} = \frac{40t - 10t^2}{10} = 4t - t^2 = \frac{d\omega}{dt}$$

$$\omega = \int_0^t \alpha \, dt = 2t^2 - \frac{t^3}{3}$$

ω is zero at

$$2t^2 - \frac{t^3}{3} = 0$$

$\Rightarrow \quad t^3 = 6t^2$

$\Rightarrow \quad t = 6\text{ s}$

As $\quad \frac{d\theta}{dt} = \omega$

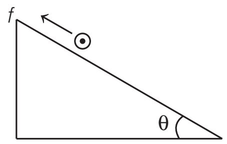
$$\Rightarrow \quad \theta = \int_0^6 \omega \, dt = \int_0^6 \left(2t^2 - \frac{t^3}{3}\right) dt$$

$$= \left[\frac{2t^3}{3} - \frac{t^4}{12}\right]_0^6$$

$$= 216\left[\frac{2}{3} - \frac{1}{2}\right] = 36 \text{ rad}$$

Number of rotation $= \frac{\theta}{2\pi} = \frac{36}{2\pi} = 5.73 < 6$

15. Linear acceleration for rolling, $a = \frac{g\sin\theta}{\sqrt{1 + \frac{K^2}{R^2}}}$



For cylinder, $\frac{K^2}{R^2} = \frac{1}{2}$

$\therefore \quad a_{\text{cylinder}} = \frac{2}{3} g\sin\theta$

For rotation, the torque

$$fR = I\alpha = (MR^2\alpha)/2$$

(where, f = force of friction)

But $R\alpha = a$

$\therefore \quad f = \frac{M}{2} a$

$\therefore \quad f = \frac{M}{2} \cdot \frac{2}{3} g\sin\theta$

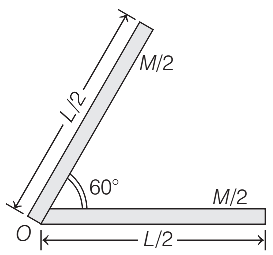
$$= \frac{M}{3} g\sin\theta$$

$\mu_s = f/N$, where N is normal reaction,

$$\therefore \quad \mu_s = \frac{\frac{M}{3} g\sin\theta}{Mg\cos\theta} = \frac{\tan\theta}{3}$$

$\therefore$ For rolling without slipping of a roller down the inclined plane, $\tan\theta \leq 3\mu_s$.

16. Since, rod is bent at the middle, so each part of it will have same length $\left(\frac{L}{2}\right)$ and mass $\left(\frac{M}{2}\right)$ as shown



Moment of inertia of each part about an axis passing through its one end

$$= \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2$$

Hence, net moment of inertia about an axis passing through its middle point O is

$$I = \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 + \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2$$

$$= \frac{1}{3}\left[\frac{ML^2}{8} + \frac{ML^2}{8}\right] = \frac{ML^2}{12}$$

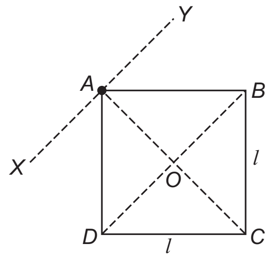
17. As, it is clear from figure,

$$AC = BD = \sqrt{l^2 + l^2} = l\sqrt{2}$$

Moment of inertia of four point masses about BD,

$$I_{BD} = m\left(\frac{l\sqrt{2}}{2}\right)^2 + m \times 0 + m\left(\frac{l\sqrt{2}}{2}\right)^2 + m \times 0$$

$$= \frac{ml^2}{2} + \frac{ml^2}{2} = ml^2$$



Applying the theorem of parallel axes,

$$I_{XY} = I_{BD} + M\,(AO)^2$$

$$= ml^2 + 4\,m\left(\frac{l}{\sqrt{2}}\right)^2$$

$$= 3\,ml^2$$

18. Mass per unit area of disc $= \frac{9\,M}{\pi R^2}$

Mass of removed portion of disc $= \frac{9\,M}{\pi R^2} \times \pi\left(\frac{R}{3}\right)^2 = M$

Moment of inertia of removed portion about an axis passing through centre of disc and perpendicular to the plane of disc, using theorem of parallel axes is

$$I_1 = \frac{M}{2}\left(\frac{R}{3}\right)^2 + M\left(\frac{2\,R}{3}\right)^2 = \frac{1}{2}MR^2$$

When portion of disc would not have been removed, then the moment of inertia of complete disc about the given axis is

$$I_2 = \frac{1}{2}MR^2$$

So moment of inertia of the disc with removed portion, about the given axis is

$$I = I_2 - I_1 = \frac{9}{2}MR^2 - \frac{1}{2}MR^2 = 4\,MR^2$$

19. When bob falls, its potential energy is converted into kinetic energy of bob and rotational kinetic energy of disc.

So, decrease in potential energy of bob = kinetic energy of bob + kinetic energy of disc

$$\Rightarrow \quad mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad \text{...(i)}$$

If there is no slip, then $v = r\omega$

Also, moment of inertia of disc,

$$I = \frac{mr^2}{2}$$

Substituting this value in Eq. (i), we get

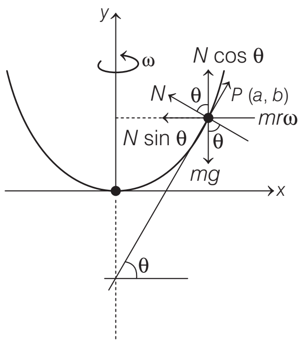
$$mgh = \frac{1}{2}m(r\omega)^2 + \frac{1}{2} \times \frac{mr^2}{2} \times \omega^2$$

$$\Rightarrow \quad mgh = \frac{3}{4}mr^2\omega^2$$

$$\Rightarrow \quad \omega^2 = \frac{4gh}{3r^2}$$

$$\text{or} \quad \omega = \frac{1}{r}\sqrt{\frac{4gh}{3}}$$

20. Various forces acting on bead at point P can be resolved as shown in figure



Normal force of rotating wire are

$$N\cos\theta = mg \quad \text{and} \quad N\sin\theta = mx\omega^2$$

$$\Rightarrow \quad \tan\theta = \frac{dy}{dx} = \frac{x\omega^2}{g} \quad \text{...(i)}$$

Given, $\quad y = 4\,Cx^2$

$$\Rightarrow \quad \frac{dy}{dx} = 8\,Cx$$

$$\left.\frac{dy}{dx}\right|_{\text{at } x = a,\, y = b} = 8\,Ca \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{x\omega^2}{g} = 8\,Ca$$

At $x = a$, we have

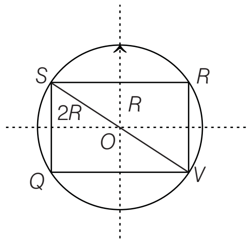
$$\frac{a\omega^2}{g} = 8\,Ca$$

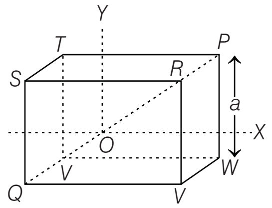
$$\Rightarrow \quad \omega = \sqrt{8\,Cg}$$

$$= 2\sqrt{2g\,C}$$

21. Use geometry of the figure to calculate mass and side length of the cube interms of M and R, respectively.

Consider the cross-sectional view of a diametric plane as shown in the following diagram.





Using geometry of the cube,

$$PQ = 2R = (\sqrt{3})\, a$$

or $$a = \frac{2R}{\sqrt{3}}$$

Volume density of the solid sphere

$$\rho = \frac{M}{(4/3)\,\pi R^3} = \frac{3}{4\pi}\left(\frac{M}{R^3}\right)$$

Mass of cube $(m) = (\rho)\,(a)^3$

$$= \left(\frac{3}{4\pi} \times \frac{M}{R^3}\right)\left[\frac{2R}{\sqrt{3}}\right]^3$$

$$= \frac{3M}{4\pi R^3} \times \frac{8R^3}{3\sqrt{3}} = \frac{2M}{\sqrt{3}\pi}$$

Moment of inertia of the cube about given axis is

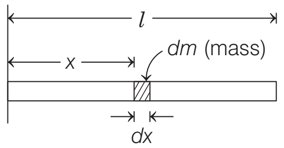
$$I_Y = \frac{ma^2}{12}\,(a^2 + a^2) = \frac{ma^2}{6}$$

$$\Rightarrow \quad I_Y = \frac{ma^2}{6}$$

$$= \frac{2M}{\sqrt{3}\pi} \times \frac{1}{6} \times \frac{4R^2}{3} = \frac{4MR^2}{9\sqrt{3}\pi}$$

22. In a uniform rod, mass per unit length remains constant. If it is denoted by λ, then

$\lambda = \dfrac{m}{l}$ = constant for all segments of rod.



To find tension at x distance from fixed end, let us assume an element of dx length and dm mass. Tension on this part due to rotation is

$$dT = Kx \qquad \ldots\text{(i)}$$

As, $$K = m\omega^2$$

For this element, $$K = (dm)\,\omega^2 \qquad \ldots\text{(ii)}$$

$$\therefore \quad dT = (dm)\,\omega^2 x \qquad \ldots\text{(iii)}$$

To find complete tension in the rod, we need to integrate Eq. (iii),

$$\int_0^T dT = \int_0^m (dm)\,\omega^2 x \qquad \ldots\text{(iv)}$$

Using linear mass density,

$$\lambda = \frac{m}{l} = \frac{dm}{dx}$$

$$\Rightarrow \quad dm = \frac{m}{l}\cdot dx \qquad \ldots\text{(v)}$$

Putting the value of Eq. (v) in Eq. (iv), we get

$$T = \int_x^l \frac{m}{l}\cdot \omega^2 x \cdot dx = \frac{m}{l}\cdot \omega^2\left[\frac{x^2}{2}\right]_x^l$$

$$\Rightarrow \quad T = \frac{m\omega^2}{2l}\,[l^2 - x^2]$$

Now, if we observe the equation, it will look like a parabolic equation and when we see from given options, the correct option is (b).

23. Kinetic energy of rotation,

$$K_{\text{rot}} = \frac{1}{2} I\omega^2 = \frac{1}{2} Mk^2 \frac{v^2}{R^2} \qquad \left(\because \omega = \frac{v}{R}\right)$$

where, k is radius of gyration.

Kinetic energy of translation,

$$K_{\text{trans}} = (1/2) Mv^2$$

Thus, total energy,

$$E = K_{\text{rot}} + K_{\text{trans}}$$

$$= \frac{1}{2} Mk^2 \frac{v^2}{R^2} + \frac{1}{2} Mv^2$$

$$= \frac{1}{2} Mv^2\left(\frac{k^2}{R^2} + 1\right)$$

$$= \frac{1}{2}\frac{Mv^2}{R^2}(k^2 + R^2)$$

Hence, $$\frac{K_{\text{rot}}}{K_{\text{total}}} = \frac{\frac{1}{2} Mk^2 \frac{v^2}{R^2}}{\frac{1}{2}\frac{Mv^2}{R^2}(k^2 + R^2)}$$

$$= \frac{k^2}{k^2 + R^2}$$

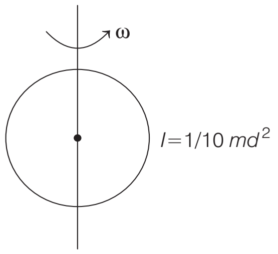
24. In given arrangement, mass of each sphere = m

Radius of each sphere = $\dfrac{d}{2}$

Distance of centre of each sphere from axis of rotation or centroidal axis = $\dfrac{2}{3}$ × Altitude of equilateral triangle of side $d = \dfrac{2}{3} \times \dfrac{\sqrt{3}}{2} \times d = \dfrac{d}{\sqrt{3}}$

Now, moment of inertia of each sphere along an axis through its centre,

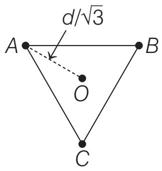
$$I_1 = \frac{2}{5}mr^2 = \frac{2}{5}m\left(\frac{d}{2}\right)^2 = \frac{1}{10}md^2$$



Now, by using parallel axes theorem, moment of inertia of each sphere around axis through O,

$$I' = I_1 + m(h^2)$$

$$= I_1 + m\left(\frac{d}{\sqrt{3}}\right)^2 = \frac{1}{10}md^2 + \frac{md^2}{3} = \frac{13}{30}md^2$$



So, moment of inertia of system of spheres about O,

$$I_0 = 3I' = 3 \times \frac{13}{30}md^2 = \frac{13}{10}md^2 \quad \ldots\text{(i)}$$

Again by using parallel axes theorem, moment of inertia of system about point A,

$$I_A = I_0 + m_{\text{system}} \times h^2$$

$$= \frac{13}{10}md^2 + 3m \times \left(\frac{d}{\sqrt{3}}\right)^2$$

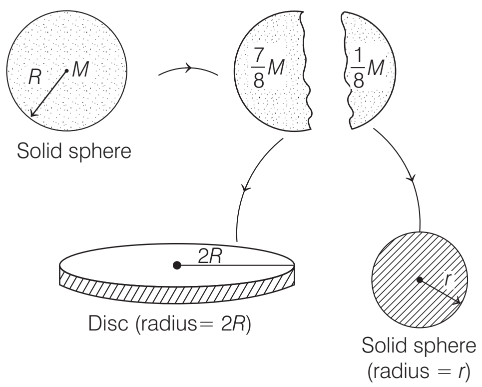
$$= \frac{23}{10}md^2 \quad \ldots\text{(ii)}$$

Hence, required ratio from Eqs. (i) and (ii), we get

$$\frac{I_0}{I_A} = \frac{13}{10}md^2 \times \frac{10}{23\,md^2}$$

$$\Rightarrow \quad \frac{I_0}{I_A} = \frac{13}{23}$$

25. The given situation is shown in the figure given below



Density of given sphere of radius R,

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{\frac{4}{3}\pi R^3}$$

Let radius of sphere formed from second part is r, then mass of second part = volume × density

$$\frac{1}{8}M = \frac{4}{3}\pi r^3 \times \frac{M}{\frac{4}{3}\pi R^3}$$

$$\therefore \quad r^3 = \frac{R^3}{8} \Rightarrow r = \frac{R}{2}$$

Now, I_1 = moment of inertia of disc (radius $2R$ and mass $\frac{7}{8}M$) about its axis

$$= \frac{\text{Mass} \times (\text{Radius})^2}{2}$$

$$= \frac{\frac{7}{8}M \times (2R)^2}{2} = \frac{7}{4}MR^2$$

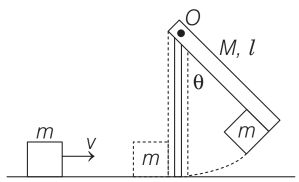
and $\quad I_2$ = moment of inertia of sphere (radius $\frac{R}{2}$ and mass $\frac{1}{8}M$) about its axis

$$= \frac{2}{5} \times \text{Mass} \times (\text{Radius})^2$$

$$= \frac{2}{5} \times \frac{1}{8}M \times \left(\frac{R}{2}\right)^2 = \frac{MR^2}{80}$$

$$\therefore \text{ Ratio } \frac{I_1}{I_2} = \frac{\frac{7}{4}MR^2}{\frac{1}{80}MR^2} = 140$$

26.



As there is no external torque, so angular momentum about pivot O is conserved.

$$\Rightarrow \quad L_i = L_f$$

$$\Rightarrow \quad mvl = (I_{\text{system}})\,\omega$$

$$\Rightarrow \quad mvl = (I_{\text{rod}} + I_{\text{block}})\,\omega$$

$$\Rightarrow \quad mvl = \left(\frac{Ml^2}{3} + ml^2\right)\omega$$

(where, ω = initial angular speed of rod and block)

$$\omega = \frac{mvl}{\left(\frac{Ml^2}{3} + ml^2\right)}$$

$$\Rightarrow \quad \omega = \frac{mv}{\left(\frac{Ml}{3} + ml\right)}$$

$$\Rightarrow \quad \omega = \frac{1 \times 6}{\left(\frac{2 \times 1}{3} + 1 \times 1\right)} = \frac{6 \times 3}{5} = \frac{18}{5} \text{ rad/s}$$

Now, by law of conservation of energy,

Initial rotational kinetic energy of rod and block = Final potential energy at angle θ of rod and block

$$\Rightarrow \quad \frac{1}{2}(I_{\text{system}}) \times \omega^2 = Mgh_1 + mgh_2$$

where, h_1 = height raised by centre of mass of rod

$$= \frac{l}{2}(1 - \cos\theta)$$

and $\quad h_2$ = height raised by block = $l(1 - \cos\theta)$.

Hence,

$$\frac{1}{2}(I_{\text{rod}} + I_{\text{block}})\,\omega^2 = Mg\frac{l}{2}(1-\cos\theta) + mgl(1-\cos\theta)$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{Ml^2}{3} + ml^2\right)\omega^2 = gl\left(\frac{M}{2} + m\right) \times (1 - \cos\theta)$$

$$\Rightarrow \quad \frac{\frac{1}{2}\left(\frac{M}{3} + m\right)l\omega^2}{\left(\frac{M}{2} + m\right)g} = 1 - \cos\theta$$

$$\Rightarrow \quad \frac{\frac{1}{2}\left(\frac{2}{3} + 1\right) \times 1 \times \left(\frac{18}{5}\right)^2}{\left(\frac{2}{2} + 1\right) \times 10} = 1 - \cos\theta$$

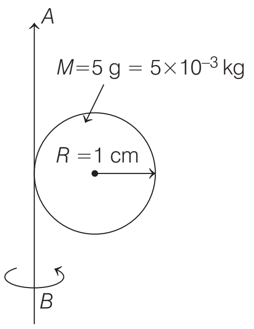
$$\Rightarrow \quad \frac{27}{50} = 1 - \cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{23}{50} = 0.46$$

$$\Rightarrow \quad \theta = \cos^{-1}(0.46) = 62.61° \simeq 63°$$

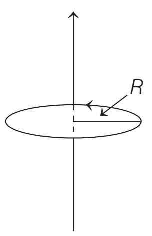
The nearest answer is 63°.

27. Moment of inertia (MI) of a disc about a tangential axis in the plane of disc can be obtained as below.



Moment of inertia of disc about its axis,

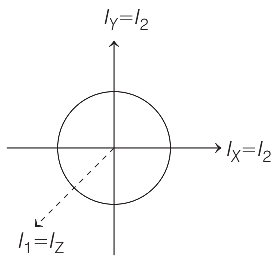
$$I_1 = \frac{MR^2}{2}$$



From perpendicular axes theorem, moment of inertia of disc about an axis along its diameter is

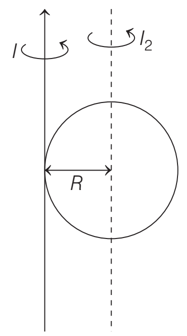
$$I_X + I_Y = I_Z \;\Rightarrow\; 2I_2 = I_1$$

$$\Rightarrow \quad I_2 = \frac{I_1}{2} = \frac{MR^2}{4}$$



So, moment of inertia about a tangential axis from parallel axes theorem is

$$I = I_2 + MR^2 = \frac{MR^2}{4} + MR^2 = \frac{5}{4}MR^2$$



Now, using torque, $\tau = I\alpha$, we have

$$\tau = I\alpha = \frac{5}{4}MR^2\left(\frac{\omega_f - \omega_i}{\Delta t}\right)$$

Here, $\quad M = 5 \times 10^{-3}$ kg, $R = 1 \times 10^{-2}$ m

$$\omega_f = 25 \text{ rps} = 25 \times 2\pi\,\frac{\text{rad}}{\text{s}} = 50\pi\,\frac{\text{rad}}{\text{s}},$$

$$\omega_i = 0,\; \Delta t = 5 \text{ s}$$

So, $$\tau = \frac{\frac{5}{4} \times 5 \times 10^{-3} \times (10^{-2})^2 \times 50\pi}{5}$$

$$\approx 2 \times 10^{-5} \text{ N-m}$$

28. Initially, centre of mass is at distance L from the top end of the swing. It shifts to $(L - l)$ distance when the person stands up on the swing.

$\therefore$ Using angular momentum conservation law, if v_0 and v_1 are the velocities before standing and after standing of the person, then

$$Mv_0L = Mv_1(L - l)$$

$$\Rightarrow \quad v_1 = \left(\frac{L}{L - l}\right)v_0 \qquad \text{...(i)}$$

Now, total work done by (person + gravitation) system will be equal to the change in kinetic energy of the person, *i.e.*

$$W_g + W_p = \text{KE}_1 - \text{KE}_0$$

$$\Rightarrow \quad -Mgl + W_p = \frac{1}{2}Mv_1^2 - \frac{1}{2}Mv_0^2$$

$$\Rightarrow \quad W_p = Mgl + \frac{1}{2}M(v_1^2 - v_0^2)$$

$$= Mgl + \frac{1}{2}M\left[\left(\frac{L}{L-l}\right)^2 v_0^2 - v_0^2\right]$$

[from Eq. (i)]

$$= Mgl + \frac{1}{2}M\,v_0^2\left[\left(\frac{L-l}{L}\right)^{-2} - 1\right]$$

$$= Mgl + \frac{1}{2}Mv_0^2\left[\left(1 - \frac{l}{L}\right)^{-2} - 1\right]$$

$$= Mgl + \frac{1}{2}Mv_0^2\left[\left(1 + \frac{2l}{L}\right) - 1\right]$$

[using $(1+x)^n = 1 + nx$ as higher terms can be neglected, if $n << 1$]

$$\Rightarrow \quad W_p = Mgl + \frac{1}{2}Mv_0^2 \times \frac{2l}{L}$$

or $$W_p = Mgl + Mv_0^2\,\frac{l}{L} \quad \text{...(ii)}$$

Here, $$v_0 = WA = \left(\sqrt{\frac{g}{L}}\right)(\theta_0 L)$$

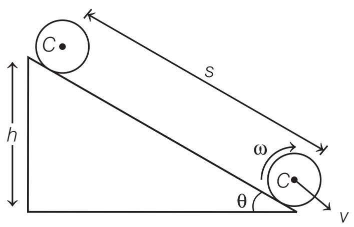
$$\Rightarrow \quad v_0 = \theta_0\sqrt{gL}$$

$\therefore$ Using this value of v_0 in Eq.(ii), we get

$$W_p = Mgl + M\theta_0^2\,gL \cdot \frac{l}{L}$$

$$\Rightarrow \quad W_p = Mgl\,(1 + \theta_0^2)$$

29. Assuming that no energy is used up against friction, the total loss in potential energy is equal to the total gain in the kinetic energy.



$$Mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2 \quad \text{[as, } v = r\omega]$$

i.e. $$Mgh = \frac{1}{2}I\left(\frac{v^2}{R^2}\right) + \frac{1}{2}Mv^2$$

or $$\frac{1}{2}v^2\left(M + \frac{I}{R^2}\right) = Mgh$$

or $$v^2 = \frac{2\,Mgh}{M + I/R^2} = \frac{2\,gh}{1 + I/MR^2}$$

If s is the distance covered along the plane, then

$$h = s\sin\theta$$

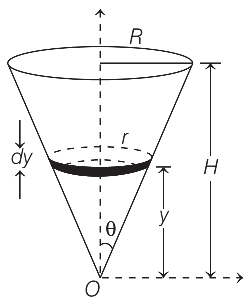
$$\therefore \quad v^2 = \frac{2gs\sin\theta}{1 + I/MR^2}$$

Now, $$v^2 = 2\,as$$

$$\therefore \quad 2\,as = \frac{2\,g\,s\sin\theta}{1 + I/MR^2}$$

or $$a = \frac{g\sin\theta}{1 + I/MR^2}$$

30. The hollow icecream cone can be assumed to be formed of several parts of rings having different radii. In general, we shall consider a ring (element) which is at distance y away from vertex O, of thickness dy, having radius r as shown in figure.



Area of cone, $A = \pi Rl = \pi R\sqrt{R^2 + H^2}$

Area of element, $dA = 2\pi r dl = 2\pi r \dfrac{dy}{\cos\theta}$

Mass of element, dm = Surface density $\times$ Area

$$= \frac{M}{\pi R\sqrt{R^2 + H^2}} \times \frac{2\pi r\,dy}{\cos\theta}$$

From above diagram, $\tan\theta = \dfrac{r}{y}$ or $r = y\tan\theta$

$$\therefore \quad dm = \frac{2M\tan\theta\,y\,dy}{R\cos\theta\sqrt{R^2 + H^2}}$$

Moment of inertia of element about its axis,

$$dI = (dm)r^2 = \left(\frac{2M\tan\theta y\,dy}{R\cos\theta\sqrt{R^2 + H^2}}\right)(y\tan\theta)^2$$

$$= \frac{2\,M\tan^3\theta y^3 dy}{R\cos\theta\sqrt{R^2 + H^2}}$$

$\therefore$ Total moment of inertia,

$$I = \int dI = \frac{2M\tan^3\theta}{R\cos\theta\sqrt{R^2 + H^2}}\int_0^H y^3 dy$$

$$= \frac{2M\tan^3\theta}{R\cos\theta\sqrt{R^2 + H^2}} \cdot \frac{H^4}{4}$$

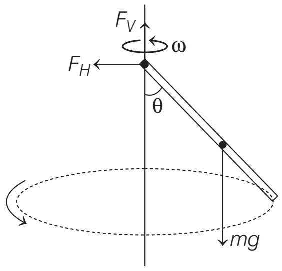
Again, from above diagram, $\tan\theta = \dfrac{R}{H}$

$$\cos\theta = \frac{H}{\sqrt{R^2 + H^2}}$$

Putting these values in the expression of I, we get

$$I = \frac{MR^2}{2}$$

31. Rod rotates around vertical axis as

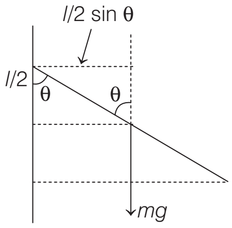


Given that,

Rate of change of angular momentum of rod = Torque provided by horizontal and vertical forces about centre of mass

$$\Rightarrow \quad \frac{ml^2}{12}\omega^2 \sin\theta\cos\theta = \text{Net torque of } F_H \text{ and } F_V \qquad \text{...(i)}$$

Here, $F_V = mg$ and its perpendicular distance from axis is $\frac{l}{2}\sin\theta$.



$$\therefore \text{ Torque of } F_V = mg\cdot\frac{l}{2}\sin\theta$$

and $\qquad F_H = \text{centripetal force} = m\omega^2\frac{l}{2}\sin\theta$

Also, distance of F_H from axis is also $\frac{l}{2}\cos\theta$.

$$\therefore \text{ Torque of } F_H = m\omega^2\frac{l}{2}\sin\theta.\frac{l}{2}\cos\theta$$

So, from Eq. (i), we have

$$mg\frac{l}{2}\sin\theta - m\omega^2\frac{l}{2}\sin\theta.\frac{l}{2}\cos\theta = \frac{ml^2}{12}\omega^2\sin\theta\cos\theta$$

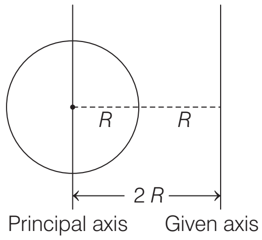
$$\Rightarrow \quad mg\frac{l}{2}\sin\theta = ml^2\omega^2\left(\frac{1}{12}+\frac{1}{4}\right)\sin\theta\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{3}{2}\left(\frac{g}{\omega^2 l}\right)$$

32. We know that, moment of inertia (MI) about the principal axis of the sphere is given by

$$I_{\text{sphere}} = \frac{2}{5}MR^2 \qquad \text{... (i)}$$

Using parallel axes theorem, moment of inertia about the given axis in the figure below will be



$$I_1 = \frac{2}{5}MR^2 + M(2R)^2$$

$$I_1 = \frac{22}{5}MR^2 \qquad \text{... (ii)}$$

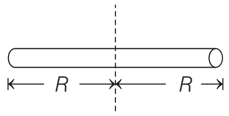
Considering both spheres at equal distance from the axis, moment of inertia due to both spheres about this axis will be

$$2I_1 = 2\times(22/5)\,MR^2$$

Now, moment of inertia of rod about its perpendicular bisector axis is given by

$$I_2 = \frac{1}{12}ML^2$$

Here, given that $\quad L = 2R$



$$\therefore \qquad I_2 = \frac{1}{12}M(2R)^2 = \frac{1}{3}MR^2 \qquad \text{... (iii)}$$

So, total moment of inertia of the system is

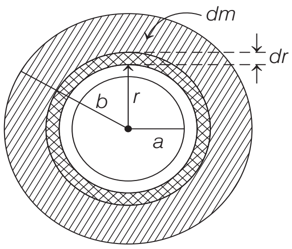
$$I = 2I_1 + I_2 = 2\times\frac{22}{5}MR^2 + \frac{1}{3}MR^2$$

$$\Rightarrow \qquad I = \left(\frac{44}{5}+\frac{1}{3}\right)MR^2 = \frac{137}{15}MR^2$$

33. Given, variation in mass per unit area (surface mass density),

$$\sigma = \frac{\sigma_0}{r} \qquad \text{...(i)}$$

Calculation of Mass of Disc



Let us divide whole disc in small area elements, one of them shown at r distance from the centre of the disc with its width as dr.

Mass of this element is $dm = \sigma\cdot dA$

$$\Rightarrow \qquad dm = \frac{\sigma_0}{r}\times 2\pi r dr \qquad \text{[from Eq. (i)]} \qquad \text{...(ii)}$$

Mass of the disc can be calculated by integrating it over the given limits of r,

$$\int_0^M dm = \int_a^b \sigma_0 \times 2\pi\times dr$$

$$\Rightarrow \qquad M = \sigma_0 2\pi(b-a) \qquad \text{...(iii)}$$

Calculation of Moment of Inertia

$$I = \int_0^M r^2 dm = \int_a^b r^2\cdot\frac{\sigma_0}{r}\times 2\pi r dr \quad \text{[from Eq. (ii)]}$$

$$= \sigma_0 2\pi \int_a^b r^2 dr$$

$$= \sigma_0 2\pi \left[\frac{r^3}{3}\right]_a^b$$

$\Rightarrow$ $$I = \frac{1}{3}\sigma_0 2\pi\ [b^3 - a^3] \quad \text{...(iv)}$$

Now, radius of gyration,

$$K = \sqrt{\frac{I}{M}} = \sqrt{\frac{\frac{2\pi\sigma_0}{3}\ (b^3 - a^3)}{2\pi\sigma_0\ (b - a)}}$$

$\Rightarrow$ $$K = \sqrt{\frac{1}{3}\ \frac{(b^3 - a^3)}{b - a}}$$

As we know, $b^3 - a^3 = (b - a)\ (b^2 + a^2 + ab)$

$\therefore$ $$K = \sqrt{\frac{1}{3}\ (b^2 + a^2 + ab)}$$

or $$K = \sqrt{\frac{a^2 + b^2 + ab}{3}}$$

34. When a spherical/circular body of radius r rolls without slipping, its total kinetic energy is

$$K_{\text{total}} = K_{\text{translation}} + K_{\text{rotation}}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I \cdot \frac{v^2}{r^2} \quad \left[\because \omega = \frac{v}{r}\right]$$

Let v be the linear velocity and R be the radius for both solid sphere and solid cylinder.

$\therefore$ Kinetic energy of the given solid sphere will be

$$K_{\text{sph}} = \frac{1}{2}mv^2 + \frac{1}{2}I_{\text{sph}}\ \frac{v^2}{R^2}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{2}{5}mR^2 \times \frac{v^2}{R^2}$$

$$= \frac{7}{10}mv^2 \quad \text{...(i)}$$

Similarly, kinetic energy of the given solid cylinder will be $K_{\text{cyl}} = \frac{1}{2}mv^2 + \frac{1}{2}I_{\text{cyl}}\ \frac{v^2}{R}$

$$= \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{mR^2}{2} \times \frac{v^2}{R^2} = \frac{3}{4}mv^2 \quad \text{...(ii)}$$

Now, from the conservation of mechanical energy,

$$mgh = K_{\text{total}}$$

$\therefore$ For solid sphere,

$$mgh_{\text{sph}} = \frac{7}{10}mv^2 \quad \text{...(iii) [using Eq. (i)]}$$

Similarly, for solid cylinder,

$$mgh_{\text{cyl}} = \frac{3}{4}mv^2 \quad \text{...(iv) [using Eq. (ii)]}$$

Taking the ratio of Eqs. (iii) and (iv), we get

$$\frac{mgh_{\text{sph}}}{mgh_{\text{cyl}}} = \frac{\frac{7}{10}mv^2}{\frac{3}{4}mv^2}$$

$\Rightarrow$ $$\frac{h_{\text{sph}}}{h_{\text{cyl}}} = \frac{7}{10} \times \frac{4}{3} = \frac{14}{15}$$

35. From question,

let height attained by ring $= h_1$

Height attained by cylinder $= h_2$

Height attained by sphere $= h_3$

As we know that for a body which is rolling up an inclined plane (without slipping), follows the law of conservation of energy.

$\therefore$ For ring, using energy conservation law at its height h_1,

$$(\text{KE})_{\text{linear}} + (\text{KE})_{\text{rotational}} = (\text{PE})$$

$\Rightarrow$ $$\frac{1}{2}m_1 v_0^2 + \frac{1}{2}I_1\omega^2 = m_1 g h_1$$

$\Rightarrow$ $$\frac{1}{2}m_1 v_0^2 + \frac{1}{2}m_1 R^2\omega^2 = m_1 g h_1$$

$(\because I = mR^2 \text{ for ring})$

$\Rightarrow$ $$gh_1 = \frac{v_0^2}{2} + \frac{v_0^2}{2} \quad (\because v_0 = \omega R)$$

$\Rightarrow$ $$h_1 = v_0^2 / g \quad \text{...(i)}$$

Similarly, for solid cylinder, applying the law of conservation of energy,

$$\frac{1}{2}m_2 v_0^2 + \frac{1}{2}I_2\omega^2 = m_2 g h_2$$

$\Rightarrow$ $$\frac{1}{2}m_2 v_0^2 + \frac{1}{2}\left(\frac{1}{2} \times m_2 \times \left(\frac{R}{2}\right)^2\right)\ \omega^2 = m_2 g h_2$$

$$\left[\because I = \frac{1}{2}mR^2 \text{ for cylinder and } R = \frac{R}{2}\right]$$

$\Rightarrow$ $$\frac{1}{2}m_2 v_0^2 + \frac{1}{2} \times \frac{1}{8}\ m_2 R^2 \times \frac{v_0^2}{(R/2)^2} = m_2 g h_2$$

$\Rightarrow$ $$\frac{1}{2}\ v_0^2 + \frac{1}{4}\ v_0^2 = gh_2$$

$\Rightarrow$ $$gh_2 = \frac{3}{4}\ v_0^2$$

$\Rightarrow$ $$h_2 = \frac{3}{4}\left(\frac{v_0^2}{g}\right) \quad \text{...(ii)}$$

Similarly, for solid sphere,

$$\frac{1}{2}m_3 v_0^2 + \frac{1}{2}I_3\omega^2 = m_3 g h_3$$

$\Rightarrow$ $$\frac{1}{2}m_3 v_0^2 + \frac{1}{2}\left[\frac{2}{5}m_3\left(\frac{R}{4}\right)^2\right]\omega^2 = m_3 g h_3$$

$$\left[\because I = \frac{2}{5}mR^2 \text{ for solid sphere and } R = \frac{R}{4}\right]$$

$$\Rightarrow \quad \frac{1}{2} m_3 v_0^2 + \frac{1}{2} \times \frac{2}{5} \times m_3 \frac{R^2}{16} \times \frac{v_0^2}{(R/4)^2} = m_3 g h_3$$

$$\Rightarrow \quad \frac{1}{2} v_0^2 + \frac{1}{5} v_0^2 = g h_3$$

$$\Rightarrow \quad g h_3 = \frac{7}{10} v_0^2$$

or $$h_3 = \frac{7}{10}\left(\frac{v_0^2}{g}\right) \quad \ldots\text{(iii)}$$

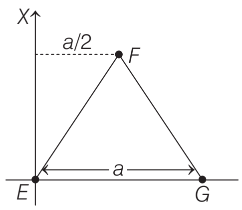
$\therefore$ Taking the ratio of h_1, h_2 and h_3 by using Eqs. (i), (ii) and (iii), we get

$$h_1 : h_2 : h_3 = \frac{v_0^2}{g} : \frac{3}{4}\frac{v_0^2}{g} : \frac{7}{10}\frac{v_0^2}{g} = 1 : \frac{3}{4} : \frac{7}{10}$$

$$\Rightarrow \quad h_1 : h_2 : h_3 = 40 : 30 : 28 = 20 : 15 : 14$$

$\therefore$ The most appropriate option is (c).

36.



Moment of inertia of system

$$= \sum_{i=1}^{3} m_i r_i^2$$

$$= m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

$$= m \times 0 + m \times a^2 + m \times \left(\frac{a}{2}\right)^2$$

$$= \frac{5}{4} m a^2 = \frac{25}{20} m a^2$$

$$= \frac{N}{20} m a^2$$

$$\therefore \quad N = 25$$

37. Given, $\mathbf{F} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ N

$$\mathbf{r} = (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}) - (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}})$$

$$= (4-1)\hat{\mathbf{i}} + (3-2)\hat{\mathbf{j}} + (-1-1)\hat{\mathbf{k}}$$

$$= (3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}) \text{ m}$$

Torque, $\tau = \mathbf{r} \times \mathbf{F}$

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix} = (3+4)\hat{\mathbf{i}} - (9+2)\hat{\mathbf{j}} + (6-1)\hat{\mathbf{k}}$$

$$= (7\hat{\mathbf{i}} - 11\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) \text{ N-m}$$

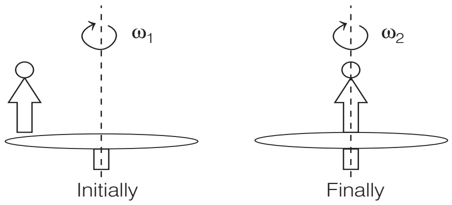
Magnitude of torque, $|\tau| = \sqrt{(7)^2 + (-11)^2 + (5)^2}$

$$= \sqrt{49 + 121 + 25}$$

$$= \sqrt{195} \text{ N-m}$$

So, the value of $x = 195$.

38.



According to conservation of angular momentum,

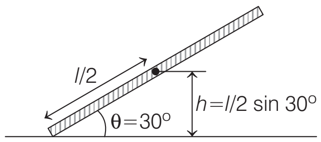
$$I_1 \omega_1 = I_2 \omega_2$$

$$\Rightarrow \quad \left(80R^2 + \frac{200R^2}{2}\right)\omega_1 = \left(\frac{200R^2}{2}\right)\omega_2$$

$$\Rightarrow \quad \frac{360R^2}{2} \times 5 = \frac{200R^2}{2} \times \omega_2$$

$$\omega_2 = 9 \text{ rpm}$$

39. When bar is released, its potential energy appears in form of rotational kinetic energy.



Now, from geometry of figure, height of centre of mass above surface,

$$h = \frac{l}{2} \cdot \sin 30° = \frac{l}{4}$$

If angular speed of bar as it hits the table is ω, then

Gravitational potential energy = Rotational kinetic energy

$$\Rightarrow \quad mgh = \frac{1}{2} I \omega^2$$

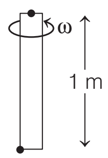
$$\Rightarrow \quad mg\frac{l}{4} = \frac{1}{2}\left(\frac{ml^2}{3}\right) \cdot \omega^2$$

$$\Rightarrow \quad \omega^2 = \frac{3}{2}\frac{g}{l} = \frac{3}{2} \times \frac{10}{1} = 15$$

$$\Rightarrow \quad \omega = \sqrt{15} \text{ s}^{-1}$$

Thus, $n = 15$

40. The given situation is shown in the following figure



Applying law of conservation of angular momentum about pivotal point,

$$L_i = L_f$$

$$\Rightarrow \quad mvl = I\omega$$

$$\Rightarrow \quad mvl = \left(\frac{1}{3} ML^2 + mL^2\right)\omega \quad \ldots\text{(i)}$$

Given, $m = 0.1$ kg, $v = 80$ ms^{-1}, $M = 0.9$ kg, $l = 1$ m

Substituting these values in Eq. (i), we get

$$0.1 \times 80 \times 1 = \left(\frac{0.9 \times 1^2}{3} + 0.1 \times 1^2\right)\omega$$

$$8 = \left(\frac{3}{10} + \frac{1}{10}\right)\omega$$

$$\Rightarrow \quad 8 = \frac{4}{10}\omega$$

$$\Rightarrow \quad \omega = 20 \text{ rad/s}$$

41. In this process, no external torque is applied, so the angular momentum will remain conserved.

From law of conservation of angular momentum,

$$(L_{\text{sys}})_i = (L_{\text{sys}})_f$$

$$\Rightarrow \quad I_1\omega_1 + 0 = (I_1 + I_2)\,\omega_2$$

$$\Rightarrow \quad \frac{MR^2}{2} \times \omega_1 = \left[\frac{MR^2}{2} + \frac{M\left(\frac{R}{2}\right)^2}{2}\right]\omega_2$$

$$\Rightarrow \quad \frac{MR^2}{2} \times \omega_1 = \left[\frac{MR^2}{2} + \frac{MR^2}{8}\right]\omega_2$$

$$\Rightarrow \quad \frac{MR^2}{2} \times \omega_1 = \frac{5\,MR^2}{8} \times \omega_2$$

$$\Rightarrow \quad \omega_1 = \frac{5}{4}\omega_2$$

$$\Rightarrow \quad \omega_2 = \frac{4}{5}\omega_1 \qquad \ldots\text{(i)}$$

Now, the loss in rotational kinetic energy,

$$(\Delta K_{\text{sys}})_{\text{loss}} = (K_{\text{sys}})_i - (K_{\text{sys}})_f$$

$$= \left[\frac{1}{2}I_1\omega_1^2 + 0\right] - \left[\frac{1}{2}(I_1 + I_2)\omega_2^2\right]$$

$$= \left[\frac{1}{2} \times \frac{MR^2}{2} \times \omega_1^2 + 0\right] - \left[\frac{1}{2} \times \left(\frac{MR^2}{2} + \frac{MR^2}{8}\right) \times \left(\frac{4}{5}\omega_1\right)^2\right]$$

$$= \frac{MR^2\omega_1^2}{4} - \frac{1}{2} \times \frac{5MR^2}{8} \times \frac{16\,\omega_1^2}{25}$$

$$= \frac{MR^2\omega_1^2}{4} - \frac{MR^2\omega_1^2}{5}$$

$$= \frac{MR^2\omega_1^2}{20}$$

Now, the % loss in rotational kinetic energy,

$$\%(\Delta K_{\text{sys}})_{\text{loss}} = \frac{(\Delta K_{\text{sys}})_{\text{loss}}}{(K_{\text{sys}})_i} \times 100\%$$

$$= \frac{\left(\frac{MR^2\omega_1^2}{20}\right)}{\left(\frac{MR^2\omega_1^2}{4}\right)} \times 100\%$$

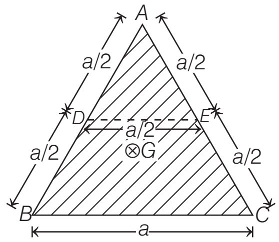
$$= \frac{4}{20} \times 100\%$$

$$= 20\%$$

and given that $\%\,(\Delta K_{\text{sys}})_{\text{loss}} = p\%$

On comparing, we get $\quad p = 20$

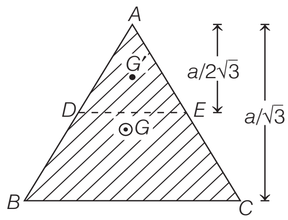
42. Let the mass of plane lamina having a shape of equilateral triangle be m and its side be a.



Then, the moment of inertia of this lamina ABC about the axis passing through its centroid G and perpendicular to its plane is

$$I_0 = \frac{ma^2}{12} \qquad \ldots\text{(i)}$$

Now, part ADE is removed.



The side of the new triangular lamina ADE is $\frac{a}{2}$.

It is also equilateral in shape.

If its centroid is G', so $AG' = \frac{a/2}{\sqrt{3}} = \frac{a}{2\sqrt{3}}$

We know that for triangular lamina ABC, the centroid is G.

So, $\quad AG = \frac{a}{\sqrt{3}}$

Now, side $G'G = AG - AG'$

$$= \frac{a}{\sqrt{3}} - \frac{a}{2\sqrt{3}}$$

$$= \frac{a}{2\sqrt{3}}$$

Now, the moment of inertia of lamina ADE about the axis passing through point G and perpendicular to its plane (using parallel axes theorem),

$$I_1 = \frac{m_1\left(\frac{a}{2}\right)^2}{12} + m_1(G'G)^2 \qquad \ldots\text{(ii)}$$

Let m_1 be the mass of lamina ADE.

$\therefore$ $$m_1 = \frac{\text{mass of lamina } ABC}{\text{area of lamina } ABC} \times \text{area of lamina}$$

$$ADE = \frac{m}{\frac{\sqrt{3}}{4}(a)^2} \times \frac{\sqrt{3}}{4}\left(\frac{a}{2}\right)^2 = \frac{m}{4}$$

So, putting all the values in Eq. (ii), we get

$$I_1 = \frac{\left(\frac{m}{4}\right)\left(\frac{a}{2}\right)^2}{12} + \left(\frac{m}{4}\right)\left(\frac{a}{2\sqrt{3}}\right)^2$$

$$= \frac{ma^2}{192} + \frac{ma^2}{48}$$

$$= \frac{5ma^2}{192} \qquad \text{...(iii)}$$

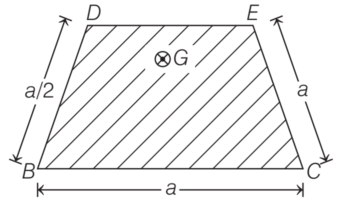
So, the moment of inertia of remaining part about the axis passing through point G and perpendicular to its plane is given by

$$I_{\text{net}} = I_0 - I_1$$

$$= \frac{ma^2}{12} - \frac{5ma^2}{192}$$

$$= \frac{11\,ma^2}{192}$$

$$= \frac{11}{16} \times \frac{ma^2}{12} = \frac{11}{16} I_0$$



It is given that, $I_{\text{net}} = \frac{N}{16} I_0$

On comparing, we get

$$N = 11$$