10

Gravitation

IN THIS CHAPTER ....

The Universal Law of Gravitation

In this universe, each body attracts other body with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. Let m_1 and m_2 be the masses of two bodies and r be the separation between them.

$$F \propto \frac{m_1 m_2}{r^2} \quad \Rightarrow \quad F = \frac{G m_1 m_2}{r^2}$$

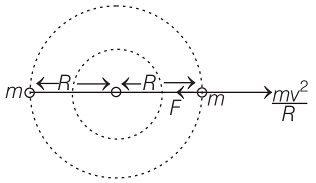
Here, G is the constant of proportionality which is called *universal gravitational constant*. The value of G is 6.67×10^{-11} N-m^{2}kg^{-2}.

Example 1. *Two particles of equal mass m go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is* **[AIEEE 2011]**

(a) $\sqrt{\frac{Gm}{R}}$ (b) $\sqrt{\frac{Gm}{4R}}$ (c) $\sqrt{\frac{Gm}{3R}}$ (d) $\sqrt{\frac{Gm}{2R}}$

Sol. (*b*) Given, masses of the two particles are $m_1 = m_2 = m$ and radius of the two particles are $r_1 = r_2 = R$. So, the gravitational force of attraction between the particles,

$$F = \frac{G \times m \times m}{R^2} \qquad \text{...(i)}$$



Centripetal force, $F = \frac{mv^2}{R}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{Gm^2}{(2R)^2} = \frac{mv^2}{R}$$

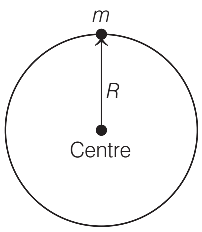
So, the speed of the each particle is $v = \sqrt{\frac{Gm}{4R}}$.

Acceleration due to Gravity

The acceleration of an object during its free fall towards the earth is called acceleration due to gravity.

If M is the mass of earth and R is the radius, the earth attracts a mass m on its surface with a force F given by

$$F = \frac{GMm}{R^2}$$



This force imparts an acceleration to the mass m which is known as acceleration due to gravity (g).

By Newton's law, we have

$$\text{Acceleration,}\quad g = \frac{F}{m} = \frac{\frac{GMm}{R^2}}{m} = \frac{GM}{R^2}$$

$$\text{On the surface of earth,}\quad g = \frac{GM}{R^2}$$

Substituting the values of G, M and R, we get

$$g = 9.81 \text{ ms}^{-2}$$

Mass of the earth, $M = 6 \times 10^{24}$ kg and radius of the earth, $R = 6.4 \times 10^6$ m.

Variation in *g* with Altitude and Depth

The value of g is variable and can vary in some cases as mentioned below

(i) **Value of acceleration due to gravity (g) at a height (h)** from the surface of the earth is given by $g' = \dfrac{gR^2}{(R+h)^2}$

If $h << R$, then $g' = g\left[1 - \dfrac{2h}{R}\right]$.

(ii) **Value of acceleration due to gravity (g) at a depth (d)** from the surface of the earth is given by

$$g' = g\left(1 - \frac{d}{R}\right)$$

At the centre of the earth, $d = R$ and hence $g' = 0$.
where, g = acceleration due to gravity on the surface.

Note It should be noted that the value of g decreases, if we move above the surface or below the surface of the earth.

Variation in the Value of *g* due to Rotation of the Earth

Due to rotation of the earth, the value of g decreases as the speed of rotation of the earth increases. The value of acceleration due to gravity at a latitude is

$$g'_\lambda = g - R\omega^2 \cos^2 \lambda$$

Following conclusions can be drawn from the above discussion

(i) The effect of centrifugal force due to rotation of the earth is to reduce the effective value of g.

(ii) The effective value of g is not truely in vertical direction.

(iii) At the equators, $\lambda = 0°$
Therefore, $g' = g - R\omega^2$ (minimum value)

(iv) At the poles, $\lambda = 90°$
Therefore, $g' = g$ (maximum value)

Example 2. *The acceleration due to gravity becomes $g/2$ (g = acceleration due to gravity on the surface of the earth) at a height equal to*

(a) $\frac{R}{4}$ (b) $\frac{R}{2}$ (c) $\frac{R}{3}$ (d) $\frac{R}{5}$

Sol. (a) The acceleration due to gravity,

$$g = \frac{GM}{R^2} \qquad \text{...(i)}$$

At a height h above the surface of the earth, the acceleration due to gravity is

$$g' = \frac{GM}{(R+h)^2} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{g}{g'} = \left(\frac{R+h}{R}\right)^2 = \left(1 + \frac{h}{R}\right)^2$$

$$\Rightarrow \quad \frac{g'}{g} = \left(1 + \frac{h}{R}\right)^{-2} = \left(1 - \frac{2h}{R}\right)$$

$$\text{Here,} \quad g' = \frac{g}{2}$$

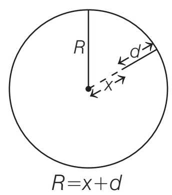
$$\therefore \quad \frac{g/2}{g} = \left(1 - \frac{2h}{R}\right)$$

$$\Rightarrow \quad \frac{2h}{R} = \frac{1}{2} \quad \text{or} \quad h = \frac{R}{4}$$

Example 3. *The value of the acceleration due to gravity is g_1 at a height $h = \frac{R}{2}$ (where, R = radius of the earth) from the surface of the earth. It is again equal to g_1 at a depth d below the surface of the earth. The ratio $\left(\frac{d}{R}\right)$ equals* **[JEE Main 2020]**

(a) $\frac{7}{9}$ (b) $\frac{1}{3}$ (c) $\frac{4}{9}$ (d) $\frac{5}{9}$

Sol. (d) Given that, acceleration due to gravity at height h from the surface of earth = acceleration due to gravity at depth d below the surface of earth = g_1



$R = x + d$
$x = R - d$

$$\Rightarrow \quad g_1 = \frac{GM}{(R+h)^2} = \frac{GM}{R^3}(R - d)$$

$$\Rightarrow \quad \frac{GM}{\left(R+\frac{R}{2}\right)^2} = \frac{GM}{R^3}(R-d) \qquad \left[\because h = \frac{R}{2}\right]$$

$$\Rightarrow \quad \frac{4GM}{9R^2} = \frac{GM}{R^2}\left(1-\frac{d}{R}\right)$$

$$\Rightarrow \quad 1-\frac{d}{R} = \frac{4}{9}$$

$$\Rightarrow \quad \frac{d}{R} = 1-\frac{4}{9} = \frac{5}{9}$$

Example 4. *A box weighs 196 N on a spring balance at the north pole. Its weight recorded on the same balance, if it is shifted to the equator is close to (Take, $g = 10\ ms^{-2}$ at the north pole and the radius of the earth = 6400 km)*

[JEE Main 2020]

(a) 195.66 N (b) 195.32 N (c) 194.66 N (d) 194.32 N

Sol. (b) Acceleration due to gravity at poles, $g_p = 10\ \text{ms}^{-2}$

Weight of box at poles, $w_p = 196$ N

So, mass of box, $m = \frac{w_p}{g_p} = 19.6$ kg

Now, due to rotation of earth acceleration due to gravity at equator,

$$g_e = g_p - R\omega^2 \qquad \text{...(i)}$$

Here, $g_p = 10\ \text{ms}^{-2}$, $R = 6400 \times 10^3$ m,

$$\omega = \frac{2\pi}{T} \text{ and } T = 24 \times 3600 \text{ s}$$

Now, substituting the given values in Eq. (i), we get

$$g_e = 10 - 0.034 = 9.966\ \text{ms}^{-2}$$

So, weight of box at equator,

$$w_e = g_e \times m = 9.966 \times 19.6 = 195.32 \text{ N}$$

Gravitational Field

The space surrounding a material body in which its gravitational force of attraction can be experienced is called its gravitational field.

Gravitational Field Intensity (E)

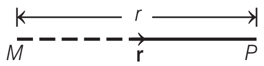
Gravitational field intensity at any point is defined as the gravitational force experienced by any test mass divided by the magnitude of test mass when placed at the desired point.

$$\mathbf{E} = \frac{\mathbf{F}_r}{m_0}$$

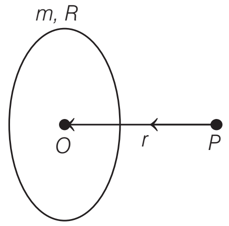
where, m_0 is a small test mass. The SI unit of gravitational intensity is Nkg^{-1}.

Gravitational Field Intensity due to Various Mass Distributions

(i) Gravitational intensity at a point P situated at a distance r from a point mass M is given by $\mathbf{E} = \frac{GM}{r^2}$



(ii) Due to a ring having a uniform mass distribution.



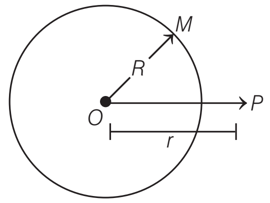
At the centre, $\mathbf{E} = 0$

On the axis, $\mathbf{E} = \frac{Gmr}{(R^2 + r^2)^{3/2}}$ [along P to O]

(iii) Due to hollow sphere having a uniform mass distribution. For inside point ($r < R$), $E = 0$. For outside point ($r \geq R$),

$$E = -\frac{GM}{r^2}$$

where, r is the distance of point from the centre.



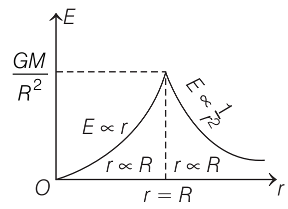
(iv) Due to a solid sphere having a uniform mass distribution.

For inside point ($r < R$),

$$E = -\frac{GMr}{R^3}$$

For outside point ($r \geq R$),

$$E = -\frac{GM}{r^2}$$



Gravitational Potential (V)

Gravitational potential at any point in a gravitational field is defined as the work done in bringing a unit mass from infinity to that point.

Gravitational potential, $V = \lim_{m_0 \to 0} \frac{W}{m_0}$

Gravitational potential due to a point mass is

$$V = -\frac{GM}{r}$$

Gravitational potential is always negative and it is maximum at infinity. It is a scalar term and its SI unit is J kg^{-1}.

Gravitational Potential due to Various Mass Distributions

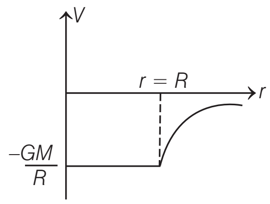
(i) Due to ring having a uniform mass distribution.

At the centre, $V = -\dfrac{GM}{R}$

On the axis, $V = -\dfrac{GM}{\sqrt{R^2 + r^2}}$

(ii) Due to a hollow sphere of radius R having a uniform mass distribution. For inside points ($r \leq R$),

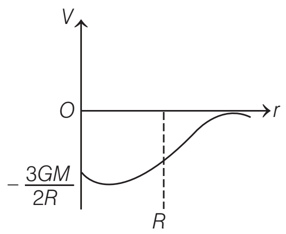
$$V = -\frac{GM}{R}$$



For outside points ($r > R$),

$$V = -\frac{GM}{r}$$

(iii) Due to a solid sphere of radius R having a uniform mass distribution.



For inside points ($r < R$),

$$V = \frac{-GM(3R^2 - r^2)}{2R^3}$$

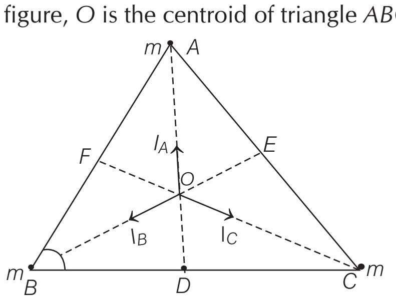
For outside points ($r \geq R$),

$$V = -\frac{GM}{r}$$

Example 5. *Three particles, each of mass m, are placed at the vertices of an equilateral triangle of side a. What is the gravitational field at the centroid of the triangle?*

(a) Zero (b) $-3\sqrt{3}\,\dfrac{Gm}{a^2}$ (c) $\dfrac{3\,Gm}{a^2}$ (d) $\dfrac{2\,Gm}{a^2}$

Sol. (a) In the figure, O is the centroid of triangle ABC,



where, $OA = \dfrac{2}{3}AD = \dfrac{2}{3}(AB\sin 60^\circ)$

$$= \frac{2}{3} \times a \times \frac{\sqrt{3}}{2} = \frac{a}{\sqrt{3}}$$

Thus, $OA = OB = OC = \dfrac{a}{\sqrt{3}}$

The gravitational intensity at O due to mass m at A is

$$E_A = \frac{Gm}{(OA)^2} = \frac{Gm}{(a/\sqrt{3})^2} \text{ along } OA$$

Similarly, the gravitational intensity at O due to mass m at B is

$$E_B = \frac{Gm}{(OB)^2} = \frac{Gm}{(a/\sqrt{3})^2} \text{ along } OB$$

and gravitational intensity at O due to mass m at C is

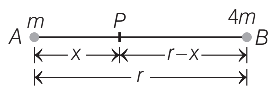
$$E_C = \frac{Gm}{(OC)^2} = \frac{Gm}{(a/\sqrt{3})^2} \text{ along } OC$$

As E_A, E_B and E_C are equal in magnitude and equally inclined to each other, the resultant gravitational intensity at O is zero.

Example 6. *Two bodies of masses m and 4 m are placed at a distance r. The gravitational potential at a point on the line joining them, where the gravitational field is zero, is* **[AIEEE 2011]**

(a) $-\dfrac{4\,Gm}{r}$ (b) $-\dfrac{6\,Gm}{r}$ (c) $-\dfrac{9\,Gm}{r}$ (d) zero

Sol. (c) Let gravitational field be zero at P as shown in figure.



$\therefore$ $\dfrac{Gm}{x^2} = \dfrac{G(4m)}{(r-x)^2}$

$\Rightarrow$ $4x^2 = (r-x)^2$

$\Rightarrow$ $2x = r - x$

$\Rightarrow$ $x = \dfrac{r}{3}$

$\therefore$ $V_P = -\dfrac{Gm}{x} - \dfrac{G(4m)}{r-x}$

$$= -\frac{Gm}{r/3} - \frac{4Gm}{r - r/3} \qquad \left[\because x = \frac{r}{3}\right]$$

$$= -\frac{3\,Gm}{r} - \frac{6\,Gm}{r} = -\frac{9\,Gm}{r}$$

Gravitational Potential Energy

Gravitational potential energy of a body or system is negative of work done by the conservative gravitational force F in bringing it from infinity to the present position. Mathematically, gravitational potential energy,

$$U = -W = -\int_{\infty}^{r} \mathbf{F} \cdot d\mathbf{r}$$

- The gravitational potential energy of two particles of masses m_1 and m_2 separated by a distance r is given by

$$U = -\frac{Gm_1 m_2}{r}$$

- The gravitational potential energy of mass m at the surface of the earth is

$$U = -\frac{GMm}{R}$$

- Difference in potential energy of mass m at a height h from the earth's surface and at the earth's surface is

$$U_{(R+h)} - U_R = \frac{mgh}{1+\frac{h}{R}} \simeq mgh \qquad \text{(if } h << R\text{)}$$

- For three particles system,

$$U = -\left[\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}\right]$$

- For n-particles system, $\frac{n(n-1)}{2}$ pairs form and total potential energy of the system is sum of potential energies of all such pairs.

Example 7. *If g is acceleration due to gravity on earth's surface, the gain of the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth, is*

(a) $2\,mgR$ (b) mgR
(c) $\frac{1}{2}mgR$ (d) $\frac{1}{4}mgR$

Sol. (c) The potential energy of an object at the surface of the earth is

$$U_1 = -\frac{GMm}{R}$$

The potential energy of the object at a height $(h = R)$ from the surface of the earth is $U_2 = -\frac{GMm}{R+h} = -\frac{GMm}{R+R}$

Gain in potential energy of the object is $\Delta U = U_2 - U_1$

$$\Delta U = -\frac{GMm}{R+R} + \frac{GMm}{R}$$

$$\Delta U = \frac{1}{2}\frac{GMm}{R}$$

As, $gR^2 = GM$

Hence, $\Delta U = \frac{1}{2}g\frac{R^2m}{R}$

$\Rightarrow$ $\Delta U = \frac{1}{2}mgR$

Example 8. *The energy required to take a satellite to a height h above earth surface (where, radius of earth $= 6.4 \times 10^3$ km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal, is* ***[JEE Main 2019]***

(a) 3.2×10^3 km (b) 1.28×10^4 km
(c) 6.4×10^3 km (d) 1.6×10^3 km

Sol. (a) The energy required for taking a satellite upto a height h from earth's surface is the difference between the energy at h height and energy at surface, then

$\Rightarrow$ $E_1 = U_f - U_i$

$$E_1 = -\frac{GM_em}{R_e+h} + \frac{GM_em}{R_e} \qquad \text{...(i)}$$

(where, U = potential energy)

$\therefore$ Orbital velocity of satellite,

$$v_o = \sqrt{\frac{GM_e}{(R_e+h)}} \qquad \text{(where, } M_e = \text{ mass of earth)}$$

So, energy required to perform circular motion,

$\Rightarrow$ $E_2 = \frac{1}{2}mv_o^2 = \frac{GM_em}{2(R_e+h)}$ ...(ii)

According to the question, $E_1 = E_2$

$\therefore$ $\frac{-GM_em}{R_e+h} + \frac{GM_em}{R_e} = \frac{GM_em}{2(R_e+h)}$

$\Rightarrow$ $3R_e = 2R_e + 2h$

$$h = \frac{R_e}{2}$$

As, radius of earth, $R_e \approx 6.4 \times 10^3$ km

Hence, $h = \frac{6.4 \times 10^3}{2}$ km or 3.2×10^3 km

Escape Velocity

It is the minimum velocity with which a body must be projected from the surface of the earth so that it escapes the gravitational field of the earth. We can also say that a body, projected with escape velocity, will be able to go to a point which is at infinite distance from the earth.

The escape velocity from the surface of a planet of mass M, radius R and acceleration due to gravity g is

$\Rightarrow$ $v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$

Substituting the values of $g = 9.81\ \text{ms}^{-2}$ and $R = 6400$ km, we get

$$v_e = 11.2\ \text{kms}^{-1}$$

Example 9. *A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 kms^{-1}, the escape velocity from the surface of the planet would be* ***[AIEEE 2008]***

(a) $1.1\ \text{kms}^{-1}$ (b) $11\ \text{kms}^{-1}$
(c) $110\ \text{kms}^{-1}$ (d) $0.11\ \text{kms}^{-1}$

Sol. (c) Mass of planet, $M_p = 10M_e$, where M_e is mass of earth.

Radius of planet, $R_p = \frac{R_e}{10}$, where R_e is radius of earth.

Escape velocity is given by $v_e = \sqrt{\frac{2GM}{R}}$

For planet, $v_p = \sqrt{\frac{2G \times M_p}{R_p}} = \sqrt{\frac{100 \times 2GM_e}{R_e}}$

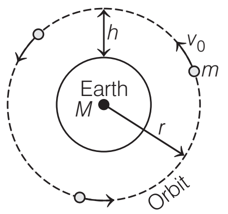
$= 10 \times v_e = 10 \times 11$
$= 110\ \text{kms}^{-1}$

Motion of a Satellite

Any body that revolves around earth or any planet is called *satellite*. These can be natural (*e.g.* moon) or artifical. The artifical satellites are man made satellites launched from the earth. The path of these satellites are elliptical with the centre of earth at a focus.

Orbital Velocity

Orbital velocity is the velocity required to put the satellite into its orbit around the earth and given as



$$\Rightarrow \quad v_o = \sqrt{\frac{GM}{r}}$$

or
$$v_o = \sqrt{\frac{GM}{R+h}}$$

where, h = height of satellite above the earth's surface.

For a satellite very close to the earth's surface, *i.e.* $h << R$ or $r \approx R$

$$v_o = \sqrt{\frac{GM}{r}} = \frac{GM}{R} = \sqrt{gR}$$

$$\therefore \quad v_e = \sqrt{2} v_o$$

Time Period

The time taken by a satellite to complete one revolution around the earth is called the *time period*. It is given by

$$T = \frac{2\pi r}{v_o} = 2\pi r\sqrt{\frac{r}{GM}}$$

$$\Rightarrow \quad T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

If $r \approx R$, then $T = 2\pi\sqrt{\frac{R}{g}}$.

Total Energy of the Satellite

Kinetic energy of satellite, $K = \frac{1}{2} m v_0^2 = \frac{GMm}{2r}$

Potential energy of satellite, $U = -\frac{GMm}{r}$

and total energy of satellite,

$$E = K + U = -\frac{GMm}{2r} = -K$$

Binding Energy of Satellite

It is the energy required to remove the satellite from its orbit and take it to infinity.

Binding energy $= -E = +\frac{GMm}{2r}$

Geostationary Satellite

If an artificial satellite revolves around the earth in an equatorial plane with a time period of 24 h in the same sense as that of the earth, then it will appear stationary to the observer on the earth. Such a satellite is known as a *geostationary satellite* or *parking satellite*.

Example 10. *A satellite is revolving in a circular orbit at a height h from the earth surface such that h << R, where R is the radius of the earth. Assuming that, the effect of earth's atmosphere can be neglected the minimum increase in the speed required, so that the satellite could escape from the gravitational field of earth is* **[JEE Main 2019]**

(a) $\sqrt{\frac{gR}{2}}$ (b) $\sqrt{gR}$

(c) $\sqrt{2gR}$ (d) $\sqrt{gR}(\sqrt{2}-1)$

Sol. (*d*) Orbital velocity of the satellite is given as $v_o = \sqrt{\frac{GM}{R+h}}$

Since, $R >> h$

$$\therefore \quad v_o = \sqrt{\frac{GM}{R}} = \sqrt{gR} \qquad \left[\because g = \frac{GM}{R^2}\right]$$

Escape velocity of the satellite,

$$v_e = \sqrt{\frac{2GM}{R+h}} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

Since, we know that in order to escape the earth's gravitational field a satellite must get escape velocity.

$\therefore$ Change in velocity,

$$\Delta v = v_e - v_o = \sqrt{gR}(\sqrt{2}-1)$$

Example 11. *A 400 kg satellite is in a circular orbit of radius $2R_E$ about the earth. Amount of energy required to transfer it to a circular orbit of radius $4R_E$ is*

(a) 3.13×10^9 J

(b) 5.29×10^2 J

(c) 3.13×10^3 J

(d) 5.29×10^9 J

Sol. (a) Initially, $E_i = -\frac{GM_E m}{4R_E}$

While finally, $E_f = -\frac{GM_E m}{8R_E}$

The change in the total energy is

$$\Delta E = E_f - E_i = \frac{GM_E m}{8R_E} = \left(\frac{GM_E}{R_E^2}\right)\frac{mR_E}{8}$$

$$\Delta E = \frac{gmR_E}{8} = \frac{9.81 \times 400 \times 6.37 \times 10^6}{8}$$

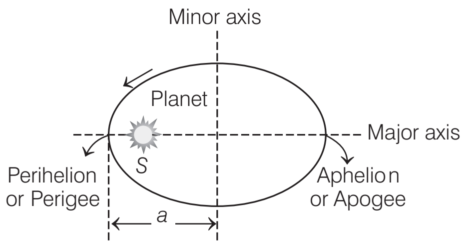
$$\Delta E = 3.13 \times 10^9 \text{ J}$$

Kepler's Laws of Planetary Motion

Kepler discovered three empirical laws which accurately describe the motion of the planets. These laws are

Kepler's First Law or Law of Orbits

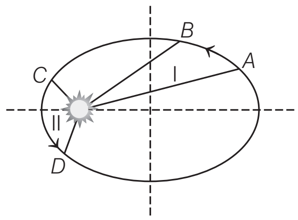
All the planets move around the sun in an elliptical orbit with sun at one of the focus of ellipse.



The point when the planet is nearest to the sun is termed as **perihelion** and the farthest one is known as **aphelion**.

Kepler's Second Law or Law of Areas

The line joining the sun to the planet sweeps out equal areas in equal intervals of time, *i.e.* areal velocity of the planet w.r.t. sun is constant.



This law indicates that a planet moves faster near the sun and slowly when away from the sun.

According to second law, area of region I = area of region II, where the planet takes same time to move from A to B and from C to D.

Kepler's Third Law or Law of Periods

The square of the planet's time period of revolution is directly proportional to the cube of semi-major axis of its orbit.

$$T^2 \propto a^3$$

where, a is the semi-major axis.

Example 12. *The maximum and minimum distances of a comet from the sun are* 8×10^{12} *m and* 1.6×10^{12} *m. If its velocity when nearest to the sun is* $60\ ms^{-1}$*. What will be its velocity (in* ms^{-1}*) when it is farthest?*

(a) 12 (b) 60
(c) 112 (d) 6

Sol. (a) By conservation of angular momentum,

$$mvr = \text{constant}$$

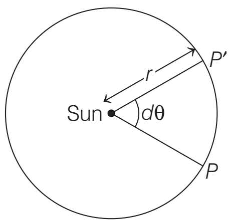
i.e. $\quad v_{\text{min}} \times r_{\text{max}} = v_{\text{max}} \times r_{\text{min}}$

$$\therefore \quad v_{\text{min}} = \frac{60 \times 1.6 \times 10^{12}}{8 \times 10^{12}} = \frac{60}{5} = 12\ \text{ms}^{-1}$$

Example 13. *If the angular momentum of a planet of mass m, moving around sun in a circular orbit is L about the centre of the sun , its areal velocity is* **[JEE Main 2019]**

(a) $\frac{4L}{m}$ (b) $\frac{2L}{m}$ (c) $\frac{L}{2m}$ (d) $\frac{L}{m}$

Sol. (c) According to Kepler's second law, the line joining the planet to sun sweeps out equal areas in equal interval of time. This means the rate of change of area with time is constant.



The area covered from P to P' is dA, which is given by

$$dA = \frac{d\theta}{2\pi} \times \pi r^2$$

$$\Rightarrow \quad dA = \frac{1}{2} r^2 d\theta \text{ or } \frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$$

where, $\frac{dA}{dt}$ = areal velocity.

$$\frac{dA}{dt} = \frac{1}{2} r^2 \omega \quad \text{or} \quad \frac{dA}{dt} = \frac{1}{2} r^2 \cdot \frac{L}{mr^2}$$

(because angular momentum, $L = mr^2\omega$)

$$\frac{dA}{dt} = \frac{L}{2m}$$

Practice Exercise

ROUND I Topically Divided Problems

Universal Law of Gravitation

1. Imagine a light planet revolving around a very massive star in a circular orbit of radius r with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-3/2}$, then T^2 is proportional to

(a) R^3 (b) $R^{5/2}$
(c) $R^{3/2}$ (d) $R^{7/2}$

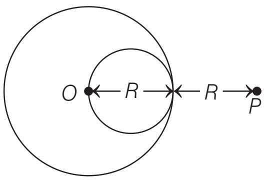
2. If a planet of given density were made larger its force of attraction for an object on its surface would increase because of planet's greater mass but would decrease because of the greater distance from the object to the centre of the planet. Which effect predominate?

(a) Increase in mass
(b) Increase in radius
(c) Both affect attraction equally
(d) None of the above

3. A uniform ring of mass M and radius r is placed directly above a uniform sphere of mass $8M$ and of same radius R. The centre of the ring is at a distance of $d = \sqrt{3}R$ from the centre of the sphere. The gravitational attraction between the sphere and the ring is

(a) $\dfrac{GM^2}{R^2}$ (b) $\dfrac{3GM^2}{2R^2}$
(c) $\dfrac{2GM^2}{\sqrt{2}R^2}$ (d) $\dfrac{\sqrt{3}GM^2}{R^2}$

4. A solid sphere of uniform density and radius R applies a gravitational force of attraction equal to F_1 on a particle placed at P, distance $2R$ from the centre O of the sphere. A spherical cavity of radius $R/2$ is now made in the sphere as shown in figure. The sphere with cavity now applies an gravitational force F_2 on same particle placed at P. The ratio F_2/F_1 will be



(a) 1/2 (b) 7/9
(c) 3 (d) 7

5. Both earth and moon are subject to the gravitational force of the sun. As observed from the sun, the orbit of the moon **[NCERT Exemplar]**

(a) will be elliptical
(b) will not be strictly elliptical because the total gravitational force on it is not central
(c) is not elliptical but will necessarily be a closed curve
(d) deviates considerably from being elliptical due to influence of planets other than earth

6. Different points in earth are at slightly different distances from the sun and hence experience different forces due to gravitation. For a rigid body, we know that if various forces act at various points in it, the resultant motion is as if a net force acts on the CM (centre of mass) causing translation and a net torque at the CM causing rotation around an axis through the CM for the earth-sun system (approximating the earth as a uniform density sphere) **[NCERT Exemplar]**

(a) the torque is zero
(b) the torque causes the earth to spin
(c) the rigid body result is not applicable since the earth is not even approximately a rigid body
(d) the torque causes the earth to move around the sun

7. Two astronauts have deserted their spaceship in a region of space far from the gravitational attraction of any other body. Each has a mass of 100 kg and they are 100 m apart. They are initially at rest relative to one another. How long will it be before the gravitational attraction brings them 1 cm closer together?

(a) 2.52 days (b) 1.41 days (c) 0.70 days (d) 0.41 days

8. Particles of masses $2M$, m and M are respectively at points A, B and C with $AB = \dfrac{1}{2}(BC)$. m is much-much smaller than M and at time $t = 0$, they are all at rest

(a) m will remain at rest **[NCERT Exemplar]**
(b) m will move towards M
(c) m will move towards $2M$
(d) m will have oscillatory motion

9. A spherical hollow is made in a lead sphere of radius R such that its surface touches the outside surface of the lead sphere and passes through the centre. The mass of the lead sphere before hollowing was M. The force of attraction that this sphere would exert on a particle of mass m which lies at a distance d (> R) from the centre of the lead sphere on the straight line joining the centres of the sphere and the hollow is

(a) $\frac{GMm}{d^2}$ (b) $\frac{GMm}{8d^2}$

(c) $\frac{GMm}{d^2}\left[1+\frac{1}{8\left(1+\frac{R}{2d}\right)}\right]$ (d) $\frac{GMm}{d^2}\left[1-\frac{1}{8\left(1-\frac{R}{2d}\right)^2}\right]$

10. A straight rod of length L extends from $x = a$ to $x = L + a$. The gravitational force it exerts on a point mass m at $x = 0$, if the mass per unit length of the rod is $A + Bx^2$, is given by **[JEE Main 2019]**

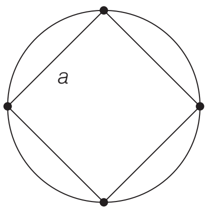
(a) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)-BL\right]$

(b) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)+BL\right]$

(c) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)+BL\right]$

(d) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)-BL\right]$

11. Four identical particles of mass M are located at the corners of a square of side a. What should be their speed, if each of them revolves under the influence of other's gravitational field in a circular orbit circumscribing the square? **[JEE Main 2019]**



(a) $1.35\sqrt{\frac{GM}{a}}$ (b) $1.16\sqrt{\frac{GM}{a}}$

(c) $1.21\sqrt{\frac{GM}{a}}$ (d) $1.41\sqrt{\frac{GM}{a}}$

Acceleration due to Gravity

12. The acceleration due to gravity on a planet is $1.96\ \text{ms}^{-2}$. If it is safe to jump from a height of 3 m on the earth, the corresponding height on the planet will be

(a) 3 m (b) 6 m (c) 9 m (d) 15 m

13. A thief stole a box full of valuable articles of weight w and while carrying it on his head jumped down from a wall of height h from the ground. Before he reaches the ground, he experienced a load

(a) zero (b) $w/2$ (c) w (d) $2w$

14. The mass of the moon is 1/8 of the earth but the gravitational pull is 1/6 of the earth. It is due to the fact that

(a) moon is the satellite of the earth
(b) the radius of the earth is 8.6 the moon
(c) the radius of the earth is $\sqrt{8/6}$ of the moon
(d) the radius of the moon is 6/8 of the earth

15. Average density of the earth **[AIEEE 2005]**

(a) does not depend on g
(b) is a complex function of g
(c) is directly proportional to g
(d) is inversely proportional to g

16. The change in the value of g at a height h above the surface of the earth is the same as at a depth d below the surface of earth. When both d and h are much smaller than the radius of earth, then which one of the following is correct? **[AIEEE 2005]**

(a) $d = \frac{h}{2}$ (b) $d = \frac{3h}{2}$ (c) $d = 2h$ (d) $d = h$

17. The earth is an approximate sphere. If the interior contained matter which is not of the same density everywhere, then on the surface of the earth, the acceleration due to gravity **[NCERT Exemplar]**

(a) will be directed towards the centre but not the same everywhere
(b) will have the same value everywhere but not directed towards the centre
(c) will be same everywhere in magnitude directed towards the centre
(d) cannot be zero at any point

18. There is a mine of depth about 2.0 km. In this mine the conditions as compared to those at the surface are

(a) lower air pressure, higher acceleration due to gravity
(b) higher air pressure, lower acceleration due to gravity
(c) higher air pressure, higher acceleration due to gravity
(d) lower air pressure, lower acceleration due to gravity

19. At a given place where, acceleration due to gravity is $g\ \text{ms}^{-2}$, a sphere of lead of density $d\ \text{kgm}^{-3}$ is gently released in a column of liquid of density $\rho\ \text{kgm}^{-3}$. If $d > \rho$, the sphere will

(a) fall vertically with an acceleration of $g\ \text{ms}^{-2}$
(b) fall vertically with no acceleration
(c) fall vertically with an acceleration $g\left(\frac{d-\rho}{d}\right)$
(d) fall vertically with an acceleration ρ/d

20. At a distance 320 km above the surface of earth, the value of acceleration due to gravity will be lower than its value on the surface of the earth by nearly (radius of earth = 6400 km)

(a) 2% (b) 6%
(c) 10% (d) 14%

21. The acceleration due to gravity at a height (1/20)th of the radius of the earth above the earth surface is 9 ms^{-2}. Its value at a point at an equal distance below the surface of the earth (in ms^{-2}) is about

(a) 8.5 (b) 9.5
(c) 9.8 (d) 11.5

22. If the value of g acceleration due to gravity at earth surface is 10 ms^{-2}, its value in ms^{-2} at the centre of the earth, which is assumed to be a sphere of radius R metre and uniform mass density is

(a) 5 (b) $10/R$
(c) $10/2R$ (d) zero

23. The height at which the acceleration due to gravity decreases by 36% of its value on the surface of the earth. (The radius of the earth is R)

(a) $\frac{R}{6}$ (b) $\frac{R}{4}$
(c) $\frac{R}{2}$ (d) $\frac{2}{3}R$

24. Two spherical planets A and B have same mass but densities in the ratio 8 : 1. For these planets, the ratio of acceleration due to gravity at the surface of A to its value at the surface of B is

(a) 1 : 4 (b) 1 : 2
(c) 4 : 1 (d) 8 : 1

25. If the radius of the earth were to shrink by 1% and its mass remaining same, the acceleration due to gravity on the earth's surface would

(a) decrease by 2%
(b) remain unchanged
(c) increase by 2%
(d) become zero

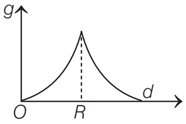
26. If the force inside the earth surface varies as r^n, where r is the distance of body from the centre of earth, then the value of n will be

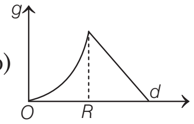
(a) –1 (b) –2
(c) 1 (d) 2

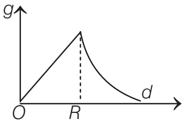
27. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where, g is the acceleration due to gravity on the surface of the earth) in terms of R (the radius of the earth) is **[AIEEE 2009]**

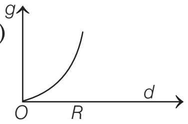
(a) $2R$ (b) $\frac{R}{\sqrt{3}}$
(c) $\frac{R}{2}$ (d) $\sqrt{2}R$

28. The variation of acceleration due to gravity g with distance d from centre of the Earth is best represented by (R = Earth's radius) **[JEE Main 2017]**

(a) 

(b) 

(c) 

(d) 

29. The depth from the surface of the earth of radius R at which the acceleration due to gravity will be 75% of the value on the surface of the earth is

(a) $R/4$ (b) $R/2$ (c) $3R/4$ (d) $R/8$

30. Two equal masses m and m are hung from a balance whose scale pans differ in height by h. If ρ is the mean density of earth, then the error in weighing machine is

(a) zero (b) $4\pi G\rho mh/3$
(c) $8\pi G\rho mh/3$ (d) $2\pi G\rho mh/3$

31. The speed of earth's rotation about its axis is ω. Its speed is increased to x times to make the effective acceleration due to gravity equal to zero at the equator, then value of x is around
(g = 10 ms^{-2}, R = 6400 km)

(a) 1 (b) 8.5 (c) 17 (d) 34

32. The bodies situated on the surface of earth at its equator, becomes weightless, when the earth has KE about it axis

(a) mgR (b) $2\,mgR/5$ (c) $MgR/5$ (d) $5\,MgR/2$

33. The height h at which the weight of a body will be the same as that at the same depth h from the surface of the earth is
(Radius of the earth is R and effect of the rotation of the earth is neglected) **[JEE Main 2020]**

(a) $\frac{\sqrt{5}}{2}(R-R)$ (b) $\frac{R}{2}$
(c) $\frac{\sqrt{5}R-R}{2}$ (d) $\frac{\sqrt{3}R-R}{2}$

34. At what height above the earth's surface, does the force of gravity decrease by 10%? The radius of the earth is 6400 km?

(a) 345.60 km (b) 687.20 km
(c) 1031.8 km (d) 12836.80 km

35. The ratio of acceleration due to gravity at a height h above the surface of the earth and at a depth h below the surface of the earth for h < radius of earth

(a) is constant
(b) increases linearly with h
(c) decreases linearly with h
(d) decreases parabolically with h

36. If a man weights 90 kg on the surface of earth, the height above the surface of the earth of radius R, where the weight is 30 kg, is

(a) $0.73\,R$ (b) $R/\sqrt{3}$
(c) $R/3$ (d) $\sqrt{3}\,R$

37. The ratio of the weights of a body on the earth's surface, so that on the surface of a planet is 9 : 4. The mass of the planet is $\frac{1}{9}$th of that of the earth. If R is the radius of the earth, what is the radius of the planet? (Take, the planets to have the same mass density) **[JEE Main 2019]**

(a) $\frac{R}{3}$ (b) $\frac{R}{4}$ (c) $\frac{R}{9}$ (d) $\frac{R}{2}$

38. The value of acceleration due to gravity at earth's surface is $9.8\ \text{ms}^{-2}$. The altitude above its surface at which the acceleration due to gravity decreases to $4.9\ \text{ms}^{-2}$, is close to (Take, radius of earth $= 6.4\times 10^6$ m) **[JEE Main 2019]**

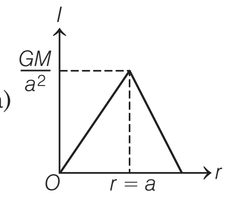
(a) 9.0×10^6 m (b) 2.6×10^6 m
(c) 6.4×10^6 m (d) 1.6×10^6 m

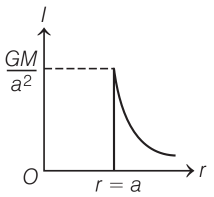
Gravitational Field, Potential and Potential Energy

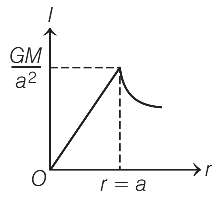
39. The gravitational field due to a mass distribution is $E = k/x^3$ in the x-direction (k is a constant). Taking the gravitational potential to be zero at infinity, its value at a distance $x/\sqrt{2}$ is

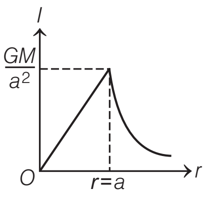
(a) k/x (b) $k/2x$
(c) k/x^2 (d) $k/2x^2$

40. Which of the following graphs represents correctly the variation of the intensity of gravitational field (I) with the distance (r) from the centre of a spherical shell of mass M and radius a?

(a)


(b)


(c)


(d)


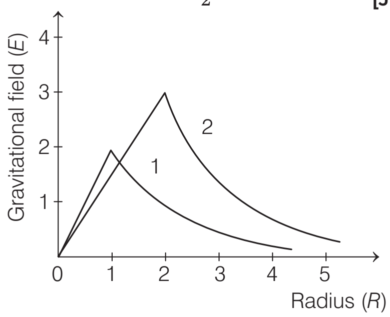
41. Two bodies of masses 2 kg and 8 kg are separated by a distance of 9 m. The point where the resultant gravitational field intensity is zero is at a distance of

(a) 4.5 m from each mass (b) 6 m from 2 kg
(c) 6 m from 8 kg (d) 2.5 m from 2 kg

42. There are two bodies of masses 100000 kg and 1000 kg separated by a distance of 1 m. At what distance (in metre) from the smaller body, the intensity of gravitational field will be zero?

(a) 1/9 (b) 1/10
(c) 1/11 (d) 10/11

43. Consider two solid spheres of radii $R_1 = 1$ m, $R_2 = 2$ m and masses M_1 and M_2, respectively. The gravitational field due to spheres 1 and 2 are shown. The value of $\frac{M_1}{M_2}$ is **[JEE Main 2020]**



(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{2}{3}$

44. In a certain region of space, the gravitational field is given by $-k/r$, where r is the distance and k is a constant. If the gravitational potential at $r = r_0$ be V_0, then what is the expression for the gravitational potential V?

(a) $k\log\left(\frac{r}{r_0}\right)$ (b) $k\log\left(\frac{r_0}{r}\right)$

(c) $V_0 + k\log\left(\frac{r}{r_0}\right)$ (d) $V_0 + k\log\left(\frac{r_0}{r}\right)$

45. A particle of mass m is placed at the centre of a uniform spherical shell of mass $3\,m$ and radius R. The gravitational potential on the surface of the shell is

(a) $-\frac{Gm}{R}$ (b) $-\frac{3\,Gm}{R}$

(c) $-\frac{4\,Gm}{R}$ (d) $-\frac{2\,Gm}{R}$

46. The kinetic energy needed to project a body of mass m from the earth's surface (radius R) to infinity is **[AIEEE 2002]**

(a) $\frac{mgR}{2}$ (b) $2\,mgR$ (c) mgR (d) $\frac{mgR}{4}$

47. Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is **[AIEEE 2002]**

(a) $\frac{GMm}{12R^2}$ (b) $\frac{GMm}{3R^2}$

(c) $\frac{GMm}{8R}$ (d) $\frac{GMm}{6R}$

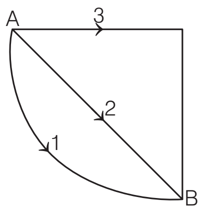
48. A mass m is placed at a point B in the gravitational field of mass M. When the mass m is brought from B to near point A, its gravitational potential energy will

(a) remain unchanged (b) increase
(c) decrease (d) become zero

49. The gravitational potential difference between the surface of a planet and a point 20 m above it is 14 J kg^{-1}. The work done in moving a 2.0 kg mass by 8.0 m on a slope of 60° from the horizontal, is equal to

(a) 7 J (b) 9.6 J
(c) 16 J (d) 32 J

50. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 respectively (as shown) in a gravitional field of point mass m, then



(a) $W_1 = W_2 = W_3$
(b) $W_1 > W_2 > W_3$
(c) $W_1 > W_2 < W_3$
(d) $W_1 < W_2 < W_3$

51. The mass of the earth is 6.00×10^{22} kg. The constant of gravitation $G = 6.67 \times 10^{-11}\,\text{Nm}^2\text{kg}^{-2}$. The potential energy of the system is -7.79×10^{28} J. The mean distance between earth and moon is

(a) 3.80×10^8 m
(b) 3.37×10^6 m
(c) 7.60×10^4 m
(d) 1.90×10^2 m

52. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth, is **[AIEEE 2004]**

(a) $2\,mgR$ (b) $\frac{1}{2}\,mgR$
(c) $\frac{1}{4}\,mgR$ (d) mgR

53. The change in potential energy when a body of mass m is raised to a height nR from the centre of earth (R = radius of earth)

(a) $mgR\dfrac{(n-1)}{n}$ (b) $nmgR$
(c) $mgR\dfrac{n^2}{n^2+1}$ (d) $mgR\dfrac{n}{n+1}$

54. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them, to take the particle far away from the sphere. (Take, $G = 6.67 \times 10^{-11}\,\text{Nm}^2/\text{kg}^2$)

(a) 13.34×10^{-10} J **[AIEEE 2005]**
(b) 3.33×10^{-10} J
(c) 6.67×10^{-9} J
(d) 6.67×10^{-10} J

55. A body of mass m rises to a height $h = R/5$ from the surface of earth, where R is the radius of earth. If g is the acceleration due to gravity at the surface of earth, the increase in potential energy is

(a) (4/5) mgh (b) (5/6) mgh
(c) (6/7) mgh (d) mgh

Motion of Satellite and Escape Velocity

56. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$? **[JEE Main 2013]**

(a) $\dfrac{5\,GmM}{6R}$ (b) $\dfrac{2\,GmM}{3R}$
(c) $\dfrac{GmM}{2R}$ (d) $\dfrac{GmM}{3R}$

57. A solid sphere of mass M and radius a is surrounded by a uniform concentric spherical shell of thickness $2a$ and $2M$. The gravitational field at distance $3a$ from the centre will be **[JEE Main 2019]**

(a) $\dfrac{GM}{9a^2}$ (b) $\dfrac{2GM}{9a^2}$
(c) $\dfrac{GM}{3a^2}$ (d) $\dfrac{2GM}{3a^2}$

58. Two planets have masses M and 16 M and their radii are a and $2a$, respectively. The separation between the centres of the planets is $10a$. A body of mass m is fired from the surface of the larger planet towards the smaller planet along the line joining their centres. For the body to be able to reach at the surface of smaller planet, the minimum firing speed needed is **[JEE Main 2020]**

(a) $2\sqrt{\dfrac{GM}{a}}$ (b) $4\sqrt{\dfrac{GM}{a}}$
(c) $\sqrt{\dfrac{GM^2}{ma}}$ (d) $\dfrac{3}{2}\sqrt{\dfrac{5\,GM}{a}}$

59. The ratio of the radii of the planets P_1 and P_2 is a. The ratio of their acceleration due to gravity is b. The ratio of the escape velocities from them will be

(a) ab (b) $\sqrt{ab}$
(c) $\sqrt{a/b}$ (d) $\sqrt{b/a}$

60. The escape velocity for a body projected vertically upwards from the surface of the earth is 11.2 kms^{-1}. If the body is projected in a direction making an angle of 45° with the vertical, the escape velocity will be

(a) 11.2 kms^{-1} (b) $11.2 \times \sqrt{2}$ kms^{-1}
(c) 11.2×2 kms^{-1} (d) $11.2/\sqrt{2}$ kms^{-1}

61. The escape velocity of a body depends upon mass as [AIEEE 2002]

(a) m^0 (b) m^1
(c) m^2 (d) m^3

62. The time period of an earth satellite in circular orbit is independent of [AIEEE 2004]

(a) the mass of the satellite
(b) radius of its orbit
(c) both the mass and radius of the orbit
(d) neither the mass of the satellite nor the radius of its orbit

63. If suddenly the gravitational force of attraction between earth and a satellite revolving around it becomes zero, then the satellite will [AIEEE 2002]

(a) continue to move in its orbit with same velocity
(b) move tangentially to the original orbit with the same velocity
(c) become stationary in its orbit
(d) move towards the earth

64. A satellite S is moving in an elliptical orbit around earth. The mass of the satellite is very small compared to the mass of the earth?

(a) The acceleration of S is always directed towards the centre of the earth
(b) The angular momentum of S about the centre of the earth changes in direction but its magnitude remains constant
(c) The total mechanical energy of S varies periodically with time
(d) The linear momentum of S remains constant in magnitude

65. A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass falling towards the earth collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same just before the collision. The subsequent motion of the combined body will be [JEE Main 2019]

(a) in the same circular orbit of radius R
(b) in an elliptical orbit
(c) such that it escapes to infinity
(d) in a circular orbit of a different radius

66. Satellites orbiting the earth have finite life and sometimes debris of satellites fall to the earth. This is because, [NCERT Exemplar]

(a) the solar cells and batteries in satellites run out
(b) the laws of gravitation predict a trajectory spiralling inwards
(c) of viscous forces causing the speed of satellite and hence height to gradually decrease
(d) of collisions with other satellites

67. The escape velocity from the earth is 11 kms^{-1}. The escape velocity from a planet having twice the radius and the same mean density as the earth would be

(a) 5.5 kms^{-1} (b) 11 kms^{-1}
(c) 15.5 kms^{-1} (d) 22 kms^{-1}

68. The mass of the moon is 1/81 of earth's mass and its radius 1/4th that of the earth. If the escape velocity from the earth's surface is 11.2 kms^{-1}, its value for the moon will be

(a) 0.15 kms^{-1} (b) 5 kms^{-1}
(c) 2.5 kms^{-1} (d) 0.5 kms^{-1}

69. Two satellites A and B have masses m and $2m$, respectively. A is in a circular orbit of radius R and B is in a circular orbit of radius $2R$ around the earth. The ratio of their kinetic energies, T_A / T_B is [JEE Main 2019]

(a) $\frac{1}{2}$ (b) 2 (c) $\sqrt{\frac{1}{2}}$ (d) 1

70. A satellite is revolving in a circular orbit at a height h from the Earth's surface (radius of Earth $R, h << R$). The minimum increase in its orbital velocity required, so that the satellite could escape from the Earth's gravitational field, is close to (Neglect the effect of atmosphere) [JEE Main 2017]

(a) $\sqrt{2gR}$ (b) $\sqrt{gR}$
(c) $\sqrt{gR/2}$ (d) $\sqrt{gR}\,(\sqrt{2}-1)$

71. Planet A has mass M and radius R and planet B has half the mass and half the radius of planet A. If the escape velocities from the planets A and B are v_A and v_B respectively, then $\frac{v_A}{v_B} = \frac{n}{4}$. The value of n is [JEE Main 2020]

(a) 1 (b) 2 (c) 3 (d) 4

72. A satellite is in an elliptical orbit around a planet P. It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is [JEE Main 2020]

(a) 1 : 6 (b) 1 : 3 (c) 1 : 2 (d) 3 : 4

73. A body is moving in a low circular orbit about a planet of mass M and radius R. The radius of the orbit can be taken to be R itself. Then, the ratio of the speed of this body in the orbit to the escape velocity from the planet is [JEE Main 2020]

(a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) 2

74. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is **[AIEEE 2004]**

(a) gx (b) $\frac{gR}{R-x}$

(c) $\frac{gR^2}{R+x}$ (d) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

75. A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass m is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is **[JEE Main 2019]**

(a) $\frac{3}{2}mv^2$ (b) $2mv^2$

(c) mv^2 (d) $\frac{1}{2}mv^2$

Kepler's Laws

76. If the radius of earth's orbit is made (1/4)th, then duration of an year will become

(a) 8 times (b) 4 times

(c) (1/8) times (d) (1/4) times

77. In our solar system, the inter-planetary region has chunks of matter (much smaller in size compared to planets) called asteroids. They **[NCERT Exemplar]**

(a) will not move around the sun since they have very small masses compared to sun

(b) will move in an irregular way because of their small masses and will drift away into outer space

(c) will move around the sun in closed orbits but not obey Kepler's laws

(d) will move in orbits like planets and obey Kepler's laws

78. The time period of a satellite of earth is 5 h. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become **[AIEEE 2003]**

(a) 10 h (b) 80 h (c) 40 h (d) 20 h

79. A geostationary satellite is orbiting around an arbitrary planet P at a height of 11 R above the surface of P, R being the radius of P. The time period of another satellite (in h) at a height of $2R$ from the surface of P is .... P has the time period of 24 h. **[JEE Main 2021]**

(a) $6\sqrt{2}$ (b) $\frac{6}{\sqrt{2}}$ (b) 3 (d) 5

80. The period of revolution of planet A around the sun is 8 times that B. The distance of A from the sun is how many times greater than that of B from the sun?

(a) 2 (b) 3 (c) 4 (d) 5

81. The largest and the shortest distance of the earth from the sun are r_1 and r_2, its distance from the sun when it is perpendicular to the major axis of the orbit drawn from the sun, is

(a) $\frac{r_1+r_2}{4}$ (b) $\frac{r_1 r_2}{r_1+r_2}$ (c) $\frac{2r_1 r_2}{r_1+r_2}$ (d) $\frac{r_1+r_2}{3}$

82. A comet of mass m moves in a highly elliptical orbit around the sun of mass M. The maximum and minimum distances of the comet from the centre of the sun are r_1 and r_2 respectively. The magnitude of angular momentum of the comet with respect to the centre of sun is

(a) $\left[\frac{GMr_1}{(r_1+r_2)}\right]^{1/2}$ (b) $\left[\frac{GMmr_1}{(r_1+r_2)}\right]^{1/2}$

(c) $\left[\frac{2Gm^2 r_1 r_2}{r_1+r_2}\right]^{1/2}$ (d) $\left[\frac{2GMm^2 r_1 r_2}{(r_1+r_2)}\right]^{1/2}$

ROUND II Mixed Bag

Only One Correct Option

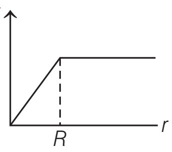
1. A rocket of mass M is launched vertically from the surface of the earth with an initial speed v. Assuming the radius of the earth to be R and negligible air resistance. The maximum height attained by the rocket above the surface of the earth,

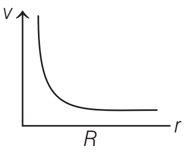
(a) $R\Big/\left(\frac{gR}{2v^2}-1\right)$ (b) $R\left(\frac{gR}{2v^2}-1\right)$

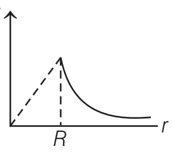
(c) $R\Big/\left(\frac{2gR}{v^2}-1\right)$ (d) $R\left(\frac{2gR}{v^2}-1\right)$

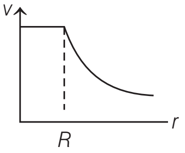
2. A spherical symmetric gravitational system of particles has a mass density $\rho = \begin{cases} \rho_0 & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$

where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a function of distance $r\,(0 < r < \infty)$ from the centre of the system is represent by

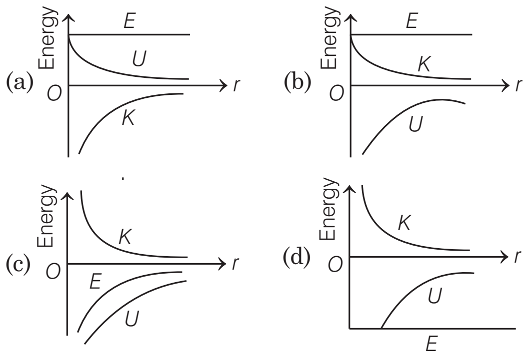
(a)


(b)


(c)


(d)


3. The correct graph representing the variation of total energy (E), kinetic energy (K) and potential energy (U) of a satellite with its distance from the centre of earth is



4. A simple pendulum has a time period T_1 when on the earth's surface and T_2 when taken to a height $2R$ above the earth's surface when R is $2R$ above earth's surface where R is the radius of the earth. The value of (T_1/T_2) is **[Kerala CET 2006]**

(a) 1/9 (b) 1/3
(c) $\sqrt{3}$ (d) 9

5. The escape velocity on the surface of earth is $11.2\ kms^{-1}$. If mass and radius of a planet is 4 and 2 times respectively, than that of earth, the escape velocity on the planet **[BVP Engg. 2006]**

(a) $11.2\ kms^{-1}$
(b) $1.12\ kms^{-1}$
(c) $22.4\ kms^{-1}$
(d) $15.8\ kms^{-1}$

6. The time period of a geostationary satellite at a height 36000 km is 24 h. A spy satellite orbits very close to earth surface (R = 6400 km). What will be its time period? **[Orissa JEE 2008]**

(a) 4 h (b) 1 h
(c) 2 h (d) 1.5 h

7. Infinite number of masses, each of 1 kg are placed along the x-axis at $x = +1\,m, \pm 2\,m_1, \pm 4\,m, \pm 8\,m, \pm 16\,m \ldots$ The magnitude of the resultant gravitational potential in terms of gravitational constant G at the origin $(x = 0)$ is **[Kerala CET 2008]**

(a) $G/2$ (b) G
(c) $2\,G$ (d) $4\,G$

8. Halley's comet has a period of 76, had a distance of closest approach to the sun equal to 8.9×10^{10} m. The comet's farthest distance from the sun, if the mass of sun is 2×10^{30} kg and $g = 6.67 \times 10^{11}$ in MKS units is

(a) 2×10^{12} m
(b) 2.7×10^{13} m
(c) 5.3×10^{12} m
(d) 5.3×10^{13} m

9. The potential energy of gravitational interaction of a point mass m and a thin uniform rod of mass M and length l, if they are located along a straight line at distance a from each other is

(a) $U = \dfrac{GMm}{a} \log_e \left(\dfrac{a+l}{a}\right)$

(b) $U = GMm \left(\dfrac{1}{a} - \dfrac{1}{a+l}\right)$

(c) $U = \dfrac{GMm}{l} \log_e \left(\dfrac{a+l}{a}\right)$

(d) $U = -\dfrac{GMm}{a}$

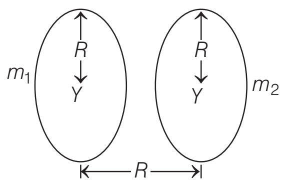
10. The work that must be done in lifting a body of weight P from the surface of the earth to a height h is

(a) $\dfrac{PRh}{R-h}$ (b) $\dfrac{R+h}{PRh}$
(c) $\dfrac{PRh}{R+h}$ (d) $\dfrac{R-h}{PRh}$

11. A spaceship is launched into a circular orbit close to earth's surface. The additional velocity that should be imparted to the spaceship in the orbit to overcome the gravitational pull is (Radius of earth = 6400 km and $g = 9.8\ ms^{-2}$)

(a) $11.2\ kms^{-1}$ (b) $8\ kms^{-1}$
(c) $3.2\ kms^{-1}$ (d) $1.5\ kms^{-1}$

12. Two identical thin rings each of radius R are coaxially placed at a distance R. If the rings have a uniform mass distribution and each has mass m_1 and m_2 respectively, then the work done in moving a mass m from centre of one ring to that of the other is

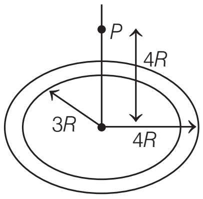


(a) $\dfrac{Gmm_1(\sqrt{2}+1)}{m_2R}$ (b) $\dfrac{Gm\,(m_1-m_2)(\sqrt{2}-1)}{\sqrt{2}\,R}$
(c) $\dfrac{Gm\sqrt{2}\,(m_1+m_2)}{R}$ (d) zero

13. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction, the speed of each particle is **[JEE Main 2014]**

(a) $\sqrt{\dfrac{GM}{R}}$ (b) $\sqrt{2\sqrt{2}\,\dfrac{GM}{R}}$
(c) $\sqrt{\dfrac{GM}{R}(1+2\sqrt{2})}$ (d) $\dfrac{1}{2}\sqrt{\dfrac{GM}{R}(1+2\sqrt{2})}$

14. A thin uniform annular disc (see figure) of mass M has outer radius $4R$ and inner radius $3R$. The work required to take a unit mass from point P on its axis to infinity is



(a) $\frac{2GM}{7R}(4\sqrt{2}-5)$ (b) $-\frac{2GM}{7R}(4\sqrt{2}-5)$

(c) $\frac{GM}{4R}$ (d) $\frac{2GM}{5R}(\sqrt{2}-1)$

15. If satellite is revolving around a planet of mass M in an elliptical orbit of semi-major axis a, find the orbital speed of the satellite when it is at a distance r from the focus.

(a) $v^2 = GM\left[\frac{2}{r}-\frac{1}{a}\right]$ (b) $v^2 = GM\left[\frac{2}{r^2}-\frac{1}{a}\right]$

(c) $v^2 = GM\left[\frac{2}{r^2}-\frac{1}{a^2}\right]$ (d) $v^2 = GM\left[\frac{2}{r}-\frac{1}{a^2}\right]$

16. The gravitational force between a point like mass M and an infinitely long, thin rod of linear mass density perpendicular to distance L from M is

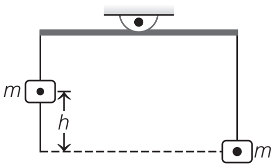
(a) $\frac{MG\lambda}{L}$ (b) $\frac{1}{2}\frac{MG\lambda}{L}$

(c) $\frac{2MG\lambda}{L^2}$ (d) infinite

17. The acceleration due to gravity on the earth's surface at the poles is g and angular velocity of the earth about the axis passing through the pole is ω. An object is weighed at the equator and at a height h above the poles by using a spring balance. If the weights are found to be same, then h is ($h << R$, where R is the radius of the earth) **[JEE Main 2020]**

(a) $\frac{R^2\omega^2}{2g}$ (b) $\frac{R^2\omega^2}{g}$ (c) $\frac{R^2\omega^2}{4g}$ (d) $\frac{R^2\omega^2}{8g}$

18. Two equal mases m and m are hung from balance whose scale pans differ in vertical height by h. Calculate the error in weighing, if any, in terms of density of earth ρ.



(a) $\frac{2}{3}\pi\rho R^3 Gm$ (b) $\frac{8}{3}\pi\rho Gmh$

(c) $\frac{8}{3}\pi\rho R^3 Gm$ (d) $\frac{4}{3}\pi\rho Gm^2 h$

19. How will you weight the sun, *i.e.* estimate its mass? You will need to know the period of one of its planets and the radius of the planetary orbit. The mean orbital radius of the earth around the sun is 1.5×10^8 km, then the mass of the sun is **[NCERT Exemplar]**

(a) 4×10^{30} kg (b) 5×10^{30}kg

(c) 2×10^{30} kg (d) 3×10^{30} kg

20. Two satellites S_1 and S_2 revolve around a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 h and 8 h respectively. The radius of orbit of S_1 is 10^4 km. When S_2 is closest to S_1, the speed of S_2 relative to S_1 is

(a) $\pi\times10^4$ km h^{-1}

(b) $2\pi\times10^4$ km h^{-1}

(c) $3\pi\times10^4$ km h^{-1}

(d) $4\pi\times10^4$ km h^{-1}

21. If a planet was suddenly stopped in its orbit, k suppose to be circular, find how much time will it take in falling onto the sun.

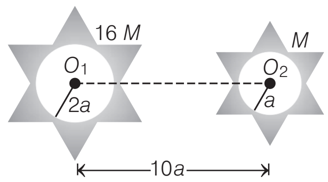
(a) $\sqrt{2}/8$ times the period of the planet's revolution

(b) $4\sqrt{2}$ times the period of the planet's revolution

(c) $3\sqrt{2}$ times the period of the planet's revolution

(d) 9 times the period of the planet's revolution

22. Distance between the centres of two stars is $10a$. The masses of these stars are M and $16M$ and their radii a and $2a$ respectively. A body of mass m is fired straight from the surface of the larger star towards the smaller star. The minimum initial speed for the body to reach the surface of smaller star is



(a) $\frac{2}{3}\sqrt{\frac{GM}{a}}$ (b) $\frac{3}{2}\sqrt{\frac{5GM}{a}}$

(c) $\frac{2}{3}\sqrt{\frac{5GM}{a}}$ (d) $\frac{3}{2}\sqrt{\frac{GM}{a}}$

23. The mass density of a planet of radius R varies with the distance r from its centre as $\rho(r) = \rho_0\left(1-\frac{r^2}{R^2}\right)$, then the gravitational field is maximum at **[JEE Main 2020]**

(a) $r = \sqrt{\frac{3}{4}}R$ (b) $r = R$

(c) $r = \sqrt{\frac{5}{9}}R$ (d) $r = \frac{1}{\sqrt{3}}R$

24. A body A of mass m is moving in a circular orbit of radius R about a planet. Another body B of mass $\frac{m}{2}$ collides with A with a velocity which is half $\left(\frac{\mathbf{v}}{2}\right)$, the instantaneous velocity $\mathbf{v}$ of A. The collision is completely inelastic, then the combined body **[JEE Main 2020]**

(a) escapes from the planet's gravitational field
(b) starts moving in an elliptical orbit around the planet
(c) falls vertically downward towards the planet
(d) continues to move in a circular orbit

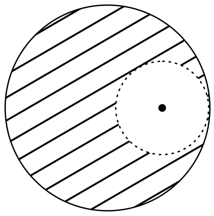
25. Two stars of masses 3×10^{31} kg each and at distance 2×10^{11} m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is (Take, gravitational constant, $(G = 6.67 \times 10^{-11}\ \text{N-m}^2\text{kg}^{-2})$ **[JEE Main 2019]**

(a) 2.8×10^5 m/s (b) 3.8×10^4 m/s
(c) 2.4×10^4 m/s (d) 1.4×10^5 m/s

26. A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius R_e. By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion, so that it becomes $\sqrt{\frac{3}{2}}$ times larger. Due to this, the farthest distance from the centre of the earth that the satellite reaches is R. Value of R is **[JEE Main 2020]**

(a) $4R_e$ (b) $2.5R_e$
(c) $3R_e$ (d) $2R_e$

27. From a solid sphere of mass M and radius R, a spherical portion of radius $R/2$ is removed as shown in the figure. Taking gravitational potential $V = 0$ at $r = \infty$, the potential at the centre of the cavity thus formed is (G = gravitational constant) **[JEE Main 2015]**



(a) $\frac{-GM}{2R}$ (b) $\frac{-GM}{R}$
(c) $\frac{-2GM}{3R}$ (d) $\frac{-2GM}{R}$

28. On the X-axis and at a distance x from the origin, the gravitational field due to a mass distribution is given by $\frac{Ax}{(x^2 + a^2)^{3/2}}$ in the x-direction. The magnitude of gravitational potential on the X-axis at a distance x, taking its value to be zero at infinity, is **[JEE Main 2020]**

(a) $\frac{A}{(x^2 + a^2)^{1/2}}$ (b) $A(x^2 + a^2)^{3/2}$
(c) $\frac{A}{(x^2 + a^2)^{3/2}}$ (d) $A(x^2 + a^2)^{1/2}$

29. The mass density of a spherical galaxy varies as $\frac{K}{r}$ over a large distance r from its centre. In that region, a small star is in a circular orbit of radius R. Then, the period of revolution T depends on R as **[JEE Main 2020]**

(a) $T \propto R$ (b) $T^2 \propto R^3$
(c) $T^2 \propto R$ (d) $T^2 \propto \frac{1}{R^3}$

30. A test particle is moving in a circular orbit in the gravitational field produced by mass density $\rho(r) = \frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T **[JEE Main 2019]**

(a) $\frac{T^2}{R^3}$ is a constant (b) $\frac{T}{R^2}$ is a constant
(c) TR is a constant (d) $\frac{T}{R}$ is a constant

31. A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 h around the planet?
(Take, mass of planet $= 8 \times 10^{22}$ kg, radius of planet $= 2 \times 10^6$ m and gravitational constant $G = 6.67 \times 10^{-11}\ \text{N-m}^2/\text{kg}^2$) **[JEE Main 2019]**

(a) 11 (b) 17 (c) 13 (d) 9

32. A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have, if the same rocket is to be launched from the surface of the moon? Assume that, the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon. **[JEE Main 2019]**

(a) $\frac{E}{64}$ (b) $\frac{E}{16}$
(c) $\frac{E}{32}$ (d) $\frac{E}{4}$

33. A satellite of mass m is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R (R = radius of the earth), it ejects a rocket of mass $\frac{m}{10}$, so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant, M is the mass of the earth) **[JEE Main 2020]**

(a) $\frac{3m}{8}\left(u+\sqrt{\frac{5\,GM}{6R}}\right)^2$ (b) $\frac{m}{20}\left(u-\sqrt{\frac{2GM}{3R}}\right)^2$

(c) $\frac{m}{20}\left(u^2+\frac{113}{200}\frac{GM}{R}\right)$ (d) $5m\left(u^2-\frac{119}{200}\frac{GM}{R}\right)$

Numerical Value Questions

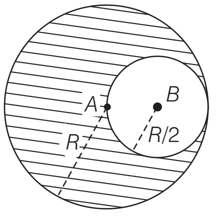
34. The radius in kilometre to which the present radius of earth ($R = 6400$ km) to be compressed, so that the escape velocity is increased to ten time is …… km. **[JEE Main 2021]**

35. If one wants to remove all the mass of the earth to infinity in order to break it up completely. The amount of energy that needs to be supplied will be $\frac{x}{5}\frac{GM^2}{R}$, where x is ……… . (Round off to the nearest integer)
(Here, M is the mass of earth, R is the radius of earth and G is the gravitational constant). **[JEE Main 2021]**

36. An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth. If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, then speed (in ms^{-1}) with which it hits the surface of the earth is $\sqrt{kgR}$. The value of k is …… . (Take, $g = 9.8\ ms^{-2}$ and $R = 6.4\times 10^6$ m)

37. An asteroid is moving directly towards the centre of the earth. When at a distance of $10\,R$ (R is the radius of the earth) from the earth's centre, it has a speed of 12 km/s. Neglecting the effect of earth's atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity from the earth is 11.2 km/s)? Give your answer to the nearest integer (in km/s) ……… . **[JEE Main 2020]**

38. Inside a fixed sphere of radius R and uniform density ρ, there is a spherical cavity of radius $\frac{R}{2}$ such that, the surface of cavity passes through the centre of the sphere as shown in the figure. A particle of mass m_0 is released from rest at centre B of the cavity. The velocity with which particle strikes the centre A of sphere is $\frac{\sqrt{2\pi G\rho R^2}}{\sqrt{\beta}}$. The value of β is ……… .



Answers

Round I

1. (b)	2. (a)	3. (d)	4. (b)	5. (b)	6. (a)	7. (b)	8. (c)	9. (d)	10. (c)
11. (b)	12. (d)	13. (a)	14. (c)	15. (c)	16. (c)	17. (d)	18. (b)	19. (c)	20. (c)
21. (b)	22. (d)	23. (b)	24. (c)	25. (c)	26. (c)	27. (a)	28. (c)	29. (a)	30. (c)
31. (c)	32. (c)	33. (c)	34. (a)	35. (c)	36. (a)	37. (d)	38. (b)	39. (c)	40. (b)
41. (c)	42. (c)	43. (c)	44. (c)	45. (c)	46. (c)	47. (d)	48. (c)	49. (b)	50. (a)
51. (a)	52. (b)	53. (a)	54. (d)	55. (b)	56. (a)	57. (c)	58. (d)	59. (b)	60. (a)
61. (a)	62. (a)	63. (b)	64. (a)	65. (b)	66. (c)	67. (d)	68. (c)	69. (d)	70. (d)
71. (d)	72. (a)	73. (b)	74. (d)	75. (c)	76. (c)	77. (d)	78. (c)	79. (c)	80. (b)
81. (c)	82. (d)								

Round II

1. (c)	2. (c)	3. (c)	4. (b)	5. (d)	6. (d)	7. (d)	8. (c)	9. (c)	10. (c)
11. (c)	12. (b)	13. (d)	14. (a)	15. (a)	16. (c)	17. (a)	18. (b)	19. (c)	20. (a)
21. (a)	22. (b)	23. (c)	24. (b)	25. (a)	26. (d)	27. (b)	28. (a)	29. (c)	30. (d)
31. (a)	32. (b)	33. (d)	34. 64	35. 3	36. 1	37. 16	38. 3		

Solutions

Round I

1. Gravitational force $= \left(\frac{GMm}{R^{3/2}}\right)$ provides the necessary centripetal force (*i.e.* $mR\omega^2$)

So, $$\frac{GMm}{R^{3/2}} = mR\omega^2 = mR\left(\frac{2\pi}{T}\right)^2 = \frac{4\pi^2 mR}{T^2}$$

or $$T^2 = \frac{4\pi^2 R^{5/2}}{GM}$$

i.e. $$T^2 \propto R^{5/2}$$

2. Let R be the original radius of a planet. Then attraction on a body of mass m placed on its surface will be

$$F = \frac{GMm}{R^2}$$

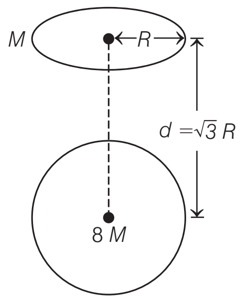
If radius of the planet is made double, *i.e.* $R' = 2R$, then mass of the planet becomes

$$M' = \frac{4}{3}\pi(2R)^3\rho = 8 \times \frac{4}{3}\pi R^3 \rho = 8M$$

New force, $$F' = \frac{GM'm}{R'^2} = \frac{G \times 8M \times m}{(2R)^2} = 2F$$

i.e. Force of attraction increases due to the increase in mass of the planet.

3. From the figure, the gravitational intensity due to the ring at a distance $d = \sqrt{3}R$ on its axis is



$$E = \frac{GMd}{(d^2 + R^2)^{3/2}} = \frac{GM \times \sqrt{3}R}{(3R^2 + R^2)^{3/2}} = \frac{\sqrt{3}GM}{8R^2}$$

Force on sphere $= (8M) \times E = (8M) \times \frac{\sqrt{3}GM}{8R^2} = \frac{\sqrt{3}GM^2}{R^2}$

4. Gravitational force due to solid sphere, $F_1 = \frac{GMm}{(2R)^2}$, where M and m are masses of the solid sphere and particle respectively and R is the radius of the sphere. The gravitational force on particle due to sphere with cavity = force due to solid sphere creating cavity, assumed to be present above at that position.

i.e. $$F_2 = \frac{GMm}{4R^2} - \frac{G(M/8)m}{(3R/2)^2} = \frac{7}{36}\frac{GMm}{R^2}$$

So, $$\frac{F_2}{F_1} = \frac{7GMm}{36R^2} \bigg/ \left(\frac{GMm}{4R^2}\right) = \frac{7}{9}$$

5. Moon is revolving around earth in almost circular orbit. Sun exerts gravitational pull on both, earth and moon. When observed from sun, the orbit of the moon will not be strictly elliptical because the total gravitational force (*i.e.* force due to earth on moon and force due to sun on moon) is not central.

6. The earth is revolving on circular orbit around sun due to gravitational force (F) which acts along the radius of circular path, towards the sun, *i.e.* angle between $\mathbf{r}$ and $\mathbf{F}$ is zero. As,

Torque, $|\tau| = |\mathbf{r} \times \mathbf{F}| = rF\sin 0° = 0$

Therefore, torque is zero.

7. Here, $m_1 = m_2 = 100$ kg, $r = 100$ m

Acceleration of first astronaut,
$$a_1 = \frac{Gm_1m_2}{r^2} \times \frac{1}{m_1} = \frac{Gm_2}{r^2}$$

Acceleration of second astronaut,
$$a_2 = \frac{Gm_1m_2}{r^2} \times \frac{1}{m_2} = \frac{Gm_1}{r^2}$$

Net acceleration of approach,
$$a = a_1 + a_2 = \frac{Gm_2}{r^2} + \frac{Gm_1}{r^2} = \frac{2Gm_1}{r^2} \quad [\because m_1 = m_2]$$
$$= \frac{2 \times (6.67 \times 10^{-11}) \times 100}{(100)^2}$$
$$= 2 \times 6.67 \times 10^{-13}\ \text{ms}^{-2}$$

As, $$s = \frac{1}{2}at^2$$

$\therefore$ $$t = \left(\frac{2s}{a}\right)^{1/2} = \left[\frac{2 \times (1/100)}{2 \times 6.67 \times 10^{-13}}\right]^{1/2} \text{second}$$

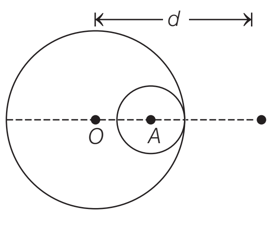
On solving, we get $t = 1.41$ days

8. Resultant force on mass m due to masses at A and B is $F = \frac{G2M \times m}{(AB)^2} - \frac{GMm}{(BC)^2}$ towards BA. Therefore, m will move towards $2M$.

9. Gravitational force between sphere of mass M and the particle of mass m at B is
$$F_1 = \frac{GMm}{d^2}$$

If M_1 is the mass of the removed part of sphere, then

$$M_1 = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3\rho = \frac{1}{8}\left(\frac{4}{3}\pi R^3\rho\right) = \frac{M}{8} \quad \left(\because \rho = \frac{M}{\frac{4}{3}\pi R^3}\right)$$



Gravitational force between the removed part and the particle of mass m at B is

$$F_2 = \frac{GM_1 m}{(d - R/2)^2} = \frac{G(M/8)m}{(d - R/2)^2} = \frac{GMm}{8(d - R/2)^2}$$

$\therefore$ Required force,

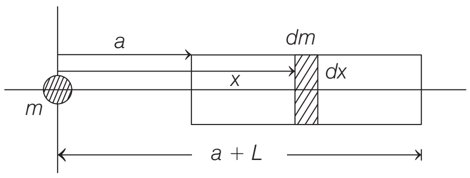
$$F = F_1 - F_2 = \frac{GMm}{d^2} - \frac{GMm}{8[d - (R/2)]^2}$$

$$= \frac{GMm}{d^2}\left[1 - \frac{1}{8\left(1 - \frac{R}{2d}\right)^2}\right]$$

10. Given, situation is

Force of attraction between mass m and an elemental mass dm of rod,

$$dF = \frac{Gmdm}{x^2} = \frac{Gm(A + Bx^2)\,dx}{x^2}$$



Total attraction force is sum of all such differential forces produced by elemental parts of rod from $x = a$ to $x = a + L$.

$$\therefore \quad F = \int dF = \int_{x=a}^{x=a+L} \frac{Gm(A + Bx^2)}{x^2}\,dx$$

$$= Gm \int_{x=a}^{x=a+L} \left(\frac{A}{x^2} + B\right) dx$$

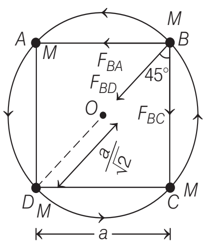
$$= Gm\left[-\frac{A}{x} + Bx\right]_{x=a}^{x=a+L}$$

$$= Gm\left(\frac{-A}{a + L} + B(a + L) + \frac{A}{a} - Ba\right)$$

$$= Gm\left(\frac{A}{a} - \frac{A}{a + L} + BL\right)$$

$$= Gm\left[A\left(\frac{1}{a} - \frac{1}{a + L}\right) + BL\right]$$

11. In given configuration of masses, net gravitational force provides the necessary centripetal force for rotation.



Net force on mass M at position B towards centre of circle is

$$F_{BO(\text{net})} = F_{BD} + F_{BA}\sin 45° + F_{BC}\cos 45°$$

$$= \frac{GM^2}{(\sqrt{2}a)^2} + \frac{GM^2}{a^2}\left(\frac{1}{\sqrt{2}}\right) + \frac{GM^2}{a^2}\left(\frac{1}{\sqrt{2}}\right)$$

[where, diagonal length BD is $\sqrt{2}a$]

$$= \frac{GM^2}{2a^2} + \frac{GM^2}{a^2}\left(\frac{2}{\sqrt{2}}\right) = \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

This force will act as centripetal force.

Distance of particle from centre of circle is $\frac{a}{\sqrt{2}}$.

Here, $F_{\text{centripetal}} = \frac{Mv^2}{r} = \frac{Mv^2}{\frac{a}{\sqrt{2}}} = \frac{\sqrt{2}Mv^2}{a} \qquad \left(\because r = \frac{a}{\sqrt{2}}\right)$

So, for rotation about the centre, $F_{\text{centripetal}} = F_{BO(\text{net})}$

$$\Rightarrow \quad \sqrt{2}\,\frac{Mv^2}{a} = \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

$$\Rightarrow \quad v^2 = \frac{GM}{a}\left(1 + \frac{1}{2\sqrt{2}}\right) = \frac{GM}{a}(1.35)$$

$$\Rightarrow \quad v = 1.16\sqrt{\frac{GM}{a}}$$

12. A person is safe, if his velocity while reaching the surface of moon from a height h' is equal to its velocity while falling from height h on earth. So,

$$\sqrt{2g'h'} = \sqrt{2gh}$$

or $\quad h' = \frac{gh}{g'} = 9.8 \times \frac{3}{1.96} = 15$ m

13. When the thief with box on his head jumped down from a wall, he along with box is falling down with acceleration due to gravity, so the apparent weight of box becomes zero, (because, $R = mg - mg = 0$), so he experiences no load till he reaches the ground.

14. As, $\quad \frac{g_m}{g_e} = \frac{\frac{GM}{8R_m^2}}{GM/R_e^2} = \frac{R_e^2}{8R_m^2} \qquad \ldots\text{(i)}$

Given, $\quad \frac{mg_m}{mg_e} = \frac{1}{6}$

$\therefore \quad \frac{g_m}{g_e} = \frac{1}{6} \qquad \ldots\text{(ii)}$

From Eqs. (i) and (ii), we get

$$\frac{R_e^2}{8R_m^2} = \frac{1}{6}$$

or $\quad R_e = R_m\sqrt{\frac{8}{6}}$

15. Mass of earth = Volume $\times$ Density

As, $\quad g = \frac{GM}{R^2}\,;\; M = \left(\frac{4}{3}\pi R^3\right)\rho$

$\therefore \quad g = \frac{4G}{3}\frac{\pi R^3}{R^2}\rho \quad$ or $\quad g = \left(\frac{4G\pi R}{3}\right)\rho$

[where, ρ = average density]

$\Rightarrow \quad g \propto \rho \quad$ or $\quad \rho \propto g$

16. Acceleration due to gravity at height h,

$$g_h = g\left(1 - \frac{2h}{R}\right) \qquad [\text{ if } h << R] \;...\text{(i)}$$

Acceleration due to gravity at depth d,

$$g_d = g\left(1 - \frac{d}{R}\right) \qquad ...\text{(ii)}$$

As per statement of the problem,

$$g_h = g_d$$

i.e. $g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{d}{R}\right)$

$\Rightarrow \quad 2h = d$

17. If the earth is an approximate sphere of non-uniform density, then the centre of gravity of earth will not be situated at the centre of earth. The distance of different points on earth will be at different distances from the centre of gravity of earth. As, $g = \frac{GM}{r^2}$ or $g \propto \frac{1}{r^2}$, so g is different on different points on the surface of earth but never zero.

18. Below the sea level the pressure is increasing with depth in mine due to presence of atmospheric air there. The acceleration due to gravity below the surface of the earth decreases with the distance from the surface of the earth, as $g' = g\left(1 - \frac{d}{R}\right)$.

19. When a sphere of mass m is released in a liquid, it falls vertically down with acceleration $= \frac{mg - F_B}{m}$

$$= \frac{\frac{4}{3}\pi r^3 dg - \frac{4}{3}\pi r^3 \rho g}{\frac{4}{3}\pi r^2 d} = \frac{(d - \rho)\, g}{d}$$

20. As, $\frac{g'}{g} = 1 - \frac{2h}{R} = 1 - \frac{2 \times 320}{6400} = 1 - \frac{1}{10} = \frac{9}{10}$

$\therefore$ % decrease in $g = \left(\frac{g - g'}{g}\right) \times 100 = \frac{1}{10} \times 100 = 10\%$

21. Given, $\quad g_h = 9 = \frac{gR^2}{(R + R/20)^2} = \frac{20 \times 20}{21 \times 21} g$

or $\quad g = \frac{9 \times 21 \times 21}{20 \times 20}$

Now, $\quad g_d = g\left(1 - \frac{d}{R}\right)$

$$= \frac{9 \times 21 \times 21}{20 \times 20}\left(1 - \frac{R/20}{R}\right)$$

$$= 9.5 \text{ ms}^{-2}$$

22. As, value of g at internal point which is at a distance x from the centre is given by

$$g' = \frac{GM}{R^3} x$$

When $x = 0$, then $g' = 0$.

23. The value of acceleration due to gravity at a height h reduces to $= 100 - 36 = 64\% = \frac{64}{100} g$

$\therefore \quad \frac{64}{100} g = \frac{gR^2}{(R + h)^2}$

or $\quad \frac{8}{10} = \frac{R}{R + h}$ or $h = \frac{R}{4}$

24. Mass of two planets is same, so

$$\frac{4}{3}\pi R_1^3 \rho_1 = \frac{4}{3}\pi R_2^3 \rho_2$$

or $\quad \frac{R_1}{R_2} = \left(\frac{\rho_2}{\rho_1}\right)^{1/3} = \left(\frac{1}{8}\right)^{1/3} = \frac{1}{2}$

$$\frac{g_1}{g_2} = \frac{GM/R_1^2}{GM/R_2^2} = \left(\frac{R_2}{R_1}\right)^2 = (2)^2 = 4$$

$\Rightarrow \quad g_1 : g_2 = 4 : 1$

25. As, $g = \frac{GM}{R^2} \quad \Rightarrow \quad g' = \frac{GM}{R'^2} = \frac{GM\,(100)^2}{(99)^2 R^2}$

% Increase in $g = \frac{(g' - g) \times 100}{g}$

$$= \left(\frac{g'}{g} - 1\right) \times 100 = \left[\left(\frac{100}{99}\right)^2 - 1\right] \times 100$$

$$= \left[\left(1 + \frac{1}{99}\right)^2 - 1\right] \times 100 \approx 2\%$$

26. Force acting on a body of mass M at a point at depth d inside the earth is

$$F = mg' = mg\left(1 - \frac{d}{R}\right)$$

$$= \frac{mGM}{R^2}\left(\frac{R - d}{R}\right) = \frac{GMm}{R^3} r \qquad (\because R - d = r)$$

So, $F \propto r$; Given $F \propto r^n$

Clearly, $\quad n = 1$

27. As, value of acceleration due to gravity above the earth surface is g.

$$g' = \frac{GM}{(R + h)^2}$$

Acceleration due to gravity at height h

$\Rightarrow \quad \frac{g}{9} = \frac{GM}{R^2} \cdot \frac{R^2}{(R + h)^2} = g\left(\frac{R}{R + h}\right)^2$

$\Rightarrow \quad \frac{1}{9} = \left(\frac{R}{R + h}\right)^2 \quad \Rightarrow \quad \frac{R}{R + h} = \frac{1}{3}$

$\Rightarrow \quad 3R = R + h \Rightarrow 2R = h$

28. Inside the earth's surface, $g = \frac{GM}{R^3} r$, *i.e.* $g \propto r$

Outside the earth's surface, $g = \frac{Gm}{r^2}$, *i.e.* $g \propto \frac{1}{r^2}$

So, till earth's surface g increases linearly with distance r, shown only in graph (c).

29. At depth d from the surface of the earth,

$$g' = g\left(1-\frac{d}{R}\right)$$

Given, $\quad g' = \frac{75}{100}g = \frac{3}{4}g$

Then, $\quad \frac{3g}{4} = g\left(1-\frac{d}{R}\right)$

On solving, we get

$$d = \frac{R}{4}$$

30. Error in weighing $= mg - mg' = mg - mg\left(1-\frac{2h}{R}\right)$

$$= \frac{mg\,2h}{R} = \frac{m2\,hg}{R}$$

$$= \frac{m\,2h}{R} \times \frac{G\frac{4}{3}\pi R^3\rho}{R^2} = \frac{8\pi G\rho mh}{3}$$

31. At equator, $g' = g - R\omega^2$

When angular velocity be $\omega' (= x\omega)$, then $g' = 0$

Since, at equator,

$$g' = g - \omega'^2 R$$

or $\quad \omega' = \sqrt{g/R} = x\omega$

or $\quad x = (\sqrt{g/R})/\omega$

or $\quad x = \frac{\sqrt{10/(6.4\times10^6)}}{2\pi} \times 24\times60\times60 = 17$

32. When there is a weightlessness in the body at the equator, then $g' = r - R\omega^2 = 0$

or $\quad \omega = \sqrt{g/R}$ and

linear velocity

$$= \omega R = (\sqrt{g/R})\,R = \sqrt{gR} \qquad \text{...(i)}$$

$\therefore$ KE of rotation of earth $= \frac{1}{2}I\omega^2 = \frac{1}{2}\times\frac{2}{5}MR^2\times\omega^2$

$$= \frac{1}{5}MgR \qquad \text{[from Eq. (i)]}$$

33. Acceleration due to gravity at height h above earth's surface is

$$g_h = g\left(\frac{R^2}{(R+h)^2}\right)$$

Also, acceleration at depth $d = h$ below surface is

$$g_d = g\left(1-\frac{d}{R}\right) = g\left(1-\frac{h}{R}\right)$$

Given, weight of body at height h = weight of body at depth h

So, $\quad mg_h = mg_d \Rightarrow \quad g_h = g_d$

$$\Rightarrow g\left(\frac{R^2}{(R+h)^2}\right) = g\left(1-\frac{h}{R}\right) \Rightarrow \frac{R^2}{(R+h)^2} = 1-\frac{h}{R}$$

$$\Rightarrow \qquad R^3 = (R-h)(R+h)^2$$

$$\Rightarrow R^3 = (R^3 - hR^2 + h^2R - h^3 + 2R^2h - 2Rh^2)$$

$$\Rightarrow h^2 + Rh - R^2 = 0$$

$$\Rightarrow \qquad h = \frac{-R \pm \sqrt{R^2 - 4(1)(-R^2)}}{2(1)}$$

$$\Rightarrow \qquad h = \frac{-R\pm\sqrt{5}R}{2}$$

As h is a positive number, $h = \frac{-R+\sqrt{5}R}{2}$ or $\frac{\sqrt{5}R-R}{2}$

34. As, $g' = g - \frac{10\,g}{100} = \frac{90}{100}g$

$\therefore \quad g' = g\frac{R^2}{(R+h)^2}$

or $\quad \frac{9}{10} = \frac{R^2}{(R+h)^2}$

or $\quad \frac{3}{\sqrt{10}} = \frac{R}{R+h}$

or $\quad h = (\sqrt{10}-3)\,R/3$

$$= \frac{(\sqrt{10}-3)\times6400}{3} = 345.60 \text{ km}$$

35. Acceleration due to gravity at height h,

$$g_1 = g\left(1-\frac{2h}{R}\right)$$

Acceleration due to gravity at depth h,

$$g_2 = g\left(1-\frac{h}{R}\right)$$

$\therefore \quad \frac{g_1}{g_2} = \frac{1-2h/R}{1-h/R} = \left(1-\frac{2h}{R}\right)\left(1-\frac{h}{R}\right)^{-1} = \left(1-\frac{h}{R}\right)$

$\therefore \frac{g_1}{g_2}$ decreases linearly with h.

36. Given, $\frac{mg'}{mg} = \frac{30}{90}$ or $\frac{g'}{g} = \frac{1}{3}$

Now, $\quad g' = g\frac{R^2}{(R+h)^2}$

or $\quad \frac{g'}{g} = \frac{R^2}{(R+h)^2} = \frac{1}{3}$ or $\frac{R}{R+h} = \frac{1}{\sqrt{3}}$

or $\quad (R+h) = \sqrt{3}\,R$

or $\quad h = (\sqrt{3}-1)\,R = 0.73\,R$

37. Let mass of given body is m, then it's weight on earth's surface $= mg_e$

where, g_e = acceleration due to gravity on earth's surface and weight on the surface of planet $= mg_p$, where g_p= acceleration due to gravity on planet's surface.

Given, $\frac{mg_e}{mg_p} = \frac{9}{4} \Rightarrow \frac{g_e}{g_p} = \frac{9}{4}$

But $g = \frac{GM}{R^2}$, so we have $\frac{\left(\frac{GM}{R^2}\right)}{\left(\frac{GM_p}{R_p^2}\right)} = \frac{9}{4}$

where, M = mass of earth, R = radius of earth,

$$M_p = \text{mass of planet} = \frac{M}{9} \qquad \text{(given)}$$

and R_p = radius of planet.

$$\Rightarrow \quad \frac{M}{M_p} \cdot \frac{R_p^2}{R^2} = \frac{9}{4} \Rightarrow 9 \cdot \left(\frac{R_p}{R}\right)^2 = \frac{9}{4}$$

$$\Rightarrow \quad \frac{R_p}{R} = \frac{1}{2} \Rightarrow \quad R_p = \frac{R}{2}$$

38. The relation of gravitational acceleration at an altitude h above the earth's surface is given as

$$g_h = g\left(1 + \frac{h}{R_e}\right)^{-2}$$

where, g is the acceleration due to gravity at earth's surface and R_e is the radius of the earth.

Given that at some height h, acceleration due to gravity,

$$g_h = 4.9 \text{ m/s}^2 \approx \frac{g}{2} \quad \text{... (i)}$$

$\therefore$ The ratio of acceleration due to gravity at earth's surface and at some altitude h is

$$\left(1 + \frac{h}{R_e}\right) = \sqrt{\frac{g}{g_h}} = \sqrt{2} \quad \text{[from Eq. (i)]}$$

$$\therefore \quad \frac{h}{R_e} = \sqrt{2} - 1 \text{ or } h = 0.414 \times R_e$$

$$h = 0.414 \times 6400 \text{ km}$$

($\because$ Given, radius of earth, $R_e = 6400 \text{ km} = 6.4 \times 10^6$ m)

or $\quad h = 2649.6 \text{ km} = 2.6 \times 10^6$ m

Thus, at 2.6×10^6 m above the earth's surface, acceleration due to gravity decreases to 4.9 m/s^2.

39. As, $\quad dV = -E\,dx$

or $\quad V = -\int_{\infty}^{x/\sqrt{2}} E\,dx = -\int_{\infty}^{x/\sqrt{2}} kx^{-3}\,dx = k/x^2$

40. Intensity of gravitational field at a point inside the spherical shell is zero

$$I = 0 \text{ upto } r = a$$

outside the shell, $I \propto \frac{1}{r^2}$ where $r > a$

41. If x is the distance of point on the line joining the two masses from mass m_2 where gravitational field intensity is zero, then

$$\frac{Gm_1}{(r-x)^2} = \frac{Gm_2}{x^2}$$

or $\quad \frac{2}{(9-x)^2} = \frac{8}{x^2}$ or $\frac{1}{9-x} = \frac{2}{x}$

On solving, $x = 6$ m

42. Let x be the distance of point from the smaller body where gravitational intensity is zero.

$$\therefore \quad \frac{Gm_1}{(1-x)^2} = \frac{Gm_2}{x^2}$$

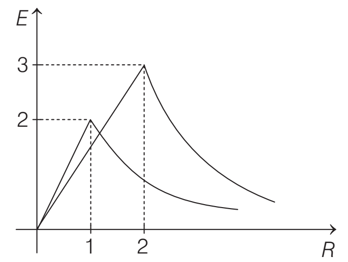
or $\quad \frac{x}{1-x} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{1000}{100000}} = \frac{1}{10}$

or $\quad 10x = 1 - x$ or $x = (1/11)$ m

43. Gravitational field of a solid sphere is maximum at its surface ($r = R$) and its value at surface,

$$E = \frac{GM}{R^2}$$

From graph given in the question,



We can observe that,

$$E_1 = \frac{GM_1}{(1)^2} = 2 \text{ and } E_2 = \frac{GM_2}{(2)^2} = 3$$

or $\quad GM_1 = 2 \quad$ ...(i)

and $\quad GM_2 = 12 \quad$ ...(ii)

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{M_1}{M_2} = \frac{2}{12} = \frac{1}{6}$$

44. Here, $I = -\frac{dV}{dr} = -k/r$ or $dV = k\frac{dr}{r}$

Integrating it, we get

$$\int_{V_0}^{V} dV = \int_{r_0}^{r} k\frac{dr}{r}$$

or $\quad V = V_0 + k\log(r/r_0)$

45. Gravitational potential on the surface of the shell is

V = Gravitational potential due to particle (V_1)

+ Gravitational potential due to shell particle (V_2)

$$= -\frac{Gm}{R} + \left(-\frac{G3m}{R}\right) = -\frac{4Gm}{R}$$

46. The minimum kinetic energy required to project a body of mass m from earth's surface to infinity is known as escape energy. Therefore,

$$\text{KE} = \frac{GM_e m}{R} = mgR \quad \left[\because gR = \frac{GM_e}{R}\right]$$

47. Gravitational potential energy of body will be

$$E = -\frac{GMm}{r}$$

where, $\quad M$ = mass of earth,

m = mass of the body

and $\quad r$ = radius of earth.

At $\quad r = 2R$,

$$E_1 = -\frac{GMm}{(2R)}$$

At $\quad r = 3R, E_2 = -\frac{GMm}{(3R)}$

Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is

$$\Delta E = \frac{GMm}{R}\left[\frac{1}{2}-\frac{1}{3}\right] = \frac{GMm}{6R}$$

48. Gravitational potential energy of a body in the gravitational field, $E = \frac{-GMm}{r}$. When r decreases negative value of E increases, *i.e.* E decreases.

49. Gravitational intensity, $I = \frac{dV}{dx} = \frac{14}{20} = 0.7\ \text{Nkg}^{-1}$

Acceleration due to gravity, $g = I = 0.7\ \text{N kg}^{-1}$

Work done under this field in displacing a body on a slope of $60°$ through a distance s

$$= m\,(g\sin 60°)\,s$$
$$= 2\times(0.7\times\sqrt{3}/2)\times 8$$
$$= 9.6\ \text{J}$$

50. Since the gravitational field is conservative field, hence the work done in taking a particle from one point to another in a gravitational field is path independent.

Hence, $W_1 = W_2 = W_3$.

51. As, $\quad U = \frac{-GMm}{r}$

or $\quad r = \frac{-GMm}{U}$

$$r = \frac{-6.67\times10^{-11}\times6\times10^{24}\times7.4\times10^{22}}{-7.79\times10^{28}}$$
$$= 3.8\times10^{8}\ \text{m}$$

52. Gravitational potential energy of body on the earth's surface is

$$U = -\frac{GM_e m}{R}$$

At a height h from earth's surface, its value is

$$U_h = -\frac{GM_e m}{(R+h)} = -\frac{GM_e m}{2R} \qquad [\text{as } h = R]$$

where, $\quad M_e$ = mass of earth, m = mass of body
and $\quad R$ = radius of earth.

$\therefore$ Gain in potential energy

$$= U_h - U = -\frac{GM_e m}{2R} - \left(-\frac{GM_e m}{R}\right)$$
$$= -\frac{GM_e m}{2R} + \frac{GM_e m}{R}$$
$$= \frac{GM_e m}{2R} = \frac{gR^2 m}{2R} \qquad \left[\text{as, } g = \frac{GM_e}{R^2}\right]$$
$$= \frac{1}{2}mgR$$

53. The change in potential energy in gravitational field is given by

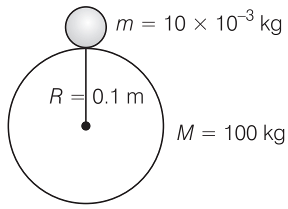
$$\Delta E = GMm\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

In this problem; $r_1 = R$ and $r_2 = nR$

$$\Delta E = GMm\left(\frac{1}{R}-\frac{1}{nR}\right)$$
$$= \frac{GMm}{R}\left(\frac{n-1}{n}\right)$$
$$= mgR\left(\frac{n-1}{n}\right) \qquad \left(\because g = \frac{Gm}{R^2}\right)$$

54. The initial potential energy of the particle,

$$U_i = -\frac{GMm}{r}$$
$$U_i = -\frac{6.67\times10^{-11}\times100\times10^{-2}}{0.1}$$
$$U_i = -\frac{6.67\times10^{-11}}{0.1} = -6.67\times10^{-10}\ \text{J}$$



We know that, work done = difference in potential energy

$\therefore \quad W = \Delta U = U_f - U_i$

$\Rightarrow \quad W = -U_i \qquad [\because U_f = 0]$

$\quad = 6.67\times10^{-10}\ \text{J}$

55. Gravitational force on a body at a distance x from the centre of earth, $F = \frac{GMm}{x^2}$

$\therefore$ Work done,

$$W = \int_R^{R+h} F\,dx = \int_R^{R+h}\frac{GMm}{x^2}\,dx$$
$$= GMm\left[-\frac{1}{x}\right]_R^{R+h}$$
$$= mgR^2\left(\frac{1}{R}-\frac{1}{R+h}\right)$$

This work done appears as increase in potential energy

$$\therefore \quad \Delta U = mgR^2\left[\frac{1}{R}-\frac{1}{R+h}\right]$$
$$= mg\,(5h)^2\left[\frac{1}{5h}-\frac{1}{6h}\right] = \frac{5}{6}mgh$$

56. From conservation of energy,

Total energy at the planet = Total energy at altitude

$$-\frac{GMm}{R} + (\text{KE})_{\text{surface}} = -\frac{GMm}{3R} + \frac{1}{2}mv_A^2 \qquad \text{...(i)}$$

In the orbit of planet, the necessary centripetal force is obtained by gravitational force.

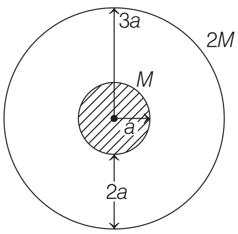
$$\therefore \qquad \frac{m v_A^2}{R+2R} = \frac{GMm}{(R+2R)^2}$$

$$\Rightarrow \qquad v_A = \frac{GM}{3R} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$(\text{KE})_{\text{surface}} = \frac{5}{6}\frac{GMm}{R}$$

57. According to question, diagram will be as follows



Gravitational field due to solid sphere of radius a at a distance, $r = 3a$, *i.e.* $(r > R)$ is

$$E_1 = \frac{GM}{r^2} = \frac{GM}{(3a)^2} = \frac{GM}{9a^2}$$

Similarly, gravitational field due to spherical shell at a distance, $r = 3a$,

i.e. $$E_2 = \frac{GM}{R^2} = \frac{G(2M)}{(3a)^2} = \frac{2GM}{9a^2}$$

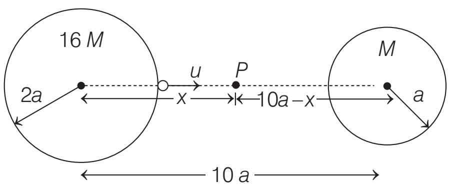
Both fields are attractive in nature, so direction will be same.

Net electric field, $E_{\text{net}} = \dfrac{GM}{9a^2} + \dfrac{2GM}{9a^2}$

$$E_{\text{net}} = \frac{GM}{3a^2}$$

58. For the body to be able to reach at the surface of smaller planet, it just has to overcome the gravitational field of bigger planet and at that point (let say P in given figure) net force will be zero. After crossing this P point, body will move towards smaller planet itself and will reach with minimum speed.

Let the distance of point P from bigger planet be x.



At point P, $\mathbf{F}_{\text{net}} = 0 \;\Rightarrow\; F_{16M} = F_M$

$$\Rightarrow \qquad \frac{G(16M)m}{x^2} = \frac{G(M)(m)}{(10a - x)^2} \;\Rightarrow\; x = 8a \qquad \text{...(i)}$$

Since, there is no net external force on system, so we can apply conservation of mechanical energy from the surface of bigger planet to point P.

i.e. $$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mu^2 + \left(-\frac{G(16M)m}{2a} - \frac{GMm}{8a}\right) = 0 + \left(\frac{-G(16M)m}{8a} - \frac{GMm}{2a}\right)$$

$$\frac{1}{2}mu^2 - \frac{65}{8}\frac{GMm}{a} = -\frac{5}{2}\frac{GMm}{a}$$

$$\frac{1}{2}mu^2 = \frac{45}{8}\frac{GMm}{a}$$

$$\Rightarrow \qquad u = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

59. As, $\dfrac{(v_e)_{P_1}}{(v_e)_{P_2}} = \dfrac{\sqrt{2g_1R_1}}{\sqrt{2g_2R_2}} = \sqrt{\dfrac{g_1}{g_2} \times \dfrac{R_1}{R_2}} = \sqrt{ab}$

60. Escape velocity of a body from the surface of earth is 11.2 kms^{-1}, which is independent of the angle of projection.

61. Escape velocity $= \sqrt{2gR_e}$

So, escape velocity is independent of m.

62. Time period of satellite, $T = 2\pi\sqrt{\dfrac{(R+h)^3}{GM_e}}$

where, $R + h$ = orbital radius of satellite

and M_e = mass of earth.

Thus, time period does not depend on mass of the satellite.

63. When gravitational force becomes zero, the centripetal force required cannot be provided.

So, the satellite will move with the velocity as it has at the instant when gravitational force becomes zero, *i.e.* it moves tangentially to the original orbit.

64. When a satellite is moving in an elliptical orbit, it's angular momentum $(= \mathbf{r} \times \mathbf{p})$ about the centre of earth does not change its direction. The linear momentum $(= m\mathbf{v})$ does not remain constant as velocity of satellite is not constant. The total mechanical energy of S is constant at all locations.

The acceleration of S (= centripetal acceleration) is always directed towards the centre of earth.

65. According to the given condition in the question, after collision the mass of combined system is doubled. Also, this system would be displaced from its circular orbit.

So, the combined system revolves around centre of mass of 'system + earth' under action of a central force. Hence, orbit must be elliptical.

66. Orbital velocity of satellite at distance r from the centre of earth, $v = \sqrt{\dfrac{GM}{r}}$

$$\text{Total energy of satellite} = \text{PE} + \text{KE} = -\frac{GMm}{r} + \frac{1}{2}mv^2$$

$$= -\frac{GMm}{r} + \frac{1}{2}m\frac{GM}{r}$$

$$= -\frac{GMm}{2r}$$

The viscous force acting on satellite decreases the energy of satellite. As a result of it, the value of r gradually decreases, consequently the height of satellite gradually decreases.

67. Escape velocity , $v_e = \sqrt{\dfrac{2GM}{R}}$

$$\Rightarrow \qquad v_e = \sqrt{\frac{2G\frac{4}{3}\pi R^3 \times d}{R}}$$

$$= \sqrt{2G\frac{4}{3}\pi R^2 \times d} = R\sqrt{\frac{8}{3}\pi Gd}$$

(Here, d = mean density of earth)

Now, $\quad v_e \propto R\sqrt{d}$

$$\therefore \qquad \frac{v_e}{v_p} = \frac{R_e}{R_p}\sqrt{\frac{d_e}{d_p}} = \frac{R_e}{2R_e}\sqrt{\frac{d_e}{d_e}}$$

$$v_p = 2v_e = 2\times 11 = 22 \text{ kms}^{-1}$$

68. Here, $\dfrac{g_m}{g_e} = \dfrac{M_m}{M_e}\times\left(\dfrac{R_e}{R_m}\right)^2 = \dfrac{1}{81}\times(4)^2 = \dfrac{16}{81}$

$$\Rightarrow \qquad g_m = \frac{16}{81}g_e$$

Also, $\quad v_e = \sqrt{2g_eR_e} = 11.2 \text{ kms}^{-1}$

$$v_m = \sqrt{2g_mR_m} = \sqrt{2\times\frac{16}{81}g_e\times\frac{1}{4}R_e}$$

$$= \frac{2}{9}\sqrt{2g_eR_e} = \frac{2}{9}\times 11.2$$

$$= 2.48 \approx 2.5 \text{ kms}^{-1}$$

69. Orbital speed of a satellite in a circular orbit is

$$v_o = \sqrt{\left(\frac{GM}{r_o}\right)}$$

where, r_o is the radius of the circular orbit.

So, kinetic energies of satellites A and B are

$$T_A = \frac{1}{2}m_Av_{OA}^2 = \frac{GMm}{2R}$$

$$T_B = \frac{1}{2}m_Bv_{OB}^2 = \frac{GM(2m)}{2(2R)} = \frac{GMm}{2R}$$

So, ratio of their kinetic energies is $\dfrac{T_A}{T_B} = 1.$

70. Given, a satellite is revolving in a circular orbit at a height h from the Earth's surface having radius of earth R, *i.e.* $h << R$.

Orbital velocity of a satellite, $v = \sqrt{\dfrac{GM}{R+h}} = \sqrt{\dfrac{GM}{R}}$

(as, $h << R$)

Velocity required to escape,

$$\frac{1}{2}mv'^2 = \frac{GMm}{R+h}$$

$$v' = \sqrt{\frac{2GM}{R+h}} = \sqrt{\frac{2GM}{R}} \qquad (h << R)$$

$\therefore$ Minimum increase in its orbital velocity required to escape from the Earth's gravitational field,

$$v' - v = \sqrt{\frac{2GM}{R}} - \sqrt{\frac{GM}{R}}$$

$$= \sqrt{2gR} - \sqrt{gR} = \sqrt{gR}\,(\sqrt{2}-1) \qquad \left(\because g = \frac{GM}{R^2}\right)$$

71. Escape velocity is given by

$$v_e = \sqrt{\frac{2GM}{R}}$$

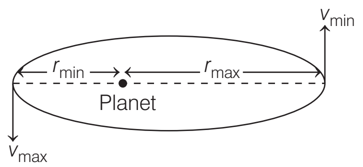
where, m = mass and R = radius of planet.

So, $\quad v_A = \sqrt{\dfrac{2GM}{R}}$

and $\quad v_B = \sqrt{\dfrac{2G\left(\frac{M}{2}\right)}{\left(\frac{R}{2}\right)}} = \sqrt{\dfrac{2GM}{R}}$

$$\Rightarrow \qquad \frac{v_A}{v_B} = 1 \text{ or } \frac{n}{4} = 1 \Rightarrow n = 4$$

72. The motion of a satellite in an elliptical orbit is shown in figure



Applying, angular momentum conservation about planet at closest and farthest points,

$$L_{\text{closest}} = L_{\text{farthest}}$$

$$mv_{\max}r_{\min} = mv_{\min}r_{\max} \quad \text{or} \quad \frac{r_{\min}}{r_{\max}} = \frac{v_{\min}}{v_{\max}}$$

Given that, $v_{\max} = 6v_{\min}$ or $\dfrac{v_{\min}}{v_{\max}} = \dfrac{1}{6}$

$$\therefore \qquad \frac{r_{\min}}{r_{\max}} = \frac{1}{6}$$

73. Orbital velocity of a body revolving around a planet,

$$v_o = \sqrt{\frac{GM}{r}} \qquad \text{...(i)}$$

where, M = mass of the planet,

r = orbital radius of revolving body = R (given)

and G = universal gravitational constant.

Escape velocity of any body from the surface of a planet,

$$v_e = \sqrt{\frac{2GM}{R}} \qquad \text{...(ii)}$$

where, M = mass of the planet,

R = radius of the planet and G = universal gravitational constant. Ratio of body's orbital velocity to the escape velocity from the planet,

$$\frac{v_o}{v_e} = \frac{\sqrt{\frac{GM}{R}}}{\sqrt{\frac{2GM}{R}}} = \frac{1}{\sqrt{2}}$$

74. The gravitational force exerted on satellite at a height x is

$$F_G = \frac{GM_e m}{(R+x)^2}$$

where, M_e = mass of earth.

Since, gravitational force provides the necessary centripetal force, so

$$\frac{GM_e m}{(R+x)^2} = \frac{mv_o^2}{(R+x)}$$

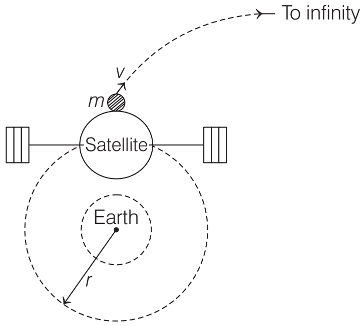
where, v_o is orbital speed of satellite.

$$\Rightarrow \quad \frac{GM_e m}{(R+x)} = mv_o^2$$

$$\text{or} \quad \frac{gR^2 m}{(R+x)} = mv_o^2 \qquad \left[\because g = \frac{GM_e}{R^2}\right]$$

$$\text{or} \quad v_o = \sqrt{\left[\frac{gR^2}{(R+x)}\right]} = \left[\frac{gR^2}{(R+x)}\right]^{1/2}$$

75. Kinetic energy must be given to an object, so that it just escapes to infinity is as shown in the figure below



Applying energy-conservation for mass m, we get

Initial energy of m at satellite = Final energy of m at infinity

$$\Rightarrow \quad U_i + K_i = U_f + K_f \qquad \text{...(i)}$$

Here, U_i = initial PE of

$$m = \frac{-G \cdot Mm}{r} = -(v^2)m$$

As, $v = \sqrt{\dfrac{GM}{r}}$ = orbital speed of the satellite,

K_i = initial kinetic energy of m,

U_f = final potential energy of m which is zero

and K_f = final kinetic energy of m which is also zero.

So, from Eq. (i), we have

$-v^2 m + K_i = 0 + 0$ or $K_i = mv^2$

76. As, $T_2 = T_1\left(\dfrac{r_2}{r_1}\right)^{3/2} = T_1\left(\dfrac{1}{4}\right)^{3/2} = \dfrac{1}{8}T_1 = \dfrac{1}{8}$ times

77. Asteroids move in circular orbits like planets under the action of central forces and obey Kepler's laws.

78. According to Kepler's law,

$$T^2 \propto r^3 \quad \Rightarrow \quad 5^2 \propto r^3 \qquad \text{...(i)}$$

$$\text{and} \quad (T')^2 \propto (4r)^3 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{25}{(T')^2} = \frac{r^3}{64r^3}$$

$$\text{or} \quad T' = \sqrt{1600} \text{ or } T' = 40 \text{ h}$$

79. From Kepler's law,

$$T^2 \propto R^3 \Rightarrow T \propto R^{3/2}$$

$$\therefore \quad \frac{T_1}{T_2} = \left(\frac{12R}{3R}\right)^{3/2} = 8$$

$$\Rightarrow \quad T_2 = \frac{T_1}{8} = \frac{24}{8} = 3 \text{ h}$$

80. As, $\quad T^2 \propto r^3$

$$\text{So,} \quad \frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3}$$

$$\text{or} \quad \frac{r_A}{r_B} = \left(\frac{T_A}{T_B}\right)^{2/3} = (8)^{2/3} = 4 \text{ or } r_A = 4\,r_B$$

$$\text{So,} \quad r_A - r_B = 4\,r_B - r_B = 3\,r_B$$

81. The earth moves around the sun in elliptical path. So, by using the properties of ellipse

$r_1 = (1+e)\,a$ and $r_2 = (1-e)\,a$

$$\Rightarrow \quad a = \frac{r_1 + r_2}{2} \text{ and } r_1 r_2 = (1-e^2)\,a^2$$

where, a = semi-major axis,

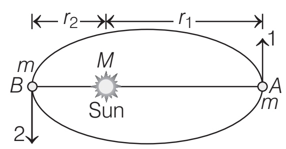
b = semi-minor axis

and e = eccentricity.

Now, required distance = semi-latus rectum = $\dfrac{b^2}{a}$

$$= \frac{a^2(1-e^2)}{a} = \frac{r_1 r_2}{(r_1+r_2)/2} = \frac{2\,r_1 r_2}{r_1 + r_2}$$

82. The gravitational force of sun on comet is radial, hence angular momentum is constant over the entire orbit. Using law of conservation of angular momentum, at locations A and B, we get



$$L = mv_1 r_1 = mv_2 r_2$$

$$\text{or} \quad v_2 = \frac{v_1 r_1}{r_2} \qquad \text{...(i)}$$

Using the principle of conservation of total energy at A and B,

$$\frac{1}{2}mv_1^2 - \frac{GMm}{r_1} = \frac{1}{2}mv_2^2 - \frac{GMm}{r_2}$$

$$\text{or} \quad v_2^2 - v_1^2 = 2\,GM\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \qquad \text{...(ii)}$$

Putting the values from Eq. (i) in Eq. (ii) and solving, we get

$$v_1 = \left[\frac{2\,GMr_2}{r_1\,(r_1 + r_2)}\right]^{1/2}$$

$$\therefore \qquad L = mv_1 r_1 = \left[\frac{2\,GMm^2 r_1 r_2}{(r_1 + r_2)}\right]^{1/2}$$

Round II

1. As, $\Delta\,\text{KE} = \Delta U$

$$\Rightarrow \qquad \frac{1}{2}mv^2 = GM_e m\left(\frac{1}{R} - \frac{1}{R+h}\right) \qquad \text{...(i)}$$

Also, $$g = \frac{GM_e}{R^2} \qquad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we have

$$h = \frac{R}{\left(\dfrac{2\,gR}{v^2} - 1\right)}$$

2. For $r \le R$,

$$\frac{mv^2}{r} = \frac{GmM}{r^2} \qquad \left(\because M = \frac{4}{3}\pi r^3 \rho_0\right)$$

$\therefore \qquad v \propto r$

i.e. v-r graph is a straight line passing through the origin.

For $r \ge R$,

$$\frac{mv^2}{r} = \frac{GM\left(\dfrac{4}{3}\pi R^3\right)\rho_0}{r^2} \Rightarrow v \propto \frac{1}{\sqrt{r}}$$

The corresponding v-r graph will be as shown in option (c).

3. We know that, total energy (E), kinetic energy (K) and potential energy (U) of moving satellite is given as

$$E = -\frac{GMm}{2r},\; K = \frac{GMm}{2r} \text{ and } U = -\frac{GMm}{r}$$

For satellite U, K and E vary with r and also U and E remain negative whereas K remains always positive.

Hence, from above expressions, graph represented in option (c) is correct.

4. As, $T = 2\pi\sqrt{\dfrac{l}{g}}$

i.e. $T \propto 1/\sqrt{g}$ (for fixed value of l)

$$\therefore \qquad \frac{T_1}{T_2} = \sqrt{\frac{g'}{g}} = \left[\frac{gR^2/(R+2R)^2}{g}\right]^{1/2} = \frac{1}{3}$$

5. Escape velocity from the surface of earth is given by

$$v_e = \sqrt{\frac{2GM_e}{R_e}} \qquad \text{...(i)}$$

Here, $v_e = 11.2\ \text{kms}^{-1}$, $M_p = 4\,M_e$

$$v_p = \sqrt{\frac{2GM_p}{R_p}} = \sqrt{\frac{2G \times 4M_e}{2R_e}} \qquad \text{...(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$v'_e = \sqrt{2} \times v_e = 1.414 \times 11.2 = 15.8\ \text{kms}^{-1}$$

6. As, $T = \dfrac{2\pi r}{v} = \dfrac{2\pi r}{[GM/r]^{1/2}} = 2\pi\left[\dfrac{r^3}{GM}\right]^{1/2}$

As per question, $24 = 2\pi\left[\dfrac{(6400 + 36000)^3}{GM}\right]^{1/2}$

and $$T' = 2\pi\left[\frac{(6400)^3}{GM}\right]^{1/2}$$

$$\therefore \qquad \frac{T'}{24} = \left[\frac{(6400)^3}{(6400 + 36000)^3}\right]^{1/2} = (0.4)^3$$

or $$T' = (0.4)^3 \times 24 = 1.53\ \text{h}$$

7. $V = \left[-\dfrac{Gm}{1} + \left(-\dfrac{Gm}{2}\right) + \left(\dfrac{Gm}{4}\right) + \ldots\right] \times 2$

$$= -2Gm\left[1 + \frac{1}{2} + \frac{1}{4} + \ldots\right] = -2Gm\left[\frac{1}{1 - \dfrac{1}{2}}\right]$$

$$= -4Gm = -4G \times 1 = -4G$$

In magnitude, $V = 4G$

8. It is self-evident that, the orbit of the comet is elliptic with sun begin at one of the focus. Now, as for elliptic orbits, according to Kepler's third law,

$$T^2 = \frac{4\pi^2 a^3}{GM} \Rightarrow a = \left(\frac{T^2 GM}{4\pi^2}\right)^{1/3}$$

or $$a = \left[\frac{(76 \times 3.14 \times 10^7) \times 6.67 \times 10^{-11} \times 2 \times 10^{30}}{4\pi^2}\right]^{1/3}$$

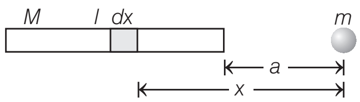
$$a = 2.7 \times 10^{12}$$

But in case of ellipse,

$$2\,a = r_{\min} + r_{\max}$$

$$\therefore \qquad r_{\max} = 2\,a - r_{\min} = 2 \times 2.7 \times 10^{12} - 8.9 \times 10^{10}$$
$$\simeq 5.3 \times 10^{12}\ \text{m}$$

9.



$$\Rightarrow \qquad dU = \frac{Gm\left(\dfrac{M}{l}\,dx\right)}{x}$$

Integrating, we get

$$\Rightarrow \qquad \int dU = \frac{GmM}{l}\int_a^{a+l}\frac{dx}{x}$$

$$\Rightarrow \qquad U = \frac{GmM}{l}\log_e\left(\frac{a+l}{a}\right)$$

10. Force on the body $= \dfrac{GMm}{x^2}$

To move it by a small distance dx,

Work done $= F\,dx = \dfrac{GMm}{x^2}\,dx$

Total work done

$$= \int F\,dx = GMm \int_R^{R+h} \frac{dx}{x^2} = \left[\frac{-GMm}{x}\right]_R^{R+h}$$

$$= GMm\left[\frac{1}{R} - \frac{1}{R+h}\right]$$

$$= GMm\left[\frac{(R+h)-R}{R(R+h)}\right] = \frac{GMmh}{R(R+h)}$$

$$= \frac{GM}{R^2} \times \frac{mhR}{R+h} = \frac{gmhR}{R+h} = \frac{PRh}{R+h}$$

11. For orbiting spaceship close to earth's surface,

$$\frac{mv_o^2}{R} = \frac{GMm}{R^2}$$

i.e. $$v_o = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

$\therefore$ $$v_o = \sqrt{(9.8 \times 6.4 \times 10^6)} \simeq 8 \text{ kms}^{-1}$$

For escaping from closed to the surface of earth,

$$\frac{GMm}{R} = \frac{1}{2} mv_e^2$$

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

$\Rightarrow$ $$v_e = \sqrt{2} \times v_o = 1.41 \times 8 = 11.2 \text{ km/s}$$

$\therefore$ The additional velocity to be imparted to the orbiting satellite for escaping is $11.2 - 8 = 3.2 \text{ kms}^{-1}$.

12. V_A = (Potential at A due to m_1) + (Potential at A due to m_2)

$\Rightarrow$ $$V_A = -\frac{Gm_1}{R} - \frac{Gm_2}{\sqrt{2}R}$$

Similarly, V_B = (Potential at B due to m_1) + (Potential at B due to m_2)

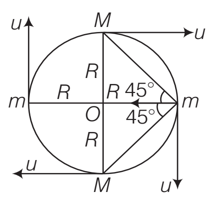
$\Rightarrow$ $$V_B = -\frac{Gm_2}{R} - \frac{Gm_1}{\sqrt{2}R}$$

Since, $W_{A\to B} = m(V_B - V_A) \Rightarrow W_{A\to B}$

$$= \frac{Gm(m_1 - m_2)(\sqrt{2}-1)}{\sqrt{2}R}$$

13. Net force acting on any one particle M

$$= \frac{GM^2}{(2R)} + \frac{GM^2}{(R\sqrt{2})^2}\cos 45° + \frac{GM}{(R\sqrt{2})^2}\cos 45°$$



$$= \frac{GM^2}{R^2}\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right)$$

This force will be equal to centripetal force.

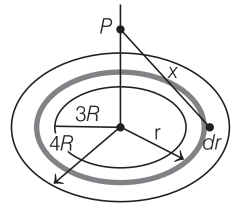
So, $$\frac{Mu^2}{R} = \frac{GM^2}{R^2}\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right)$$

$\Rightarrow$ $$u = \sqrt{\frac{GM}{4R}(1 + 2\sqrt{2})}$$

$$= \frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$$

Hence, speed of each particle in a circular motion is $\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$.

14. As, $W_{\text{ext}} = U_\infty - U_\rho$



$$W_{\text{ext}} = 0 - \int -\left(-\frac{G\rho M}{x} \cdot dx\right)$$

$$W_{\text{ext}} = G\int \frac{M}{\pi \times 7R^2} \cdot \frac{2\pi r dx}{\sqrt{16R^2 + r^2}}$$

$$= \frac{2GM}{7R^2}\int \frac{r\,dr}{\sqrt{16R^2 + r^2}}$$

$$W_{\text{ext}} = \frac{2GM}{7R^2}\int \frac{2\,dz}{2} = \frac{2GM}{7R^2}[z]$$

$$= \frac{2GM}{7R^2}[\sqrt{16R^2 + r^2}]_{3R}^{4R}$$

$$= \frac{2GM}{7R^2}[4\sqrt{2}R - 5R]$$

$$= \frac{2GM}{7R}(4\sqrt{2} - 5)$$

15. As in case of elliptic orbit of a satellite, mechanical energy $E = -\left(\frac{GMm}{2a}\right)$ remains constant, at any position of satellite in the orbit.

i.e. $$\text{KE} + \text{PE} = -\frac{GMm}{2a} \quad \ldots\text{(i)}$$

Now, if at position r, v is the orbital speed of satellite

$$\text{KE} = \frac{1}{2}mv^2$$

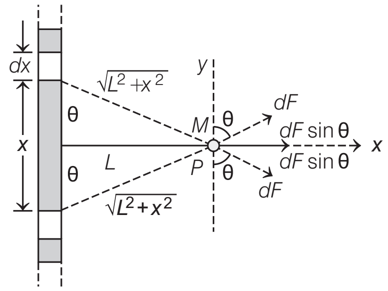
and $$\text{PE} = -\frac{GMm}{r} \quad \ldots\text{(ii)}$$

So, from Eqs. (i) and (ii), we have

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2a},$$

i.e. $$v^2 = GM\left[\frac{2}{r} - \frac{1}{a}\right]$$

16.



Let the mass M be placed symmetrically.

$$\Rightarrow \quad F_{\text{net}} = \int_{-\infty}^{\infty} dF \sin\theta \quad (dF\cos\theta \text{ are cancelled out})$$

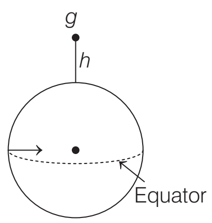
$$= \int_{-\infty}^{\infty} \frac{GM(\lambda\, dx)}{X^2 + L^2} \frac{L}{\sqrt{X^2 + L^2}}$$

$$\Rightarrow \quad F_{\text{net}} = GM\lambda L \int_{-\infty}^{\infty} \frac{dx}{(X^2 + L^2)^{3/2}}$$

$$\Rightarrow \quad F_{\text{net}} = \frac{GM\lambda L}{L^2}(2) \Rightarrow F_{\text{net}} = \frac{2GM\lambda}{L^2}$$

17. Weight of the object near the equator $= m(g - R\omega^2)$ and at height h near the poles

$$= mg\frac{R^2}{(R+h)^2}$$



According to question, both the weights are same.

$$\Rightarrow \quad m(g - R\omega^2) = \frac{mgR^2}{(R+h)^2}$$

$$(g - R\omega^2) = gR^2\left[\frac{1}{R^2 + h^2 + 2Rh}\right]$$

$$= \frac{gR^2}{R^2}\left[\frac{1}{1 + \frac{h^2}{R^2} + \frac{2Rh}{R^2}}\right]$$

$$= g\left[\frac{1}{1 + \frac{2h}{R}}\right] \qquad (\because h << R)$$

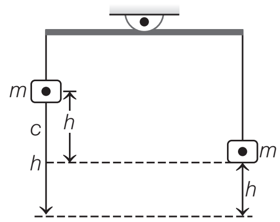
$$= g\left[1 - \frac{2h}{R}\right]$$

$$\Rightarrow \quad R\omega^2 = \frac{2gh}{R}$$

$$\Rightarrow \quad h = \frac{R^2\omega^2}{2g}$$

18. As with height g varies as

$$g' = \frac{g}{[1 + h/R]^2} = g\left[1 - \frac{2h}{R}\right]$$



and in according with figure $h_1 > h_2$, W_1 will be lesser than W_2 and

$$i.e., \quad W_2 - W_1 = mg_2 - mg_1 = 2mg\left[\frac{h_1}{R} - \frac{h_2}{R}\right]$$

$$\text{or} \quad W_2 - W_1 = 2m\frac{GM}{R^2}\frac{h}{R}$$

$$\left[\text{As, } g = \frac{GM}{R^2} \text{ and } (h_1 - h_2) = h\right]$$

$$\text{or} \quad W_2 - W_1 = \frac{2mhG}{R^3}\left(\frac{4}{3}\pi R^3\rho\right)$$

$$= \frac{8}{3}\pi\rho Gmh \qquad \left[\text{as, } M = \frac{4}{3}\pi R^3\rho\right]$$

19. Mean orbital radius of the earth around the sun

$r = 1.5 \times 10^8$ km $= 1.5 \times 10^{11}$ m

Time period of earth around the sun = 1 yr

$= 365$ days $= 365 \times 24 \times 60 \times 60$ s

As required centripetal force is obtained from the gravitational force, therefore

Centripetal force = Gravitational force

$$\frac{mv^2}{r} = \frac{GM_s m}{r^2}$$

$$v^2 = \frac{GM_s}{r}$$

$$(r\omega)^2 = \frac{GM_s}{r} \qquad (\because v = r\omega)$$

$$\text{or} \quad \omega^2 = \frac{GM_s}{r^3}$$

$$\text{But} \quad \omega = \frac{2\pi}{T}$$

$$\therefore \quad \left(\frac{2\pi}{T}\right)^2 = \frac{GM_s}{r^3}$$

$$\text{or} \quad M_s = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{4 \times (3.14)^2 \times (1.5 \times 10^{11})^3}{6.67 \times 10^{-11}(365 \times 24 \times 60 \times 60)^2}$$

$$\approx 2 \times 10^{30} \text{ kg}$$

20. For satellite, $\frac{mv^2}{R} = \frac{GMm}{R^2}$

$$\Rightarrow \quad v^2 = \frac{GM}{R}$$

$$v = \frac{2\pi R}{T}$$

$$\Rightarrow \quad v^2 = \frac{4\pi^2 R^2}{T^2} = \frac{GM}{R}$$

$$\therefore \quad T^2 = \frac{4\pi^2 R^3}{GM}$$

If T_1 and T_2 are the time periods for satellites S_1 and S_2 respectively.

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$$

$$\Rightarrow \quad R_2 = \left(\frac{T_2}{T_1}\right)^{2/3} x R_1$$

$$T_1 = 1 \text{ h}, T_2 = 8 \text{ h}, R_1 = 10^4 \text{ km}$$

$$R_2 = \left(\frac{8}{1}\right)^{2/3} \times 10^4 \text{ km} = 4 \times 10^4 \text{ km}$$

$$v_1 = \frac{2\pi R_1}{T_1} = \frac{2\pi \times 10^4}{1} = 2\pi \times 10^4 \text{ kmh}^{-1}$$

$$v_2 = \frac{2\pi R_2}{T_2} = \frac{2\pi \times 4 \times 10^4}{8} = \pi \times 10^4 \text{ kmh}^{-1}$$

Relative velocity of S_2 with respect to S_1 is

$$v = v_2 - v_1 = (\pi \times 10^4 - 2\pi \times 10^4) \text{ kmh}^{-1}$$

$$|v| = \pi \times 10^4 \text{ kmh}^{-1}$$

21. If the mass of sun is M and radius of the planet's orbit is r, then as $v_o = \sqrt{GM/r}$

$$T = \frac{2\pi r}{v_0} = 2\pi r \times \sqrt{\frac{r}{GM}}$$

$$i.e. \quad T^2 = \frac{4\pi^2 r^3}{GM} \qquad \ldots\text{(i)}$$

Now, if the planet (When stopped in the orbit) has velocity v when it is at a distance x from the sun, by conservation of mechanical energy, we get

$$\frac{1}{2}mv^2 + \left(-\frac{GMm}{x}\right) = 0 - \frac{GMm}{r}$$

$$\text{or} \quad \left(-\frac{dx}{dt}\right) = \frac{2GM}{r}\left[\frac{r-x}{x}\right]$$

$$i.e. \quad -\frac{dx}{dt} = \sqrt{\frac{2GM}{r}}\sqrt{\frac{(r-x)}{x}}$$

$$\text{or} \quad \int_0^t dt = -\sqrt{\frac{r}{2GM}}\int_r^0 \left[\frac{x}{(r-x)}\right] dx$$

Substituting $x = r\sin^2\theta$ and solving the RHS, we get

$$t = \sqrt{\frac{r}{2GM}} \times \left(\frac{\pi r}{2}\right) \qquad \text{(After integrating)}$$

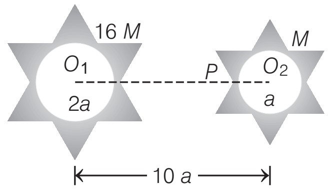
Now, Eq. (i) reduces to

$$t = \frac{1}{\sqrt{4\sqrt{2}}} T, \; i.e. \; t = \left(\frac{\sqrt{2}}{8}\right) T$$

22. First we have to find a point where the resultant field due to both is zero. Let the point P be at a distance x from centre of bigger star.

$$\Rightarrow \quad \frac{G(16M)}{x^2} = \frac{GM}{(10a - x)^2}$$

$$\Rightarrow \quad x = 8a \qquad \text{(from } O_1\text{)}$$



i.e. Once the body reaches P, the gravitational pull of attraction due to M takes the lead to make m move towards it automatically as the gravitational pull of attraction due to 16 M vanishes *i.e.*, a minimum KE or velocity has to be imparted to m from surface of 16 M such that it is just able to overcome the gravitational pull of 16 M. By law of conservation of energy,

(Total mechanical energy at A)

= (Total mechanical energy at P)

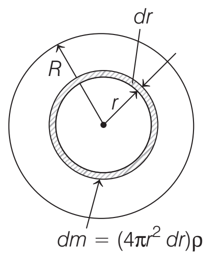
$$\Rightarrow \quad \frac{1}{2}mv_{\min}^2 + \left[\frac{G(16M)m}{2a} - \frac{G(16M)m}{8a}\right]$$

$$= 0 + \left[\frac{GMm}{2a} - \frac{G(16M)m}{8a}\right]$$

$$\Rightarrow \quad \frac{1}{2}mv_{\min}^2 = \frac{GMm}{8a}(45)$$

$$\Rightarrow \quad v_{\min} = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

23. Gravitational field intensity at a distance x from centre of planet is $E = \frac{GM}{x^2}$, where M = mass enclosed within a spherical volume of radius x.



$$\text{So,} \quad E = \frac{G}{x^2}\int_0^x (4\pi r^2 dr)[\rho(r)] \qquad \ldots\text{(i)}$$

$$\text{Here,} \quad M = \int_0^x dm = \int_0^x 4\pi r^2 dr \cdot \rho(r)$$

So, from Eq. (i), we get

$$E = \frac{G}{x^2}\int_0^x 4\pi r^2 \cdot \rho_0\left(1 - \frac{r^2}{R^2}\right) dr$$

$$= \frac{G}{x^2}\int_0^x 4\pi\rho_0\left(r^2 - \frac{r^4}{R^2}\right)dr$$

$$= \frac{4\pi G\rho_0}{x^2}\left[\frac{r^3}{3} - \frac{r^5}{5R^2}\right]_0^x$$

$$= \frac{4\pi G\rho_0}{x^2}\left(\frac{x^3}{3} - \frac{x^5}{5R^2}\right)$$

$$= 4\pi G\rho_0\left(\frac{x}{3} - \frac{x^3}{5R^2}\right)$$

For gravitational field to be maximum, $\frac{dE}{dx} = 0$

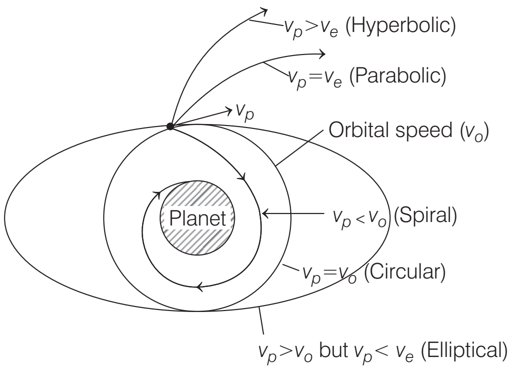
$$\Rightarrow \quad \frac{d}{dx}\left[4\pi G\rho_0\left(\frac{x}{3} - \frac{x^3}{5R^2}\right)\right] = 0$$

$$\Rightarrow \quad 4\pi G\rho_0\left(\frac{1}{3} - \frac{3x^2}{5R^2}\right) = 0$$

$$\Rightarrow \quad x = \sqrt{\frac{5}{9}}\,R$$

So, E is maximum at a distance of $\sqrt{\frac{5}{9}}\,R$ from centre of planet.

24. For a particle or satellite with speed v_p around a planet whose escape speed is v_e and orbital speed of particle v_o, we have following possibilities



Now, by momentum conservation in given collision, we have, $p_i = p_f$

$$\Rightarrow \quad mv_A + \frac{m}{2}\left(\frac{v_A}{2}\right) = \left(m + \frac{m}{2}\right)v_p \qquad \text{...(i)}$$

Body A is orbiting in orbit of radius $R + h$, hence for a small value of h,

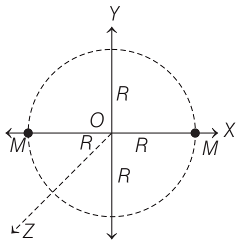
$$v_A = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R}} = \frac{v_e}{\sqrt{2}}$$

So, final speed of combined mass system,

$$v_p = \frac{5}{6}v_o = \frac{5}{6}\times\frac{1}{\sqrt{2}}v_e$$

As v_p is less than v_o, so it must spiral towards planet. Most nearest option is (b), if we take spiral close to elliptical.

25. Let us assume that, stars are moving in XY-plane with origin as their centre of mass as shown in the figure below



According to question, mass of each star,

$$M = 3\times 10^{31}\ \text{kg}$$

and diameter of circle, $2R = 2\times 10^{11}$ m

$$\Rightarrow \quad R = 10^{11}\ \text{m}$$

Potential energy of meteorite at O,

$$U_{\text{total}} = -\frac{2\,GMm}{r}$$

If v is the velocity of meteorite at O, then

kinetic energy K of the meteorite is $K = \frac{1}{2}mv^2$.

To escape from this dual star system, total mechanical energy of the meteorite at infinite distance from stars must be at least zero.

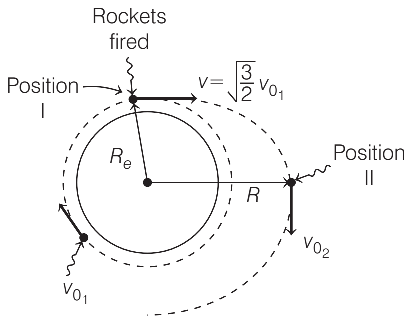
By conservation of energy, we have

$$\frac{1}{2}mv^2 - \frac{2\,GMm}{R} = 0$$

$$\Rightarrow \quad v^2 = \frac{4\,GM}{R} = \frac{4\times 6.67\times 10^{-11}\times 3\times 10^{31}}{10^{11}}$$

$$\Rightarrow \quad v = 2.83\times 10^5 \simeq 2.8\times 10^5\ \text{m/s}$$

26.



Using law of conservation of energy for position I and position II of satellite, we have

$$(\text{PE+ KE})_{\text{Position I}} = (\text{PE+ KE})_{\text{Position II}}$$

$$\Rightarrow \quad \frac{-GMm}{R_e} + \frac{1}{2}mv_1^2 = \frac{-GMm}{R} + \frac{1}{2}mv_2^2$$

$$\Rightarrow \quad \frac{-GMm}{R_e} + \frac{1}{2}m\left(\frac{3}{2}v_{0_1}^{\,2}\right) = \frac{-GMm}{R} + \frac{1}{2}mv_{0_2}^{\,2}$$

As, orbital speed, $v_0 = \sqrt{\frac{GM}{R}}$

(where, R = orbital radius)

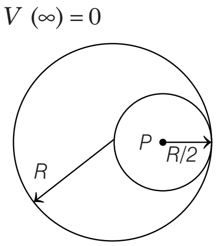
Therefore, $\dfrac{-GMm}{R_e}+\dfrac{1}{2}m\left(\dfrac{3}{2}\dfrac{GM}{R_e}\right)$

$$=-\frac{GMm}{R}+\frac{1}{2}m\left(\frac{GM}{R}\right)$$

$$\Rightarrow \qquad \frac{-1}{4}\left(\frac{GMm}{R_e}\right)=\frac{-1}{2}\left(\frac{GMm}{R}\right)$$

$$\Rightarrow \qquad R=2R_e$$

27. Consider the cavity formed in a solid sphere as shown in figure.



According to the question,we can write potential at an internal point P due to complete solid sphere,

$$V_s=-\frac{GM}{2R^3}\left[3R^2-\left(\frac{R}{2}\right)^2\right]$$

$$=\frac{-GM}{2R^3}\left[3R^2-\frac{R^2}{4}\right]$$

$$=\frac{-GM}{2R^3}\left[\frac{11\,R^2}{4}\right]=\frac{-11\,GM}{8R}$$

Mass of removed part $=\dfrac{M}{(4/3)\times\pi R^3}\times\dfrac{4}{3}\pi\left(\dfrac{R}{2}\right)^3=\dfrac{M}{8}$

Potential at point P due to removed part,

$$V_c=\frac{-3}{2}\times\frac{GM/8}{R/2}=\frac{-3GM}{8R}$$

Thus, potential due to remaining part at point P,

$$V_P=V_s-V_c=\frac{-11GM}{8R}-\left(-\frac{3GM}{8R}\right)$$

$$=\frac{(-11+3)GM}{8R}=\frac{-GM}{R}$$

28. Here, gravitational field, $E_G=\dfrac{Ax}{(x^2+a^2)^{3/2}}$

and also gravitational potential at infinity, $V_\infty=0$

Now, $\quad dV=-\mathbf{E}_G\cdot d\mathbf{x}$

$$\int_{V_\infty}^{V_x}dV=-\int_\infty^x \mathbf{E}_G\cdot d\mathbf{x}$$

$$[V]_{V_\infty}^{V_x}=-\int_\infty^x\frac{Ax}{(x^2+a^2)^{3/2}}\,dx\,\cos 0^\circ$$

$$V_x-V_\infty=-\int_\infty^x\frac{Ax}{(x^2+a^2)^{3/2}}\,dx\times 1$$

$$V_x-0=-\int_\infty^x\frac{Ax}{(x^2+a^2)^{3/2}}\,dx$$

$$V_x=-A\int_\infty^x\frac{x}{(x^2+a^2)^{3/2}}\,dx \qquad \text{...(i)}$$

Let $x^2+a^2=t$, then $2xdx+0=dt$

$$x\,dx=\frac{dt}{2}$$

At $x=\infty$, $(\infty)^2+a^2=t$

$$t=\infty \qquad \text{(lower limit)}$$

Again, at $x=x$

$$x^2+a^2=t \Rightarrow t=x^2+a^2 \qquad \text{(upper limit)}$$

From Eq. (i), we get

$$V_x=-A\int_\infty^{x^2+a^2}\frac{dt/2}{(t)^{3/2}}$$

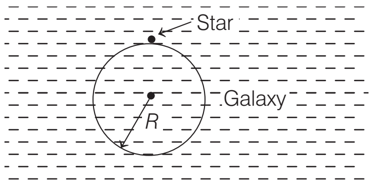
$$=-\frac{A}{2}\int_\infty^{x^2+a^2}(t)^{-3/2}dt$$

$$=-\frac{A}{2}\left[\frac{(t)^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}\right]_\infty^{x^2+a^2}=-\frac{A}{2}\left[\frac{(t)^{-\frac{1}{2}}}{-\frac{1}{2}}\right]_\infty^{x^2+a^2}$$

$$=A\left[\frac{1}{(t)^{\frac{1}{2}}}\right]_\infty^{x^2+a^2}=A\left[\frac{1}{(x^2+a^2)^{\frac{1}{2}}}-\frac{1}{(\infty)^{\frac{1}{2}}}\right]$$

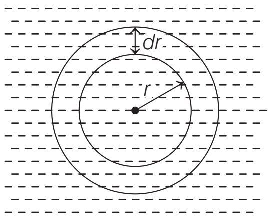
$$=A\left[\frac{1}{(x^2+a^2)^{\frac{1}{2}}}-0\right]=\frac{A}{(x^2+a^2)^{\frac{1}{2}}}$$

29. Force on star is provided by the mass enclosed inside region of radius R as shown below



where, R = orbital radius of star.

Now, consider a spherical region of radius r and thickness dr as shown below



Mass enclosed in this region is given by

$$dm=\text{density}\times\text{volume}=\frac{k}{r}\cdot 4\pi r^2\cdot dr$$

So, total mass enclosed in a region of radius R is given by

$$M=\int_0^R dm=\int_0^R\frac{k}{r}\cdot 4\pi r^2 dr$$

$$=4\pi k\left[\frac{r^2}{2}\right]_0^R=2k\pi R^2$$

Gravitational pull of this mass provides necessary centripetal force for rotation of star.

So, $\frac{GMm}{R^2} = \frac{mv^2}{R} \Rightarrow v^2 = \frac{GM}{R} = \frac{G2k\pi R^2}{R}$

$\Rightarrow \quad v^2 = 2Gk\pi R \Rightarrow v = \sqrt{2Gk\pi R}$

Time period of rotation,

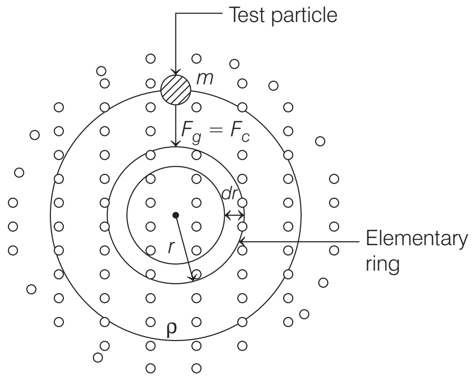
$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{2Gk\pi R}} \Rightarrow T \propto \sqrt{R} \text{ or } T^2 \propto R$$

30. Let the mass of the test particle be m and its orbital linear speed be v.

Force of gravity of the mass-density would provide the necessary centripetal pull on test particle.

$\Rightarrow$ Gravitational force (F_g) = Centripetal force (F_c)

Let us now assume an elementary ring at a distance r from the centre of the mass density, such that mass of this elementary ring be M.



$\Rightarrow \quad \frac{GMm}{R^2} = \frac{mv^2}{R}$

where, $M = \int_0^R \rho \cdot 4\pi r^2 \cdot dr$

$\Rightarrow \quad \frac{G\left(\int_0^R \frac{K}{r^2} \cdot 4\pi r^2 \cdot dr\right)}{R^2} = \frac{v^2}{R} \qquad \left[\because \text{ given, } \rho(r) = \frac{K}{r^2}\right]$

$\Rightarrow \quad G \cdot 4\pi K = v^2$

$\therefore$ Orbital speed of mass, $m = v = \sqrt{G4\pi K}$ ... (i)

Time period of rotation of the test particle,

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{G \cdot 4\pi K}} \qquad [\because \text{ using Eq. (i)}]$$

Hence, $\frac{T}{R} = \sqrt{\frac{\pi}{GK}}$ = a constant

31. A satellite or spaceship in a circular orbit at a distance $(R + h)$ from centre of a planet experiences a gravitational force given by

$$F_g = \frac{GmM}{(R+h)^2}$$

where, M = mass of planet,

m = mass of spaceship,

R = radius of planet

and h = height of spaceship above surface.

This gravitational pull provides necessary centripetal pull for orbital motion of spaceship.

So, $F_g = F_{\text{centripetal}} \Rightarrow \frac{GmM}{(R+h)^2} = \frac{mv^2}{(R+h)}$

where, v = orbital speed of spaceship.

Orbital speed of spaceship,

$$v = \sqrt{\frac{GM}{(R+h)}} \qquad \text{...(i)}$$

Here, $G = 6.67 \times 10^{-11}$ N-m^2/kg^2,

$M = 8 \times 10^{22}$ kg

$R + h = (2 \times 10^6 + 20 \times 10^3)$ m

$= 2.02 \times 10^6$ m

So, substituting these values in Eq. (i), we get

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 8 \times 10^{22}}{2.02 \times 10^6}} = 1.6 \times 10^3 \text{ ms}^{-1}$$

Time period of rotation of spaceship will be

$$T = \frac{2\pi(R+h)}{v} \Rightarrow T = \frac{2\pi \times 2.02 \times 10^6}{1.6 \times 10^3}$$

$$\approx 8 \times 10^3 \text{ s} = \frac{8 \times 10^3}{60 \times 60} \text{ (h)} = 2.2 \text{ h}$$

So, number of revolutions made by spaceship in 24 h,

$$n = \frac{24}{T} = \frac{24}{2.2} \approx 11 \text{ rev}$$

32. Given, volume of earth (V_e) is 64 times of volume of moon (V_m), *i.e.*

$$\frac{V_e}{V_m} = 64 = \frac{\frac{4}{3}\pi R_e^3}{\frac{4}{3}\pi R_m^3}$$

where, R_e and R_m are the radii of earth and moon, respectively.

Then, $\frac{R_e}{R_m} = 4$ ...(i)

Also, since the density of moon and earth are equal, *i.e.* $\rho_m = \rho_e$

$\Rightarrow \frac{M_e}{V_e} = \frac{M_m}{V_m}$, where M_e and M_m are the mass of the earth and moon, respectively.

$\Rightarrow \quad \frac{M_e}{M_m} = \frac{V_e}{V_m} = 64$ ...(ii)

The minimum energy or escape energy delivered by the rocket launcher, so that the rocket never returns to earth is $E_e = \frac{GM_e m}{R_e} = E$

where, m is the mass of the rocket.

Similarly, minimum energy that a launcher should have to escape or to never return, if rocket is launched from surface of the moon is

$$E_m = \frac{GM_m m}{R_m}$$

$\therefore$ Ratio of escape energies E_e and E_m is

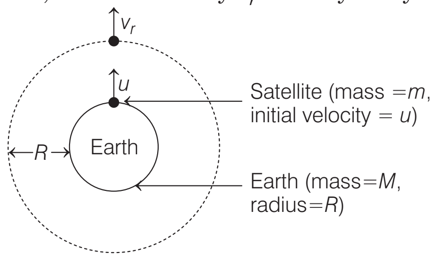
$$\frac{E_e}{E_m} = \frac{\left(\frac{GM_e m}{R_e}\right)}{\left(\frac{GM_m m}{R_m}\right)} = \frac{M_e}{M_m} \cdot \frac{R_m}{R_e}$$

$$= 64 \times \frac{1}{4} = 16$$

[using Eqs. (i) and (ii)]

$$\therefore \qquad E_m = \frac{E_e}{16} = \frac{E}{16}$$

33. Let us assume that, when satellite reaches in orbit at height R, it has a velocity v_r radially away from earth.



According to conservation of energy,
Total energy of satellite at surface of earth = Total energy at height R

$\Rightarrow (\text{KE} + \text{PE})_{\text{initial}} = (\text{KE} + \text{PE})_{\text{final}}$

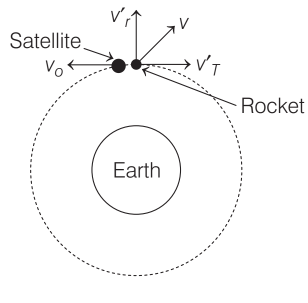
$$\Rightarrow \quad \frac{1}{2}mu^2 + \left(\frac{-GMm}{R}\right) = \frac{1}{2}mv_r^2 + \left(\frac{-GMm}{2R}\right)$$

So, radial velocity of satellite at height R,

$$v_r = \left(u^2 - \frac{GM}{R}\right)^{1/2} \qquad \text{...(i)}$$

To provide satellite an orbital velocity $\left(v_o = \sqrt{\frac{GM}{2R}}\right)$, a rocket $\left(\text{mass} = \frac{m}{10}\right)$ is fired.

Rocket must be given a velocity v such that it destroyed or opposes radial momentum of satellite and also provides orbital velocity to the satellite.



Now, we apply conservation of momentum to calculate radial (v'_r) and tangential (v'_T) components of rocket's velocity.

When rocket is ejected from satellite, mass of satellite will be

$$m - \frac{m}{10} = \frac{9m}{10}$$

By applying conservation of tangential momentum of rocket and satellite system, we have

$$\frac{m}{10} \times v'_T + \frac{9m}{10}(-v_o) = 0 \qquad \text{...(ii)}$$

Similarly, by applying conservation of radial momentum of rocket and satellite system, we have

$$\frac{m}{10} \times v'_r + \frac{9m}{10}(v_r) = 0 \qquad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii), we have

$$v'_T = 9\sqrt{\frac{GM}{2R}} \text{ and } v'_r = 10\sqrt{u^2 - \frac{GM}{R}}$$

So, kinetic energy of the rocket,

$$K = \frac{1}{2}\left(\frac{m}{10}\right) \cdot v^2 = \frac{1}{2}\left(\frac{m}{10}\right)\left(\sqrt{v_T'^{\,2} + v_r'^{\,2}}\right)^2$$

$$= \frac{1}{2}\left(\frac{m}{10}\right)(v_T'^2 + v_r'^2)$$

$$= \frac{1}{2}\left(\frac{m}{10}\right)\left[81 \times \frac{GM}{2R} + 100 \times \left(u^2 - \frac{GM}{R}\right)\right]$$

$$= 5m\left(u^2 - \frac{119}{200}\frac{GM}{R}\right)$$

34. Escape velocity of earth, $v_e = \sqrt{\frac{2GM}{R}}$ ...(i)

$$\Rightarrow \qquad 10v_e = \sqrt{\frac{2GM}{R'}} \qquad \text{...(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$10 = \sqrt{\frac{R}{R'}}$$

$$\Rightarrow \qquad R' = \frac{R}{100} = \frac{6400}{100} = 64 \text{ km}$$

35. Initial potential energy, $U_i = -\frac{3}{5}\frac{GM^2}{R}$

Final potential energy, $U_f = 0$

$\therefore$ Energy supplied $= U_f - U_i$

$$= 0 - \left(-\frac{3}{5}\frac{GM^2}{R}\right)$$

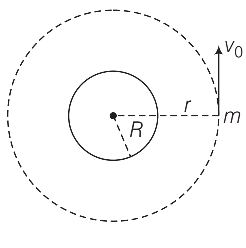
$$= \frac{3}{5}\frac{GM^2}{R}$$

$$\therefore \qquad x = 3$$

36. For a satellite in circular orbit,

$$\frac{mv_0^2}{r} = \frac{GMm}{r^2}$$

or $$v_0^2 = \frac{GM}{R+h}$$



Given, $$v_0 = \frac{v_e}{2}$$

$$\Rightarrow \quad \sqrt{\frac{GM}{R+h}} = \frac{1}{2}\sqrt{\frac{2GM}{R}}$$

$$\frac{GM}{R+h} = \frac{GM}{2R}$$

$\Rightarrow \quad h = R$

When the satellite stops, $K = 0$

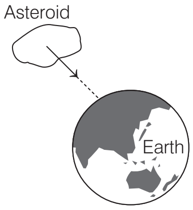
$\therefore \quad U_i - U_f = K_f$

$$\Rightarrow \quad -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right) = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

$\therefore \quad k = 1$

37.



Applying energy conservation, we have

Total energy of asteroid at 12 km = Total energy of asteroid at surface of earth

$$\Rightarrow \quad U_1 + K_1 = U_2 + K_2$$

$$\Rightarrow \quad \frac{-GM_e m}{10R_e} + \frac{1}{2}mv_0^2 = \frac{-GM_e m}{R_e} + \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{9}{10}\frac{GM_e m}{R_e} + \frac{1}{2}mv_0^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{9}{10} \times \frac{2GM_e}{R_e} + v_0^2 = v^2$$

$$\Rightarrow \quad \frac{9}{10} \times (\sqrt{v_e})^2 + v_0^2 = v^2$$

$$\Rightarrow \quad \frac{9}{10} \times (11.2)^2 + (12)^2 = v^2$$

$$\Rightarrow \quad v^2 = \sqrt{\frac{9}{10} \times (11.2)^2 + (12)^2} \Rightarrow v \approx 16 \text{ kms}^{-1}$$

38. $V_A = \begin{bmatrix}\text{Potential due to the} \\ \text{complete sphere at } A\end{bmatrix} - \begin{bmatrix}\text{Potential due to the} \\ \text{spherical cavity at } A\end{bmatrix}$

$$= -\frac{3}{2}\frac{GM}{R} - \left(-\frac{GM'}{r}\right)$$

$$M = \frac{4}{3}\pi R^3 \rho$$

$$M' = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \rho = \frac{1}{8}\left(\frac{4}{3}\pi R^3 \rho\right), r = \frac{R}{2}$$

$$\therefore \quad V_A = \frac{G}{R}\left[\frac{\pi\rho R^3}{3} - 2\pi\rho R^3\right] = -\frac{5}{3}\pi G\rho R^2$$

Now, $\quad V_B = \begin{bmatrix}\text{Potential due to the} \\ \text{complete sphere at } B\end{bmatrix} - \begin{bmatrix}\text{Potential due to the} \\ \text{spherical cavity at } B\end{bmatrix}$

$$= -\frac{GM}{2R^3}(3R^2 - r^2) - \left[-\frac{3}{2}\frac{GM'}{r}\right]$$

or $\quad V_B = -\frac{11}{8}\frac{GM}{R} + \frac{3GM'}{R}$

$$= \frac{G}{R}\left[\pi\rho R^3 - \frac{11\pi\rho R^3}{6}\right]$$

$$= -\frac{4}{3}\pi G\rho R^2$$

Now, $\quad V_B - V_A = \frac{1}{3}\pi G\rho R^2$

$$\frac{1}{2}m_0 v^2 = U_B - U_A = m_0(V_B - V_A)$$

$$\Rightarrow \quad v = \sqrt{2(V_B - V_A)}$$

or $\quad v = \frac{\sqrt{2\pi G\rho R^2}}{\sqrt{3}}$

$\Rightarrow \quad \beta = 3$