JEE Type Solved Examples :
Single Option Correct Type Questions

- This section contains **6 multiple choice examples**. Each example has four choices (a), (b), (c) and (d), out of which **ONLY ONE** is correct.

Ex. 1 *Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n are*

(a) 7, 6 (b) 6, 3 (c) 5, 1 (d) 8, 7

Sol. (*b*) Since, $2^m - 2^n = 56 = 8 \times 7 = 2^3 \times 7$

$\Rightarrow \quad 2^n(2^{m-n} - 1) = 2^3 \times 7$

$\Rightarrow \quad n = 3$ and $2^{m-n} = 8 = 2^3 \Rightarrow n = 3$ and $m - n = 3$

$\Rightarrow \quad n = 3$ and $m - 3 = 3 \quad \Rightarrow n = 3$ and $m = 6$

Ex. 2 *If* $aN = \{ax : x \in N\}$ *and* $bN \cap cN = dN$, *where* $b, c \in N$ *are relatively prime, then*

(a) $d = bc$ (b) $c = bd$
(c) $b = cd$ (d) None of these

Sol. (a) bN = The set of positive integral multiples of b
cN = The set of positive integral multiples of c

$\therefore bN \cap cN$ = The set of positive integral multiples of bc

$= bcN$ [$\because$ b and c are prime]

$\therefore \quad d = bc$

Ex. 3 *In a town of 10000 families, it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy newspapers A and B, 3% buy newspapers B and C and 4% buy newspapers A and C. If 2% families buy all the three newspapers, then number of families which buy A only is*

(a) 3100 (b) 3300 (c) 2900 (d) 1400

Sol. (*b*) $n(A) = 40\%$ of $10000 = 4000$

$n(B) = 20\%$ of $10000 = 2000$

$n(C) = 10\%$ of $10000 = 1000$

$n(A \cap B) = 5\%$ of $10000 = 500$

$n(B \cap C) = 3\%$ of $10000 = 300$

$n(C \cap A) = 4\%$ of $10000 = 400$

$n(A \cap B \cap C) = 2\%$ of $10000 = 200$

We want to find $n(A \cap B^c \cap C^c) = n[A \cap (B \cup C)^c]$

$= n(A) - n[A \cap (B \cup C)] = n(A) - n[(A \cap B) \cup (A \cap C)]$
$= n(A) - [n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)]$
$= 4000 - [500 + 400 - 200] = 4000 - 700 = 3300$

Ex. 4 *Let R be the relation on the set R of all real numbers defined by aRb iff* $|a - b| \le 1$. *Then, R is*

(a) reflexive and symmetric (b) symmetric only
(c) transitive only (d) anti-symmetric only

Sol. (a) $\because |a - a| = 0 < 1 \Rightarrow a\,R\,a, \forall a \in R$

$\therefore$ R is reflexive.

Again, $aRb \Rightarrow |a - b| \le 1$

$\Rightarrow \quad |b - a| \le 1 \Rightarrow bRa$

$\therefore$ R is symmetric.

Again, $1\,R\,2$ and $2R1$ but $2 \ne 1$

$\therefore$ R is not anti-symmetric.

Further, $1R2$ and $2R3$ but $1 \not R 3$ [$\because |1 - 3| = 2 > 1$]

$\therefore$ R is not transitive.

Ex. 5 *The relation R defined on A = {1, 2, 3} by aRb, if* $|a^2 - b^2| \le 5$. *Which of the following is false?*

(a) $R = \{(1, 1), (2, 2), (3, 3), (2, 1), (1, 2), (2, 3), (3, 2)\}$
(b) $R^{-1} = R$
(c) Domain of $R = \{1, 2, 3\}$
(d) Range of $R = \{5\}$

Sol. (*d*) Let $a = 1$

Then, $|a^2 - b^2| \le 5 \Rightarrow |1 - b^2| \le 5$

$\Rightarrow \quad |b^2 - 1| \le 5 \Rightarrow b = 1, 2$

Let $a = 2$

Then, $|a^2 - b^2| \le 5$

$\Rightarrow \quad |4 - b^2| \le 5 \Rightarrow |b^2 - 4| \le 5$

$\therefore \quad b = 1, 2, 3$

Let $a = 3$

Then, $|a^2 - b^2| \le 5$

$\Rightarrow \quad |9 - b^2| \le 5 \Rightarrow |b^2 - 9| \le 5 \Rightarrow b = 2, 3$

$\therefore \quad R = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)\}$

$R^{-1} = \{(y, x) : (x, y) \in R\}$

$= \{(1, 1), (2, 1), (1, 2), (2, 2), (3, 2), (2, 3), (3, 3)\} = R$

Domain of $R = \{x : (x, y) \in R\} = \{1, 2, 3\}$

Range of $R = \{y : (x, y) \in R\} = \{1, 2, 3\}$

Ex. 6 *If* $f(x) = \dfrac{1}{(1 - x)}$, $g(x) = f\{f(x)\}$ *and* $h(x) = f[f\{f(x)\}]$. *Then the value of* $f(x) \cdot g(x) \cdot h(x)$ *is*

(a) 6 (b) −1 (c) 1 (d) 2

Sol. (*b*) $\because g(x) = f\{f(x)\} = f\left(\dfrac{1}{1 - x}\right) = \dfrac{1}{1 - \dfrac{1}{1 - x}} = \dfrac{x - 1}{x}$

and $h(x) = f[f\{f(x)\}] = f(g(x))$

$= \dfrac{1}{1 - g(x)} = \dfrac{1}{1 - \dfrac{x - 1}{x}} = x$

$\therefore f(x) \cdot g(x) \cdot h(x) = \dfrac{1}{(1 - x)} \cdot \dfrac{(x - 1)}{x} \cdot x = -1$

JEE Type Solved Examples :
More than One Correct Option Type Questions

- This section contains **3 multiple choice examples**. Each example has four choices (a), (b), (c) and (d) out of which **MORE THAN ONE** may be correct.

Ex. 7 *If I is the set of integers and if the relation R is defined over I by aRb, iff $a-b$ is an even integer, $a, b \in I$, the relation R is*

(a) reflexive (b) anti-symmetric
(c) symmetric (d) equivalence

Sol. (a, c, d)

$aRb \Leftrightarrow a-b$ is an even integer, $a, b \in I$

$a - a = 0$ (even integer)

$\therefore \quad (a, a) \in R, \ \forall \ a \in I$

$\therefore$ R is reflexive relation.

Let $(a, b) \in R \Rightarrow (a-b)$ is an even integer.

$\Rightarrow \quad -(b-a)$ is an even integer.

$\Rightarrow \quad (b-a)$ is an even integer.

$\Rightarrow \quad (b, a) \in R$

$\therefore$ R is symmetric relation.

Now, let $(a, b) \in R$ and $(b, c) \in R$

Then, $(a-b)$ is an even integer and $(b-c)$ is an even integer.

So, let $\quad a - b = 2x_1, x_1 \in I$

and $\quad b - c = 2x_2, x_2 \in I$

$\therefore \quad (a-b)+(b-c) = 2(x_1+x_2)$

$\Rightarrow \quad (a-c) = 2(x_1+x_2) \Rightarrow a - c = 2x_3$

$\therefore$ $(a-c)$ is an even integer.

$\therefore$ aRb and $bRc \Rightarrow aRc$ So, R is transitive relation.

Hence, R is an equivalence relation.

A relation R on given set A is said to be anti-symmetric iff $(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b, \forall \ a, b \in A$.

$\therefore$ Given relation is not anti-symmetric relation.

Ex. 8 *If $f(x) = \dfrac{a-x}{a+x}$, the domain of $f^{-1}(x)$ contains*

(a) $(-\infty, \infty)$ (b) $(-\infty, -1)$
(c) $(-1, \infty)$ (d) $(0, \infty)$

Sol. (b, c, d)

Let $\quad y = f(x) = \dfrac{a-x}{a+x} \Rightarrow ay + xy = a - x$

$\therefore \quad x = \dfrac{a(1-y)}{(1+y)} = f^{-1}(y) \Rightarrow f^{-1}(x) = \dfrac{a(1-x)}{(1+x)}$

$\therefore$ $f^{-1}(x)$ is not defined for $x = -1$.

Domain of $f^{-1}(x)$ belongs to $(-\infty, -1) \cup (-1, \infty)$.

Now, for $a = -1$, given function $f(x) = -1$, which is constant.

Then, $f^{-1}(x)$ is not defined.

$\therefore \quad a \neq -1$

Ex. 9 *If $f(x) = \dfrac{\sin([x]\pi)}{x^2+x+1}$, where $[\cdot]$ denotes the greatest integer function, then*

(a) f is one-one
(b) f is not one-one and non-constant
(c) f is constant function (d) f is zero function

Sol. (c, d)

$\because \quad \sin([x]\pi) = 0$

$\therefore \quad f(x) = 0 \quad [\because [x]$ is an integer$]$

$\Rightarrow$ $f(x)$ is a constant function and also $f(x)$ is a zero function.

JEE Type Solved Examples :
Passage Based Questions

- This section contains **2 solved passages** based upon each of the passage **3 multiple choice** examples have to be answered. Each of these examples has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Passage I
(Ex. Nos. 10 to 12)

If $A = \{x : |x| < 2\}$, $B = \{x : |x-5| \le 2\}$,
$C = \{x : |x| > x\}$ *and* $D = \{x : |x| < x\}$

10. *The number of integral values in $A \cup B$ is*

(a) 4 (b) 6
(c) 8 (d) 10

11. *The number of integral values in $A \cap C$ is*

(a) 1 (b) 2
(c) 3 (d) 0

12. *The number of integral values in $A \cap D$ is*

(a) 2 (b) 4
(c) 6 (d) 0

Sol. (Ex. Nos. 10 to 12)

$$A = \{x : |x| < 2\} = \{x : -2 < x < 2\} = (-2, 2)$$
$$B = \{x : |x - 5| \le 2\} = \{x : -2 \le x - 5 \le 2\}$$
$$= \{x : 3 \le x \le 7\} = [3, 7]$$
$$C = \{x : |x| > x\} = \{x : x < 0\} = (-\infty, 0)$$
and $D = \{x : |x| < x\} = \phi$

10. (*c*) $A \cup B = (-2, 2) \cup [3, 7]$

Integral values in $A \cup B$ are $-1, 0, 1, 3, 4, 5, 6, 7$.

$\therefore$ Number of integral values in $A \cup B$ is 8.

11. (*a*) $A \cap C = (-2, 2) \cap (-\infty, 0) = (-2, 0)$

Integral value in $A \cap C$ is -1.

$\therefore$ Number of integral values in $A \cap C$ is 1.

12. (*d*) $A \cap D = (-2, 2) \cap \phi = \phi$

$\therefore$ Number of integral values in $A \cap D$ is 0.

Passage II
(Ex. Nos. 13 to 15)

If $A = \{x : x^2 - 2x + 2 > 0\}$ *and* $B = \{x : x^2 - 4x + 3 \le 0\}$

13. *$A \cap B$ equals*

(a) $[1, \infty)$ (b) $[1, 3]$
(c) $(-\infty, 3]$ (d) $(-\infty, 1) \cup (3, \infty)$

14. *$A - B$ equals*

(a) $(-\infty, \infty)$ (b) $(1, 3)$
(c) $(3, \infty)$ (d) $(-\infty, 1) \cup (3, \infty)$

15. *$A \cup B$ equals*

(a) $(-\infty, 1)$ (b) $(3, \infty)$
(c) $(-\infty, \infty)$ (d) $(1, 3)$

Sol. (Ex. Nos. 13 to 15)

$$A = \{x : x^2 - 2x + 2 > 0\} = \{x : (x - 1)^2 + 1 > 0\} = (-\infty, \infty)$$
$$B = \{x : x^2 - 4x + 3 \le 0\} = \{x : (x - 1)(x - 3) \le 0\}$$
$$= \{x : 1 \le x \le 3\} = [1, 3]$$

13. (*b*) $A \cap B = (-\infty, \infty) \cap [1, 3] = [1, 3]$

14. (*d*) $A - B = (-\infty, \infty) - [1, 3] = (-\infty, 1) \cup (3, \infty)$

15. (*c*) $A \cup B = (-\infty, \infty) \cup [1, 3] = (-\infty, \infty)$

JEE Type Solved Examples :
Single Integer Answer Type Questions

- This section contains **2 examples.** The answer to each example is a **single digit integer** ranging from **0** to **9** (both inclusive).

Ex. 16 *If $f : R^+ \to A$, where $A = \{x : -5 < x < \infty\}$ is defined by $f(x) = x^2 - 5$ and if $f^{-1}(13) = \{-\lambda\sqrt{(\lambda - 1)}, \lambda\sqrt{(\lambda - 1)}\}$, the value of λ is*

Sol. (*3*) $f^{-1}(13) = \{x : f(x) = 13\} = \{x : x^2 - 5 = 13\}$

$$= \{x : x^2 = 18\} = \{x : x = \pm 3\sqrt{2}\}$$
$$= \{-3\sqrt{2}, 3\sqrt{2}\}$$
$$= \{-\lambda\sqrt{(\lambda - 1)}, \lambda\sqrt{(\lambda - 1)}\} \quad \text{[given]}$$
$\therefore$ $\lambda = 3$

Ex. 17 *If $A = \{2, 3\}$, $B = \{4, 5\}$ and $C = \{5, 6\}$, then $n\{(A \times B) \cup (B \times C)\}$ is*

Sol. (*8*) $\because$ $A \times B = \{2, 3\} \times \{4, 5\}$

$$= \{(2, 4), (2, 5), (3, 4), (3, 5)\}$$
and $B \times C = \{4, 5\} \times \{5, 6\}$
$$= \{(4, 5), (4, 6), (5, 5), (5, 6)\}$$
$\therefore$ $(A \times B) \cup (B \times C) = \{(2, 4), (2, 5), (3, 4), (3, 5), (4, 5), (4, 6), (5, 5), (5\ 6)\}$

Now, $n\{(A \times B) \cup (B \times C)\} = 8$

JEE Type Solved Examples :
Matching Type Questions

- This section contains **1 examples.** Example 18 have three statements (A, B and C) given in Column I and four statements (p, q, r and s) in **Column II.** Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II.**

Ex. 18

	Column I		Column II
(A)	$R = \{(x, y) : x < y ; x, y \in N\}$	(p)	Reflexive
(B)	$S = \{(x, y) : x + y = 10 ; x, y \in N\}$	(q)	Symmetric
(C)	$T = \{(x, y) : x = y$ or $x - y = 1 ; x, y \in N\}$	(r)	Transitive
(D)	$U = \{(x, y) : x^y = y^x; x, y \in N\}$	(s)	Equivalence

Sol. (A) → (r); (B) → (q); (C) → (p); (D) → (p, q, r, s)

(A) $\because \quad R = \{(x, y) : x < y; x, y \in N\}$

$x \not< x \ \therefore (x, x) \notin R$

So, R is not reflexive.

Now, $(x, y) \in R \Rightarrow x < y \not\Rightarrow y < x \Rightarrow (y, x) \notin R$

$\therefore$ R is not symmetric.

Let $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow \quad x < y$ and $y < z \Rightarrow x < z \Rightarrow (x, z) \in R$

$\therefore R$ is transitive.

(B) $\because \ S = \{(x, y) : x + y = 10 ; x, y \in N\}$

$\therefore \quad x + x = 10 \Rightarrow 2x = 10 \Rightarrow x = 5$

So, each element of N is not related to itself by the relation $x + y = 10$.

$\therefore$ S is not reflexive.

Now, $(x, y) \in S \Rightarrow x + y = 10 \Rightarrow y + x = 10$

$\Rightarrow \quad (y, x) \in S$

$\therefore$ S is symmetric relation.

Now, let $(3, 7) \in S$ and $(7, 3) \in S \Rightarrow (3, 3) \notin S$

$\therefore$ S is not transitive.

(C) $\because \quad T = \{(x, y) : x = y$ or $x - y = 1; x, y \in N\}$

$\therefore \quad x = x$

So, $\quad (x, x) \in T, \forall \ x \in N$

$\therefore \quad T$ is reflexive.

Let $\quad (3, 2) \in T$ and $3 - 2 = 1$

$\not\Rightarrow \quad 2 - 3 = -1 \Rightarrow (2, 3) \notin T$

$\therefore \quad T$ is not symmetric.

Now, let $\quad (3, 2) \in T$ and $(2, 1) \in T$

$\therefore \quad 3 - 2 = 1$ and $2 - 1 = 1$

Then, $\quad (3, 1) \notin T \qquad [\because 3 - 1 = 2 \neq 1]$

$\therefore \quad T$ is not transitive.

(D) $U = \{(x, y) : x^y = y^x; x, y \in N\}$

$\because \quad x^x = x^x$

$\therefore \quad (x, x) \in U$

$\therefore$ U is reflexive.

Now, $(x, y) \in U \Rightarrow x^y = y^x$

$\Rightarrow \quad y^x = x^y \Rightarrow (y, x) \in U$

$\therefore$ U is symmetric.

Now, let $(x, y) \in U$ and $(y, z) \in U$

$\Rightarrow \quad x^y = y^x$ and $y^z = z^y$

Now, $\quad (x^y)^z = (y^x)^z$

$\Rightarrow \quad (x^z)^y = (y^z)^x \Rightarrow (x^z)^y = (z^y)^x$

$\Rightarrow \quad (x^z)^y = (z^x)^y \Rightarrow x^z = z^x \Rightarrow (x, z) \in U$

$\therefore \quad U$ is transitive.

Hence, U is an equivalence relation.

JEE Type Solved Examples :
Statement I and II Type Questions

- **Directions** Example numbers 19 and 20 are Assertion-Reason type examples. Each of these examples contains two statements:
Statement-1 (Assertion) and
Statement-2 (Reason)
Each of these examples also has four alternative choices, only one of which is the correct answer. You have to select the correct choice as given below:
(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(c) Statement-1 is true; Statement-2 is false
(d) Statement-1 is false; Statement-2 is true

Ex. 19 **Statement-1** *If $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then $B = C$.*

Statement-2 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Sol. (*a*) We have, $B = B \cup (A \cap B)$

$= B \cup (A \cap C) \qquad [\because A \cap B = A \cap C]$

$= (A \cup C) \cap (B \cup C)$ $\quad [\because A \cup B = A \cup C]$

$= (A \cap B) \cup C$

$= (A \cap C) \cup C$ $\quad [\because A \cap B = A \cap C]$

$= C$

Hence, Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation of Statement-1.

● ***Ex. 20*** **Statement-1** *If U is universal set and $B = U - A$, then $n(B) = n(U) - n(A)$.*

Statement-2 *For any three arbitrary sets A, B and C, if $C = A - B$, then $n(C) = n(A) - n(B)$.*

Sol. (c) $\because$ $\quad B = U - A = A'$

$\therefore \quad n(B) = n(A') = n(U) - n(A)$

So, Statement-1 is true.

But for any three arbitrary sets A, B and C, we cannot always have

$$n(C) = n(A) - n(B)$$

if $\quad C = A - B$

As it is not specified A is universal set or not. In case not conclude

$$n(C) = n(A) - n(B)$$

Hence, Statement-2 is false.

Subjective Type Examples

- In this section, there are **12 subjective** solved examples.

● ***Ex. 21.*** *If $A = A \cup B$, prove that $B = A \cap B$.*

Sol. $\because A = A \cap B$

$\therefore \quad A \subseteq A \cup B$ and $A \cup B \subseteq A$

Now, let $x \in B \Leftrightarrow x \in A \cup B$ $\quad$ [by definition of union]

$\Leftrightarrow \quad x \in A$ $\quad [\because A \subseteq A \cup B]$

$\Leftrightarrow \quad x \in A \cap B$ $\quad [\because A \subseteq A \cup B$, then also $A \subseteq A \cap B]$

$\therefore \quad B \subseteq A \cap B$ and $A \cap B \subseteq B$

Hence, $\quad A \cap B = B$ $\quad$ **Hence proved.**

● ***Ex. 22*** *Find the smallest and largest sets of Y such that $Y \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$.*

Sol. Smallest set of Y has three elements and largest set of Y has five elements, since RHS set has five elements.

$\therefore$ Smallest set of Y is $\{3, 5, 9\}$

and largest set of Y is $\{1, 2, 3, 5, 9\}$.

● ***Ex. 23*** *If P, Q and R are the subsets of a set A, then prove that $R \times (P^c \cup Q^c)^c = (R \times P) \cap (R \times Q)$.*

Sol. We know that from De-Morgan's law,

$$A^c \cap B^c = (A \cup B)^c \quad \text{...(i)}$$

Replacing A by P^c and B by Q^c, then Eq. (i) becomes

$$(P^c)^c \cap (Q^c)^c = (P^c \cup Q^c)^c$$

$\Rightarrow \quad P \cap Q = (P^c \cup Q^c)^c$ $\quad [\because (A^c)^c = A]$ ...(ii)

$\therefore R \times (P^c \cup Q^c)^c = R \times (P \cap Q)$ $\quad$ [from Eq. (ii)]

$= (R \times P) \cap (R \times Q)$ [by cartesian product]

Hence, $R \times (P^c \cup Q^c)^c = (R \times P) \cap (R \times Q)$

● ***Ex. 24*** *Check the following relations R and ρ for reflexive, symmetry and transitivity.*

(i) aRb iff b is divisible by a, where a and b are natural numbers.

(ii) $\alpha\rho\beta$ iff α is perpendicular to β, where α and β are straight lines in a plane.

Sol. (i) The relation R is reflexive, since a is divisible by a, R is not symmetric because b is divisible by a but a is not divisible by b. i.e., $aRb \not\Rightarrow bRa$

Again, R is transitive, since b is divisible by a and c is divisible by b, then always c is divisible by a.

(ii) The relation ρ is not reflexive as no line can be perpendicular to itself. The relation ρ is symmetric, since a line α is perpendicular to β, then β is perpendicular to α and the relation ρ is not transitive, since a line α is perpendicular to β and if β is perpendicular to γ (new line), then α is not perpendicular to γ (since, α is parallel to γ).

● ***Ex. 25*** *Let $f : [0, 1] \to [0, 1]$ be defined by $f(x) = \dfrac{1 - x}{1 + x}$; $0 \le x \le 1$ and $g : [0, 1] \to [0, 1]$ be defined by $g(x) = 4x(1 - x), 0 \le x \le 1$.*

Determine the functions *fog* and *gof*.

Note that $[0, 1]$ stands for the set of all real members x that satisfy the condition $0 \le x \le 1$.

Sol. $(fog)x = f\{g(x)\} = f\{4x(1 - x)\}$ $\quad [\because g(x) = 4x(1 - x)]$

$$= \frac{1 - 4x(1 - x)}{1 + 4x(1 - x)} \quad \left[\because f(x) = \frac{1 - x}{1 + x}\right]$$

$$= \frac{1 - 4x + 4x^2}{1 + 4x - 4x^2} = \frac{(2x - 1)^2}{1 + 4x - 4x^2}$$

and $(gof)x = g\{f(x)\} = g\left\{\frac{1-x}{1+x}\right\}$ $\left[\because f(x) = \frac{1-x}{1+x}\right]$

$$= 4\left(\frac{1-x}{1+x}\right)\left(1 - \frac{1-x}{1+x}\right) = 4\left(\frac{1-x}{1+x}\right)\left(\frac{2x}{1+x}\right)$$

$$= \frac{8x(1-x)}{(1+x)^2}$$

● ***Ex. 26*** *If A, B are two sets, prove that* $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$.

Hence or otherwise prove that

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

where, $n(A)$ denotes the number of elements in A.

Sol. Let $x \in A \cup B \Rightarrow x \in A$ or $x \in B$

$\Leftrightarrow (x \in A$ and $x \notin B)$ or $(x \in B$ and $x \notin A)$

or $(x \in A$ and $x \in B)$ [from definition of union]

$\Leftrightarrow$ $x \in (A - B)$ or $x \in (B - A)$ or $x \in A \cap B$

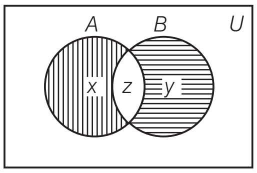
$\Leftrightarrow$ $x \in (A - B) \cup (B - A) \cup (A \cap B)$

$\therefore$ $A \cup B \subseteq (A - B) \cup (B - A) \cup (A \cap B)$

and $(A - B) \cup (B - A) \cup (A \cap B) \subseteq A \cup B$

Hence, $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

Let the common elements in A and B are z and only element of A are x (represented by vertical lines in the Venn diagram) and only element of B are y (represented by horizontal lines in the Venn diagram)



$\therefore$ $n(A) =$ Total elements of $A = x + z$

$n(B) =$ Total elements of $B = y + z$

$n(A \cap B) =$ Common elements in A and $B = z$

Now, $n(A \cup B) =$ Total elements in complete region of A and B

$$= x + y + z$$
$$= (x + z) + (y + z) - z$$
$$= n(A) + n(B) - n(A \cap B)$$

Hence, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

● ***Ex. 27*** *Let* $A = \{\theta : 2\cos^2\theta + \sin\theta \le 2\}$ *and* $B = \{\theta : \pi/2 \le \theta \le 3\pi/2\}$. Then find $A \cap B$.

Sol. $\because$ $2\cos^2\theta + \sin\theta \le 2$

$\therefore$ $2(1 - \sin^2\theta) + \sin\theta \le 2$

$\Rightarrow$ $2\sin^2\theta - \sin\theta \ge 0$

$\Rightarrow$ $\sin\theta(2\sin\theta - 1) \ge 0$

$\Rightarrow$ $\sin\theta\left(\sin\theta - \frac{1}{2}\right) \ge 0$

$\therefore$ $\sin\theta \le 0$ and $\sin\theta \ge \frac{1}{2}$

Now, the values of θ which lie in the interval $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$.

$\left[\because B = \left\{\theta : \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}\right\}\right]$

So, θ satisfy $\sin\theta \le 0$ in the interval $\pi \le \theta \le \frac{3\pi}{2}$

and θ satisfy $\sin\theta \ge \frac{1}{2}$ in the interval $\frac{\pi}{2} \le \theta \le \frac{5\pi}{6}$.

$\therefore$ $A \cap B = \left\{\theta : \pi \le \theta \le \frac{3\pi}{2}\right\}$

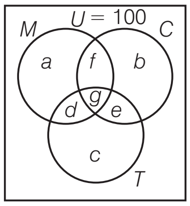
and $A \cap B = \left\{\theta : \frac{\pi}{2} \le \theta \le \frac{5\pi}{6}\right\}$

Hence, $A \cap B = \left\{\theta : \frac{\pi}{2} \le \theta \le \frac{5\pi}{6} \text{ or } \pi \le \theta \le \frac{3\pi}{2}\right\}$

$$= \left\{\theta : \theta \in \left[\frac{\pi}{2}, \frac{5\pi}{6}\right] \cup \left[\pi, \frac{3\pi}{2}\right]\right\}$$

● ***Ex. 28*** *An investigator interviewed* 100 *students to determine their preferences for the three drinks; milk* (M)*, coffee* (C) *and tea* (T)*. He reported the following:* 10 *students has all the three drinks M, C, T;* 20 *had M and C;* 30 *had C and T,* 25 *had M and T;* 12 *had M only;* 5 *had C only and* 8 *had T only. Using a Venn diagram, find how many did not take any of the three drinks?*

Sol. Given, M, C and T are the sets of drinks; milk, coffee and tea, respectively. Let us denote the number of drinks (students) contained in the bounded region as shown in the diagram by a, b, c, d, e, f and g, respectively.



Then, $g = 10$

$g + f = 20 \Rightarrow f = 10$ $[\because g = 10]$

$g + e = 30 \Rightarrow e = 20$

$d + g = 25 \Rightarrow d = 15$

and $a = 12, b = 5, c = 8$

Thus, total number of students taking drinks M or C or T

$$= a + b + c + d + e + f + g$$
$$= 12 + 5 + 8 + 15 + 20 + 10 + 10 = 80$$

Hence, the number of students taking none of them drinks

$$= 100 - 80 = 20$$

● **Ex. 29** *In a certain city, only two newspapers A and B are published. It is known that 25% of the city population reads A and 20% reads B, while 8% reads A and B. It is also known that 30% of those who read A but not B, look into advertisements and 40% of those who read B but not A, look into advertisements while 50% of those who read both A and B, look into advertisements. What per cent of the population read on advertisement?*

Sol. Let C = Set of people who read paper A

and D = Set of people who read paper B

Given, $n(C) = 25, n(D) = 20, n(C \cap D) = 8$

$\therefore \quad n(C \cap D') = n(C) - n(C \cap D)$

$= 25 - 8 = 17$

But total number of people who read A but not B = 30%

$\therefore$ Percentage of people reading A but not B = 30% of 17

$$= \frac{30 \times 17}{100} = \frac{51}{10}$$

and $n(C' \cap D) = n(D) - n(C \cap D) = 20 - 8 = 12$

Also, total number of people who read B but not A = 40%

$\therefore$ Percentage of people reading B but not A = 40% of 12

$$= \frac{40 \times 12}{100} = \frac{24}{5}$$

and given total people who read A and B = 50%

$\therefore$ Total number of people who read A and B = 50% of 8

$$= \frac{50 \times 8}{100} = 4$$

$\therefore$ Percentage of people reading an advertisement

$$= \frac{51}{10} + \frac{24}{5} + 4 = 13.9\%$$

● **Ex. 30** *An analysis of 100 personal injury claims made upon a motor insurance company revealed that loss or injury in respect of an eye, an arm, a leg occurred in 30, 50 and 70 cases, respectively. Claims involving this loss or injury to two of these members numbered 44. How many claims involved loss or injury to all the three, we must assume that one or another of three members was mentioned in each of the 100 claims?*

Sol. Let the set of people having injuries in eyes, arms or legs be denoted by E, A and L, respectively. Then, according to the problem, we have

$$n(E \cup A \cup L) = 30, n(E) = 30$$

$$n(A) = 50, n(L) = 70$$

and $n[(E \cap A \cap L') \cup (E \cap A' \cap L) \cup (E' \cap A \cap L)] = 44$

or $n(E \cap A \cap L') + n(E \cap A' \cap L) + n(E' \cap A \cap L) = 44$

[$\because$ each case is mutually exclusive]

or $n(E \cap A) - n(E \cap A \cap L) + n(E \cap L) - n(E \cap A \cap L) + n(E \cap L) - n(E \cap A \cap L) = 44$

$\Rightarrow \quad n(E \cap A) + n(E \cap L) + n(A \cap L) - 3n(E \cap A \cap L) = 44 \quad \text{...(i)}$

$\because \quad n(E \cup A \cup L) = 100$

$\therefore n(E) + n(A) + n(L) - n(E \cap A) - n(A \cap L) - n(E \cap L) + n(E \cap A \cap L) = 100$

$\Rightarrow \; 30 + 50 + 70 - \{44 + 3n(E \cap A \cap L)\} + n(E \cap A \cap L) = 100$ [from Eq. (i)]

$\Rightarrow \quad 6 - 2n(E \cap A \cap L) = 0$

$\therefore \quad n(E \cap L \cap A) = 3$

Hence, there are three claims involved in loss or injury to all the three.

Aliter

Let E = Set of people having injuries in eyes

$\therefore \; n(E) = 30$

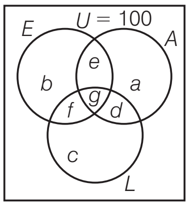
A = Set of people having injuries in arms

$\therefore \; n(A) = 50$

and L = Set of people having injuries in legs

$\therefore \; n(L) = 70$

Let us denote the number of injuries contained in the bounded region as shown in the diagram by a, b, c, d, e, f and g, respectively.



Then, $b + e + f + g = 30 \quad \text{...(i)}$

$a + d + e + g = 50 \quad \text{...(ii)}$

$c + d + f + g = 70 \quad \text{...(iii)}$

$d + e + f = 44 \quad \text{...(iv)}$

and $a + b + c + d + e + f + g = 100 \quad \text{...(v)}$

On adding Eqs. (i), (ii) and (iii), we get

$$a + b + c + 2(d + e + f) + 3g = 150$$

$\Rightarrow 100 - d - e - f - g + 2(d + e + f) + 3g = 150$

[from Eq. (v)]

$\Rightarrow \quad d + e + f + 2g = 50$

$\Rightarrow \quad 44 + 2g = 50$ [from Eq. (iv)]

$\therefore \quad g = 3$

Hence, there are three claims involved loss or injury to all the three.

● **Ex. 31** *N is the set of natural number. The relation R is defined on $N \times N$ as follows:*

$$(a, b) \, R \, (c, d) \Leftrightarrow ad(b + c) = bc(a + d)$$

Prove that R is an equivalence relation.

Sol. **Reflexive**

Since, $(a, b) R (a, b) \Leftrightarrow ab(b+a) = ba(a+b), \forall\ a, b \in N$ is true.

Hence, R is reflexive.

Symmetric $(a, b) R (c, d)$

$$\Leftrightarrow \quad ad(b+c) = bc(a+d)$$
$$\Leftrightarrow \quad bc(a+d) = ad(b+c)$$
$$\Leftrightarrow \quad cb(d+a) = da(c+b)$$
$$\Leftrightarrow \quad (c, d) R (a, b)$$

Hence, R is symmetric.

Transitive

Since, $(a, b) R (c, d) \Leftrightarrow ad(b+c) = bc(a+d)$

$$\Leftrightarrow \quad \frac{b+c}{bc} = \frac{a+d}{ad}$$
$$\Leftrightarrow \quad \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$$
$$\Leftrightarrow \quad \frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$$
$$\therefore \quad (a, b) R (c, d) \Leftrightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d} \quad \text{...(i)}$$

and similarly $(c, d) R (e, f) \Leftrightarrow \dfrac{1}{c} - \dfrac{1}{d} = \dfrac{1}{e} - \dfrac{1}{f}$ ...(ii)

From Eqs. (i) and (ii),

$$(a,b)R(c,d) \text{ and } (c,d)R(e,f) \Leftrightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{e} - \frac{1}{f}$$
$$\Leftrightarrow (a, b) R (e, f)$$

So, R is transitive. Hence, R is an equivalence relation.

● **Ex. 32** *The sets S and E are defined as given below:*

$S = \{(x, y) : |x-3| < 1 \text{ and } |y-3| < 1\}$ *and*

$E = \{(x, y) : 4x^2 + 9y^2 - 32x - 54y + 109 \le 0\}$.

Show that $S \subset E$.

Sol. **Graph of S**

$\because |x-3| < 1 \Rightarrow -1 < (x-3) < 1 \Rightarrow 2 < x < 4$

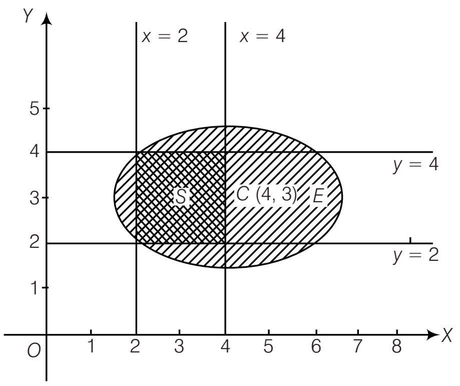
Similarly, $|y-3| < 1 \Rightarrow 2 < y < 4$

So, S consists of all points inside the square (not on $x \ne 2, 4$ and $y \ne 2, 4$) bounded by the lines $x = 2, y = 2, x = 4$ and $y = 4$.

Graph of E

$$\because \quad 4x^2 + 9y^2 - 32x - 54y + 109 \le 0$$
$$\Rightarrow \quad 4(x^2 - 8x) + 9(y^2 - 6y) + 109 \le 0$$
$$\Rightarrow \quad 4(x-4)^2 + 9(y-3)^2 \le 36$$
$$\Rightarrow \quad \frac{(x-4)^2}{3^2} + \frac{(y-3)^2}{2^2} \le 1$$

So, E consists of all points inside and on the ellipse with centre $(4, 3)$ and semi-major and semi-minor axes are 3 and 2, respectively.



From the above graph, it is evident that the double hatched (which is S) is within the region represented by E. i.e., $S \subset E$