8. INTRODUCTION TO TRIGONOMETRY

1. If $\sqrt{3}\sin\theta = \cos\theta$, find the value of $\frac{\sin\theta.\tan\theta.(1+\cot\theta)}{\sin\theta+\cos\theta}$.

   [**Ans :** $\frac{1}{\sqrt{3}}$]

2. In ΔPQR, if $\angle Q = 90°$ and $\sin R = \frac{3}{5}$, then find the value of cos P.

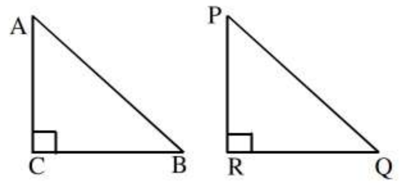
   [**Ans :** $\cos P = \frac{3}{5}$]

3. If 5 sin P = 12 cos P, then find the value of sec P.

   [**Ans :** $\sec P = \frac{13}{5}$]

4. If $\angle B$ and $\angle Q$ are acute angles such that sin B = sin Q, then prove that $\angle B = \angle Q$.

   [Sol. : Let us consider two right triangles ABC and PQR in which sin B = sin Q.



We have : $\sin B = \frac{AC}{AB}$

and $\sin Q = \frac{PR}{PQ}$

since $\sin B = \sin Q$, so

$$\frac{AC}{AB} = \frac{PR}{PQ}$$

$$\Rightarrow \quad \frac{AC}{PR} = \frac{AB}{PQ} = k \text{ (say)} \qquad \text{............(1)}$$

Now, from ΔABC, we have :

$$BC = \sqrt{AB^2 - AC^2}$$

and from ΔPQR, we have :

$$QR = \sqrt{PQ^2 - PR^2}$$

So, $$\frac{BC}{QR} = \sqrt{\frac{AB^2 - AC^2}{PQ^2 - PR^2}} = \frac{\sqrt{k^2PQ^2 - k^2PR^2}}{\sqrt{PQ^2 - PR^2}} = k \qquad \text{........(2)}$$

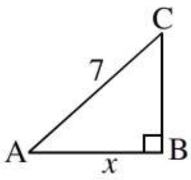
FRom eqns. (1) and (2), we have :

$$\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$$

So, $\Delta ABC \sim \Delta PQR$ (SSS similarity)

$\Rightarrow$ $\angle B = \angle Q$ (Corresponding angles), proved.]

5. In ΔABC, AB = x units, AC = 7 units and $\angle B = 90°$.



Evaluate : $\sqrt{7-x}.\tan C + \sqrt{7+x}.\cot A - 14\cos A + 21\sin C + \sqrt{49+x^2}.\cos B$

[Sol. : In ΔABC, AB = x units, AC = 7 units and $\angle B = 90°$.

So, $BC = \sqrt{AC^2 - AB^2} = \sqrt{7^2 - x^2} = \sqrt{49 - x^2}$

Now, $\tan C = \dfrac{AB}{BC} = \dfrac{x}{\sqrt{49-x^2}}$,

$\cot A = \dfrac{AB}{BC} = \dfrac{x}{\sqrt{49-x^2}}$,

$\cos A = \dfrac{AB}{AC} = \dfrac{x}{7}$,

$\sin C = \dfrac{AB}{AC} = \dfrac{x}{7}$

and $\cos B = \cos 90° = 0$.

Therefor, $\sqrt{7-x}\tan C + \sqrt{7+x}\cot A - 14\cos A + 21\sin C + \sqrt{49-x^2}.\cos B$

$$= \sqrt{7-x} \times \frac{x}{\sqrt{49-x^2}} + \sqrt{7+x} \times \frac{x}{\sqrt{49-x^2}} - 14 \times \frac{x}{7} + 21 \times \frac{x}{7} + \sqrt{49-x^2} \times 0$$

$$= \frac{x}{\sqrt{7+x}} + \frac{x}{\sqrt{7-x}} + x$$

$$= \frac{x\sqrt{7-x} + x\sqrt{7+x} + x\sqrt{49-x^2}}{\sqrt{(7+x)(7-x)}}.$$

$$= \frac{x\sqrt{7-x} + x\sqrt{7+x} + x\sqrt{49-x^2}}{\sqrt{49-x^2}}$$]

6. If 3 tan A = 4, then prove that :

i) $\sqrt{\dfrac{\sec A - \operatorname{cosec} A}{\sec A + \operatorname{cosec} A}} = \dfrac{1}{\sqrt{7}}$ ii) $\sqrt{\dfrac{1-\sin A}{1+\cos A}} = \dfrac{1}{2\sqrt{2}}$

7. If $\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$, find the values of all other trigonometric ratios.

[Ans : $\cos\theta = \frac{b}{\sqrt{a^2+b^2}}, \operatorname{cosec}\theta = \frac{\sqrt{a^2+b^2}}{a}, \tan\theta = \frac{a}{b}, \cot\theta = \frac{b}{a}, \sec\theta = \frac{\sqrt{a^2+b^2}}{b}$]

8. Given a right angled ΔABC, right angled at B in which $\tan A = \frac{15}{8}$ and $\tan C = \frac{8}{15}$, then find the value of sin A. cos C + cos A. sin C.

[Ans : 1]

9. If $\cot\theta = \frac{2x}{\sqrt{9-4x^2}}$, then evaluate :

i) $\sin\theta + \cos\theta$ ii) $\operatorname{cosec}\theta - \tan\theta$

[Ans : i) $\frac{\sqrt{9-4x^2}+2x}{3}$ ii) $\frac{4x^2+6x-9}{2x\sqrt{9-4x^2}}$]

10. If A and B are acute angles such that :tan $A = \frac{1}{3}$, tan B = $\frac{1}{2}$ and tan (A + B) = $\frac{\tan A + \tan B}{1 - \tan A \tan B}$, show that A + B = 45°.

[Sol: $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$= \frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\times\frac{1}{2}} = \frac{\frac{2+3}{6}}{\frac{6-1}{6}} = \frac{5}{6}\times\frac{6}{5} = 1$$

$= \tan 45°$ $[\because \tan 45° = 1]$

A + B = 45°.]

11. In ΔABC, right angled at C, if tanA = $\frac{1}{\sqrt{3}}$, show that sin A. cos B + cos A. sin B = 1.

[Sol. : We have : $\tan A = \frac{1}{\sqrt{3}}$.

$\Rightarrow$ tan A = tan 30°

$\Rightarrow$ A = 30°.

But in ΔABC, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow$ $30° + \angle B + 90° = 180°$ $[\because \angle C = 90°, \text{Given}]$

$B \quad 180 \quad 120 \quad 60$.

So, sin A cos B + cos A sin B = sin 30°. cos 60° + cos 30°. sin 60°

$$=\frac{1}{2}\times\frac{1}{2}+\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}$$

$$=\frac{1}{4}+\frac{3}{4}=\frac{4}{4}=1.$$

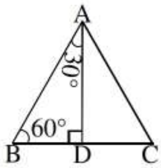
Hence proved.]

12. Find the value of tan 30° geometrically.

[Sol. : Let ABC be an equilateral triangle of side 2a.

Draw $AD \perp BC$ (see figure)

$\Delta ABD \cong \Delta ACD$(By RHS)



So, BD = DC

$\angle BAD = \angle CAD$

$$\Rightarrow \quad \angle BAD = \frac{1}{2}\times 60° = 30°$$

BD = DC $\Rightarrow$ $BD = \frac{1}{2}BC = \frac{1}{2}\times 2a = a$

Also, $AD^2 = AB^2 - BD^2$

$\Rightarrow$ $AD^2 = (2a)^2 - a^2 = 3a^2$

$\Rightarrow$ $AD = a\sqrt{3}$ ·

Now, from rt. triangle ABD, we get

$$\tan 30° = \frac{BD}{AD} = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}.$$

13. State which of the trigonometric ratios of an acute angle A have values:
i) always less than 1   ii) always greater than 1

[Ans :i) sine and cosine

ii) secant and cosecant]

14. Given that cos (A + B) = cos A. cos B – sin A. sin B and sin (A + B) = sin A cos B + cos A. sin B, find the values of cos 75° and sin 75° by taking suitable values of A and B.

[Ans: $\cos 75° = \frac{\sqrt{3}-1}{2\sqrt{2}}$, $\sin 75° = \frac{\sqrt{3}+1}{2\sqrt{2}}$]

15. If $k + 1 = \sec^2\theta(1+\sin\theta)(1-\sin\theta)$, then find the value of k.

[Sol : k = 0]

16. Prove the following identity : $\dfrac{\sin^4\theta+\cos^4\theta}{1-2\sin^2\theta.\cos^2\theta}=1.$

17. Prove that : $\dfrac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta}=2+\dfrac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}.$

18. Prove that : $\dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}-\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\dfrac{4}{\cot\theta-\tan\theta}=\dfrac{4\tan\theta}{1-\tan^2\theta}.$

19. Prove that : $\dfrac{\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}\theta-\cot\theta}=1+2\cot^2\theta+2\operatorname{cosec}\theta.\cot\theta;\ 0<\theta<90°.$

20. If x = cotA + cosA and y = cotA – cosA; prove that $\left(\dfrac{x-y}{x+y}\right)^2+\left(\dfrac{x-y}{2}\right)^2=1.$

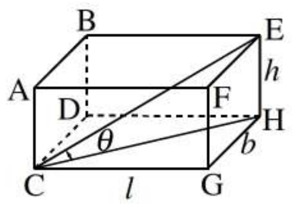
21. If m = cosA – sinA and n = cosA + sinA, then show that

$$\frac{m}{n}-\frac{n}{m}=\frac{4\sin A\cos A}{\cos^2 A-\sin^2 A}=-\frac{4}{\cot A-\tan A}.$$

22. If x = tanA + sinA and y = tanA – sinA, prove that : $\left(\dfrac{x+y}{x-y}\right)^2-\left(\dfrac{x+y}{2}\right)^2=1.$

23. If $l\sin\theta+m\cos\theta+n=0$ and $l'\sin\theta+m'\cos\theta+n'=0$, then prove that $(mn'-m'n)^2+(nl'-ln')^2=(lm'-l'm)^2$.

24. Shown below is a cuboid. Its length is l units,breadth b units and height h units.



i) Express $\cos\theta$ in terms of l, b and h.

ii) If the figure was a cube, what would be the value of $\cos\theta$? Show your work.

[Sol. : i) Finds the length of CH using the pythagoras' theorem in ΔCGH as :

$$CH=\sqrt{(CG^2+GH^2)}=\sqrt{(l^2+b^2)}\text{units}$$

Finds the length of CE using the Pythagoras' theorem in ΔCHE as :

$$CE=\sqrt{(CH^2+EH^2)}=\sqrt{(l^2+b^2+h^2)}\ \text{units}$$

Finds $\cos\theta$ as :

$$\cos\theta = \frac{CH}{CE} = \frac{\sqrt{l^2+b^2}}{\sqrt{l^2+b^2+h^2}}$$

$$\Rightarrow \qquad \cos\theta = \sqrt{\frac{l^2+b^2}{l^2+b^2+h^2}}$$

ii) Applies l = b = h for a cube and solves for $\cos\theta$ as :

$$\cos\theta = \sqrt{\frac{l^2+b^2}{l^2+b^2+h^2}} = \sqrt{\frac{2}{3}}$$]

25. The teacher asked the students to correctly complete the following sentence about the rhombus.

"A rhombus has a side length of l units and one of its angles is equal to θ. The rato of the lengths of the two diagonals is dependent on ________."

Ashima : only l

Bilal : only θ.

Chris : both l and θ.

Duleep : neither l nor θ.

Who asnwered the question correctly? Show your work and give valid reasons.

[Sol. : Draws a rhombus, say ABCD, and connects diagonals AC and BD bisecting at a point, say E.

In ΔEAD, applies the properties of the rhombus to get

i) $\angle AED = 90°$, ii) $AE = \frac{AC}{2}$,

iii) $DE = \frac{BD}{2}$, iv) $\angle EAD = \frac{\theta}{2}$

Allies trigonometric ratio to get

$$\tan\frac{\theta}{2} = \frac{AE}{DE} = \frac{AC}{BD}$$

Writes that the ratio of the diagonals $\frac{AC}{BD}$ is only dependent on θ and not l. Writes that Bilal answered it correctly.]