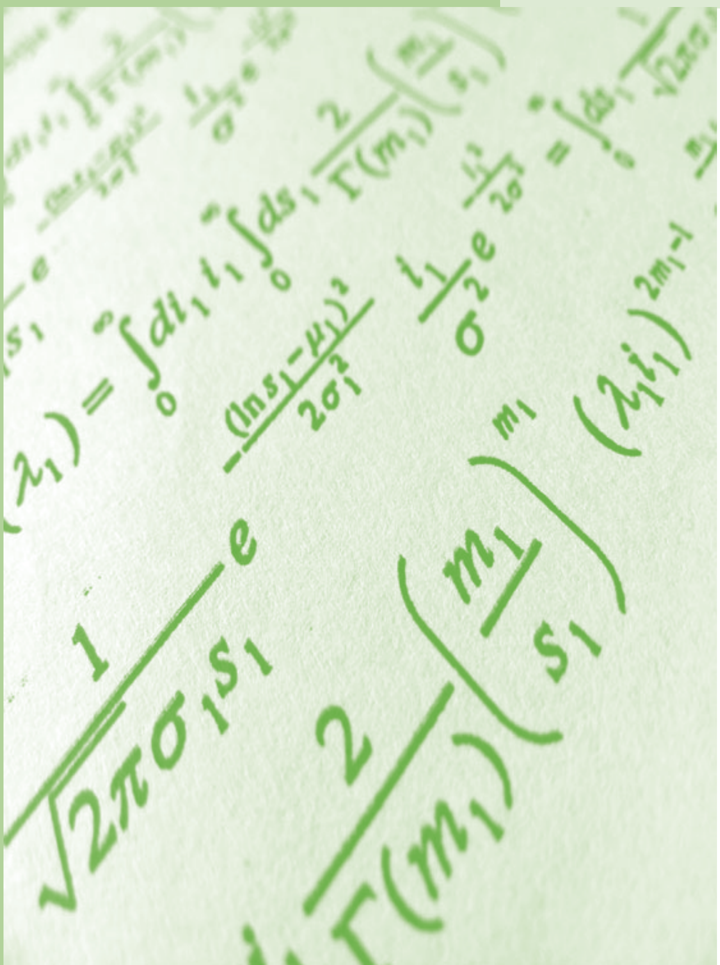
Chapter

10

Matrices



REMEMBER

Before beginning this chapter, you should be able to:

- State terms such as matrix, order of a matrix, etc.
- Apply basic operations on matrices

KEY IDEAS

After completing this chapter, you would be able to:

- Obtain order of a matrix
- Apply operations on matrices such as addition, subtraction, multiplication and study their properties
- Calculate determinant
- Obtain solution of simultaneous linear equations in two variable using matrix method

INTRODUCTION

A *matrix* is a rectangular arrangement of a set of elements in the form of horizontal and vertical lines. The elements can be numbers (real or complex) or variables. Matrices is the plural of matrix.

Horizontal line of elements is called a row and the vertical line of elements is called a column. The rectangular array of elements in a matrix are enclosed by brackets [ ] or parenthesis ( ).

Generally we use capital letters to denote matrices.

Examples:

1. $A = \begin{bmatrix} 2 & 3 & -2 \\ 1 & -1 & 0 \end{bmatrix}$ is a matrix having 2 rows and 3 columns.

Here the elements of matrix are numbers.

2. $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a matrix having 2 rows and 2 columns.

Here the elements of matrix are variables.

ORDER OF A MATRIX

If a matrix A has 'm' rows and 'n' columns, then $m \times n$ is called the order (or type) of matrix, and is denoted as $A_{m \times n}$.

Examples:

1. $A = \begin{bmatrix} 1 & 2 & -3 \\ -2 & 4 & 1 \end{bmatrix}$ is a matrix consisting of 2 rows and 3 columns. So its order is 2×3.

2. $B = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$ is a matrix consisting of 3 rows and 1 column. So its order is 3×1.

So in general, a set of mn elements can be arranged as a matrix having m rows and n columns as shown below.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \cdot & a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{24} & \cdot & a_{2n} \\ a_{31} & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & a_{ij} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{m1} & \cdot & \cdot & \cdot & \cdot & a_{mn} \end{bmatrix}_{m \times n} \quad \text{or} \quad A = [a_{ij}]_{m \times n}$$

In the above matrix a_{ij} represents an element of the matrix occurring in ith row and jth column.

In general, a_{ij} is called (i, j)th element of the matrix.

Hence in a particular matrix, we can note that (3, 4)th element is the element occurring in 3rd row and 4th column.

(1, 3)rd element is the element occurring is 1st row and 3rd column.

Example: Let $P = \begin{bmatrix} 2 & 3 & 51 \\ 4 & -2 & -3 \\ 5 & -31 & 1 \end{bmatrix}$

In this matrix we have,

(1, 1)th element = 2; (1, 2)th element = 3

(1, 3)th element = 51; (2, 1)th element = 4

(2, 2)th element = −2; (2, 3)th element = −3

(3, 1)th element = 5; (3, 2)th element = −31; (3, 3)th element = 1.

In compact form any matrix A can be represented as

$$A = [a_{ij}]_{m\times n} \text{ where } 1 \le i \le m,$$
$$1 \le j \le n.$$

Various Types of Matrices

Rectangular Matrix

In a matrix if number of rows is not equal to number of columns, then the matrix is called a rectangular matrix.

Examples:

1. $A = \begin{bmatrix} 2 & 5 \\ 3 & 2 \\ 1 & 3 \end{bmatrix}_{3\times 2}$; A has 3 rows and 2 columns.
2. $B = \begin{bmatrix} -2 & 3 & 1 & -4 \\ 5 & -1 & 0 & 1 \end{bmatrix}_{2\times 4}$; B has 2 rows and 4 columns.

Row Matrix

A matrix which has only one row is called a row matrix.

Examples:

1. $[5 \quad -3 \quad 2 \quad 1]$ is a row matrix of order 1×4.
2. $[4 \quad 3 \quad 13 \quad -4 \quad -31]$ is a row matrix of order 1×5.

In general, order of any row matrix is $1 \times n$, where n is number of columns and $n = 2, 3, 4, \ldots$.

Column Matrix

A matrix which has only one column is called a column matrix.

Examples:

1. $\begin{bmatrix} 5 \\ -3 \\ 2 \\ -1 \end{bmatrix}_{4\times 1}$ is a column matrix of order 4×1.

2. $\begin{bmatrix} -3 \\ 5 \\ 20 \\ -2006 \\ 2 \\ -1 \end{bmatrix}_{6\times 1}$ is a column matrix of order 6×1.

In general, order of any column matrix is $m \times 1$, where m is number of rows in the matrix and $m = 2, 3, 4, \ldots$.

Null Matrix or Zero Matrix

If every element of a matrix is zero, then the matrix is called null matrix or zero matrix. A zero matrix of order $m \times n$ is denoted by $O_{m\times n}$ or in short by O.

Examples:

1. $O = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{2\times 4}$ **2.** $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}_{3\times 2}$ **3.** $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$

Square Matrix

In a matrix, if number of rows is equal to number of columns, then the matrix is called a square matrix. A matrix of order $n \times n$ is termed as a square matrix of order n.

Examples:

1. $\begin{bmatrix} -2 & 3 \\ -1 & 1 \end{bmatrix}$ is a square matrix of order 2.

2. $\begin{bmatrix} a & b & -3 \\ 4 & c & -2 \\ y & x & z \end{bmatrix}$ is a square matrix of order 3.

Principal Diagonal of a Square Matrix In a square matrix A of order n, the elements a_{ii} (i.e., a_{11}, a_{22}, …, a_{nn}) constitute principal diagonal. The elements a_{ii} are called elements of principal diagonal.

Examples:

1. $A = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$

The elements 2 and 5 constitute the principal diagonal of A.

2. $P = \begin{bmatrix} -3 & 4 & 5 \\ 6 & -2 & 1 \\ a & 3 & b \end{bmatrix}$

The elements -3, -2 and b constitute principal diagonal of P.

Diagonal Matrix

In a square matrix, if all the non-diagonal elements are zeroes and at least one principal diagonal element is non-zero, then the matrix is called diagonal matrix.

Examples:

1. $\begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix}$ is a diagonal matrix of order 2.

2. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ is a diagonal matrix of order 3.

Scalar Matrix

In a matrix, if all the diagonal elements are equal and rest of the elements are zeroes, then the matrix is called scalar matrix.

Examples:

1. $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ is a scalar matrix of order 2.

2. $\begin{bmatrix} 31 & 0 & 0 \\ 0 & 31 & 0 \\ 0 & 0 & 31 \end{bmatrix}$ is a scalar matrix of order 3.

Identity Matrix or Unit Matrix

In a square matrix, if all the principal diagonal elements are unity and rest of the elements are zeroes, then the square matrix is called identity matrix or unit matrix. It is denoted by I.

Examples:

1. $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an identity matrix of order 2.

2. $I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ is an identity matrix of order 4.

Comparable Matrices

Two matrices A and B can be compared, only when they are of same order.

Example: Consider two matrices A and B given by

$$A = \begin{bmatrix} -1 & 2 \\ 3 & 21 \\ 10 & -4 \end{bmatrix}_{3\times 2} \quad \text{and } B = \begin{bmatrix} -3 & 10 \\ 4 & -3 \\ -2 & 1 \end{bmatrix}_{3\times 2}$$

Matrices A and B can be compared as both of them are of order 3×2.

Equality of Two Matrices

Two matrices are said to be equal only when,

1. they are of same order and
2. corresponding elements of both the matrices are equal.

Example: If $\begin{bmatrix} 2 & -3 \\ 1 & 5 \\ 6 & -2 \end{bmatrix}$ and $\begin{bmatrix} 2 & -3 \\ a & 2 \\ 6 & b \end{bmatrix}$ are equal matrices, then $a = 1$ and $b = -2$.

OPERATIONS ON MATRICES

Multiplication of a Matrix by a Scalar

If every element of a matrix A is multiplied by a number (real or complex) k, the matrix obtained is k times.

A and is denoted by kA and the operation is called scalar multiplication.

EXAMPLE 10.1

If $A = \begin{bmatrix} 2 & 3 & -1 \\ 5 & 6 & 1 \end{bmatrix}$, then find **(a)** $-A$ **(b)** $3A$ **(c)** $\frac{1}{4}A$.

SOLUTION

(a) $-A = -\begin{bmatrix} 2 & 3 & -1 \\ 5 & 6 & 1 \end{bmatrix} = \begin{bmatrix} -1\times 2 & -1\times 3 & -1\times(-1) \\ -1\times 5 & -1\times 6 & -1\times 1 \end{bmatrix} = \begin{bmatrix} -2 & -3 & 1 \\ -5 & -6 & -1 \end{bmatrix}$.

(b) $3A = 3\begin{bmatrix} 2 & 3 & -1 \\ 5 & 6 & 1 \end{bmatrix} = \begin{bmatrix} 3\times 2 & 3\times 3 & 3(-1) \\ 3\times 5 & 3\times 6 & 3\times 1 \end{bmatrix} = \begin{bmatrix} 6 & 9 & -3 \\ 15 & 18 & 3 \end{bmatrix}$.

(c) $\frac{1}{4}A = \frac{1}{4}\begin{bmatrix} 2 & 3 & -1 \\ 5 & 6 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{4}\times 2 & \frac{1}{4}\times 3 & \frac{1}{4}\times(-1) \\ \frac{1}{4}\times 5 & \frac{1}{4}\times 6 & \frac{1}{4}\times 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} & \frac{-1}{4} \\ \frac{5}{4} & \frac{3}{2} & \frac{1}{4} \end{bmatrix}$.

Notes

1. If a and b are any two scalars and P is a matrix, then $a(bP) = (ab)P$.
2. If m and n are any two scalars and A is a matrix, then $(m + n)\,A = mA + nA$.

Addition of Matrices

1. Two matrices A and B can be added only when they are of same order.

Example: Let $A = \begin{bmatrix} -3 & 2 & 1 \\ 5 & 6 & -5 \end{bmatrix}$, $B = \begin{bmatrix} -13 & 21 & 33 \\ -52 & 4 & 49 \end{bmatrix}$.

Here both matrices A and B are of order 2×3. So, they can be added.

2. The **sum** matrix of two matrices A and B is obtained by adding the corresponding elements of A and B and the **sum** matrix of same order as that of A or B.
3. Here $A+B=\begin{bmatrix} -3+(-13) & 2+21 & 1+33 \\ 5+(-52) & 6+4 & -5+49 \end{bmatrix}=\begin{bmatrix} -16 & 23 & 34 \\ -47 & 10 & 44 \end{bmatrix}$.

Note If two matrices are of different orders, then their sum is not defined.

Properties of Matrix Addition

1. Matrix addition is commutative, i.e., if A and B are two matrices of same order, then $A + B = B + A$.
2. Matrix addition is associative, i.e., if A, B and C are three matrices of same order, then $A + (B + C) = (A + B) + C$.
3. Additive identity:

   If $O_{m\times n}$ is a null matrix of order $m \times n$ and A is any matrix of order $m \times n$, then $A + O = O + A = A$.

   So, O is called additive identity.
4. Additive inverse:

   If $A_{m\times n}$ is any matrix of order $m \times n$, then $A + (-A) = (-A) + A = O$.

   So, $-A$ is called additive inverse of the matrix A.
5. If k is a scalar and A and B are two matrices of same order, then $k(A + B) = kA + kB$.

Matrix Subtraction

1. Matrix subtraction is possible only when both the matrices are of same order.
2. The difference of two matrices of same type (or order) A and B, i.e., $A - B$, is obtained by subtracting corresponding element of B from that of A.
3. The difference matrix is of the same order as that of A or B.

EXAMPLE 10.2

If $A=\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ and $B=\begin{bmatrix} -3 & 1 \\ 4 & -2 \end{bmatrix}$ then find $A - B$.

SOLUTION

$$A-B=\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}-\begin{bmatrix} -3 & 1 \\ 4 & -2 \end{bmatrix}$$

$$A-B=\begin{bmatrix} 2-(-3) & 3-1 \\ -1-4 & 4-(-2) \end{bmatrix}=\begin{bmatrix} 5 & 2 \\ -5 & 6 \end{bmatrix}.$$

Transpose of a Matrix

For a given matrix A, the matrix obtained by interchanging its rows and columns is called transpose of the matrix A and is denoted by A^T.

Examples:

1. If $A=\begin{bmatrix} 2 & -1 & 3 & 4 \\ 5 & -6 & 11 & -1 \end{bmatrix}$, then

Transpose of $A = A^T = \begin{bmatrix} 2 & 5 \\ -1 & -6 \\ 3 & 11 \\ 4 & -1 \end{bmatrix}$.

Here we can note that order of A is 2×4, while that of A^T is 4×2.

2. $A = \begin{bmatrix} -1 & 3 \\ 4 & 2 \end{bmatrix}$, then $A^T = \begin{bmatrix} -1 & 4 \\ 3 & 2 \end{bmatrix}$.

3. $A = \begin{bmatrix} -1 \\ 2004 \\ 53 \\ -47 \end{bmatrix}$, then $A^T = [-1 \quad 2004 \quad 53 \quad -47]$.

4. If $A = [5003]$, then $A^T = [5003]$.

Notes

1. If the order of a matrix is $m \times n$, then order of transpose of the matrix is $n \times m$.
2. $(A^T)^T = A$.

Example: Let $A = \begin{bmatrix} -2 & 5 \\ 3 & -1 \\ -4 & 0 \end{bmatrix}$

$$A^T = \begin{bmatrix} -2 & 5 \\ 3 & -1 \\ -4 & 0 \end{bmatrix}^T = \begin{bmatrix} -2 & 3 & -4 \\ 5 & -1 & 0 \end{bmatrix}$$

$$(A^T)^T = \begin{bmatrix} -2 & 3 & -4 \\ 5 & -1 & 0 \end{bmatrix}^T = \begin{bmatrix} -2 & 5 \\ 3 & -1 \\ -4 & 0 \end{bmatrix} = A.$$

3. If A and B are two matrices of same order, then $(A + B)^T = A^T + B^T$.
4. If k is a scalar and A is any matrix, then $(kA)^T = kA^T$.

Symmetric Matrix

A square matrix is said to be symmetric if the transpose of the given matrix is equal to the matrix itself.

Hence a square matrix A is symmetric.

$$\Rightarrow \quad A = A^T.$$

Examples:

1. If $A = \begin{bmatrix} -2 & 3 \\ 3 & 4 \end{bmatrix}$ then $A^T = \begin{bmatrix} -2 & 3 \\ 3 & 4 \end{bmatrix}^T = \begin{bmatrix} -2 & 3 \\ 3 & 4 \end{bmatrix}$.

Thus we can observe that $A^T = A$, so A is a symmetric matrix.

2. Similarly for $P = \begin{bmatrix} -3 & 5 & 43 \\ 5 & 4 & 4 \\ 43 & 4 & 2 \end{bmatrix}$,

$$P^T = \begin{bmatrix} -3 & 5 & 43 \\ 5 & 4 & 4 \\ 43 & 4 & 2 \end{bmatrix}^T = \begin{bmatrix} -3 & 5 & 43 \\ 5 & 4 & 4 \\ 43 & 4 & 2 \end{bmatrix} = P.$$

So, P is a symmetric matrix.

Skew-symmetric Matrix

A square matrix A is said to be skew-symmetric if $A^T = -A$, i.e., transpose of the matrix is equal to its additive inverse.

Example: If $A = \begin{bmatrix} 0 & 2003 \\ -2006 & 0 \end{bmatrix}$, then $A^T = \begin{bmatrix} 0 & -2006 \\ 2006 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2006 \\ -2006 & 0 \end{bmatrix} = -A.$

So, A is a skew-symmetric matrix.

Notes

1. For a square matrix $A, \frac{1}{2}(A + A^T)$ is always a symmetric matrix.
2. For a square matrix $A, \frac{1}{2}(A - A^T)$ is always a skew-symmetric matrix.

Multiplication of Matrices

Two matrices A and B can be multiplied only if the number of columns in A is equal to the number of rows in B.

Suppose order of matrix A is $m \times q$. Then order of matrix B, such that AB exists, should be of the form $q \times n$. Further order of the product matrix AB will be $m \times n$.

Now consider a matrix A of order 2×3 and another matrix B of the order 3×4. As the number of columns in $A(=3)$ is equal to number of rows in $B(=3)$. So AB exists and it is of the order 2×4. We can obtain the product matrix AB as follows:

(1, 1)th element of AB

= sum of products of elements of first row of A with the corresponding elements of first column of B.

(1, 2)th element of AB

= sum of products of elements of first row of A with the corresponding elements of second column of B.

(2, 1)th element of AB

= sum of products of elements of second row of A with the corresponding elements of first column of B and so on.

In general (i, j)th element of AB

= sum of products of elements of ith row in A with the corresponding elements of jth column in B.

Following example will clearly illustrate the method.

Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 7 \\ 3 & 5 \end{bmatrix}_{3\times 2}$, $B = \begin{bmatrix} -5 & 6 & 4 \\ 9 & 11 & 8 \end{bmatrix}_{2\times 3}$

As A is of order 3×2 and B is of order 2×3, AB will be of the order 3×3.

$$\therefore \quad AB = \begin{bmatrix} 2\times(-5)+(-1)\times 9 & 2\times 6+(-1)\times 11 & 2\times 4+(-1)\times 8 \\ 1\times(-5)+7\times 9 & 1\times 6+7\times 11 & 1\times 4+7\times 8 \\ 3\times(-5)+5\times 9 & 3\times 6+5\times 11 & 3\times 4+5\times 8 \end{bmatrix}_{3\times 3}$$

$$= \begin{bmatrix} -10+(-9) & 12+(-11) & 8+(-8) \\ -5+63 & 6+77 & 4+56 \\ -15+45 & 18+55 & 12+40 \end{bmatrix}_{3\times 3}$$

$$= \begin{bmatrix} -19 & 1 & 0 \\ 58 & 83 & 60 \\ 30 & 73 & 52 \end{bmatrix}_{3\times 3}.$$

In general if $A = [a_{ip}]$ is a matrix of order $m \times q$ and $B = [b_{pj}]$ is a matrix of order $q \times n$, then the product matrix $AB = Q = [x_{ij}]$ will be of the order $m \times n$ and is given by $x_{ij} = \sum_{p=1}^{q} a_{ip} b_{pj}$.

This evaluation is made clear in the following.

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} & \cdot & x_{1n} \\ x_{21} & \cdot & \cdot & \cdot & \cdot & \cdot \\ x_{31} & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & x_{ij} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ x_{m1} & x_{m2} & x_{m3} & \cdot & \cdot & x_{mn} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & \cdot & a_{1q} & \cdot & a_{1n} \\ a_{21} & & \cdot & \cdot & \cdot & \cdot \\ a_{i1} & \cdot & \cdot & a_{iq} & \cdot & a_{in} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{m1} & \cdot & \cdot & \cdot & \cdot & a_{mn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdot & b_{1q} & \cdot & b_{1n} \\ b_{21} & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ b_{i1} & \cdot & \cdot & b_{iq} & \cdot & b_{in} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ b_{m1} & \cdot & \cdot & b_{mq} & \cdot & b_{mn} \end{bmatrix}.$$

Properties of Matrix Multiplication

1. In general, matrix multiplication is not commutative, i.e., $AB \neq BA$.

Example: Let $A = \begin{bmatrix} -3 & 1 \\ 0 & 2 \end{bmatrix} B = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} -3\times 2+1\times 0 & -3\times(-1)+1\times 1 \\ 0\times 2+2\times 0 & 0(-2)+2\times 1 \end{bmatrix} = \begin{bmatrix} -6 & 4 \\ 0 & 2 \end{bmatrix}.$$

$$BA = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2(-3)+(-1)\times 0 & 2\times 1+(-1)\times 2 \\ 0(-3)+1\times 0 & 0\times 1+1\times 2 \end{bmatrix} = \begin{bmatrix} -6 & 0 \\ 0 & 2 \end{bmatrix}.$$

So, we can observe that $AB \neq BA$.

2. Matrix multiplication is associative, i.e., $A(BC) = (AB)C$.

3. Matrix multiplication is distributive over addition, i.e.,
  (i) $A(B + C) = AB + AC$,
  (ii) $(B + C)A = BA + CA$.

4. For any two matrices A and B if $AB = O$, then it is not necessarily imply that $A = O$ or $B = O$ or both A and B are zero.

Example: Let $A = \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & -12 \\ -4 & 6 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}\begin{bmatrix} 8 & -12 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} 2(8)+4(-4) & 2(-12)+4(6) \\ 4(8)+8(-4) & 4(-12)+8(6) \end{bmatrix}$$

$$= \begin{bmatrix} 16-16 & -24+24 \\ 32-32 & -48+48 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Hence we can observe that, though $AB = O$, $A \neq O$ and $B \neq O$.

5. For any three matrices A, B and C if $AB = AC$, then it is not necessarily imply that $B = C$ or $A = O$ (But in case of any three real numbers a, b and c, if $ab = ac$ and $a \neq 0$, then it is necessary that $b = c$).

Example: Let $A = \begin{bmatrix} 5 & 10 \\ 10 & 20 \end{bmatrix}$, $B = \begin{bmatrix} 10 & 5 \\ 15 & 10 \end{bmatrix}$ and $C = \begin{bmatrix} -10 & 35 \\ 25 & -5 \end{bmatrix}$.

$$AB = \begin{bmatrix} 5 & 10 \\ 10 & 20 \end{bmatrix}\begin{bmatrix} 10 & 5 \\ 15 & 10 \end{bmatrix} = \begin{bmatrix} 5\times 10+10\times 15 & 5\times 5+10\times 10 \\ 10\times 10+20\times 15 & 10\times 5+20\times 10 \end{bmatrix}$$

$$= \begin{bmatrix} 50+150 & 25+100 \\ 100+300 & 50+200 \end{bmatrix} = \begin{bmatrix} 200 & 125 \\ 400 & 250 \end{bmatrix}.$$

$$AC = \begin{bmatrix} 5 & 10 \\ 10 & 20 \end{bmatrix}\begin{bmatrix} -10 & 35 \\ 25 & -5 \end{bmatrix} = \begin{bmatrix} 5(-10)+10(25) & 5(35)+10(-5) \\ 10(-10)+20(25) & 10(35)+20(-5) \end{bmatrix}$$

$$= \begin{bmatrix} -50+250 & 175-50 \\ -100+500 & 350-100 \end{bmatrix} = \begin{bmatrix} 200 & 125 \\ 400 & 250 \end{bmatrix}.$$

Here $AB = AC$, but $B \neq C$.

6. If A is a square matrix of order n and I is the identity matrix of order n, then $AI = IA = A$, i.e., I is the multiplicative identity matrix.

Example:

$$A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AI = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} = A.$$

$$IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} = A.$$

$\therefore$ $AI = IA = A$; Here I is called identity matrix.

7. If matrix A is multiplied by a null matrix, then the resultant matrix is null matrix, i.e., $AO = OA = O$.

8. If A and B are two matrices such that AB exists, then $(AB)^T = B^T A^T$.

Note If $A_1, A_2, A_3, \ldots, A_n$ are n matrices, then $(A_1 A_2 A_3 \ldots A_n)^T = A_n{}^T A^T{}_{n-1} \ldots A_1{}^T$

9. If A is any square matrix, then $(A^T)^n = (A^n)^T$.

10. If A and B are any two square matrices, then

(i) $(A + B)^2 = (A + B)(A + B) = A(A + B) + B(A + B)$
$= A^2 + AB + BA + B^2$.

(ii) $(A - B)^2 = (A - B)(A - B) = A(A - B) - B(A - B)$
$= A^2 - AB - BA + B^2$.

(iii) $(A + B)(A - B) = A(A - B) + B(A - B)$
$= A^2 - AB + BA - B^2$.

EXAMPLE 10.3

If $A = \begin{pmatrix} 0 & 2009 \\ -2009 & 0 \end{pmatrix}$, then find $[A^{2009} + (A^T)^{2009}]$.

(a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ **(b)** $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ **(c)** $\begin{pmatrix} 0 & 2009 \\ -2009 & 0 \end{pmatrix}$ **(d)** None of these

SOLUTION

Find A^2 then calculate A^{2009}.

EXAMPLE 10.4

If $A = \begin{pmatrix} 2 & 3 \\ 5 & 1 \end{pmatrix}$, $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $AB = -13I$, then find the value of $a + b - c + d$.

(a) 5 **(b)** 3 **(c)** 2 **(d)** 1

SOLUTION

$A\,B = -13I$

$$\begin{pmatrix} 2 & 3 \\ 5 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = -13\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2a+3c & 2b+3d \\ 5a+c & 5b+d \end{pmatrix} = \begin{pmatrix} -13 & 0 \\ 0 & -13 \end{pmatrix}$$

$$\Rightarrow \quad 2a + 3c = -13, \qquad (1)$$

$$5a + c = 0 \qquad (2)$$

$$2b + 3d = 0 \qquad (3)$$

$$5b + d = -13 \qquad (4)$$

Solving Eqs. (1) and (2), we get $a = 1$ and $c = -5$. Solving Eqs. (3) and (4), we get $b = -3$ and $d = 2$.

$$a + b - c + d = 1 - 3 + 5 + 2 = 5.$$

EXAMPLE 10.5

If $A = \begin{pmatrix} 3 & -6 \\ -1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}$ are two matrices, then find $AB + BA$.

(a) I **(b)** O **(c)** A **(d)** B

SOLUTION

$$AB = \begin{pmatrix} 3 & -6 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 6-6 & 18-18 \\ -2+2 & -6+6 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

$$BA = \begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 3 & -6 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 6-6 & -12+12 \\ 3-3 & -6+6 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

$$AB + BA = O.$$

We have learnt about the order of matrix, different kinds of matrices, some operations like transpose, addition, subtraction, multiplication of matrices. We also learnt different properties of matrix multiplication.

In this chapter we will learn how to find determinant and inverse of a 2×2 square matrix. We also learn about how to apply the concept of matrices to solve a system of linear equations in two variables.

DETERMINANT

For a given 2×2 square matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the real number $(ad - bc)$ is defined as the determinant of A and is denoted by $|A|$ or $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$.

Example: If $A = \begin{bmatrix} 2 & -5 \\ 6 & 3 \end{bmatrix}$, then determinant of $A = |A| = \begin{vmatrix} 2 & -5 \\ 6 & 3 \end{vmatrix} = 2(3) - (-5) \times 6 = 36.$

Singular Matrix

If determinant of a square matrix is zero, then the matrix is called a singular matrix.

Example: For the square matrix $A = \begin{bmatrix} 6 & 9 \\ 2 & 3 \end{bmatrix}$, $|A| = \begin{vmatrix} 6 & 9 \\ 2 & 3 \end{vmatrix} = 6 \times 3 - 9 \times 2 = 18 - 18 = 0.$

So, A is a singular matrix.

Non-singular Matrix

If determinant of a square matrix is not equal to zero, then the matrix is called non-singular matrix.

Example: For the square matrix $A = \begin{bmatrix} 2 & -4 \\ 5 & 3 \end{bmatrix}$, $|A| = \begin{vmatrix} 2 & -4 \\ 5 & 3 \end{vmatrix} = 2(3) - (-4) \times 5 = 6 + 20 = 26 \neq 0.$

So, A is a non-singular matrix.

EXAMPLE 10.6

If $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$, $ps = 15$ and det $A = 21$, then find the value of qr.

(a) 6 **(b)** −6 **(c)** 5 **(d)** −8

SOLUTION

$$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

det $A = ps - qr = 21 \quad \Rightarrow \quad qr = ps - 21$

$qr = 15 - 21 \quad \Rightarrow \quad qr = -6.$

Multiplicative Inverse of a Square Matrix

For every non-singular square matrix A of order n, there exists a non-singular square matrix B of same order, such that $AB = BA = I$. (Note that I is unit matrix of order n). Here B is called multiplicative inverse of A and is denoted as $A^{-1} \quad \Rightarrow \quad B = A^{-1}$.

Note If $AB = KI$, then $A^{-1} = \dfrac{1}{K}B$.

Multiplicative Inverse of a 2 × 2 Square Matrix

For a 2×2 square matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, we can show that $A^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \dfrac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Notes

1. For a singular square matrix $|A| = 0$, and so its multiplicative inverse doesn't exist. Conversely if a matrix A doesn't have multiplicative inverse, then $|A| = 0$.

2. If A is a square matrix and K is any scalar, then $(KA)^{-1} = \frac{1}{K}A^{-1}$.

3. For any two square matrices A and B of same order $(AB)^{-1} = B^{-1}A^{-1}$.

Method for Finding Inverse of a 2 × 2 Square Matrix

We know that for a square matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$,

$$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

From this formula we can find A^{-1} using the following steps.

1. Find whether $|A| = 0$ or not. If $|A| = 0$, then the given matrix is singular, so A^{-1} doesn't exist. If $|A| \neq 0$, then the matrix has a multiplicative inverse and can be found by the following steps (2), (3) and (4).
2. Interchange the elements of principal diagonal.
3. Multiply the other two elements by -1.
4. Multiply each element of the matrix by $\frac{1}{|A|}$.

EXAMPLE 10.7

Find the inverse of the matrix $A = \begin{bmatrix} 2 & -4 \\ 3 & -5 \end{bmatrix}$.

SOLUTION

$$|A| = \begin{vmatrix} 2 & -4 \\ 3 & -5 \end{vmatrix} = -10 + 12 = 2 \neq 0.$$

$\therefore$ A is non-singular and A^{-1} exists.

$$\therefore \quad A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -5 & 4 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{5}{2} & \frac{4}{2} \\ -\frac{3}{2} & \frac{2}{2} \end{bmatrix} = \begin{bmatrix} -\frac{5}{2} & 2 \\ -\frac{3}{2} & 1 \end{bmatrix}.$$

EXAMPLE 10.8

If $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, then find $A^{-1} + A$.

(a) I **(b)** $2I$ **(c)** $3I$ **(d)** $4I$

SOLUTION

$$A=\begin{bmatrix}1 & 2\\ 1 & 3\end{bmatrix},\ A^{-1}=\begin{bmatrix}3 & -2\\ -1 & 1\end{bmatrix}$$

$$\Rightarrow \quad A^{-1}+A=\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}=4I.$$

EXAMPLE 10.9

If $A=\begin{bmatrix}x^2 & y\\ 5 & -4\end{bmatrix}$ and $A=A^{-1}$, then find $\begin{bmatrix}x^3 & y+x\\ 1 & 2x^2+y\end{bmatrix}^{-1}$.

(a) $\dfrac{1}{41}\begin{bmatrix}8 & 1\\ -1 & 5\end{bmatrix}$ **(b)** $\dfrac{1}{41}\begin{bmatrix}5 & 8\\ 1 & -1\end{bmatrix}$ **(c)** $\begin{bmatrix}5 & 1\\ -1 & 8\end{bmatrix}$ **(d)** $\dfrac{1}{41}\begin{bmatrix}5 & 1\\ -1 & 8\end{bmatrix}$

HINTS

(i) Use $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ and $A=A^{-1}$, then $a=d$ and $ad-bc=-1$.

(ii) Substitute the values of x and y. Find the inverse of that matrix.

Solution of Simultaneous Linear Equations in Two Variables

The concept of matrices and determinants can be applied to solve a system of linear equations in two or more variables. Here we present two such methods. First one is Matrix Inversion method and the second one is Cramer's method.

Matrix Inversion Method

Let us try to understand the method through an example.

EXAMPLE 10.10

Solve the simultaneous linear equations

$$2x-5y=1,\ 5x+3y=18.$$

SOLUTION

Given system of linear equations can be written in matrix form as shown below:

$$\begin{bmatrix}2x-5y\\ 5x+3y\end{bmatrix}=\begin{bmatrix}1\\ 18\end{bmatrix}.$$

The LHS matrix can be further written as product of two matrices as shown below.

$$\begin{bmatrix}2 & -5\\ 5 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1\\ 18\end{bmatrix} \quad \text{or} \quad AX=B. \tag{1}$$

Here $A = \begin{bmatrix} 2 & -5 \\ 5 & 3 \end{bmatrix}$ is called coefficient matrix, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ is called variable matrix and $B = \begin{bmatrix} 1 \\ 18 \end{bmatrix}$ is called constant matrix. Now we need to find values of x and y, i.e., the matrix X.

To find X pre-multiplying both the sides of Eq. (1) with A^{-1}.

$$\Rightarrow \quad A^{-1}(AX) = A^{-1}B$$

or $(A^{-1}A)X = A^{-1}B$ [since $A(BC) = (AB)C$]

or $IX = A^{-1}B$, [$\because A^{-1}A = I$] or $X = A^{-1}B$, [$IX = X$]

$$X = A^{-1}B$$

So to find X we have to find inverse of coefficient matrix (i.e., A) and multiply it with B.

$$\therefore \quad X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ 5 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 18 \end{bmatrix}$$

$$= \frac{1}{2\times 3 - (-5)\times 5} \begin{bmatrix} 3 & 5 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 18 \end{bmatrix}$$

$$= \frac{1}{31} \begin{bmatrix} 3\times 1 + 5\times 18 \\ -5\times 1 + 2\times 18 \end{bmatrix}$$

$$= \frac{1}{31} \begin{bmatrix} 93 \\ 31 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}.$$

Thus $X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad \Rightarrow \quad x = 3, y = 1.$

Thus in general any system of linear equations $px + qy = a$, and $rx + sy = b$ can be represented in matrix form (i.e., $AX = B$) as $\begin{bmatrix} p & q \\ r & s \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$.

Here A is coefficient matrix $= \begin{bmatrix} p & q \\ r & s \end{bmatrix}$, X is variables matrix $= \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} a \\ b \end{bmatrix}$ is constant matrix.

$$X = A^{-1}B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}^{-1} \begin{bmatrix} a \\ b \end{bmatrix}.$$

Notes

1. Matrix inversion method is applicable only when the coefficient matrix A is non-singular, i.e., $|A| \neq 0$. If $|A| = 0$, then A^{-1} doesn't exist and so the method is not applicable.
2. This method can be extended for a system of linear equations in more than 2 variables.

Cramer's Rule or Cramer's Method

This is another method of solving system of linear equations using concept of determinants. Unlike matrix inversion method, in this method we don't need to find the inverse of coefficient matrix.

EXAMPLE 10.11

Solve the system of linear equations $3x + 4y = 2$, $5x - 3y = 13$ by Cramer's method.

SOLUTION

The system of equations can be written in matrix form (i.e., $AX = B$) as shown below:

$$\begin{bmatrix} 3 & 4 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{2}{13} \end{bmatrix}.$$

Here, $A = \begin{bmatrix} 3 & 4 \\ 5 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 13 \end{bmatrix}$.

To find the solution by Cramer's method we define two matrices B_1 and B_2. The matrix B_1 is obtained by replacing first column of matrix A by the column in B. similarly B_2 is obtained by replacing column 2 of matrix A by the column in B.

That is, $B_1 = \begin{bmatrix} 2 & 4 \\ 13 & -3 \end{bmatrix}$, $B_2 = \begin{bmatrix} 3 & 2 \\ 5 & 13 \end{bmatrix}$.

Now, $x = \dfrac{|B_1|}{|A|}$

$$= \frac{\begin{vmatrix} 2 & 4 \\ 13 & -3 \end{vmatrix}}{\begin{vmatrix} 3 & 4 \\ 5 & -3 \end{vmatrix}} = \frac{2(-3) - 4(13)}{3(-3) - 4(5)}$$

$$= \frac{-6 - 52}{-9 - 20} = \frac{58}{29} = 2$$

and,

$$y = \frac{|B_2|}{|A|} = \frac{\begin{vmatrix} 3 & 2 \\ 5 & 13 \end{vmatrix}}{\begin{vmatrix} 3 & 4 \\ 5 & -3 \end{vmatrix}}$$

$$= \frac{3 \times 13 - 2 \times 5}{3(-3) - 4 \times 5} = \frac{29}{-29} = -1.$$

Thus in general for a system of linear equations $px + qy = a$, $rx + sy = b$, solution by Cramer's method is

$$x = \frac{\begin{vmatrix} a & q \\ b & s \end{vmatrix}}{\begin{vmatrix} p & q \\ r & s \end{vmatrix}},\ y = \frac{\begin{vmatrix} p & a \\ r & b \end{vmatrix}}{\begin{vmatrix} p & q \\ r & s \end{vmatrix}}.$$

Notes

1. If the coefficient matrix A is singular, then $|A| = 0$, and so the method is not applicable.
2. This method can be extended to system of linear equations in more than two variables.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. Who gave the name 'matrix' to a rectangular arrangement of certain numbers in some rows and columns?

2. If $a_{ij} = 0$ $(i \neq j)$ and $a_{ij} = 4(i = j)$, then the matrix $A = [a_{ij}]_{n\times n}$ is a _______ matrix.

3. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & m \end{bmatrix}$ is a scalar matrix, then $x + m$ = _______.

4. The order of column matrix containing n rows is _______.

5. If $P = \begin{bmatrix} 3 & 0 \\ 0 & \lambda \end{bmatrix}$ is scalar matrix then λ = _______.

6. If $\begin{bmatrix} 4 & -3 \\ 2 & 16 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 2 & 2^t \end{bmatrix}$ then t = _______.

7. If $A = \begin{pmatrix} 2 & 3 \\ 6 & 9 \end{pmatrix}$, then find $|A|$.

8. The product of two matrices, i.e., $AB = I$, then B is called the _______ of A and written as _______.

9. If $(A + B^T)^T$ is a matrix of order 4×3, then the order of matrix B is _______.

10. If $\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} + \begin{bmatrix} -y & 4 \\ 7 & x \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 7 & 6 \end{bmatrix}$ then x = _______ and y = _______.

11. Is $A = \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$ singular?

12. If A is any square matrix, then $\frac{1}{2}(A - A^T)$ is a _______ matrix.

13. If the determinant of a square matrix is non-zero, then the matrix is called a _______ matrix.

14. $(AB)^{-1}$ = _______.

15. The inverse of matrix A, if $A^2 = I$, is _______.

16. The additive inverse of $\begin{bmatrix} -1 & 3 & 4 \\ 5 & -7 & 8 \end{bmatrix}$ is _______.

17. If $A \times \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$, then the order of A is _______.

18. If the order of matrices A, B and C are 3×4, 7×3 and 4×7 respectively, then the order of $(AC)B$ is _______.

19. Express the equations $2x - y + 6 = 0$ and $6x + y + 8 = 0$, in the matrix equation form.

20. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then A^{-1} = _______.

21. If $\begin{vmatrix} 2 & -4 \\ 9 & d-3 \end{vmatrix} = 4$, then d = _______.

22. If $AB = KI$, where $K \in R$, then A^{-1} = _______.

23. If $p = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then P^{-1} = _______.

24. The value of $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ = _______.

25. If K is real number, then $(KA)^{-1}$ = _______.

26. The matrix $A = \begin{bmatrix} a & d \\ c & a \end{bmatrix}$ is singular then a = _______.

27. If A and B commute, then $(A + B)^2$ = _______.

28. If $|A| = 5$, $|B_1| = 5$ and $|B_2| = 25$, then find the values of x and y in Cramer's method.

29. If $A = [s\ 2]$ and $B = \begin{bmatrix} x \\ y \end{bmatrix}$ then AB = _______.

30. The matrix obtained by multiplying each of the given matrix A with -1 is called the _______ of A and is denoted by _______.

Short Answer Type Questions

31. If $A = [a_{ij}]_{2\times2}$ such that $a_{ij} = i - j + 3$, then find A.

32. If $A + B^T = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$ and $A^T - B = \begin{bmatrix} 7 & 8 \\ -1 & 3 \end{bmatrix}$, then find matrices A and B.

33. If $\begin{pmatrix} \frac{1}{2} & -\frac{3}{5} \\ \frac{4}{6} & -\frac{1}{7} \end{pmatrix} = \begin{pmatrix} -a & b \\ c & -d \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then find a, b, c and d.

34. If $B = \begin{bmatrix} -1 & 0 \\ 2 & 4 \end{bmatrix}$ and $f(x) = x^2 - 4x + 5$, then find $f(B)$.

35. If $A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 5 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 2 \\ 0 & 5 \end{bmatrix}$ then find $A(B + C)$.

36. If $A \times \begin{bmatrix} -3 & 4 \\ 5 & 10 \end{bmatrix} = [13 \quad 6]$, then find A.

37. Two friends Jack and Jill attend IIT entrance test which has three sections; Mathematics, Physics and Chemistry. Each question in Mathematics, Physics and Chemistry carry 5 marks, 8 marks and 3 marks respectively. Jack attempted 10 questions in Mathematics, 12 in Physics and 6 in Chemistry while Jill attempted 18, 5 and 9 questions in Mathematics, Physics and Chemistry respectively. Assuming that all the questions attempted were correct, find the individual marks obtained by the boys by showing the above information as a matrix product.

38. If A and B are two matrices such that $A + B = \begin{bmatrix} 3 & 8 \\ 11 & 6 \end{bmatrix}$ and $A - B = \begin{bmatrix} 5 & 2 \\ -3 & -6 \end{bmatrix}$, then find the matrices A and B.

39. Compute the product

$$\begin{pmatrix} -5 & 1 \\ 6 & -1 \\ -2 & 0 \end{pmatrix}\begin{pmatrix} 4 & -5 & -1 \\ 5 & 6 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ -4 \\ 1 \end{pmatrix}.$$

40. If $A = \begin{pmatrix} 7 & 2 \\ 18 & 5 \end{pmatrix}$, then show that $A - A^{-1} = 12I$.

41. If $A = \begin{pmatrix} 9 & -7 \\ -4 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & -7 \\ -4 & -9 \end{pmatrix}$, then find AB and hence find A^{-1}.

42. Given $A = \begin{pmatrix} 3 & p \\ 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 5 & 6 \end{pmatrix}$. If $AB = BA$, then find p.

43. If $A = \begin{bmatrix} 2 & -5 \\ 0 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then find the matrix X such that $4A - 2X + I = O$.

44. If $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -5 & 9 \\ 3 & 4 \end{bmatrix}$, and $C = \begin{bmatrix} -3 & 6 \\ 2 & 1 \end{bmatrix}$, then find $2A + 3B - 4C$.

45. Find the possible orders possible of matrices A and B if they have 18 and 19 elements respectively.

Essay Type Questions

46. If $A = \begin{pmatrix} 4 & 3 \\ -2 & 1 \end{pmatrix}$, then find $A + 10\,A^{-1}$.

47. Solve the following simultaneous equations using Cramer's method:

$$\frac{3x - 5y}{18} = 1,\ 2y - 4x + 10 = 0.$$

48. If $A = \begin{pmatrix} 2 & 7 \\ 1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & -7 \\ -2 & 3 \end{pmatrix}$ find AB, $(AB)^{-1}$, A^{-1}, B^{-1} and $B^{-1}A^{-1}$. What do you notice?

49. Solve the following system of linear equations using matrix inversion method:

$$5x - 3y = -13,\ 2x + 5y = 1.$$

50. If $A = \begin{pmatrix} 4 & -3 \\ 5 & 2 \end{pmatrix}$, then show that $A + 23\,A^{-1} = 6I$.

CONCEPT APPLICATION

Level 1

1. If $\begin{vmatrix} 2 & -3 \\ p-4 & 2p-1 \end{vmatrix} = -6$, then $p =$

(a) $\frac{8}{7}$ (b) $\frac{7}{8}$

(c) 5 (d) 0

2. If $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b+c \\ b-c & d \end{pmatrix} = \begin{pmatrix} 4 & -5 \\ 3 & 2 \end{pmatrix}$, then $(a - b) + (c - d) =$

(a) −2 (b) 9

(c) 2 (d) −1

3. If $\begin{vmatrix} 5 & -3 \\ 6 & -a \end{vmatrix} = 4$, then $5a - 4 =$

(a) 0 (b) 10

(c) 14 (d) $\frac{14}{5}$

4. If $\begin{bmatrix} 2 & -3 \\ 5x+4 & 4 \end{bmatrix}$ has a multiplicative inverse, then x cannot be

(a) $\frac{3}{4}$ (b) $\frac{4}{5}$

(c) $\frac{-3}{4}$ (d) $\frac{-4}{3}$

5. If $A = \begin{bmatrix} 8 & 7 \\ -9 & -8 \end{bmatrix}$, then $A^{-1} =$ ____________.

(a) A (b) $-A$

(c) $2A$ (d) $\begin{bmatrix} 8 & 7 \\ -(-9) & -8 \end{bmatrix}$

6. Given $A = \begin{bmatrix} 4 & -2 \\ 2a-1 & 5a-3 \end{bmatrix}$ and if A does not have multiplicative inverse, then $12a - 13 =$

(a) 6 (b) $\frac{7}{12}$

(c) $\frac{12}{7}$ (d) −6

7. If $A = \begin{pmatrix} 7 & 2 \\ -3 & 9 \end{pmatrix}$, $B = \begin{pmatrix} p & 2 \\ -3 & 5 \end{pmatrix}$ and $AB = BA$, then find p.

(a) −2 (b) 1

(c) 3 (d) p does not have a unique value

8. Given $A = \begin{pmatrix} 5 & -3 \\ -2 & 1 \end{pmatrix}$, then $A^{-1} =$ ____________.

(a) $\begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & -3 \\ -2 & -5 \end{pmatrix}$

(c) $\begin{pmatrix} -1 & -3 \\ 2 & -5 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 3 \\ 2 & -5 \end{pmatrix}$

9. If $\begin{vmatrix} 7a-5b & 3c \\ -1 & 2 \end{vmatrix} = 0$, then which of the following is true?

(a) $14a + 3c = 5b$ (b) $14a - 3c = 5b$

(c) $14a + 3c = 10b$ (d) $14a + 10b = 3c$

10. If a square matrix A is skew-symmetric, then which of the following is correct?

(a) A^T is skew-symmetric

(b) A^{-1} is skew-symmetric

(c) A^{2007} is skew-symmetric

(d) All of these

11. If $|A| = 47$, then find $|A^T|$.

(a) −47 (b) 47

(c) 0 (d) Cannot be determined

12. There are 25 software engineers and 10 testers in Infosys and 15 software engineers and 8 testers in Wipro. In both the companies, a software engineer is paid ₹5000 per month and a tester is paid ₹3000 per month. Find the total amount paid by each of the companies per month by representing the data in matrix form.

(a) $\begin{pmatrix} 155000 \\ 99000 \end{pmatrix}$ (b) $\begin{pmatrix} 23000 \\ 24000 \end{pmatrix}$

(c) $\begin{pmatrix} 50000 \\ 30000 \end{pmatrix}$ (d) $\begin{pmatrix} 155000 \\ 100000 \end{pmatrix}$

13. If det(A) = 5, then find det($15A$) where A is of order 2×2.

(a) 225 (b) 75

(c) 375 (d) 1125

14. If $A = \begin{bmatrix} \operatorname{cosce}\alpha & \tan\alpha \\ \cot\alpha & -\sin\alpha \end{bmatrix}$, then A is a/an ________.

(a) singular matrix

(b) scalar matrix

(c) symmetric matrix

(d) non-singular matrix

15. Which of the following statements is true?

(a) A singular matrix has an inverse.

(b) If a matrix doesn't have multiplicative inverse, it need not be a singular matrix.

(c) If a, b are non-zero real numbers, then $\begin{bmatrix} a+b & a-b \\ b-a & a+b \end{bmatrix}$ is a non-singular matrix.

(d) $\begin{bmatrix} 5 & 2 \\ 3 & -1 \end{bmatrix}$ is a singular matrix.

16. What is the condition that is to be satisfied for the identity $(P + Q)(P - Q) = P^2 - Q^2$ to be true for any two square matrices P and Q?

(a) The identity is always true.

(b) $PQ \neq QP$.

(c) Both PQ and QP are not null matrices.

(d) P, Q and PQ are symmetric.

17. Solve the simultaneous equations:

$$2x - 3y = 11 \text{ and } 5x + 4y = 16$$

(a) $x = 5,\ y = -\dfrac{1}{3}$ (b) $x = 2,\ y = \dfrac{2}{3}$

(c) $x = -1,\ y = 4$ (d) $x = 4,\ y = -1$

18. If I is a 2×2 identity matrix, then $|(3I)^{30}|^{-1} =$

(a) $\dfrac{1}{3^{30}}$ (b) $\dfrac{1}{3^{60}}$

(c) 3^{30} (d) 3^{60}

19. If A is a 2×2 square matrix, such that det $A = 9$, then det($9A$) =

(a) $\dfrac{1}{3^{30}}$ (b) 9

(c) 81 (d) 729

20. Which of the following statement (s) is true?

(a) Inverse of a square matrix is not unique.

(b) If A and B are two square matrices, then $(AB)^T = A^T B^T$.

(c) If A and B are two square matrices, then $(AB)^{-1} = A^{-1} B^{-1}$.

(d) If A is a non-singular square matrix, then its inverse can be uniquely expressed as sum of a symmetric and a skew-symmetric matrix.

21. If A and B are two square matrices such that $AB = A$ and $BA = B$, then find $(A^{2006} B^{2006})^{-1}$.

(a) $A^{-1} B^{-1}$ (b) $B^{-1} A^{-1}$

(c) AB (d) Cannot be determined

22. If $A = \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix}$, then find the determinant of AB.

(a) 10 (b) 20

(c) 12 (d) 15

23. If the trace of the matrix A is 4 and the trace of matrix B is 7, then find the trace of matrix AB.

(a) 4 (b) 7

(c) 28 (d) Cannot be determined

24. If the trace of the matrix A is 5, and the trace of the matrix B is 7, then find trace of the matrix $(3A + 2B)$.

(a) 12 (b) 29

(c) 19 (d) None of these

25. If $A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$, then find A^n. (where $n \in N$)

(a) $\begin{bmatrix} 3n & 0 \\ 0 & 3n \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $I_{2 \times 2}$

26. If A is a 2×2 scalar matrix and 7 is the one of the elements in its principal diagonal, then the inverse of A is

(a) $\begin{bmatrix} \frac{-1}{7} & 0 \\ 0 & \frac{-1}{7} \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -7 \\ -7 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{7} & 0 \\ 0 & \frac{1}{7} \end{bmatrix}$

27. $A_1, A_2, A_3, \ldots, A_n$ and $B_1, B_2, B_3, \ldots, B_n$ are non-singular square matrices order n such that $A_1B_1 = I_n$, $A_2B_2 = I_n$, $A_3B_3 = I_n$, $\ldots$, $A_n\,B_n = I_n$, then $(A_1\, A_2\, A_3, \ldots, A_n)^{-1} =$

(a) $B_1\ B_2\ B_3 \ldots B_n$

(b) $B_1^{-1}\ B_2^{-1}\ B_3^{-1} \ldots B_n^{-1}$

(c) $B_n\ B_{n-1}\ B_{n-2} \ldots B_1$

(d) $B_{n-1}\ B_{n-1}^{-1}\ B_{n-2}^{-1} \ldots B_1^{-1}$

28. If $A = \begin{bmatrix} 5 & 6 \\ 9 & 9 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ p & 3 \end{bmatrix}$ and $AB = BA$, then $p =$

(a) $\frac{9}{2}$ (b) $\frac{-2}{9}$

(c) $\frac{-9}{2}$ (d) $\frac{2}{9}$

29. The inverse of a scalar matrix A of order 2×2, where one of the principal diagonal elements is 5, is

(a) $5I$ (b) I

(c) $\frac{1}{5}I$ (d) $\frac{1}{25}I$

30. If $A = \begin{bmatrix} 3 & -2 \\ 6 & 4 \end{bmatrix}$, then $AA^{-1} =$ ________.

(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

Level 2

31. If $A = \begin{pmatrix} 4 & 22 \\ -1 & -6 \end{pmatrix}$, then find $A + A^{-1}$.

(a) $\begin{bmatrix} 8 & -11 \\ -1 & -6 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & 33 \\ \frac{1}{2} & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 7 & 33 \\ -\frac{3}{2} & -8 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & 33 \\ -\frac{3}{2} & -4 \end{bmatrix}$

32. If $A = \begin{bmatrix} 4 & p \\ 3 & -4 \end{bmatrix}$ and $A - A^{-1} = 0$, then $p =$ ________.

(a) 4 (b) 3

(c) −5 (d) 5

33. If $\begin{pmatrix} 11 & -4 \\ 8 & -3 \end{pmatrix}\begin{pmatrix} -x & 4 \\ -8 & y \end{pmatrix} = -\begin{pmatrix} 2 & 3 \\ 6 & 9 \end{pmatrix}$, then find $2x - y$.

(a) −5 (b) 5

(c) 0 (d) 14

34. If $\begin{bmatrix} a^x \\ a^{-x} \end{bmatrix}[1 \quad 2] = \begin{bmatrix} p & a^{-2} \\ q & \log_2 2 \end{bmatrix}, (a > 0)$, then $a^{p-q} =$ ______.

(a) $2^{\frac{3}{2}}$ (b) $2^{\frac{-3}{2}}$

(c) 1 (d) $4^{\frac{3}{2}}$

35. If the matrix $\begin{bmatrix} 2^a & 32 \\ 36 & 12^b \end{bmatrix}$ is singular and if $k = \frac{2a}{ca+1}$, then find c.

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$

(c) $\frac{4}{3}$ (d) $\frac{3}{4}$

36. If $A = \begin{bmatrix} 7 & 6 \\ -8 & -7 \end{bmatrix}$, then find $(A^{12345})^{-1}$.

(a) A^T (b) A

(c) I (d) Cannot be determined

37. The inverse of a diagonal matrix, whose principal diagonal elements are l, m is__________.

(a) $\begin{bmatrix}\frac{1}{l} & 0\\ 0 & \frac{1}{m}\end{bmatrix}$ (b) $\begin{bmatrix}l & 0\\ 0 & m\end{bmatrix}$

(c) $\begin{bmatrix}l^2 & 0\\ 0 & m^2\end{bmatrix}$ (d) $\begin{bmatrix}2l & 0\\ 0 & 2m\end{bmatrix}$

38. If $\begin{bmatrix}4^b & 288\\ 72 & 18^a\end{bmatrix}$ is a singular matrix and $2b = a + \frac{1}{c}$, then c is ________.

(a) 4 (b) $\frac{1}{4}$

(c) $\frac{1}{6}$ (d) 6

39. If A is a non-singular square matrix such that $A^2 - 7A + 5I = 0$, then $A^{-1} =$ ________.

(a) $7A - I$ (b) $\frac{7}{5}I - \frac{1}{5}A$

(c) $\frac{7}{5}I + \frac{1}{5}A$ (d) $\frac{A}{5} - \frac{7}{5}$

40. If $A = a\begin{bmatrix}-1 & 0\\ -1 & 2\end{bmatrix}$nd $B = \begin{bmatrix}3 & 2\\ -1 & 0\end{bmatrix}$, then find $B^{-1} \cdot A^{-1}$.

(a) $\begin{bmatrix}\frac{-1}{2} & \frac{3}{4}\\ \frac{-1}{2} & \frac{-5}{4}\end{bmatrix}$ (b) $\begin{bmatrix}\frac{-1}{2} & \frac{1}{2}\\ \frac{-5}{4} & \frac{3}{4}\end{bmatrix}$

(c) $\begin{bmatrix}\frac{1}{2} & \frac{-1}{2}\\ \frac{-5}{4} & \frac{3}{4}\end{bmatrix}$ (d) $\begin{bmatrix}\frac{1}{2} & \frac{-3}{4}\\ \frac{-1}{2} & \frac{-5}{4}\end{bmatrix}$

41. If

$P = \begin{bmatrix}\sec\alpha & \tan\alpha\\ -\cot\alpha & \cos\alpha\end{bmatrix}$ and $Q = \begin{bmatrix}-\cos\alpha & \tan\alpha\\ -\cot\alpha & -\sec\alpha\end{bmatrix}$,

then $2P^{-1} + Q =$

(a) $\begin{bmatrix}\cos\alpha & -\tan\alpha\\ \cot\alpha & \sec\alpha\end{bmatrix}$ (b) $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

(c) $\begin{bmatrix}-\cos\alpha & \tan\alpha\\ -\cot\alpha & -\sec\alpha\end{bmatrix}$ (d) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

42. If A is a skew-symmetric matrix such that $AB = aI$, then find $(A^{-1})^T$.

(a) $-1B$ (b) $(-aB^T)$

(c) $\frac{B}{a}$ (d) $-\frac{B}{a}$

43. If $A = \begin{bmatrix}8 & -7\\ 9 & -8\end{bmatrix}$, then $(A^{2007})^{-1} =$ ______.

(a) I (b) $2A$

(c) A (d) $2007I$

44. If the matrix $\begin{pmatrix}10 & -9\\ 5x+7 & 5\end{pmatrix}$ is non-singular, then the range of x.

(a) $\frac{113}{45}$ (b) $R - \left\{\frac{-113}{45}\right\}$

(c) $R - \left\{\frac{113}{45}\right\}$ (d) $\frac{-113}{45}$

45. If $AB = BA$, then prove that $ABAB = A^2B^2$. The following are the steps involved in proving the above result. Arrange them in the sequential order.

(A) $ABAB = A(BA)B$

(B) $(AA)(BB)$

(C) $A(AB)B$

(D) A^2B^2

(a) ABCD

(b) ACBD

(c) BCAD

(d) ADBC

46. The following are the steps in finding the matrix B, if $B + \begin{pmatrix}2 & 3\\ 4 & 5\end{pmatrix} = \begin{pmatrix}5 & 4\\ 3 & 2\end{pmatrix}$. Arrange them in sequential order.

(A) $\therefore \begin{pmatrix}p & q\\ r & s\end{pmatrix} + \begin{pmatrix}2 & 3\\ 4 & 5\end{pmatrix} = \begin{pmatrix}5 & 4\\ 3 & 2\end{pmatrix}$

(B) Let $B = \begin{pmatrix}p & q\\ r & s\end{pmatrix}$

(C) $\begin{pmatrix}p+2 & q+3\\ r+4 & s+5\end{pmatrix} = \begin{pmatrix}5 & 4\\ 3 & 2\end{pmatrix}$

(D) $p + 2 = 5,\ q + 3 = 4,\ r + 4 = 3,\ s + 5 = 2$
$\Rightarrow\ p = 3, q = 1, r = -1, s = -3$

(E) $\therefore\ B = \begin{pmatrix} 3 & 1 \\ -1 & -3 \end{pmatrix}$

(a) BACDE
(b) BADCE
(c) BDCAE
(d) BADEC

47. $(AB)^{-1} =$ ________.

(a) $A^{-1}B^{-1}$
(b) $B^{-1}A$
(c) AB^{-1}
(d) $B^{-1}A^{-1}$

48. If $\begin{vmatrix} 2 & -4 \\ 9 & d-3 \end{vmatrix} = 4$, then $d =$ ________.

(a) 13 (b) 26
(c) −13 (d) −26

49. If $A = \begin{pmatrix} 2 & 3 \\ 6 & 9 \end{pmatrix}$, then find $|A|$.

(a) 0 (b) 1
(c) 2 (d) 3

Level 3

50. The number of integral values of x for which the determinant of the matrix $\begin{bmatrix} 5x+14 & -2 \\ 7x+8 & x \end{bmatrix}$ is always less than 1 is

(a) 3 (b) 4
(c) 5 (d) 6

51. If $A = \begin{bmatrix} x^2 & y^2 \\ \log_{1024} a & -9 \end{bmatrix}$, $a = 16^{25}$ and if $A = A^{-1}$, then $\begin{bmatrix} x^2 & y \\ 1 & x^2+y \end{bmatrix}^{-1} =$

(a) $\frac{1}{65}\begin{bmatrix} 7 & 2 \\ -1 & 9 \end{bmatrix}$ (b) $\frac{1}{65}\begin{bmatrix} 7 & -2 \\ 1 & 9 \end{bmatrix}$

(c) $\frac{1}{65}\begin{bmatrix} 7 & 2 \\ 1 & 9 \end{bmatrix}$ (d) $\frac{1}{65}\begin{bmatrix} 9 & 2 \\ -1 & 7 \end{bmatrix}$

52. If $A = \begin{pmatrix} 5 & 5 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 5 & 5 \end{pmatrix}$ and $A^n = \begin{pmatrix} 5^{200} & 5^{200} \\ 0 & 0 \end{pmatrix}$, then find n.

(a) 100 (b) 50
(c) 25 (d) None of these

53. If $\begin{pmatrix} 7 & -6 \\ 8 & -7 \end{pmatrix}^{2008} = \begin{pmatrix} 7 & -6 \\ 8 & -7 \end{pmatrix}^{2010}$, then $\begin{pmatrix} 7 & -6 \\ 8 & -7 \end{pmatrix}^{2009}$ is ________.

(a) $\begin{pmatrix} 7 & -6 \\ 8 & -7 \end{pmatrix}^{2008} + \begin{pmatrix} 7 & -6 \\ 8 & -7 \end{pmatrix}^{2010}$

(b) $\frac{1}{2}\left[\begin{pmatrix} 7 & -6 \\ 8 & -7 \end{pmatrix}^{2010} - \begin{pmatrix} 7 & -6 \\ 8 & -7 \end{pmatrix}^{2008}\right]$

(c) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 7 & -6 \\ 8 & -7 \end{pmatrix}$

54. If $A = \begin{bmatrix} \sin\theta & \tan\theta \\ \tan\theta & \sin\theta \end{bmatrix}$ has no multiplicative inverse, then ________.

(a) $q = 0°$ (b) $q = 45°$
(c) $q = 60°$ (d) $q = 30$

55. A and B are two square matrices of same order. If $AB = B^{-1}$, then $A^{-1} =$ ________.

(a) BA (b) A^2
(c) B^2 (d) B

56. $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$, where p, q, r and s are positive integers. If A is symmetric matrix, $|A| = 20$ and $p = q$, then find how many values are possible for s.

(a) 1 (b) 2
(c) 3 (d) 4

57. If $a = \begin{pmatrix} 2 & 0 \\ 5 & -3 \end{pmatrix}, B = \begin{pmatrix} -2 & 1 \\ 3 & -1 \end{pmatrix}$, then find the trace of $(AB^T)^T$.

(a) 10 (b) 14

(c) −4 (d) −18

58. If A is a 2×3 matrix and B is 3×2 matrix, then the order of $(AB)^T$ is equal to the order of ________.

(a) AB (b) $A^T B^T$

(c) BA (d) All of these

59. If $A_{2\times3}$, $B_{4\times3}$ and $C_{2\times4}$ are three matrices, then which of the following is/are defined?

(a) AC^TB (b) B^TC^TA

(c) AB^TC (d) All of these

60. If $A = \begin{bmatrix} 2 & 4 \\ k & -2 \end{bmatrix}$ and $A^2 = O$, then find the value of k.

(a) −4 (b) −3

(c) −2 (d) −1

61. If $A = \begin{bmatrix} a & b & c \\ x & y & z \\ l & m & n \end{bmatrix}$ is a skew-symmetric matrix, then which of the following is equal to $x + y + z$?

(a) $a + b + c$ (b) $l + m + n$

(c) $a - b - m$ (d) $c - l - n$

62. If $P = \begin{bmatrix} 0 & 4 & -2 \\ x & 0 & -y \\ 2 & -8 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then $x - y =$ ______.

(a) 8 (b) 4

(c) −12 (d) −8

63. In solving simultaneous linear equations by Crammer's method, $B_1 = \begin{bmatrix} 3 & 3 \\ 4 & 1 \end{bmatrix}$ and $B_2 = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, then det A = ________. (A is the coefficient matrix).

(a) −1 (b) −2

(c) 3 (d) 4

64. $M = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ is a sin gular matrix. Its determinant is equal to its trace, then $p =$ ________.

(a) $-q$ (b) r

(c) 0 (d) $-s$

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. James Joseph Sylvester
2. scalar
3. -2
4. $n \times 1$
5. $\lambda = 3$
6. 4.
7. 0
8. multiplicative inverse, A^{-1}
9. 4×3
10. 5 and 1
11. Given matrix is singular matrix
12. skew-symmetric
13. non-singular
14. $B^{-1}A^{-1}$
15. A
16. $\begin{bmatrix} 1 & -3 & -4 \\ -5 & 7 & -8 \end{bmatrix}$
17. 3×2
18. 3×3
19. $\begin{bmatrix} 2 & -1 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -8 \end{bmatrix}$
20. $\dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
21. -13
22. $\dfrac{1}{K}B$
23. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
24. 1
25. $\dfrac{1}{K}A^{-1}$
26. $\sqrt{bc}$
27. $A^2 + 2AB + B^2$
28. $x, y = 3, 5$ respectively
29. $(sx + 2y)_{1\times 1}$
30. additive inverse, $(-A)$.

Short Answer Type Questions

31. $\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$
32. $A = \begin{bmatrix} 4 & 1 \\ 6 & 4 \end{bmatrix}; B = \begin{bmatrix} -3 & -2 \\ 2 & 1 \end{bmatrix}$
33. $a = -\dfrac{1}{2}, b = -\dfrac{3}{5}, c = \dfrac{4}{6}$ and $d = \dfrac{1}{7}$
34. $\begin{bmatrix} 10 & 0 \\ -2 & 5 \end{bmatrix}$
35. $\begin{bmatrix} -13 & -5 \\ 9 & 25 \end{bmatrix}$
36. $\begin{bmatrix} -2 & \dfrac{7}{5} \end{bmatrix}$
37. Marks obtained by Jack = 164

    Marks obtained by Jill = 157
38. $A = \begin{bmatrix} 4 & 5 \\ 4 & 0 \end{bmatrix}; B = \begin{bmatrix} -1 & 3 \\ 7 & 6 \end{bmatrix}$
39. $\begin{bmatrix} -193 \\ 232 \\ -78 \end{bmatrix}$
41. $AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

    $A^{-1} = B = \begin{bmatrix} -3 & -7 \\ -4 & -9 \end{bmatrix}$

42. $p = 0$

43. $\begin{bmatrix} \frac{9}{2} & -10 \\ 0 & \frac{5}{2} \end{bmatrix}$

44. $\begin{bmatrix} 3 & 7 \\ 3 & 16 \end{bmatrix}$

45. The orders possible for A are 1×18, 2×9, 3×6, 6×3, 9×2, 18×1 the orders possible for B are 1×19 and 19×1

Essay Type Questions

46. $5I$

47. $x = 1, y = -3$

48. $(AB)^{-1} = B^{-1} \cdot A^{-1}$

49. $x = 1, y = -3$

CONCEPT APPLICATION

Level 1

1. (a)	**2.** (d)	**3.** (b)	**4.** (d)	**5.** (a)	**6.** (d)	**7.** (c)	**8.** (b)	**9.** (c)	**10.** (d)
11. (b)	**12.** (a)	**13.** (d)	**14.** (d)	**15.** (c)	**16.** (d)	**17.** (d)	**18.** (b)	**19.** (d)	**20.** (d)
21. (b)	**22.** (b)	**23.** (d)	**24.** (b)	**25.** (c)	**26.** (d)	**27.** (c)	**28.** (a)	**29.** (c)	**30.** (c)

Level 2

31. (c)	**32.** (c)	**33.** (a)	**34.** (b)	**35.** (a)	**36.** (b)	**37.** (a)	**38.** (b)	**39.** (b)	**40.** (c)
41. (b)	**42.** (d)	**43.** (c)	**44.** (b)	**45.** (b)	**46.** (a)	**47.** (d)	**48.** (c)	**49.** (a)	

Level 3

50. (b)	**51.** (a)	**52.** (a)	**53.** (d)	**54.** (a)	**55.** (c)	**56.** (c)	**57.** (b)	**58.** (a)	**59.** (b)
60. (d)	**61.** (c)	**62.** (b)	**63.** (a)	**64.** (d)					

CONCEPT APPLICATION

Level 1

1. $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$

2. Apply matrix multiplication concept and then equate the corresponding elements.

3. $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$

4. If, multiplicative inverse of A exists, then determinant of $A \neq 0$.

5. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, A^{-1}

$$= \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

6. If, multiplicative inverse of A does not exist, then $|A| = 0$.

7. Apply matrix multiplication concept.

8. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

9. $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$

10. Apply the properties of skew-symmetric.

11. $\det A = \det A^T$.

12. Write the given data in matrix form.

13. $\det(kA) = k^2|A|$ (where A is 2×2 matrix).

14. Find determinant of A.

16. If P, Q and PQ are symmetric, then $PQ = QP$.

17. Write the equations in matrix form.

18. If A is an $n \times n$ matrix, then $|kA| = k^n|A|$ and $\det I = 1$.

19. If A is an $n \times n$ matrix, then $|kA| = k^n|A|$.

20. Recall the properties of matrices.

21. (i) If $AB = A$, $BA = B$ then $A^2 = A$; $B^2 = B$.

    (ii) $(AB)^{-1} = B^{-1}A^{-1}$.

22. $\det(AB) = \det A \cdot \det B$.

23. Trace $(AB) \neq \text{trace}(A) \cdot \text{trace}(B)$.

24. Trace $(kA + mB) = k(\text{trace } A) + m(\text{trace } B)$.

25. (i) $A = 3I$.

    (ii) $A^n = 3^n I^n$.

26. (i) Refer the definition of scalar matrix and write A.

    (ii) $A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

27. (i) $AB = I \quad \Rightarrow \quad A^{-1} = B$.

    (ii) $(AB)^{-1} = B^{-1} A^{-1}$.

28. Find AB and BA.

29. Write matrix A and find A^{-1}.

30. $A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Level 2

31. $A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

32. $A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

33. Apply matrix multiplication concept.

34. Multiply the left side matrices and equate the corresponding elements.

35. (i) If the matrix is singular, then its determinant is zero. Using this condition we can obtain the values of a and b.

    (ii) Substitute the values of a and b in $b = \frac{2a}{ca + 1}$, then find the value of 'c'.

36. (i) Calculate A^2.

(ii) Find A^2, A^3, … and observe.

37. The inverse of diagonal matrix $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ is $\begin{bmatrix} \frac{1}{a} & 0 \\ 0 & \frac{1}{b} \end{bmatrix}$.

38. If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a singular matrix, then $ad - bc = 0$.

39. Pre-multiply the given equation with A^{-1} and use the relation $A^{-1} A = I$, $A^{-1}I = A^{-1}$.

40. First find AB, after that find $(AB)^{-1}$.

We know that $(AB)^{-1} = B^{-1} A^{-1}$.

41. Find P^{-1} and then proceed.

42. (i) A is skew-symmetric matrix, $A^T = -A$.

(ii) According to the problem, the inverse of A is $\frac{1}{a}B$.

43. Calculate A^2, then find $(A^{2007})^{-1}$.

44. Determinant is non-zero.

45. The required sequential order is ACBD.

46. The required sequential order is BACDE.

47. $(AB)^{-1} = B^{-1}A^{-1}$.

48. Given $\begin{vmatrix} 2 & -4 \\ 9 & d-3 \end{vmatrix} = 2(d-3) - (-4 \times 9) = 4$

$$2d - 6 + 36 = 4,$$

$$2d = -26,\ d = -13.$$

49. Given $A = \begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}, |A| = \begin{vmatrix} 2 & 3 \\ 6 & 9 \end{vmatrix} = (18 - 18)$

$$= 0 \left(\because \begin{vmatrix} a & b \\ c & d \end{vmatrix} = (ad - bc) \right).$$

Level 3

50. (i) Find the determinant of matrix.

(ii) Solve the inequation.

51. (i) Put $a = 16^{25}$ in $\log_{1024} a$ and simplify.

(ii) Then, find x and y values using the relation $A = A^{-1}$.

(iii) Find the inverse of the given matrix.

52. (i) Simplify the matrix A.

(ii) Find A^2, A^3, …, A^n.

53. (i) Let $= A = \begin{pmatrix} 7 & -6 \\ 8 & -7 \end{pmatrix}$.

(ii) Calculate A^2.

54. Since A has no multiplicative inverse, $|A| = 0$.

$\Rightarrow \quad \sin^2\theta - \tan^2\theta = 0$

$\Rightarrow \quad \sin^2\theta = \tan^2\theta$

$\Rightarrow \quad \sin\theta = \tan\theta \quad \Rightarrow \quad \theta = 0°$.

55. $A\ B = B^{-1}$

$(AB)\ B = B^{-1} \cdot B$

$AB^2 = I$

$A^{-1} \cdot A \cdot B^2 = A^{-1} \cdot I$

$I \cdot B^2 = A^{-1}$

$\therefore\ A^{-1} = B^2$.

56. Given $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$, A is symmetric, $p = \theta$.

$$\therefore \quad A = \begin{bmatrix} p & p \\ p & s \end{bmatrix}$$

$|A| = ps - p^2 = 20$

$p(s - p) = 20$.

S No	$p \times (s - p)$	s	$ps - p^2$
1	1×20	21	20
2	2×10	12	20
3	4×5	9	20

$\therefore$ There are 3 possible values for s.

57. $A = \begin{pmatrix} 2 & 0 \\ 5 & -3 \end{pmatrix}$ $B = \begin{pmatrix} -2 & 1 \\ 3 & -1 \end{pmatrix}$

$$B^T = \begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix}$$

$$A \times B^T = \begin{pmatrix} 2 & 0 \\ 5 & -3 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -4+0 & 6-0 \\ -10-3 & 15+3 \end{pmatrix} = \begin{pmatrix} -4 & 6 \\ -13 & 18 \end{pmatrix}.$$

$$(AB^T)^T = \begin{pmatrix} -4 & -13 \\ 6 & 18 \end{pmatrix}.$$

Trace of $(AB^T)^T = -4 + 18 = 14$.

58. The product AB is of the order 2×2. Since AB is a square matrix, its transpose $(AB)^T$ is also of the same order. The product BA is of the 3×3. Order of A^T is 3×2 and that of B^T is 2×3.

$\therefore$ The order of $B^T \cdot A^T$ is 2×2.

59. Given A is 2×3 matrix

B is 4×3 matrix

C is 2×4 matrix

C^T is 4×2 matrix

$\therefore$ $C^T B$ is not possible

$C^T = 4 \times 2$ $A = 2 \times 3$

$\therefore$ $C^T A$ is a matrix of order 4×3

B^T 3×4; $C^T A$ is 4×3

$B^T (C^T A)$ is a matrix of order 3×3

Option (b) follows.

60. $A = \begin{bmatrix} 2 & 4 \\ k & -2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & 4 \\ k & -2 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ k & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4k & 8-8 \\ 2k-2k & 4k+4 \end{bmatrix}$$

$A^2 = 0$.

Given, $\Rightarrow \begin{bmatrix} 4+4k & 0 \\ 0 & 4k+4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore$ $4 + 4k = 0 \Rightarrow k = -1$.

61. If $A^T = -A$ then A is called skew-symmetric matrix.

$$A = \begin{bmatrix} a & b & c \\ x & y & z \\ l & m & n \end{bmatrix}$$

$$-A = \begin{bmatrix} -a & -b & -c \\ -x & -y & -z \\ -l & -m & -n \end{bmatrix}$$

$$A^T = \begin{bmatrix} a & x & l \\ b & y & m \\ c & z & n \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & x & l \\ b & y & m \\ c & z & n \end{bmatrix} = \begin{bmatrix} -a & -b & -c \\ -x & -y & -z \\ -l & -m & -n \end{bmatrix}$$

$\Rightarrow$ $x = -b$, $z = -m$.

In a skew-symmetric matrix principle diagonal elements should be zero.

$\therefore$ $a = y = n = 0$

$\therefore$ $x + y + z = -b + a - m$.

62. $P^T = -P$

$$\Rightarrow \begin{bmatrix} 0 & x & 2 \\ 4 & 0 & -8 \\ -2 & -y & 0 \end{bmatrix} = \begin{bmatrix} 0 & -4 & 2 \\ -x & 0 & y \\ -2 & 8 & 0 \end{bmatrix}$$

$\therefore$ $x = -4$, $y = -8$

$\Rightarrow$ $x - y = -4 + 8 = 4$.

63. Given $B_1 = \begin{bmatrix} 3 & 3 \\ 4 & 1 \end{bmatrix}$ and $B_2 = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

$$\Rightarrow A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow |A| = 2 - 3 = -1.$$

64. $M = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$

trace of $M = p + s$

$p + s = 0$ (Trace $= |M| = 0$)

$p = -s$.