SURE SHOT QUESTIONS 2026

Chapter – 01 (Questions)

Electric Charges and Fields

➢ Questions

1. Two point charges of +1 μC and +4 μC are kept 30 cm apart. How far from the +1 μC charge on the line joining the two charges, will the net electric field be zero?

2. (a) Derive an expression for the electric field E due to a dipole of length '2a' at a point distance r from the centre of the dipole on the axial line.
(b) Draw a graph of E versus r for r >> a.

3. Three point charges, 1 pC each,, are kept at the vertices of an equilateral triangle of side 10 cm. Find the net electric field at the centroid of triangle.

4. A particle of charge $2\mu C$ and mass 1.6 g is moving with a velocity $4\hat{i}ms^{-1}$. At t = 0 the particle enters in a region having an electric field $\vec{E}$ (in N C^{-1}) = $80\hat{i} + 60\hat{j}$. Find the velocity of the particle at t = 5s.

5. Four point charges of $1\ \mu C, -2\ \mu C, 1\ \mu C\ and\ -2\ \mu C$ are placed at the corners A, 6, C and D respectively, of a square of side 30 cm. Find the net force acting on a charge of 4 μC placed at the centre of the square.

6. Derive an expression for the electric field due to dipole of dipole moment $\vec{p}$ at a point on its perpendicular bisector.

OR

Derive the expression for electric field at a point on the equatorial line of an electric dipole.

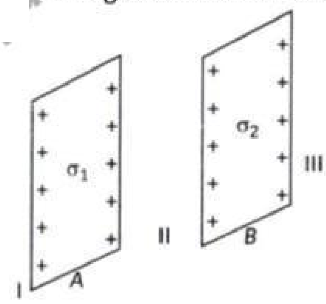
OR

Find resultant electric field due to an electric dipole of dipole moment 2aq (2a being the separation between the charges $\pm q$) at a point distance x on its equator.

7. (i) Define the term 'electric flux'. Write its SI unit.
(ii) What is the flux due to electric field $\vec{E} = 3\times10^3\,\hat{i}\,N/C$ through a square of side 10 cm, when it is held normal to $\vec{E}$?

8. A point charge (+Q) is kept in the vicinity of an uncharged conducting plate. Sketch the electric field lines between the charge and the plate.

9. Two infinitely large plane thin parallel sheets having surface charges densities σ_1 and σ_2 $(\sigma_1 > \sigma_2)$ are shown in the figure. Write the magnitudes and directions of the net electric fields in the regions marked II and III.

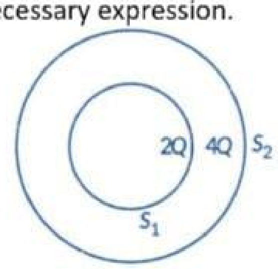


10. An electric field is uniform and acts along +x direction in the region of positive x. It is also uniform with the same magnitude but acts in -x direction in the region of negative x. The value of the field is E = 200 N C^{-1} for x > 0 and E = -200 N C^{-1} for x < 0. A right circular cylinder of length 20 cm and radius 5 cm has its centre at the origin and its axis along the x-axis so that one flat face is at x = +10 cm and the others is at x = -10 cm.
Find:
(i) The net outward flux through the cylinder.
(ii) The net charge present inside the cylinder.

11. Consider two hollow concentric spheres S_1 and S_2, enclosing charges 2Q and 4Q respectively as shown in figure.

(i) Find out the ratio of the electric flux through them.

(ii) How will the electric flux through the sphere S_1 change if a medium of dielectric constant 'ε_r' is introduced in the space inside S_1 in place of air? Deduce the necessary expression.



12. State Gauss's law on electrostatics and derive an expression for the electric field due to a long straight thin uniformly charged wire (linear charge density λ) at a point lying at a distance r from the wire.

13. Using Gauss law, derive expression for electric field due to a spherical shell of uniform charge distribution σ and radius R at a point lying at a distance x from the centre of shell, such that

14. Two large charged plane sheets of charge densities σ and -2σ C/m^2 are arranged vertically with a separation of d between them. Deduce expressions for the electric field at points (i) to the left of the first sheet, (ii) to the right of the second sheet, and (iii) between the two sheets.

15. Consider a uniform electric field $\vec{E} = 3 \times 10^3 \hat{\imath} N/C$. Calculate the flux of this Field through a square surface of area 10 cm^2 when

(i) its plane is parallel to the $y - z$ plane

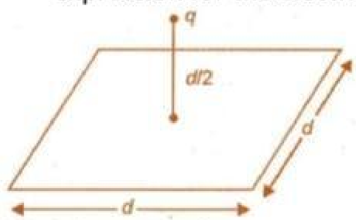
(ii) the normal to its plane makes a $60°$ angle with the $x - axis$.

16. (a) Derive an expression for the electric field at any point on the equatorial line of an electric dipole.

(b) The identical point charges, q each, are kept 2 m apart in air. A third point charge Q of unknown magnitude and sign is placed on the line joining the charges such that the system remains in equilibrium. Find the position and nature of Q.

17. A charge is distributed uniformly over a ring of radius 'a'. Obtain an expression for the electric intensity E at a point on the axis of the ring. Hence show that for points at large distances from the ring, it behaves like a point charge.

18. (a)Define Electric flux. Is it a scalar or a vector quantity? A point charge q is at a distance of d/2 directly above the centre of a square of side d, as shown in figure. Use gauss's law to obtain the expression for the electric flux through the square.

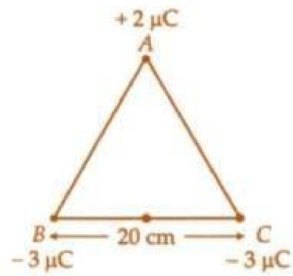


(b) If the point charge is now moved to distance 'd' from the centre of the square and the side of the square is doubled, explain how the electric flux will be affected.

19. State Coulomb's law and express it in vector form.

20. Two free point charges +4e and +e are placed at distance 'a' apart. Where should a third point charge q is placed between them such that the entire system may be in equilibrium? What should be the magnitude and sign of q? What type of equilibrium will it be?

21. Three point charges of $+2\mu C, -3\mu C$ and $-3\mu C$ are kept at the vertices A, B and C respectively of an equilateral triangle of side 20 cm as shown in figure. What should be the sign and magnitude of the charge to be placed at the midpoint (M) of side BC so that the charge at A remains in equilibrium?



22. Two point charges $+4\mu C$ and $+ 1\mu C$ are separated by distance of 2 m in air. Find the point on the line joining charges at which the net electric field of the system is zero?

23. Derive an expression for the electric field at a point on the axial position of an electric dipole.

24. Find the expression for electric field intensity at a point on the axis of a uniformly charged ring.

25. Define electric flux. Write its SI unit.

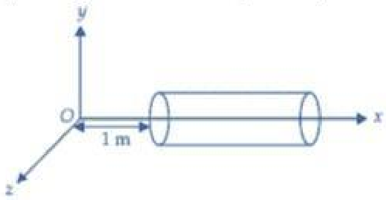
26. Derive an expression for torque on an electric dipole in a uniform electric field.

27. State Gauss theorem and use it to find the electric field at a point due to an infinitely large thin plane sheet has a uniform surface charge density

28. A hollow cylindrical box of length 1 m and area of cross-section 25 cm^2 is placed in a three dimensional coordinate system as shown in the figure. The electric field in the region is given by $\vec{E} = 50x\hat{\imath}$, where E is in NC^{-1} and x is in metres. Find

(i) net flux through the cylinder.

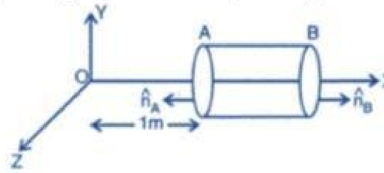
(ii) charge enclosed by the cylinder.



29. A hollow cylindrical box of length 1 m and area of cross section 25 cm^2 is placed in a three dimensional coordinate system as shown in the figure. The electric field in the region is given by $\vec{E} = 5x\hat{\imath}$, where E is in NC^{-1} and x is in metres. Find

(i) Net flux through the cylinder.

(ii) Charge enclosed by the cylinder.

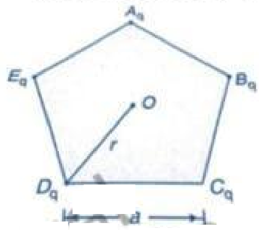


30. Derive an expression for the electric field due to an infinitely long straight uniformly charged wire

31. Five Charges, q each are placed at the corners of a regular pentagon of side a.

(i) What will be the electric field at O if the charge from one of the corners say (A) is removed?

(ii) What will be the electric field at O if the charge q at A is replaced by -q?



32. Given a uniform field $\vec{E} = 5 \times 10^3 \hat{\imath}\ N/C$, Find the flux of this field through a square of side 10 cm on a side whose plane is parallel to the y-z plane. What would be the flux through the same square if the plane makes a 30^0 angle with the x axis?

SURE SHOT QUESTIONS 2026

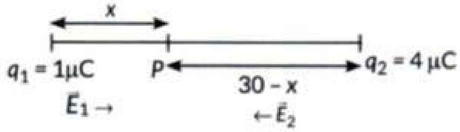
Chapter – 01 (Solutions)

Electric Charges and Fields

➢ Solutions

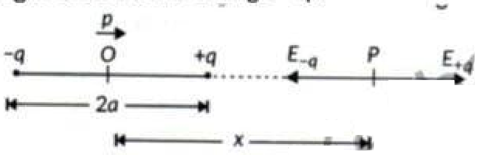
1. Ans. Let at point P, the net electric field is zero,

then $\frac{1}{4\pi\varepsilon_0}\frac{q_1}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(30-x)^2}$



$\frac{1}{x^2} = \frac{4}{(30-x)^2} \Rightarrow x = 10cm$

2. Ans. (a) Electric field at an axial point of an electric dipole. Let us consider an electric dipole consisting of charges +q and -q, separated by distance 2a and placed in vacuum. Let P be a point on the axial line at distance r from the centre O of the dipole on right side of the charge +q.



Electric field at an axial point of dipole

$\vec{E}_{-q} = \frac{-q}{4\pi\varepsilon_0(r+a)^2}\hat{p}$ (towards left)

Where $\hat{p}$ is a unit vector along the dipole axis from -q to +q.

Electric field due to charge +q at point P is

$\vec{E}_{+q} = \frac{q}{4\pi\varepsilon_0(r-a)^2}\hat{p}$ (towards right)

Hence the resultant electric field at point P is

$\overrightarrow{E_{axial}} = \vec{E}_{+q} + \vec{E}_{-q} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p}$

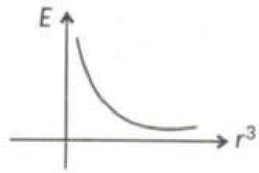
$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{4ar}{(r^2-a^2)^2}\hat{p}$

or $\overrightarrow{E_{axial}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2pr}{(r^2-a^2)^2}\hat{p}$ Here p = q x 2a = dipole moment

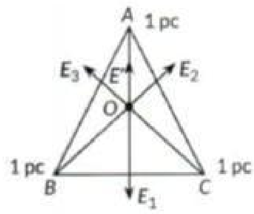
For r >> a, a^2 can be neglected as compared to r^2.

Or $\overrightarrow{E_{axial}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3}\hat{p}$ (towards right)

(b)



3. Ans.



Let O be the centriod of triangle.

Angle between E_3 and E_2 is 120°.

Resultant of E_3 and E_2, E' =

$\sqrt{E_3^2 + E_2^2 + 2E_3E_2\cos 120^\circ}$

$= \sqrt{E^2 + E^2 + 2E^2\left(-\frac{1}{2}\right)} = E$

Since all charges are of same magnitude, thus,

$E_1 = E_2 = E_3 = E$

E' and E_1 are same in magnitude but opposite in direction.

So, net electric field at $O = E' - E_1 = 0$

4. Ans. Given q = 2 μ C, m = 1.6 = 1.6 x 10-3 kg, $u = 4\hat{i}ms^{-1}, \vec{E} = 80\hat{i} + 60\hat{j}$ and t = 5 s.

$F = m\vec{a}$ (From Newton's law)

Or $q\vec{E} = m\vec{a}$

$\Rightarrow 2\times 10^{-6}(80\hat{i} + 60\hat{j}) = (1.6\times 10^{-3})\vec{a}$

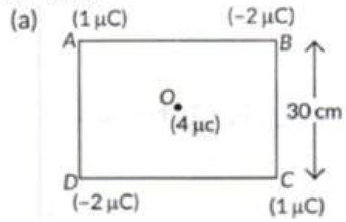
$\Rightarrow \vec{a} = 100\times 10^{-3}\hat{i} + 75\times 10^{-3}\hat{j}$

Now from equation of motion,

$\vec{v} = \vec{u} + \vec{a}t = 4\hat{i} + (100\times 10^{-3}\hat{i} + 75\times 10^{-3}\hat{j})5 = 4.5\hat{i} + 0.375\hat{j}$

5. Ans.

(a)



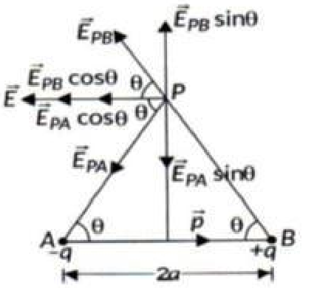
Let O be the centre of the square.

Force between charges at O and at B and charges at 0 and D will cancel each other.

Force between charges at 0 and A, and between charges at 0 and C will cancel each other.

So, net force on 4 μC placed at the centre of square will be zero.

6. Ans. Electric field on the equatorial line of an electric dipole : Electric field at any point on the perpendicular bisector of an electric dipole at distance r from its centre is



$$E_{net} = E_x = E_{PA}\cos\theta + E_{PB}\cos\theta$$

(Vertical component cancel each other)

Or $E_{net} = 2E_{PA}\cos\theta \quad (E_{PA} = E_{PB})$

$$E_{net} = 2.\frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+a^2)}.\frac{a}{(r^2+a^2)^{1/2}}$$

$$E_{net} = \frac{1}{4\pi\varepsilon_0}\frac{q.2a}{(r^2+a^2)^{3/2}} \text{ or } E_{net} = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2+a^2)^{3/2}}$$

Directed antiparallel to dipole moment $\vec{p}$. For short dipole, when r >>>> a, then electric field at point P is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

In vectorial form, the electric field intensity at point P on the perpendicular bisector of short electric dipole is

then given by $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{-\vec{p}}{r^3}.\hat{r}$

7. Ans. (i) Electric flux: Total number of electric field lines crossing a surface normally is called electric flux. SI unit of electric flux is N m^2 C^{-1}.

(ii) The area of a surface can be represented as a vector along normal to the surface.

Here, $\vec{E} = 3\times10^3\,\hat{i}\,NC^{-1}$

Area of the square $\Delta S = 10\times10cm^2$

ΔS = 100 cm² = 10^{-2} m²

Since normal to the square is along x-axis, we

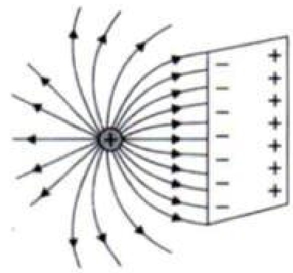
$\Delta S = 10^{-2}\,\hat{i}\,m^2$

Electric flux through the square,

$$\phi = \vec{E}.\Delta\vec{S} = (3\times10^3\,\hat{i}).(10^{-2}\,\hat{j})$$

$$\phi = 30\ Nm^2C^{-1}$$

8. Ans.



9. Ans. (a) Surface charge density on the inner surface

$$= \frac{q}{4\pi r_1^2}$$

On the outer surface = $\frac{Q-q}{4\pi r_2^2}$

(b) For a spherical Gaussian surface x > r_2

$$\oint \vec{E}\,\vec{ds} = \frac{Q-q}{\varepsilon_0}$$

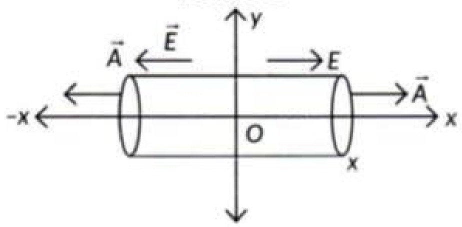
$$E\times4\pi x^2 = \frac{Q-q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon}\frac{Q-q}{x^2}$$

10. Ans. (i) Given l = 20 cm, r = 5 cm = 0.05 m

Net flux,

$$\phi = \int E.dA + \int E.dA = 200\pi(0.05)^2\cos0\times2$$
$$= \pi Nm^2C^{-1}$$



(ii) The net charge enclosed, $q = \phi_1\varepsilon_0$

$$\pi Nm^2C^{-1}\times8.85\times10^{-12}C^2N^{-1}m^2$$
$$= 27.789\times10^{-12}C$$

11. Ans. (i) Charge enclosed by sphere S_1 = 2Q

By Gauss law, electric flux through sphere S_1 is

$$\phi_1 = 2Q/\varepsilon_0$$

Charge enclosed by sphere,

$S_2 = 2Q + 4Q = 6Q$

$\phi_2 = 6Q/\varepsilon_0$

The ratio of the electric flux is

$\phi_1 : \phi_2 = 2:6 = 1:3$

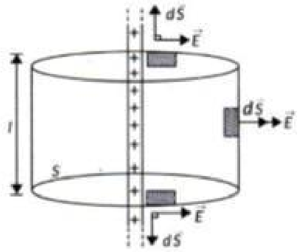
(ii) When a medium of dielectric constant ε_r is introduced in sphere S_1, the flux through S_1 would be $\phi'_1 = \frac{2Q}{\varepsilon_0 \varepsilon_r}$

12. Ans. According to Gauss's law, total flux over a closed surface S in vacuum is $\frac{1}{\varepsilon_0}$ times the total charge enclosed by closed surface S

$$\phi = \oint_S \vec{E}.\vec{ds} = \frac{q_{enclosed}}{\varepsilon_0}$$

Electric field intensity due to line charge or infinite long uniformly charged wire at point P at distance r from it is obtained as:

Assume a cylindrical gaussian surface S with charged wire on its axis and point P on its surface, then net electric flux through surface S is



$$\phi = \oint_S \vec{E}\cdot d\vec{S} = \int_{\text{upper plane face}} EdS\cos 90^\circ + \int_{\text{curved surface}} EdS\cos 0^\circ + \int_{\text{lower plane face}} EdS\cos 90^\circ$$

Or $\phi = 0 + EA + 0$ or $\phi = E.2\pi r l$

But by Gauss's theorem $\phi = \frac{q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$

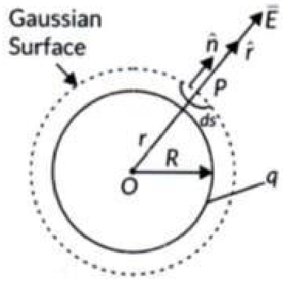
Where q is the charge on length l of wire enclosed by cylindrical surface S, and λ is uniform linear charge density of wire.

$$\therefore E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0} \text{ or } E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

13. Ans. Consider a thin spherical shell of radius R carrying charge q. To find the electric field outside the shell, we consider a spherical Gaussian surface of radius r (> R), concentric with given shell. The electric field $\vec{E}$ is same at every point of Gaussian surface and directed radially outwards (as is unit vector $\hat{n}$ so that $\theta = 0^0$)

According to Gauss's theorem,

$$\oint_S \vec{E}.d\vec{s} = \oint_S \vec{E}.\hat{n}ds = \frac{q}{\varepsilon_0}$$



Or $E\oint_S ds = \frac{q}{\varepsilon_0}$

$$\therefore E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

Vectorially, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$

Special cases

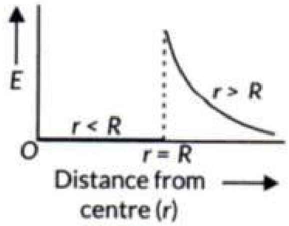
At the point on the surface of the shell, r = R

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

If σ is the surface charge density on the shell then

$q = 4\pi R^2 \sigma$

$$\therefore E = \frac{1}{4\pi\varepsilon_0}.\frac{4\pi R^2\sigma}{R^2} = \frac{\sigma}{\varepsilon_0}$$



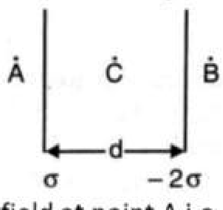
If the point P lies inside the spherical shell then the Gaussian surface encloses no cha

14. Soln.

(i) Electric field due to plane sheet toward right = $\frac{\sigma}{\varepsilon_0}$

Where, σ is charge density.

Towards left = $\frac{-\sigma}{\varepsilon_0}$



Electric field at point A i.e., to the left of first sheet and due to large plane sheet.

$$\vec{E}_A = -\frac{\sigma}{\varepsilon_0} + \left(+\frac{2\sigma}{\varepsilon_0}\right) = \frac{\sigma}{\varepsilon_0}$$

(ii) Electric field at point B i.e., to the right of second sheet,

$$\vec{E}_B = \frac{\sigma}{\varepsilon_0} - \frac{2\sigma}{\varepsilon_0} = \frac{-\sigma}{\varepsilon_0}$$

(iii) Electric field at point C i.e., between two plates,

$$\vec{E}_C = \frac{\sigma}{\varepsilon_0} + \frac{2\sigma}{\varepsilon_0} = \frac{3\sigma}{\varepsilon_0}.$$

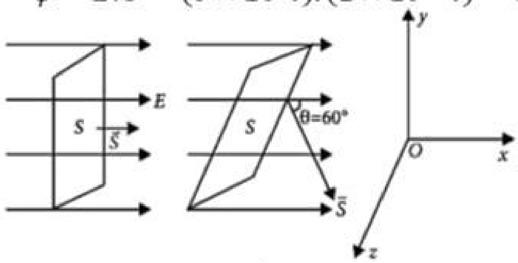
15. Soln. Given electric field $\vec{E} = 3\times10^3 \hat{i}\, NC^{-1}$

Magnitude of area, S = 10 cm^2 = 1 x 10^{-3} m^2

(i) When the surface is parallel to y-z plane, the normal to plane is along x-axis.

In this case $\theta = 0$; so electric flux,

$$\phi = \vec{E}.\vec{S} = (3\times10^3\hat{i}).(1\times10^{-3}\hat{i}) = 3\, N\, m^2\, C^{-1}$$



(ii) In this case $\theta = 60^0$, so electric flux,

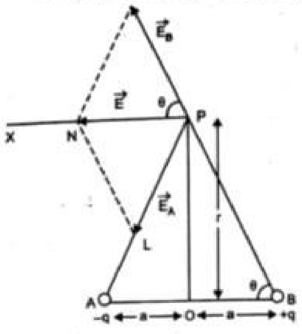
$$\phi = E.S\cos\theta$$

$$= 3\times10^3\times1\times10^{-3}\cos 60^0 = 3\times\frac{1}{2}$$

$$= 1.5 Nm^2C^{-1}.$$

16. Soln.

(a) Consider an electric dipole of charges -q and +q separated by a distance 2a and placed in a free space. Let P be a point on equatorial line of dipole at a distance r from the centre of a dipole.



Let $\vec{E}_A$ and $\vec{E}_B$ be the electric field at point P due to charges -q and +q

Then resultant electric field at point P is

$$\vec{E} = \vec{E}_A + \vec{E}_B$$

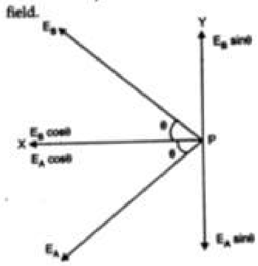
Now,

$$|\vec{E}_A| = \frac{1}{4\pi\varepsilon_0}.\frac{q}{AP^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+a^2)}(along\ PA)$$

$$|\vec{E}_B| = \frac{1}{4\pi\varepsilon_0}.\frac{q}{BP^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+a^2)}(along\ BP)$$

The, resultant intensity is the vector sum of E_A and E_B.

E_A and E_B can be resolved into two components. The Y-components cancel out each other. And X-component will add up to give the resultant Field.



$$\therefore |\vec{E}| = E_A\cos\theta + E_B\cos\theta$$

Now in right, triangle ORB

$$\cos\theta = \frac{OB}{BP} = \frac{a}{\sqrt{r^2+a^2}}$$

$$\therefore E = 2\times\frac{1}{4\pi\varepsilon_0}\times\frac{q}{r^2+a^2}\frac{a}{(r^2+a^2)^{1/2}}$$

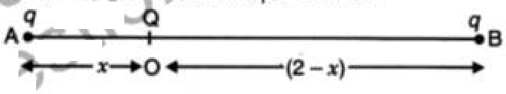
$$= \frac{2qa}{4\pi\varepsilon_0(r^2+a^2)^{3/2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2+a^2)^{3/2}}$$

$$[\because 2qa = p]$$

This is the required expression.

(b) Let the two charges of +q each placed at point A and B at a distance 2 m apart in air.



Suppose, the third charge Q (unknown magnitude and charge) is placed at a point O, on the line joining the other two charges, such that OA = x and OB = 2 – x.

For the system to be in equilibrium, net force on each 3 charges must be zero.

If we assume that charge Q placed at O is positive, the force on it at O may be zero. But the force on charge q at point A or B will not be zero. It is because, the forces on a charge q due to the other two charges will act in same direction. If charge Q is negative, then the forces on q due to other two charges will act in opposite direction.

Hence, Q will be negative in nature.

For charge (-Q) to be in equilibrium

Force on charge (-q) due to charge (+q) at point A should be equal and opposite to charge (+Q) at B

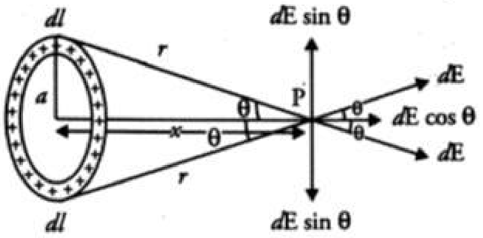
$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{x^2} = \frac{1}{4\pi\varepsilon_0}.\frac{Qq}{(2-x)^2}$$

Or $\quad (2-x)^2 = x^2$

$\Rightarrow \quad x = (2-x) \Rightarrow x = 1m$

Therefore, for the system to be in equilibrium a charge -Q is placed at a mid point between the two charges of +q each.

17. Soln. Suppose we have a ring of radius a that carries a uniformly distributed positive charge q.



As the total charge q is uniformly distributed, the charge dq on the element dl is

$$dq = \frac{q}{2\pi a}.dl$$

∴ The magnitude of the electric field produced by the element dl at the axial point P is

$$dE = k.\frac{dq}{r^2} = \frac{kq}{2\pi a}.\frac{dl}{r^2}$$

The electric field dE has two components.

(i) The axial components $dE\cos\theta$ and

(ii) The perpendicular component $dE\sin\theta$.

Since the perpendicular component of any two diametrically opposite elements are equal and opposite, they cancel out in pairs. Only the axial components will add up to produce the resultant field.

E at point P is given by

$$E = \int_0^{2\pi a} dE\cos\theta$$

[∵ Only the axial components contribute towards E]

$$E = \int_0^{2\pi a} \frac{kq}{2\pi a}.\frac{dl}{r^2}.\frac{x}{r}\left[\because \cos\theta = \frac{x}{r}\right]$$

$$= \frac{kqx}{2\pi a}.\frac{1}{r^3}\int_0^{2\pi a} dl$$

$$= \frac{kqx}{2\pi a}.\frac{1}{r^3}(l)_0^{2\pi a}$$

$$= \frac{kqx}{2\pi a}.\frac{1}{(x^2+a^2)^{3/2}}.2\pi a$$

$[\because r^2 = x^2 + a^2]$

$$E = \frac{kqx}{(x^2+a^2)^{3/2}}$$

[Where $k = \frac{1}{4\pi\varepsilon_0}$a = constant]

$$= \frac{1}{4\pi\varepsilon_0}.\frac{qx}{(x^2+a^2)^{3/2}}$$

If x >> a, then $x^2 + a^2 \approx x^2$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(x^2)^{3/2}}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$$

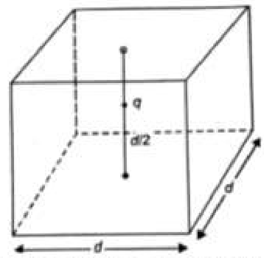
This expression is similar to electric field due to a point charge.

18. Soln. (a) Electric flux: Electric flux through an area is defined as the product of electric field strength E and area dS perpendicular to the field. It represents the field lines crossing the area. It is a scalar quantity. Imagine a cube of edge d, enclosing the charge. The square surface is one of the six faces of this cube. According to Gauss' theorem in electrostatics,

Total electric flux through the cube = $\frac{q}{\varepsilon_0}$

This is the total flux through all six surface

∴ Electric flux through the square surface = $\frac{q}{\sigma\varepsilon_0}$



(b) On moving the charge to distance d from the centre of square and making side of square 2d, does not change the flux at all because flux is independent of side of square or distance of charge in this case.

19. Soln. The force of attraction or repulsion between two point charges q_1 and q_2 separated by a distance r is directly proportional to product of magnitude of charges and inversely proportional to square of distance between charges, written as:

$F \propto q_1q_2$ and $F \propto \frac{1}{r^2}$

$$\therefore F \propto \frac{q_1q_2}{r^2} or F = k\frac{q_1q_2}{r^2}$$

Where k is a constant of proportionality, called *electrostatic force constant.* The value of k depends on the nature of the medium between the two charges and the system of units chosen to measure F, q_1, q_2 and r.

For the two charges located in free space and in SI units, we have

$$k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9 Nm^2C^{-2}$$

Where ε_0 is called *permittivity* of free space. So we can express Coulomb's law in SI units as

$$F = \frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{r^2}$$

In vector form, Coulomb's law may be expressed as

$\vec{F}_{21}$ =Force on charge q_2 due to q_1

$$= \frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{r^2}\hat{r}_{12}$$

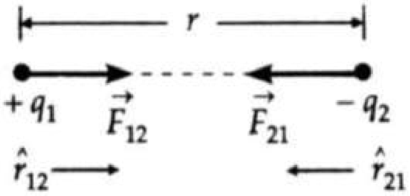
Where $\hat{r}_{12} = \frac{\vec{r}_{12}}{r}$, is a unit vector in the direction from q_1 and q_2.

Similarly, $\vec{F}_{12}$ =Force on charge q_1 due to q_2

$$= \frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{r^2}\hat{r}_{21}$$

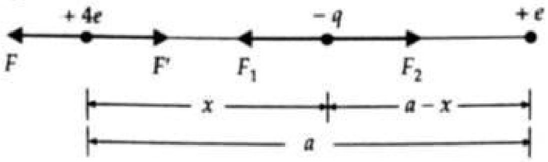
Where $\hat{r}_{21} = \frac{\vec{r}_{21}}{r}$, is a unit vector in the direction from q_2 to q_1.

The coulombian forces between unlike charges $(q_1q_2 < 0)$ are attractive, as shown in fig.



Attractive coulombian forces for $q_1q_2 < 0$.

20. Soln. Suppose the charges are placed as shown in fig.



As the charge +e exerts repulsion F on charge +4e, so for the equilibrium of charge +4e, the charge -q must exert attraction F' on +4e. This requires the charge q to be negative.

For equilibrium of charge +4e,

$$F = F'$$

$$\frac{1}{4\pi\varepsilon_0}\frac{4e\times e}{a^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4e\times q}{x^2}$$

Or $$q = \frac{ex^2}{a^2}$$

For equilibrium of charge -q,

Attraction F_1 between +4e and -q

= Attraction F_2 between +e and -q

$$\therefore \frac{1}{4\pi\varepsilon_0}\frac{4e\times q}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{e\times q}{(a-x)^2}$$

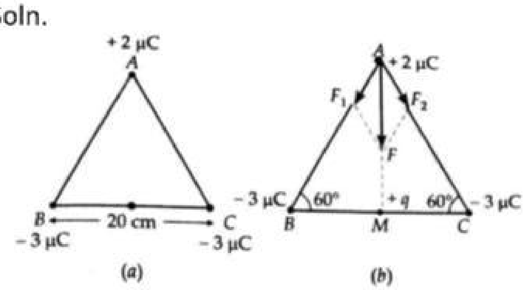
Or $$x^2 = 4(a-x)^2$$

$$\therefore x = 2a/3$$

Hence $$q = \frac{ex^2}{a^2} = \frac{e}{a^2}\cdot\frac{4a^2}{9} = \frac{4e}{9}.$$

The equilibrium of the negative charge q will be unstable.

21. Soln.



As shown in fig. the force exerted on charge $+2\mu C$ by charge at B,

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

$$= \frac{9\times10^9\times2\times10^{-6}\times3\times10^{-6}}{(0.20)^2}$$

$$= 1.35N, along AB$$

Force exerted on charge $+2\mu C$ by charge at C,

$$F_2 = \frac{9\times10^9\times2\times10^{-6}\times3\times10^{-6}}{(0.20)^2}$$

$$= 1.35N, along AC$$

Resultant force of F_1 and F_2

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_1\cos 60^0}$$

$$= \sqrt{1.35^2 + 1.35^2 + 2\times1.35\times1.35\times0.5}$$

$$= 1.35\times\sqrt{3} = 2.34N, along AM$$

For the charge at A to be equilibrium, the charge q to be placed at point M must be a positive charge so that it exerts a force on $+2\mu C$ charge along MA.

Now, $$AM = \sqrt{20^2 - 10^2}$$

$$= \sqrt{300} = 10\sqrt{3}cm$$

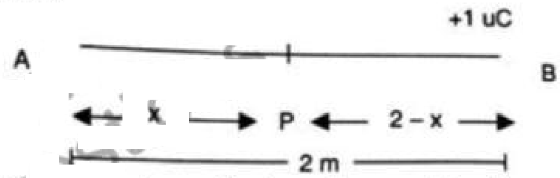
$$= 0.1\times\sqrt{3}m$$

Net force on charge at A will be zero if

$$\frac{9\times10^9\times q\times2\times10^{-6}}{(0.1\times\sqrt{3})^2} = 2.34$$

Or $$q = \frac{2.34\times0.01\times3}{18\times10^3} = 3.9\times10^{-6}C = 3.9\mu C.$$

22. Soln.



Distance between the two charges, AB = 2 m

$\therefore$ $PA = xm, PB = 2 - xm$

Net electric field at point P = E

Electric field at point P caused by $+4\mu C$ charge,

$$E_1 = \frac{4\times10^{-6}}{4\pi\varepsilon_0(PA)^2} N/C \text{ along PA}$$

Magnitude of electric field at point P caused by $+1\mu C$ charge,

$$E_2 = \frac{+1\times10^{-6}}{4\pi\varepsilon_0(PB)^2} N/C \text{ along PB}$$

Now, $$\frac{4\times10^{-6}}{4\pi\varepsilon_0(x)^2} = \frac{+1\times10^{-6}}{4\pi\varepsilon_0(2-x)^2}$$

Cross multiply and solve:

$$4\pi\varepsilon_0(2-x)^2\,4\times10^{-6} = 4\pi\varepsilon_0(x)^2\,1\times10^{-6}$$

$$(2-x)^2\,4\times10^{-6} = (x)^2\,1\times10^{-6}$$

$$(2-x)^2\,4 = (x)^2\,1$$

$$16 - 16x + 3x^2 = 0$$

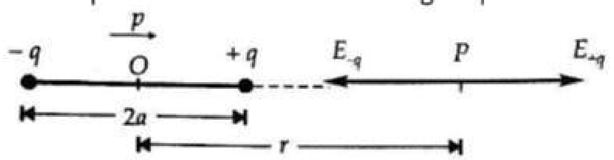
Now on factorizing,

$$4(4-x) - 3x(4-x) = 0$$

$$(4-3x)(4-x) = 0$$

$x = \frac{4}{3}, x = 4$, possible point on the line will be $x = \frac{4}{3}$.

23. Soln. Electric field at an axial point of an electric dipole. As shown in fig. consider an electric dipole consisting of charges +q and -q, separated by distance 2a and placed in vacuum. Let P be a point on the axial line at distance r from the centre O of the dipole on the side of the charge +q.



Electric field due to charge -q at point P is

$$\vec{E}_{-q} = \frac{-q}{4\pi\varepsilon_0 (r+a)^2}\hat{p} \qquad \text{(towards left)}$$

Where $\hat{p}$ is a unit vector along the dipole axis from -q to +q.

Electric field due to charge +q at point P is

$$\vec{E}_{+q} = \frac{q}{4\pi\varepsilon_0 (r-a)^2}\hat{p} \qquad \text{(towards right)}$$

Hence the resultant electric field at point P is

$$\vec{E}_{axial} = \vec{E}_{+q} + \vec{E}_{-q}$$
$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p}$$
$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{4ar}{(r^2-a^2)^2}\hat{p}$$

Or $\overrightarrow{E_{axial}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2pr}{(r^2-a^2)^2}\hat{p}$

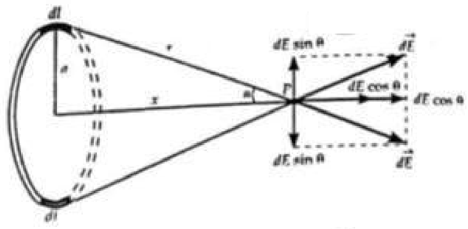
Here $p = q \times 2a$ =dipole moment.

For r >> a, a^2 can be neglected compared to r^2.

$$\therefore \overrightarrow{E_{axial}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3}\hat{p} \qquad \text{(towards right)}$$

Clearly, electric field at any axial point of the dipole acts along the dipole axis from negative to positive charge i.e., in the direction of dipole moment $\vec{p}$.

24. Soln.



$\therefore$ The magnitude of the field $d\vec{E}$ produced by the element dl at the field point P is

$$d\vec{E} = k\frac{dq}{r^2} = \frac{kq}{2\pi a}\cdot\frac{dl}{r^2}$$

As shown in fig. the field dE has two components:

1. The *axial component* dE $\cos\theta$, and
2. *The perpendicular component* dE $\sin\theta$.

Since the perpendicular components of any two diametrically opposite elements are equal and opposite, they all cancel out in pairs. Only the axial components will add up to produce the resultant field $\vec{E}$ at point P, which is given by

$$E = \int_0^{2\pi a} dE\cos\theta \qquad [\because \text{Only the axial components contribute towards E}]$$

$$= \int_0^{2\pi a}\frac{kq}{2\pi a}\cdot\frac{dl}{r^2}\cdot\frac{x}{r} = \frac{kqx}{2\pi a}\cdot\frac{1}{r^3}\int_0^{2\pi a} dl \qquad \left[\because \cos\theta = \frac{x}{r}\right]$$

$$= \frac{kqx}{2\pi a}\cdot\frac{1}{r^3}[l]_0^{2\pi a} = \frac{kqx}{2\pi a}\cdot\frac{1}{(x^2+a^2)^{3/2}}\cdot 2\pi a \qquad [\because r^2 = x^2+a^2]$$

Or $E = \frac{kqx}{(x^2+a^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qx}{(x^2+a^2)^{3/2}}$.

25. Soln. The electric flux through a given area held inside an electric field is the measure of the total number of electric lines of force passing normally through that area.

$$\Delta\phi_E = E\Delta\cos\theta = \vec{E}.\vec{\Delta S}$$

SI unit of electric flux = Vm.

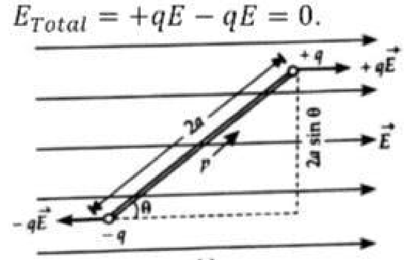
26. Soln. Torque on a dipole in a uniform electric field. As shown in fig. consider an electric dipole consisting of charges +q and -q and of length 2a placed in a uniform electric field $\vec{E}$ making an angle θ with it. It has a dipole moment of magnitude,

$$p = q\times 2a$$

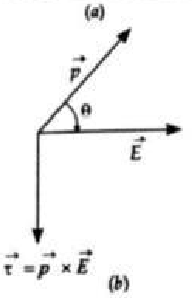
Force exerted on charge +q by field $\vec{E} = q\vec{E}$ (along $\vec{E}$)

Force exerted on charge -q by field $\vec{E} = -q\vec{E}$ (opposite to $\vec{E}$)

$$\vec{E}_{Total} = +q\vec{E} - q\vec{E} = 0.$$



(a)



(b)

Hence the net translating force on a dipole in a uniform electric field is zero. But the two equal and opposite forces act at different points of the dipole. They form a couple which exerts a torque.

Torque = Either force x Perpendicular distance between the two forces

$\tau = qE \times 2a\sin\theta = (q \times 2a)E\sin\theta$

Or $\tau = pE\sin\theta$

$(p = q \times 2a)$

As the direction of torque $\vec{\tau}$ is perpendicular to both $\vec{p}$ and $\vec{E}$, so we can write

$\vec{\tau} = \vec{p} \times \vec{E}$

The direction of vector $\vec{\tau}$ is that in which a right handed screw would advance when rotated from $\vec{p}$ to $\vec{E}$. As shown in fig. the direction of vector $\vec{\tau}$ is perpendicular to and points into the plane of paper.

When the dipole is released, the torque $\vec{\tau}$ tends to align the dipole with the field $\vec{E}$ i.e., tends to reduce angle θ to 0. When the dipole gets aligned with $\vec{E}$, the torque $\vec{\tau}$ becomes zero.

Clearly, the torque on the dipole will be maximum when the dipole is held perpendicular to $\vec{E}$. Thus

$$\tau_{max} = pE\sin 90^0 = pE.$$

Dipole moment. We know that the torque,

$$\tau = pE\sin\theta$$

If E = 1 unit, $\theta = 90^0$, then $\tau = p$

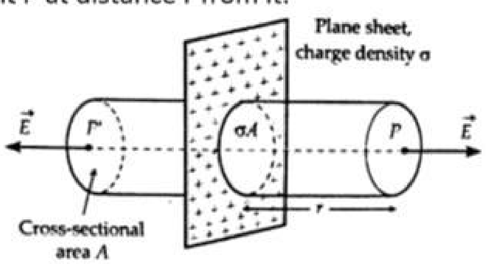
Hence *dipole moment may be defined as the toque acting on an electric dipole, placed perpendicular to a uniform electric field of unit strength.*

27. Soln. *Gauss theorem states that total flux through a closed surface is* $1/\varepsilon_0$ *times the net charge enclosed by the closed surface.*

Mathematically, it can be expressed as

$$\phi_E = \int_S \vec{E}.\vec{dS} = \frac{q}{\varepsilon_0}$$

Electric field due to a uniformly charged infinite plane sheet. As shown in fig. consider a thin, infinite plane sheet of charge with uniform surface charge density σ. We wish to calculate its electric field at a point P at distance r from it.



By symmetry, electric field E points outwards normal to the sheet. Also, it must have same magnitude and opposite direction at two points P and P' equidistant from the sheet and on opposite sides. We choose cylindrical Gaussian surface of cross-sectional area A and length 2r with its perpendicular to the sheet.

As the lines of force are parallel to the curved surface of the cylinder, the flux through the curved surface is zero. The flux through the plane – end faces of the cylinder is

$$\phi_E = EA + EA = 2EA$$

Charge enclosed by the Gaussian surface,

$$q = \sigma A$$

According to Gauss's theorem,

$$\phi_E = \frac{q}{\varepsilon_0}$$

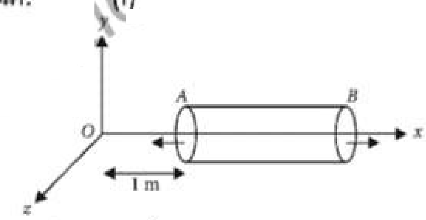
$$\therefore \quad 2EA = \frac{\sigma A}{\varepsilon_0} \quad or \quad E = \frac{\sigma}{2\varepsilon_0}$$

Clearly, E is independent of r, the distance from the plane sheet.

(i) If the sheet is *positively* charged $(\sigma > 0)$, the field is directed *away* from it.

If the sheet is *negatively* charged $(\sigma < 0)$, the field is directed towards it.

28. Soln. (i)



Given, $\vec{E} = 50x\hat{i}$ and $A = 25cm^2 = 25 \times 10^{-4}m^2$ As the electric field is only along the x-axis, s, flux will pass only through the cross – section of cylinder.

Magnitude of electric field at cross-section A,

$E_A = 50 \times 1 = 50NC^{-1}$

Magnitude of electric field at cross – section B,

$E_B = 50 \times 2 = 100NC^{-1}$

The corresponding electric fluxes are

$$\phi_A = \vec{E_A}.\vec{A} = 50 \times 25 \times 10^{-4} \cos 180^0 = -0.125Nm^2C^{-1}$$

$$\phi_B = \vec{E_B}.\vec{A} = 100 \times 25 \times 10^{-4} \cos 0^0 = 0.25Nm^2C - 1$$

So, the next flux through the cylinder,

$\phi = \phi_A + \phi_B = -0.125 + 0.25 = 0.125Nm^2C^{-1}$

(ii) Using gauss's law

$$\oint \vec{E}.\vec{dA} = \frac{q}{\varepsilon_0} \Rightarrow 0.125 = \frac{q}{8.85 \times 10^{-12}}$$

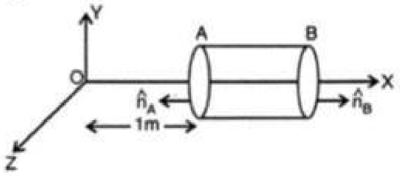
$$\Rightarrow q = 8.85 \times 0.125 \times 10^{-12} = 1.1 \times 10^{-12}C$$

29. Soln. (i) Given:

$\vec{E} = 50x\hat{i}$

And $\Delta S = 25cm^2 or 25 \times 10^{-4}m^2$

As the electric field is only along the x-axis , hence, flux will pass only through the cross-section of cylinder.



Magnitude of electric field at cross-section A,

$$E_A = 50 \times 1 = 50 N/C \text{ .}$$

Magnitude of electric field at cross-section B,

$$E_B = 50 \times 2 = 100 N/C$$

The corresponding electric fluxes are

$$\oint \phi_A = \vec{E}.\vec{\Delta S}$$
$$= 50 \times 25 \times 10^{-4} \times \cos 180^0$$
$$= -0.125 Nm^2/C^2$$

$$\oint \phi_B = \vec{E}.\vec{\Delta S}$$
$$= 100 \times 25 \times 10^{-4} \times \cos 0^0$$
$$= 0.25 Nm^2/C^2$$

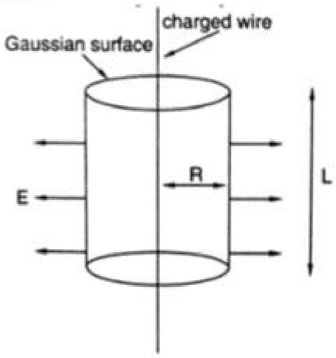
So, the net flux through the cylinder,

$$\oint \phi = \oint \phi_A + \oint \phi_B$$
$$= 0.125 + 0.25$$
$$= 0.375 Nm^2/C^2$$

(ii) Using the Gauss's law,

$$\phi = \oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$$
$$\Rightarrow 0.375 = \frac{q}{8.85 \times 10^{-12}}$$
$$\Rightarrow q = 8.85 \times 10^{-12} \times 0.375$$
$$= 3.3 \times 10^{-12} C.$$

30. Soln. In a long straight wire with uniform charge per unit length λ, there should be electric field generated by charge distribution for cylindrical symmetry. Also, field to point will radially be away from the wire.



In this, cylindrical gaussian surface is co-axial with the wire of radius R and length L where symmetry implies to electric field generated by wire that will be perpendicular to curved surface of cylinder, so as per Gauss' law,

$$E(\mathrm{R}) \times 2\pi \mathrm{RL} = \frac{\lambda L}{\varepsilon_0}$$

Where, E(R) is electric field strength which acts as perpendicular distance R from the wire.

In figure, left part shows electric flux through Gaussian surface while right part shows total charge enclosed by cylinder which is divided by ε_0.

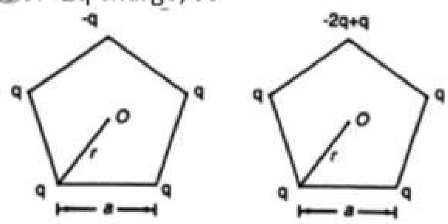
Further, $$E(\mathrm{R}) = \frac{\lambda}{2\pi\varepsilon_0 R}$$

Here, the field points are radially away from the wire then $\lambda > 0$, and radially towards the wire when $\lambda < 0$

31. Soln. (i) If a charge q is removed from point A, a negative charge is developed at A where electric field will be

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \text{ which is along } \overline{OA}.$$

(ii) If a charge q is replaced by charge -q at point A, there generates a net electric field at point O as a result of -2q charge, so



$$E = \frac{1}{4\pi\varepsilon_0}\frac{-2q}{r^2}$$
$$E = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2} along \overline{OA}$$

32. Soln. Flux of this field through a square of 10 cm

Flux of the square with normal making 30^0 angle

$$\phi = EA\cos\theta$$
$$\phi = 5 \times 10^3 \times 10^{-2} \cos 0^0\, NC^{-1}m^2$$
$$\phi = 50 NC^{-1}m^2$$
$$\phi = 5 \times 10^3 \times 10^{-2} \cos 60^0 NC^{-1}m^2$$
$$= 25 NC^{-1}m^2$$