04

Kinematics II (Projectile Motion)

IN THIS CHAPTER ....

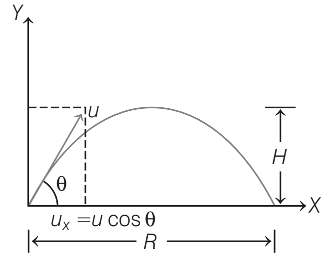
An object which is projected in the air with some velocity and makes a random angle with the horizontal surface is known *projectile*. The path followed by the projectile (called *trajectory*) is a parabola and the motion exhibited by it is called *projectile motion*. It is a two-dimensional motion.

Equation of Trajectory

Let us consider a projectile launched, so that its initial velocity u makes an angle θ with the horizontal. In the following diagram, horizontal direction is taken as X-axis and vertical direction is taken as the Y-axis.

$$\mathbf{u} = u_x\hat{\mathbf{i}} + u_y\hat{\mathbf{j}}$$

$$\Rightarrow \qquad \mathbf{u} = u\cos\theta\,\hat{\mathbf{i}} + u\sin\theta\,\hat{\mathbf{j}}$$



It can be seen that the X-axis is parallel to the horizontal, Y-axis is parallel to the vertical and u lies in the XY-plane. The constant acceleration a is given as

$$a = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}$$

where, $a_x = 0$ (Since, there is no acceleration along X-axis)

$a_y = -g$ (Since, the acceleration is downwards and equal to g)

Velocity after time t can be given as

$$\mathbf{v}_t = v\cos\theta\hat{\mathbf{i}} + (u\sin\theta - gt)\,\hat{\mathbf{j}}$$

Speed of the projectile at any time t is $v = \sqrt{v_x^2 + v_y^2}$ and displacement at time t will be

$$= ut\cos\theta\hat{\mathbf{i}} + \left(ut\sin\theta - \frac{1}{2}gt^2\right)$$

The direction of **v** with the X-axis is given by

$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$.

Coordinates of the projectile after time t is given by

$$\Rightarrow \quad x = u\cos\theta\, t \quad \text{...(i)}$$

$$\Rightarrow \quad y = u\sin\theta\, t - \frac{1}{2}gt^2 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), eliminating t, we get

$$y = u\sin\theta \frac{x}{u\cos\theta} - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$$

$$\Rightarrow \quad y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \quad \text{...(iii)}$$

The equation between x and y represents the path of the projectile known as **trajectory**. The Eq. (iii) shows that it is the equation of parabola of the form

$$y = bx + cx^2$$

where, $b = \tan\theta$ = constant and

$$c = \frac{g}{2u^2\cos^2\theta} = \text{constant.}$$

Example 1. *A particle moves according to the equation* $v = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$
At the initial point, $x = 0 = y$, *the radius of the trajectory is*

(a) $\frac{bx^2}{2a}$ (b) $\frac{2a}{bx^2}$ (c) $\frac{x^2}{a}$ (d) $\frac{a}{x^2}$

Sol. (a) Distance = speed × time

Given, $v = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$

At the initial point, $x = at, \frac{dy}{dt} = bx$

$$\Rightarrow \quad t = \frac{x}{a} \text{ or } dt = \frac{dx}{a}$$

$$\Rightarrow \quad dy = bx\,dt = bx\frac{dx}{a}$$

On integrating the above expression, using $\int x^n dx = \frac{x^{n+1}}{n+1}$

$$dy = \frac{b}{a}\int x\,dx$$

Thus, $$y = \frac{bx^2}{2a}$$

Example 2. *A biker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an initial speed of 15* ms^{-1}. *Neglecting air resistance, the speed with which the stone hits the ground is*

(a) 10 ms^{-1} (b) 99 ms^{-1} (c) 15 ms^{-1} (d) 30 ms^{-1}

Sol. (b) We choose the origin of the X-axis and Y-axis at the edge of the cliff. At $t = 0$ s, the stone is thrown. Consider the positive direction of X-axis to be along the initial velocity and the positive direction of Y-axis to be the vertically upward direction. The x and y- components of the motion can be treated independently. The equations of motion are

$$x = x_0 + u_x t$$

$$y = y_0 + u_y t + \frac{1}{2}a_y t^2$$

Here, $x_0 = y_0 = 0, u_y = 0, a_y = -g = -9.8\text{ ms}^{-2}, u_x = 15\text{ ms}^{-1}$

The stone hits the ground when, $y = -490$ m

$$-490 = -\frac{1}{2}(9.8)t^2$$

$$\Rightarrow \quad t = 10\text{ s}$$

The velocity components are, $u_x = u_{0x}$ and $u_y = u_{0y} - gt$, so that when the stone hits the ground

$$u_x = 15\text{ ms}^{-1}$$

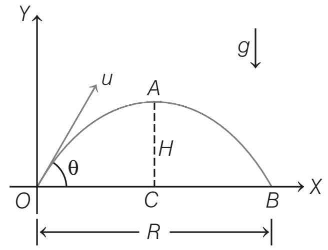
$$u_y = 0 - 9.8 \times 10 = -98\text{ ms}^{-1}$$

∴ The speed of the stone is

$$\sqrt{u_x^2 + u_y^2} = \sqrt{15^2 + 98^2} = 99\text{ ms}^{-1}$$

Important Terms in Projectile Motion

Consider a body is projected from the point O with an initial velocity u at an angle θ with the horizontal and it hits the ground at same level at point B.



The distance OB is called the *horizontal range* (R) or simply range and the vertical height AC is known as the *maximum height* (H). The total time taken by the particle in tracing the path OAB is called the *time of flight* (T).

Time of Flight (T)

Time of flight for the projectile is given as $T = \frac{2u\sin\theta}{g}$

Example 3. *A very broad elevator is going up vertically with a constant acceleration of 2* ms^{-2}. *At the instant, when its velocity is 4* ms^{-1}, *a ball is projected from the floor of the lift with a speed of 4* ms^{-1} *relative to the floor at an elevation of 30°. If* $g = 10$ ms^{-2}, *then what is the time taken by the ball to return to the floor?*

(a) $\frac{1}{2}$ s (b) $\frac{1}{3}$ s

(c) $\frac{1}{4}$ s (d) 1 s

Sol. (*b*) Here, $u = 4\ \text{ms}^{-1}, \theta = 30°$

Acceleration of the ball relative to the lift $= 10 + 2 = 12\ \text{ms}^{-2}$ acting in the negative y-direction or vertically downwards. It means, here $g' = 12\ \text{ms}^{-2}$.

Time of flight, $T = \dfrac{2u\sin\theta}{g'} = \dfrac{2\times 4\times \sin 30°}{12} = \dfrac{1}{3}\ \text{s}$

Horizontal Range (R)

The horizontal range R is the horizontal distance covered by the projectile is given as,

$$\Rightarrow \qquad R = \frac{u^2 \sin 2\theta}{g}$$

Range will be maximum, if

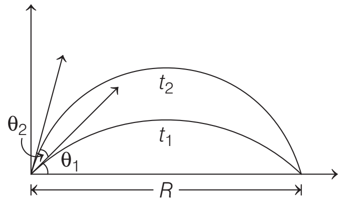
$$\sin 2\theta = 1$$
$$\sin 2\theta = \sin 90°$$
$$2\theta = 90°$$
or
$$\theta = 45°$$
$$R_{\max} = \frac{u^2}{g} \qquad (\text{at } \theta = 45°)$$

Example 4. *A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is* **[JEE Main 2019]**

(a) $\dfrac{R}{4g}$ (b) $\dfrac{R}{g}$

(c) $\dfrac{R}{2g}$ (d) $\dfrac{2R}{g}$

Sol. (*d*)



Given, range of the fired shell, $R = R$

and time of flights are t_1 and t_2.

Let θ_1 and θ_2 be the two angles at which shell is fired. As, range in both cases is same, *i.e.*

$$R_1 = R_2 = R$$

Here, $$R_1 = \frac{u^2 \sin 2\theta_1}{g}$$

and $$R_2 = \frac{u^2 \sin 2\theta_2}{g}$$

$$\Rightarrow \quad R = \frac{u^2 \sin 2\theta_1}{g} = \frac{u^2 \sin 2\theta_2}{g} \qquad \ldots\text{(i)}$$

$$\Rightarrow \quad \sin 2\theta_1 = \sin 2\theta_2$$

$$\Rightarrow \quad \sin 2\theta_1 = \sin(180° - 2\theta_2) \qquad [\because \sin(180° - \theta) = \sin\theta]$$

$$\Rightarrow \quad 2(\theta_1 + \theta_2) = 180° \text{ or } \theta_1 + \theta_2 = 90°$$

$$\Rightarrow \quad \theta_2 = 90° - \theta_1 \qquad \ldots\text{(ii)}$$

So, time of flight in first case,

$$t_1 = \frac{2u\sin\theta_1}{g} \qquad \ldots\text{(iii)}$$

and time of flight in second case,

$$t_2 = \frac{2u\sin\theta_2}{g} = \frac{2u\sin(90° - \theta_1)}{g} = \frac{2u\cos\theta_1}{g} \qquad \ldots\text{(iv)}$$

From Eqs. (iii) and (iv), we get

$$t_1t_2 = \frac{2u\sin\theta_1}{g} \times \frac{2u\cos\theta_1}{g}$$

$$\Rightarrow \quad t_1t_2 = \frac{4u^2\sin\theta_1\cos\theta_1}{g^2}$$

$$\Rightarrow \quad t_1t_2 = \frac{2u^2\sin 2\theta_1}{g^2} \qquad (\because \sin 2\theta = 2\sin\theta\cos\theta) \ldots\text{(v)}$$

From Eq. (i), we get

$$\therefore \quad t_1\,t_2 = \frac{2R}{g} \qquad \left(\because R = \frac{u^2\sin 2\theta_1}{g}\right)$$

Example 5. *A body is projected at $t = 0$ with a velocity $10\ ms^{-1}$ at an angle of $60°$ with the horizontal. The radius of curvature of its trajectory at $t = 1s$ is R. Neglecting air resistance and taking acceleration due to gravity $g = 10\ ms^{-2}$, the value of R is* **[JEE Main 2019]**

(a) 10.3 *m* (b) 2.8 *m*

(c) 5.1 *m* (d) 2.5 *m*

Sol. (*b*) Components of velocity at an instant of time t of a body projected at an angle θ is

$$v_x = u\cos\theta + g_x t \quad \text{and} \quad v_y = u\sin\theta + g_y t$$

Here, components of velocity at $t = 1\ \text{s}$, is

$$v_x = u\cos 60° + 0 \qquad [\text{as } g_x = 0]$$
$$= 10 \times \frac{1}{2} = 5\ \text{m/s}$$

and
$$v_y = u\sin 60° + (-10)\times(1)$$
$$= 10 \times \frac{\sqrt{3}}{2} + (-10)\times(1)$$
$$= 5\sqrt{3} - 10$$

$$\Rightarrow \quad |v_y| = |10 - 5\sqrt{3}|\ \text{m/s}$$

Now, angle made by the velocity vector at time of $t = 1\ \text{s}$

$$|\tan\alpha| = \left|\frac{v_y}{v_x}\right| = \frac{|10 - 5\sqrt{3}|}{5}$$

$$\Rightarrow \quad \tan\alpha = |2 - \sqrt{3}|$$

or $$\alpha = 15°$$

$\therefore$ Radius of curvature of the trajectory of the projected body,

$$R = v^2 / g\cos\alpha$$
$$= \frac{(5)^2 + (10 - 5\sqrt{3})^2}{10 \times 0.97}$$
$$[\because v^2 = v_x^2 + v_y^2 \text{ and } \cos 15° = 0.97]$$

$$\Rightarrow \quad R = 2.77\ \text{m} \approx 2.8\ \text{m}$$

Maximum Height (H)

Maximum height attained by the projectile is given as,

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

Example 6. *Two stones having different masses m_1 and m_2 are projected at angles θ and $(90° - \theta)$ with same velocity from the same point. The ratio of their maximum heights is*

(a) 1 : 1 *(b)* 1 : tan θ *(c)* tan θ : 1 *(d)* $\tan^2 \theta : 1$

Sol. (*d*) Maximum height, $H_1 = \frac{u^2 \sin^2 \theta}{2g}$ ...(i)

and $H_2 = \frac{u^2 \sin^2 (90° - \theta)}{2g}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{H_1}{H_2} = \frac{\sin^2 \theta}{\sin^2 (90° - \theta)}$$

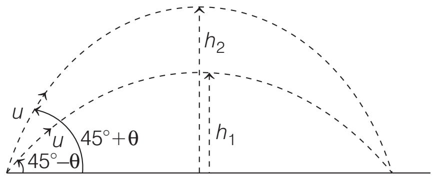
$$\frac{H_1}{H_2} = \frac{\tan^2 \theta}{1}$$

$\therefore$ $H_1 : H_2 = \tan^2 \theta : 1$

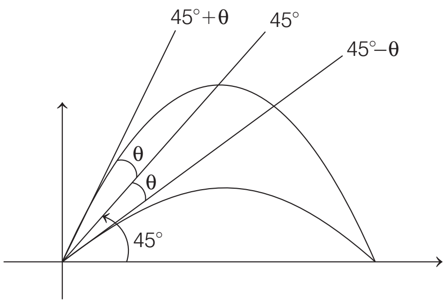
Example 7. *Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights h_1 and h_2. Which of the following is correct?* **[JEE Main 2019]**

(a) $R^2 = 4h_1h_2$ *(b)* $R^2 = 16h_1h_2$ *(c)* $R^2 = 2h_1h_2$ *(d)* $R^2 = h_1h_2$

Sol. (*b*)



As maximum range occurs at $\theta = 45°$ for a given initial projection speed, we take angles of projection of two particles as



$$\theta_1 = 45° + \theta, \theta_2 = 45° - \theta$$

where, θ is angle of projectiles with 45° line. So, range of projectiles will be

$$R = R_1 = R_2 = \frac{u^2 \sin 2(\theta_1)}{g}$$

$\Rightarrow$ $$R = \frac{u^2 \sin 2(45° + \theta)}{g}$$

$\Rightarrow$ $$R = \frac{u^2 \sin(90° + 2\theta)}{g}$$

$\Rightarrow$ $$R = \frac{u^2 \cos 2\theta}{g}$$

$\Rightarrow$ $$R^2 = \frac{u^4 \cos^2 2\theta}{g^2}$$ ...(i)

Maximum heights achieved in two cases are

$$h_1 = \frac{u^2 \sin^2(45° + \theta)}{2g}$$

and $$h_2 = \frac{u^2 \sin^2(45° - \theta)}{2g}$$

So, $$h_1h_2 = \frac{u^4 \sin^2(45° + \theta) \sin^2(45° - \theta)}{4g^2}$$

Using $2 \sin A \cdot \sin B = \cos(A - B) - \cos(A + B)$, we have

$$\sin(45° + \theta) \sin(45° - \theta) = \frac{1}{2}(\cos 2\theta - \cos 90°)$$

$$\Rightarrow \sin(45° + \theta) \sin(45° - \theta) = \frac{\cos 2\theta}{2} \quad [\because \cos 90° = 0]$$

So, we have

$$h_1h_2 = \frac{u^4\left(\frac{\cos 2\theta}{2}\right)^2}{4g^2} \Rightarrow h_1h_2 = \frac{u^4 \cos^2 2\theta}{16g^2}$$ ...(ii)

From Eqs. (i) and (ii), we get

$\Rightarrow$ $$h_1h_2 = \frac{R^2}{16} \Rightarrow R^2 = 16\,h_1h_2$$

Important Points Regarding Projectile Motion

- The equation of trajectory of projectile can be written in the terms of range of projectile as $y = x\left(1 - \frac{x}{R}\right)\tan\theta$.
- A projectile has the same range at angles of projection θ and $(90° - \theta)$, though time of flight, maximum height and trajectories are different. This is also true for a range of projectile for $\theta_1 = (45° - \alpha)$ and $\theta_2 = (45° + \alpha)$ and is equal to $\frac{u^2 \cos 2\alpha}{g}$.
- There are two unique times at which the projectile is at the same height $h \,(< H)$ and the sum of these two times equals the time of flight T. Since, $h = (u \sin\theta)\,t - \frac{1}{2}gt^2$ is quadratic in time, so it has two unique roots t_1 and t_2 (say) such that sum of roots $(t_1 + t_2)$ is $\frac{2u \sin\theta}{g}$ and product (t_1t_2) is $\frac{2h}{g}$. The time lapse $(t_1 - t_2)$ between these two events is $(t_1 - t_2)^2 = (t_1 + t_2)^2 - 4t_1t_2$

$$t_1 - t_2 = \sqrt{\frac{4u^2 \sin^2\theta}{g^2} - \frac{8h}{g}}$$

- In case of projectile motion, range R is n times the maximum height H.

i.e. $R = nH$

Then, $\frac{u^2 \sin 2\theta}{g} = n \frac{u^2 \sin^2\theta}{2g}$

i.e. $\tan\theta = \frac{4}{n}$ or $\theta = \tan^{-1}\left(\frac{4}{n}\right)$

- If air resists the projectile motion, then
  (i) Time taken by projectile during upward motion < Time taken during downward motion.
  (ii) The values of height attained and range of a projectile decrease both.
  (iii) The projectile returns to the ground with less speed. At its trajectory, its horizontal velocity also decreases.
  (iv) Time of flight also decreases.
  (v) The angle which the projectile makes with the ground, increases.
- If K' is the kinetic energy at the point of launch, then kinetic energy at the highest point is

$$K' = \frac{1}{2} mv_x^2 = \frac{1}{2} mu^2 \cos^2\theta \Rightarrow K' = K\cos^2\theta$$

- For complementary angles ϕ and $90° - \phi$, if T_ϕ and $T_{90°-\phi}$ are the times of flight and R is the range, then

$$T_\phi T_{90°-\phi} = \frac{2R_\phi}{g} = \frac{2R_{90°-\phi}}{g} = \frac{2R}{g}$$

e.g. $T_{1°} T_{89°} = \frac{2R_{1°}}{g} = \frac{2R_{89°}}{g}$

- The velocity of the projectile is minimum at the highest point ($= u\cos\theta$) and is maximum at the point of projection or at the point of striking the ground.
- At the maximum point of projectile motion, the velocity is not zero, but is $u\cos\theta$ which acts in the horizontal direction. The angle between velocity and acceleration varies from $0° < \theta < 180°$.
- Path of a projectile w.r.t. other projectile is a straight line.
- In oblique projection of a projectile, the following physical quantities remains constant during motion.
  (i) horizontal component of velocity ($u\cos\theta$)
  (ii) acceleration due to gravity (g)
  (iii) total energy of the projectile.

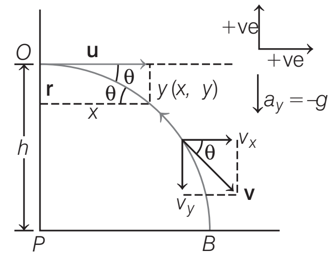
  The following physical quantities which change during the motion are
  (i) speed and velocity
  (ii) direction of motion
  (iii) linear momentum
  (iv) kinetic energy and potential energy
  (v) angle between direction of motion and acceleration due to gravity. (It changes from $90° + \theta$ to $90° - \theta$).

Special Cases of Projectile Motion

Projection from a Height

Case I Projectile Projected in Horizontal Direction

Let us consider that a projectile is projected with a velocity **u**. The following observations are taken from point O at a height h from the ground.



Here, $u_x = u$,
$u_y = 0$, $a_x = 0$
$a_y = -g$

(a) Horizontal motion, $x = ut$ ...(i)

Vertical motion, $-h = 0(t) - \frac{1}{2}gt^2$ ...(ii)

or $t = \sqrt{\frac{2h}{g}}$

(b) Horizontal range, $(R) = u \times t = u\sqrt{2h/g}$

(c) Let at time t, the coordinates of the position of projectile be (x, y), then

$$x = ut \text{ and } y = 0 - \frac{1}{2}gt^2$$

Therefore, at time t, position vector

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = ut\hat{\mathbf{i}} - \frac{1}{2}gt^2\hat{\mathbf{j}}$$

$$|\mathbf{r}| = \sqrt{x^2 + y^2} = \sqrt{(ut)^2 + \left(-\frac{1}{2}gt^2\right)^2}$$

and $\tan\theta = y/x$

(d) Let at time t, the horizontal and vertical velocities of projectile be v_x and v_y, respectively.

Hence, $v_x = u$

and $v_y = 0 + (-gt) = -gt$

$\mathbf{r} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = u\hat{\mathbf{i}} + (-gt)\hat{\mathbf{j}}$

and $v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + (-gt)^2}$

and $\tan\theta = \frac{v_y}{v_x}$

Case II Projectile Projected at an Angle θ above Horizontal

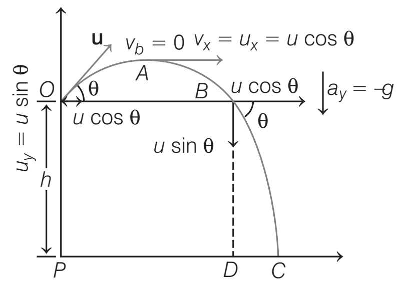
$$u_x = u\cos\theta, a_x = 0$$
$$u_y = u\sin\theta, a_y = -g$$

From equation of horizontal motion,

$$x = u\cos\theta t \qquad \text{...(i)}$$

Equation of vertical motion will be

$$-h = u\sin\theta t - \frac{1}{2}gt^2 \qquad \text{...(ii)}$$



$$gt^2 - 2u\sin\theta t - 2h = 0 \qquad \text{...(iii)}$$

or $$t = \frac{u\sin\theta}{g} \pm \sqrt{\frac{u^2\sin^2\theta}{g^2} \times \frac{2h}{g}}$$

Horizontal distance covered in time T,

$$PC = (u\cos\theta)T$$

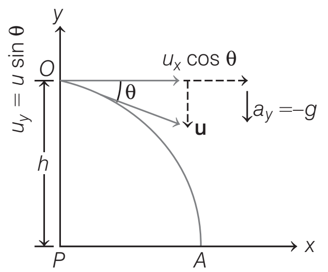
and horizontal distance covered during this time,

$$OB = \frac{u^2\sin 2\theta}{g}$$

In such case for range PC to become maximum, θ should be 45°.

***Case* III Projection at an Angle θ below Horizontal**

$$u_x = u\cos\theta, a_x = 0$$
$$u_y = -u\sin\theta, a_y = -g$$



From equation of motion,

$$s = ut + \frac{1}{2}at^2$$

$$-h = (-u\sin\theta)t + \frac{1}{2}(-g)t^2$$

or $$gt^2 + (2u\sin\theta)t - 2h = 0$$

On solving this equation, value of t can be obtained

$$t = \frac{-2u\sin\theta}{2g} \pm \frac{\sqrt{4u^2\sin^2\theta + 8gh}}{2g}$$

Neglect –ve root of t.

In this time, the horizontal distance covered on the Earth

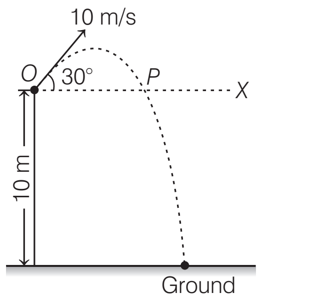
$$PA = (u\cos\theta)t$$

Example 8. *A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 m/s at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground?*

[$g = 10$ m/s^2, $\sin 30° = \frac{1}{2}$, $\cos 30° = \sqrt{3}/2$]

(a) 5.20 *m* *(b)* 4.33 *m* *(c)* 2.60 *m* *(d)* 8.66 *m*

Sol. (*d*) The ball will be at point *P* when it is at a height of 10 m from the ground. So, we have to find the distance *OP*, which can be calculated directly by considering it as a projectile on a levelled plane (*OX*).



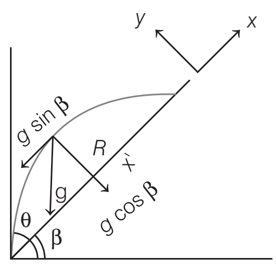
$$OP = R = \frac{u^2\sin 2\theta}{g} = \frac{10^2 \times \sin(2 \times 30°)}{10}$$

$$= \frac{10\sqrt{3}}{2} = 5\sqrt{3} = 8.66 \text{ m}$$

Projectile Motion on an Inclined Plane

When a projectile is projected from an inclined plane, we consider two axes x and y, *i.e.* along and perpendicular to the inclined plane. Different cases of projectile motion on an inclined plane are shown below.

***Case* I Motion up the Plane** In xy-plane,



$$u_x = u\cos(\theta - \beta)$$
$$u_y = u\sin(\theta - \beta)$$
$$a_x = -g\sin\beta,$$
$$a_y = -g\cos\beta$$

Time of flight,

$$\Rightarrow \quad T = \frac{2u\sin(\theta - \beta)}{g\cos\beta}$$

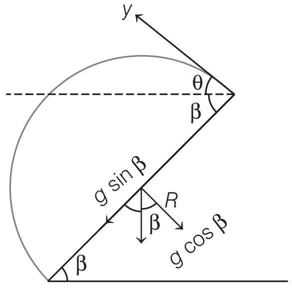
Range of inclined plane,

$$R = \frac{2u^2\cos\theta\sin(\theta - \beta)}{g\cos^2\beta}$$

Maximum range on inclined plane,

$$R_{\max} = \frac{u^2}{g(1+\sin\beta)}$$

***Case* II Motion down the Plane** Let the particle be thrown with a velocity u at an angle θ with the horizontal as shown in the figure.



Time of flight (T) and Range (R) on inclined plane is given as,

$$\Rightarrow \quad T = \frac{2u\sin(\theta+\beta)}{g\cos\beta}$$

$$R = \frac{u^2}{g}\left[\frac{\sin(2\theta+\beta)+\sin\beta}{1-\sin^2\beta}\right]$$

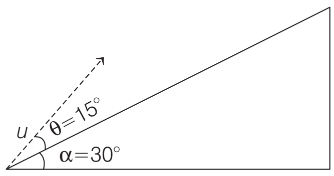
Since, θ is variable and the maximum value of sine function is 1.

For R to be maximum, $\sin(2\theta+\beta) = 1$

and $\quad R_{\max} = \dfrac{u^2}{g}\left(\dfrac{1+\sin\beta}{1-\sin^2\beta}\right)$

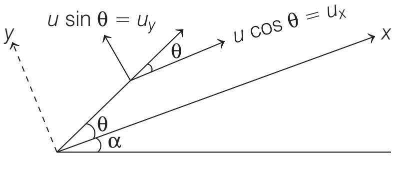
$$= \frac{u^2}{g(1-\sin\beta)} \text{ down the plane}$$

Example 9. *A plane is inclined at an angle $\alpha = 30°$ with respect to the horizontal. A particle is projected with a speed $u = 2\ ms^{-1}$, from the base of the plane, making an angle $\theta = 15°$ with respect to the plane as shown in the following figure. The distance from the base, at which the particle hits the plane is close to [Take, $g = 10\ ms^{-2}$]*



(a) 26 cm *(b)* 20 cm
(c) 18 cm *(d)* 14 cm

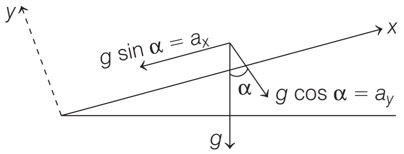
Sol. (*b*) When a projectile is projected at an angle θ with an inclined plane making angle α with the horizontal, then



Components of u along and perpendicular to plane are

$$u_x = u\cos\theta \quad \text{and} \quad u_y = u\sin\theta$$

We can also resolve acceleration due to gravity into its components along and perpendicular to plane as shown below.



So, we can now apply formula for range, *i.e.* net horizontal displacement of the particle as

$$R = u_xT + \frac{1}{2}a_xT^2 \qquad \text{...(i)}$$

where, T = time of flight.

Using formula for time of flight, we have

$$T = \frac{2u_y}{a_y} = \frac{2u\sin\theta}{g\cos\alpha} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

Range up the inclined plane is

$$R = u_xT + \frac{1}{2}a_xT^2 = u\cos\theta\left(\frac{2u\sin\theta}{g\cos\alpha}\right) - \frac{1}{2}g\sin\alpha\left(\frac{2u\sin\theta}{g\cos\alpha}\right)^2$$

Here, $u = 2\ \text{ms}^{-1}$, $g = 10\ \text{ms}^{-2}$, $\theta = 15°$, $\alpha = 30°$

So, $\quad T = \dfrac{2u\sin\theta}{g\cos\alpha} = \dfrac{2\times 2\sin 15°}{10\times\cos 30°}$

$$= \frac{2\times 2\times 0.258\times 2}{10\times 1.732} = 0.1191$$

Now, $R = 2\times\cos 15°\times 0.1191 - \frac{1}{2}\times 10\sin 30°\,(0.1191)^2$

$$= 2\times 0.965\times 0.1191 - \frac{5}{2}(0.1191)^2$$

$$= 0.229 - 0.0354 = 0.1936 \text{ m}$$

$$\approx 0.20 \text{ m} = 20 \text{ cm}$$

Example 10. *A cannon fired from under a shelter inclined at an angle α to the horizontal. The cannon is at point A at distant R from the base (B) of the shelter. The initial velocity of the cannon is v_0 and its trajectory lies in the plane. The maximum range R_{max} of the shell is*

(a) $\dfrac{v_0^2}{g}\sin 2\alpha$

(b) $\dfrac{g}{v_0^2}\sin 2(\phi-\alpha)$

(c) $\dfrac{v_0^2}{g}\sin 2\left(\alpha + \sin^{-1}\dfrac{\sqrt{gR\sin 2\alpha}}{v_0}\right)$

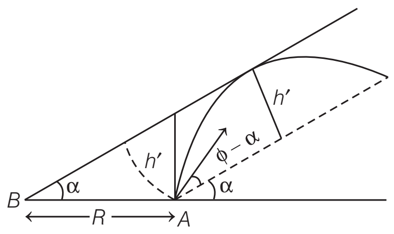
(d) $\dfrac{v_0^2}{2g}\sin 2\left(\alpha + \sin^{-1}\dfrac{\sqrt{R\sin 2\alpha}}{g}\right)$

Sol. (*c*) For h' to be maximum,

$$h' = R\sin\alpha = \frac{v_0^2\sin^2(\phi-\alpha)}{2g\cos\alpha}$$

$$gR \sin 2\alpha = v_0^2 \sin^2(\phi - \alpha)$$

$$\Rightarrow \qquad \phi = \alpha + \sin^{-1} \frac{\sqrt{gR \sin 2\alpha}}{g}$$



$$\text{Range, } R_{\max} = \frac{v_0^2 \sin 2\phi}{g}$$

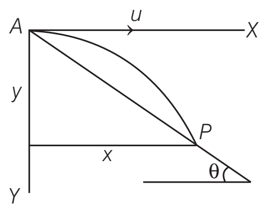
$$= \frac{v_0^2}{g} \sin 2\left(\alpha + \sin^{-1} \frac{\sqrt{gR \sin 2\alpha}}{v_0}\right)$$

Example 11. *A particle is projected horizontally with a speed u from the top of a plane inclined at an angle θ with the horizontal. How far from the point of projection will the particle strike the plane?*

(a) $\frac{2u^2}{g} \tan\theta \sec\theta$ (b) $\frac{2u}{g} \tan^2\theta \sec\theta$

(c) $\frac{2u^2}{g} \tan\theta \cos\theta$ (d) $\frac{2u}{g} \tan\theta \cos^2\theta$

Sol. (a) Take X and Y-axes as shown in figure below. Suppose that the particle strikes the plane at a point P with coordinates (x, y). Consider the motion between A and P.



Motion in x-direction,

$$\text{Initial velocity} = u$$
$$\text{Acceleration} = 0$$
$$x = ut \qquad \text{...(i)}$$

Motion in y- direction,

$$\text{Initial velocity} = 0$$
$$\text{Acceleration} = g$$
$$y = \frac{1}{2} g t^2 \qquad \text{...(ii)}$$

Eliminating t from Eqs. (i) and (ii), we get

$$y = \frac{1}{2} g \frac{x^2}{u^2}$$

Also, $\quad y = x \tan\theta$

Thus, $\frac{g x^2}{2u^2} = x \tan\theta$ giving $x = 0$ or $\frac{2u^2 \tan\theta}{g}$

Clearly, the point P corresponds to $x = \frac{2u^2 \tan\theta}{g}$, then

$$y = x \tan\theta - \frac{2u^2 \tan^2\theta}{g}$$

The distance, $AP = l = \sqrt{x^2 + y^2}$

$$= \frac{2u^2}{g} \tan\theta \sqrt{1 + \tan^2\theta}$$

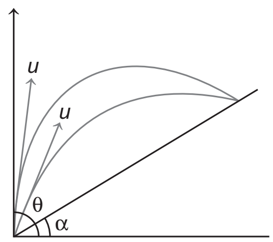
$$= \frac{2u^2}{g} \tan\theta \sec\theta$$

Example 12. *Two bodies are projected from the same point with equal speeds in such directions that they both strike the same point on a plane whose inclination is α. If θ be the angle of projection of the first body with the horizontal, then the ratio of their time of flight is*

(a) $\frac{\cos(\theta - \alpha)}{\cos\beta}$ (b) $\frac{\sin(\theta - \alpha)}{\cos\theta}$

(c) $\frac{\sin(\theta - \alpha)}{\sin\beta}$ (d) $\frac{\cos(\theta - \alpha)}{\sin\theta}$

Sol. (b) Range, $R = \frac{u^2}{g \cos^2\theta} [\sin(2\theta - \alpha) - \sin\alpha]$



Range of both the bodies is same.

Therefore,

$$\sin(2\theta - \alpha) = \sin(2\theta' - \alpha)$$
$$2\theta' - \alpha = \pi - (2\theta - \alpha)$$
$$\theta' = \frac{\pi}{2} - (\theta - \alpha)$$

Now, $\quad T = \frac{2u \sin(\theta - \alpha)}{g \cos\alpha} \qquad \text{...(i)}$

and $\quad T' = \frac{2u \sin(\theta' - \alpha)}{g \cos\alpha} \qquad \text{...(ii)}$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{T}{T'} = \frac{\sin(\theta - \alpha)}{\sin(\theta' - \alpha)}$$

$$= \frac{\sin(\theta - \alpha)}{\sin\left\{\frac{\pi}{2} - (\theta - \alpha) - \alpha\right\}}$$

$$= \frac{\sin(\theta - \alpha)}{\sin\left(\frac{\pi}{2} - \theta\right)} = \frac{\sin(\theta - \alpha)}{\cos\theta}$$

Practice Exercise

ROUND I Topically Divided Problems

Equation of Trajectory

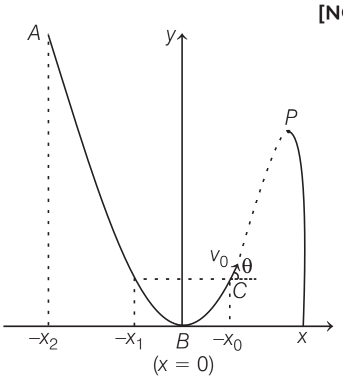
1. The height y and distance x along the horizontal for a body projected in the xy-plane are given by $y = 8t - 5t^2$ and $x = 6t$. The initial speed of projection is

(a) 8 m/s (b) 9 m/s
(c) 10 m/s (d) (10/3) m/s

2. A particle moves in the xy-plane with velocity $v_x = 8t - 2$ and $v_y = 2$. If it passes through the point $x = 14$ and $y = 4$ at $t = 2$ s, find the equation (x-y relation) of the path.

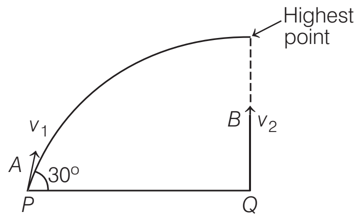
(a) $x = y^2 - y + 2$
(b) $x = 2y^2 + 2y - 3$
(c) $x = 3y^2 + 5$
(d) Cannot be found from above data

3. A particle slides down a frictionless parabolic ($y = x^2$) track ($A - B - C$) starting from rest at point A. Point B is at the vertex of parabola and point C is at a height less than that of point A. After C, the particle moves freely in air as a projectile. If the particle reaches highest point at P, then **[NCERT Exemplar]**



(a) KE at P = KE at B
(b) height at P = height at A
(c) total energy at P = total energy at A
(d) time of travel from A to B = time of travel from B to P.

4. A projectile A is thrown at an angle of 30° to the horizontal from point P. At the same time, another projectile B is thrown with velocity v^2 upwards from the point Q vertically below the highest point. For B to collide with A, $\frac{v_2}{v_1}$ should be



(a) 1 (b) 2
(c) $\frac{1}{2}$ (d) 4

5. A projectile is fired with a velocity v at an angle θ with the horizontal. The speed of the projectile when its direction of motion makes an angle β with the horizontal is

(a) $v \cos\theta$ (b) $v \cos\theta \cos\beta$
(c) $v \cos\theta \sec\beta$ (d) $v \cos\theta \tan\beta$

6. A ball is projected with velocity u at an angle α with horizontal plane. Its speed when it makes an angle β with the horizontal is

(a) $u \cos\alpha$ (b) $\frac{u}{\cos\beta}$
(c) $u \cos\alpha \cos\beta$ (d) $\frac{u \cos\alpha}{\cos\beta}$

7. A projectile is thrown with a velocity of 10 m/s at an angle 60° with horizontal. The interval between the moment when speed is $\sqrt{5g}$ m/s, is ($g = 10\,\text{m/s}^2$)

(a) 1 s (b) 3 s
(c) 2 s (d) 4 s

8. A body of mass m is thrown upward at an angle θ with the horizontal with velocity v. While rising up, the velocity of the mass after t second will be

(a) $\sqrt{(v\cos\theta)^2 + (v\sin\theta)^2}$
(b) $\sqrt{(v\cos\theta - v\sin\theta)^2 - gt}$
(c) $\sqrt{v^2 + g^2t^2 - (2v\sin\theta)\,gt}$
(d) $\sqrt{v^2 + g^2t^2 - (2v\cos\theta)\,gt}$

9. The equation of motion of a projectile are given by $x = 36t$ and $2y = 96t - 9.8t^2$ m. The angle of projectile will be

(a) $\sin^{-1}\left(\frac{4}{5}\right)$ (b) $\sin^{-1}\left(\frac{3}{5}\right)$

(c) $\sin^{-1}\left(\frac{4}{3}\right)$ (d) $\sin^{-1}\left(\frac{3}{4}\right)$

10. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0, then (Take, $g = 10\,\text{ms}^{-2}$) **[JEE Main 2019]**

(a) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}\,\text{ms}^{-1}$

(b) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\,\text{ms}^{-1}$

(c) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}\,\text{ms}^{-1}$

(d) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\,\text{ms}^{-1}$

Terms in the Projectile Motion

11. The horizontal range of a projectile fired at an angle of 15° is 50 m. If it is fired with the same speed at an angle of 45°, its range will be **[NCERT Exemplar]**

(a) 60 m (b) 71 m (c) 100 m (d) 141 m

12. A particle is projected from horizontal making an angle 60° with initial velocity $40\,\text{ms}^{-1}$. The time taken by the particle to make angle 45° from horizontal, is

(a) 15 s (b) 2.0 s (c) 20 s (d) 1.5 s

13. A particle is projected from the ground at an angle of 60° with horizontal with speed $u = 20\,\text{ms}^{-1}$. The radius of curvature of the path of the particle, when its velocity makes an angle of 30° with horizontal is ($g = 10\,\text{ms}^{-2}$)

(a) 10.6 m (b) 12.8 m (c) 15.4 m (d) 24.2 m

14. A bomb is dropped on an enemy post by an aeroplane flying horizontally with a velocity of $60\,\text{kmh}^{-1}$ and at a height of 490 m. At the time of dropping the bomb, how far the aeroplane should be from the enemy post so that the bomb may directly hit the target?

(a) $\frac{400}{3}$ m (b) $\frac{500}{3}$ m (c) $\frac{1700}{3}$ m (d) 498 m

15. A stone is thrown at an angle θ to be the horizontal reaches a maximum height H, then the time of flight of stone will be

(a) $\sqrt{\frac{2H}{g}}$ (b) $2\sqrt{\frac{2H}{g}}$

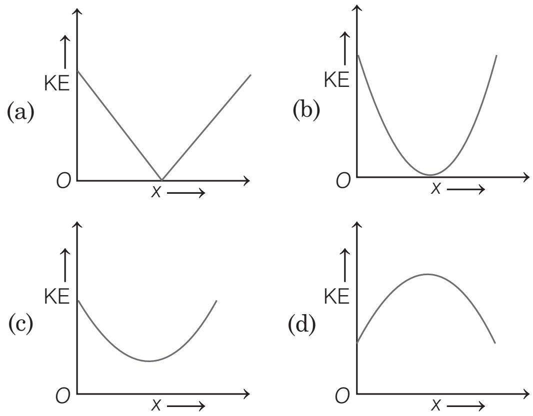
(c) $\frac{2\sqrt{2H\sin\theta}}{g}$ (d) $\frac{\sqrt{2H\sin\theta}}{g}$

16. An arrow is shot into air. Its range is 200 m and its time of flight is 5 s. If $g = 10\,\text{m/s}^2$, then horizontal component of velocity and the maximum height will be respectively

(a) 20 m/s, 62.50 m (b) 40 m/s, 31.25 m

(c) 80 m/s, 62.5 m (d) None of these

17. A ball is thrown up with a certain velocity at an angle θ to the horizontal. The kinetic energy (KE) of the ball varies in the horizontal displacement x as



18. It was calculated that a shell when fired from a gun with a certain velocity and at an angle of elevation $\frac{5\pi}{36}$ rad should strike a given target. In actual practice, it was found that a hill just prevented the trajectory. At what angle of elevation, should the gun be to hit the target?

(a) $\frac{5\pi}{36}$ rad (b) $\frac{11\pi}{36}$ rad

(c) $\frac{7\pi}{36}$ rad (d) $\frac{13\pi}{36}$ rad

19. A cricket ball is hit at 30° with the horizontal with kinetic energy E_k. What is the kinetic energy at the highest point?

(a) $\frac{E_k}{2}$ (b) $\frac{3E_k}{4}$ (c) $\frac{E_k}{4}$ (d) Zero

20. A particle is projected with a velocity of 30 m/s, at an angle $\theta_0 = \tan^{-1}\left(\frac{3}{4}\right)$. After 1 s, the particle is moving at an angle θ to the horizontal, where $\tan\theta$ will be equal to ($g = 10\,\text{m/s}^2$)

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

21. Two stones thrown at different angles have same initial velocity and same range. If H is the maximum height attained by one stone thrown at an angle of 30°, then the maximum height attained by the other stone is

(a) $\frac{H}{2}$ (b) H (c) $2H$ (d) $3H$

22. A projectile shot into air at some angle with the horizontal has a range of 200 m. If the time of flight is 5 s, then the horizontal component of the velocity of the projectile at the highest point of trajectory is

(a) 40 ms^{-1}
(b) 0 ms^{-1}
(c) 9.8 ms^{-1}
(d) equal to the velocity of projection of the projectile

23. The kinetic energy of a projectile at the highest, point is half of the initial kinetic energy. What is the angle of projection with the horizontal?

(a) 30° (b) 45° (c) 60° (d) 90°

24. A ball is projected from a certain point on the surface of a planet at a certain angle with the horizontal surface. The horizontal and vertical displacement x and y vary with time t in second as $x = 10\sqrt{3}\, t$ and $y = 10\,t - t^2$. The maximum height attained by the ball is

(a) 100 m (b) 75 m (c) 50 m (d) 25 m

25. For a projectile thrown into space with a speed v, the horizontal range is $\frac{\sqrt{3}v^2}{2g}$. The vertical range is $\frac{v^2}{8g}$. The angle which the projectile makes with the horizontal initially is

(a) 15° (b) 30°
(c) 45° (d) 60°

26. A projectile of mass m is thrown with a velocity v making an angle of 45° with the horizontal. The change in momentum from departure to arrival along vertical direction, is

(a) $2mv$ (b) $\sqrt{2}\, mv$
(c) mv (d) $\frac{mv}{2}$

27. A body is projected at an angle θ to the horizontal with kinetic energy E_k. The potential energy at the highest point of the trajectory is

(a) E_k (b) $E_k \cos^2\theta$
(c) $E_k \sin^2\theta$ (d) $E_k \tan^2\theta$

28. The horizontal range of an oblique projectile is equal to the distance through which a projectile has to fall freely from rest to acquire a velocity equal to the velocity of projection in magnitude. The angle of projection is

(a) 15° (b) 60° (c) 45° (d) 30°

29. A projectile is fired at an angle of 30° to the horizontal such that the vertical component of its initial velocity is 80 ms^{-1}. Its time of flight is T. Its velocity at $t = \frac{T}{4}$ has a magnitude of nearly

(a) 200 ms^{-1} (b) 300 ms^{-1}
(c) 140 ms^{-1} (d) 100 ms^{-1}

30. Two projectiles A and B are thrown with velocities v and $\frac{v}{2}$, respectively. They have the same range. If projectile B is thrown at an angle of 15° to the horizontal, then projectile A must have been thrown at an angle

(a) $\sin^{-1}\left(\frac{1}{16}\right)$ (b) $\sin^{-1}\left(\frac{1}{4}\right)$
(c) $2\sin^{-1}\left(\frac{1}{4}\right)$ (d) $\frac{1}{2}\sin^{-1}\left(\frac{1}{8}\right)$

31. The velocity of projection of an oblique projectile is $(6\hat{\mathbf{i}} + 8\hat{\mathbf{j}}) \text{ ms}^{-1}$. The horizontal range of the projectile is

(a) 4.9 m (b) 9.6 m (c) 19.6 m (d) 14 m

32. A projectile is thrown with velocity v making an angle θ with the horizontal. It just crosses the tops of two poles, each of height h, after 1 s and 3 s respectively. The time of flight of the projectile is

(a) 1 s (b) 3 s (c) 4 s (d) 7.8 s

33. Two particles are simultaneously projected in opposite directions horizontally from a given point in space whose gravity g is uniform. If u_1 and u_2 be their initial speeds, then the time t after which their velocites are mutually perpendicular is given by

(a) $\frac{\sqrt{u_1 u_2}}{g}$ (b) $\frac{\sqrt{u_1^2 + u_2^2}}{g}$
(c) $\frac{\sqrt{u_1 (u_1 + u_2)}}{g}$ (d) $\frac{\sqrt{u_2 (u_1 + u_2)}}{g}$

34. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball? **[NCERT Exemplar]**

(a) 40 m (b) 45 m (c) 500 m (d) 50 m

35. A piece of marble is projected from Earth's surface with velocity of 50 ms^{-1}. 2 s later, it just clears a wall 5 m high. What is the angle of projection?

(a) 45° (b) 30°
(c) 60° (d) None of these

36. The ceiling of a long hall is 25 m high. Then, the maximum horizontal distance that a ball thrown with a speed of 40 m/s can go without hitting the ceiling of the hall, is **[NCERT Exemplar]**

(a) 95.5 m (b) 105.5 m
(c) 100 m (d) 150.5 m

37. A particle leaves the origin with an initial velocity $\mathbf{v} = (3.00\, \hat{\mathbf{i}}) \text{ ms}^{-1}$ and a constant acceleration $\mathbf{a} = (-1.00\, \hat{\mathbf{i}} - 0.50\, \hat{\mathbf{j}}) \text{ ms}^{-2}$. When the particle reaches its maximum x-coordinate, what is its y-component of velocity?

(a) -2.0 ms^{-1} (b) -1.0 ms^{-1}
(c) -1.5 ms^{-1} (d) 1.0 ms^{-1}

38. Two stones are projected so as to reach the same distance from the point of projection on a horizontal surface. The maximum height reached by one exceeds the other by an amount equal to half the sum of the height attained by them, then angle of projection of the stone which attains smaller height is

(a) 45° (b) 60°
(c) 30° (d) $\tan^{-1}(3/4)$

39. The angle of projection of a projectile for which the horizontal range and maximum height are equal to

(a) $\tan^{-1}(2)$ (b) $\tan^{-1}(4)$
(c) $\cot^{-1}(2)$ (d) 60°

40. A particle is projected with velocity $2\sqrt{gh}$, so that it just clears two walls of equal height h, which are at a distance of $2h$ from each other. What is the time interval of passing between the two walls?

(a) $\frac{2h}{g}$ (b) $\sqrt{\frac{gh}{g}}$ (c) $\sqrt{\frac{h}{g}}$ (d) $2\sqrt{\frac{h}{g}}$

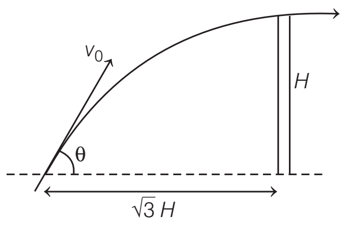
41. A particle is projected from the ground with an initial speed of v at an angle θ with horizontal. The average velocity of the particle between its point of projection and highest point of trajectory is

(a) $\frac{v}{2}\sqrt{1+2\cos^2\theta}$ (b) $\frac{v}{2}\sqrt{1+\cos^2\theta}$
(c) $\frac{v}{2}\sqrt{1+3\cos^2\theta}$ (d) $v\cos\theta$

42. The maximum range of a bullet fired from a toy pistol mounted on a car at rest is $R_0 = 40$ m. What will be the acute angle of inclination of the pistol for maximum range when the car is moving in the direction of firing with uniform velocity $v = 20$ m/s, on a horizontal surface? $(g = 10\,\text{m/s}^2)$ **[JEE Main 2013]**

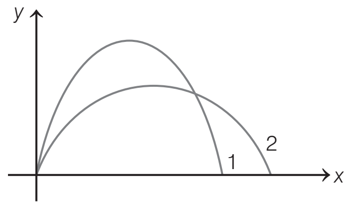
(a) 30° (b) 60°
(c) 75° (d) 45°

43. A projectile is thrown at an angle θ such that it is just able to cross a vertical wall as its highest point as shown in the figure. The angle θ at which the projectile is thrown is given by



(a) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\tan^{-1}\sqrt{3}$
(c) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (d) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

44. Trajectories of two projectiles are shown in figure. Let T_1 and T_2 be the time periods and u_1 and u_2 their speeds of projection, then



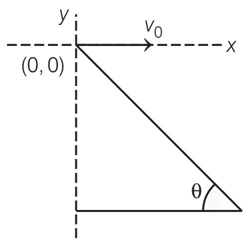
(a) $T_2 > T_1$ (b) $T_1 = T_2$
(c) $u_1 > u_2$ (d) $u_1 < u_2$

Special Cases of Projectile Motion

45. A body of mass m thrown horizontally with velocity v, from the top of tower of height h touches the level ground at distance of 250 m from the foot of the tower. A body of mass 2 m thrown horizontally with velocity $\frac{v}{2}$, from the top of tower of height $4h$ will touch the level ground at a distance x from the foot of the tower. The value of x is

(a) 250 m (b) 500 m
(c) 125 m (d) $250\sqrt{2}$ m

46. A man standing on a hill top projects a stone horizontally with speed v_0 as shown in figure. Taking the coordinate system as given in the figure. The coordinates of the point where the stone will hit the hill surface



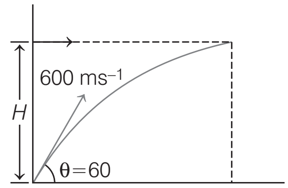
(a) $\left(\frac{2v_0^2\tan\theta}{g}, \frac{-2v_0^2\tan^2\theta}{g}\right)$

(b) $\left(\frac{2v_0^2}{g}, \frac{2v_0^2\tan^2\theta}{g}\right)$

(c) $\left(\frac{2v_0^2\tan\theta}{g}, \frac{2v_0^2}{g}\right)$

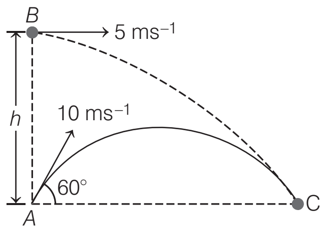
(d) $\left(\frac{2v_0^2\tan^2\theta}{g}, \frac{2v_0^2\tan\theta}{g}\right)$

47. A fighter plane enters inside the enemy territory, at time $t = 0$ with velocity $v_0 = 250\,\text{ms}^{-1}$ and moves horizontally with constant acceleration $a = 20\,\text{ms}^{-2}$ (see figure). An enemy tank at the border, spot the plane and fire shots at an angle $\theta = 60°$ with the horizontal and with velocity $u = 600\,\text{ms}^{-1}$. At what altitude H of the plane, it can be hit by the shot?



(a) $1500\sqrt{3}$ m (b) 125 m
(c) 1400 m (d) 2473 m

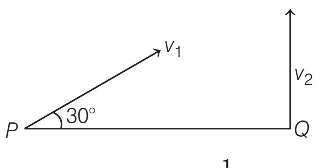
48. A particle A is projected from the ground with an initial velocity of $10\ \text{ms}^{-1}$ at an angle of 60° with horizontal. From what height h should an another particle B be projected horizontally with velocity $5\ \text{ms}^{-1}$, so that both the particles collide with velocity $5\ \text{ms}^{-1}$ on the ground at point C, if both are projected simultaneously? ($g = 10\ \text{ms}^{-2}$)



(a) 10 m (b) 30 m
(c) 15 m (d) 25 m

49. A particle is projected with speed v at an angle $\theta \left(0 < \theta < \frac{\pi}{2}\right)$ above the horizontal from a height H above the ground. If v = speed with which particle hits the ground and t = time taken by particle to reach ground, then
(a) as θ increases, v decreases and t increases
(b) as θ increases, v increases and t increases
(c) as θ increases, v remains same and t increases
(d) as θ increases, v remains same and t decreases

50. A projectile A is thrown at an angle 30° to the horizontal from point P. At the same time, another projectile B is thrown with velocity v_2 upwards from the point Q vertically below the highest point A. If projectile B collides with projectile A, then the ratio $\frac{v_2}{v_1}$ should be



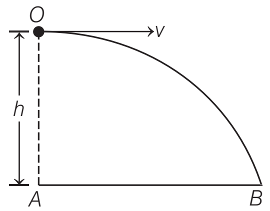
(a) $\frac{\sqrt{3}}{2}$ (b) 2 (c) $\frac{1}{2}$ (d) $\frac{2}{\sqrt{3}}$

51. An aircraft, diving at an angle of 53.0° with the vertical releases a projectile at an altitude of 730 m. The projectile hits the ground 5.00 s after being released. What is the speed of the aircraft?
(a) $282\ \text{ms}^{-1}$
(b) $202\ \text{ms}^{-1}$
(c) $182\ \text{ms}^{-1}$
(d) $102\ \text{ms}^{-1}$

52. A bomber plane moves horizontally with a speed of $500\ \text{ms}^{-1}$ and a bomb releases from it, strikes the ground in 10 s. Angle at which it strikes the ground will be ($g = 10\ \text{ms}^{-2}$)
(a) $\tan^{-1}\left(\frac{1}{5}\right)$
(b) $\tan\left(\frac{1}{5}\right)$
(c) $\tan^{-1}(1)$
(d) $\tan^{-1}(5)$

53. A plane surface is inclined making an angle θ with the horizontal. From the bottom of this inclined plane, a bullet is fired with velocity v. The maximum possible range of the bullet on the inclined plane is
(a) $\frac{v^2}{g}$ (b) $\frac{v^2}{g(1+\sin\theta)}$
(c) $\frac{v^2}{g(1-\sin\theta)}$ (d) $\frac{v^2}{g(1+\sin\theta)^2}$

54. An aeroplane is flying in a horizontal direction with a velocity $600\ \text{kmh}^{-1}$ at a height of 1960 m. When it is vertically above the point A on the ground, a body is dropped from it. The body strikes the ground at point B. Calculate the distance AB.



(a) 3.33 km (b) 333 km
(c) 33.3 km (d) 3330 km

55. A ball is projected up an incline of 30° with a velocity of $30\ \text{ms}^{-1}$ at an angle of 30° with reference to the inclined plane from the bottom of the inclined plane. If $g = 10\ \text{ms}^{-2}$, then the range on the inclined plane is
(a) 12 m (b) 60 m
(c) 120 m (d) 600 m

ROUND II Mixed Bag

Only One Correct Option

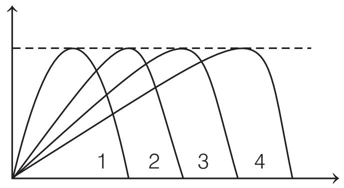
1. A ball projected from ground at an angle of 45° just clears a wall in front. If point of projection is 4 m from the foot of wall and ball strikes the ground at a distance of 6 m on the other side of the wall, the height of the wall is **[JEE Main 2013]**

(a) 4.4 m (b) 2.4 m
(c) 3.6 m (d) 1.6 m

2. A shell is fired from a cannon with a velocity v at angle θ with horizontal. At the highest point, it explodes into two pieces of equal mass. One of the pieces retraces its path to the cannon. The speed of the other piece just after explosion is

(a) $3v\cos\theta$ (b) $2v\cos\theta$
(c) $\frac{3}{2}v\cos\theta$ (d) $\frac{\sqrt{3}}{2}v\cos\theta$

3. Figure shows four paths for a kicked football ignoring the effects of air on the flight. Rank the paths according to the initial horizontal velocity component highest first.



(a) 1, 2, 3, 4 (b) 2, 3, 4, 1
(c) 3, 4, 1, 2 (d) 4, 3, 2, 1

4. Two second after projection, a projectile is travelling in a direction inclined at 30° to the horizontal. After 1 more second, it is travelling horizontally (use $g = 10\,\text{ms}^{-2}$). The initial velocity of its projection is

(a) $10\,\text{ms}^{-1}$ (b) $10\sqrt{3}\,\text{ms}^{-1}$
(c) $20\,\text{ms}^{-1}$ (d) $20\sqrt{3}\,\text{ms}^{-1}$

5. A projectile is given an initial velocity of $(\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$ m/s, where $\hat{\mathbf{i}}$ is along the ground and $\hat{\mathbf{j}}$ is along the vertical. If $g = 10\,\text{m/s}^2$, the equation of its trajectory is **[JEE Main 2013]**

(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$
(c) $4y = 2x - 5x^2$ (d) $4y = 2x - 25x^2$

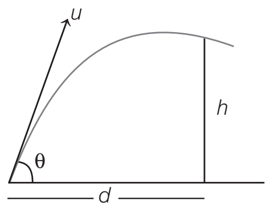
6. A boy can throw a stone upto a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone upto will be **[AIEEE 2012]**

(a) $20\sqrt{2}$ m (b) 10 m
(c) $10\sqrt{2}$ (d) 20 m

7. A particle of mass m is projected with a velocity v making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height h is **[AIEEE 2011]**

(a) $\frac{\sqrt{3}}{2}\frac{mv^2}{g}$ (b) zero
(c) $\frac{mv^3}{\sqrt{2}g}$ (d) $\frac{\sqrt{3}}{16}\frac{mv^3}{g}$

8. If a stone is to hit at a point which is at a distance d away and at a height h above the point from where the stone starts, then what is the value of initial speed u, if the stone is launched at an angle θ?



(a) $\frac{g}{\cos\theta}\sqrt{\frac{d}{2(d\tan\theta - h)}}$ (b) $\frac{d}{\cos\theta}\sqrt{\frac{g}{2(d\tan\theta - h)}}$
(c) $\sqrt{\frac{gd^2}{h\cos^2\theta}}$ (d) $\sqrt{\frac{gd^2}{(d-h)}}$

9. Two projectiles A and B are projected with same speed at angles 15° and 75° respectively to the maximum height and have same horizontal range. If h be the maximum height and T total time of flight of a projectile, then

(a) $h_A > h_B$ (b) $h_A = h_B$
(c) $T_A < T_B$ (d) $T_A > T_B$

10. A projectile has the same range R for two angles of projections. If T_1 and T_2 be the times of flight in the two cases, then (using θ as the angle of projection corresponding to T_1)

(a) $T_1 T_2 \propto R$ (b) $T_1 T_2 \propto R^2$
(c) $T_1/T_2 = \cot\theta$ (d) $T_1/T_2 = 1$

11. Two particles are projected in air with speed v_0 at angles θ_1 and θ_2 (both acute) to the horizontal, respectively. If the height reached by the first particle is greater than that of the second, then tick the right choices. **[NCERT Exemplar]**

(a) Angle of projection : $\theta_1 < \theta_2$
(b) Time of flight : $T_1 > T_2$
(c) Horizontal range : $R_1 > R_2$
(d) Total energy : $U_1 > U_2$

12. The trajectory of a projectile in vertical plane is $y = ax - bx^2$, where a and b are constants and x and y are respectively horizontal and vertical distances of the projectile from the point of projection. The maximum height attained by the particle and the angle of projection from the horizontal are

(a) $\frac{b^2}{4b}, \tan^{-1}(b)$ (b) $\frac{a^2}{b}, \tan^{-1}(2b)$

(c) $\frac{a^2}{4b}, \tan^{-1}(a)$ (d) $\frac{2a^2}{b}, \tan^{-1}(a)$

13. The speed of projection of a projectile is increased by 10%, without changing the angle of projection. The percentage increase in the range will be

(a) 10% (b) 20%

(c) 15% (d) 5%

14. A body of mass 1 kg is projected with velocity 50 m/s at an angle of 30° with the horizontal. At the highest point of its path, a force 10 N starts acting on body for 5 s vertically upward besides gravitational force, what is horizontal range of the body? (Take, $g = 10\,\text{m/s}^2$)

(a) $125\sqrt{3}$ m (b) $200\sqrt{3}$ m

(c) 500 m (d) $250\sqrt{3}$ m

15. A particle is projected with a velocity $200\,\text{ms}^{-1}$ at an angle of 60°. At the highest point, it explodes into three particles of equal masses. One goes vertically upwards with a velocity $100\,\text{ms}^{-1}$, the second particle goes vertically downwards. What is the velocity of third particle?

(a) $120\,\text{ms}^{-1}$ making 60° angle with horizontal

(b) $200\,\text{ms}^{-1}$ making 30° angle with horizontal

(c) $300\,\text{ms}^{-1}$

(d) $200\,\text{ms}^{-1}$

16. Two stones are projected with the same velocity in magnitude but making different angles with the horizontal. Their ranges are equal. If the angle of projection of one is $\frac{\pi}{3}$ and its maximum height is y_1, the maximum height of the other will be

(a) $3y_1$ (b) $2y_1$

(c) $\frac{y_1}{2}$ (d) $\frac{y_1}{3}$

17. A car is travelling at a velocity of $10\,\text{kmh}^{-1}$ on a straight road. The driver of the car throws a parcel with a velocity of $10\sqrt{2}\,\text{kmh}^{-1}$ when the car is passing by a man standing on the side of the road. If the parcel is to reach the man, the direction of throw makes the following angle with direction of the car

(a) 135° (b) 45°

(c) $\tan^{-1}(\sqrt{2})$ 60° (d) $\tan\left(\frac{1}{\sqrt{2}}\right)$

18. A particle of mass m is projected with a velocity v at an angle of 60° with horizontal. When the particle is at its maximum height, the magnitude of its angular momentum about the point of projection is

(a) zero (b) $\frac{3mv^3}{16g}$

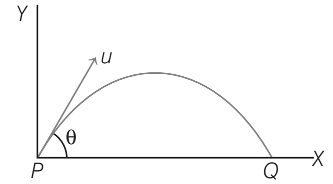
(c) $\frac{\sqrt{3}mv^2}{16g}$ (d) $\frac{3mv^2}{3g}$

19. A stone is projected with a velocity $20\sqrt{2}\,\text{ms}^{-1}$ at an angle of 45° to the horizontal. The average velocity of stone during its motion from starting point to its maximum height is (Take, $g = 10\,\text{ms}^{-2}$)

(a) $5\sqrt{5}\,\text{ms}^{-1}$ (b) $10\sqrt{5}\,\text{ms}^{-1}$

(c) $20\,\text{ms}^{-1}$ (d) $20\sqrt{5}\,\text{ms}^{-1}$

20. Average torque on a projectile of mass m, initial speed u and angle of projection θ, between initial and final position P and Q as shown in figure about the point of projection is



(a) $mu^2 \sin\theta$ (b) $mu^2 \cos\theta$

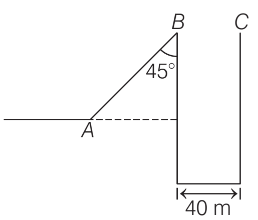
(c) $\frac{1}{2}mu^2 \sin 2\theta$ (d) $\frac{1}{2}mu^2 \cos 2\theta$

21. After one second, the velocity of a projectile makes an angle of 45° with the horizontal. After another one second, it is travelling horizontally. The magnitude of its initial velocity and angle of projection are ($g = 10\,\text{ms}^{-2}$)

(a) $14.62\,\text{ms}^{-1}$, $\tan^{-1}(2)$ (b) $22.36\,\text{ms}^{-1}$, $\tan^{-1}(2)$

(c) $14.62\,\text{ms}^{-1}$, 60° (d) $22.36\,\text{ms}^{-1}$, 60°

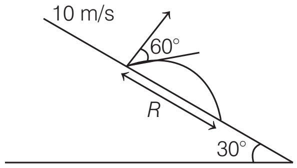
22. A body is projected up over a smooth inclined plane with a velocity v_0 from the point A as shown in the figure. The angle of inclination is 45° and top B of the plane is connected to a well of diameter 40 m. If the body just manages to cross the well, what is the value of v_0? Length of the inclined plane is $20\sqrt{2}$ m, and $g = 10\,\text{ms}^{-2}$.



(a) $20\,\text{ms}^{-1}$ (b) $20\sqrt{2}\,\text{ms}^{-1}$

(c) $40\,\text{ms}^{-1}$ (d) $40\sqrt{2}\,\text{ms}^{-1}$

23. A projectile is launched with a speed of 10 m/s at an angle 60° with the horizontal from a sloping surface of inclination 30°. The range R is.
(Take, $g = 10\,\text{m/s}^2$)

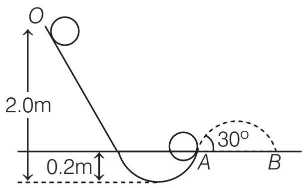


(a) 4.9 m (b) 13.3 m (c) 9.1 m (d) 12.6 m

24. A tennis ball (treated as hollow spherical shell) starting from O rolls down a hill. At point A, the ball becomes air borne leaving at an angle of 30° with the horizontal. The ball strikes the ground at B. What is the value of the distance AB?

(Moment of inertia of a spherical shell of mass m and radius R about its diameter $= \frac{2}{3} mR^2$)

[JEE Main 2013]



(a) 1.87 m (b) 2.08 m (c) 1.57 m (d) 1.77 m

25. A projectile is fired with a velocity v at right angle to the slope which is inclined at an angle θ with the horizontal. What is the range covered by the projectile?

(a) $\frac{2v^2}{g}\tan\theta$ (b) $\frac{v^2}{g}\tan\theta$

(c) $\frac{2v^2}{g}\sec\theta$ (d) $\frac{2v^2}{g}\tan\theta\sec\theta$

Numerical Value Questions

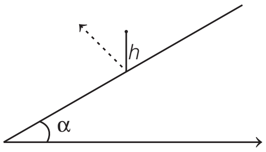
26. An aeroplane in a level flight at 144 kmh^{-1} is at an altitude of 1000 m. The value of distance (in m) from which the body should be released to hit the given target will be ……… .

27. A ball thrown by one player reaches the other in 2s. The maximum value of height (in m) attained by the ball above the point of projection will be (Take, $g = 10\,\text{ms}^{-2}$) …… .

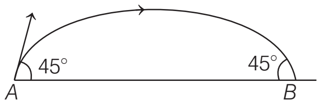
28. Two stones are projected with the same velocity but making different angles with the horizontal. Their ranges are equal. If angle of projection of one is 30° and its maximum height is y, then the maximum height of other will be ny, where value of n will be …… .

29. When the angle of projection is 75°, a ball falls 10 m shorter of the target. When the angle of projection is 45°, it falls 10 m ahead of the target. Both are projected from the same point with the same speed in the same direction, the distance of the target (in m) from the point of projection is ……… .

30. A marble starts falling from rest on a smooth inclined plane of inclination α. After covering a distance h, the ball rebounds off the plane. The distance from the impact point where the ball rebounds for the second time $nh\sin\alpha$. Here, the value of n is …… .



31. The projectile motion of a particle of mass 5g is shown in the figure.



The initial velocity of the particle is $5\sqrt{2}\,\text{ms}^{-1}$ and the air resistance is assumed to be negligible. The magnitude of the change in momentum between the points A and B is $x \times 10^{-2}$ kg-ms^{-1}. The value of x, to the nearest integer,is ……… . **[JEE Main 2021]**

Answers

Round I

1. (c)	**2.** (a)	**3.** (c)	**4.** (c)	**5.** (c)	**6.** (d)	**7.** (a)	**8.** (c)	**9.** (a)	**10.** (c)
11. (c)	**12.** (d)	**13.** (c)	**14.** (b)	**15.** (b)	**16.** (b)	**17.** (c)	**18.** (d)	**19.** (b)	**20.** (d)
21. (d)	**22.** (a)	**23.** (b)	**24.** (d)	**25.** (b)	**26.** (b)	**27.** (c)	**28.** (a)	**29.** (c)	**30.** (d)
31. (b)	**32.** (c)	**33.** (a)	**34.** (d)	**35.** (b)	**36.** (d)	**37.** (c)	**38.** (c)	**39.** (b)	**40.** (d)
41. (c)	**42.** (b)	**43.** (c)	**44.** (d)	**45.** (a)	**46.** (a)	**47.** (d)	**48.** (c)	**49.** (c)	**50.** (c)
51. (b)	**52.** (a)	**53.** (b)	**54.** (a)	**55.** (b)					

Round II

1. (c)	**2.** (a)	**3.** (d)	**4.** (d)	**5.** (b)	**6.** (d)	**7.** (d)	**8.** (b)	**9.** (c)	**10.** (a)
11. (b)	**12.** (c)	**13.** (b)	**14.** (d)	**15.** (c)	**16.** (d)	**17.** (b)	**18.** (b)	**19.** (b)	**20.** (c)
21. (b)	**22.** (b)	**23.** (b)	**24.** (b)	**25.** (a)	**26.** 571.43	**27.** 5	**28.** 3	**29.** 30	**30.** 8
31. 5									

Solutions

Round I

1. $v_y = \dfrac{dy}{dt} = 8 - 10\,t,\ v_x = \dfrac{dx}{dt} = 6$

At $t = 0$, $v_y = 8$ m/s and $v_x = 6$ m/s

$\therefore \quad v = \sqrt{v_x^2 + v_y^2} = 10$ m/s

2. $v_x = 8t - 2$

or $\quad \dfrac{dx}{dt} = 8t - 2$

or $\quad \int_{14}^{x} dx = \int_{2}^{t} (8t - 2)dt$

or $\quad x - 14 = [4t^2 - 2t]_2^t = 4t^2 - 2t - 12$

or $\quad x = 4t^2 - 2t + 2 \quad$ ...(i)

Further, $v_y = 2$

or $\quad \dfrac{dy}{dt} = 2$

$\therefore \quad \int_{4}^{y} dy = \int_{2}^{t} 2\,dt$

or $\quad y - 4 = [2t]_2^t = 2t - 4$

or $\quad y = 2t$

or $\quad t = \dfrac{y}{2} \quad$ ...(ii)

Substituting the value of t from Eq. (ii) in Eq. (i), we have

$$x = y^2 - y + 2$$

3. Since $y = x^2$, the motion is in two dimensions. Velocity at B is greater than at P. In the given motion of a particle, the law of conservation of energy is obeyed. Therefore, total energy at P = total energy at A. As vertical distance $AB > BP$, time of travel from A to B is greater than that from B to P.

4. Equating velocities along the vertical,

$$v_2 = v_1 \sin 30°$$

or $\quad \dfrac{v_2}{v_1} = \dfrac{1}{2}$

5. As, $v' \cos\beta = v \cos\theta$

($\because$ horizontal component of velocities are always equal.)

or $\quad v' = v \cos\theta \sec\beta$

6. As, $v \cos\beta = u \cos\alpha$

($\because$ horizontal component of velocities are always equal.)

$\therefore \quad v = \dfrac{u \cos\alpha}{\cos\beta}$

7. $v^2 = v_y^2 + v_x^2$

or $\quad 5\,g = (u_y - gt)^2 + u_x^2$

or $\quad 50 = (5\sqrt{3} - 10\,t)^2 + (5)^2$

$\therefore \quad (5\sqrt{3} - 10\,t) = \pm 5$

$$t_1 = \frac{5\sqrt{3} - 5}{10}$$

and $\quad t_2 = \dfrac{5\sqrt{3} + 5}{10}$

$\therefore \quad t_2 - t_1 = 1$ s

8. Instantaneous velocity of rising mass after t s will be

$$v_t = \sqrt{v_x^2 + v_y^2}$$

where, $v_x = v\cos\theta$ = Horizontal component of velocity

$v_y = v\sin\theta - gt$ = Vertical component of velocity

$$v_t = \sqrt{(v\cos\theta)^2 + (v\sin\theta - gt)^2}$$

$$v = \sqrt{v^2 + g^2 t^2 - (2v\sin\theta)\,gt}$$

9. $x = 36\,t$

$\therefore \quad v_x = \dfrac{dx}{dt} = 36$

$y = 48\,t - 4.9\,t^2$

$\therefore \quad v_y = \dfrac{dy}{dt} = 48 - 9.8\,t$

At $t = 0$, $\quad v_x = 36$ m/s

and $\quad v_y = 48$ m/s

So, angle of projection, $\theta = \tan^{-1}\left(\dfrac{v_y}{v_x}\right)$

$= \tan^{-1}\left(\dfrac{48}{36}\right)$

$= \tan^{-1}\left(\dfrac{4}{3}\right)$

or $\quad \theta = \sin^{-1}\left(\dfrac{4}{5}\right)$

10. Given, $g = 10$ m/s^2

Equation of trajectory of the projectile,

$$y = 2x - 9x^2 \quad \ldots\text{(i)}$$

In projectile motion, equation of trajectory is given by

$$y = x\tan\theta_0 - \frac{g\,x^2}{2v_0^2\cos^2\theta_0} \quad \ldots\text{(ii)}$$

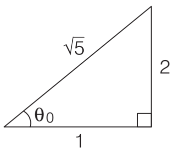
By comparison of Eqs. (i) and (ii), we get

$$\tan\theta_0 = 2 \quad \ldots\text{(iii)}$$

and $\quad \dfrac{g}{2v_0^2\cos^2\theta_0} = 9 \;$ or $\; v_0^2 = \dfrac{g}{9\times 2\cos^2\theta_0} \quad \ldots$(iv)

From Eq. (iii), we can get value of $\cos\theta$ and $\sin\theta$

$$\cos\theta_0 = \frac{1}{\sqrt{5}} \text{ and } \sin\theta_0 = \frac{2}{\sqrt{5}} \quad \ldots\text{(v)}$$



Using value of $\cos\theta_0$ from Eq. (v) to Eq. (iv), we get

$$v_0^2 = \frac{10\times(\sqrt{5})^2}{2\times(1)^2\times 9} = \frac{10\times 5}{2\times 9}$$

$\Rightarrow \quad v_0^2 = \dfrac{25}{9}$ or $v_0 = \dfrac{5}{3}$ m/s $\quad \ldots$(vi)

From Eq. (v), we get

$$\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

11. Here, $\quad 50 = \dfrac{u^2\sin 2\times 15^\circ}{g}$

or $\quad \dfrac{u^2}{g} = \dfrac{50}{\sin 30^\circ} = \dfrac{50}{1/2} = 100$

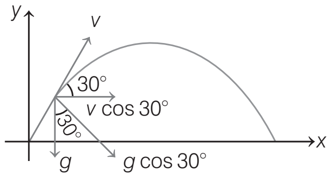
$R = \dfrac{u^2\sin 2\times 45^\circ}{g} = \dfrac{u^2}{g} = 100\,\text{m}$

12. At 45°, $v_x = v_y$

or $\quad u_x = u_y - gt$

$\therefore \quad t = \dfrac{u_y - u_x}{g} = \dfrac{40(\sin 60^\circ - \sin 30^\circ)}{9.8} = 1.5$ s

13. Let v be the velocity of particle when it makes 30° with horizontal, then



$v\cos 30^\circ = u\cos 60^\circ$

or $\quad v = \dfrac{u\cos 60^\circ}{\cos 30^\circ} = \dfrac{(20)\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \dfrac{20}{\sqrt{3}}\,\text{ms}^{-1}$

Now, $\quad g\cos 30^\circ = \dfrac{v^2}{R}$

or $\quad R = \dfrac{v^2}{g\cos 30^\circ} = \dfrac{\left(\frac{20}{\sqrt{3}}\right)^2}{(10)\frac{\sqrt{3}}{2}} = 15.4$ m

14. $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2\times 490}{9.8}} = \sqrt{100} = 10$ s

$x = vt = \left(60\times\dfrac{5}{18}\right)\text{ms}^{-1}\times 10\text{ s} = \dfrac{500}{3}$ m

15. $H = \dfrac{u^2\sin^2\theta}{2\,g}$ and $T = \dfrac{2u\sin\theta}{g}$

$\Rightarrow \quad T^2 = \dfrac{4u^2\sin^2\theta}{g^2}$

$\therefore \quad \dfrac{T^2}{H} = \dfrac{8}{g}$

$\Rightarrow \quad T = \sqrt{\dfrac{8\,H}{g}} = 2\sqrt{\dfrac{2\,H}{g}}$

16. $T = \dfrac{2\,u_y}{g}$

$\therefore \quad u_y = \dfrac{gT}{2} = 25$ m/s

Now, $\quad H = \dfrac{u_y^2}{2\,g} = \dfrac{(25)^2}{20} = 31.25$ m

Further, $\quad R = u_x T$

$\therefore \quad u_x = \dfrac{R}{T} = 40$ m/s

17. As x increases, kinetic energy initially decreases till the highest point of projectile, because vertical component of velocity is decreasing (horizontal velocity remains same).

At maximum height, kinetic energy becomes minimum but not zero, because of some horizontal velocity. Thus after that, kinetic energy (KE) again increases with horizontal displacement (x) as magnitude of y-component of velocity increases on falling down. This is depicted in option (c).

18. For same target, there are two angle of projection, if one is θ, then other is $\left(\frac{\pi}{2}-\theta\right)$, hence required angle

$$= \frac{\pi}{2} - \frac{5\pi}{36} = \frac{18\pi - 5\pi}{36}$$

$$= \frac{13\pi}{36} \text{ rad}$$

19. As, $E_k' = E_k \cos^2 30° = \frac{3E_k}{4}$

20. Given, $u_x = u\cos\theta_0 = 30 \times \frac{4}{5} = 24$ m/s

and $\quad u_y \sin\theta_0 = 30 \times \frac{3}{5} = 18$ m/s

After 1 s, u_x will remain as it is and u_y will decrease by 10 m/s or it will remain 8 m/s.

$\therefore \quad \tan\theta = \frac{v_y}{u_x} = \frac{8}{24} = \frac{1}{3}$

21. Since, range is given to be same, therefore the other angle is $(90° - 30°)$, *i.e.* 60°.

$$H = \frac{v^2 \sin^2 30°}{2g} = \frac{1}{4}\left[\frac{v^2}{2g}\right]$$

$$H' = \frac{v^2 \sin^2 60°}{2g} = \frac{3}{4}\left[\frac{v^2}{2g}\right]$$

$$\frac{H'}{H} = \frac{3}{4} \times \frac{4}{1} = 3 \quad \text{or} \quad H' = 3H$$

22. $R = \frac{v^2 \sin 2\theta}{g} = 200, T = \frac{2v\sin\theta}{g} = 5$

Dividing, we get

$$\frac{v^2 \times 2\sin\theta\cos\theta}{g} \times \frac{g}{2v\sin\theta} = \frac{200}{5} = 40$$

or $\quad v\cos\theta = 40 \text{ ms}^{-1}$

It may be noted here that the horizontal component of the velocity of projection remains the same during the flight of the projectile.

23. As the kinetic energy of a projectile at the highest, point is equal to half of the initial kinetic energy, so

$$(\text{KE})_H = \frac{1}{2}(\text{KE})_i$$

$$\frac{1}{2}mv^2\cos^2\theta = \frac{1}{2}\left(\frac{1}{2}mv^2\right) = \frac{1}{4}mv^2$$

or $\quad \cos^2\theta = \frac{1}{2}$ or $\cos\theta = \frac{1}{\sqrt{2}}$

or $\quad \theta = 45°$

24. $\because y = 10t - t^2$ (Given)

$$v_y \frac{d}{dt}(y) = \frac{d}{dt}(10t) - \frac{d}{dt}(t^2) = 10 - 2t$$

At maximum height, $v_y = 0$

$\therefore \quad 10 - 2t = 0$

or $\quad 2t = 10$ or $t = 5$ s

$\therefore \quad y = (10 \times 5 - 5 \times 5) = 25$ m

25. As, $\frac{v^2 \sin 2\theta}{g} = \frac{\sqrt{3}v^2}{2g}$

or $\quad \sin 2\theta = \frac{\sqrt{3}}{2}$

or $\quad 2\theta = 60°$

or $\quad \theta = 30°$

Let us cross check with the help of data for vertical range.

$$\frac{v^2 \sin^2\theta}{2g} = \frac{v^2}{8g} \quad \text{or} \quad \sin^2\theta = \frac{1}{4}$$

or $\quad \sin\theta = \frac{1}{2}$

or $\quad \theta = 30°$

26. Change in momentum is the product of force and time.

$$\Delta p = mg \times \frac{2u\sin\theta}{g} \qquad \left(\because F = \frac{\Delta p}{\Delta t}\right)$$

$$= 2mv\sin\theta = 2mv\sin 45° \qquad [\because \text{Here } u = v]$$

$$= \frac{2mv}{\sqrt{2}} = \sqrt{2}\, mv$$

27. Let v be the velocity of projection and θ the angle of projection.

Kinetic energy at highest point

$$= \frac{1}{2}mv^2\cos^2\theta \quad \text{or} \quad E_k\cos^2\theta$$

Potential energy at highest point

$$= E_k - E_k\cos^2\theta$$
$$= E_k(1 - \cos^2\theta)$$
$$= E_k\sin^2\theta$$

28. Using, $v^2 - u^2 = 2as$, we get

$$s = \frac{v^2}{2g}$$

Now, $\quad \frac{v^2 \sin 2\theta}{g} = \frac{v^2}{2g}$

or $\quad \sin 2\theta = \frac{1}{2}$

or $\quad \sin 2\theta = \sin 30° \qquad \left[\because \sin 30° = \frac{1}{2}\right]$

or $\quad \theta = 15°$

29. $u_y = u\sin 30°$

$$\Rightarrow u = \frac{u_y}{\sin 30°} = \frac{80°}{1/2} = 160 \text{ ms}^{-1}$$

$\therefore u_x = u\cos 30° = 160 \times \frac{\sqrt{3}}{2} = 80\sqrt{3}\ \text{ms}^{-1}$

$$T = \frac{2u_y}{g} = \frac{2 \times 80}{10} = 16\ \text{s}$$

At $\quad t = \frac{T}{4} = 4\ \text{s},\ v_x = 80\sqrt{3}\ \text{ms}^{-1}$

$$v_y = 80 - 10 \times 4 = 40\ \text{ms}^{-1}$$

$\therefore \quad v = \sqrt{(80\sqrt{3})^2 + (40)^2} = 144.2\ \text{ms}^{-1}$

$$\approx 140\ \text{ms}^{-1}$$

30. As, $R = \frac{v^2 \sin 2\theta}{g}$

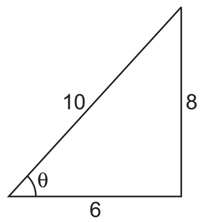
In the given problem, $v^2 \sin 2\theta = \text{constant}$

$$v^2 \sin 2\theta = \left(\frac{v}{2}\right)^2 \sin 30° = \frac{v^2}{8} \quad \left[\because v = \frac{v}{2} \text{ and } \theta = 15°\right]$$

or $\quad \sin 2\theta = \frac{1}{8}$ or $2\theta = \sin^{-1}\left(\frac{1}{8}\right)$

or $\quad \theta = \frac{1}{2}\sin^{-1}\left(\frac{1}{8}\right)$

31. Here, $\mathbf{v} = 6\hat{\mathbf{i}} + 8\hat{\mathbf{j}}\ \text{ms}^{-1}$



Comparing with $\mathbf{v} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$, we get

$$v_x = 6\ \text{ms}^{-1}$$

and $\quad v_y = 8\ \text{ms}^{-1}$

Also, $\quad v^2 = v_x^2 + v_y^2$

$$= 36 + 64 = 100$$

or $\quad v = 10\ \text{ms}^{-1}$

$$\sin\theta = \frac{8}{10} \text{ and } \cos\theta = \frac{6}{10}$$

$$R = \frac{v^2 \sin 2\theta}{g} = \frac{2v^2 \sin\theta\cos\theta}{g}$$

$$R = 2 \times 10 \times 10 \times \frac{8}{10} \times \frac{6}{10} \times \frac{1}{10}\ \text{m} = 9.6\ \text{m}$$

32. $h = v\sin\theta t - \frac{1}{2}gt^2$

or $\quad \frac{1}{2}gt^2 - v\sin\theta t + h = 0$

$$t_1 + t_2 = -\frac{-v\sin\theta}{\frac{1}{2}g}$$

or $\quad t_1 + t_2 = \frac{2v\sin\theta}{g} = T$

$$T = (1 + 3)\ \text{s} = 4\ \text{s}$$

33. Since, $v_1 \perp v_2$

$\therefore \quad v_1 \cdot v_2 = 0$

or $\quad (u_1\hat{\mathbf{i}} - gt\hat{\mathbf{j}}) \cdot (-u_2\hat{\mathbf{i}} - gt\hat{\mathbf{j}}) = 0$

$\therefore \quad g^2t^2 = u_1u_2$

or $\quad t = \frac{\sqrt{u_1u_2}}{g}$

34. Horizontal range of a projectile is given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

If $\theta = 45°$, then R is maximum and is equal to

$$R_{\max} = \frac{u^2}{g}$$

Given, $\quad R_{\max} = 100\ \text{m}$

$\therefore \quad 100 = \frac{u^2}{g} \quad$ ...(i)

When cricketer throws the ball vertically upward, then ball goes upto height H.

Using equation of motion,

$$v^2 = u^2 + 2as$$

$$(0)^2 = u^2 + 2(-g)H$$

or $\quad H = \frac{u^2}{2g} = \frac{1}{2}\left(\frac{u^2}{g}\right)$

$= \frac{1}{2} \times 100 \quad$ [using Eq. (i)]

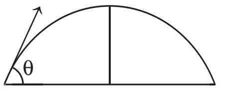
$= 50\ \text{m}$

35. Horizontal component $= u\cos\theta$

Vertical component $= u\sin\theta$

$$g = -10\ \text{ms}^{-2},\ u = 50\ \text{ms}^{-1},\ h = 5\ \text{m},\ t = 2\ \text{s}$$

$$h = u_y t + \frac{1}{2}gt^2$$



$\therefore \quad 5 = 50\sin\theta - \frac{1}{2} \times 10 \times 4$

or $\quad 5 = 50\sin\theta - 20$

or $\quad \sin\theta = \frac{25}{50} = \frac{1}{2}$

$\therefore \quad \theta = 30°$

36. Given, initial velocity $(u) = 40$ m/s

Height of the hall $(H) = 25$ m

Let the angle of projection of the ball be θ, when maximum height attained by it is 25 m.

Maximum height attained by the ball,

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$25 = \frac{(40)^2 \sin^2\theta}{2 \times 9.8}$$

or $\sin^2\theta = \frac{25 \times 2 \times 9.8}{1600} = 0.3063$

or $\sin\theta = 0.5534 = \sin 33.6°$

or $\theta = 33.6°$

$\therefore$ Horizontal range $(R) = \frac{u^2 \sin 2\theta}{g}$

$$= \frac{(40)^2 \sin 2 \times 33.6°}{9.8}$$

$$= \frac{1600 \times \sin 67.2°}{9.8}$$

$$= \frac{1600 \times 0.9219}{9.8}$$

$$= 150.5 \text{ m}$$

37. The velocity of the particle at any time t

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}\, t$$

The x-component is $v_x = v_{ox} + a_x t$

The y-component is $v_y = v_{oy} + a_x = (-0.5\, t)$ ms^{-1}

When the particle reaches its maximum x-coordinate, $v_x = 0$.

i.e. $3 - t = 0 \Rightarrow t = 3$ s

The y-component of the velocity of this time is

$$v_y = -0.5 \times 3 = -1.5 \text{ ms}^{-1}$$

38. As, $H_1 - H_2 = \frac{H_1 + H_2}{2}$

or $H_1 = 3H_2$

$\therefore$ $\frac{u^2 \sin^2\theta}{2g} = 3\left\{\frac{u^2 \sin^2(90° - \theta)}{2g}\right\}$

$\tan^2\theta = 3$

$\therefore$ $\tan\theta = \sqrt{3}$

or $\theta = 60°$

Therefore, the other angle is $(90° - \theta)$ or $30°$.

Hence, when angle of projection is 30°, then stone attains smaller height.

39. Given, $R = H$

$$\frac{u^2 \sin 2\alpha}{g} = \frac{u^2 \sin^2\alpha}{2g}$$

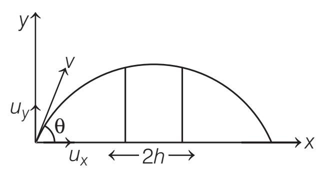
or $2\sin\alpha\cos\alpha = \frac{\sin^2\alpha}{2}$

or $\frac{\sin\alpha}{\cos\alpha} = 4$

or $\tan\alpha = 4$

$\therefore$ $\alpha = \tan^{-1}(4)$

40. Let Δt be the time interval, then



$2h = (u_x)(\Delta t)$

or $u_x = \frac{2h}{\Delta t}$ ...(i)

Further, $h = u_y t - \frac{1}{2} g t^2$

or $g t^2 - 2u_y t + 2h = 0$

$\therefore$ $t_1 = \frac{2u_y + \sqrt{4u_y^2 - 8gh}}{2g}$

and $t_2 = \frac{2u_y - \sqrt{4u_y^2 - 8gh}}{2g}$

$$\Delta t = t_1 - t_2 = \frac{\sqrt{4u_y^2 - 8gh}}{g}$$

or $u_y^2 = \frac{g^2(\Delta t)^2}{4} + 2gh$

Given, $u_x^2 + u_y^2 = (2\sqrt{gh})^2$

$\therefore$ $\frac{4h^2}{(\Delta t)^2} + \frac{g^2(\Delta t)^2}{4} + 2gh = 4gh$

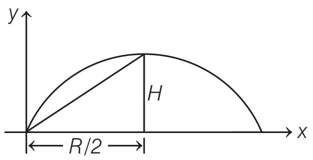
$$\frac{g^2}{4}(\Delta t)^4 - 2gh(\Delta t)^2 + 4h^2 = 0$$

$$(\Delta t)^2 = \frac{2gh \pm \sqrt{4g^2h^2 - 4g^2h^2}}{g^2/2} = \frac{4h}{g}$$

or $\Delta t = 2\sqrt{\frac{h}{g}}$

41. Average velocity $= \frac{\text{Displacement}}{\text{Time}}$

$$v_{av} = \frac{\sqrt{H^2 + \frac{R^2}{4}}}{T/2} \quad \text{...(i)}$$



Here, H = maximum height $= \frac{v^2 \sin^2\theta}{2g}$,

R = range $= \frac{v^2 \sin 2\theta}{g}$,

and T = time of light $= \frac{2v\sin\theta}{g}$

Substituting in Eq. (i), we get

$$v_{av} = \frac{v}{2}\sqrt{1 + 3\cos^2\theta}$$

42. According to question,

$$R_{max} = R_0 = \frac{u^2}{g} = 40$$

$\Rightarrow$ $u^2 = 40 \times g = 40 \times 10 = 400$

$\Rightarrow$ $u = 20$ m/s

When the car is moving : We will take ground as a frome of reference,

In ground frame; Range

$$R = \frac{2u_x u_y}{g}$$

$$\Rightarrow \quad R = \frac{2\,(20 + u\cos\theta)\, u\sin\theta}{g}$$

Range R will be maximum, if $\frac{dR}{d\theta} = 0$

$$\Rightarrow \quad \frac{2[20u\cos\theta + u^2(\cos^2\theta - \sin^2\theta)]}{g} = 0$$

$$\Rightarrow \quad 20\cos\theta + u(\cos^2\theta - \sin^2\theta) = 0$$

$$\Rightarrow \quad 20\cos\theta + (2\cos^2\theta - 1)u = 0$$

$$\Rightarrow \quad 20\cos\theta + 40\cos^2\theta - 20 = 0 \qquad [\because u = 20 \text{ m/s}]$$

$$\Rightarrow \quad 2\cos^2\theta + \cos\theta - 1 = 0$$

$$\Rightarrow \quad \cos\theta = -1, \frac{1}{2}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 60°$$

43. $\frac{R/2}{H} = \frac{\sqrt{3}\,H}{H} = \sqrt{3}$

or $$\frac{(v_0^2 \sin\theta\cos\theta)/g}{(v_0^2\sin^2\theta)/2\,g} = \sqrt{3}$$

$$2\cot\theta = \sqrt{3}$$

$$\tan\theta = \frac{2}{\sqrt{3}}$$

or $$\theta = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

44. Maximum height and time of flight depend on the vertical component of initial velocity.

$$H_1 = H_2 \Rightarrow u_{y_1} = u_{y_2}$$

Here $$T_1 = T_2$$

Range $$R = \frac{u^2 \sin 2\theta}{g} = \frac{2(u\sin\theta)\,(u\cos\theta)}{g}$$

$$= \frac{2u_x u_y}{g}$$

$$R_2 > R_1$$

$\therefore$ $$u_{x_2} > u_{x_1} \text{ or } u_2 > u_1$$

45. $t = \sqrt{\frac{2h}{g}}$

Distance from the foot of the tower

$$d = vt = v\sqrt{\frac{2h}{g}} = 250 \text{ m}$$

When velocity $= \frac{v}{2}$ and height of tower $= 4h$, then

distance $x = \frac{v}{2}\sqrt{\frac{2(4h)}{g}}$

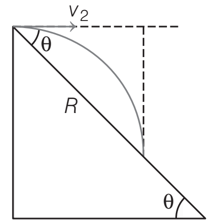
$$x = v\sqrt{\frac{2h}{g}} = 250 \text{ m}$$

46. Range of the projectile on an inclined plane (down the plane) is,

$$R = \frac{u^2}{g\cos^2\beta}[\sin(2\alpha + \beta) + \sin\beta]$$

Here, $$u = v_0,\ \alpha = 0 \text{ and } \beta = \theta$$

$\therefore$ $$R = \frac{2v_0^2 \sin\theta}{g\cos^2\theta}$$



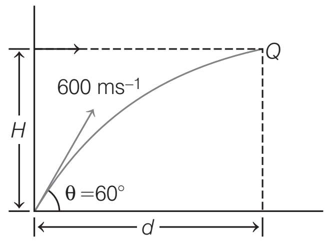
Now, $$x = R\cos\theta = \frac{2v_0^2\tan\theta}{g}$$

and $$y = -R\sin\theta = -\frac{2v_0^2\tan^2\theta}{g}$$

47. If it is being hit, then

$$d = v_0 t + \frac{1}{2}at^2 = (u\cos\theta)t$$

($\because$ acceleration in horizontal direction is zero)



or $$t = \frac{u\cos\theta - v_0}{a/2}$$

$\therefore$ $$t = \frac{600 \times \frac{1}{2} - 250}{10} = 5 \text{ s}$$

$$H = (u\sin\theta)t - \frac{1}{2} \times gt^2$$

$$= 600 \times \frac{\sqrt{3}}{2} \times 5 - \frac{1}{2} \times 10 \times 25$$

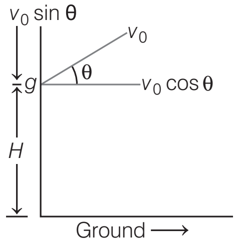
$$H = 2473 \text{ m}$$

48. Horizontal component of velocity of A is $10\cos 60°$ or 5 ms^{-1} which is equal to the velocity of B in horizontal direction. They will collide at C, if time of flight of the particles are equal or $t_A = t_B$

$$\frac{2u\sin\theta}{g} = \sqrt{\frac{2h}{g}} \qquad \left(\because h = \frac{1}{2}gt_B^2\right)$$

or $$h = \frac{2u^2\sin^2\theta}{g} = \frac{2(10)^2\left(\frac{\sqrt{3}}{2}\right)^2}{10} = 15 \text{ m}$$

49. From figure,



$$H = (-v_0 \sin\theta)t + \frac{1}{2}gt^2$$

$$v_x = v_0 \cos\theta$$

$$v_y^2 = (v_0 \sin\theta)^2 + 2gH$$

$$v = \sqrt{v_x^2 + v_y^2} \text{ at ground}$$

$$v = \sqrt{v_0^2 + 2gH}$$

It means speed is independent of angle of projection.

Also, $\frac{1}{2}gt^2 = H + t\, v_0 \sin\theta$

From this we can say that as θ increases, t increases.

50. Vertical component of velocity of A should be equal to vertical velocity of B.

or $v_1 \sin 30° = v_2$

or $\frac{v_1}{2} = v_2$

$\therefore$ $\frac{v_2}{v_1} = \frac{1}{2}$

51. Since, the projectile is released, therefore, its initial velocity is same as the velocity of the plane at the time of release.

Take the origin at the point of release.

Let x and $y(= -730 \text{ m})$ be the coordinates of the point on the ground where the projectile hits and let t be the time when it hits. Then,

$$y = -v_0\, t \cos\theta - \frac{1}{2}gt^2$$

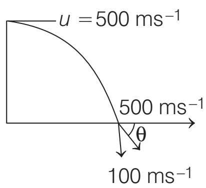
where, $\theta = 53.0°$

This equation gives $v_0 = -\dfrac{y + \frac{1}{2}gt^2}{t\cos\theta}$

$$= \frac{-730 + \frac{1}{2}(9.8)(5)^2}{5\cos 53°} = 202 \text{ ms}^{-1}$$

52. Horizontal component of velocity, $v_x = 500 \text{ ms}^{-1}$ and vertical component of velocity while striking the ground

$$v_y = 0 + 10 \times 10 = 100 \text{ ms}^{-1}$$



$\therefore$ Angle with which it strikes the ground,

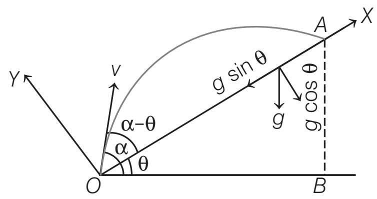
$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{100}{500}\right)$$

$$\theta = \tan^{-1}\left(\frac{1}{5}\right)$$

53. As, $v_x = v\cos(\alpha - \theta); v_y = v\sin(\alpha - \theta)$

$$a_x = -g\sin\theta; a_y = -g\cos\theta$$

If T is the time of flight, then



$$T = \frac{2v\sin(\alpha - \theta)}{g\cos\theta}$$

Again, $OB = v\cos\alpha \times T$

Now, $\cos\theta = \frac{OB}{OA}$

or $OA = \frac{OB}{\cos\theta}$

or $OA = \frac{v\cos\alpha \cdot T}{\cos\theta}$

or $OA = v\cos\alpha \times \frac{2v\sin(\alpha-\theta)}{g\cos\theta} \times \frac{1}{\cos\theta}$

or $OA = \frac{v^2}{g\cos^2\theta}[2\sin(\alpha-\theta)\cos\alpha]$

or $OA = \frac{v^2}{g\cos^2\theta}[\sin(2\alpha-\theta) + \sin(-\theta)]$

or $OA = \frac{v^2}{g\cos^2\theta}[\sin(2\alpha-\theta) - \sin\theta]$

Clearly, the range $R(= OA)$ will be maximum when $\sin(2\alpha - \theta)$ is maximum, *i.e.* 1. This would mean

$$2\alpha - \theta = \frac{\pi}{2} \quad \text{or} \quad \alpha = \frac{\theta}{2} + \frac{\pi}{4}$$

Maximum range up the inclined plane,

$$R_{\max} = \frac{v^2}{g\cos^2\theta}(1 - \sin\theta) = \frac{v^2(1-\sin\theta)}{g(1-\sin^2\theta)}$$

$$= \frac{v^2(1-\sin\theta)}{g(1-\sin\theta)(1+\sin\theta)} = \frac{v^2}{g(1+\sin\theta)}$$

54. From $h = \frac{1}{2}gt^2$,

We have $t_{OB} = \sqrt{\frac{2h_{OA}}{g}} = \sqrt{\frac{2 \times 1960}{9.8}} = 20 \text{ s}$

Horizontal distance, $AB = vt_{OB}$

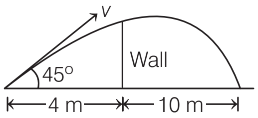
$$= \left(600 \times \frac{5}{18}\right)(20) = 3333.33 \text{ m} = 3.33 \text{ km}$$

55. $R = \dfrac{2 \times 30 \times 30 \sin 30° \cos 60°}{10 \cos^2 30°}$

$= 180 \times \dfrac{1}{2} \times \dfrac{1}{2} \times \dfrac{2 \times 2}{3}$ m $= 60$ m

Round II

1. As range $= 10 = \dfrac{u^2 \sin 2\theta}{g} \Rightarrow u^2 = 10\,g$



$\therefore \quad u = 10$ m/s $\quad$ (as $g = 10$ m/s^2)

$$y = x \tan \theta - \frac{1}{2} \frac{gx^2}{2v_0^2 \cos^2 \theta}$$

$$= 4 \tan 45° - \frac{1}{2} \frac{g \times 16}{2\, v_0^2 \cos^2 45°}$$

$$= 4 \times 1 - \frac{1}{2} \frac{10 \times 16}{2 \times 10 \times 10 \times \frac{1}{2}}$$

$$= 4 - 0.8$$

$$= 3.2 \approx 3.6 \text{ m}$$

2. According to law of conservation of linear momentum at the highest point,

$$mv \cos \theta = \frac{m}{2}(-v \cos \theta) + \frac{m}{2} v_1$$

or $\quad v_1 = 3\, v \cos \theta$

3. $R = \dfrac{u^2 \sin 2\theta}{g} = \dfrac{2\, u_x\, u_y}{g}$

$\therefore$ Range $\propto$ horizontal initial velocity component (u_x)

In path 4, range is maximum of football and has maximum horizontal velocity component in this path.

4. Time of flight, $T = \dfrac{2u_y}{g}$

$$u_y = \frac{gT}{2} = \frac{10 \times 6}{2} = 30 \text{ ms}^{-1}$$

Vertical velocity after 2s, $v_y = u_y - gT$

$= 30 - 10 \times 2$

$= 30 - 20 = 10$ ms^{-1}

According to question,

$$\tan 30° = \frac{v_y}{u_x} = \frac{10}{u_x}$$

or $\quad u_x = \dfrac{10}{\tan 30°} = 10\sqrt{3}$ ms^{-1}

$\therefore \quad u = \sqrt{u_x^2 + u_y^2} = \sqrt{(10\sqrt{3})^2 + (30)^2}$

$= 20\sqrt{3}$ ms^{-1}

5. Initial velocity, $\mathbf{v} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$ m/s

Magnitude of velocity, $\mathbf{v} = \sqrt{(1)^2 + (2^2)} = \sqrt{5}$ m/s

Equation of trajectory of projectile,

$$y = x \tan \theta - \frac{gx^2}{2\,v^2}(1 + \tan^2 \theta) \qquad \left(\tan \theta = \frac{y}{x} = \frac{2}{1} = 2\right)$$

$$\therefore \quad y = x \times 2 - \frac{10\,(x)^2}{2\,(\sqrt{5})^2}[1 + (2)^2]$$

$$= 2x - \frac{10\,(x^2)}{2 \times 5}(1 + 4) = 2x - 5x^2$$

6. Maximum speed with which the boy can throw stone is

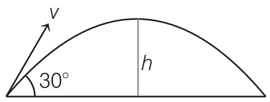
$$u = \sqrt{2gh} = \sqrt{2 \times 10 \times 10} = 10\sqrt{2} \text{ m/s}$$

Range is maximum when projectile is thrown at an angle of 45°, thus

$$R_{\text{max}} = \frac{u^2}{g} = \frac{(10\sqrt{2})^2}{10} = 20 \text{ m}$$

7. Angular momentum of the projectile as given by

$$L = mv_h r_\perp = m\,(v \cos \theta)\, h$$



where, h is the maximum height.

$$= m\,(v \cos \theta)\left(\frac{v^2 \sin^2 \theta}{2g}\right)$$

$$L = \frac{mv^3 \sin^2 \theta \cos \theta}{2g} = \frac{\sqrt{3}}{16} \frac{mv^3}{g}$$

8. $h = (u \sin \theta)t - \dfrac{1}{2} gt^2$

$d = (u \cos \theta)t \quad$ or $\quad t = \dfrac{d}{u \cos \theta}$

$$h = u \sin \theta \cdot \frac{d}{u \cos \theta} - \frac{1}{2} g \cdot \frac{d^2}{u^2 \cos^2 \theta}$$

$$u = \frac{d}{\cos \theta} \sqrt{\frac{g}{2(d \tan \theta - h)}}$$

9. For $\theta_A = 15°$ and $\theta_B = 75°$, $R_A = R_B$

$$u_A = u_B,$$

But $\quad \dfrac{h_A}{h_B} = \dfrac{u_A^2 \sin^2 \theta_A}{u_B^2 \sin^2 \theta_B} = \left(\dfrac{\sin 15°}{\sin 75°}\right)^2 < 1$

or $\quad h_A < h_B$

Again, $\quad \dfrac{T_A}{T_B} = \dfrac{u_A \sin \theta_A}{u_B \sin \theta_B} = \dfrac{\sin 15°}{\sin 75°} < 1$

or $\quad T_A < T_B$

10. Horizontal range is same when angle of projection is θ and $(90° - \theta)$.

$\therefore \quad R = \dfrac{u^2 \sin 2\theta}{g} = \dfrac{2\,u^2 \sin \theta \cos \theta}{g}$

When angle of projection is θ,

$$T_1 = \frac{2\,u \sin \theta}{g}$$

When angle of projection is $(90° - \theta)$,

$$T_2 = \frac{2u \sin(90° - \theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\therefore \quad T_1 T_2 = \frac{4u^2 \sin\theta\cos\theta}{g^2} = \left(\frac{2u^2\sin\theta\cos\theta}{g}\right)\left(\frac{2}{g}\right) = \frac{2R}{g}$$

$$\therefore \quad T_1 T_2 \propto R$$

and $\quad \dfrac{T_1}{T_2} = \dfrac{2u\sin\theta/g}{2u\cos\theta/g} = \tan\theta$

11. Height, $h = \dfrac{v_0^2 \sin^2\theta}{2g}$, $i.e. \, h \propto \sin^2\theta$

$$\therefore \quad \frac{h_1}{h_2} = \frac{\sin^2\theta_1}{\sin^2\theta_2} > 1$$

So, $\quad \sin^2\theta_1 > \sin^2\theta_2$

or $\quad \theta_1 > \theta_2$

Time of flight, $T = \dfrac{2v_0\sin\theta}{g}$

or $\quad T \propto \sin\theta$

$$\therefore \quad \frac{T_1}{T_2} = \frac{\sin\theta_1}{\sin\theta_2} > 1$$

or $\quad T_1 > T_2$

Horizontal range, $\quad R = \dfrac{u^2 \sin 2\theta}{g}$

or $\quad R \propto \sin 2\theta$

$$\therefore \quad \frac{R_1}{R_2} = \frac{\sin 2\theta_1}{\sin 2\theta_2} \le 1$$

or $\quad R_1 \le R_2$

Total energy of each particle will be equal to KE of each particle at the time of its projection.

12. $y = ax - bx^2$

For height of y to be maximum,

$$\frac{dy}{dx} = 0$$

or $\quad a - 2bx = 0$

or $\quad x = \dfrac{a}{2b}$

$$\therefore \quad y_{\max} = a\left(\frac{a}{2b}\right) - b\left(\frac{a}{2b}\right)^2 = \frac{a^2}{4b}$$

and $\quad \left(\dfrac{dy}{dx}\right)_{x=0} = a = \tan\theta$

where, θ = angle of projection

$\therefore \quad \theta = \tan^{-1}(a)$.

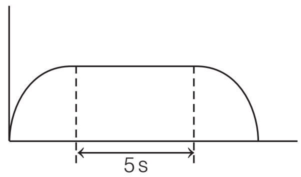
13. $H = \dfrac{u^2\sin^2\theta}{2g}$

$$dH = \frac{2u\sin^2\theta}{2g} du$$

$$\therefore \quad \frac{dH}{H} = \frac{2du}{u} = 2 \times \frac{1}{10}$$

$\therefore$ % increase in $H = \dfrac{dH}{H} \times 100 = \dfrac{2}{10} \times 100 = 20\%$

14. For 5 s, weight of the body is balanced by the given force. Hence, it will move in a straight line as shown below.



$$R = \frac{u^2 \sin 2\theta}{g} + (u\cos\theta)(5)$$

$$= \frac{(50)^2 \cdot \sin 60°}{10} + (50 \times \cos 30°)(5) = 250\sqrt{3} \text{ m}$$

15. If a particle is projected with velocity u at an angle θ with the horizontal, the velocity of the particle at the highest point is

$$v = u\cos\theta = 200\cos 60° = 100 \text{ ms}^{-1}$$

If m is the mass of the particle, then its initial momentum at highest point in the horizontal direction $= mv = m \times 100$. It means at the highest point, initially the particle has no momentum in vertically upward or downward direction. Therefore, after explosion, the final momentum of the particles going upward and downward must be zero.

Hence, the final momentum after explosion is the momentum of the third particle, in the horizontal direction. If the third particle moves with velocity v', then its momentum becomes $\dfrac{mv'}{3}$. According to law of conservation of linear momentum, we have

$$\frac{mv'}{3} = m \times 100 \text{ or } v' = 300 \text{ ms}^{-1}.$$

16. Given, $\theta_1 = \pi/3 = 30°$

Horizontal range is same if $\theta_1 + \theta_2 = 90°$

$$\therefore \quad \theta_2 = 90° - 30° = 60°$$

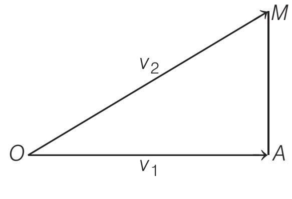
$$y_1 = \frac{u^2 \sin^2 30°}{2g}$$

and $\quad y_2 = \dfrac{u^2\sin^2 60°}{2g}$

$$\therefore \quad \frac{y_2}{y_1} = \frac{\sin^2 30°}{\sin^2 60°} = \left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)^2 = \frac{1}{3}$$

or $\quad y_2 = \dfrac{y_1}{3}$

17. Let v_1 be the velocity of the car and v_2 be the velocity of the parcel. The parcel is thrown at an angle θ from O, it reaches the man at M.



$$\therefore \qquad \cos\theta = \frac{v_1}{v_2} = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}} = \cos 45^\circ$$

So, $\theta = 45^\circ$

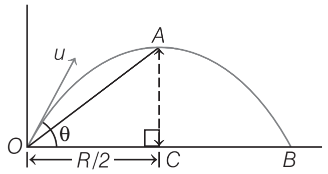
18. Maximum height, $H = \dfrac{v^2 \sin^2 60^\circ}{2g} = \dfrac{v^2}{2g} \times \dfrac{3}{4} = \dfrac{3v^2}{8g}$

Momentum of particle at highest point,

$$p = mv\cos 60^\circ = \frac{mv}{2}$$

Angular momentum $= pH = \dfrac{mv}{2} \times \dfrac{3v^2}{8g} = \dfrac{3mv^3}{16g}$

19. When projectile is at A, the given situation can be shown as



$$OC = \frac{R}{2} = \frac{1}{2}\frac{u^2}{g}\sin 2\theta$$

$$= \frac{1}{2} \times \frac{(20\sqrt{2})^2}{10}\sin 2 \times 45^\circ = 40 \text{ m}$$

$$AC = H = \frac{u^2 \sin^2\theta}{2g} = \frac{(20\sqrt{2})^2}{2 \times 10}\sin^2 45^\circ = 20 \text{ m}$$

$\therefore$ Displacement, $OA = \sqrt{OC^2 + CA^2} = \sqrt{40^2 + 20^2}$

Time of projectile from O to A

$$= \frac{1}{2}\left(\frac{2u\sin\theta}{g}\right) = \frac{u\sin\theta}{2g}$$

$$= \frac{(20\sqrt{2})\sin 45^\circ}{10} = 2 \text{ s}$$

$$\therefore \text{ Average velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{\sqrt{40^2 + 20^2}}{2}$$

$$= 10\sqrt{5} \text{ ms}^{-1}$$

20. Time of flight, $t = \dfrac{2u\sin\theta}{g}$

Horizontal range, $R = \dfrac{u^2 \sin 2\theta}{g}$

Change in angular momentum,

$$|d\mathbf{L}| = (\mathbf{L}_f - \mathbf{L}_i) \text{ about point of projection}$$

$$= (mu\sin\theta) \times \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{mu^3 \sin\theta \sin 2\theta}{g}$$

$$\text{Torque}|\tau| = \frac{\text{Change in angular momentum}}{\text{Time of flight}}$$

$$= \left|\frac{d\mathbf{L}}{T}\right| = \frac{mu^3 \sin\theta \cdot 2\sin\theta \cdot \cos\theta}{g} \times \frac{g}{2u\sin\theta}$$

or $\quad \tau = \dfrac{1}{2}mu^2 \sin 2\theta$

21. Time of flight of this particle, $T = 4$ s. If u is its initial speed and θ is the angle of projection, then

$$T = 4 = \frac{2u\sin\theta}{g}$$

or $\qquad u\sin\theta = 2g \qquad \ldots\text{(i)}$

After 1 s, the velocity vector particle makes an angle of 45° with horizontal, so $v_x = v_y$

i.e., $\qquad u\cos\theta = (u\sin\theta) - gt$

or $\qquad u\cos\theta = 2g - g \qquad (\because t = 1 \text{ s})$

or $\qquad u\cos\theta = g \qquad \ldots\text{(ii)}$

Squaring and adding Eqs. (i) and (ii), we have

$$u^2 = 5g^2 = 5(10)^2 = 500$$

or $\qquad u = \sqrt{500} = 22.36 \text{ ms}^{-1}$

Dividing Eq. (i) by Eq. (ii), we have

$$\tan\theta = 2 \text{ or } \theta = \tan^{-1}(2)$$

22. Let v be the velocity acquired by the body at B which will be moving and making an angle 45° with the horizontal direction. As the body just crosses the well, so $\dfrac{v^2}{g} = 40$

or $\qquad v^2 = 40g = 40 \times 10 = 400$

or $\qquad v = 20 \text{ ms}^{-1}$

Taking motion of the body from A to B along the inclined plane, we have

$$u = v_0,\ a = -g\sin 45^\circ = -\frac{10}{\sqrt{2}} \text{ ms}^{-2},$$

$$s = 20\sqrt{2} \text{ m},\ v = 20 \text{ ms}^{-1}$$

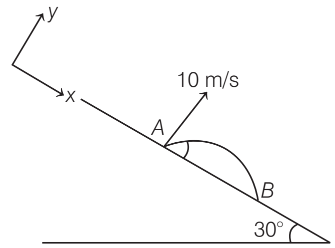
As $\qquad v^2 = u^2 + 2as$

$$\therefore \qquad 400 = v_0^2 + 2\left(-\frac{10}{\sqrt{2}}\right) \times 20\sqrt{2}$$

or $\qquad v_0^2 = 400 + 400 = 800$

or $\qquad v = 20\sqrt{2} \text{ ms}^{-1}$

23. At $B, S_y = 0$



$$\therefore \quad u_y t + \frac{1}{2}a_y t^2 = 0$$

or $\qquad t = -\dfrac{2u_y}{a_y} = \dfrac{-2(10)}{-10 \times \sqrt{3}/2} = \dfrac{4}{\sqrt{3}} \text{ s}$

Now, $\qquad AB = R = \dfrac{1}{2}a_x t^2$

$$= \frac{1}{2}\left(10 \times \frac{1}{2}\right)\left(\frac{16}{3}\right) = 13.33 \text{ m}$$

24. Total mechanical energy at $O = mgh = 2\,mg$

Now, total mechanical energy at A

$$= \frac{1}{2} mv^2 \left(1 + \frac{k^2}{R^2}\right) + 0.2\, mgh$$

Since, total mechanical energy is conserved

$$\Rightarrow \quad \frac{1}{2} mv^2 \left(1 + \frac{k^2}{R^2}\right) + 0.2\, mgh = 2\, mg$$

$$\Rightarrow \quad \frac{1}{2} v^2 \left(1 + \frac{\left(R\sqrt{\frac{2}{5}}\right)^2}{R^2}\right) + 0.2\, g = 2\, g$$

$$\left[\because k = \sqrt{\frac{2}{5}}\, R \text{ for spherical sphere}\right]$$

$$\Rightarrow \quad \frac{1}{2} v^2 \left[1 + \frac{2}{5}\right] + 2 = 20 \Rightarrow v^2 \left[\frac{7}{5}\right] = 36$$

$$\Rightarrow \quad v = \sqrt{\frac{36 \times 5}{7}} = \sqrt{\frac{180}{7}} = 5.1 \text{ m/s}$$

Now, range of projectile

$$\text{Range} = \frac{u^2 \sin 2\theta}{g} = \frac{(5.1)^2 \sin 60^\circ}{10} \approx 2.08 \text{ m}$$

25. We know that the range of projectile projected with velocity u, making an angle θ with the horizontal direction up the inclined plane, whose inclination with the horizontal direction is θ_0, is

$$R = \frac{u^2}{g \cos^2 \theta_0} [\sin(2\theta - \theta_0) - \sin \theta_0]$$

Here, $\quad u = v, \theta = (90^\circ + \theta), \theta_0 = \theta$

$$\therefore \quad R = \frac{v^2}{g \cos^2 \theta_0} \{\sin[2(90^\circ + \theta)] - \theta\} - \sin \theta\}$$

$$= \frac{v^2}{g \cos^2 \theta_0} [\sin(180^\circ + \theta) - \sin \theta]$$

$$= -\frac{v^2}{g \cos^2 \theta_0} 2 \sin \theta = -\frac{2v^2}{g} \tan \theta \sec \theta$$

$$\Rightarrow \quad R = \frac{2v^2}{g} \tan \theta \text{ (in magnitude)}$$

26. Horizontal velocity, $u = \dfrac{144 \times 1000}{60 \times 60} = 40 \text{ ms}^{-1}$

Time of flight, $T = \sqrt{2h/g} = \sqrt{2 \times 1000/9.8}$

Horizontal range $= 40\sqrt{2 \times 1000/9.8} = 571.43\,\text{m}$

27. Given, $T = \dfrac{2u \sin \theta}{g} = 2 \quad \text{or} \quad \dfrac{u \sin \theta}{g} = 1$

Maximum height,

$$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{g}{2}\left(\frac{u^2 \sin^2 \theta}{g^2}\right) = \frac{10}{2} \times 1^2 = 5 \text{ m}$$

28. As horizontal range of the two stones is same. So the sum of angles of projection of two stones must be 90°, $30^\circ + \theta = 90^\circ$ or $\theta = 60^\circ$.

According to question, $y = \dfrac{u^2 \sin^2 30^\circ}{2g} = \dfrac{u^2 (1/2)^2}{2g}$

And $\quad y' = \dfrac{u^2 \sin^2 60^\circ}{2g} = \dfrac{u^2 (3\sqrt{3}/2)^2}{2g}$

$\therefore \quad \dfrac{y'}{y} = 3 \text{ or } y' = 3y$

29. If range is R then, $\dfrac{u^2 \sin 2 \times 75^\circ}{g} = R - 10$ ...(i)

and $\quad \dfrac{u^2 \sin 2 \times 45^\circ}{g} = R + 10$

or $\quad \dfrac{u^2}{g} = R + 10$

From Eq. (i), $(R + 10) \sin 150^\circ = R - 10$

or $\quad (R + 10)\dfrac{1}{2} = R - 10$

or $\quad R = 30\,\text{m}$

30. Velocity before strike, $u = \sqrt{2gh}$

Component of acceleration along the inclined plane $= g \sin \alpha$ and the perpendicular component $= g \cos \alpha$

Using $\quad s = ut + \dfrac{1}{2} at^2,$

For vertical direction, we get

$$0 = v \cos \alpha t - \frac{1}{2} g \cos \alpha t^2$$

and for horizontal direction,

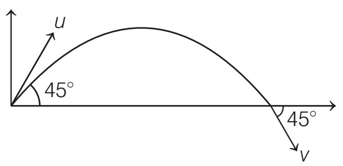
$$x = u \sin \alpha t + \frac{1}{2} g \sin \alpha t^2$$

$$= u \sin \alpha \frac{2u}{g} + \frac{1}{2} g \sin \alpha \left(\frac{2u}{g}\right)^2 \quad \left(\because t = \frac{2u}{g}\right)$$

$$= \frac{2u^2 \sin \alpha}{g} + \frac{2u^2 \sin \alpha}{g}$$

$$= \frac{4u^2 \sin \alpha}{g} = 4 \times \frac{2gh \times \sin \alpha}{g} = 8h \sin \alpha$$

31.



$|\mathbf{u}| = |\mathbf{v}|$

$\mathbf{u} = u \cos 45 \hat{\mathbf{i}} + u \sin 45 \hat{\mathbf{j}}$ ...(i)

$\mathbf{v} = v \cos 45 \hat{\mathbf{i}} - v \sin 45 \hat{\mathbf{j}}$ ...(ii)

$|\Delta \mathbf{p}| = |m(\mathbf{v} - \mathbf{u})|$

$\Delta \mathbf{p} = 2mu \sin 45^\circ$ [from Eqs. (i) and (ii)]

$$= 2 \times 5 \times 10^{-3} \times 5\sqrt{2} \times \frac{1}{\sqrt{2}}$$

$$= 50 \times 10^{-3} = 5 \times 10^{-2}$$

$\therefore \quad x = 5$