# Matrices Exercise 1 :
## Single Option Correct Type Questions

- This section contains **30 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct

**1.** If $A^5 = O$ such that $A^n \neq I$ for $1 \leq n \leq 4$, then $(I - A)^{-1}$ is equal to

(a) $A^4$ (b) $A^3$
(c) $I + A$ (d) None of these

**2.** Let $A = \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$ and suppose that $\det(A) = 2$, then $\det(B)$ equals, where $B = \begin{bmatrix} 4x & 2a & -p \\ 4y & 2b & -q \\ 4z & 2c & -r \end{bmatrix}$

(a) $-2$ (b) $-8$ (c) $-16$ (d) 8

**3.** If both $A - \frac{1}{2} I$ and $A + \frac{1}{2} I$ are orthogonal matrices, then

(a) $A$ is orthogonal
(b) $A$ is skew-symmetric matrix of even order
(c) $A^2 = \frac{3}{4} I$
(d) None of the above

**4.** Let $a = \lim_{x \to 1} \left( \frac{x}{\ln x} - \frac{1}{x \ln x} \right)$, $b = \lim_{x \to 0} \left( \frac{x^3 - 16x}{4x + x^2} \right)$, $c = \lim_{x \to 0} \left( \frac{\ln(1 + \sin x)}{x} \right)$ and $d = \lim_{x \to -1} \frac{(x+1)^3}{3[\sin(x+1) - (x+1)]}$, then $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

(a) idempotent (b) involutory
(c) non-singular (d) nilpotent

**5.** Let $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$. If $\theta$ is the angle between the two non-zero column vectors $X$ such that $AX = \lambda X$ for some scalar $\lambda$, then $\tan \theta$ is equal to

(a) 3 (b) 5
(c) 7 (d) 9

**6.** If a square matrix $A$ is involutory, then $A^{2n+1}$ is equal to

(a) $I$ (b) $A$
(c) $A^2$ (d) $(2n + 1) A$

**7.** If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then $\lim_{n \to \infty} \frac{A^n}{n}$ is (where $\theta \in R$)

(a) a zero matrix (b) an identity matrix
(c) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$

**8.** The rank of the matrix $\begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{bmatrix}$ is (where $a = -6$)

(a) 1 (b) 2 (c) 3 (d) 4

**9.** $A$ is an involutory matrix given by $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$, the inverse of $\frac{A}{2}$ will be

(a) $2A$ (b) $\frac{A^{-1}}{2}$ (c) $\frac{A}{2}$ (d) $A^2$

**10.** Let $A$ be a $n$th order square matrix and $B$ be its adjoint, then $|AB + kI_n|$, is (where $k$ is a scalar quantity)

(a) $(|A| + k)^{n-2}$ (b) $(|A| + k)^n$
(c) $(|A| + k)^{n-1}$ (d) $(|A| + k)^{n+1}$

**11.** If $A$ and $B$ are two square matrices such that $B = -A^{-1}BA$, then $(A + B)^2$ is equal to

(a) $O$ (b) $A^2 + B^2$
(c) $A^2 + 2AB + B^2$ (d) $A + B$

**12.** If matrix $A = [a_{ij}]_{3 \times 3}$, matrix $B = [b_{ij}]_{3 \times 3}$, where $a_{ij} + a_{ji} = 0$ and $b_{ij} - b_{ji} = 0$, then $A^4 \cdot B^3$ is

(a) skew-symmetric matrix (b) singular
(c) symmetric (d) zero matrix

**13.** Let $A$ be a $n \times n$ matrix such that $A^n = \alpha A$, where $\alpha$ is a real number different from 1 and $-1$. The matrix $A + I_n$ is

(a) singular (b) invertible
(c) scalar matrix (d) None of these

**14.** If $A = \begin{bmatrix} \frac{-1 + i\sqrt{3}}{2i} & \frac{-1 - i\sqrt{3}}{2i} \\ \frac{1 + i\sqrt{3}}{2i} & \frac{1 - i\sqrt{3}}{2i} \end{bmatrix}$, $i = \sqrt{-1}$ and $f(x) = x^2 + 2$, then $f(A)$ equals to

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\left( \frac{3 - i\sqrt{3}}{2} \right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
(c) $\left( \frac{5 - i\sqrt{3}}{2} \right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $(2 + i\sqrt{3}) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

**15.** The number of solutions of the matrix equation $X^2 = I$ other than $I$ is

(a) 0 (b) 1
(c) 2 (d) more than 2

**16.** If $A$ and $B$ are square matrices such that $A^{2006}=0$ and $AB=A+B$, then det $(B)$ equals to

(a) $-1$ (b) 0
(c) 1 (d) None of these

**17.** If $P=\begin{bmatrix}\cos\frac{\pi}{6} & \sin\frac{\pi}{6}\\ -\sin\frac{\pi}{6} & \cos\frac{\pi}{6}\end{bmatrix}$, $A=\begin{bmatrix}1&1\\0&1\end{bmatrix}$ and $Q=PAP^T$, then $P^TQ^{2007}P$ is equal to

(a) $\begin{bmatrix}1 & \sqrt{3}/2\\ 0 & 2007\end{bmatrix}$ (b) $\begin{bmatrix}1 & 2007\\ 0 & 1\end{bmatrix}$
(c) $\begin{bmatrix}\sqrt{3}/2 & 2007\\ 0 & 1\end{bmatrix}$ (d) $\begin{bmatrix}\sqrt{3}/2 & -1/2\\ 1 & 2007\end{bmatrix}$

**18.** There are two possible values of $A$ in the solution of the matrix equation $\begin{bmatrix}2A+1 & -5\\ -4 & A\end{bmatrix}^{-1}\begin{bmatrix}A-5 & B\\ 2A-2 & C\end{bmatrix}=\begin{bmatrix}14 & D\\ E & F\end{bmatrix}$, where $A, B, C, D, E, F$ are real numbers. The absolute value of the difference of these two solutions, is

(a) $\frac{8}{3}$ (b) $\frac{11}{3}$ (c) $\frac{1}{3}$ (d) $\frac{19}{3}$

**19.** If $f(\theta)=\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta & -\sin\theta\\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta\\ \sin\theta & -\cos\theta & 0\end{bmatrix}$, then $f\left(\frac{\pi}{7}\right)$ is

(a) symmetric (b) skew-symmetric
(c) singular (d) non-singular

**20.** In a square matrix $A$ of order 3 the elements $a_{ii}$'s are the sum of the roots of the equation $x^2-(a+b)x+ab=0$; $a_{i,\,i+1}$'s are the product of the roots, $a_{i,\,i-1}$'s are all unity and the rest of the elements are all zero.

The value of the det $(A)$ is equal to

(a) 0 (b) $(a+b)^3$
(c) $a^3-b^3$ (d) $(a^2+b^2)(a+b)$

**21.** If $A$ and $B$ are two non-singular matrices of the same order such that $B^r=I$ for some positive integer $r>1$. Then $A^{-1}B^{r-1}A-A^{-1}B^{-1}A$ is equal to

(a) $I$ (b) $2I$ (c) 0 (d) $-I$

**22.** If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{bmatrix}$, $B=\begin{bmatrix}1 & 0\\ -1 & 1\end{bmatrix}$, $C=ABA^T$, then $A^TC^nA$, $n\in I^+$ equals to

(a) $\begin{bmatrix}-n & 1\\ 1 & 0\end{bmatrix}$ (b) $\begin{bmatrix}1 & -n\\ 0 & 1\end{bmatrix}$ (c) $\begin{bmatrix}0 & 1\\ 1 & -n\end{bmatrix}$ (d) $\begin{bmatrix}1 & 0\\ -n & 1\end{bmatrix}$

**23.** If $A$ is a square matrix of order 3 such that $|A|=2$, then $|(\text{adj } A^{-1})^{-1}|$ is

(a) 1 (b) 2 (c) 4 (d) 8

**24.** If $A$ and $B$ are different matrices satisfying $A^3=B^3$ and $A^2B=B^2A$, then

(a) det $(A^2+B^2)$ must be zero
(b) det $(A-B)$ must be zero
(c) det $(A^2+B^2)$ as well as det $(A-B)$ must be zero
(d) atleast one of det $(A^2+B^2)$ or det $(A-B)$ must be zero

**25.** If $A$ is a skew-symmetric matrix of order 2 and $B$, $C$ are matrices $\begin{bmatrix}1 & 4\\ 2 & 9\end{bmatrix}$, $\begin{bmatrix}9 & -4\\ -2 & 1\end{bmatrix}$ respectively, then

$A^3(BC)+A^5(B^2C^2)+A^7(B^3C^3)+\ldots+A^{2n+1}(B^nC^n)$, is

(a) a symmetric matrix (b) a skew-symmetric matrix
(c) an identity matrix (d) None of these

**26.** If $A=\begin{bmatrix}a & b & c\\ x & y & z\\ p & q & r\end{bmatrix}$, $B=\begin{bmatrix}q & -b & y\\ -p & a & -x\\ r & -c & z\end{bmatrix}$ and if $A$ is invertible, then which of the following is not true?

(a) $|A|=|B|$
(b) $|A|=-|B|$
(c) $|\text{adj } A|=|\text{adj } B|$
(d) $A$ is invertible $\Leftrightarrow B$ is invertible

**27.** Let three matrices $A=\begin{bmatrix}2 & 1\\ 4 & 1\end{bmatrix}$, $B=\begin{bmatrix}3 & 4\\ 2 & 3\end{bmatrix}$ and $C=\begin{bmatrix}3 & -4\\ -2 & 3\end{bmatrix}$, then tr $(A)$ + tr $\left(\frac{ABC}{2}\right)$ + tr $\left(\frac{A(BC)^2}{4}\right)$ + tr $\left(\frac{A(BC)^3}{8}\right)+\ldots+\infty$ equals to

(a) 4 (b) 9 (c) 12 (d) 6

**28.** If $A$ is non-singular and $(A-2I)(A-4I)=O$, then $\frac{1}{6}A+\frac{4}{3}A^{-1}$ is equal to

(a) $O$ (b) $I$
(c) $2I$ (d) $6I$

**29.** If $A=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & a & 1\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}1/2 & -1/2 & 1/2\\ -4 & 3 & b\\ 5/2 & -3/2 & 1/2\end{bmatrix}$, then

(a) $a=1, b=-1$ (b) $a=2, b=-\frac{1}{2}$
(c) $a=-1, b=1$ (d) $a=\frac{1}{2}, b=\frac{1}{2}$

**30.** Given the matrix $A=\begin{bmatrix}x & 3 & 2\\ 1 & y & 4\\ 2 & 2 & z\end{bmatrix}$. If $xyz=60$ and $8x+4y+3z=20$, then $A$ (adj $A$) is equal to

(a) $64I$ (b) $88I$ (c) $68I$ (d) $34I$

# Matrices Exercise 2

## More than One Correct Option Type Questions

- This section contains **15 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **MORE THAN ONE** may be correct.

**31.** If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, then

(a) $A^3 = 9A$ (b) $A^3 = 27A$
(c) $A + A = A^2$ (d) $A^{-1}$ does not exist

**32.** A square matrix $A$ with elements from the set of real numbers is said to be orthogonal if $A' = A^{-1}$. If $A$ is an orthogonal matrix, then

(a) $A'$ is orthogonal (b) $A^{-1}$ is orthogonal
(c) adj $A = A'$ (d) $|A^{-1}| = 1$

**33.** Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then

(a) $A^2 - 4A - 5I_3 = O$ (b) $A^{-1} = \frac{1}{5}(A - 4I_3)$
(c) $A^3$ is not invertible (d) $A^2$ is invertible

**34.** $D$ is a $3 \times 3$ diagonal matrix. Which of the following statements are not true?

(a) $D^T = D$
(b) $AD = DA$ for every matrix $A$ of order $3 \times 3$
(c) $D^{-1}$ if exists is a scalar matrix
(d) None of the above

**35.** The rank of the matrix $\begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{bmatrix}$, is

(a) 2, if $a = -6$ (b) 2, if $a = 1$
(c) 1, if $a = 2$ (d) 1, if $a = -6$

**36.** If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then

(a) adj (adj $A$) $= A$ (b) $|$ adj (adj $(A)$) $| = 1$
(c) $|$ adj $(A)$ $| = 1$ (d) None of these

**37.** If $B$ is an idempotent matrix and $A = I - B$, then

(a) $A^2 = A$ (b) $A^2 = I$
(c) $AB = O$ (d) $BA = O$

**38.** If $A$ is a non-singular matrix, then

(a) $A^{-1}$ is symmetric if $A$ is symmetric
(b) $A^{-1}$ is skew-symmetric if $A$ is symmetric
(c) $|A^{-1}| = |A|$
(d) $|A^{-1}| = |A|^{-1}$

**39.** Let $A$ and $B$ are two matrices such that $AB = BA$, then for every $n \in N$

(a) $A^n B = BA^n$
(b) $(AB)^n = A^n B^n$
(c) $(A + B)^n = {}^nC_0 A^n + {}^nC_1 A^{n-1} B + \ldots + {}^nC_n B^n$
(d) $A^{2n} - B^{2n} = (A^n - B^n)(A^n + B^n)$

**40.** If $A$ and $B$ are $3 \times 3$ matrices and $|A| \neq 0$, which of the following are true?

(a) $|AB| = 0 \Rightarrow |B| = 0$
(b) $|AB| = 0 \Rightarrow B = 0$
(c) $|A^{-1}| = |A|^{-1}$
(d) $|A + A| = 2|A|$

**41.** If $A$ is a matrix of order $m \times m$ such that $A^2 + A + 2I = O$, then

(a) $A$ is non-singular (b) $A$ is symmetric
(c) $|A| \neq 0$ (d) $A^{-1} = -\frac{1}{2}(A + I)$

**42.** If $A^2 - 3A + 2I = 0$, then $A$ is equal to

(a) $I$ (b) $2I$
(c) $\begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix}$

**43.** If $A$ and $B$ are two matrices such that their product $AB$ is a null matrix, then

(a) det $A \neq 0 \Rightarrow B$ must be a null matrix
(b) det $B \neq 0 \Rightarrow A$ must be a null matrix
(c) atleast one of the two matrices must be singular
(d) if neither det $A$ nor det $B$ is zero, then the given statement is not possible

**44.** If $D_1$ and $D_2$ are two $3 \times 3$ diagonal matrices where none of the diagonal elements is zero, then

(a) $D_1 D_2$ is a diagonal matrix
(b) $D_1 D_2 = D_2 D_1$
(c) $D_1^2 + D_2^2$ is a diagonal matrix
(d) None of the above

**45.** Let, $C_k = {}^nC_k$ for $0 \leq k \leq n$ and $A_k = \begin{bmatrix} C_{k-1}^2 & 0 \\ 0 & C_k^2 \end{bmatrix}$ for $k \geq 1$ and

$A_1 + A_2 + A_3 + \ldots + A_n = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}$, then

(a) $k_1 = k_2$ (b) $k_1 + k_2 = 2$
(c) $k_1 = {}^{2n}C_n - 1$ (d) $k_2 = {}^{2n}C_{n+1}$

# Matrices Exercise 3 :
## Passage Based Questions

- This section contains **6 passages**. Based upon each of the passage **3 multiple choice questions** have to be answered. Each of these questions has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

### Passage I (Q. Nos. 46 to 48)

Suppose $A$ and $B$ be two non-singular matrices such that $AB = BA^m$, $B^n = I$ and $A^p = I$, where $I$ is an identity matrix.

**46.** If $m = 2$ and $n = 5$, then $p$ equals to

(a) 30 (b) 31
(c) 33 (d) 81

**47.** The relation between $m, n$ and $p$, is

(a) $p = mn^2$ (b) $p = m^n - 1$
(c) $p = n^m - 1$ (d) $p = m^{n-1}$

**48.** Which of the following ordered triplet $(m, n, p)$ is false?

(a) (3, 4, 80) (b) (6, 3, 215)
(c) (8, 3, 510) (d) (2, 8, 255)

### Passage II (Q. Nos. 49 to 51)

Let $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ is an orthogonal matrix and $abc = \lambda (< 0)$.

**49.** The value of $a^2b^2 + b^2c^2 + c^2a^2$, is

(a) $2\lambda$ (b) $-2\lambda$
(c) $\lambda^2$ (d) $-\lambda$

**50.** The value of $a^3 + b^3 + c^3$, is

(a) $\lambda$ (b) $2\lambda$
(c) $3\lambda$ (d) None of these

**51.** The equation whose roots are $a, b, c$, is

(a) $x^3 - 2x^2 + \lambda = 0$ (b) $x^3 - \lambda x^2 + \lambda x + \lambda = 0$
(c) $x^3 - 2x^2 + 2\lambda x + \lambda = 0$ (d) $x^3 \pm x^2 - \lambda = 0$

### Passage III (Q. Nos. 52 to 53)

Let $A = [a_{ij}]_{3 \times 3}$. If tr is arithmetic mean of elements of $r$th row and $a_{ij} + a_{jk} + a_{ki} = 0$ holds for all $1 \le i, j, k \le 3$.

**52.** $\sum_{1 \le i} \sum_{j \le 3} a_{ij}$ is not equal to

(a) $t_1 + t_2 + t_3$ (b) zero
(c) $(\det(A))^2$ (d) $t_1 t_2 t_3$

**53.** Matrix $A$ is

(a) non-singular
(b) symmetric
(c) skew-symmetric
(d) neither symmetric nor skew-symmetric

### Passage IV (Q. Nos. 54 to 56)

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ be a square matrix and $C_1, C_2, C_3$ be three column matrices satisfying $AC_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AC_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ and $AC_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ of matrix $B$. If the matrix $C = \frac{1}{3}(A \cdot B)$.

**54.** The value of $\det(B^{-1})$, is

(a) 2 (b) $\frac{1}{2}$ (c) 3 (d) $\frac{1}{3}$

**55.** The ratio of the trace of the matrix $B$ to the matrix $C$, is

(a) $-\frac{9}{5}$ (b) $-\frac{5}{9}$ (c) $-\frac{2}{3}$ (d) $-\frac{3}{2}$

**56.** The value of $\sin^{-1}(\det A) + \tan^{-1}(9 \det C)$, is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) $\pi$

### Passage V (Q. Nos. 57 to 59)

If $A$ is symmetric and $B$ skew-symmetric matrix and $A + B$ is non-singular and $C = (A + B)^{-1}(A - B)$.

**57.** $C^T(A + B)C$ equals to

(a) $A + B$ (b) $A - B$
(c) $A$ (d) $B$

**58.** $C^T(A - B)C$ equals to

(a) $A + B$ (b) $A - B$ (c) $A$ (d) $B$

**59.** $C^T AC$ equals to

(a) $A + B$ (b) $A - B$
(c) $A$ (d) $B$

### Passage VI (Q. Nos. 60 to 61)

Let $A$ be a square matrix of order 3 satisfies the matrix equation $A^3 - 6A^2 + 7A - 8I = O$ and $B = A - 2I$. Also, $\det A = 8$.

**60.** The value of $\det(\text{adj}(I - 2A^{-1}))$ is equal to

(a) $\frac{25}{16}$ (b) $\frac{125}{64}$
(c) $\frac{64}{125}$ (d) $\frac{16}{25}$

**61.** If $\text{adj}\left(\left(\frac{B}{2}\right)^{-1}\right) = \left(\frac{p}{q}\right)B$, where $p, q \in N$, the least value of $(p + q)$ is equal to

(a) 7 (b) 9 (c) 29 (d) 41

# Matrices Exercise 4 :
## Single Integer Answer Type Questions

- This section contains **10 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

**62.** Let $A, B, C, D$ be (not necessarily) real matrices such that $A^T = BCD; B^T = CDA; C^T = DAB$ and $D^T = ABC$ for the matrix $S = ABCD$, the least value of $k$ such that $S^k = S$ is

**63.** If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ and a function $f(x)$ is defined as $f(x) = \det(A^T A^{-1})$ and if $f(f(f(f \dots f(x))))$ is $(n \geq 2)\ \lambda$, the value of $2^\lambda$ is

**64.** If the matrix $A = \begin{bmatrix} \lambda_1^2 & \lambda_1\lambda_2 & \lambda_1\lambda_3 \\ \lambda_2\lambda_1 & \lambda_2^2 & \lambda_2\lambda_3 \\ \lambda_3\lambda_1 & \lambda_3\lambda_2 & \lambda_3^2 \end{bmatrix}$ is idempotent, the value of $\lambda_1^2 + \lambda_2^2 + \lambda_3^2$ is

**65.** Let $A$ be a $3 \times 3$ matrix given by $A = [a_{ij}]$. If for every column vector $X$, $X^T AX = O$ and $a_{23} = -1008$, the sum of the digits of $a_{32}$ is

**66.** Let $X$ be the solution set of the equation $A^x = I$, where $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ and $I$ is the corresponding unit matrix and $x \subseteq N$, the minimum value of $\Sigma(\cos^x \theta + \sin^x \theta)$, $\theta \in R - \left\{\frac{n\pi}{2}, n \in I\right\}$ is

**67.** If $A$ is an idempotent matrix and $I$ is an identity matrix of the same order, then the value of $n$, such that $(A + I)^n = I + 127\ A$ is

**68.** Suppose $a, b, c \in R$ and $abc = 1$, if $A = \begin{bmatrix} 3a & b & c \\ b & 3c & a \\ c & a & 3b \end{bmatrix}$ is such that $A^T A = 4^{1/3}\ I$ and $|A| > 0$, the value of $a^3 + b^3 + c^3$ is

**69.** If $A = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$ and $(A^8 + A^6 + A^4 + A^2 + I)\ V = \begin{bmatrix} 0 \\ 11 \end{bmatrix}$, where $V$ is a vertical vector and $I$ is the $2 \times 2$ identity matrix and if $\lambda$ is sum of all elements of vertical vector $V$, the value of $11\lambda$ is

**70.** Let the matrix $A$ and $B$ be defined as $A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 7 & 3 \end{bmatrix}$, then the absolute value of $\det(2A^9\ B^{-1})$ is

**71.** Let $A = \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix}$ and $(A + I)^{70} - 70A = \begin{bmatrix} a-1 & b-1 \\ c-1 & d-1 \end{bmatrix}$, the value of $a + b + c + d$ is

# Matrices Exercise 5 :
## Matching Type Questions

- This section contains **4 questions**. Question 72 has four statements (A, B, C and D) given in **Column I** and four statements (p, q, r and s) in **Column II** and questions 73 to 75 have four statements (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II**.

**72.** Suppose $a, b, c$ are three distinct real numbers and $f(x)$ is a real quadratic polynomial such that

$$\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 + 3a \\ 3b^2 + 3b \\ 3c^2 + 3c \end{bmatrix}.$$

| | Column I | | Column II |
|---|---|---|---|
| (A) | $x$-coordinate(s) of the point of intersection of $y = f(x)$ with the $X$-axis is | (p) | –2 |
| (B) | Area (in sq units) bounded by $y = \frac{3}{2} f(x)$ and the $X$-axis is | (q) | 1 |
| (C) | Maximum value of $f(x)$ is | (r) | 2 |
| (D) | Length (in unit) of the intercept made by $y = f(x)$ on the $X$-axis is | (s) | 4 |

**73.** If $A$ is non-singular matrix of order $n \times n$,

| | Column I | | Column II |
|---|---|---|---|
| (A) | adj $(A^{-1})$ is | (p) | $A\,(\det A)^{n-2}$ |
| (B) | det (adj $(A^{-1})$) is | (q) | $(\det A)^{n-1}$ (adj $A$) |
| (C) | adj (adj $A$) is | (r) | $\dfrac{\text{adj (adj } A)}{(\det A)^{n-1}}$ |
| (D) | adj ($A$ det ($A$)) is | (s) | $(\det A)^{1-n}$ |
| | | (t) | $\dfrac{A}{(\det A)}$ |

**74.**

| | Column I | | Column II |
|---|---|---|---|
| (A) | If $A$ is a diagonal matrix of order $3 \times 3$ is commutative with every square matrix of order $3 \times 3$ under multiplication and tr $(A) = 12$, then $\|A\|$ is divisible by | (p) | 3 |
| | | (q) | 4 |
| (B) | Let $a, b, c \in R^+$ and the system of equations $(1-a)x + y + z = 0$, $x + (1-b)y + z = 0$, $x + y + (1-c)z = 0$ has infinitely many solutions. If $\lambda$ be the minimum value of $a\,b\,c$, then $\lambda$ is divisible by | (r) | 6 |
| (C) | Let $A = [a_{ij}]_{3\times 3}$ be a matrix whose elements are distinct integers from 1, 2, 3, ..., 9. The matrix is formed so that the sum of the numbers is every row, column and each diagonal is a multiple of 9. If number of all such possible matrices is $\lambda$, then $\lambda$ is divisible by | (s) | 8 |
| (D) | If the equations $x + y = 1$, $(c+2)x + (c+4)y = 6$, $(c+2)^2 x + (c+4)^2 y = 36$ are consistent and $c_1, c_2$ $(c_1 > c_2)$ are two values of $c$, then $c_1\,c_2$ is divisible by | (t) | 9 |

**75.**

| | Column I | | Column II |
|---|---|---|---|
| (A) | If $C$ is skew-symmetric matrix of order $n$ and $X$ is $n \times 1$ column matrix, then $X^T CX$ is | (p) | invertible |
| (B) | If $A$ is skew - symmetric, then $I - A$ is, where $I$ is an identity matrix of order $A$. | (q) | singular |
| (C) | If $S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $A = \begin{bmatrix} b+c & c-a & b-a \\ c-b & c+a & a-b \\ b-c & a-c & a+b \end{bmatrix}$ $(a, b, c \neq 0)$, then $SAS^{-1}$ is | (r) | symmetric |
| (D) | If $A, B, C$ are the angles of a triangle, then the matrix $A = \begin{bmatrix} \sin 2A & \sin C & \sin B \\ \sin C & \sin 2B & \sin A \\ \sin B & \sin A & \sin 2C \end{bmatrix}$ is | (s) | non-singular |
| | | (t) | non-invertible |

# Matrices Exercise 6 :
## Statement I and II Type Questions

- **Directions** (Q. Nos. 76 to 85) are Assertion-Reason type questions. Each of these questions contains two statements:
**Statement-1** (Assertion) and **Statement-2** (Reason) Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice as given below.
  (a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
  (b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
  (c) Statement1 is true, Statement-2 is false
  (d) Statement-1 is false, Statement-2 is true

**76.** **Statement-1** If matrix $A = [a_{ij}]_{3\times 3}$, $B = [b_{ij}]_{3\times 3}$, where $a_{ij} + a_{ji} = 0$ and $b_{ij} - b_{ji} = 0$, then $A^4 B^5$ is non-singular matrix.
**Statement-2** If $A$ is non-singular matrix, then $|A| \neq 0$.

**77.** **Statement-1** If $A$ and $B$ are two square matrices of order $n \times n$ which satisfy $AB = A$ and $BA = B$, then $(A+B)^7 = 2^6\,(A+B)$.
**Statement-2** $A$ and $B$ are unit matrices.

**78.** **Statement-1** For a singular matrix $A$, if $AB = AC \Rightarrow B = C$
**Statement-2** If $|A| = 0$, then $A^{-1}$ does not exist.

**79.** **Statement-1** If $A$ is skew-symmetric matrix of order 3, then its determinant should be zero.
**Statement-2** If $A$ is square matrix, $\det(A) = \det(A') = \det(-A')$.

**80.** Let $A$ be a skew-symmetric matrix, $B = (I - A)(I + A)^{-1}$ and $X$ and $Y$ be column vectors conformable for multiplication with $B$.
**Statement-1** $(BX)^T\,(BY) = X^T Y$
**Statement-2** If $A$ is skew-symmetric, then $(I + A)$ is non-singular.

**81.** **Statement-1** Let a $2\times 2$ matrix $A$ has determinant 2. If $B = 9A^2$, the determinant of $B^T$ is equal to 36.

**Statement-2** If $A$, $B$ and $C$ are three square matrices such that $C = AB$, then $|C| = |A||B|$.

**82.** **Statement-1** If $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$, then

$A^3 + A^2 + A = I$.

**Statement-2** If $\det(A - \lambda I) = C_0\lambda^3 + C_1\lambda^2 + C_2\lambda + C_3 = 0$, then $C_0A^3 + C_1A^2 + C_3A + C_3I = O$.

**83.** **Statement-1** $A = [a_{ij}]$ be a matrix of order $3\times 3$, where $a_{ij} = \dfrac{i-j}{i+2j}$ cannot be expressed as a sum of symmetric and skew-symmetric matrix.

**Statement-2** Matrix $A = [a_{ij}]_{n\times n}$, $a_{ij} = \dfrac{i-j}{i+2j}$ is neither symmetric nor skew-symmetric.

**84.** **Statement-1** If $A, B, C$ are matrices such that $|A_{3\times 3}| = 3$, $|B_{3\times 3}| = -1$ and $|C_{2\times 2}| = 2$, $|2ABC| = -12$.

**Statement-2** For matrices $A, B, C$ of the same order $|ABC| = |A||B||C|$.

**85.** **Statement-1** The determinant of a matrix $A = [a_{ij}]_{n\times n}$, where $a_{ij} + a_{ji} = 0$ for all $i$ and $j$ is zero.

**Statement-2** The determinant of a skew-symmetric matrix of odd order is zero.

# Matrices Exercise 7 :
## Subjective Type Questions

- In this section, there are **12 subjective** questions.

**86.** If $S$ is a real skew-symmetric matrix, the show that $I - S$ is non-singular and matrix $A = (I+S)(I-S)^{-1} = (I-S)^{-1}(I+S)$ is orthogonal.

**87.** If $M$ is a $3\times 3$ matrix, where $\det M = I$ and $MM^T = I$, where $I$ is an identity matrix, prove that $\det(M - I) = 0$.

**88.** If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, $B = \begin{bmatrix} \cos 2\beta & \sin 2\beta \\ \sin 2\beta & -\cos 2\beta \end{bmatrix}$, where $0 < \beta < \dfrac{\pi}{2}$, then prove that $BAB = A^{-1}$. Also, find the least value of $\alpha$ for which $BA^4B = A^{-1}$.

**89.** Find the product of two matrices

$A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$

Show that, $AB$ is the zero matrix if $\theta$ and $\phi$ differ by an odd multiple of $\dfrac{\pi}{2}$.

**90.** Show that the matrix $\begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$ is orthogonal,

if $l_1^2 + m_1^2 + n_1^2 = \Sigma l_1^2 = 1 = \Sigma l_2^2 = \Sigma l_3^2$ and $l_1l_2 + m_1m_2 + n_1n_2 = \Sigma l_1l_2 = 0 = \Sigma\, l_2l_3 = \Sigma l_3l_1$.

**91.** A finance company has offices located in every division, every district and every taluka in a certain state in India. Assume that there are five divisions, thirty districts and 200 talukas in the state. Each office has one head clerk, one cashier, one clerk and one peon. A divisional office has, in addition, one office superintendent, two clerks, one typist and one poen. A district office, has in addition, one clerk and one peon. The basic monthly salaries are as follows:

Office superintendent ₹ 500, Head clerk ₹ 200, cashier ₹ 175, clerks and typist ₹ 150 and peon ₹ 100. Using matrix notation find

(i) the total number of posts of each kind in all the offices taken together,

(ii) the total basic monthly salary bill of each kind of office

(iii) the total basic monthly salary bill of all the offices taken together.

**92.** In a development plan of a city, a contractor has taken a contract to construct certain houses for which he needs building materials like stones, sand etc. There are three firms $A$, $B$, $C$ that can supply him these materials. At one time these firms $A$, $B$, $C$ supplied him 40, 35 and 25 truck loads of stones and 10, 5 and 8 truck loads of stone and sand, respectively. If the cost of one truck load of stone and sand are ₹ 1200 and 500 respectively, find the total amount paid by the contractor to each of these firms $A$, $B$, $C$ separately.

**93.** Show that the matrix $A = \begin{bmatrix} 1 & a & \alpha & a\alpha \\ 1 & b & \beta & b\beta \\ 1 & c & \gamma & c\gamma \end{bmatrix}$ is of rank 3 provided no two of $a, b, c$ are equal and no two of $\alpha, \beta, \gamma$ are equal.

**94.** By the method of matrix inversion, solve the system.

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} x & u \\ y & v \\ z & w \end{bmatrix} = \begin{bmatrix} 9 & 2 \\ 52 & 15 \\ 0 & -1 \end{bmatrix}$$

**95.** If $x_1 = 3y_1 + 2y_2 - y_3$, $\quad y_1 = z_1 - z_2 + z_3$

$x_2 = -y_1 + 4y_2 + 5y_3$, $y_2 = z_2 + 3z_3$

$x_3 = y_1 - y_2 + 3y_3$, $\quad y_3 = 2z_1 + z_2$

express $x_1, x_2, x_3$ in terms of $z_1, z_2, z_3$.

**96.** For what values of $k$ the set of equations
$2x-3y+6z-5t=3, y-4z+t=1,$
$4x-5y+8z-9t=k$ has
(i) no solution? (ii) infinite number of solutions?

**97.** Let $A, B, U, V$ and $X$ be the matrices defined as follows.

$$A=\begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}, B=\begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}, U=\begin{bmatrix} f \\ g \\ h \end{bmatrix}, V=\begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}, X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

If $AX=U$ has infinitely many solutions, show that $BX=V$ cannot have a unique solution. If $afd \neq 0$, show that $BX=V$ has no solution.

# Matrices Exercise 8 :
## Questions Asked in Previous 13 Year's Exam

- This section contains questions asked in **IIT-JEE, AIEEE, JEE Main & JEE Advanced** from year **2005** to year **2017**.

**98.** $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}; I=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and

$A^{-1}=\frac{1}{6}[A^2+cA+dI]$ where $c, d \in R$, the pair of values $(c, d)$ **[IIT- JEE 2005, 3M]**

(a) $(6, 11)$ (b) $(6, -11)$ (c) $(-6, 11)$ (d) $(-6, -11)$

**99.** If $P=\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, A=\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q=PAP^T$, the $P(Q^{2005})P^T$ equal to **[IIT- JEE 2005, 3M]**

(a) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} \sqrt{3}/1 & 2005 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2005 \\ \sqrt{3}/2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & \sqrt{3}/2 \\ 0 & 2005 \end{bmatrix}$

**100.** If $A=\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, which one of the following holds for all $n \geq 1$, (by the principal of mathematical induction) **[AIEEE 2005, 3M]**

(a) $A^n=nA+(n-1)I$ (b) $A^n=2^{n-1}A+(n-1)I$
(c) $A^n=nA-(n-1)I$ (d) $A^n=2^{n-1}A-(n-1)I$

**101.** If $A^2-A+I=0$, then $A^{-1}$ is equal to **[AIEEE 2005, 3M]**

(a) $A^{-2}$ (b) $A+I$ (c) $I-A$ (d) $A-I$

**102.** If $A=\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$, $U_1, U_2$ and $U_3$ are column matrices satisfying $AU_1=\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, AU_2=\begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$ and $AU_3=\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ and $U$ is $3\times 3$ matrix when columns are $U_1, U_2, U_3$, then answer the following questions

(i) The value of $|U|$ is
(a) 3 (b) $-3$ (c) $3/2$ (d) 2

(ii) The sum of the elements of $U^{-1}$ is
(a) $-1$ (b) 0 (c) 1 (d) 3

(iii) The value of $(3\ 2\ 0)\ U\begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$, is **[IIT- JEE 2006, 5+5+5M]**
(a) 5 (b) $5/2$ (c) 4 (d) $3/2$

**103.** Let $A=\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B=\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, $a, b \in N$. Then, **[AIEEE 2006, 4½M]**

(a) there cannot exist any $B$ such that $AB=BA$
(b) there exist more than one but finite number of $B$'s such that $AB=BA$
(c) there exists exactly one $B$ such that $AB=BA$
(d) there exist infinitely among $B$'s such that $AB=BA$

**104.** If $A$ and $B$ are square matrices of size $n\times n$ such that $A^2-B^2=(A-B)(A+B)$, which of the following will be always true? **[AIEEE 2006, 3M]**

(a) $A=B$ (b) $AB=BA$
(c) Either of $A$ or $B$ is a zero matrix
(d) Either of $A$ or $B$ is identity matrix

**105.** Let $A=\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. If $|A^2|=25$, then $|\alpha|$ equals to **[AIEEE 2007, 3M]**

(a) $5^2$ (b) 1 (c) $1/5$ (d) 5

**106.** Let $A$ and $B$ be $3\times 3$ matrices of real numbers, where $A$ is symmetric, $B$ is skew-symmetric and $(A+B)(A-B)=(A-B)(A+B)$. If $(AB)^t=(-1)^k AB$, where $(AB)^t$ is the transpose of matrix $AB$, the value of $k$ is **[IIT- JEE 2008, 1½M]**

(a) 0 (b) 1 (c) 2 (d) 3

**107.** Let $A$ be a square matrix all of whose entries are integers. Which one of the following is true? **[AIEEE 2008, 3M]**

(a) If det $A \neq \pm 1$, then $A^{-1}$ exists and all its entries are non-integers
(b) If det $A=\pm 1$, then $A^{-1}$ exists and all its entries are integers
(c) If det $A=\pm 1$, then $A^{-1}$ need not exist
(d) If det $A=\pm 1$, then $A^{-1}$ exists but all its entries are not necessarily integers

**108.** Let $A$ be a $2 \times 2$ matrix with real entries. Let $I$ be the $2 \times 2$ identity matrix. Denote by $\operatorname{tr}(A)$, the sum of diagonal entries of $A$. Assume that $A^2 = I$. **[AIEEE 2008, 3M]**

**Statement-1** If $A \neq I$ and $A \neq -I$, then $\det A = -1$.

**Statement-2** If $A \neq I$ and $A \neq -1$, then $\operatorname{tr}(A) \neq 0$.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(c) Statement-1 is true, Statement-2 is false
(d) Statement-1 is false, Statement-2 is true

**109.** Let $A$ be the set of all $3 \times 3$ symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0. **[IIT- JEE 2009, 4+4+4M]**

(i) The number of matrices in $A$ is
(a) 12 (b) 6
(c) 9 (d) 3

(ii) The number of matrices $A$ for which the system of linear equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has a unique solution, is
(a) less than 4 (b) atleast 4 but less than 7
(c) atleast 7 but less than 10
(d) atleast 10

(iii) The number of matrices $A$ in which the system of linear equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is inconsistent is
(a) 0 (b) more than 2
(c) 2 (d) 1

**110.** Let $A$ be a $2 \times 2$ matrix

**Statement-1** $\operatorname{adj}(\operatorname{adj} A) = A$

**Statement-2** $|\operatorname{adj} A| = |A|$ **[AIEEE 2009, 4M]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(c) Statement-1 is true, Statement-2 is false
(d) Statement-1 is false, Statement-2 is true

**111.** The number of $3 \times 3$ matrices $A$ whose are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is **[IIT- JEE 2010, 3M]**
(a) 0 (b) $2^9 - 1$
(c) 168 (d) 2

**112.** Let $p$ be an odd prime number and $T_p$ be the following set of $2 \times 2$ matrices.

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix};\ a, b, c, \in \{0, 1, 2, \dots, p-1\} \right\}$$

**[IIT- JEE 2010, 3+3+3M]**

(i) The number of $A$ in $T_p$ such that $A$ is either symmetric or skew-symmetric or both and $\det(A)$ divisible by $p$, is
(a) $(p-1)^2$ (b) $2(p-1)$
(c) $(p-1)^2 + 1$ (d) $2p - 1$

(ii) The number of $A$ in $T_p$ such that the trace of $A$ is not divisible by $p$ but $\det(A)$ is divisible by $p$, is

[**Note** The trace of a matrix is the sum of its diagonal entries]
(a) $(p-1)(p^2 - p + 1)$ (b) $p^3 - (p-1)^2$
(c) $(p-1)^2$ (d) $(p-1)(p^2 - 2)$

(iii) The number of $A$ in $T_p$ such that $\det(A)$ is not divisible by $p$, is
(a) $2p^2$ (b) $p^3 - 5p$ (c) $p^3 - 3p$ (d) $p^3 - p^2$

**113.** Let $k$ be a positive real number and let

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}.$$

If $\det(\operatorname{adj} A) + \det(\operatorname{adj} B) = 10^6$, then $[k]$ is equal to **[IIT- JEE 2010, 3M]**

**Note** adj $M$ denotes the adjoint of a square matrix $M$ and $[k]$ denotes the largest integer less than or equal to $k$}.

**114.** The number of $3 \times 3$ non-singular matrices, with four entries as 1 and all other entries as 0, is **[AIEEE 2010, 8M]**
(a) 5 (b) 6
(c) atleast 7 (d) less than 4

**115.** Let $A$ be a $2 \times 2$ matrix with non-zero entries and let $A^2 = I$, where $I$ is $2 \times 2$ identity matrix. Define $\operatorname{Tr}(A)$ = sum of diagonal elements of $A$ and $|A|$ = determinant of matrix $A$.

**Statement-1** $\operatorname{Tr}(A) = 0$

**Statement-2** $|A| = 1$ **[AIEEE 2010, 4M]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

**116.** Let $M$ and $N$ be two $3 \times 3$ non-singular skew-symmetric matrices such that $MN = NM$. If $P^T$ denotes the transpose of $P$, then $M^2 N^2 (M^T N)^{-1} (MN^{-1})^T$ is equal to **[IIT- JEE 2011, 4M]**
(a) $M^2$ (b) $-N^2$ (c) $-M^2$ (d) $MN$

**117.** Let $a$, $b$ and $c$ be three real numbers satisfying

$$[a\ b\ c]\begin{bmatrix}1 & 9 & 7\\ 8 & 2 & 7\\ 7 & 3 & 7\end{bmatrix}=[0\ 0\ 0]. \qquad \ldots(E)$$

(i) If the point $P(a, b, c)$, with reference to $(E)$, lies on the plane $2x+y+z=1$, then the value of $7a+b+c$ is

(a) 0 (b) 12 (c) 7 (d) 6

(ii) Let $\omega$ be a solution of $x^3-1=0$ with $\text{Im}(\omega)>0$. If $a=2$ with $b$ and $c$ satisfying $(E)$, the value of $\dfrac{3}{\omega^a}+\dfrac{1}{\omega^b}+\dfrac{3}{\omega^c}$ is equal to

(a) $-2$ (b) 2 (c) 3 (d) $-3$

(iii) Let $b=6$ with $a$ and $c$ satisfying $(E)$. If $\alpha$ and $\beta$ are the roots of the quadratic equation $ax^2+bx+c=0$, then $\sum_{n=0}^{\infty}\left(\dfrac{1}{\alpha}+\dfrac{1}{\beta}\right)^n$ is **[IIT- JEE 2011, 3+3+3M]**

(a) 6 (b) 7

(c) $\dfrac{6}{7}$ (d) $\infty$

**118.** Let $\omega\neq 1$ be a cube root of unity and $S$ be the set of all non-singular matrices of the form $\begin{bmatrix}1 & a & b\\ \omega & 1 & c\\ \omega^2 & \omega & 1\end{bmatrix}$, where each of $a$, $b$ and $c$ is either $\omega$ or $\omega^2$. The number of distinct matrices in the set $S$ is **[IIT- JEE 2011, 3M]**

(a) 2 (b) 6

(c) 4 (d) 8

**119.** Let $M$ be a $3\times 3$ matrix satisfying $M\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix}$,

$$M\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix}\text{ and } M\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}.$$

The sum of the diagonal entries of $M$ is **[IIT- JEE 2011, 4M]**

**120.** Let $A$ and $B$ are symmetric matrices of order 3.

**Statement-1** $A(BA)$ and $(AB)A$ are symmetric matrices.

**Statement-2** $AB$ is symmetric matrix, if matrix multiplication of $A$ with $B$ is commutative.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(b) Statement-1 is true, Statement-2 is false

(c) Statement-1 is false, Statement-2 is true

(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1 **[AIEEE 2011, 4M]**

**121.** Let $P=[a_{ij}]$ be a $3\times 3$ matrix and $Q=[b_{ij}]$, where $b_{ij}=2^{i+j}a_{ij}$ for $1\leq i, j\leq 3$. If the determinant of $P$ is 2, the determinant of the matrix $Q$ is **[IIT- JEE 2012, 3M]**

(a) $2^{11}$ (b) $2^{12}$

(c) $2^{13}$ (d) $2^{10}$

**122.** If $P$ is a $3\times 3$ matrix such that $P^T=2P+I$, where $P^T$ is the transpose of $P$ and $I$ is the $3\times 3$ identity matrix, then there exists a column matrix $X=\begin{bmatrix}x\\y\\z\end{bmatrix}\neq\begin{bmatrix}0\\0\\0\end{bmatrix}$ such that **[IIT- JEE 2012, 3M]**

(a) $PX=\begin{bmatrix}0\\0\\0\end{bmatrix}$ (b) $PX=X$ (c) $PX=2X$ (d) $PX=-X$

**123.** If the adjoint of a $3\times 3$ matrix $P$ is $\begin{bmatrix}1 & 4 & 4\\ 2 & 1 & 7\\ 1 & 1 & 3\end{bmatrix}$, then the possible value(s) of the determinant of $P$ is (are) **[IIT- JEE 2012, 4M]**

(a) $-2$ (b) $-1$ (c) 1 (d) 2

**124.** If $A=\begin{pmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 3 & 2 & 1\end{pmatrix}$, $u_1$ and $u_2$ are the column matrices such that $Au_1=\begin{pmatrix}1\\0\\0\end{pmatrix}$ and $Au_2=\begin{pmatrix}0\\1\\0\end{pmatrix}$, then $u_1+u_2$ is equal to **[AIEEE 2012, 4M]**

(a) $\begin{pmatrix}-1\\-1\\0\end{pmatrix}$ (b) $\begin{pmatrix}1\\-1\\-1\end{pmatrix}$ (c) $\begin{pmatrix}-1\\1\\0\end{pmatrix}$ (d) $\begin{pmatrix}-1\\1\\-1\end{pmatrix}$

**125.** Let $P$ and $Q$ be $3\times 3$ matrices with $P\neq Q$. If $P^3=Q^3$ and $P^2Q=Q^2P$, the determinant of $(P^2+Q^2)$ is equal to **[AIEEE 2012, 4M]**

(a) 0 (b) $-1$ (c) $-2$ (d) 1

**126.** If $P=\begin{bmatrix}1 & \alpha & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$ is the adjoint of a $3\times 3$ matrix $A$ and $|A|=4$, then $\alpha$ is equal to **[JEE Main 2013, 4M]**

(a) 11 (b) 5 (c) 0 (d) 4

**127.** For $3\times 3$ matrices $M$ and $N$, which of the following statement(s) is (are) **not** correct?

(a) $N^TMN$ is symmetric or skew-symmetric, according as $M$ is symmetric or skew-symmetric

(b) $MN-NM$ is skew-symmetric for all symmetric matrices $M$ and $N$

(c) $MN$ is symmetric for all symmetric matrices $M$ and $N$

(d) $(\text{adj}\,M)(\text{adj}\,N)=\text{adj}\,(MN)$ for all invertible matrices $M$ and $N$ **[JEE Advanced 2013, 4M]**

**128.** Let $\omega$ be a complex cube root of unity with $\omega \neq 1$ and $P = [p_{ij}]$ be a $n \times n$ matrix with $p_{ij} = \omega^{i+j}$. Then, $p^2 \neq 0$, when $n$ is equal to **[JEE Advanced 2013, 3M]**

(a) 55 (b) 56 (c) 57 (d) 58

**129.** If $A$ is a $3 \times 3$ non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then $BB'$ equals to **[JEE Main 2014, 4M]**

(a) $B^{-1}$ (b) $(B^{-1})'$ (c) $I + B$ (d) $I$

**130.** Let $M$ be a $2 \times 2$ symmetric matrix with integer entries. Then, $M$ is invertible, if

(a) the first column of $M$ is the transpose of the second row of $M$

(b) the second row of $M$ is the transpose of the first column of $M$

(c) $m$ is a diagonal matrix with non-zero entries in the main diagonal

(d) the product of entries in the main diagonal of $M$ is not the square of an integer **[JEE Advanced 2014, 3M]**

**131.** Let $M$ and $N$ be two $3 \times 3$ matrices such that $MN = NM$. Further, if $M \neq N^2$ and $M^2 = N^4$, then

(a) determinant of $(M^2 + MN^2)$ is 0

(b) there is a $3 \times 3$ non-zero matrix $U$ such that $(M^2 + MN^2)\,U$ is the zero matrix

(c) determinant of $(M^2 + MN^2) \geq 1$

(d) for a $3 \times 3$ matrix $U$, if $(M^2 + MN^2)\,U$ equals the zero matrix, then $U$ is the zero matrix

**[JEE Advanced 2014, 3M]**

**132.** If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where $I$ is $3 \times 3$ identity matrix, then the ordered pair $(a, b)$ is equal to **[JEE Main 2015, 4M]**

(a) $(2, 1)$ (b) $(-2, -1)$ (c) $(2, -1)$ (d) $(-2, 1)$

**133.** Let $X$ and $Y$ be two arbitrary $3 \times 3$ non-zero, skew-symmetric matrices and $Z$ be an arbitrary $3 \times 3$ non-zero, symmetric matrix. Then, which of the following matrices is (are) skew-symmetric?

**[JEE Advanced 2015, 4M]**

(a) $Y^3Z^4 - Z^4Y^3$ (b) $X^{44} + Y^{44}$

(c) $X^4Z^3 - Z^3X^4$ (d) $X^{23} + Y^{23}$

**134.** If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A\,\text{adj}A = AA^T$, then $5a + b$ is equal to **[JEE Main 2016, 4M]**

(a) 5 (b) 13

(c) 4 (d) $-1$

**135.** Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix such that $PQ = kI$, where $k \in R, k \neq 0$ and $I$ is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and det. (Q) $= \frac{k^2}{2}$, then **[JEE Advanced 2016, 4M]**

(a) $\alpha = 0, k = 8$ (b) $4\alpha - k + 8 = 0$

(c) det (Padj (Q)) $= 2^9$ (d) det (Q adj (P)) $= 2^{13}$

**136.** Let $z = \frac{-1 + \sqrt{3}i}{2}$, where $i = \sqrt{-1}$, and $r, s = \{1, 2, 3\}$.

Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and $I$ be the identity matrix of oreder 2. Then the total number of ordered pairs $(r, s)$ for which $p^2 = -I$ is **[JEE Advanced 2016, 3M]**

(a) $\frac{1}{2}|\mathbf{a} - \mathbf{b}|$ (b) $\frac{1}{2}|\mathbf{a} + \mathbf{b}|$

(c) $|\mathbf{a} - \mathbf{b}|$ (d) $|\mathbf{a} + \mathbf{b}|$

**137.** Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3. If $Q = \left[q_{ij}\right]$ is a matrix such that $P^{50} - Q = I$, then $\frac{q_{31} + q_{32}}{q_{21}}$ equals **[JEE Advanced 2016, 3M]**

(a) 52 (b) 103

(c) 201 (d) 205

**138.** If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to **[JEE Main 2017, 4M]**

(a) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$ (b) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

(c) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ (d) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

# Answers

## Chapter Exercises

**1.** (d) **2.** (c) **3.** (b) **4.** (d) **5.** (c) **6.** (b)
**7.** (a) **8.** (a) **9.** (a) **10.** (b) **11.** (b) **12.** (b)
**13.** (b) **14.** (d) **15.** (d) **16.** (b) **17.** (b) **18.** (d)
**19.** (d) **20.** (d) **21.** (c) **22.** (d) **23.** (c) **24.** (d)
**25.** (b) **26.** (a) **27.** (d) **28.** (b) **29.** (a) **30.** (c)
**31.** (a, d) **32.** (a, b, d) **33.** (a, b, d) **34.** (b, c)
**35.** (b, d) **36.** (a, b, c) **37.** (a, c, d) **38.** (a, d)
**39.** (a,c,d) **40.** (a, c) **41.** (a, c, d) **42.** (a,b,c,d)
**43.** (c, d) **44.** (a, b, c) **45.** (a, c)
**46.** (b) **47.** (b) **48.** (c) **49.** (b) **50.** (d) **51.** (d)
**52.** (d) **53.** (c) **54.** (d) **55.** (a) **56.** (c) **57.** (a)
**58.** (b) **59.** (c) **60.** (a) **61.** (a)
**62.** (3) **63.** (2) **64.** (1) **65.** (9) **66.** (2) **67.** (7)
**68.** (9) **69.** (1) **70.** (2) **71.** (6)
**72.** (A) $\rightarrow$ (p, r); (B) $\rightarrow$ (s); (C) $\rightarrow$ (q); (D) $\rightarrow$ (s)
**73.** (A) $\rightarrow$ (r, t); (B) $\rightarrow$ (s); (C) $\rightarrow$ (p); (D) $\rightarrow$ (q)
**74.** (A) $\rightarrow$ (q, s); (B) $\rightarrow$ (p, t); (C) $\rightarrow$ (p, q, r, s); (D) $\rightarrow$ (q, s)
**75.** (A) $\rightarrow$ (q, t); (B) $\rightarrow$ (p, s); (C) $\rightarrow$ (p, r, s); (D) $\rightarrow$ (q, r, t)
**76.** (d) **77.** (c) **78.** (d) **79.** (c) **80.** (a) **81.** (d)
**82.** (d) **83.** (d) **84.** (d) **85.** (a)
**88.** $\alpha = \dfrac{2\pi}{3}$ **89.** $\begin{bmatrix} \cos\theta\cos\phi\cos(\theta\sim\phi) & \cos\theta\sin\phi\cos(\theta\sim\phi) \\ \sin\theta\cos\phi\cos(\theta\sim\phi) & \sin\theta\sin\phi\cos(\theta\sim\phi) \end{bmatrix}$

**91.** (i) Number of posts in all the offices taken together are 5 office superintendents; 235 head clerks; 235 cashiers; 275 clerks; 5 typists and 270 peons.
(ii) Total basic monthly salary bill of each division or district and taluka offices an ₹1675, ₹875 and ₹625, respectively.
(iii) Total basic monthly salary bill of all the offices taken together is ₹ 159625.

**92.** ₹53000; ₹44500; ₹34000, respectively
**94.** $x = 1, u = -1, y = 3, v = 2, z = 5, w = 1$
**95.** $x_1 = z_1 - 2z_2 + 9z_3, x_2 = 9z_1 + 10z_2 + 11z_3, x_3 = 7z_1 + z_2 - 2z_3$
**96.** (i) $k \neq 7$ (ii) $k = 7$
**98.** (c) **99.** (a) **100.** (c)
**101.** (c) **102.** (i) (a), (ii) (b), (iii) (a) **103.** (b) **104.** (b)
**105.** (c) **106.** (b,d) **107.** (d) **108.** (c)
**109.** (i) (a), (ii) (b), (iii) (b)
**110.** (b) **111.** (a)
**112.** (i) (d), (ii) (c), (iii) (d) **113.** (4) **114.** (c)
**115.** (b) **116.** (c)
**117.** (i) (d), (ii) (a), (iii) (b)
**118.** (a) **119.** (9) **120.** (a) **121.** (c)
**122.** (d) **123.** (a, d) **124.** (b) **125.** (a) **126.** (a) **127.** (c,d)
**128.** (a,b,d) **129.** (d) **130.** (c, d) **131.** (a, b) **132.** (b) **133.** (c, d)
**134.** (a) **135.** (b,c) **136.** (1) **137.** (b) **138.** (c)

# Solutions

**1.** $\because \quad A^4(I-A) = A^4I - A^5 = A^4 - 0 = A^4 \neq I$

$$A^3(I-A) = A^3I - A^4 = A^3 - A^4 \neq I$$

$$(I+A)(I-A) = I^2 - A^2 = I - A^2 \neq I$$

**2.** $\because$ $\det(B) = \begin{vmatrix} 4x & 2a & -p \\ 4y & 2b & -q \\ 4z & 2c & -r \end{vmatrix} = -8 \begin{vmatrix} x & a & p \\ y & b & q \\ z & c & r \end{vmatrix}$

$$= -8 \begin{vmatrix} x & y & z \\ a & b & c \\ p & q & r \end{vmatrix} = 8 \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} \quad \text{[by property]}$$

$$= -8 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = -8 \det(A) = -16$$

**3.** $\because$ $\left(A - \frac{1}{2}I\right)\left(A - \frac{1}{2}I\right)^T = I$ ...(i)

and $\left(A + \frac{1}{2}I\right)\left(A + \frac{1}{2}I\right)^T = I$ ...(ii)

$\Rightarrow$ $\left(A - \frac{1}{2}I\right)\left(A^T - \frac{I}{2}\right) = I$

and $\left(A + \frac{1}{2}I\right)\left(A^T + \frac{1}{2}I\right) = I$

$\Rightarrow$ $A + A^T = 0$ [subtracting the two results]

$\Rightarrow$ $A^T = -A$

$\therefore$ $A$ is skew-symmetric matrix.

From first result, we get

$$AA^T = \frac{3}{4}I$$

$\Rightarrow$ $A^2 = -\frac{3}{4}I$

$\therefore$ $|A^2| = \left|-\frac{3}{4}I\right|$

$\therefore$ $|A|^2 = \left(-\frac{3}{4}\right)^n$

$\Rightarrow$ $n$ is even.

**4.** $\because$ $a = \lim_{x \to 1}\left(\frac{x}{\ln x} - \frac{1}{x \ln x}\right) = \lim_{x \to 1}\left(\frac{x^2 - 1}{x \ln x}\right)$

$$= \lim_{x \to 1}\left(\frac{2x}{1 + \ln x}\right) \quad \text{[by L'Hospital's Rule]}$$

$$b = \lim_{x \to 0}\left(\frac{x^3 - 16x}{4x + x^2}\right)$$

$$= \lim_{x \to 0}\left(\frac{x(x+4)(x-4)}{x(x+4)}\right) = \lim_{x \to 0}(x - 4) = -4$$

$$c = \lim_{x \to 0} \frac{\ln(1 + \sin x)}{x}$$

$$= \lim_{x \to 0} \frac{\ln(1 + \sin x)}{\sin x} \cdot \lim_{x \to 0} \frac{\sin x}{x} = 1 \cdot 1 = 1$$

$$d = \lim_{x \to -1} \frac{(x+1)^3}{3[\sin(x+1) - (x+1)]}$$

$$= \lim_{x \to -1} \frac{3(x+1)^2}{3[\cos(x+1) - 1]} \quad \text{[using L'Hospital's Rule]}$$

$$= -\lim_{x \to -1} \frac{1}{\dfrac{[1 - \cos(x+1)]}{(x+1)^2}} = -2$$

Let, $A = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} \Rightarrow A^2 = 0$

**5.** $\because$ $(A - \lambda I)X = 0$

$\therefore$ $|A - \lambda I| = 0$

$\Rightarrow$ $\begin{vmatrix} 1 - \lambda & 4 \\ 3 & 2 - \lambda \end{vmatrix} = 0$

$\Rightarrow$ $\lambda^2 - 3\lambda - 10 = 0$

$\therefore$ $\lambda = -2, 5$

For $\lambda = -2 \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$

For $\lambda = 5 \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\therefore$ $\cos\theta = \dfrac{4 \cdot 1 + (-3) \cdot 1}{\sqrt{(16+9)}\sqrt{(1+1)}} = \dfrac{1}{5\sqrt{2}}$

$\therefore$ $\tan\theta = \sqrt{(\sec^2\theta - 1)} = \sqrt{49} = 7$

**6.** $\because A^{2n+1} = (A^2)^n \cdot A = (I)^n \cdot A = IA = A$

**7.** $\because$ $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$\therefore$ $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$

$$\Rightarrow \lim_{n \to \infty} \frac{A^n}{n} = \begin{bmatrix} \lim\limits_{n \to \infty} \dfrac{\cos n\theta}{n} & \lim\limits_{n \to \infty} \dfrac{\sin n\theta}{n} \\ -\lim\limits_{n \to \infty} \dfrac{\sin n\theta}{n} & \lim\limits_{n \to \infty} \dfrac{\cos n\theta}{n} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

= a zero matrix [$\because -1 < \sin\infty < 1$ and $-1 < \cos\infty < 1$]

**8.** Let $A = \begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & -10 \\ 1 & -2 & -5 \end{bmatrix}$ [$\because a = -6$]

Applying $R_2 \to R_2 + 2R_1$ and $R_3 \to R_3 + R_1$, then

$$A = \begin{bmatrix} -1 & 2 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \rho(A) = 1$$

**9.** $\because$ $A$ is involutory

$\therefore$ $A^2 = I \Rightarrow A = A^{-1}$

$\therefore$ $\left(\frac{A}{2}\right)^{-1} = 2A^{-1} = 2A$

**10.** $\because \quad B = \text{adj } A$

$\Rightarrow \quad AB = A(\text{adj } A) = |A| I_n$

$\therefore \quad AB + kI_n = |A| I_n + kI_n = (|A| + k) I_n$

$\Rightarrow \quad |AB + kI_n| = |(|A| + k) I_n| = (|A| + k)^n$

**11.** $\because \quad B = -A^{-1}BA$

$\Rightarrow \quad AB = -BA$

$\Rightarrow \quad AB + BA = 0$

Now, $(A+B)^2 = (A+B)(A+B)$

$= A^2 + AB + BA + B^2$

$= A^2 + 0 + B^2$

$= A^2 + B^2$

**12.** Since, $A$ is skew-symmetric.

$\therefore \quad |A| = 0$

$\Rightarrow \quad |A^4 B^3| = |A^4||B^3| = |A|^4 |B|^3 = 0$

**13.** Let $B = A + I_n$

$\therefore \quad A = B - I_n$

Given, $A^n = \alpha A$

$\Rightarrow \quad (B - I_n)^n = \alpha (B - I_n)$

$\Rightarrow \quad B^n - {}^nC_1 B^{n-1} + {}^nC_2 B^{n-2} + \ldots + (-1)^n I_n$

$= \alpha B - \alpha I_n$

$\Rightarrow \quad B(B^{n-1} - {}^nC_1 B^{n-2} + {}^nC_2 B^{n-3} + \ldots + (-1)^{n-1} I_n - \alpha I_n)$

$= [(-1)^{n+1} - \alpha] I_n \neq 0 \quad [\because \alpha \neq \pm 1]$

Hence, $B$ is invertible.

**14.** $\because \quad \omega = \dfrac{-1 + i\sqrt{3}}{2}$ and $\omega^2 = \dfrac{-1 - i\sqrt{3}}{2}$

Also, $\omega^3 = 1$ and $\omega + \omega^2 = -1$

Thus, $A = \begin{bmatrix} -i\omega & -i\omega^2 \\ i\omega^2 & i\omega \end{bmatrix}$

$\therefore \quad A^2 = \begin{bmatrix} -i\omega & -i\omega^2 \\ i\omega^2 & i\omega \end{bmatrix}\begin{bmatrix} -i\omega & -i\omega^2 \\ i\omega^2 & i\omega \end{bmatrix} = \begin{bmatrix} -\omega^2 + \omega & 0 \\ 0 & -\omega^2 + \omega \end{bmatrix}$

Now, $f(A) = A^2 + 2I = \begin{bmatrix} -\omega^2 + \omega & 0 \\ 0 & -\omega^2 + \omega \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} -\omega^2 + \omega + 2 & 0 \\ 0 & -\omega^2 + \omega + 2 \end{bmatrix}$

$= (-\omega^2 + \omega + 2)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (2 + i\sqrt{3})\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

**15.** $\because \quad X^2 = I \Rightarrow (X^{-1}X)X = X^{-1}I$

$\Rightarrow \quad IX = X^{-1}$

$\Rightarrow \quad X = X^{-1}$

which is self invertible involutory matrix.

There are many such matrices which are inverse of their own.

**16.** $\because \quad AB = A + B$

$\Rightarrow \quad B = AB - A = A(B - I)$

$\Rightarrow \quad \det(B) = \det(A) \cdot \det(B - I) = 0$

$[\because A^{2006} = 0 \Rightarrow \det A^{2006} = 0]\ [\therefore \det A = 0]$

**17.** We have, $P^T = P^{-1} \quad [\because PP^T = I]$

Now, $Q = PAP^T = PAP^{-1}$

$\therefore \quad Q^{2007} = PA^{2007}P^{-1}$

$\therefore \quad P^T Q^{2007} P = P^{-1}(PA^{2007}P^{-1})P$

$= A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$

$\left[\because A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}, \ldots\right]$

**18.** $\because \quad \begin{bmatrix} A-5 & B \\ 2A-2 & C \end{bmatrix} = \begin{bmatrix} 2A+1 & -5 \\ -4 & A \end{bmatrix}\begin{bmatrix} 14 & D \\ E & F \end{bmatrix}$

$\Rightarrow \quad A - 5 = 28A + 14 - 5E$

$\Rightarrow \quad 5E = 27A + 19 \quad$ ...(i)

$2A - 2 = -56 + AE$

$\Rightarrow \quad AE = 2A + 54 \quad$ ...(ii)

From Eq. (i), we get

$5AE = 27A^2 + 19A$

$\Rightarrow \quad 5(2A + 54) = 27A^2 + 19A \quad$ [from Eq. (ii)]

$\Rightarrow \quad 27A^2 + 9A - 270 = 0$

$\Rightarrow \quad 9(A - 3)(3A + 10) = 0$

$\therefore \quad A = 3, A = -\dfrac{10}{3}$

$\therefore$ Absolute value of difference

$= \left| 3 + \dfrac{10}{3} \right| = \dfrac{19}{3}$

**19.** $\because |f(\theta)| = \begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta & -\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$

On multiplying in $R_3$ by $\cos\theta$ and then take common $\cos\theta$ from $C_1$, then

$|f(\theta)| = \begin{vmatrix} \cos\theta & \cos\theta\sin\theta & -\sin\theta \\ \sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos^2\theta & 0 \end{vmatrix}$

Applying $R_2 \to R_2 - R_3$, we get

$|f(\theta)| = \begin{vmatrix} \cos\theta & \cos\theta\sin\theta & -\sin\theta \\ 0 & 1 & \cos\theta \\ \sin\theta & -\cos^2\theta & 0 \end{vmatrix} = 1$

Applying $C_2 \to C_2 - \sin\theta\, C_1$, then

$|f(\theta)| = \begin{vmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & \cos\theta \\ \sin\theta & -1 & 0 \end{vmatrix} = 1$

$\therefore \quad f\left(\dfrac{\pi}{7}\right)$ is non-singular matrix.

**20.** $\because\ a_{11} = a_{22} = a_{33} = a + b,$

$a_{12} = a_{23} = ab, a_{21} = a_{32} = 1, a_{13} = a_{31} = 0$

$$\therefore \quad A = \begin{bmatrix} a+b & ab & 0 \\ 1 & a+b & ab \\ 0 & 1 & a+b \end{bmatrix}$$

$$\Rightarrow |A| = \begin{vmatrix} a+b & ab & 0 \\ 1 & a+b & ab \\ 0 & 1 & a+b \end{vmatrix}$$

$$= (a+b)[(a+b)^2 - ab] - ab(a+b) = (a+b)(a^2+b^2)$$

**21.** Given, $B^r = I \Rightarrow B^r B^{-1} = IB^{-1}$

$\Rightarrow \quad B^{r-1} = B^{-1}$

$\therefore \quad A^{-1}B^{r-1}A = A^{-1}B^{-1}A$

$\Rightarrow \quad A^{-1}B^{r-1}A - A^{-1}B^{-1}A = 0$

**22.** Here, $A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$

$\Rightarrow \quad AA^T = I$

$\because \quad C = ABA^T \Rightarrow A^TC = BA^T$

Now, $A^TC^nA = A^TC \cdot C^{n-1}A$

$= BA^TC^{n-1}A = BA^TC\, C^{n-2}A$

$= B^2A^TC^{n-2}A$

... ... ...

$= B^{n-1}A^TCA = B^{n-1}(BA^T)A$

$= B^nA^TA = B^nI = B^n = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$

**23.** $\because\ |\text{adj } A^{-1}| = |A^{-1}|^2 = \dfrac{1}{|A|^2}$

$\therefore\ |(\text{adj } A^{-1})^{-1}| = \dfrac{1}{|\text{adj } A^{-1}|} = |A|^2 = 2^2 = 4$

**24.** $\because\ A^3 - A^2B = B^3 - B^2A$

$\Rightarrow \quad A^2(A-B) = B^2(B-A)$

or $\quad (A^2+B^2)(A-B) = 0$

or $\det(A^2+B^2) \cdot \det(A-B) = 0$

Either $\det(A^2+B^2) = 0$ or $\det(A-B) = 0$

**25.** Let, $A = \begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix},$

$$BC = \begin{bmatrix} 1 & 4 \\ 2 & 9 \end{bmatrix}\begin{bmatrix} 9 & -4 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$\therefore\ B^2C^2 = (BC)^2 = I^2 = I$

Similarly, $B^2C^2 = B^3C^3 = \dots = B^nC^n = I$

Let, $D = A^3(BC) + A^5(B^2C^2) + A^7(B^3C^3) + \dots + A^{2n+1}(B^nC^n)$

$= A^3 + A^5 + A^7 + \dots + A^{2n+1}$

$= A(A^2 + A^4 + A^6 + \dots + A^{2n})$

Let, $A = \begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix}$

$\Rightarrow \quad A^2 = \begin{bmatrix} -a^2 & 0 \\ 0 & -a^2 \end{bmatrix} = -a^2 I$

$\therefore \quad D = IA(-a^2 + a^4 - a^6 + \dots + (-1)^n a^{2n})$ $\quad [a > 0]$

$= A(-a^2 + a^4 - a^6 + \dots + (-1)^n a^{2n})$

Hence, $D$ is skew-symmetric.

**26.** $\because\ |B| = \begin{vmatrix} q & -b & y \\ -p & a & -x \\ r & -c & z \end{vmatrix}$

Applying $R_2 \to (-1)R_2$, then

$$|B| = \begin{vmatrix} q & -b & y \\ p & -a & x \\ r & -c & z \end{vmatrix}$$

Applying $C_2 \to (-1)C_2$, then

$$|B| = \begin{vmatrix} q & b & y \\ p & a & x \\ r & c & z \end{vmatrix} = |B^T| = \begin{vmatrix} q & p & r \\ b & a & c \\ y & x & z \end{vmatrix}$$

$$= -\begin{vmatrix} b & a & c \\ q & p & r \\ y & x & z \end{vmatrix} \quad [R_1 \leftrightarrow R_2]$$

$$= \begin{vmatrix} b & a & c \\ y & x & z \\ q & p & r \end{vmatrix} \quad [R_2 \leftrightarrow R_3 0]$$

$$= -\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = -|A|$$

$\Rightarrow \quad |B| = -|A|$

Also, $|\text{adj } B| = |B|^2$

$= |A|^2 = |\text{adj } A|$ $\quad [\because |A| \neq 0, \text{ then } |B| \neq 0]$

**27.** $\because \quad BC = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$$\therefore \quad \text{tr}(A) + \text{tr}\left(\frac{ABC}{2}\right) + \text{tr}\left(\frac{A(BC)^2}{4}\right) + \text{tr}\left(\frac{A(BC)^3}{8}\right) + \dots$$

$$= \text{tr}(A) + \text{tr}\left(\frac{A}{2}\right) + \text{tr}\left(\frac{A}{2^2}\right) + \text{tr}\left(\frac{A}{2^3}\right) + \dots \text{ upto } \infty$$

$$= \text{tr}(A) + \frac{1}{2}\text{tr}(A) + \frac{1}{2^2}\text{tr}(A) + \dots \text{ upto } \infty$$

$$= \frac{\text{tr}(A)}{1 - \left(\frac{1}{2}\right)} = 2\,\text{tr}(A) = 2(2+1) = 6$$

**28.** We have, $\quad (A - 2I)(A - 4I) = 0$

$\Rightarrow \quad A^2 - 4A - 2A + 8I^2 = 0$

$$\Rightarrow \quad A^2 - 6A + 8I = 0$$
$$\Rightarrow \quad A^{-1}(A^2 - 6A + 8I) = A^{-1}0$$
$$\Rightarrow \quad A - 6I + 8A^{-1} = 0$$
$$\Rightarrow \quad \frac{1}{6}A + \frac{4}{3}A^{-1} = I$$

**29.** We have, $AA^{-1} = I$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & 6 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & b+1 \\ 0 & 1 & 2(b+1) \\ 4(1-a) & 3(a-1) & ab+2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing, we get

$$b + 1 = 0, ab + 2 = 1, a - 1 = 0$$
$$\therefore \quad a = 1, b = -1$$

**30.** $\because \quad A\,(\text{adj } A) = |A|\,I \quad$ ...(i)

Now, $$|A| = \begin{vmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{vmatrix}$$
$$= x(yz - 8) - 3(z - 8) + 2(2 - 2y)$$
$$= xyz - (8x + 4y + 3z) + 28$$
$$= 60 - 20 + 28 = 68$$

From Eq. (i), $A\,(\text{adj } A) = 68I$

**31.** Here, $|A| = 0$

$\therefore A^{-1}$ does not exist.

Now, $$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 3A$$

$$\therefore \quad A^3 = A^2 \cdot 2 = 3A \cdot A = 3A^2 = 3(3A) = 9A$$

**32.** $\because \quad A' = A^{-1} \Rightarrow AA' = I \quad$ ...(i)

Now, $(A')' A' = I$

$\therefore A'$ is orthogonal

From Eq. (i), $(AA')^{-1} = I^{-1}$

$$\Rightarrow \quad (A')^{-1} A^{-1} = I$$
$$\Rightarrow \quad (A^{-1})' (A^{-1}) = I$$

$\therefore A^{-1}$ is orthogonal

Since, $\text{adj } A = A^{-1}|A| \neq A'$

and $|A^{-1}| = \dfrac{1}{|A|} = \pm 1 \quad$ [for orthogonal $|A| = \pm 1$]

**33.** $\because$ $$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

We have, $A^2 - 4A - 5I_3$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$$\Rightarrow \quad 5I_3 = A^2 - 4A = A(A - 4I_3)$$
$$\Rightarrow \quad I_3 = \frac{1}{5}A(A - 4I_3)$$
$$\therefore \quad A^{-1} = \frac{1}{5}(A - 4I_3)$$

Since, $|A| = 5$

$$\therefore \quad |A^3| = |A|^3 = 125 \neq 0$$

$\Rightarrow A^3$ is invertible

Similarly, $A^2$ is invertible.

**34.** Let, $D = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} = D^T$ and let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$$\therefore \quad DA = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = \begin{bmatrix} aa_1 & aa_2 & aa_3 \\ bb_1 & bb_2 & bb_3 \\ cc_1 & cc_2 & cc_3 \end{bmatrix}$$

$$AD = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} = \begin{bmatrix} a_1a & a_2b & a_3c \\ b_1a & b_2b & b_3c \\ c_1a & c_2b & c_3c \end{bmatrix} \neq DA$$

and $$D^{-1} = \begin{bmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix}$$

$$|D^{-1}| = \frac{1}{abc} \neq 0 \quad [\because a \neq 0, b \neq 0, c \neq 0]$$

**35.** Let $$A = \begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{bmatrix}$$

Applying $R_2 \to R_2 + 2R_1$ and $R_3 \to R_3 + R_1$, then

$$A = \begin{bmatrix} -1 & 2 & 5 \\ 0 & 0 & a+6 \\ 0 & 0 & a+6 \end{bmatrix}$$

Applying $R_3 \to R_3 - R_2$, then

$$A = \begin{bmatrix} -1 & 2 & 5 \\ 0 & 0 & a+6 \\ 0 & 0 & 0 \end{bmatrix}$$

For $a = -6$, $\rho(A) = 1$

For $a = 1, 2$, $\rho(A) = 2$

**36.** Here, $|A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix}$

$$= 3(-3+4) + 3(2-0) + 4(-2+0) = 1 \neq 0$$

$\because$ $\operatorname{adj}(\operatorname{adj} A) = |A|^{3-2} A = A$ ...(i)

and $|\operatorname{adj}(A)| = |A|^{3-1} = |A|^2 = 1^2 = 1$

Also, $|\operatorname{adj}(\operatorname{adj}(A))| = |A| = 1$ [from Eq. (i)]

**37.** $\because$ $A = I - B$

$\Rightarrow$ $A^2 = I^2 + B^2 - 2B = I - B = A$ [$\because B$ is idempotent]

and $AB = B - B^2 = B - B = 0$ [null matrix]

and $BA = B - B^2 = B - B = 0$ [null matrix]

**38.** $\because$ $|A| \neq 0 \Rightarrow A^{-1}$ is also symmetric, if $A$ is symmetric

and $|A^{-1}| = \dfrac{1}{|A|} = |A|^{-1}$

**39.** $\because$ $A^2B = A(AB) = A(BA) = (AB)A = (BA)A = BA^2$

Similarly, $A^3B = BA^3$

In general, $A^nB = BA^n, \forall\, n \geq 1$

and $(A+B)^n = {}^nC_0A^n + {}^nC_1A^{n-1}B + {}^nC_2A^{n-2}B^2 + \ldots + {}^nC_nB^n$

Also, $(A^n - B^n)(A^n + B^n) = A^nA^n + A^nB^n - B^nA^n - B^nB^n$

$= A^{2n} - B^{2n}$ [$\because AB = BA$]

**40.** $|AB| = 0 \Rightarrow |A||B| = 0$

$\therefore$ $|B| = 0$ as $|A| \neq 0$

Also, $|A^{-1}| = |A|^{-1}$

**41.** Here, $A(A+I) = -2I$ ...(i)

$\Rightarrow$ $|A(A+I)| = |-2I| = (-2)^m \neq 0$

Thus, $|A| \neq 0$,

also, $I = -\dfrac{1}{2}A(A+I)$ [from Eq. (i)]

$\therefore$ $A^{-1} = -\dfrac{1}{2}(A+I)$

**42.** $\because$ $A^2 - 3A + 2I = 0$ ...(i)

$\Rightarrow$ $A^2 - 3AI + 2I^2 = 0$

$\Rightarrow$ $(A-I)(A-2I) = 0$

$\therefore$ $A = I$ or $A = 2I$

Characteristic Eq. (i) is

$$\lambda^2 - 3\lambda + 2 = 0 \Rightarrow \lambda = 1, 2$$

It is clear that alternate (c) and (d) have the characteristic equation $\lambda^2 - 3\lambda + 2 = 0$.

**43.** $\because$ $AB = 0$

$\Rightarrow$ $|AB| = 0 \Rightarrow |A||B| = 0$

or $(\det A)(\det B) = 0$

$\Rightarrow$ Either $\det A = 0$ or $\det B = 0$

Hence, atleast one of the two matrices must be singular otherwise this statement is not possible.

**44.** Let $D_1 = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$ and $D_2 = \begin{bmatrix} d_4 & 0 & 0 \\ 0 & d_5 & 0 \\ 0 & 0 & d_6 \end{bmatrix}$

$\therefore D_1D_2 = \begin{bmatrix} d_1d_4 & 0 & 0 \\ 0 & d_2d_5 & 0 \\ 0 & 0 & d_3d_6 \end{bmatrix} = D_2D_1$

and $D_1^2 + D_2^2 = \begin{bmatrix} d_1^2 & 0 & 0 \\ 0 & d_2^2 & 0 \\ 0 & 0 & d_3^2 \end{bmatrix} + \begin{bmatrix} d_4^2 & 0 & 0 \\ 0 & d_5^2 & 0 \\ 0 & 0 & d_6^2 \end{bmatrix}$

$$= \begin{bmatrix} d_1^2 + d_4^2 & 0 & 0 \\ 0 & d_2^2 + d_5^2 & 0 \\ 0 & 0 & d_3^2 + d_6^2 \end{bmatrix}$$

**45.** $A_1 + A_2 + A_3 + \ldots + A_n = \begin{bmatrix} C_0^2 & 0 \\ 0 & C_1^2 \end{bmatrix} + \begin{bmatrix} C_1^2 & 0 \\ 0 & C_2^2 \end{bmatrix} + \begin{bmatrix} C_2^2 & 0 \\ 0 & C_3^2 \end{bmatrix} + \ldots + \begin{bmatrix} C_{n-1}^2 & 0 \\ 0 & C_n^2 \end{bmatrix}$

$$= \begin{bmatrix} C_0^2 + C_1^2 + C_2^2 + \ldots + C_{n-1}^2 & 0 \\ 0 & C_1^2 + C_2^2 + C_3^2 + \ldots + C_n^2 \end{bmatrix}$$

$$= \begin{bmatrix} {}^{2n}C_n - 1 & 0 \\ 0 & {}^{2n}C_n - 1 \end{bmatrix} = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \quad \text{[given]}$$

$\therefore$ $k_1 = k_2 = {}^{2n}C_n - 1$

## Passage (Q. Nos. 46 to 48)

$\because$ $AB = BA^m$

$\Rightarrow$ $B = A^{-1}BA^m$

$\therefore$ $B^n = \underbrace{(A^{-1}BA^m)(A^{-1}BA^m)\ldots(A^{-1}BA^m)}_{n\text{ times}}$

$= A^{-1}\underbrace{BA^{m-1}BA^{m-1}\ldots BA^{m-1}BA^{m-1}}_{n\text{ times}}A$ ...(i)

Given, $AB = BA^m$

$\Rightarrow$ $AAB = ABA^m = BA^{2m} \Rightarrow AAAB = BA^{3m}$

Similarly, $A^xB = BA^{mx}\ \forall\, m \in N$

From Eq. (i), we get

$$B^n = A^{-1}BA^{m-1}\underbrace{BA^{m-1}BA^{m-1}\ldots BA^{m-1}BA^{m-1}}_{(n-1)\text{ times}}A$$

$$= A^{-1}B(A^{m-1}B)A^{m-1}\underbrace{BA^{m-1}\ldots BA^{m-1}BA^{m-1}}_{(n-2)\text{ times}}A$$

$$= A^{-1}BBA^{(m-1)m}A^{m-1}\underbrace{BA^{m-1}\ldots BA^{m-1}BA^{m-1}}_{(n-2)\text{ times}}A$$

$$= A^{-1}B^2A^{(m^2-1)}\underbrace{BA^{m-1}\ldots BA^{m-1}BA^{m-1}}_{(n-2)\text{ times}}A$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

$$= A^{-1}B^m(A)^{(m^n-1)}A$$

$I = A^{-1}IA^{(m^n-1)}A$ [$\because B^n = I$]

$I = A^{-1}A^{(m^n-1)}A = A^{-1}A^{m^n}$

$\Rightarrow$ $I = A^{(m^n-1)}$

$\therefore$ $p = m^n - 1$ ...(ii) [$\because A^p = I$]

**46.** Put $m = 2, n = 5$ in Eq. (ii), we get

$$p = 2^5 - 1 = 31$$

**47.** From Eq. (ii), we get

$$p = m^n - 1$$

**48.** From Eq. (ii), we get

$$510 \neq 8^3 - 1$$

## Passage (Q. Nos. 49 to 51)

$\because A$ is an orthogonal matrix

$\therefore \quad AA^T = I$

$$\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} = 1\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a^2+b^2+c^2 & ab+bc+ca & ab+bc+ca \\ ab+bc+ca & a^2+b^2+c^2 & ab+bc+ca \\ ab+bc+ca & ab+bc+ca & a^2+b^2+c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By equality of matrices, we get

$$a^2 + b^2 + c^2 = 1 \quad \text{...(i)}$$

$$ab + bc + ca = 0 \quad \text{...(ii)}$$

$$(a+b+c)^2 + a^2 = b^2 + c^2 + 2(ab + bc + ca)$$

$$= 1 + 0 = 1$$

$$\therefore \quad a + b + c = \pm 1 \quad \text{...(iii)}$$

**49.** $\because a^2b^2 + b^2c^2 + c^2a^2 = (ab + bc + ca)^2 - 2abc(a + b + c)$

$$= 0 - 2abc(\pm 1) = \mp 2\lambda \quad [\because abc = \lambda]$$

$$= -2\lambda \quad [\because \lambda < 0]$$

**50.** $\because a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$\Rightarrow \quad a^3 + b^3 + c^3 - 3\lambda = (\pm 1)(1 - 0)$

[from Eqs. (i), (ii) and (iii) and $abc = \lambda$]

$\Rightarrow \quad a^3 + b^3 + c^3 = 3\lambda \pm 1$

**51.** Equation whose roots are $a, b, c$ is

$$x^3 - (a+b+c)x^2 + (ab+bc+ca)x - abc = 0$$

$\Rightarrow \quad x^3 - (\pm 1)x^2 + 0 - \lambda = 0$

$\therefore \quad x^3 \pm x^2 - \lambda = 0$

## Passage (Q. Nos. 52 to 53)

$$\therefore \quad A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$\Rightarrow \quad t_1 = \dfrac{a_{11} + a_{12} + a_{13}}{3} = 0, \quad [\because a_{ij} + a_{jk} + a_{ki} = 0]$

$$t_2 = \frac{a_{21} + a_{22} + a_{23}}{3} = 0$$

and $\quad t_3 = \dfrac{a_{31} + a_{32} + a_{33}}{3} = 0$

**52.** $\displaystyle\sum_{1 \le i,} \sum_{j \le 3} a_{ij} = 3(t_1 + t_2 + t_3) = 0 = t_1 + t_2 + t_3$

$$\neq t_1t_2t_3 \quad [\because t_1 = 0, t_2 = 0, t_3 = 0]$$

and $\quad \det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$= \begin{vmatrix} 0 & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{vmatrix} = 0$$

$\therefore \quad (\det A)^2 = 0$

**53.** $\because \quad a_{11} + a_{11} + a_{11} = 0, a_{11} + a_{12} + a_{21} = 0,$

$a_{11} + a_{13} + a_{31} = 0, a_{22} + a_{22} + a_{22} = 0,$

$a_{22} + a_{12} + a_{21} = 0, a_{22} + a_{23} + a_{32} = 0,$

$a_{33} + a_{13} + a_{31} = 0, a_{33} + a_{23} + a_{32} = 0$

and $a_{33} + a_{12} + a_{21} = 0$, we get

$$a_{11} = a_{22} = a_{33} = 0$$

and $a_{12} = -a_{21}, a_{23} = -a_{32}, a_{13} = -a_{31}$

Hence, $A$ is skew-symmetric matrix.

## Passage (Q. Nos. 54 to 56)

Let $\quad B = \begin{bmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{bmatrix}$

$\therefore \quad C_1 = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}, C_2 = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}$ and $C_3 = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix}$

$\Rightarrow \quad AC_1 = \begin{bmatrix} \alpha_1 \\ 2\alpha_1 + \alpha_2 \\ 3\alpha_1 + 2\alpha_2 + \alpha_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow \quad \alpha_1 = 1, \alpha_2 = -2, \alpha_3 = 1$

$\Rightarrow \quad AC_2 = \begin{bmatrix} \beta_1 \\ 2\beta_1 + \beta_2 \\ 3\beta_1 + 2\beta_2 + \beta_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$

$\Rightarrow \quad \beta_1 = 2, \beta_2 = -1, \beta_3 = -4$

and $\quad AC_3 = \begin{bmatrix} \gamma_1 \\ 2\gamma_1 + \gamma_2 \\ 3\gamma_1 + 2\gamma_2 + \gamma_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$

$\Rightarrow \quad \gamma_1 = 2, \gamma_2 = -1, \gamma_3 = -3$

$\therefore \quad B = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix}$

$\Rightarrow \quad \det B = \begin{vmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{vmatrix}$

$$= 1(3 - 4) - 2(6 + 1) + 2(8 + 1) = 3$$

and $\quad C = \dfrac{1}{3}\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix}$

$$= \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 0 & 3 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad \det C = \begin{vmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ 0 & 1 & 1 \\ 0 & 0 & \frac{1}{3} \end{vmatrix} = \frac{1}{9}$$

**54.** $\det(B^{-1}) = \dfrac{1}{\det B} = \dfrac{1}{3}$

**55.** $\dfrac{\text{Trace of } B}{\text{Trace of } C} = \dfrac{(-3)}{\left(\frac{5}{3}\right)} = -\dfrac{9}{5}$

**56.** $\sin^{-1}(\det A) + \tan^{-1}(9\det C) = \sin^{-1}(1) + \tan^{-1}(1)$

$$= \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$$

## Passage (Q. Nos. 57 to 59)

Given, $A^T = A$, $B^T = -B$, $\det(A+B) \neq 0$

and $\quad C = (A+B)^{-1}(A-B)$

$\Rightarrow \quad (A+B)C = A - B \quad$ ...(i)

Also, $\quad (A+B)^T = A - B \quad$ ...(ii)

and $\quad (A-B)^T = A + B \quad$ ...(iii)

**57.** $C^T(A+B)C = C^T[(A+B)C]$

$= C^T(A-B)$ [from Eq. (i)]

$= C^T(A+B)^T$ [from Eq. (ii)]

$= [(A+B)C]^T$

$= (A-B)^T$ [from Eq. (i)]

$= A + B$ [from Eq. (iii)]

**58.** $C^T(A-B)C = [C^T(A+B)^T]C$ [from Eq. (ii)]

$= [(A+B)C]^T C$

$= (A-B)^T C$ [from Eq. (i)]

$= (A+B)C$ [from Eq. (iii)]

$= A - B$ [from Eq. (i)]

**59.** $C^T AC = C^T\left(\dfrac{A+B+A-B}{2}\right)C$

$= \dfrac{1}{2}C^T(A+B)C + \dfrac{1}{2}C^T(A-B)C$

$= \dfrac{1}{2}(A+B) + \dfrac{1}{2}(A-B)$ [from Q13 and Q15]

$= A$

## Passage (Q. Nos. 60 to 61)

$\because \quad B = A - 2I$

$\therefore \quad A^{-1}B = I - 2A^{-1} \quad$ ...(i)

**60.** $\det[\text{adj}(I - 2A^{-1})] = \det[\text{adj}(A^{-1}B)]$ [from Eq. (i)]

$= |\text{adj}(A^{-1}B)|$

$= |A^{-1}B|^2 = (|A^{-1}||B|)^2 = \left(\dfrac{|B|}{|A|}\right)^2 \quad$ ...(ii)

From Eq. (i), we get $B = A - 2I$

$\therefore \quad B^3 = (A-2I)^3 = A^3 - 6A^2 + 12A - 8I$

$= 5A \quad [\because A^3 - 6A^2 + 7A - 8I = 0]$

$\Rightarrow \quad |B^3| = |5A|$

$\Rightarrow \quad |B|^3 = 5^3|A|$

$\Rightarrow \quad |B|^3 = 5^3 \times 8$

$\Rightarrow \quad |B|^3 = (10)^3$

$\therefore \quad |B| = 10$

From Eq. (ii), we get

$$\det[\text{adj}(I - 2A^{-1})] = \left(\frac{|B|}{|A|}\right)^2 = \left(\frac{10}{8}\right)^2 = \frac{25}{16}$$

**61.** $\text{adj}\left[\left(\dfrac{B}{2}\right)^{-1}\right] = \dfrac{\frac{B}{2}}{\left|\frac{B}{2}\right|} = \dfrac{\frac{B}{2}}{\frac{1}{8}|B|} = \dfrac{4B}{|B|} = \dfrac{4}{10}B \quad [\because |B| = 10]$

$= \dfrac{2}{5}B = \dfrac{p}{q}B$ [given]

$\therefore \quad p = 2$ and $q = 5$

Hence, $\quad p + q = 7$

**62.** $S = ABCD = A(BCD) = AA^T \quad$ ...(i)

$\therefore \quad S^3 = (ABCD)(ABCD)(ABCD)$

$= (ABC)(DAB)(CDA)(BCD)$

$= D^T C^T B^T A^T = (BCD)^T A^T$

$= (A^T)^T A^T = AA^T = S$

$\Rightarrow \quad S^3 = S$

Hence, least value of $k$ is 3.

**63.** $\because \quad A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$

$\therefore \quad \det A = \begin{vmatrix} 1 & \tan x \\ -\tan x & 1 \end{vmatrix} = (1 + \tan^2 x) = \sec^2 x$

$\Rightarrow \quad \det A^T = \det A = \sec^2 x$

Now, $\quad f(x) = \det(A^T A^{-1}) = (\det A^T)(\det A^{-1})$

$= (\det A^T)(\det A)^{-1} = \dfrac{\det A^T}{\det A} = 1$

$\therefore \quad \underbrace{\lambda = f(f(f(f \ldots f(x))))}_{n \text{ times}} = 1 \quad [\because f(x) = 1]$

Hence, $\quad 2^\lambda = 2^1 = 2$

**64.** $\because A^2 = A \cdot A = \begin{bmatrix} \lambda_1^2 & \lambda_1\lambda_2 & \lambda_1\lambda_3 \\ \lambda_2\lambda_1 & \lambda_2^2 & \lambda_2\lambda_3 \\ \lambda_3\lambda_1 & \lambda_3\lambda_2 & \lambda_3^2 \end{bmatrix}\begin{bmatrix} \lambda_1^2 & \lambda_1\lambda_2 & \lambda_1\lambda_3 \\ \lambda_2\lambda_1 & \lambda_2^2 & \lambda_2\lambda_3 \\ \lambda_3\lambda_1 & \lambda_3\lambda_2 & \lambda_3^2 \end{bmatrix}$

$$= \begin{bmatrix} \lambda_1^2(\lambda_1^2 + \lambda_2^2 + \lambda_3^2) & \lambda_1\lambda_2(\lambda_1^2 + \lambda_2^2 + \lambda_3^2) & \lambda_1\lambda_3(\lambda_1^2 + \lambda_2^2 + \lambda_3^2) \\ \lambda_1\lambda_2(\lambda_1^2 + \lambda_2^2 + \lambda_3^2) & \lambda_2^2(\lambda_1^2 + \lambda_2^2 + \lambda_3^2) & \lambda_2\lambda_3(\lambda_1^2 + \lambda_2^2 + \lambda_3^2) \\ \lambda_1\lambda_3(\lambda_1^2 + \lambda_2^2 + \lambda_3^2) & \lambda_3\lambda_2(\lambda_1^2 + \lambda_2^2 + \lambda_3^2) & \lambda_3^2(\lambda_1^2 + \lambda_2^2 + \lambda_3^2) \end{bmatrix}$$

$$= (\lambda_1^2 + \lambda_2^2 + \lambda_3^2)\, A$$

Given, $A$ is idempotent

$\Rightarrow \quad A^2 = A$

$\therefore \quad \lambda_1^2 + \lambda_2^2 + \lambda_3^2 = 1$

**65.** Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and given $X^T AX = O$

$$\Rightarrow \quad [x_1\ x_2\ x_3] \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = O$$

$$\Rightarrow \quad [x_1\ x_2\ x_3] \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{bmatrix} = O$$

$$\Rightarrow \quad a_{11}x_1^2 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{21}x_1x_2 + a_{22}x_2^2 + a_{23}x_2x_3 + a_{31}x_1x_3 + a_{32}x_2x_3 + a_{33}x_3^2 = 0$$

$$\Rightarrow \quad a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + (a_{12} + a_{21})\, x_1x_2 + (a_{23} + a_{32})\, x_2x_3 + (a_{31} + a_{13})\, x_3x_1 = 0$$

it is true for every $x_1, x_2, x_3$, then

$$a_{11} = a_{22} = a_{33} = 0 \text{ and } a_{12} = -a_{21}, a_{23} = -a_{32}, a_{13} = -a_{31}$$

Now, as $a_{23} = -1008 \Rightarrow a_{32} = 1008$

$\therefore$ Sum of digits $= 1 + 0 + 0 + 8 = 9$

**66.** $\because A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

$$\therefore \quad A^2 = A \cdot A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$\Rightarrow \quad A^2 = I \Rightarrow A^4 = A^6 = A^8 = \dots = I$

Now, $\quad A^x = I$

$\Rightarrow \quad x = 2, 4, 6, 8, \dots$

$$\therefore \sum (\cos^x \theta + \sin^x \theta) = (\cos^2 \theta + \sin^2 \theta) + (\cos^4 \theta + \sin^4 \theta) + (\cos^6 \theta + \sin^6 \theta) + \dots$$

$$= (\cos^2 \theta + \cos^4 \theta + \cos^6 \theta + \dots) + (\sin^2 \theta + \sin^4 \theta + \sin^6 \theta + \dots)$$

$$= \frac{\cos^2 \theta}{1 - \cos^2 \theta} + \frac{\sin^2 \theta}{1 - \sin^2 \theta}$$

$$= \cot^2 \theta + \tan^2 \theta \geq 2$$

Hence, minimum value of $\sum (\cos^x \theta + \sin^x \theta)$ is 2.

**67.** $\because A$ is idempotent matrix

$\therefore \quad A^2 = A$

$\Rightarrow \quad A = A^2 = A^3 = A^4 = A^5 = \dots \quad$ ...(i)

Now, $(A + I)^n = (I + A)^n$

$$= I + {}^nC_1\, A + {}^nC_2\, A^2 + {}^nC_3\, A^3 + \dots + {}^nC_n A^n$$

$$= I + ({}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n)\, A$$

[from Eq.(i)]

$\Rightarrow \quad (A + I)^n = I + (2^n - 1)A \quad$ ...(ii)

Given, we get

$(A + I)^n = I + 127\, A \quad$ ...(iii)

From Eqs (ii) and (iii), we get

$$2^n - 1 = 127$$

$\Rightarrow \quad 2^n = 128 = 2^7$

$\therefore \quad n = 7$

**68.** $\because \quad A = \begin{bmatrix} 3a & b & c \\ b & 3c & a \\ c & a & 3b \end{bmatrix}$

$$\therefore \det(A) = \begin{vmatrix} 3a & b & c \\ b & 3c & a \\ c & a & 3b \end{vmatrix} = 29abc - 3(a^3 + b^3 + c^3)$$

Or

$|A| = 29abc - 3(a^3 + b^3 + c^3) \quad$ ...(i)

Given, $\quad A^T A = 4^{1/3}\, I$

$\Rightarrow \quad |A^T A| = |4^{1/3}\, I|$

$\Rightarrow \quad |A^T||A| = (4^{1/3})^3\, |I|$

$\Rightarrow \quad |A||A| = 4 \cdot 1$

$\Rightarrow \quad |A|^2 = 4$

$\therefore \quad |A| = 2 \quad [\because |A| > 0]$

From Eq.(i), we get

$$2 = 29abc - 3\,(a^3 + b^3 + c^3)$$

$\Rightarrow \quad 2 = 29 - 3\,(a^3 + b^3 + c^3) \quad [\because abc = 1]$

$\therefore \quad a^3 + b^3 + c^3 = 9$

**69.** $\because \quad A = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = 3I$$

$\Rightarrow \quad A^4 = (A^2)^2 = 9I, A^6 = 27I, A^8 = 81\,I$

Now, $\quad (A^8 + A^6 + A^4 + A^2 + I)\, V = (121)\, IV = (121)\, V \quad$ ...(i)

Given, $(A^8 + A^6 + A^4 + A^2 + I)\, V = \begin{bmatrix} 0 \\ 11 \end{bmatrix} \quad$ ...(ii)

From Eqs.(i) and (ii), $(121)\, V = \begin{bmatrix} 0 \\ 11 \end{bmatrix} \Rightarrow V = \begin{bmatrix} 0 \\ \frac{1}{11} \end{bmatrix}$

$\therefore$ Sum of elements of $\quad V = 0 + \frac{1}{11} = \frac{1}{11} = \lambda \quad$ [given]

$\therefore \quad 11\,\lambda = 1$

**70.** $\because A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 7 & 3 \end{bmatrix}$

$\therefore \quad \det A = -1$ and $\det B = 2$

Now, $\det(2A^9\, B^{-1}) = 2^2 \cdot \det(A^9) \cdot \det(B^{-1})$

$$= 2^2 \cdot (\det A)^9 \cdot (\det B)^{-1}$$

$$= 2^2 \cdot (-1)^9 \cdot (2)^{-1} = -2$$

Hence, absolute value of $\det\ (2A^9 B^{-1}) = 2$

**71.** $\because \quad A = \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix}$

$\therefore \quad A^2 = A \cdot A = \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

$\Rightarrow \quad A^2 = A^3 = A^4 = A^5 = \ldots = 0$

Now, $(A + I)^{70} = (I + A)^{70}$

$= I + {}^{70}C_1\, A + {}^{70}C_2\, A^2 + {}^{70}C_3\, A^3 + .. + {}^{70}C_{70}\, A^{70}$

$= I + 70\, A + 0 + 0 + \ldots = I + 70A$

$\Rightarrow (A + I)^{70} - 70A = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a-1 & b-1 \\ c-1 & d-1 \end{bmatrix}$ [given]

$\therefore \quad a - 1 = 1, b - 1 = 0, c - 1 = 0, d - 1 = 1$

$\Rightarrow \quad a = 2, b = 1, c = 1, d = 2$

Hence, $a + b + c + d = 6$

**72.** (A) $\rightarrow$ (p, r); (B) $\rightarrow$ (s); (C) $\rightarrow$ (q); (D) $\rightarrow$ (s)

On comparing, we get

$\{4f(-1) - 3\}a^2 + \{4f(1) - 3\}a + f(2) = 0$

$\{4f(-1) - 3\}b^2 + \{4f(1) - 3\}b + f(2) = 0,$

and $\{4f(-1) - 3\}c^2 + \{4f(1) - 3\}C + f(2) = 0$

It is clear that $a, b, c$ are the roots of

$\{4f(-1) - 3\}x^2 + \{4f(1) - 3\}x + f(2) = 0$, then

$4f(-1) - 3 = 0, 4f(1) - 3 = 0, f(2) = 0$

$\Rightarrow \quad f(-1) = \frac{3}{4}, f(1) = \frac{3}{4}, f(2) = 0$

Let $\quad f(x) = (x - 2)(ax + b)$

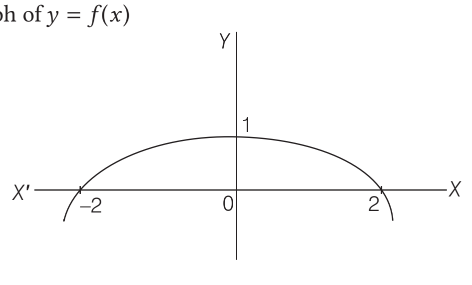
Now, $\quad f(-1) = \frac{3}{4} \Rightarrow (-3)(-a + b) = \frac{3}{4} \Rightarrow a - b = \frac{1}{4}$

$f(1) = \frac{3}{4} \Rightarrow (-1)(a + b) = \frac{3}{4} \Rightarrow a + b = -\frac{3}{4}$

$\therefore \quad a = -\frac{1}{4}, b = -\frac{1}{2}$

$\Rightarrow \quad f(x) = \frac{1}{4}(4 - x^2)$

Graph of $y = f(x)$




**(A)** $x$-coordinates of the point intersection of $y = f(x)$ with the $X$-axis are $-2$ and 2.

**(B)** Area $= \frac{3}{2}\int_{-2}^{2} \frac{1}{4}(4 - x^2)dx = \frac{3}{4}\int_0^2 (4 - x^2)dx$

$= \frac{3}{4}\left[4x - \frac{x^3}{3}\right]_0^2 = \frac{3}{4} \times \frac{16}{3} = 4$

**(C)** Maximum value of $f(x)$ is 1.

**(D)** Length of intercept on the $X$-axis is 4.

**73.** (A) $\rightarrow$ (r, t); (B) $\rightarrow$ (s); (C) $\rightarrow$ (p); (D) $\rightarrow$ (q)

**(A)** adj $(A^{-1}) = (A^{-1})^{-1}$ det $(A^{-1}) = \dfrac{A}{\det(A)}$

Also, $\dfrac{\text{adj (adj } A)}{(\text{adj } A)^{n-1}} = \dfrac{A[\det(A)]^{n-2}}{(\det A)^{n-1}} = \dfrac{A}{\det(A)}$

**(B)** $\det(\text{adj } (A^{-1})) = (\det\, A^{-1})^{n-1}$

$= \dfrac{1}{(\det A)^{n-1}} = (\det A)^{1-n}$

**(C)** adj [adj $A$] $= A(\det A)^{n-2}$

**(D)** adj $(A \det A) = (\det A)^{n-1}$ (adj $A$)

**74.** (A)$\rightarrow$(q, s); (B)$\rightarrow$(p, t); (C)$\rightarrow$(p, q, r, s); (D)$\rightarrow$(q, s)

**(A)** A diagonal matrix is commutative with every square matrix, if it is scalar matrix, so every diagonal element is 4.

Therefore, $|A| = \begin{vmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{vmatrix} = 64$

**(B)** $\begin{vmatrix} 1-a & 1 & 1 \\ 1 & 1-b & 1 \\ 1 & 1 & 1-c \end{vmatrix} = 0$

Applying $R_1 \rightarrow R_1 - R_3$ and $R_2 \rightarrow R_2 - R_3$, then

$\begin{vmatrix} -a & 0 & c \\ 0 & -b & c \\ 1 & 1 & 1-c \end{vmatrix} = 0$

$\Rightarrow \quad -a(-b + bc - c) - 0 + c(b) = 0$

$ab + bc + ca = abc$ ...(i)

Now, $\quad AM \geq GM$

$\Rightarrow \quad \dfrac{ab + bc + ca}{3} \geq (ab \cdot bc \cdot ca)^{\frac{1}{3}}$

$\Rightarrow \quad \dfrac{abc}{3} \geq (abc)^{\frac{2}{3}}$ [from Eq. (i)]

$\Rightarrow \quad (abc)^{\frac{1}{3}} \geq 3$

$\therefore \quad abc \geq 27$

Hence, $\quad \lambda = 27$

**(C)** $\because \quad A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Given, $\sum_{k=1}^{3} a_{ik} = 9\lambda_i, \forall\ i \in \{1, 2, 3\};$

$\sum_{k=1}^{3} a_{kj} = 9\mu_j, \forall\ j \in \{1, 2, 3\}$ and

$a_{11} + a_{22} + a_{33} = 9\upsilon$; where $\lambda_i, \mu_j, \nu \in \{1, 2\}$

Following types of matrices are possible:

$A = \begin{bmatrix} 1 & & \\ & 3 & \\ & & 5 \end{bmatrix}; B = \begin{bmatrix} 2 & & \\ & 3 & \\ & & 4 \end{bmatrix}; C = \begin{bmatrix} 7 & & \\ & 3 & \\ & & 8 \end{bmatrix};$

$D = \begin{bmatrix} 6 & & \\ & 3 & \\ & & 9 \end{bmatrix}; E = \begin{bmatrix} 1 & & \\ & 6 & \\ & & 2 \end{bmatrix}; F = \begin{bmatrix} 3 & & \\ & 6 & \\ & & 9 \end{bmatrix};$

$$G=\begin{bmatrix}4&&\\&6&\\&&8\end{bmatrix};\ H=\begin{bmatrix}5&&\\&6&\\&&7\end{bmatrix};\ I=\begin{bmatrix}1&&\\&9&\\&&8\end{bmatrix};$$

$$\mathcal{J}=\begin{bmatrix}2&&\\&9&\\&&7\end{bmatrix};\ K=\begin{bmatrix}3&&\\&9&\\&&6\end{bmatrix};\ L=\begin{bmatrix}4&&\\&9&\\&&5\end{bmatrix}$$

Now, if we interchange 1 and 5 to obtain

$$A_1=\begin{bmatrix}5&4&9\\7&3&8\\6&2&1\end{bmatrix}$$

Also, $$A^T=\begin{bmatrix}1&8&9\\2&3&4\\6&7&5\end{bmatrix}$$

and $$A_1^T=\begin{bmatrix}5&7&6\\4&3&2\\9&8&1\end{bmatrix}$$

Then, from $A$ we get four matrices $A, A_1, A^T, A_1^T$.

Similarly, from $B, C, D, \ldots, K, L$ we get 4 matrices.

Thus, total $12 \times 4 = 48$ matrices. Hence, $\lambda = 48$.

**(D)** For consistent, $\begin{vmatrix}1&1&1\\c+2&c+4&6\\(c+2)^2&(c+4)^2&36\end{vmatrix}=0$

Applying $C_2 \to C_2 - C_1$, we get

$$\begin{vmatrix}1&0&1\\c+2&2&6\\(c+2)^2&4c+12&36\end{vmatrix}=0$$

$$\Rightarrow \quad 2\begin{vmatrix}1&0&1\\c+2&1&6\\(c+2)^2&2c+6&36\end{vmatrix}=0$$

$\Rightarrow \quad -12c-0+1[(c+2)(2c+6)-(c+2)^2]=0$

$\Rightarrow \quad c^2-6c+8=0$

$\Rightarrow \quad c=2, 4$

$\therefore \quad c_1=4, c_2=2$

$\Rightarrow \quad c_1^{c_2}=4^2=16$

**75.** (A) $\to$(q, t); (B) $\to$(p, s); (C) $\to$(p, r, s); (D) $\to$(q, r, t)

(A) Here, $X$ is a $n \times 1$ matrix, $C$ is $n \times n$ matrix and $X^T$ is a $1 \times n$ matrix.

Hence, $X^TCX$ is a $1 \times 1$ matrix.

Let $X^TCX=[\lambda]$, then

$$(X^TCX)^T = X^TC^T(X^T)^T = X^T(-C)X = -X^TCX$$

$\therefore \quad [\lambda]=-[\lambda]$

$\Rightarrow \quad \lambda=0$

$\Rightarrow \quad X^TCX=O$

i.e., $X^TCX$ is null matrix.

(B) Consider the homogeneous system

$(I-A)X=O$

$\Rightarrow \quad AX=IX=X \qquad \ldots$ (i)

Now, $\quad (AX)^T = X^TA^T \Rightarrow X^T=-X^TA$

$\Rightarrow \quad X^TX=-X^TAX=-X^TX \qquad$ [from Eq. (i)]

$\Rightarrow \quad 2X^TX=O \quad \Rightarrow \quad |X|=O$

$(I-A)X=O$ has only trivial solution

$\therefore \quad I-A$ is non-singular

$\Rightarrow$ $(I-A)$ is invertible

(C) $\because \quad S=\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}$

$$\Rightarrow \quad S^{-1}=\frac{1}{2}\begin{bmatrix}-1&1&1\\1&-1&1\\1&1&-1\end{bmatrix}$$

We have, $SA=\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}\begin{bmatrix}b+c&c-a&b-a\\c-b&c+a&a-b\\b-c&a-c&a+b\end{bmatrix}$

$$=\begin{bmatrix}0&2a&2a\\2b&0&2b\\2c&2c&0\end{bmatrix}$$

$$\therefore \quad SAS^{-1}=\begin{bmatrix}0&2a&2a\\2b&0&2b\\2c&2c&0\end{bmatrix}\frac{1}{2}\begin{bmatrix}-1&1&1\\1&-1&1\\1&1&-1\end{bmatrix}$$

$$=\begin{bmatrix}0&a&a\\b&0&b\\c&c&0\end{bmatrix}\begin{bmatrix}-1&1&1\\1&-1&1\\1&1&-1\end{bmatrix}=\begin{bmatrix}2a&0&0\\0&2b&0\\0&0&2c\end{bmatrix}$$

$\therefore \quad |SAS^{-1}|=8abc \neq 0$

(D) $\because \quad A=\begin{bmatrix}\sin 2A&\sin C&\sin B\\\sin C&\sin 2B&\sin A\\\sin B&\sin A&\sin 2C\end{bmatrix}$

$$|A|=\begin{bmatrix}2ak\cos A&ck&bk\\ck&2bk\cos B&ak\\bk&ak&2ck\cos C\end{bmatrix}$$

$$=k^3\begin{bmatrix}a\cos A+a\cos A&a\cos B+b\cos A&a\cos C+c\cos A\\a\cos B+b\cos A&b\cos B+b\cos B&b\cos C+c\cos B\\a\cos C+c\cos A&b\cos C+c\cos B&c\cos C+c\cos C\end{bmatrix}$$

$$=k^3\begin{vmatrix}a&\cos A&0\\b&\cos B&0\\c&\cos C&0\end{vmatrix}\times\begin{vmatrix}\cos A&a&0\\\cos B&b&0\\\cos C&c&0\end{vmatrix}=k^3\cdot 0\cdot 0=0$$

**76.** Since, matrix $A$ is skew-symmetric

$\therefore \quad |A|=0$

$\therefore \quad |A^4B^5|=0$

$\Rightarrow A^4B^5$ is singular matrix.

Statement-1 is false and Statement-2 is true.

**77.** $\because \quad AB = A, BA = B \Rightarrow A^2 = A$ and $B^2 = B$

$\therefore \quad (A+B)^2 = A^2 + B^2 + AB + BA = A + B + A + B$

$= 2(A+B)$

$(A+B)^3 = (A+B)^2 . (A+B)$

$= 2(A+B)^2 = 2^2 (A+B)$

$\therefore \quad (A+B)^7 = 2^6 (A+B)$

Statement-1 is true and Statement-2 is false.

**78.** $A^{-1}$exists only for non-singular matrix

$$AB = AC \Rightarrow A^{-1}(AB) = A^{-1}(AC)$$

$\Rightarrow \quad (A^{-1} A) B = (A^{-1} A) C$

$\Rightarrow \quad IB = IC$

$\Rightarrow \quad B = C$, if $A^{-1}$ exist

$\therefore \quad |A| \neq 0$

Statement-1 is false and Statement-2 is true.

**79.** Statement-2 is false

$\because \quad \det(A^{-1}) \neq \det(-A')$

$[\because \det(-A') = (-1)^3 \det(A') = -\det(A')]$

but in Statement-1

$$A' = -A \Rightarrow A = -A'$$

$\therefore \quad \det(A) = \det(-A')$

$= -\det A' = -\det(A)$

$\Rightarrow \quad 2\det(A) = 0$

$\therefore \quad \det(A) = 0$

Then, Statement-1 is true.

**80.** $\because (BX)^T (BY) = \{(I-A)(I+A)^{-1} X\}^T (I-A)(I+A)^{-1} Y$

$= X^T \{(I+A)^{-1}\}^T (I-A)^T (I-A)(I+A)^{-1} Y$

$= X^T (I+A^T)^{-1} (I-A^T)(I-A)(I+A)^{-1} Y$

$= X^T (I-A)^{-1} (I+A)(I-A)(I+A)^{-1} Y$

$= X^T (I-A)^{-1} (I-A)(I+A)(I+A)^{-1} Y$

$[\because A^T = -A$ and $(I-A)(I+A) = (I+A)(I-A)]$

$= X^T \cdot I \cdot I \cdot Y = X^T Y$

Both Statements are true; Statement-2 is correct explanation for Statement-1.

**81.** $\because \quad |A| = 2$

and $\quad B = 9A^2$ (given)

$\therefore \quad |B| = |9A^2| = 9^2 |A|^2$

$= 81 \times 4 = 324 \Rightarrow |B^T| = |B| = 324$

Hence, Statement-1 is false but Statement-2 is true.

**82.** $\therefore \det(A - \lambda I) = \begin{vmatrix} 1-\lambda & -1 & -1 \\ 1 & -1-\lambda & 0 \\ 1 & 0 & -1-\lambda \end{vmatrix} = 0$

$\Rightarrow \quad (1-\lambda)(1+\lambda)^2 - 1 - \lambda - 1 - \lambda = 0$

$\Rightarrow \quad \lambda^3 + \lambda^2 + \lambda + 1 = 0$

$\Rightarrow \quad A^3 + A^2 + A + I = 0$

$\Rightarrow \quad A^3 + A^2 + A = -I$

Statement-1 is false but Statement-2 is true.

**83.** $A = \begin{bmatrix} 0 & -\frac{1}{5} & -\frac{2}{7} \\ \frac{1}{4} & 0 & -\frac{1}{8} \\ \frac{2}{5} & \frac{1}{7} & 0 \end{bmatrix}$

which is neither symmetric nor skew-symmetric. Infact every square matrix can be expressed as a sum of symmetric and skew-symmetric matrix. Hence, Statement-1 is false and Statement-2 is true.

**84.** $ABC$ is not defined, as order of $A$, $B$ and $C$ are such that they are not conformable for multiplication.

Hence, Statement-1 is false and Statement-2 is true.

**85.** $\because \quad A^T = -A$

$\Rightarrow \quad |A^T| = |-A|$

$= (-1)^5 |A| = -|A|$

$\Rightarrow \quad |A| = -|A|$

$\Rightarrow \quad 2|A| = 0$

$\therefore \quad |A| = 0$

Both Statements are true but Statement-2 is a correct explanation of Statement-1.

**86.** $\because S$ is skew-symmetric matrix

$\therefore \quad S^T = -S \quad$ ...(i)

First we will show that $I - S$ is non-singular. The equality $|I - S| = 0 \Rightarrow I$ is a characteristic root of the matrix $S$ but this is not possible, for a real skew-symmetric matrix can have zero or purely imaginary numbers as its characteristic roots. Thus, $|I - S| \neq 0$ i.e., $I - S$ is non-singular.

We have,

$A^T = \{(I+S)(I-S)^{-1}\}^T = \{(I-S)^{-1}(I+S)\}^T$

$= ((I-S)^{-1})^T (I+S)^T = (I+S)^T \{(I-S)^{-1}\}^T$

$= ((I-S)^T)^{-1} (I+S)^T = (I+S)^T ((I-S)^T)^{-1}$

$= (I^T - S^T)^{-1} (I^T + S^T) = (I^T + S^T)(I^T - S^T)^{-1}$

$= (I+S)^{-1}(I-S) = (I-S)(I+S)^{-1}$ [from Eq. (i)]

$\therefore \quad A^T A = (I+S)^{-1}(I-S)(I+S)(I-S)^{-1}$

$= (I-S)(I+S)^{-1}(I-S)^{-1}(I+S)$

$= (I+S)^{-1}(I+S)(I-S)(I-S)^{-1}$

$= (I-S)(I-S)^{-1}(I+S)^{-1}(I+S)$

$= I \cdot I = I \cdot I = I = I$

Hence, $A$ is orthogonal.

**87.** $\because \quad MM^T = I \quad$ ...(i)

Let $\quad B = M - I \quad$ ...(ii)

$\therefore \quad B^T = M^T - I^T = M^T - M^T M \quad$ [from Eq. (i)]

$= M^T (I - M) = -M^T B \quad$ [from Eq. (ii)]

Now, $\det(B^T) = \det(-M^T B)$

$= (-1)^3 \det(M^T) \det(B) = -\det(M^T) \det(B)$

$\Rightarrow \quad \det(B) = -\det(M) \det(B) = -\det(B)$

$\therefore \quad \det(B) = 0$

$\Rightarrow \quad \det(M - I) = 0$

**88.** $\because \quad BAB = A^{-1}$

$\Rightarrow \quad ABAB = I$

$\Rightarrow \quad (AB)^2 = I$

Now, $AB = \begin{bmatrix} \cos(\alpha + 2\beta) & \sin(\alpha + 2\beta) \\ \sin(\alpha + 2\beta) & -\cos(\alpha + 2\beta) \end{bmatrix}$

and $\quad (AB)^2 = (AB)(AB) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \quad [\because (AB)(AB) = I]$

Also, $\quad BA^4B = A^{-1}$

or $\quad A^4B = B^{-1}A^{-1} = (AB)^{-1} = AB$

or $\quad A^4 = A \quad$ ...(i)

Now, $\quad A^2 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$

Similarly, $A^4 = \begin{bmatrix} \cos 4\alpha & -\sin 4\alpha \\ \sin 4\alpha & \cos 4\alpha \end{bmatrix}$

Hence, from Eq. (i)

$$\begin{bmatrix} \cos 4\alpha & -\sin 4\alpha \\ \sin 4\alpha & \cos 4\alpha \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

or $\quad 4\alpha = 2\pi + \alpha$

$\therefore \quad \alpha = \dfrac{2\pi}{3}$

**89.** $AB = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\theta\cos^2\phi + \cos\theta\cos\phi\sin\theta\sin\phi & \cos^2\theta\cos\phi\sin\phi + \sin^2\phi\sin\theta\cos\theta \\ \cos^2\phi\cos\theta\sin\theta + \sin^2\theta\sin\phi\cos\phi & \cos\theta\cos\phi\sin\theta\sin\phi + \sin^2\theta\sin^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\phi(\cos\theta\cos\phi + \sin\theta\sin\phi) & \cos\phi\sin\phi(\cos\theta\cos\phi + \sin\theta\sin\phi) \\ \sin\theta\cos\phi(\cos\theta\cos\phi + \sin\theta\sin\phi) & \sin\theta\sin\phi(\cos\theta\cos\phi + \sin\theta\sin\phi) \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\phi\cos(\theta - \phi) & \cos\theta\sin\phi\cos(\theta - \phi) \\ \sin\theta\cos\phi\cos(\theta - \phi) & \sin\theta\sin\phi\cos(\theta - \phi) \end{bmatrix}$$

Clearly, $AB$ is the zero matrix, if $\cos(\theta \sim \phi) = 0$ i.e., $\theta - \phi$ is an odd multiple of $\dfrac{\pi}{2}$.

**90.** Let $\quad A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$

$\therefore \quad A^T = \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix}$

Now, $AA^T = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \times \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix}$

$$= \begin{bmatrix} \Sigma l_1^2 & \Sigma l_1 l_2 & \Sigma l_3 l_1 \\ \Sigma l_1 l_2 & \Sigma l_2^2 & \Sigma l_2 l_3 \\ \Sigma l_3 l_1 & \Sigma l_2 l_3 & \Sigma l_3^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Hence, matrix $A$ is orthogonal.

**91.** Let us use the symbols Div, Dis, Tal for division, district, taluka respectively and O, H, C, Cl, T and P for office superintendent, Head clerk, Cashier, Clerk, Typist and Peon respectively.

Then, the number of offices can be arranged as elements of a row matrix $A$ and the composition of staff in various offices can be arranged in a $3 \times 6$ matrix $B$ (say).

$$\therefore \quad A = \begin{array}{c} \begin{array}{ccc} \text{Div} & \text{Dis} & \text{Tal} \end{array} \\ \begin{bmatrix} 5 & 30 & 200 \end{bmatrix} \end{array}$$

and $$B = \begin{array}{c} \begin{array}{cccccc} \text{O} & \text{H} & \text{C} & \text{Cl} & \text{T} & \text{P} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 2+1 & 1 & 1+1 \\ 0 & 1 & 1 & 1+1 & 0 & 1+1 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

or $$B = \begin{array}{c} \begin{array}{cccccc} \text{O} & \text{H} & \text{C} & \text{Cl} & \text{T} & \text{P} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 3 & 1 & 2 \\ 0 & 1 & 1 & 2 & 0 & 2 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

The basic monthly salaries of various types of employees of these offices correspond to the elements of the column matrix $C$.

$$\therefore \quad C = \begin{array}{c} \text{O} \\ \text{H} \\ \text{C} \\ \text{Cl} \\ \text{T} \\ \text{P} \end{array} \begin{bmatrix} 500 \\ 200 \\ 175 \\ 150 \\ 150 \\ 100 \end{bmatrix}$$

(i) Total number of Posts $= AB$

$$= \begin{array}{c} \begin{array}{ccc} \text{Div} & \text{Dis} & \text{Tal} \end{array} \\ \begin{bmatrix} 5 & 30 & 200 \end{bmatrix} \end{array} \times \begin{array}{c} \begin{array}{cccccc} \text{O} & \text{H} & \text{C} & \text{Cl} & \text{T} & \text{P} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 3 & 1 & 2 \\ 0 & 1 & 1 & 2 & 0 & 2 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

$$= \begin{array}{c} \begin{array}{cccccc} \text{O} & \text{H} & \text{C} & \text{Cl} & \text{T} & \text{P} \end{array} \\ \begin{bmatrix} 5 & 235 & 235 & 275 & 5 & 270 \end{bmatrix} \end{array}$$

i.e., Required number of posts in all the offices taken together are 5 office Suprintendents, 235 Head Clerks, 235 Cashiers, 275 Clerks, 5 Typists and 270 Peons.

(ii) The total basic monthly salary bill of each kind of office $= BC$

$$= \begin{array}{c} \begin{array}{cccccc} \text{O} & \text{H} & \text{C} & \text{Cl} & \text{T} & \text{P} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 3 & 1 & 2 \\ 0 & 1 & 1 & 2 & 0 & 2 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array} \times \begin{bmatrix} 500 \\ 200 \\ 175 \\ 150 \\ 150 \\ 100 \end{bmatrix} \begin{array}{c} \text{O} \\ \text{H} \\ \text{C} \\ \text{Cl} \\ \text{T} \\ \text{P} \end{array}$$

$$= \begin{bmatrix} 500 + 200 + 175 + 3 \times 150 + 1 \times 150 + 2 \times 100 \\ 0 + 1 \times 200 + 1 \times 175 + 2 \times 150 + 0 + 2 \times 100 \\ 0 + 1 \times 200 + 1 \times 175 + 1 \times 150 + 0 + 1 \times 100 \end{bmatrix}$$

$$= \begin{bmatrix} 1675 \\ 875 \\ 625 \end{bmatrix}$$

i.e., The total basic monthly salary bill of each divisional, district and taluka offices are ₹ 1675, ₹ 875 and ₹ 625, respectively.

(iii) The total basic monthly salary bill of all the offices taken together

$$= ABC = A(BC)$$

$$= [5 \quad 30 \quad 200] \times \begin{bmatrix} 1675 \\ 875 \\ 625 \end{bmatrix}$$

$$= [5 \times 1675 + 30 \times 875 + 200 \times 625]$$

$$= [159625]$$

Hence, total basic monthly salary bill of all the offices taken together is ₹ 159625.

**92.** The total load of stone and sand supplied by $A$ can be represented by row matrix $X_1$ and cost of one truck load of stone and sand can be represented by column matrix $Y_1$.

$$\therefore \quad X_1 = [40 \quad 10],\ Y_1 = \begin{bmatrix} 1200 \\ 500 \end{bmatrix}$$

Total amount paid by contractor to $A = X_1 Y_1$

$$= [40 \quad 10] \begin{bmatrix} 1200 \\ 500 \end{bmatrix}$$

$$= [48000 + 5000]$$

$$= [53000]$$

$\therefore$ Amount paid by contractor to $A$ is ₹ 53000.

Similarly for $B$, $X_2 = [35 \quad 5]$, $Y_2 = \begin{bmatrix} 1200 \\ 500 \end{bmatrix}$

Total amount paid by contractor to $B = X_2 Y_2$

$$= [35 \quad 5] \begin{bmatrix} 1200 \\ 500 \end{bmatrix} = [42000 + 2500]$$

$$= [44500]$$

$\therefore$ Amount paid by contractor to $B$ is ₹ 44500.

Similarly for $C$,

$$X_3 = [25 \quad 8],\ Y_3 = \begin{bmatrix} 1200 \\ 500 \end{bmatrix}$$

Total amount paid by contractor to $C = X_3 Y_3$

$$= [25 \quad 8] \begin{bmatrix} 1200 \\ 500 \end{bmatrix}$$

$$= [30000 + 4000] = [34000]$$

$\therefore$ Amount paid by contractor to $C$ is ₹ 34000.

**93.** We have, $A = \begin{bmatrix} 1 & a & \alpha & a\alpha \\ 1 & b & \beta & b\beta \\ 1 & c & \gamma & c\gamma \end{bmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$A = \begin{bmatrix} 1 & a & \alpha & a\alpha \\ 0 & b-a & \beta-\alpha & b\beta - a\alpha \\ 0 & c-a & \gamma-\alpha & c\gamma - a\alpha \end{bmatrix}$$

Applying $C_2 \to C_2 - aC_1$, $C_3 \to C_3 - \alpha C_1$ and $C_4 \to C_4 - a\alpha C_1$, we get

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & b-a & \beta-\alpha & b\beta - a\alpha \\ 0 & c-a & \gamma-\alpha & c\gamma - a\alpha \end{bmatrix}$$

Applying $C_4 \to C_4 - \alpha C_2 - bC_3$, we get

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & b-a & \beta-\alpha & 0 \\ 0 & c-a & \gamma-\alpha & (c-b)(\gamma-\alpha) \end{bmatrix}$$

For $\rho(A) = 3$

$$c - a \neq 0, \gamma - \alpha \neq 0, c - b \neq 0, b - a \neq 0, \beta - \alpha \neq 0$$

i.e., $a \neq b, b \neq c, c \neq a$ and $\alpha \neq \beta, \beta \neq \gamma, \gamma \neq \alpha$

**94.** We have, $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x & u \\ y & v \\ z & w \end{bmatrix} = \begin{bmatrix} 9 & 2 \\ 52 & 15 \\ 0 & -1 \end{bmatrix}$

or $\quad AX = B$

or $\quad X = A^{-1}B \quad$ ...(i)

Where, $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x & u \\ y & v \\ z & \omega \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 2 \\ 52 & 15 \\ 0 & -1 \end{bmatrix}$

$$\therefore \quad |A| = 1(-5-7) - 1(-2-14) + 1(2-10)$$

$$= -12 + 16 - 8 = -4 \neq 0$$

Let $C$ be the matrix of cofactors of elements of $|A|$.

$$\therefore \quad C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \begin{vmatrix} 5 & 7 \\ 1 & -1 \end{vmatrix} & -\begin{vmatrix} 2 & 7 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 5 & 7 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & 7 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} -12 & 16 & -8 \\ 2 & -3 & 1 \\ 2 & -5 & 3 \end{bmatrix}$$

$$\therefore \quad \text{adj } A = C' = \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{\text{adj } A}{|A|} = -\frac{1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$$

Now, $A^{-1}B = -\dfrac{1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix} \times \begin{bmatrix} 9 & 2 \\ 52 & 15 \\ 0 & -1 \end{bmatrix}$

$$= -\frac{1}{4}\begin{bmatrix} -4 & 4 \\ -12 & -8 \\ -20 & -4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 3 & 2 \\ 5 & 1 \end{bmatrix}$$

From Eq. (i) $\quad X = A^{-1}B$

$$\Rightarrow \quad \begin{bmatrix} x & u \\ y & v \\ z & w \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 3 & 2 \\ 5 & 1 \end{bmatrix}$$

On equating the corresponding elements, we have

$$x = 1, u = -1$$
$$y = 3, v = 2$$
$$z = 5, w = 1$$

**95.** Since, $x_1 = 3y_1 + 2y_2 - y_3$

$$\Rightarrow \quad [x_1] = [3 \quad 2 \quad -1]\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Putting the values of $y_1, y_2, y_3$, we get

$$[x_1] = [3 \quad 2 \quad -1]\begin{bmatrix} z_1 - z_2 + z_3 \\ 0 + z_2 + 3z_3 \\ 2z_1 + z_2 + 0 \end{bmatrix}$$

$$= [3 \quad 2 \quad -1]\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 3 \\ 2 & 1 & 0 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$= [3 + 0 - 2 \quad -3 + 2 - 1 \quad 3 + 6 + 0]\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$= [1 \quad -2 \quad 9]\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$[x_1] = [z_1 - 2z_2 + 9z_3]$$

$\therefore \quad x_1 = z_1 - 2z_2 + 9z_3 \quad$ ...(i)

Further, $\quad x_2 = -y_1 + 4y_2 + 5y_3$

$$\Rightarrow \quad [x_2] = [-1 \quad 4 \quad 5]\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Putting the values of $y_1, y_2, y_3$, we get

$$[x_2] = [-1 \quad 4 \quad 5]\begin{bmatrix} z_1 - z_2 + z_3 \\ 0 + z_2 + 3z_3 \\ 2z_1 + z_2 + 0 \end{bmatrix}$$

$$= [-1 \quad 4 \quad 5]\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 3 \\ 2 & 1 & 0 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$= [-1 + 0 + 10 \quad 1 + 4 + 5 \quad -1 + 12 + 0]\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$= [9 \quad 10 \quad 11]\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = [9z_1 + 10z_2 + 11z_3]$$

Hence, $\quad x_2 = 9z_1 + 10z_2 + 11z_3 \quad$ ...(ii)

Further, $\quad x_3 = y_1 - y_2 + 3y_3$

$$\therefore \quad [x_3] = [1 \quad -1 \quad 3]\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Putting the values of $y_1, y_2, y_3$, we get

$$\Rightarrow \quad [x_3] = [1 \quad -1 \quad 3]\begin{bmatrix} z_1 - z_2 + z_3 \\ 0 + z_2 + 3z_3 \\ 2z_1 + z_2 + 0 \end{bmatrix}$$

$$= [1 \quad -1 \quad 3]\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 3 \\ 2 & 1 & 0 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$= [1 - 0 + 6 \quad -1 - 1 + 3 \quad 1 - 3 + 0]\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$= [7 \quad 1 \quad -2]\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = [7z_1 + z_2 - 2z_3]$$

$\therefore \quad x_3 = 7z_1 + z_2 - 2z_3 \quad$ ...(iii)

Hence, from Eqs. (i), (ii) and (iii), we get

$x_1 = z_1 - 2z_2 + 9z_3,\ x_2 = 9z_1 + 10z_2 + 11z_3,\ x_3 = 7z_1 + z_2 - 2z_3$

**96.** Given equations can be written as,

$$2x - 3y + 6z = 5t + 3$$
$$y - 4z = 1 - t$$
$$4x - 5y + 8z = 9t + k$$

which is of the form $\quad AX = B$.

Let $C$ be the augmented matrix, then

$$C = [A : B] = \left[\begin{array}{ccc|c} 2 & -3 & 6 & 5t + 3 \\ 0 & 1 & -4 & 1 - t \\ 4 & -5 & 8 & 9t + k \end{array}\right]$$

Applying $R_3 \to R_3 - 2R_1$, then

$$C = \left[\begin{array}{ccc|c} 2 & -3 & 6 & 5t + 3 \\ 0 & 1 & -4 & 1 - t \\ 0 & 1 & -4 & -t + k - 6 \end{array}\right]$$

Applying $R_3 \to R_3 - R_2$, then

$$C = \left[\begin{array}{ccc|c} 2 & -3 & 6 & 5t + 3 \\ 0 & 1 & -4 & 1 - t \\ 0 & 0 & 0 & k - 7 \end{array}\right]$$

(i) For no solution

$$R_A \neq R_C$$

$\therefore \quad k \neq 7$

(ii) For infinite number of solutions

$$R_A = R_C$$

$\therefore \quad k = 7$

**97.** $AX = U$ has infinite many solutions

$\Rightarrow \quad |A| = 0 = |A_1| = |A_2| = |A_3|$

Now, $\quad |A| = 0$

$$\Rightarrow \quad \begin{vmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{vmatrix} = 0 \Rightarrow (ab - 1)(c - d) = 0$$

$\Rightarrow \quad ab = 1$ or $c = d \quad$ ...(i) and $$|A_1| = 0$$

$$\Rightarrow \begin{vmatrix} f & 1 & 0 \\ g & b & d \\ h & b & c \end{vmatrix} = 0$$

$$\Rightarrow fb(c-d) - gc + hd = 0$$

$$\Rightarrow fb(c-d) = gc - hd \quad \text{...(ii)}$$

$$\Rightarrow |A_2| = 0$$

$$\Rightarrow \begin{vmatrix} a & f & 0 \\ 1 & g & d \\ 1 & h & c \end{vmatrix} = 0$$

$$\Rightarrow a(gc - dh) - f(c-d) = 0$$

$$\Rightarrow a(gc - dh) = f(c-d) \quad \text{...(iii)}$$

$$|A_3| = 0$$

$$\Rightarrow \begin{vmatrix} a & 1 & f \\ 1 & b & g \\ 1 & b & h \end{vmatrix} = 0$$

$$\Rightarrow (h-g)(ab-1) = 0$$

$$\Rightarrow h = g \text{ or } ab = 1 \quad \text{...(iv)}$$

Taking $c = d \Rightarrow h = g$ and $ab \neq 1$ (from Eqs. (i), (ii) and (iv))

Now, taking $BX = V$,

Then, $$|B| = \begin{vmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{vmatrix} = 0$$

[$\because$ In view of $c = d$ and $g = h$, $c_2$ and $c_3$ are identical]

$\Rightarrow$ $BX = V$ has no unique solution.

and $$|B_1| = \begin{vmatrix} a^2 & 1 & 1 \\ 0 & d & c \\ 0 & g & h \end{vmatrix} = 0 \quad [\because c = d,\ g = h]$$

$$|B_2| = \begin{vmatrix} a & a^2 & 1 \\ 0 & 0 & c \\ f & 0 & h \end{vmatrix} = a^2 fc = a^2 df \quad [\because c = d]$$

and $$|B_3| = \begin{vmatrix} a & 1 & a^2 \\ 0 & d & 0 \\ f & g & 0 \end{vmatrix} = -a^2 df$$

If $a^2 df \neq 0$, then $|B_2| = |B_3| \neq 0$

Hence, no solution exist.

**98.** Given, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$, $A^{-1} = \frac{1}{6}\begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix}$$

$$cA = \begin{bmatrix} c & 0 & 0 \\ 0 & c & c \\ 0 & -2c & 4c \end{bmatrix}; dI = \begin{bmatrix} d & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & d \end{bmatrix}$$

$\therefore$ By $A^{-1} = \frac{1}{6}[A^2 + cA + dI]$

$\Rightarrow$ $6 = 1 + c + d$ [By equality of matrices]

$\therefore (-6, 11)$ satisfy the relation.

**99.** If $Q = PAP^T$

then $P^T Q = AP^T$ [$\because PP^T = I$]

$$\Rightarrow P^T Q^{2005} P = AP^T Q^{2004} P$$

$$= A^2 P^T Q^{2003} P = A^3 P^T Q^{2002} P$$

$$= A^{2004} P^T (QP)$$

$$= A^{2004} P^T (PA) \quad [Q = PAP^T \Rightarrow QP = PA]$$

$$= A^{2005}$$

$$\therefore A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

**100.** $$A^2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$\therefore A^n = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$$

$$nA = \begin{bmatrix} n & 0 \\ n & n \end{bmatrix}, (n-1)I = \begin{bmatrix} n-1 & 0 \\ 0 & n-1 \end{bmatrix}$$

$$nA - (n-1)I = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix} = A^n$$

**101.** $$A^2 - A + I = 0$$

$$\Rightarrow I = A - A^2 \Rightarrow I = A(I - A)$$

$$\Rightarrow A^{-1} I = A^{-1}(A(I - A)) \Rightarrow A^{-1} = I - A$$

**102.** (i) Let $U_1$ be $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ so that $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

Similarly, $$U_2 = \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}, U_3 = \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$$

Hence, $$U = \begin{pmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{pmatrix}$$

$$\therefore |U| = 3$$

(ii) $\because \quad \text{Adj}U = \begin{pmatrix} -1 & -2 & 0 \\ -7 & -5 & -3 \\ 9 & -6 & 3 \end{pmatrix}$

$\therefore \quad U^{-1} = \frac{\text{Adj}U}{|U|} = \frac{\text{Adj}U}{3}$

$\Rightarrow$ sum of the elements of

$U^{-1} = \frac{1}{3}(-1-2+0-7-5-3+9+6+3) = 0$

(iii) The value of

$$(3\ 2\ 0)\ U \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} = (3\ 2\ 0) \begin{pmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -6 & -3 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$$

$$= (-1\ 4\ 4) \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$$

$$= (-3+8+0) = 5$$

**103.** $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$

$\Rightarrow \quad AB = \begin{pmatrix} a & 2b \\ 3a & 4b \end{pmatrix}$

and $\quad BA = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} a & 2a \\ 3b & 4b \end{pmatrix}$

Hence, $AB = BA$ only when $a = b$.

**104.** $A^2 - B^2 = (A-B)(A+B)$

$\Rightarrow \quad A^2 - B^2 = A^2 + AB - BA - B^2$

$\Rightarrow \quad AB = BA$

**105.** $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix} \Rightarrow |A \cdot A| = |A||A| = (25\alpha)^2 = 25$

$\Rightarrow \quad \alpha^2 = \frac{1}{25}$

$\Rightarrow \quad \alpha = \pm\frac{1}{5}$

**106.** $\because \quad A^t = A, B^t = -B$

Given, $\quad (A+B)(A-B) = (A-B)(A+B)$

$\Rightarrow \quad A^2 - AB + BA - B^2 = A^2 + AB - BA - B^2$

$\Rightarrow \quad AB = BA$

Also, given $\quad (AB)^t = (-1)^k AB$

$\Rightarrow \quad B^t A^t = (-1)^k AB$

$\Rightarrow \quad -BA = (-1)^k AB$

$\Rightarrow \quad (-1) = (-1)^k \quad [\because AB = BA]$

$\therefore \quad k = 1, 3, 5, \ldots$

**107.** Let $A = \begin{bmatrix} 2 & 1 \\ 0 & 1/2 \end{bmatrix}$

$\det A = \begin{vmatrix} 2 & 1 \\ 0 & 1/2 \end{vmatrix} = 1$

and $\quad A^{-1} = \begin{bmatrix} 1/2 & -1 \\ 0 & 2 \end{bmatrix}$

and let $\quad A = \begin{bmatrix} 3 & 0 \\ -3 & -1/3 \end{bmatrix}$,

$\det A = \begin{vmatrix} 3 & 0 \\ -3 & -1/3 \end{vmatrix} = -1$

and $\quad A^{-1} = \begin{bmatrix} 1/3 & 0 \\ -3 & -3 \end{bmatrix}$

**108.** Let $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ or $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

Then, $A^2 = I$

$\therefore \quad \det A = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = -1$ and $\text{tr}(A) = 0$

**109.** (i) If two zero's are the entries in the diagonal, then

$^3C_2 \times {}^3C_1 = 9$

If the entries in the principal diagonal is1, then

$^3C_1 = 3$

$\Rightarrow$ Total matrix $= 9 + 3 = 12$

(ii) $\begin{bmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 1 \end{bmatrix}$ either $b = 0$ or $c = 0 \Rightarrow |A| \neq 0$

$\Rightarrow$ 2 matrices

$\begin{bmatrix} 0 & a & b \\ a & 1 & c \\ b & c & 0 \end{bmatrix}$ either $a = 0$ or $c = 0 \Rightarrow |A| \neq 0$

$\Rightarrow$ 2 matrices

$\begin{bmatrix} 1 & a & b \\ a & 0 & c \\ b & c & 0 \end{bmatrix}$ either $a = 0$ or $b = 0 \Rightarrow |A| \neq 0$

$\Rightarrow$ 2 matrices

$$\begin{bmatrix} 1 & a & b \\ a & 1 & c \\ b & c & 1 \end{bmatrix}$$

If $a = b = 0 \Rightarrow |A| = 0$

If $a = c = 0 \Rightarrow |A| = 0$

If $b = c = 0 \Rightarrow |A| = 0$

$\Rightarrow$ There will be only 6 matrices.

(iii) The six matrix $A$ for which $|A| = 0$ are

$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow$ inconsistent

$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow$ inconsistent

$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow$ infinite solutions

$$\begin{bmatrix}1&1&0\\1&1&0\\0&0&1\end{bmatrix} \Rightarrow \text{inconsistent}$$

$$\begin{bmatrix}1&0&1\\0&1&0\\1&0&1\end{bmatrix} \Rightarrow \text{inconsistent}$$

$$\begin{bmatrix}1&0&0\\0&1&1\\0&1&1\end{bmatrix} \Rightarrow \text{infinite solutions}$$

**110.** $|\text{adj } A| = |A|^{n-1} = |A|^{2-1} = |A|$

$$\text{adj}(\text{adj } A) = |A|^{n-2} A$$

$$= |A|^{2-2} A = |A|^0 A = A$$

**111.** Three planes cannot meet only at two distinct points.

Hence, number of matrices = 0

**112.** If $A$ is symmetric matrix, then $b = c$

$\therefore$ $\det(A) = \begin{vmatrix} a & b \\ b & a \end{vmatrix} = a^2 - b^2 = (a+b)(a-b)$

$a, b, c \in \{0, 1, 2, 3, \dots, p-1\}$

Number of numbers of type

$$np = 1$$
$$np + 1 = 1$$
$$np + 2 = 1$$
$$\dots\dots\dots\dots$$
$$\dots\dots\dots\dots$$
$$np + (p-1) = 1 \ \forall \ n \in I$$

(i) as $\det(A)$ is divisible by $p \Rightarrow$ either $a + b$ divisible by $p$ corresponding number of ways $= (p-1)$ [excluding zero] or $(a-b)$ is divisible by $p$ corresponding number of ways $= p$

Total Number of ways $= 2p - 1$

(ii) as $\text{Tr}(A)$ not divisible by $p \Rightarrow a \neq 0$

$\det(A)$ is divisible by $p \Rightarrow a^2 - bc$ divisible by $p$

Number of ways of selection of $a, b, c$

$$= (p-1)[(p-1) \times 1] = (p-1)^2$$

(iii) Total number of $A = p \times p \times p = p^3$

Number of $A$ such that $\det(A)$ divisible by $p$

$$= (p-1)^2 + \text{ number of } A \text{ in which } a = 0$$

$$= (p-1)^2 + p + p - 1 = p^2$$

Required number $= p^3 - p^2$

**113.** $|A| = (2k-1)(-1+4k^2) + 2\sqrt{k}\,(2\sqrt{k} + 4k\sqrt{k})$

$$+ 2\sqrt{k}\,(4k\sqrt{k} + 2\sqrt{k})(2k-1)(4k^2-1)$$

$$+ 4k + 8k^2 + 8k^2 + 4k$$

$$= (2k-1)(4k^2-1) + 8k + 16k^2$$

$$= 8k^3 - 4k^2 - 2k + 1 + 8k + 16k^2$$

$$= 8k^3 + 12k^2 + 6k + 1$$

$|B| = 0$ as $B$ is skew-symmetric matrix of odd order.

$\Rightarrow$ $(8k^3 + 12k^2 + 6k + 1)^2 = (10^3)^2$

$\Rightarrow$ $(2k+1)^3 = 10^3$

$\Rightarrow$ $2k + 1 = 10$

$\Rightarrow$ $k = 4.5$

$\Rightarrow$ $[k] = 4$

**114.** First row with exactly one zero

$\therefore$ Total number of cases = 6

First row 2 zeroes, we get more cases.

$\therefore$ Total we get more than 7.

**115.** Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $abcd \neq 0$

$$A^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$\Rightarrow$ $A^2 = \begin{pmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{pmatrix}$

$\Rightarrow$ $a^2 + bc = 1, bc + d^2 = 1$

$$ab + bd = ac + cd = 0$$

$$c \neq 0 \text{ and } b \neq 0$$

and $\quad a + d = 0$

Trace $\quad A = a + d = 0$

$$|A| = ad - bc = -a^2 - bc = 1$$

**116.** $MN = NM$

$M^2N^2 (M^TN)^{-1} (MN^{-1})^T M^2N^2N^{-1} (M^T)^{-1} (N^{-1})^T \cdot M^T$

$= M^2N \cdot (M^T)^{-1} (N^{-1})^T M^T = -M^2 \cdot N(M)^{-1} (N^T)^{-1} M^T$

$= + M^2NM^{-1}N^{-1}M^T = -M \cdot NMM^{-1}N^{-1} M$

$= -MNN^{-1}M = -M^2$

### Note

A skew-symmetric matrix of order 3 cannot be non-singular hence the question is wrong.

**117.** (i) $a + 8b + 7c = 0;\ \ 9a + 2b + 3c = 0$

$7a + 7b + 7c = 0$

Solving these equations, we get

$b = 6a$

$\Rightarrow$ $c = -7a$

Now, $\quad 2x + y + z = 0$

$\Rightarrow$ $2a + 6a + (-7a) = 1$

$\Rightarrow$ $a = 1, b = 6, c = -7$

$\therefore$ $7a + b + c = 7 + 6 - 7 = 6$

(ii) $\because a = 2$ with $b$ and $c$ satisfying $(E)$

$\therefore$ $2 + 8b + 7c = 0, 18 + 2b + 3c = 0$

and $2 + b + c = 0$

we get $\quad b = 12$ and $c = -14$

Hence, $\dfrac{3}{\omega^a} + \dfrac{1}{\omega^b} + \dfrac{3}{\omega^c} = \dfrac{3}{\omega^2} + \dfrac{1}{\omega^{12}} + \dfrac{3}{\omega^{-14}}$

$$= \frac{3\omega}{\omega^3} + \frac{1}{1} + 3\omega^{14}$$

$$= 3\omega + 1 + 3\omega^2$$

$$= 1 + 3(\omega + \omega^2)$$

$$= 1 + 3(-1) = -2$$

(iii) $\because b = 6$, with $a$ and $c$ satisfying $(E)$

$\therefore \quad a + 48 + 7c = 0, 9a + 12 + 3c = 0, a + 6 + c = 0$

we get $\quad a = 1, c = -7$

Given, $\alpha, \beta$ are the roots of $ax^2 + bx + c = 0$

$$\therefore \quad \alpha + \beta = -\frac{b}{a} = -6,$$

$$\alpha\beta = \frac{c}{a} = -7$$

$$\text{Now, } \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-6}{-7} = \frac{6}{7}$$

$$\therefore \quad \sum_{n=0}^{\infty}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n = \sum_{n=0}^{\infty}\left(\frac{6}{7}\right)^n$$

$$= 1 + \left(\frac{6}{7}\right) + \left(\frac{6}{7}\right)^2 + \dots \infty$$

$$= \frac{1}{1 - 6/7} = 7$$

**118.** For the given matrix to be non singular

$$\therefore \quad \begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix} \neq 0$$

$\Rightarrow \quad 1 - (a + c)\omega + ac\omega^2 \neq 0$

$\Rightarrow \quad (1 - a\omega)(1 - c\omega) \neq 0$

$\Rightarrow \quad a \neq \omega^2$ and $c \neq \omega^2$

$\because a, b$ and $c$ are complex cube roots of unity.

$\therefore a$ and $c$ can take only one value i.e., $\omega$ while $b$ can take two values i.e., $\omega$ and $\omega^2$.

$\therefore$ Total number of distinct $= 2$

**119.** Let $M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$$M\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow b = -1, e = 2, h = 3$$

$$M\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \Rightarrow a = 0, d = 3, g = 2$$

$$M\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix} \Rightarrow g + h + i = 12 \Rightarrow i = 7$$

$\therefore$ Sum of diagonal elements $= a + e + i = 0 + 2 + 7 = 9$

**120.** Since, $A$ and $B$ are symmetric matrices

$\therefore \quad A' = A$ and $B' = B$

**Statement-1** Let $\quad P = A(BA)$

$$\therefore \quad P' = (A(BA))' = (BA)'A'$$
$$= (A'B')A'$$
$$= (AB)A \qquad [\because A' = A, B' = B]$$
$$= A(BA) \qquad \text{[By associative law]}$$
$$= P$$

$\Rightarrow A(BA)$ is symmetric

Now, let $\quad Q = (AB)A$

$$Q' = ((AB)A)'$$
$$= A'(AB)' = A'(B'A')$$
$$= A(BA) \qquad [\because A' = A, B' = B]$$
$$= (AB)A \qquad \text{[By associative law]}$$
$$= Q$$

$\Rightarrow \quad (AB)A$ is symmetric.

$\therefore \quad$ Statement-1 is true.

**Statement - 2** $(AB)' = B'A' = BA \qquad [\because A' = A, B' = B]$

$\qquad = AB \qquad [\because AB = BA]$

$\Rightarrow AB$ is symmetric matrix

$\therefore$Statement-2 is true.

Hence, both Statements are true, Statement-2 is not a correct explanation for Statement-1.

**121.** We have, $|Q| = \begin{vmatrix} 2^2a_{11} & 2^3a_{12} & 2^4a_{13} \\ 2^3a_{21} & 2^4a_{22} & 2^5a_{23} \\ 2^4a_{31} & 2^5a_{32} & 2^6a_{33} \end{vmatrix}$

$$= 2^2 \cdot 2^3 \cdot 2^4 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 2^2a_{31} & 2^2a_{32} & 2^2a_{33} \end{vmatrix}$$

$$= 2^9 \cdot 2 \cdot 2^2 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 2^{12}|P|$$

$\therefore \quad |Q| = 2^{12} \times 2 = 2^{13}$

**122.** $\because \quad P^T = 2P + I \qquad \dots$ (i)

$\therefore \quad (P^T)^T = (2P + I)^T$

$\Rightarrow \quad P = 2P^T + I \qquad \dots$ (ii)

From Eqs. (i) and (ii), we get

$$P = 2(2P + I) + I$$

$\Rightarrow \quad P = -I$

$\therefore \quad PX = -IX = -X$

**123.** Given, $\text{adj } P = \begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$

$$\Rightarrow \quad |\text{adj} P| = \begin{vmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{vmatrix}$$

$$= 1(-4) - 4(-1) + 4(1) = 4$$

$\Rightarrow \quad |P|^{3-1} = 4$

$\Rightarrow \quad |P| = \pm 2$

**124.** Let $u_1 + u_2 = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$\text{Now,} \quad Au_1 + Au_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\Rightarrow \quad A(u_1 + u_2) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ 2x + y \\ 3x + 2y + z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$\therefore \quad x = 1, 2x + y = 1$

and $\quad 3x + 2y + z = 0$

$\Rightarrow \quad x = 1, y = -1, z = -1$

Hence, $\quad u_1 + u_2 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$.

**125.** Given, $P^3 = Q^3$ ...(i)

and $\quad P^2Q = Q^2P$ ... (ii)

Subtracting Eq. (i) and (ii), we get

$$P^3 - P^2Q = Q^3 - Q^2P$$

$$P^2 (P - Q) = - Q^2 (P - Q)$$

$$\Rightarrow \quad (P^2 + Q^2)(P - Q) = O$$

$$\Rightarrow \quad |(P^2 + Q^2)(P - Q)| = |O|$$

$$\Rightarrow \quad |P^2 + Q^2||P - Q| = 0$$

$$\therefore \quad |P^2 + Q^2| = 0 \qquad [\because P \neq Q]$$

**126.** Given, adj $A = P$

$$\therefore \quad |\text{adj } A| = |P|$$

$$\Rightarrow \quad |A|^{3-1} = |P| \qquad [\because |A| = 4]$$

$$\Rightarrow \quad 16 = |P|$$

$$\Rightarrow \quad 16 = \begin{vmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{vmatrix}$$

$$\Rightarrow \quad 16 = 1(0) - \alpha (4 - 6) + 3(4 - 6)$$

$$\Rightarrow \quad 16 = 2\alpha - 6$$

$$\Rightarrow \quad 2\alpha = 22$$

$$\therefore \quad \alpha = 11$$

**127.** (a) $(N^TMN)^T = N^TM^T(N^T)^T = N^TM^TN = N^TMN$

or $-N^TMN$ According as $M$ is symmetric or skew-symmetric.

$\therefore$ Correct.

(b) $(MN - NM)^T = (MN)^T - (NM)^T = N^TM^T - M^TN^T$

$= NM - MN \qquad [\because M, N$ are symmetric$]$

$= -(MN - NM)$

$\therefore$ correct

(c) $(MN)^T = N^TM^T = NM \neq MN \qquad [\because M, N$ are symmetric$]$

$\therefore$ Incorrect.

(d) $(\text{adj} M)(\text{adj} N) = \text{adj}(NM) \neq \text{adj}(MN)$

$\therefore$ Incorrect.

**128.** $P = [p_{ij}]_{n \times n} = [\omega^{i+j}]_{n \times n} = \begin{bmatrix} \omega^2 & \omega^3 & \omega^4 & \dots \omega^{1+n} \\ \omega^3 & \omega^4 & \omega^5 & \dots \omega^{2+n} \\ \omega^4 & \omega^5 & \omega^6 & \dots \omega^{3+n} \\ \vdots & \vdots & \vdots & \vdots \\ \omega^{n+1} & \omega^{n+2} & \omega^{n+3} & \dots \omega^{2n} \end{bmatrix}$

$$\therefore \quad P^2 = \begin{bmatrix} 0 & 0 & 0 & \dots 0 \\ 0 & 0 & 0 & \dots 0 \\ 0 & 0 & 0 & \dots 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots 0 \end{bmatrix} = 0$$

If $n$ is multiple of 3, so for $P^2 \neq 0$, $n$ should not be a multiple of 3, i.e. $n$ can take values 55, 56 and 58.

**129.** $B = A^{-1} A'$

$$B' = (A^{-1} A')' = A(A^{-1})'$$

Now, $\quad BB' = (A^{-1}A') A (A^{-1})' = A^{-1} (A' A)(A^{-1})'$

$= A^{-1} (AA')(A^{-1})' \qquad [\because A' A = AA']$

$= (A^{-1}A) A' (A^{-1})'$

$= (IA')(A^{-1})' = A' (A^{-1})' = (A^{-1} A)' = I' = I$

**130.** Let $M = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$, where $a, b, c \in I$

$M$ is invertible if $\begin{vmatrix} a & b \\ b & c \end{vmatrix} \neq 0$

$$\Rightarrow \quad ac - b^2 \neq 0$$

(a) $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ c \end{bmatrix} \Rightarrow a = b = c \Rightarrow ac - b^2 = 0$

$\therefore$ Option (a) is incorrect

(b) $[b \;\; c] = [a \;\; b] \Rightarrow a = b = c \Rightarrow ac - b^2 = 0$

$\therefore$ Option (b) is incorrect

(c) $M = \begin{bmatrix} a & 0 \\ 0 & c \end{bmatrix}$, then $|M| = ac \neq 0$

$\therefore$ $M$ is invertible

$\therefore$ Option (c) is correct.

(d) As $ac \neq (\text{Integer})^2 \Rightarrow ac \neq b^2$

$\therefore$ Option (d) is correct.

**131.** Given, $MN = NM$, $M \neq N^2$ and $M^2 = N^4$

Then, $\quad M^2 = N^4$

$$\Rightarrow \quad (M + N^2)(M - N^2) = 0$$

$$\therefore \quad M + N^2 = 0 \qquad [\because M \neq N^2]$$

$$\Rightarrow \quad |M + N^2| = 0$$

(a) $|M^2 + MN^2| = |M||M + N^2| = 0$

$\therefore$ Option (a) is correct.

(b) $(M^2 + MN^2) U = M(M + N^2) U = 0$

$\therefore$ Option (b) is correct.

(c) $\because$ $| M^2 + MN^2 | = 0$ from option (a)

$\therefore$ $| M^2 + MN^2| \ngeq 1$

$\therefore$ Option (c) is incorrect.

(d) If $AX = 0$ and $| A | = 0$, then $X$ can be non-zero.

**132.** $\because$ $AA^T = 9I$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9 & 0 & a+4+2b \\ 0 & 9 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

On comparing, we get

$$a + 2b + 4 = 0 \quad \text{... (i)}$$
$$2a - 2b + 2 = 0 \quad \text{... (ii)}$$

From Eqs. (i) and (ii), we get

$a = -2,$

$b = -1$

$\therefore$ Ordered pair is $(-2, -1)$.

**133.** $\because$ $X^T = -X, Y^T = -Y, Z^T = Z$

(a) $(Y^3 Z^4 - Z^4 Y^3)^T = (Y^3 Z^4)^T - (Z^4 Y^3)^T$

$= (Z^4)^T (Y^3)^T - (Y^3)^T (Z^4)^T$

$= (Z^T)^4 (Y^T)^3 - (Y^T)^3 (Z^T)^4$

$= -Z^4Y^3 + Y^3 Z^4$

$= Y^3Z^4 - Z^4Y^3$

Option (a) is incorrect.

(b) $X^{44} + Y^{44}$ is symmetric matrix. Option $b$ is incorrect.

(c) $(X^4 Z^3 - Z^3X^4)^T = (X^4Z^3)^T - (Z^3 X^4)^T$

$= (Z^3)^T (X^4)^T - (X^4)^T (Z^3)^T$

$= (Z^T)^3 (X^T)^4 - (X^T)^4 (Z^T)^3$

$= Z^3X^4 - X^4Z^3$

$= -(X^4Z^3 - Z^3X^4)$

$\therefore$ Option (c) is correct.

(d) $X^{23} + Y^{23}$ is skew-symmetric matrix. Option (d) is correct.

**134.** $\because$ $A \text{ adj } A = AA^T$

$\Rightarrow$ $A^{-1}(A \text{ adj } A) = A^{-1}(AA^T)$

$\Rightarrow$ $(A^{-1}A) \text{ adj } A = (A^{-1}A)A^T$

$\Rightarrow$ $I(\text{adj } A) = IA^T$

or $\text{adj } A = A^T$

or $\begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix} = \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$

$\Rightarrow$ $5a = 2$ and $b = 3$

$\therefore$ $5a + b = 5$

**135.** $\because$ $PQ = kI \Rightarrow \dfrac{P.Q}{k} = I \Rightarrow P^{-1} = \dfrac{Q}{k}$ ...(i)

Also $|P| = 12\alpha + 20$ ...(ii)

and given $q_{23} = \dfrac{-k}{8}$

Comparing the third element of $2^{nd}$ row on both sides,

we get $\dfrac{1}{(12\alpha+20)}(-(3\alpha+4)) = \dfrac{1}{k} \times \dfrac{-k}{8}$

$\Rightarrow$ $24\alpha + 32 = 12\alpha + 20$

$\alpha = -1$ ...(iii)

From (ii), $|P| = 8$ ...(iv)

Also $PQ = kI$

$\Rightarrow$ $|PQ| = |kI|$

$\Rightarrow$ $|P||Q| = k^3$

$\Rightarrow$ $8 \times \dfrac{k^2}{2} = k^3$ $\left(\because |P| = 8, |Q| = \dfrac{k^2}{2}\right)$

$\therefore$ $k = 4$ ...(v)

(b) $4\alpha - k + 8 = -4 - 4 + 8 = 0$

(c) $\det(P \text{ adj}(Q)) = |P| \, |\text{adj } Q| = |P| |Q|^2 = 8 \times 8^2 = 2^9$

(d) $\det(Q \text{ adj}(P)) = |Q| \, |\text{adj } P| = |Q| |P|^2 = 8 \times 8^2 = 2^9$

**136.** $\because Z = \dfrac{-1+\sqrt{3}i}{2} = \omega$ ...(i)

$\Rightarrow$ $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$

Now, $P = \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix}$

$\therefore$ $P^2 = \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix}\begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix}$

$= \begin{bmatrix} \omega^{2r} + \omega^{4s} & \omega^{2r}((-\omega)^r + \omega^r) \\ \omega^{2s}((-\omega)^r + \omega^r) & \omega^{4s} + \omega^{2r} \end{bmatrix}$

$= \begin{bmatrix} \omega^{2r} + \omega^{s} & \omega^{2s}((-\omega)^r + \omega^r) \\ \omega^{2s}((-\omega)^r + \omega^s) & \omega^{s} + \omega^{2r} \end{bmatrix}$ $(\because \omega^3 = 1)$

$\because$ $P^2 = -I = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ ...(ii)

Form Eqs. (i) and (ii), we get

$\omega^{2r} + \omega^s = -1$

and $\omega^{2s}((-\omega)^r + \omega^r) = 0$

$\Rightarrow r$ is odd and $s = r$ but not a multiple of 3. Which is possible when $r = s = 1$

$\therefore$ Only one pair is there.

**137.** $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} = I + \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & 0 \\ 16 & 4 & 0 \end{bmatrix} = I + A$

Let $A = \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & 0 \\ 16 & 4 & 0 \end{bmatrix}$

$\Rightarrow \quad A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 16 & 0 & 0 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow A^n$ is a null matrix $\forall n \geq 3$

$\therefore \quad P^{50} = (I + A)^{50} = I + 50A + \frac{50 \times 49}{2} A^2$

$\Rightarrow \quad Q + \mathcal{J} = I + 50A + 25 \times 49 A^2$

or $\quad Q = 50A + 25 \times 49 A^2$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 200 & 0 & 0 \\ 800 & 200 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 19600 & 0 & 0 \end{bmatrix}$$

$\therefore \quad \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 200 & 0 & 0 \\ 20400 & 200 & 0 \end{bmatrix}$

On comparing, we get

$$q_{21} = q_{32} = 200, q_{31} = 20400$$

$\therefore \quad \frac{q_{31} + q_{32}}{q_{21}} = \frac{20400 + 200}{200}$

$$= 102 + 1 = 103$$

**138.** $\because A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$

$\therefore \quad 3A^2 + 12A = 3\begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix} + 12\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

$\Rightarrow \quad \text{adj}\,(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$