Shortcuts and Important Results to Remember

1 $||z_1| - |z_2|| \le |z_1 + z_2| \le |z_1| + |z_2|$

Thus, $|z_1| + |z_2|$ is the greatest possible value of $|z_1 + z_2|$ and $||z_1| - |z_2||$ is the least possible value of $|z_1 + z_2|$.

If $\left| z \pm \frac{b}{z} \right| = a$, then the greatest and least values of $|z|$ are $\frac{a + \sqrt{(a^2 + 4|b|)}}{2}$ and $\frac{-a + \sqrt{(a^2 + 4|b|)}}{2}$, respectively.

$$\left| z_1 + \sqrt{(z_1^2 - z_2^2)} \right| + \left| z_2 - \sqrt{(z_1^2 - z_2^2)} \right| = |z_1 + z_2| + |z_1 - z_2|$$

$|z_1 + z_2| = |z_1| + |z_2| \Leftrightarrow \arg(z_1) = \arg(z_2)$

i.e. z_1 and z_2 are parallel.

$|z_1 + z_2| = |z_1| - |z_2| \Leftrightarrow \arg(z_1) - \arg(z_2) = \pi$

6 $|z_1 + z_2| = |z_1 - z_2| \Leftrightarrow \arg(z_1) - \arg(z_2) = \pm\pi/2$

7 If $|z_1| = |z_2|$ and $\arg(z_1) + \arg(z_2) = 0$, then z_1 and z_2 are conjugate complex numbers of each other.

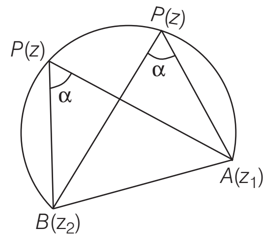
The equation $|z - z_1|^2 + |z - z_2|^2 = k, k \in R$ will represent a circle with centre at $\frac{1}{2}(z_1 + z_2)$ and radius is $\frac{1}{2}\sqrt{2k - |z_1 - z_2|^2}$ provided $k \ge \frac{1}{2}|z_1 - z_2|^2$.

9 Area of triangle whose vertices are z, iz and $z + iz$, where $i = \sqrt{-1}$, is $\frac{1}{2}|z|^2$.

10 Area of triangle whose vertices are z, ωz and $z + \omega z$ is $\frac{\sqrt{3}}{4}|z|^2$, where ω is cube root of unity.

11 $\arg(z) - \arg(-z) = \pi$ or $-\pi$ according as $\arg(z)$ is positive or negative.

12 If $\arg\left(\frac{z - z_1}{z - z_2}\right) = \alpha$ (fixed), then the locus of z is a segment of circle.



If $\arg\left(\frac{z - z_1}{z - z_2}\right) = \pm\pi/2$, the locus of z is a circle with z_1 and z_2 as the vertices of diameter.

14 If $\arg\left(\frac{z - z_1}{z - z_2}\right) = 0$ or π, the locus of z is a straight line passing through z_1 and z_2.

15 If three complex numbers are in AP, they lie on a straight line in the complex plane.

16 If three points z_1, z_2, z_3 connected by relation $a z_1 + b z_2 + c z_3 = 0$, where $a + b + c = 0$, the three points are collinear.

17 If z_1, z_2, z_3 are vertices of a triangle, its centroid

$$z_0 = \frac{z_1 + z_2 + z_3}{3}, \text{ circumcentre } z_1 = \frac{\sum |z_1|^2 (z_2 - z_3)}{\sum \bar{z}_1(z_2 - z_3)},$$

$$\text{orthocentre } z = \frac{\sum \bar{z}_1(\bar{z}_2 - \bar{z}_3) + \sum |z_1|^2 (z_2 - z_3)}{\sum (z_1\bar{z}_2 - \bar{z}_1 z_2)}$$

$$\text{and its area} = \frac{1}{4}\left| \begin{vmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{vmatrix} \right|.$$

18 If $|z_1| = n_1, |z_2| = n_2, |z_3| = n_3, \ldots, |z_m| = n_m$,

then $\left| \frac{n_1^2}{z_1} + \frac{n_2^2}{z_2} + \frac{n_3^2}{z_3} + \ldots + \frac{n_m^2}{z_m} \right| = |z_1 + z_2 + z_3 + \ldots + z_m|$.