Sets, Relations and Functions Exercise 1 :
Single Option Correct Type Questions

- This section contains **39 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct

1. If A and B are two sets, then $A \cap (A \cup B)$ equals
(a) A (b) B
(c) ϕ (d) None of these

2. If R is a relation from a set A to a set B and S is a relation from a set B to set C, then the relation SoR
(a) is from A to C (b) is from C to A
(c) does not exist (d) None of these

3. Let $R = \{(1, 3), (2, 2), (3, 2)\}$ and $S = \{(2, 1), (3, 2), (2, 3)\}$ be two relations on set $A = \{(1, 2, 3)\}$. Then, RoS is equal
(a) $\{(2, 3), (3, 2), (2, 2)\}$ (b) $\{(1, 3), (2, 2), (3, 2), (2, 1), (2, 3)\}$
(c) $\{(3, 2), (1, 3)\}$ (d) $\{(2, 3) (3, 2)\}$

4. If X and Y are two sets, then $X \cap (Y \cap X)'$ equals
(a) X (b) Y
(c) ϕ (d) None of these

5. For real numbers x and y, we write $x\ R\ y \Leftrightarrow x - y + \sqrt{2}$ is an irrational number. Then, the relation R is
(a) reflexive (b) symmetric
(c) transitive (d) None of these

6. Let $f(x) = (x + 1)^2 - 1, (x \geq -1)$. Then, the set $S = \{x : f(x) = f^{-1}(x)\}$ is
(a) $\left\{0, -1, \dfrac{-3 + i\sqrt{3}}{2}, \dfrac{-3 - i\sqrt{3}}{2}\right\}, i = \sqrt{-1}$
(b) $\{0, 1, -1\}$ (c) $\{0, -1\}$
(d) empty

7. The number of elements of the power set of a set containing n elements is
(a) 2^{n-1} (b) 2^n (c) $2^n - 1$ (d) 2^{n+1}

8. Which one of the following is not true?
(a) $A - B \subseteq A$ (b) $B' - A' \subseteq A$
(c) $A \subseteq A - B$ (d) $A \cap B' \subseteq A$

9. If $A = \{1, 2, 3\}$ and $B = \{3, 8\}$, then $(A \cup B) \times (A \cap B)$ is
(a) $\{(3, 1), (3, 2), (3, 3), (3, 8)\}$ (b) $\{(1, 3), (2, 3), (3, 3), (8, 3)\}$
(c) $\{(1, 2), (2, 2), (3, 3), (8, 8)\}$ (d) $\{(8, 3), (8, 2), (8, 1), (8, 8)\}$

10. Let $A = \{p, q, r\}$. Which of the following is not an equivalence relation on A?
(a) $R_1 = \{(p, q), (q, r), (p, r), (p, p)\}$
(b) $R_2 = \{(r, q), (r, p), (r, r), (q, q)\}$
(c) $R_3 = \{(p, p), (q, q), (r, r), (p, q)\}$
(d) None of the above

11. Let $A = \{x : x$ is a multiple of 3$\}$ and $B = \{x : x$ is a multiple of 5$\}$, then $A \cap B$ is given by
(a) $\{3, 6, 9\}$ (b) $\{5, 10, 15, 20, \dots\}$
(c) $\{15, 30, 45, \dots\}$ (d) None of these

12. Let $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$, then $A \cup (B \cap C)$ is
(a) $\{3\}$ (b) $\{1, 2, 3, 4\}$
(c) $\{1, 2, 5, 6\}$ (d) $\{1, 2, 3, 4, 5, 6\}$

13. Let $A = \{x, y, z\}$, $B = \{u, v, w\}$ and $f : A \to B$ be defined by $f(x) = u$,
$f(y) = v, f(z) = w$. Then, f is
(a) surjective but not injective
(b) injective but not surjective
(c) bijective
(d) None of the above

14. If $A = \{2, 4\}$ and $B = \{3, 4, 5\}$, then $(A \cap B) \times (A \cup B)$ is
(a) $\{(2, 2), (3, 4), (4, 2), (5, 4)\}$ (b) $\{(2, 3), (4, 3), (4, 5)\}$
(c) $\{(2, 4), (3, 4), (4, 4), (4, 5)\}$ (d) $\{(4, 2), (4, 3), (4, 4), (4, 5)\}$

15. In the set $X = \{a, b, c, d\}$, which of the following functions in X?
(a) $R_1 = \{(b, a) (a, b), (c, d), (a, c)\}$
(b) $R_2 = \{(a, d) (d, c), (b, b), (c, c)\}$
(c) $R_3 = \{(a, b) (b, c), (c, d), (b, d)\}$
(d) $R_4 = \{(a, a) (b, b), (c, c), (a, d)\}$

16. The composite mapping fog of the map $f : R \to R$, $f(x) = \sin x$ and $g : R \to R$, $g(x) = x^2$ is
(a) $x^2 \sin x$ (b) $(\sin x)^2$ (c) $\sin x^2$ (d) $\sin x / x^2$

17. Which of the following is the empty set?
(a) $\{x : x$ is a real number and $x^2 - 1 = 0\}$
(b) $\{x : x$ is a real number and $x^2 + 1 = 0\}$
(c) $\{x : x$ is a real number and $x^2 - 9 = 0\}$
(d) $\{x : x$ is a real number and $x^2 = x + 2\}$

18. In order that a relation R defined on a non-empty set A is an equivalence relation. It is sufficient, if R
(a) is reflexive
(b) is symmetric
(c) is transitive
(d) possesses all the above three properties

19. Let $A = \{p, q, r, s\}$ and $B = \{1, 2, 3\}$. Which of the following relations from A to B is not a function?
(a) $R_1 = \{(p, 1), (q, 2), (r, 1), (s, 2)\}$
(b) $R_2 = \{(p, 1), (q, 2), (r, 1), (s, 1)\}$
(c) $R_3 = \{(p, 1), (q, 2), (r, 2), (r, 2)\}$
(d) $R_4 = \{(p, 2), (q, 3), (r, 2), (s, 2)\}$

20. n/m means that n is factor of m, then the relation f is
(a) reflexive and symmetric
(b) transitive and symmetric
(c) reflexive, transitive and symmetric
(d) reflexive, transitive and not symmetric

21. The solution of $8x \equiv 6 \pmod{14}$ are
(a) [8],[6] (b) [8],[14]
(c) [6],[13] (d) [8],[14],[16]

22. Let A be a set containing 10 distinct elements, the total number of distinct functions from A to A is
(a) 10! (b) 10^{10} (c) 2^{10} (d) $2^{10}-1$

23. Let A and B be two non-empty subsets of set X such that A is not a subset of B, then
(a) A is a subset of the complement of B
(b) B is a subset of A
(c) A and B are disjoint
(d) A and the complement of B are non-disjoint

24. f and h are function from $A \to B$, where $A = \{a, b, c, d\}$ and $B = \{s, t, u\}$ defined as follows
$f(a) = t, f(b) = s, f(c) = s$
$f(d) = u, h(a) = s, h(b) = t$
$h(c) = s, h(a) = u, h(d) = u$
Which one of the following statement is true?
(a) f and h are functions
(b) f is a function and h is not a function
(c) f and h are not functions
(d) None of the above

25. Let I be the set of integer and $f : I \to I$ be defined as $f(x) = x^2, x \in I$, the function is
(a) bijection (b) injection
(c) surjection (d) None of these

26. Which of the four statements given below is different from others?
(a) $f : A \to B$
(b) $f : x \to f(x)$
(c) f is a mapping of A into B
(d) f is a function of A into B

27. The number of surjections from $A = \{1, 2, \ldots, n\}, n \geq 2$ onto $B = \{a, b\}$ is
(a) ${}^nP_2$ (b) $2^n - 2$
(c) $2^n - 1$ (d) None of these

28. Let $f : R \to R$ be defined by $f(x) = 3x - 4$, then $f^{-1}(x)$ is
(a) $\frac{1}{3}(x+4)$ (b) $\frac{1}{3}x - 4$
(c) $3x + 4$ (d) not defined

29. $f : R \to R$ is a function defined by $f(x) = 10x - 7$. If $g = f^{-1}$, then $g(x)$ equals
(a) $\frac{1}{10x-7}$ (b) $\frac{1}{10x+7}$ (c) $\frac{x+7}{10}$ (d) $\frac{x-7}{10}$

30. Let R be a relation defined by $R = \{(a, b) : a \geq b\}$, where a and b are real numbers, then R is
(a) reflexive, symmetric and transitive
(b) reflexive, transitive but not symmetric
(c) symmetric, transitive but not reflexive
(d) neither transitive, nor reflexive, not symmetric

31. If sets A and B are defined as
$A = \{(x, y) : y = e^x, x \in R\}$ and $B = \{(x, y) : y = x, x \in R\}$.
(a) $B \subset A$ (b) $A \subset B$
(c) $A \cap B = \phi$ (d) $A \cup B$

32. If $f : A \to B$ is a bijective function, then $f^{-1}of$ is equal to
(a) fof^{-1}
(b) f
(c) f^{-1}
(d) I_A(the identity map of the set A)

33. If $f(y) = \frac{y}{\sqrt{(1-y^2)}}, g(y) = \frac{y}{\sqrt{(1+y^2)}}$, then $(fog)y$ is equal to
(a) $\frac{y}{\sqrt{(1-y^2)}}$ (b) $\frac{y}{\sqrt{(1+y^2)}}$ (c) y (d) $\frac{(1-y^2)}{\sqrt{(1-y^2)}}$

34. If $f : R \to R$ is defined by $f(x) = 2x + |x|$, then $f(3x) - f(-x) - 4x$ equals
(a) $f(x)$ (b) $-f(x)$ (c) $f(-x)$ (d) $2f(x)$

35. Let R and S be two non-void relations on a set A. Which of the following statement is false?
(a) R and S are transitive $\Rightarrow R \cup S$ is transitive.
(b) R and S are transitive $\Rightarrow R \cap S$ is symmetric.
(c) R and S are symmetric $\Rightarrow R \cup S$ is symmetric.
(d) R and S are reflexive $\Rightarrow R \cap S$ is reflexive.

36. Let $f : R \to R, g : R \to R$ be two functions given by $f(x) = 2x - 3, g(x) = x^3 + 5$. Then, $(fog)^{-1}(x)$ is equal to
(a) $\left(\frac{x+7}{2}\right)^{1/3}$ (b) $\left(x - \frac{7}{2}\right)^{1/3}$ (c) $\left(\frac{x-2}{7}\right)^{1/3}$ (d) $\left(\frac{x-7}{2}\right)^{1/3}$

37. If $f(x) = ax + b$ and $g(x) = cx + d$, then $f(g(x)) = g(f(x))$ $\Leftrightarrow$
(a) $f(a) = g(c)$ (b) $f(b) = g(b)$
(c) $f(d) = g(b)$ (d) $f(c) = g(a)$

38. If $f : R \to R, g : R \to R$ be two given functions, then $f(x) = 2\min(f(x) - g(x), 0)$ equals
(a) $f(x) + g(x) - |g(x) - f(x)|$
(b) $f(x) + g(x) + |g(x) - f(x)|$
(c) $f(x) - g(x) + |g(x) - f(x)|$
(d) $f(x) - g(x) - |g(x) - f(x)|$

39. Let $f : R \to R, g : R \to R$ be two given functions, such that f is injective and g is surjective, then which of the following is injective?
(a) gof (b) fog (c) gog (d) fof

Sets, Relations and Functions Exercise 2 :
More than One Correct Option Type Questions

- This section contains **3 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **MORE THAN ONE** may be correct.

40. Let L be the set of all straight lines in the Euclidean plane. Two lines l_1 and l_2 are said to be related by the relation R iff l_1 is parallel to l_2. Then, the relation R is

(a) reflexive (b) symmetric
(c) transitive (d) equivalence

41. Let $X = \{1, 2, 3, 4, 5, 6\}$ and $Y = \{1, 3, 5, 7, 9\}$. Which of the following is/are relations from X to Y?

(a) $R_1 = \{(x, y) : y = 2 + x, x \in X, y \in Y\}$
(b) $R_2 = \{(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)\}$
(c) $R_3 = \{(1, 1), (1, 3), (3, 5), (3, 7), (5, 7)\}$
(d) $R_4 = \{(1, 3), (2, 5), (2, 4), (7, 9)\}$

42. Let the function $f : R - \{-b\} \to R - \{1\}$ be defined by $f(x) = \dfrac{x + a}{x + b} (a \neq b)$, then

(a) f is one-one but not onto
(b) f is onto but not one-one
(c) f is both one-one and onto
(d) $f^{-1}(2) = a - 2b$

Sets, Relations and Functions Exercise 3 :
Passage Based Questions

- This section contains **2 passages**. Based upon each of the passage **3 multiple choice questions** have to be answered. Each of these questions has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Passage I
(Q. Nos. 43 to 45)

Let f and g be real valued functions defined as

$$f(x) = \begin{cases} 7x^2 + x - 8, & x \le 1 \\ 4x + 5, & 1 < x \le 7 \\ 8x + 3, & x > 7 \end{cases} \quad g(x) = \begin{cases} |x|, & x < -3 \\ 0, & -3 \le x < 2 \\ x^2 + 4, & x \ge 2 \end{cases}$$

43. The value of $(gof)(0) + (fog)(-3)$ is

(a) -8 (b) 0
(c) 8 (d) 16

44. The value of $2(fog)(7) - (gof)(6)$ is

(a) 9 (b) 11
(c) 13 (d) 15

45. The value of $4(gof)(2) - (fog)(9)$ is

(a) 0 (b) 2 (c) 5 (d) 9

Passage II
(Q. Nos. 46 to 48)

R_1 on Z defined by $(a, b) \in R_1$ iff $|a - b| \le 7$, R_2 on Q defined by $(a, b) \in R_2$ iff $ab = 4$ and R_3 on R defined by $(a, b) \in R_3$ iff $a^2 - 4ab + 3ab^2 = 0$.

46. Relation R_1 is

(a) reflexive and symmetric (b) symmetric and transitive
(c) reflexive and transitive (d) equivalence

47. Relation R_2 is

(a) reflexive (b) symmetric
(c) transitive (d) equivalence

48. Relation R_3 is

(a) reflexive (b) symmetric
(c) transitive (d) equivalence

Sets, Relations and Functions Exercise 4 :
Single Integer Answer Type Questions

- This section contains **6 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

49. In a group of 45 students, 22 can speak Hindi only and 12 can speak English only . If $(2\lambda + 1)$ student can speak both Hindi and English, the value of λ is

50. If $A = \left\{x \mid \cos x > -\dfrac{1}{2} \text{ and } 0 \le x \le \pi\right\}$ and $B = \left\{x \mid \sin x > \dfrac{1}{2} \text{ and } \dfrac{\pi}{3} \le x \le \pi\right\}$ and if $\pi\lambda \le A \cap B < \pi\mu$, the value of $(\lambda + \mu)$ is

51. If $S = R$, $A = \{ x : -3 \le x < 7 \}$ and $B = \{ x : 0 < x < 10 \}$, the number of positive integers in $A \Delta B$ is

52. Two finite sets have m and n elements. The total number of subsets of the first set is 48 more than the total number of subsets of the second set. The value of $m - n$ is

53. If two sets A and B are having 99 elements in common, the number of elements common to each of the sets $A \times B$ and $B \times A$ are $121 \lambda^2$, the value of λ is

Sets, Relations and Functions Exercise 5 :
Matching Type Questions

- This section contains **2 questions**. Questions 54 and 55 have three statements (A, B and C) given in **Column I** and four statements (p, q, r and s) in **Column II** and questions 70 and 71 have four statements (A, B, C and D) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II**.

54. The functions defined have domain R.

	Column I		Column II
(A)	$7x + 1$	(p)	onto $[-1, 1]$ but not one-one $[0, \pi]$
(B)	$\cos x$	(q)	one-one on $[0, \pi]$ but not onto R
(C)	$\sin x$	(r)	one-one and onto R
(D)	$1 + \ln x$	(s)	one-one on $(0, \infty)$

55. The domain of the function $f(x)$ is denoted by D_f.

	Column I		Column II
(A)	$f(x) = \sqrt{(3 - x)} + \sin^{-1}\left(\frac{3 - 2x}{5}\right)$, then D_f is	(p)	$\bigcup_{k \in I} [2k\pi, (2k + 1)\pi]$
(B)	$f(x) = \log_{10} (1 - \log_{10} (x^2 - 5x + 16))$, then D_f is	(q)	$[-4, -\pi] \cup [0, \pi]$
(C)	$f(x) = \cos^{-1}\left(\frac{2}{2 + \sin x}\right)$, then D_f is	(r)	$(2, 3)$
(D)	$f(x) = \sqrt{(\sin x)} + \sqrt{(16 - x^2)}$, then D_f is	(s)	$[-1, 3]$

Sets, Relations and Functions Exercise 6 :
Statement I and II Type Questions

- **Directions** Question numbers 56 to 59 are Assertion-Reason type questions. Each of these questions contains two statements :
  - (a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
  - (b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
  - (c) Statement-1 is true, Statement-2 is false.
  - (d) Statement-1 is false, Statement-2 is true.

56. **Statement-1** If a set A has n elements, then the number of binary relations on $A = n^{n^2}$.

Statement-2 Number of possible relations from A to $A = 2^{n^2}$.

57. **Statement-1** If $A = \{x \mid g(x) = 0\}$ and $B = \{x \mid f(x) = 0\}$, then $A \cap B$ be a root of $\{f(x)\}^2 + \{g(x)\}^2 = 0$.

Statement-2 $x \in A \cap B \Rightarrow x \in A$ or $x \in B$.

58. **Statement-1** $P(A) \cap P(B) = P(A \cap B)$, where $P(A)$ is power set of set A.

Statement-2 $P(A) \cup P(B) = P(A \cup B)$

59. **Statement-1** If Sets A and B have three and six elements respectively, then the minimum number of elements in $A \cup B$ is 6.

Statement-2 $A \cap B = 3$.

Sets, Relations and Functions Exercise 7 :
Subjective Type Questions

- In this section, there are **15 subjective** questions.

60. Let $A = \{x : x \text{ is a natural number}\}$,
$B = \{x : x \text{ is an even natural number}\}$,
$C = \{x : x \text{ is an odd natural number}\}$
and $D = \{x : x \text{ is a prime number}\}$.

Find
(i) $A \cap B$ (ii) $A \cap C$
(iii) $B \cap D$ (iv) $C \cap D$

61. Let U be the set of all people and $M = \{\text{Males}\}$,
$S = \{\text{College students}\}$,
$T = \{\text{Teenagers}\}$, $W = \{\text{People having height more than five feet}\}$.
Express each of the following in the notation of set theory.
(i) College student having heights more than five feet.
(ii) People who are not teenagers and have their height less five feet.
(iii) All people who are neither males nor teenagers nor college students.

62. The set X consists of all points within and on the unit circle $x^2 + y^2 = 1$, whereas the set Y consists of all points on and inside the rectangular boundary $x = 0$, $x = 1$, $y = -1$ and $y = 1$. Determine $X \cup Y$ and $X \cap Y$. Illustrate your answer by diagrams.

63. In a group of children, 35 play football out of which 20 play football only, 22 play hockey; 25 play cricket out of which 11 play cricket only. Out of these 7 play cricket and football but not hockey, 3 play football and hockey but not cricket and 12 play football and cricket both.
How many play all the three games? How many play cricket and hockey but not football, how many play hockey only? What is the total number of children in the group?

64. Of the members of three athletic team in a certain school, 21 are on the basketball team, 26 on the hockey team and 29 on the football team. 14 play hockey and basketball, 15 play hockey and football, 12 play football and basketball and 8 play all the three games. How many members are there in all?

65. In a survey of 200 students of higher secondary school, it was found that 120 studied Mathematics; 90 studies Physics and 70 studied Chemistry; 40 studied Mathematics and Physics; 3 studied Physics and Chemistry; 50 studied Chemistry and Mathematics and 20 studied none of these subjects. Find the number of students who studied all the three subjects.

66. In a survey of population of 450 people, it is found that 205 can speak English, 210 can speak Hindi and 120 people can speak Tamil. If 100 people can speak both Hindi and English; 80 people can speak both English and Tamil, 35 people can speak Hindi and Tamil and 20 people can speak all the three languages, find the number of people who can speak English but not a Hindi or Tamil. Find also the number of people who can speak neither English nor Hindi nor Tamil.

67. A group of 123 workers went to a canteen for cold drinks, ice-cream and tea, 42 workers took ice-cream, 36 tea and 30 cold drinks. 15 workers purchased ice-cream and tea, 10 ice-cream and cold drinks, and 4 cold drinks and tea but not ice-cream, 11 took ice-cream and tea but not cold drinks. Determine how many workers did not purchase anything?

68. Let n be a fixed positive integer. Define a relation R on I (the set of all integers) as follows:
$a\,R\,b$ iff $n|(a-b)$ i.e., iff $(a-b)$ is divisible by n. Show that R is an equivalence relations on I.

69. N is the set of positive integers. The relation R is defined on $N \times N$ as follows:
$$(a, b)\,R\,(c, d) \Leftrightarrow ad = bc$$
Prove that R is an equivalence relation.

70. The following relations are defined on the set of real numbers.
(i) $a\,R\,b \Leftrightarrow |a-b| > 0$
(ii) $a\,R\,b \Leftrightarrow |a| = |b|$
(iii) $a\,R\,b \Leftrightarrow |a| \geq |b|$
(iv) $a\,R\,b \Leftrightarrow 1 + ab > 0$
(v) $a\,R\,b \Leftrightarrow |a| \leq b$
Find whether these relations are reflexive, symmetric or transitive.

71. Let $A = \{x : -1 \leq x \leq 1\} = B$ for each of the following functions from A to B. Find whether it is surjective, injective or bijective
(i) $f(x) = \dfrac{x}{2}$
(ii) $g(x) = |x|$
(iii) $h(x) = x|x|$
(iv) $k(x) = x^2$
(v) $l(x) = \sin \pi x$

72. If the functions f and g defined from the set of real numbers R to R such that $f(x) = e^x$ and $g(x) = 3x - 2$, then find functions fog and gof. Also, find the domain of the functions $(fog)^{-1}$ and $(gof)^{-1}$.

73. If $f(x) = \dfrac{x^2 - x}{x^2 + 2x}$, then find the domain and range of f. Show that f is one-one. Also, find the function $\dfrac{d(f^{-1}(x))}{dx}$ and its domain.

74. If the functions f, g and h are defined from the set of real numbers R to R such that

$$f(x) = x^2 - 1,\ g(x) = \sqrt{(x^2 + 1)},$$

$$h(x) = \begin{cases} 0, & \text{if } x \le 0 \\ x, & \text{if } x \ge 0 \end{cases}.$$

Then, find the composite function $hofog$ and determine whether the function fog is invertible and h is the identity function.

Sets, Relations and Functions Exercise 8 :
Questions Asked in Previous 13 Year's Exam

- This section contains questions asked in **IIT-JEE, AIEEE, JEE Main** & **JEE Advanced** from year **2005** to year **2017**.

75. Let $R = \{(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9)\}$ be a relation on the set $A = \{3, 6, 9, 12\}$.
The relation is **[AIEEE 2005, 3M]**

(a) an equivalence relation
(b) reflexive and symmetric only
(c) reflexive and transitive only
(d) reflexive only

76. Let W denotes the words in the English dictionary. Define the relation R by $R = \{(x, y) \in W \times W\}$ the words x and y have atleast one letter in common, then R is **[AIEEE 2006, 3M]**

(a) not reflexive, symmetric and transitive
(b) reflexive, symmetric and not transitive
(c) reflexive, symmetric and transitive
(d) reflexive, not symmetric and transitive

77. Let R be the real line, consider the following subsets of the plane $R \times R$ such that **[AIEEE 2008, 3M]**

$S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$

$T = \{(x, y) : x - y \text{ is an integer}\}$.

Which one of the following is true?

(a) Both S and T are equivalence relations on R
(b) S is an equivalence relation on R but T is not
(c) T is an equivalence relation on R but S is not
(d) Neither S nor T is an equivalence relations on R

78. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then **[AIEEE 2009, 4M]**

(a) $A \cap B = \phi$ (b) $A = B$
(c) $A = C$ (d) $B = C$

79. Let $S = \{1, 2, 3, 4\}$. The total number of unordered pair of disjoint subsets of S is equal to **[IIT-JEE 2010, 5M]**

(a) 25 (b) 34 (c) 42 (d) 41

80. Consider the following relations.

$R = \{(x, y) \mid x, y$ are real numbers and $x = wy$ for some rational number $w\}$

$S = \left\{\left(\dfrac{m}{n}, \dfrac{p}{q}\right) \middle| m, n, p \text{ and } q \text{ are integers such that } n, q \ne 0 \text{ and } qm = pn\right\}$, then **[AIEEE 2010, 4M]**

(a) neither R nor S is an equivalence relation
(b) S is an equivalence relation but R is not an equivalence relation
(c) R and S both are equivalence relations
(d) R is an equivalence relation but S is not an equivalence relation

81. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then, **[IIT-JEE 2011, 3M]**

(a) $P \subset Q$ and $A - P \ne \phi$ (b) $Q \not\subset P$
(c) $P \not\subset Q$ (d) $P = Q$

82. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in R$. Then, the set of all x satisfying $(fogogof)(x) = (gogof)(x)$, where $(fog)(x) = f(g(x))$ is **[IIT-JEE 2011, 3M]**

(a) $\pm\sqrt{n\pi}, n \in \{0, 1, 2, \ldots\}$
(b) $\pm\sqrt{n\pi}, n \in \{1, 2, 3, \ldots\}$
(c) $\dfrac{\pi}{2} + 2n\pi, n \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$
(d) $2n\pi, n \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

83. Let R be the set of real numbers.

Statement-1 $A = \{(x, y) \in R \times R : y - x$ is an integer$\}$ is an equivalence relation on R.

Statement-2 $B = \{(x, y) \in R \times R : x = \alpha y$ for some rational number $\alpha\}$ is an equivalence relation on R. **[AIEEE 2011, 4M]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(b) Statement-1 is true, Statement-2 is false
(c) Statement-1 is false, Statement-2 is true
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

84. Let A and B be two sets containing 2 elements and 4 elements, respectively. The number of subsets of $A \times B$ having 3 or more elements, is **[JEE Main 2013, 4M]**
(a) 220 (b) 219 (c) 211 (d) 256

85. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n = N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to **[JEE Main 2014, 4M]**
(a) X (b) Y (c) N (d) $Y - X$

86. Let A and B be two sets containing four and two elements, respectively. Then, the number of subsets of the set $A \times B$, each having atleast three elements is **[JEE Main 2015, 4M]**
(a) 275 (b) 510 (c) 219 (d) 256

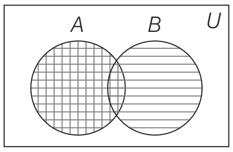
Answers

Chapter Exerises

1. (a) **2.** (a) **3.** (a) **4.** (d) **5.** (a) **6.** (c)
7. (b) **8.** (c) **9.** (b) **10.** (d) **11.** (c) **12.** (b)
13. (c) **14.** (d) **15.** (b) **16.** (c) **17.** (b) **18.** (d)
19. (c) **20.** (d) **21.** (c) **22.** (b) **23.** (d) **24.** (b)
25. (d) **26.** (b) **27.** (b) **28.** (a) **29.** (c) **30.** (b)
31. (b) **32.** (d) **33.** (c) **34.** (d) **35.** (a) **36.** (d)
37. (c) **38.** (d) **39.** (d) **40.** (a,b,c,d) **41.** (a,b,c)
42. (c,d) **43.** (b) **44.** (a) **45.** (d) **46.** (a) **47.** (b)
48. (a) **49.** (5) **50.** (1) **51.** (3) **52.** (2) **53.** (9)
54. (A) $\to$ (r); (B) $\to$ (q); (C) $\to$ (p); (D) $\to$ (s)
55. (A) $\to$ (s); (B) $\to$ (r); (C) $\to$ (p); (D) $\to$ (q)
56. (b) **57.** (c) **58.** (c) **59.** (a)
60. (i) B (ii) C (iii) $\{2\}$ (iv) $\{x : x$ is an odd prime, natural number$\}$
61. (i) $S \cap W$ (ii) $T' \cap W'$ (iii) $(M \cup T \cup S)'$
62. $X \cup Y = \{(x, y) : x^2 + y^2 \le 1$ or $0 \le x \le 1$ and $-1 \le y \le 1\}$
$X \cap Y = \{(x, y) : x^2 + y^2 \le 1$ and $x \ge 0\}$
63. 5, 2, 12, 60 **64.** 43 **65.** 20 **66.** 45, 110
67. 44
70. (i) Not reflexive, symmetric, not transitive
(ii) Reflexive, symmetric, transitive
(iii) Reflexive, not symmetric, transitive
(iv) Reflexive, symmetric, not transitive
(v) Not reflexive, not symmetric, transitive
71. (i) Injective (ii) Injective
(iii) Bijective (iv) Not injective
(v) Surjective
72. $(fog)x = e^{3x-2}$; $x \in R$ $(gof)x = 3e^x - 2$; $x \in R$
Domain of $(fog)^{-1}(x) = (0, \infty)$.
Domain of $(gof)^{-1}(x) = (-2, \infty)$.
73. $\dfrac{3}{(1-x)^2}$, $R - \{1\}$
$\dfrac{df^{-1}(x)}{dx} = \dfrac{3}{(1-x)^2}$, Domain of $\dfrac{df^{-1}(x)}{dx} = R - \{1\}$
74. $(hofog)\, x = \begin{cases} 0, & x^2 \le 0 \\ x^2, & x^2 \ge 0 \end{cases}$, h is not an identity function and fog is not invertible.
75. (c) **76.** (b) **77.** (c) **78.** (d) **79.** (d) **80.** (b)
81. (d) **82.** (a) **83.** (a) **84.** (b) **85.** (b) **86.** (c)

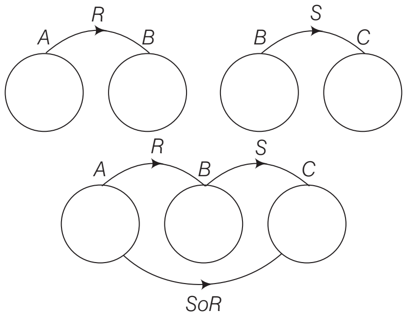
Solutions

1. By Venn diagram,



It is clear that $A \cap (A \cup B) = A$

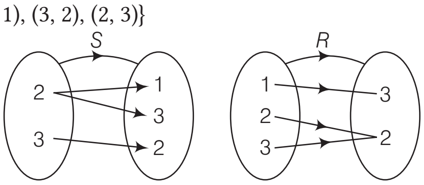
2.



SoR is the relation from A to C.

3. $R = \{(1, 3), (2, 2), (3, 2)\}$

$S = \{(2, 1), (3, 2), (2, 3)\}$



$RoS = \{(2, 3), (2, 2), (3, 2)\}$

4. $X \cap (Y \cap X)' = X \cap (Y' \cup X')$

$= (X \cap Y') \cup (X \cap X')$

$= (X \cap Y') \cup \phi = X \cap Y'$

$= X - (X \cap Y)$

5. $xRy \Leftrightarrow (x - y + \sqrt{2})$ is an irrational number.

Let $(x, x) \in R$.

Then, $x - x + \sqrt{2} = \sqrt{2}$ which is an irrational number.

$\therefore \quad x\ R\ x, \forall\ x \in R$

$\therefore$ R is an reflexive relation.

$x\ R\ y \Rightarrow (x - y + \sqrt{2})$ is an irrational number.

$\Rightarrow\ -(y - x - \sqrt{2})$ is an irrational number.

$\Rightarrow (y - x - \sqrt{2})$ is an irrational number.

$y\ R\ x \Rightarrow (y - x + \sqrt{2})$ is an irrational number.

So, $xRy \not\Rightarrow yRx \therefore R$ is not a symmetric relation.

Let $(1, 2) \in R$, then $(1 - 2 + \sqrt{2})$ is an irrational number.

$\Rightarrow \qquad (\sqrt{2} - 1)$ is an irrational number.

and $(2, 3) \in R$, then $(2\ - 3 + \sqrt{2})$ is an irrational number.

$\Rightarrow \qquad (\sqrt{2} - 1)$ is an irrational number.

$(1, 3) \in R \Rightarrow (1 - 3 + \sqrt{2})$ is an irrational number.

$\Rightarrow \qquad (\sqrt{2} - 2)$ is an irrational number.

So, $(1, 2) \in R$ and $(2, 3) \in R \not\Rightarrow (1, 3) \in R$ (by any way)

$\therefore$ R is not transitive relation.

6. $f(x) = (x + 1)^2 - 1 \qquad [\because\ x \geq 1]$

$= x^2 + 1 + 2x - 1 = x^2 + 2x$

$S = \{x : f(x) \equiv f^{-1}(x)\}$

S is the set of point of intersection of $(y = x)$ and tf.

Now, solve $\qquad y = x$ and $f(x) = x^2 + 2x$

$x^2 + 2x = x$

$x^2 + x = 0$

$x(x + 1) = 0$

$x = 0$ or $x = -1$

7. Let set A contains n elements.

Power set of A is the set of all subsets.

$\therefore$ Number of subsets of $A = {}^nC_o + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$

$\therefore$ Power set of A contains 2^n elements.

8. By Venn diagram, it is clear that

$A - B \subseteq A$ and $B' - A' \subseteq A$ and $A \cap B' \subseteq A$

but $A \not\subseteq A - B$

9. $A = \{1, 2, 3\}$

$B = \{3, 8\}$

$A \cup B = \{1, 2, 3, 8\}$

$A \cap B = \{3\}$

$(A \cup B) \times (A \cap B) = \{1, 2, 3, 8\} \times \{3\}$

$= \{(1, 3), (2, 3), (3, 3) (8, 3)\}$

10. $A = \{p, q, r\}$

$R_1 = \{(p, q), (q, r), (p, r), (p, p)\}$

$(q, q) \notin R_1$, so R_1 is not reflexive relation.

So, R_1 is not an equivalence relation.

$R_2 = \{(r, q), (r, p), (r, r), (q, q)\}$

Here, $(p, p) \notin R_2$, so R_2 is not reflexive relation.

So, R_2 is not an equivalence relation.

$R_3 = \{(p, p), (q, q), (r, r), (p, q)\}$

R_3 is an reflexive relation.

$(p, a) \in R_3$ but $(q, p) \notin R_3$

R_3 is not symmetric relation.

So, R_3 is not equivalence relation.

11. $A = \{x : x$ is a multiple of $3\}$

$A = \{x : x = 3m, m \in N\}$

$B = \{x : x$ is a multiple of $5\}$

$B = \{x : x = 5n, n \in N\}$

$A \cap B = \{x : x$ is a multiple of both 3 and $5\}$

$= \{15, 30, 45, \ldots\}$

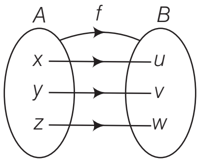
12. $A = \{1, 2, 3\}, B = \{3, 4\}, \ C = \{4, 5, 6\}$

$\Rightarrow \quad B \cap C = \{4\}$

and $A \cup (B \cap C) = \{1, 2, 3, 4\}$

13. $A = \{x, y, z\}, B = \{u, v, w\}$

Now, $f : A \to B$



f is one-one and f is onto.

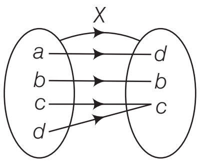
14.
$$A = \{2, 4\}$$
$$B = \{3, 4, 5\}$$
$$A \cap B = \{4\}$$
$$A \cup B = \{2, 3, 4, 5\}$$
$$(A \cap B) \times (A \cup B) = \{(4, 2), (4, 3), (4, 4), (4, 5)\}$$

15. $X = \{a, b, c, d\}$

$$R_1 = \{(b, a), (a, b), (c, d), (a, c)\}$$
$$(a, b) \in R_1 \text{ and } (a, c) \in R,$$

$\therefore R_1$ is not a function.

$$R_2 = \{(a, d), (d, c), (b, b), (c, c)\}$$



Hence, R_2 is a function.

16. $f : R \to R$

$\Rightarrow \quad f(x) = \sin x$ and $g : R \to R$

$\Rightarrow \quad g(x) = x^2$

Range of g is $R^+ \cup \{0\}$, which is the subset of domain of f.

$\therefore$ Composition of fog is possible.

$$fog = f(g(x)) = f(x^2)$$
$$= \sin x^2$$

17. $x^2 - 1 = 0$

$\Rightarrow \quad x = -1, 1$

$\therefore$ x is real, q $\quad x^2 + 1 = 0$

$\Rightarrow \quad x = \pm i$

$\therefore$ x is not real, $\quad x^2 - 9 = 0$

$\Rightarrow \quad x = \pm 3$

$\therefore$ x is real $\quad x^2 - x - 2 = 0$

$\Rightarrow \quad x = 2, -1$

$\therefore$ x is real.

18. By definition for equivalent relation.

R should be reflexive, symmetric, transitive.

19. $\because$ x - coordinates of two brackets are same.

20. $\frac{n}{m}$ means that n is a factor of m.

So, f is reflexive.

$\because$ A number is a factor of itself.

Now, if n is a factor of m, then m is not a factor of n

$\therefore$ f is not symmetric. Let n is a factor of m and m is a factor of s, then it is true that n is a factor of s.

$\therefore$ f is transitive.

21. $\lambda = \dfrac{8x - 6}{14}$, where $\lambda \in I_+$

$\therefore \quad 8x = 14\lambda + 6 \ \Rightarrow \ x = \dfrac{14\lambda + 6}{8}$

$\Rightarrow \quad x = \dfrac{7\lambda + 3}{4} = \lambda + \dfrac{3}{4}(\lambda + 1)$, when $\lambda \in I$

and here greatest common divisor of 8 and 14 is 2, so there are two required solutions.

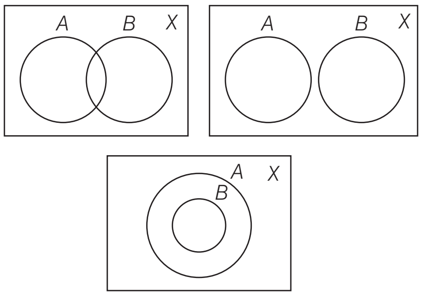
for $\lambda = 3$ and $\lambda = 7$, $x = 6, 13$ or $x = [6][13]$

22. $n(A) = 10$

Total number of distinc Functions from A to $A = 10^{10}$.

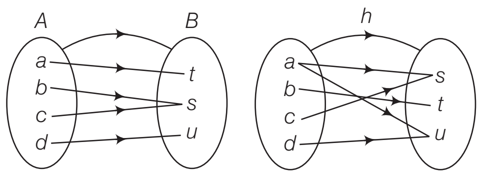
23. $A \subseteq X$ and $B \subseteq X$ and $A \subseteq B$

In all 3 possible cases,



24. $A = \{a, b, c, d\}$

$B = \{s, t, u\}$



It is clear that f is a function.

But in relation h, a have h image s and u.

So, h is not a function.

25. $f(x) = x^2, x \in Z$

$$f(1) = 1$$
$$f(-1) = 1$$

$\therefore$ f is not one-one

Range of f is set of whole number.

Which is a subset of Z.

$\therefore f$ is not onto.

26. It is obvious.

27. $A = \{1, 2, ..., n\}\ n \geq 2$

$B = \{a, b\}$

Number of into functions from A to $B = 2$

Total Numer of functions from A to $B = [n(B)]^{n(A)} = 2^n$

$\therefore$ Total Number of onto functions from A to $B = 2^n - 2$

28. $f : R \to R$

$\Rightarrow \quad f(x) = 3x - 4$

f is one-one onto function.

$\therefore$ Let $\quad y = 3x - 4$

$$x = \frac{y+4}{3}$$

Replace x by $y \Rightarrow y = \frac{x+4}{3} = f^{-1}(x)$

29. $f : R \to R$

$\Rightarrow \quad f(x) = 10x - 7$

It is clear that f is one-one and onto.

$\therefore$ Let $\quad y = 10x - 7$

$\therefore \quad x = \frac{y+7}{10} = f^{-1}(y)$

$$g(x) = f^{-1}(x) = \frac{x+7}{10}$$

30. $R = \{(a, b) : a \geq b\}$

We know that, $a \geq a$

$\therefore \quad (a, a) \in R, \forall a \in R$

R is a reflexive relation.

Let $(a, b) \in R$

$\Rightarrow \quad a \geq b$

$\not\Rightarrow \quad b \leq a$

$\not\Rightarrow \quad (b, a) \in R$

So, R is not symmetric relation.

Now, let $(a, b) \in R$ and $(b, c) \in R$.

$\Rightarrow \quad a \geq b$ and $b \geq c$

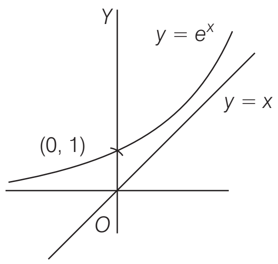
$\Rightarrow \quad a \geq c$

$\Rightarrow \quad (a, c) \in R$

$\therefore$ R is a transitive relation.

31. $A = \{(x, y) : y = e^x, x \in R\}$

$B = \{(x, y) : y = x, x \in R\}$

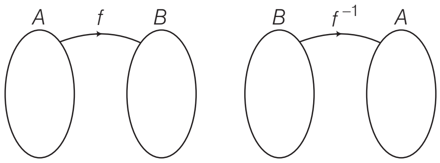


$\therefore \quad A \subset B$

32. $f : A \to B$

f is a function, then f^{-1} is also a bijective function.

Composite function $(f^{-1} of) = I_A$



33. $f(y) = \frac{y}{\sqrt{(1-y^2)}}, g(y) = \frac{y}{\sqrt{(1+y^2)}}$

and $\quad (fog)y = f(g(y)) = f\left(\frac{y}{\sqrt{(1+y^2)}}\right)$

$$= \frac{\frac{y}{\sqrt{(1+y^2)}}}{\sqrt{1 - \frac{y^2}{(1+y^2)}}} = \frac{\frac{y}{\sqrt{(1+y^2)}}}{\frac{1}{\sqrt{(1+y^2)}}} = y$$

34. $f : R \to R$

$$f(x) = 2x + |x|$$

When $x \geq 0$, then $f(x) = 2x + x = 3x$

When $x < 0$, then $f(x) = 2x - x = x$

Now, when $x \geq 0$

$f(3x) - f(-x) - 4x = 3(3x) - (-x) - 4x = 9x + x - 4x$

$= 6x \quad [\because x \geq 0]$

$= 2(3x) = 2f(x) \quad [\therefore -x \leq 0]$

When $x < 0$,

$f(3x) - f(-x) - 4x = 3x - (-3x) - 4x = 2x = 2f(x)$

35. Let $\quad A = \{1, 2, 3\}, R = \{(1, 1)\ (1, 2)\}$

and $S = \{(2, 2), (2, 3)\}$

be the transitive relation on A.

Then, $R \cup S = \{(1, 1,)\ (1, 2)(2, 2)\ (2, 3)\}$

$R \cup S$ is not transitive, because $(1, 2) \in R \cup S$

and $(2, 3) \in R \cup S$ but $(1, 3) \notin R \cup S$.

36. $f : R \to R$

$g : R \to R$

$f(x) = 2x - 3$

$g(x) = x^3 + 5$

$\Rightarrow \quad (fog)(x) = f(g(x)) = f(x^3 + 5) = 2(x^3 + 5) - 3$

$= 2x^3 + 7$

Now, $\quad$ let $y = 2x^3 + 7$

$2x^3 = y - 7$

$$x = \left(\frac{y-7}{2}\right)^{1/3}$$

Replacing x by y, we get

$$y = \left(\frac{x-7}{2}\right)^{1/3}$$

$\therefore \quad (fog)^{-1}(x) = \left(\frac{x-7}{2}\right)^{1/3}$

37.
$$f(x) = ax + b$$
$$g(x) = cx + d$$
$$f(g(x)) = g(f(x))$$
$$f(cx + d) = g(ax + b)$$
$$a(cx + d) + b = c(ax + b) + d$$
$$acx + ad + b = acx + bc + d$$
$$ad + b = cb + d$$
$$f(d) = g(b)$$

38. $f : R \to R, g : R \to R$

$f(x) = 2 \min f(x) - g(x), 0)$

Let $f(x) - g(x) > 0$, then

$F(x) = f(x) - g(x) - |f(x) - g(x)|$ and $f(x) - g(x) < 0$, then

$F(x) = 2[f(x) - g(x)] = [f(x) - g(x)] - |f(x) - g(x)|$

39. $f : R \to R$ and $g : R \to R$ such that f is injective and $+g$ is surjective.

Then, g may be one-one or many-one.

If g is one-one, then gof is one-one.

fog is one-one

gog is one-one

But if g is many-one, then *gof* is not one-one.

fog is not one-one.

gog is many-one

Now, *fof* is one-one

40. Relation R on the set of all straight lines in the plane is of parallel line.

A line is parallel to itself. So, R is reflexive.

If l_1 is parallel to l_2, then l_2 is parallel to l_1.

$\therefore$ R is symmetric relation. $[l_1, l_2 \in L]$

Let $l_1, l_2, l_3 \in L$

l_1 is parallel to l_2 and l_2 is parallel to l_3.

Then, l_1 is parallel to l_3.

$\therefore$ R is transitive relation.

So, R is equivalence relation.

41. $X = \{1, 2, 3, 4, 5\}$

$Y = \{1, 3, 5, 7, 9\}$

(a) $R_1 = \{(x, y) : y = 2 + x, x \in X, y \in Y\}$

$x = 1$	$y = 2$
$x = 2$	$y = 4$
$x = 3$	$y = 5$
$x = 4$	$y = 6$
$x = 5$	$y = 7$

So, R_1 is a relation from X to Y.

(b) $R_2 = \{(1, 1), (2, 1), (3, 3) (4, 3), (5, 5)\}$

$R_2 \subseteq X \times Y$

(c) $R_3 = \{(1, 1), (1, 3), (3, 5) (5, 7)\}$

$R_3 \subseteq X \times Y$

(d) $R_4 \not\subseteq X \times Y$

42. $f : R - \{-b\} \to R - \{1\}$

$$f(x) = \frac{x + a}{x + b} \qquad [a \neq b]$$

Let $x_1, x_2 \in D_f$

$$f(x_1) = f(x_2)$$

$\Rightarrow \quad \dfrac{x_1 + a}{x_1 + b} = \dfrac{x_2 + a}{x_2 + b}$

$\Rightarrow x_1x_2 + bx_1 + ax_2 + ab = x_1x_2 + ax_1 + bx_2 + ab$

$\Rightarrow \quad b(x_1 - x_2) = a(x_1 - x_2)$

$\Rightarrow \quad (x_1 - x_2)(b - a) = 0$

$\Rightarrow \quad x_1 = x_2 \qquad [\because a \neq b]$

$\therefore f$ is one-one function.

Now, let $\quad y = \dfrac{x + a}{x + b}$

$$xy + by = x + a$$
$$x(y - 1) = a - by$$
$$x = \frac{a - by}{y - 1} \text{ and } f^{-1}(y) = \frac{a - by}{y - 1}$$

$\because \quad y \in R - \{1\}$

$\therefore \quad x$ is defined, $\forall y \in R - \{1\}$

$$f^{-1}(2) = \frac{a - 2b}{2 - 1} = a - 2b$$

Sol. (Q. Nos. 43 to 45)

43. $(gof)(0) = g(f(0)) = g(7(0)^2 + 0 - 8)$

$= g(-8) = |-8| = 8$

and $\quad (fog)(-3) = f(g(-3)) = f(0) = 7(0)^2 + 0 - 8 = -8$

$\therefore \quad (gof)(0) + (fog)(-3) = -8 + 8 = 0$

44. $(fog)(7) = f(g(7)) = f(7^2 + 4) = f(53)$

$= 8(53) + 3 = 427$

and $\quad (gof)(6) = g(f(6)) = g(4 \times 6 + 5) + g(29)$

$= (29)^2 + 4 = 845$

$\therefore \quad 2(fog)(7) - (gof)(6) = 2 \times 427 - 845 = 9$

45. $(gof)(2) = g(f(2)) = g(4 \times 2 + 5) = g(13)$

$= (13)^2 + 4 = 173$

and $\quad (fog)(g) = f(g(9)) = f(9^2 + 4) = f(85)$

$= 8 \times 85 + 3 = 683$

$\therefore \quad 4(gof)(2) - (fog)(9) = 4 \times 173 - 683 = 9$

Sol. (Q. Nos. 46-48)

46. We have, $(a, b) \in R_1$ iff $|a - b| \leq 7$, where $a, b \in z$

Reflexivity Let $a \in z$

$\Rightarrow \quad a - a = 0$

$\Rightarrow \quad |a - a| \leq 7$

$\Rightarrow \quad 0 \leq 7$

$\Rightarrow \quad (a, a) \in R_1$

$\therefore$The relation R_1 is reflexive.

Symmetry

$(a, b) \in R_1$

$\Rightarrow \quad |a - b| \leq 7 \Rightarrow |-(b - a)| \leq 7$

$\Rightarrow \quad |b - a| \leq 7 \Rightarrow (b, a) \in R_1$

$\therefore$The relation R_1 is symmetric.

Transitivity We have $(2, 6), (6, 10) \in R_1$ because

$|2 - 6| = 4 \leq 7$ and $|6 - 10| = 4 \leq 7$

Also, $\quad |2 - 10| = 8 \not\leq 7$

$\therefore \quad (2, 10) \notin R_1$

Hence, the relation R_1 is not transitive.

47. We have $(a, b) \in R_2$ iff $ab = 4$, where $a, b \in Q$

Reflexivity $5 \in Q$ and $(5)(5) = 25 \neq 4$

$\therefore \quad (5, 5) \notin R_2$

The relation R_2 is not reflexive.

Symmetry

$(a, b) \in R_2$

$\Rightarrow \quad ab = 4 \Rightarrow ba = 4$

$\Rightarrow \quad (b, a) \in R_2$

$\therefore$ The relation R_2 is symmetric.

Transitivity We have $\left(8, \frac{1}{2}\right), \left(\frac{1}{2}, 8\right) \in R_2$ because

$$8\left(\frac{1}{2}\right) = 4 \text{ and } \left(\frac{1}{2}\right)(8) = 4$$

Also, $\quad 8(8) = 64 \neq 4$

$\therefore \quad (8, 8) \notin R_2$

$\therefore$ The relation R_2 is not transitive.

48. We have, $(a, b) \in R_3$ iff $a^2 - 4ab + 3b^2 = 0$

where $a, b \in R$

Reflexivity

$\therefore \quad a^2 - 4a \cdot a + 3d^2 = 4a^2 - 4a^2 = 0$

$\therefore \quad (a, a) \in R_3$

$\therefore$ The relation R_3 is reflexive.

Symmetry

$(a, b) \in R_3$

$\Rightarrow \quad a^2 - 4ab + 3b^2 = 0$, we get $a = b$ and $a = 3b$

and $\quad (b, a) \in R_3$

$\Rightarrow \quad b^2 - 4ab + 3a^2 = 0$

we get $b = a$ and $b = 3a$

$\therefore \quad (a, b) \in R_3 \not\Rightarrow (b, a) \in R_3$

$\therefore$ The relation R_3 is not symmetric.

Transitivity We have $(3, 1), \left(1, \frac{1}{3}\right) \in R_3$

because $(3)^2 - 4(3)(1) + 3(1)^2 = 9 - 12 + 3 = 0$

and $\quad (1)^2 - 4(1)\left(\frac{1}{3}\right) + 3\left(\frac{1}{3}\right)^2 = 1 - \frac{4}{3} + \frac{1}{3} = 0$

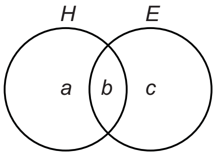
Also, $\left(3, \frac{1}{3}\right) \notin R_3$, because

$$(3)^2 - 4 \cdot (3)\left(\frac{1}{3}\right) + 3\left(\frac{1}{3}\right)^2 = 9 - 4 + \frac{1}{3} = \frac{16}{3} \neq 0$$

$\therefore$ The relation R_3 is not transitive.

49. Given, $a = 22$,

$c = 12$



and $\quad a + b + c = 45$

$\Rightarrow \quad 22 + b + 12 = 45$

$\therefore \quad b = 11 = 2\lambda + 1$

$\Rightarrow \quad \lambda = 5$

50. $\therefore \quad \cos x > -\frac{1}{2}$ and $0 \leq x \leq \pi$

$\Rightarrow \quad -\frac{2\pi}{3} < x < \frac{2\pi}{3}$ and $0 \leq x \leq \pi$

$\Rightarrow \quad 0 \leq x < \frac{2\pi}{3}$

$\therefore \quad A = \left[0, \frac{2\pi}{3}\right]$

Again, $\quad \sin x > \frac{1}{2}$ and $\frac{\pi}{3} \leq x \leq \pi$

$\Rightarrow \quad \frac{\pi}{6} < x < \frac{5\pi}{6}$ and $\frac{\pi}{3} \leq x \leq \pi$

$\Rightarrow \quad \frac{\pi}{3} \leq x < \frac{5\pi}{6}$

$B = \left[\frac{\pi}{3}, \frac{5\pi}{6}\right)$

Now, $\quad A \cap B = \left[\frac{\pi}{3}, \frac{2\pi}{3}\right)$

$\frac{\pi}{3} \leq A \cap B < \frac{2\pi}{3}$

Here $\quad \lambda = \frac{1}{3}$ and $\mu = \frac{2}{3}$

$\lambda + \mu = 1$

51. Here, $\quad A = [-3, 7), B = (0, 10)$

and $\quad S = (-\infty, \infty)$

$\therefore \quad A - B = [-3, 0]$ and $B - A = [7, 10)$

$\therefore \quad A \Delta B = (A - B) \cup (B - A) = [-3, 0] \cup [7, 10)$

$\therefore$ Positive integers are 7, 8, 9.

Number of positive integers $= 3$

52. As $2^m - 2^n = 48 = 16 \times 3 = 2^4 \times 3$

$\Rightarrow \quad 2^n(2^{m-n} - 1) = 2^4 (2^2 - 1)$

$\therefore \quad n = 4$ and $m - n = 2$

$n = 4$ and $m = 6$

Now, $\quad m - n = 2$

53. $n((A \times B) \cap (B \times A)) = n((A \cap B) \times (B \cap A))$

$= n(A \cap B) \cdot n(B \cap A)$

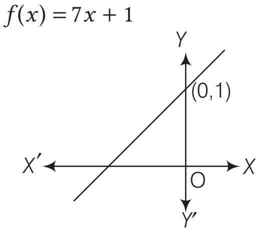
$= n(A \cap B) \cdot n(A \cap B)$

$= 99 \times 99 = 121 \times 9^2$

$\therefore \quad \lambda = 9$

54. (A) $y = 7x + 1$

$f(x) = 7x + 1$



Let $\quad x_1, x_2 \in D_f$,

then $\quad f(x_1) = f(x_2)$

$\Rightarrow \quad 7x_1 + 1 = 7x_2 + 1 \Rightarrow x_1 = x_2$

f is one-one, $\quad \forall x \in R$

Now, $y = 7x + 1 \Rightarrow x = \frac{y-1}{7}$

for each $y \in R$, we get $x \in R$

f is onto function

(B) $y = \cos x$

for $x \in [0, \pi]$, $y \in [-1, 1]$

$\therefore f$ is one-one on $[0\ \pi]$,

$$\forall\ x \in R, y \in [-1, 1]$$

y is not onto R.

(C) $y = \sin x$ or $f(x) = \sin x$

for $x \in [0, \pi]$, $y \in [0, 1]$

$$f\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \text{ and } f\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$\therefore f$ is not one-one on $(0, \pi)$,

$$\forall\ x \in R \text{ and } y \in [-1, 1]$$

$\therefore f$ is onto $[-1, 1]$.

(D) $y = 1 + \text{In}\, x$ and $f(x) = 1 + \text{In}\, x$

y is defined for $x \in (0, \infty)$

Let $x_1, x_2 \in D_f$

then $f(x_1) = f(x_2)$

$\Rightarrow 1 + \text{In}\, x_1 = 1 + \text{In}\, x_2$

$\Rightarrow x_1 = x_2$

$\therefore f$ is one-one, $\forall\ x \in (0, \infty)$

55. **(A)** Let $y = \sqrt{3-x} + \sin^{-1}\left(\frac{3-2x}{5}\right)$

For y to be defined $3 - x \geq 0$ on $-1 \leq \frac{3-2x}{5} \leq 1$

$$x \leq 3 \qquad \text{...(i)}$$

$$-5 \leq 3 - 2x \leq 5$$

and $-1 \leq x \leq 4$ ...(ii)

From Eqs. (i) and (ii), we get

$$x \in [-1, 3]$$

(B) Let $y = \log_{10}\{1 - \log_{10}(x^2 - 5x + 16)\}$ for y to be defined

$x^2 - 5x + 16 > 0$ and $1 - \log_{10}(x^2 - 5x + 16) > 0$

$\left(x - \frac{5}{2}\right)^2 + \frac{39}{4} > 0$ and $\log_{10}(x^2 - 5x + 16) < 1$

which is true, $\forall\ x \in R$ ...(i)

$\Rightarrow x^2 - 5x + 16 < 10$

$\Rightarrow x^2 - 5x + 6 < 0 \Rightarrow (x-3)(x-2) < 0$

$\Rightarrow 2 < x < 3$ ...(ii)

From Eqs. (i) and (ii), $x \in (2, 3)$

(C) Let $y = \cos^{-1}\frac{2}{2+\sin x}$, for y to be defined

$$-1 \leq \frac{2}{2+\sin x} \leq 1 \qquad \begin{bmatrix} \because -1 < \sin x \leq 1 \\ 1 < 2 + \sin x \leq 3 \end{bmatrix}$$

Multiplying by $(2 + \sin x)$

$$-(2 + \sin x) \leq 2 \leq 2 + \sin x$$

$\Rightarrow -2 - \sin x \leq 2$	$2 \leq 2 + \sin x$
$\Rightarrow -\sin x \leq 4$	$\sin x \geq 0$
$\Rightarrow \sin x \geq -4$	$2n\pi \leq x \leq (2n+1)\pi, n \in z$ ...(i)

We know that $\sin x \in [-1, 1]$

$\therefore \quad x \in R$ ... (ii)

From Eqs. (i) and (ii); $x \in [2k\pi, (2k+1)\pi]$

Domain $= \bigcup\limits_{k \in I} [2k\pi, (2k + 1\pi)]$

(D) $y = \sqrt{\sin x} + \sqrt{16 - x^2}$ for y to be defined

$\sin x \geq 0$	$16 - x^2 \geq 0$
$x \in [2k\pi, (2k+1)\pi], k \in I$ ...(i)	$-4 \leq x \leq 4$ ...(ii)

From Eqs. (i) and (ii), we get

$$x \in [-4, -\pi] \cup [0, \pi]$$

56. Let

$A = \{a_1, a_2, a_3, \ldots, a_n\}$

Then, the number of binary relations on $A = n^{(n \times n)} = n^{n^2}$

and number of relations form A to $A = 2^{n \times n} = 2^{n^2}$

Both statements are true but Statement-2 is not a correct explanation for Statement-1.

57. Let $\alpha \in (A \cap B) \Rightarrow \alpha \in A$ and $\alpha \in B$

$\Rightarrow g(\alpha) = 0$

and $f(\alpha) = 0$

$\Rightarrow \{f(\alpha)\}^2 + \{g(\alpha)\}^2 = 0$

$\Rightarrow \alpha$ is a root of $\{f(x)\}^2 + \{g(x)\}^2 = 0$

Hence, Statement-1 is true and Statement-2 is false.

58. Let $x \in P(A \cap B)$

$\Leftrightarrow x \subseteq (A \cap B)$

$\Leftrightarrow x \subseteq A$ and $x \subseteq B$

$\Leftrightarrow x \in P(A)$ and $x \in P(B)$

$\Leftrightarrow x \in P(A) \cap P(B)$

$\therefore P(A \cap B) \subseteq P(A) \cap P(B)$

and $P(A) \cap P(B) \subseteq P(A \cap B)$

Hence, $P(A) \cap P(B) = P(A \cap B)$

Now, consider sets $A = \{1\}, B = \{2\} \Rightarrow A \cup B = \{1, 2\}$

$\therefore P(A) = \{\phi, \{1\}\}, P(B) = \{\phi, \{2\}\}$.

and $P(A \cup B) = \{\phi\ \{1\}, \{2\}, \{1, 2\} \neq P(A) \cup P(B)\}$

Hence, Statement-1 is true and Statement-2 is false.

59. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 3 + 6 - n(A \cap B) = 9 - n(A \cap B)$

As maximum number of element in $(A \cap B) = 3$

$\therefore$ Minimum number of elements in $(A \cap B) = 9 - 3 = 6$

Both statements are true; Statement-2 is a correct explanation for Statement-1.

60. $A = \{x : x$ is a natural number$\}$

$B = \{x : x$ is an even natural number$\}$

$C = \{x : x$ is an odd natural number$\}$

$D = \{x : x$ is a prime number$\}$

(i) $A \cap B = \{x : x = 2n, n \in N\} = B$

(ii) $A \cap C = \{x : x$ is an odd natural number$\} = C$

(iii) $B \cap D = \{x : x$ is prime natural number$\} = \{2\}$

(iv) $C \cap D = \{x : x$ is odd prime natural number$\}$

61. U = Set of all people

M = {Males}

S = {College students}

T = {Teenagers}

W = {People having height more than 5 feet}

(i) College students having heights more than 5 feet $= S \cap W$

(ii) People who are not teenagers and having their heights less than 5 feet $= T' \cap W'$

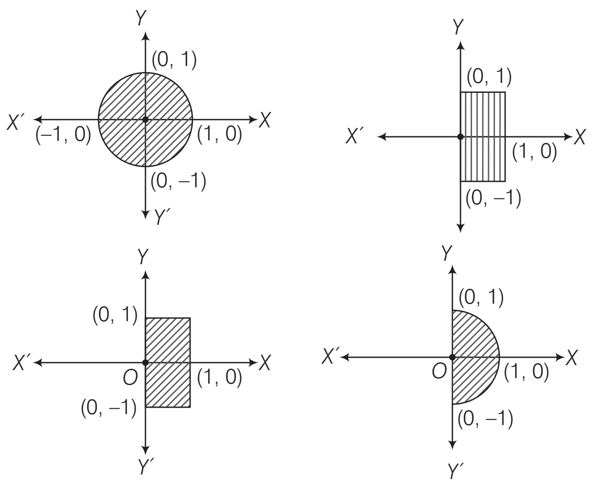
(iii) All people who are neither males nor teenagers nor college students $= (M \cup T \cup S)'$

62. $X = \{(x, y) : x^2 + y^2 \le 1\}$

$Y = \{(x, y) : 0 \le x \le 1, -1 \le y \le 1\}$

$X \cup Y = \{(x, y) : x^2 + y^2 \le 1 \text{ or } 0 \le x \le 1 \text{ and } -1 \le y \le 1\}$

$X \cap Y = \{(x, y) : x^2 + y^2 \le 1 \text{ and } x \ge 0\}$

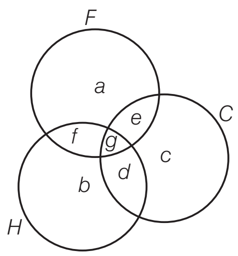


63. Given, $a = 20$ ...(i)

$e + f + g = 15$ ...(ii)

$b + d + f + g = 22$ ...(iii)

$c = 11$ ...(iv)



$c + d + e + g = 25$ ...(v)

$\Rightarrow$ $d + e + g = 14$ ...(vi)

$e = 7$ ...(vii)

$f = 3$ ...(viii)

From Eqs. (vii), (viii) and (ix),

$e + g = 12$ ...(ix)

$e = 7, f = 3, g = 5$

From Eq. (vi), $d = 2$

From Eq. (iii) $b + 2 + 3 + 5 = 22$

$\therefore$ $b = 12$

Hence, $a = 20, b = 12, c = 11, d = 2, e = 7, f = 3, g = 5$

Number of children play all the three games $= g = 5$

Number of children play cricket and hockey but not football $= d = 2$

Number of children play hockey only $= b = 12$

Total number of children in the group
$= a + b + c + d + e + f + g = 60$

64. $a + f + e + g = 21$ ...(i)

$b + d + f + g = 26$ ...(ii)

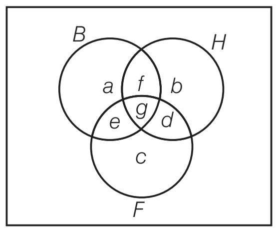
$c + d + e + g = 29$ ...(iii)

$f + g = 14$ ...(iv)

$g + d = 15$ ...(v)

$e + g = 12$ ...(vi)

$g = 8$ ...(vii)



From Eqs. (vii) and (vi), $e = 4$

Form Eqs. (vii) and (v), $d = 7$

From Eqs. (vii) and (iv), $f = 6$

From Eq. (iii), $c + 7 + 4 + 8 = 29 \Rightarrow c = 29 - 19 = 10 = c$

From Eq. (ii), $b + 7 + 6 + 8 = 26 \Rightarrow b = 26 - 21 \Rightarrow b = 5$

From Eq. (i), $a + 6 + 4 + 8 = 21 \Rightarrow a = 21 - 18 \Rightarrow a = 3$

$n(B) + n(H) + n(F) = a + b + c + d + e + f + g$
$= 3 + 5 + 10 + 7 + 4 + 6 + 8 = 43$

65. $a + e + f + g = 120$ ...(i)

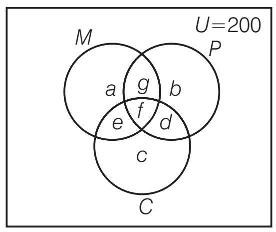
$b + d + f + g = 90$ ...(ii)

$e + f + c + d = 70$ ...(iii)

$g + f = 40$ ...(iv)

$f + d = 30$ ...(v)

$e + f = 50$ ...(vi)



$U - (a + b + c + d + e + f + g) = 20$

$\Rightarrow$ $a + b + c + d + e + f + g = 180$ ...(vii)

From Eqs. (i) and (iv), $a + e = 80$ ...(viii)

From Eqs. (ii) and (iv), $b + d = 50$ ...(ix)

From Eqs. (iii) and (v), $e + c = 40$ ...(x)

from Eqs (viii), (ix) & (x), $a + b + c + d + e + e = 197$ ...(xi)

from (xi), (vii) and (iv), $197 - e + 40 = 180$

$$170 - e + 40 = 180$$

$$e = 210 - 180 = 30$$

From Eq. (vi), $e + f = 50$

$\Rightarrow$ $30 + f = 50$

$\Rightarrow$ $f = 20$

66. $b + e + f + g = 205$ ...(i)

$a + d + f + g = 210$ ...(ii)

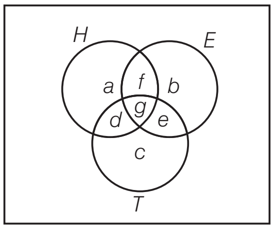
$c + d + e + g = 120$ ...(iii)

$f + g = 100$ ...(iv)

$e + g = 800$ ...(v)

$d + g = 35$ ... (vi)

$g = 20$ ...(vii)



From Eqs. (vi) and (vii), $d = 15$

From Eqs. (vii) and (v), $e = 60$

From Eqs. (vii) and (iv), $f = 80$

From Eq. (i), $b + 60 + 80 + 20 = 205 \Rightarrow b = 205 - 160$

$\Rightarrow b = 45$ = Can speak English but not Hindi or Tamil.

From Eq. (ii) $a + 15 + 80 + 20 = 210$

$\Rightarrow$ $a + 115 = 210 \Rightarrow a = 95$

From Eq. (iii), $c + 15 + 60 + 20 = 120$

$\Rightarrow$ $c = 120 - 95 \Rightarrow c = 25$

People who can speak neither E nor H nor T

$$= 450 - (95 + 45 + 25 + 15 + 60 + 80 + 20)$$

$$= 450 - 340 = 110$$

67. $c + f + g + e = 42$ ...(i)

$b + d + g + e = 36$ ...(ii)

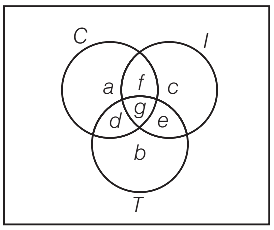
$a + f + d + g = 30$ ...(iii)

$g + e = 15$ ...(iv)

$f + g = 10$ ...(v)

$d = 4$ ...(vi)

$e = 11$ ...(vii)



From (iv) and (vii), $g + 11 = 15 \Rightarrow g = 4$ ...(viii)

From (v) and (viii), $f + 4 = 10 \Rightarrow f = 6$ ...(ix)

From (i), $c + 6 + 4 + 11 = 42 \Rightarrow c = 21$ ...(x)

From (ii), $b + 4 + 4 + 11 = 36 \Rightarrow b = 17$ ...(xi)

From (iii), $a + 6 + 4 + 4 = 30 \Rightarrow a = 16$ ...(xii)

Number of required persons

$$= 123 - (16 + 17 + 21 + 4 + 11 + 6 + 4)$$

$$= 123 - 79$$

$$= 44$$

68. aRb iff $n \mid (a - b)$ | i.e. $(a - b)$ is divisible by n.

Reflexivity $a - a = 0$ which is divisible by n.

So, $(a, a) \in R, \forall a \in I$

$\therefore$ R is reflexive relation.

Symmetry Let $(a, b) \in R$

Then, $(a, b) \in R \Rightarrow (a - b)$ is divisible by n.

$\Rightarrow$ $-(b - a)$ is divisible by n.

$\Rightarrow$ $(b - a)$ is divisible by n.

$\Rightarrow$ $(b, a) \in R$

$\therefore$ R is symmetric relation.

Transitivity Let $(a, b) \in R, (b, c) \in R$, then $(a - b)$ and $(b - c)$ are divisible by n.

$\Rightarrow$ $a - b = nk_1$ and $b - c = nk_2$ $[k_1, k_2 \in I]$

$\Rightarrow$ $(a - b) + (b - c) = n(k_1 + k_2)$

$\Rightarrow$ $a - c = n(k_1 + k_2)$

$\Rightarrow$ $(a - c)$ is divisible by n.

$\Rightarrow$ $(a, c) \in R$

$\therefore$ R is transitive relation.

$\therefore$ R is an equivalence relation.

69. R defined on $N \times N$ such that

$$(a, b)\ R\ (c, d) \Leftrightarrow ad = bc$$

Reflexivity Let $(a, b) \in N \times N$

$\Rightarrow$ $a, b \in N \Rightarrow ab = ba$

$\Rightarrow$ $(a, b)\ R\ (a, b)$

$\therefore$ R is reflexive on, $N \times N$.

Symmetry Let $(a, b), (c, d) \in N \times N$,

then $(a, b)\ R\ (c, d) \Rightarrow ad = bc$

$\Rightarrow$ $cb = da$

$\Rightarrow$ $(c, d)\ R\ (a, b)$

$\therefore$ R is symmetric on $N \times N$.

Transitivity Let $(a, b), (c, d), (e, f) \in N \times N$

Then, $(a, b)\ R\ (c, d) \Rightarrow ad = bc$ ...(i)

$(c, d)\ R\ (e, f) \Rightarrow cf = de$ ...(ii)

From Eqs. (i) and (ii), $(ad)(cf) = (bc)(de)$

$\Rightarrow$ $af = be$

$\Rightarrow$ $(a, b)\ R\ (e, f)$

$\therefore$ R is transitive relation on $N \times N$.

$\therefore$ R is equivalence relation on $N \times N$.

70. (i) $aRb \Leftrightarrow |a-b| > 0$

Reflexivity $a - a = 0$

$\therefore \quad (a, a) \notin R$

$\therefore$ R is not reflexive

Symmetry $(a, b) \in R \Rightarrow |a - b| > 0$

$\Rightarrow \quad |-(b-a) > 0|$

$\Rightarrow \quad |b - a| > 0$

$\Rightarrow \quad (b, a) \in R$

$\therefore$ R is symmetric relation

Transitivity $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow \quad |a-b| > 0$ and $|b - c| > 0$

$\Rightarrow \quad |a-b| + |b-c| > 0 \quad$ [by addition]

Now, let $a > b$ and $b > c$, then $a > c$

$|a-b| + |b-c| = a - b + b - c = a - c > 0$

$\Rightarrow \quad |a - c| > 0$

If $a < b$ and $b > c$, then

$|a-b| + |b-c| = -(a-b) + (b-c) = -a + 2b - c$

$\not\Rightarrow \quad |a - c| > 0$

$\therefore$ R is not transitive relation.

(ii) $aRb \Leftrightarrow |a| = |b|$

Reflexivity We have, $|a| = |a|$

$\Rightarrow \quad aRa \forall a$

$\therefore$ R is reflecxive relation.

Symmetry $aRb \Rightarrow |a| = |b|$

$\Rightarrow \quad |b| = |a|$

$\Rightarrow \quad bRa$

$\therefore$ R is symmetric relation.

Transitivity $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow \quad |a| = |b|$ and $|b| = |c|$

$\Rightarrow \quad |a| = |c|$

$\Rightarrow \quad (a, c) \in R$

$\therefore$ R is transitive relation.

(iii) $aRb \Leftrightarrow |a| \geq |b|$

Reflexivity For any $a \in R$, we have $|a| \geq |a|$

So, $\quad aRa \ \forall \ a$

$\therefore$ R is reflexive relation.

Symmetry $aRb \Rightarrow |a| \geq |b|$

$\Rightarrow \quad |b| \leq |a|$

$\therefore$ R is not symmetric relation.

Transitivity aRb and $bRc \Rightarrow |a| \geq |b|$ and $|b| \geq |c|$

$\Rightarrow \quad |a| \geq |c|$

$\Rightarrow \quad a \, R \, c$

$\therefore$ R is transitive relation.

(iv) $aRb \Leftrightarrow 1 + ab > 0, \forall \, a, b \in R$

Reflexivity Let $a \notin R \Rightarrow 1 + a \cdot a = 1 + a^2 > 0$

$\Rightarrow \quad (a, a) \in R$

$\therefore$ R is reflexive on R.

Symmetry Let $(a, b) \in R$, then $(a, b) \in R$

$\Rightarrow \quad 1 + ab > 0$

$\Rightarrow \quad 1 + ba > 0$

$\Rightarrow \quad (b, a) \in R$

$\therefore$ R is symmetric on R.

Transitivity We observe that $\left(1, \frac{1}{2}\right) \in R$ and $\left(\frac{1}{2}, -1\right) \in R$ but $(1, -1) \notin R$ because

$$1 + (1)(-1) = 0 \not> 0$$

$\therefore$ R is not transitive on R.

(v) $aRb \Leftrightarrow |a| \leq b$

Reflexivity Let $-1 \in R$, then $|-1| \not\leq (-1)$

$\therefore$ R is not reflexive relation

Symmetry Now, let $-3 R 4$, then $|4| \not\leq -3$

$\Rightarrow \quad 4 \not R -3$

$\therefore R$ is not symmetric relation

Transitivity aRb and $bRc \Rightarrow |a| \leq b$ and $|b| \leq c$

Then, $\quad |a| \leq c \ \Rightarrow aRc$

$\therefore$ R is transitive relation.

71. $A = \{x : -1 \leq x \leq\}$

$B = \{x : -1 \leq x \leq\}$

(i) $f(x) = \dfrac{x}{2}$

Let $\quad x_1, x_2 \in A$

$\therefore \quad f(x_1) = f(x_2) \Rightarrow \dfrac{x_1}{2} = \dfrac{x_2}{2}$

$\Rightarrow \quad x_1 = x_2$

$\therefore$ f is one-one function.

Now, let $\quad y = \dfrac{x}{2} \Rightarrow x = 2y$

$\Rightarrow \quad -1 \leq y \leq 1$

$\Rightarrow \quad -2 \leq 2y \leq 2 \Rightarrow -2 \leq x \leq 2$

Let $\quad x \in [-1, 1]$

$\therefore$ There are some value of y for which x does not exist. So, f not onto.

(ii) $g(x) = |x|$

For $\quad x = -1, g(-1) = 1$

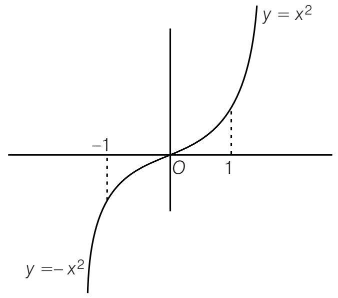
and for $\quad x = 1, g(1) = 1$

$\therefore$ f is not one-one function

Let $\quad y = |x|$, then $y \geq 0$

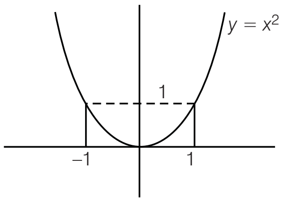
$\therefore$ f is not onto.

(iii) $h(x) = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$



From figure, it is clear tat h is one-one and onto i.e., bijective.

(iv) $k(x) = x^2$



$$k(1) = 1$$

and $k(-1) = 1$

So, k is many-one function.

From figure, $y \in (0, 1)$

$\therefore$ y is not onto function.

(v) $y = l(x) = \sin \pi x$

for $x = 1$, $l(1) = \sin \pi = 0$

for $x = -1$, $l(-1) = \sin(-\pi) = 0$

$\therefore l$ is not one-one.

Now, $-1 \le x \le 1$

$\Rightarrow$ $-\pi \le \pi\ x \le \pi$

$\Rightarrow$ $-1 \le \sin \pi\ x \le 1$

$\therefore$ y is onto function.

Hence, l is surjective function.

72. $(fog)x = f(3x - 2) = e^{3x-2}$

and $(gof)x = g(e^x) = 3e^x - 2$

Let $(fog)x = y \Rightarrow e^{3x-2} = y$

$\Rightarrow$ $3x - 2 = \log_e y \Rightarrow x = \dfrac{2 + \log_e y}{3}$

$\Rightarrow$ $(fog)^{-1}(y) = \dfrac{2 + \log_e y}{3}$

$\Rightarrow$ $y > 0$ So, domain of $(fog)^{-1}$ is $(0, \infty)$.

Now, again let $(gof)x = 3e^x - 2$

$\Rightarrow$ $y = 3e^x - 2 \Rightarrow e^x = \dfrac{y+2}{3}$

$\therefore$ $x = \log_e\left(\dfrac{y+2}{3}\right)$

$\Rightarrow$ $(gof)^{-1}(y) = \log_e\left(\dfrac{y+2}{3}\right)$

Clearly, $y + 2 > 0 \Rightarrow y > -2$

$\therefore$ Domain of $(gof)^{-1}$ is $(-2, \infty)$.

73. $f(x) = \dfrac{x^2 - x}{x^2 + 2x}$ ...(i)

$f(x) = \dfrac{x(x-1)}{x(x+2)}$

$f(x) = \dfrac{(x-1)}{(x+2)}, x \ne 0$ ...(ii)

$D_f = \{x : x^2 + 2x \ne 0\}$ [from Eq. (i)]

$= \{x : x \in R - \{0, -2\}\}$

Now, let $y = \dfrac{x-1}{x+2}$

$\Rightarrow$ $yx + 2y = x - 1 \Rightarrow x(y-1) = -(1+2y)$

$\Rightarrow$ $x = \dfrac{1+2y}{1-y}$

Now, for $y = 1$, x is not defined.

Now, $x = 0, f(x) = -\dfrac{1}{2}$

$\therefore$ $R_f = R - \left\{1, -\dfrac{1}{2}\right\}$

Now, let $x_1, x_2 \in D_f$

Then, $f(x_1) = f(x_2) \Rightarrow \dfrac{x_1 - 1}{x_1 + 2} = \dfrac{x_2 - 1}{x_2 + 2}$

$\Rightarrow$ $x_1x_2 + 2x_1 - x_2 - 2 = x_1x_2 - x_1 + 2x_2 - 2$

$\Rightarrow$ $x_1 = x_2$

$\therefore f$ is one-one function.

Now, let $y = \dfrac{x-1}{x+2}$

Then, $x = \dfrac{1+2y}{1-y}$

$\Rightarrow$ $f^{-1}(y) = \dfrac{1+2y}{1-y}$ $[\because f(x) = y \Rightarrow x = f^{-1}(y)]$

Replace y by x, we get $f^{-1}(x) = \dfrac{1+2x}{1-x}$

$\Rightarrow$ $\dfrac{d}{dx}\{f^{-1}(x)\} = \dfrac{(1-x)\,2 - (1+2x)(-1)}{(1-x)^2}$

$= \dfrac{2 - 2x + 1 + 2x}{(1-x)^2}$

$\Rightarrow$ $\dfrac{d}{dx}\{f^{-1}(x)\} = \dfrac{3}{(1-x)^2}$

$\therefore$ Domain of $\dfrac{d}{dx}\{f^{-1}(x)\} = R - \{1\}$

74. $f(x) = x^2 - 1$

$$g(x) = \sqrt{x^2+1};\ h(x) = \begin{cases} 0, \text{ if } & x \le 0 \\ x, \text{ if } & x \ge 0 \end{cases}$$

$\therefore$ $(hofog)(x) = (hof)\{g(x)\}$

$= (hof)\sqrt{x^2+1}$

$= h\{f\sqrt{(x^2+1)}\}$

$= h\{\sqrt{(x^2+1)}^2 - 1\} = h(x^2 + 1 - 1)$

$= h(x^2) = x^2$ $[\because x^2 \ge 0]$

and $(fog)(x) = f\{g(x)\}$

$= f(\sqrt{x^2+1}) = \left(\sqrt{x^2+1}\right)^2 - 1 = x^2 + 1 - 1 = x^2$

Let $y = (fog)x = x^2, \forall\ x \in R$

If $x = 1$, then $y = 1$

If $x = -1$, then $y = 1$

So, fog is not one-one, so it is not invertible $h(x) = \begin{cases} 0, & x \le 0 \\ x, & x \ge 0 \end{cases}$

For $x = -1$, $h(-1) = 0$ and for $x = -2$, $h(-2) = 0$

$\therefore$ h is not identity function.

75. Here, (3, 3), (6, 6), (9, 9), (12, 12) So, it is Reflexive and

(3, 6), (6, 12), (3, 12) So, it is Transitive

Here, reflexive and transitive only.

76. Clearly, $(x, x) \in R, \forall\ x \in W$

So, R is reflexive.

Let $(x, y) \in R$, then $(y, x) \in R$ as x and y have atleast one letter in common. So, R is symmetric. But R is not transitive.

e.g. Let x = INDIA, y = BOMBAY and z = JUHU

Then, $(x, y) \in R$ and $(y, z) \in R$ but $(x, z) \notin R$

77. $T = \{(x, y) : x - y \in I\}$

As $0 \in I$, so T is a reflexive relation.

If $x - y \in I \Rightarrow y - x \in I$

$\therefore T$ is symmetric also.

If $x - y = I$ and $y - z = I2$

Then, $x - z = (x - y) + (y - z) = I_1 + I_2 \in I$

$\therefore$ T is also transitive.

Hence, T is an equivalence relation. Clearly, $x \neq x + 1 \Rightarrow (x, x) \notin S$

$\therefore S$ is not reflexive.

78. $\because$ $A \cap B = A \cap C \Rightarrow B = C$ and $A \cup B = A \cup C \Rightarrow B = C$

Hence, $B = C$

79. For disoint sets, $A \cap B = \phi$

Each element in either A or B or neither.

$\therefore$ Total ways $= 3^4 = 81$; $A = B$ iff $A = B = \phi$

Otherwise, A and B are interchangable

$\therefore$ Number of unordered pair for disoint subsets of

$$S = \frac{3^4 + 1}{2} = 41$$

80. xRy need not implies yRx.

$$S : \frac{m}{n} S \frac{p}{q} \Leftrightarrow qm = pm \Rightarrow \frac{m}{s} S \frac{m}{n} \text{ is reflexive.}$$

$$\frac{m}{n} S \frac{p}{q} \Rightarrow \frac{p}{q} S \frac{m}{n} \text{ is symmetric.}$$

and $$\frac{m}{n} S \frac{p}{q}, \frac{p}{q} S \frac{r}{t} \Rightarrow qm = pn,\ pt = qr$$

$$\Rightarrow \quad mt = nr \Rightarrow \frac{m}{n} S \frac{r}{t} \text{ is transitive.}$$

$\therefore$ S is an equivalence relation.

81. $P : \sin\theta - \cos\theta = \sqrt{2}\cos\theta \Rightarrow \tan\theta = \sqrt{2} + 1$

$$Q : \sin\theta + \cos\theta = \sqrt{2}\sin\theta \Rightarrow \tan\theta = \frac{1}{\sqrt{2} - 1} = \sqrt{2} + 1$$

$\therefore$ $P = Q$

82. $\because (fogogof)(x) = (gogof)(x)$

$\therefore$ $(\sin\sin x^2)^2 = \sin\sin x^2 \Rightarrow \sin\sin x^2 = 0$ or 1

$\Rightarrow$ $x = +\sqrt{n\pi}, n \in \{0, 1, 2, 3, \ldots\}$

83. **Statement-1** $A = \{(x, y) \in R \times R : y - x \text{ is an integer}\}$

(a) **Reflexive** $xRy : (x - x)$ is an integer

which is true.

Hence it is reflexive.

(b) **Symmetric** $xRy : (x - y)$ is an integer.

$\Rightarrow -(y - x)$ is also an integer.

$\therefore$ $(y - x)$ is also an integer.

$\Rightarrow$ $y\ R\ x$

Hence, it is symmetric.

(c) **Transitive** $x\ R\ y$ and $y\ R\ z$

$\Rightarrow (x - y)$ and $(y - z)$ are integere and.

$\Rightarrow (x - y) + (y - z)$ is an integer.

$\Rightarrow (x - z)$ is an integer.

$\Rightarrow xRz$

$\therefore$ It is transitive

Hence, it is equvalence relation.

Statement-2

$B = \{(x, y) \in R \times R : x = \alpha y$ for some reational number $\alpha\}$

If $\alpha = 1$, then

$xRy : x = y$ (To check equivalence)

(a) **Reflexive** $xRx : x = x$ (True)

$\therefore$Reflexive

(b) **Symmetric** $xRy : x = y \Rightarrow y = x \Rightarrow yRx$

$\therefore$ Symmetric

(c) **Transitive** xRy and $yRz \Rightarrow x = y$

and $y = z \Rightarrow x = z \Rightarrow xRz$

$\therefore$ Transitive

Hence, it is equivalence relation.

$\therefore$Both are true but Statement-2 is not correct explanation of Statement-2

84. $\because A \times B$ has 8 elements.

$\therefore$Number of subsets $= 2^8 = 256$

Number of subsets with zero element $= {}^8C_0 = 1$

Number of subsets with one element $= {}^8C_1 = 8$

Number of subsets with one elements $= {}^8C_2 = 28$

Hence, Number of subsets of $A \times B$ having 3 or more elements

$= 256 - (1 + 8 + 28) = 256 - 37 = 219$

85. Since, $4^n - 3n - 1 = (1 + 3)^n - 3n - 1$

$= (1 + {}^nC_1 \cdot 3 + {}^nC_2 \cdot 3^2 + {}^nC_3 \cdot 3^3 + \ldots + {}^nC_n \cdot 3^n) - 3n - 1$

$= 3^2({}^nC_2 + {}^nC_3 \cdot 3 + \ldots + {}^nC_n \cdot 3^{n-2})$

$\Rightarrow 4^n - 3n - 1$ is a multiple of 9 for $n \geq 2$

For $n = 1$, $4^n - 3n - 1 = 4 - 3 - 1 = 0$

For $n = 2$, $4^n - 3n - 1 = 16 - 6 - 1 = 9$

$\therefore 4^n - 3n - 1$ is multiple of 9 for all $n \in N$.

It is clear that X contains elements, which are multiples of 9 and Y contains all multiples of 9.

$\therefore$ $X \subseteq Y$ i.e., $X \cup Y = Y$

86. $n(A) = 4, n(B) = 2 \Rightarrow n(A \times B) = 8$

The number of subsets of $A \times B$ having at least three elements

$= {}^8C_3 + {}^8C_4 + {}^8C_5 + \ldots + {}^8C_8$

$= 2^8 - ({}^8C_0 + {}^8C_1 + {}^8C_2)$

$= 256 - (1 + 8 + 28) = 219$