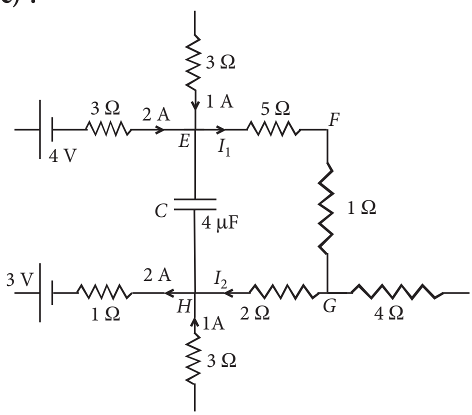
VERY SIMILAR
PRACTICE TEST 4

Hints and Explanations

1. (c) :



At steady state, no current flows through capacitor C.

According to Kirchhoff's law,

(i) at junction E, $I_1 = 3\text{A}$ (ii) at junction H, $I_2 = 1\text{A}$

(iii) Potential difference across capacitor,

$V_E - V_H = 6I_1 + 2I_2$

or $V = (6 \times 3) + (2 \times 1) = 18 + 2 = 20$ V

$\therefore$ Energy $= \frac{1}{2}\quad{}^{2}$

or $U = \frac{1}{2} \times (4 \times 10^{-6}) \times (20)^2 = 8 \times 10^{-4}$ J

2. (c) : When the like poles are tied together, the net magnetic moment is $(m_1 + m_2)$ and the moment of inertia is $(I_1 + I_2)$.

$\therefore$ The time period $T_1 = 2\pi\sqrt{\frac{I_1 + I_2}{(m_1 + m_2)B}}$.

When the unlike poles are tied together, the net magnetic moment is $(m_1 - m_2)$, while the moment of inertia (being a scalar quantity) remains unchanged.

$\therefore$ The time period $T_2 = 2\pi\sqrt{\frac{I_1 + I_2}{(m_1 - m_2)B}}$.

Thus,

$$\frac{T_2^2}{T_1^2} = \frac{(m_1 + m_2)}{(m_1 - m_2)} \Rightarrow \frac{m_1}{m_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2} = \frac{\upsilon_1^2 + \upsilon_2^2}{\upsilon_1^2 - \upsilon_2^2}.$$

Given, $\upsilon_1 = 12$ per minute and $\upsilon_2 = 4$ per minute.

$$\therefore \quad \frac{m_1}{m_2} = \frac{12^2 + 4^2}{12^2 - 4^2} = \frac{144 + 16}{144 - 16} = \frac{160}{128} = \frac{5}{4}.$$

3. (c) : The angular velocity is given as

$\omega^2 = \omega_0^2 + 2\,\alpha\,(\theta - \theta_0)$

when the fan is switch off $\theta = 2\pi n$, $\theta_0 = 0$, $\omega = \frac{\omega_0}{4}$

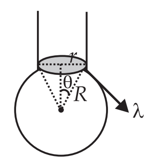
$$\left(\frac{\omega_0}{4}\right)^2 = \omega_0^2 - 2\alpha(2\pi n) \Rightarrow 2\alpha(2\pi n) = \frac{15}{16}\omega_0^2$$

$$2\pi n = \frac{15}{16}\left(\frac{\omega_0^2}{2\alpha}\right)$$

When the fan comes to rest

$$0 = \omega_0{}^2 - 2\alpha(2\pi n') \Rightarrow 2\pi n' = \left(\frac{\omega_0^2}{2\alpha}\right) \text{ or } n' = \frac{16}{15}n$$

4. (a) : The vertical force due to the surface tension on the drop



$= T\,2\pi r \sin\theta$

$= T\,2\pi r\frac{r}{R} = \frac{2\pi r^2 T}{R}$

When the drop detaches from the dropper, then

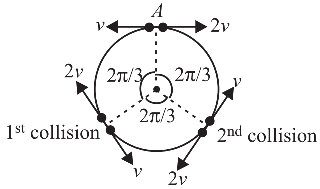
$$\frac{2\pi r^2 T}{R} = mg = \frac{4}{3}\pi R^3 \rho g \quad \text{or} \quad R^4 = \frac{3}{2}\frac{r^2 T}{\rho g}$$

or $R = \left(\frac{3}{2}\frac{r^2 T}{\rho g}\right)^{1/4}$

Substituting the given values, we get

$$R = \left(\frac{3 \times (5 \times 10^{-4})^2 \times 0.11}{2 \times 10^3 \times 10}\right)^{1/4} = 1.4 \times 10^{-3} \text{ m}$$

5. (c) : At first collision, particle having speed $2v$ will rotate by 240° $\left(\text{or } \frac{4\pi}{3}\right)$ while other particle having speed v will rotate by 120° $\left(\text{or } \frac{2\pi}{3}\right)$. At first collision, they will exchange their velocities. Now as shown in figure, after two collisions they will again reach at point A.



6. **(a) :** The entropy change of the block L is

$$\Delta S_L = (1.5\text{ kg})(386\text{ J/kg K})\int_{333\text{ K}}^{313\text{ K}}\frac{dT}{T}$$

$$= (1.5\text{ kg})(386\text{ J/kg K})\ln\frac{313\text{ K}}{333\text{ K}} = -35.8\text{ J/K}$$

The entropy change of the block R is

$$\Delta S_R = (1.5\text{ kg})(386\text{ J/kg K})\int_{293\text{ K}}^{313\text{ K}}\frac{dT}{T}$$

$$= (1.5\text{ kg})(386\text{ J/kg K})\ln\left(\frac{313\text{ K}}{293\text{ K}}\right) = +38.2\text{ J/K}$$

Thus, the net entropy change for the two block system during irreversible process is

$\Delta S_{\text{irrev}} = \Delta S_L + \Delta S_R = -35.8\text{ J/K} + 38.2\text{ J/K} = 2.4\text{ J/K}$

7. **(b) :** Kinetic energy of neutron = 0.0327 eV

or $K = 0.0327 \times 1.6 \times 10^{-19}$ J

or $\frac{1}{2}mv^2 = 0.0327\times1.6\times10^{-19}$

or $v^2 = \dfrac{2\times0.0327\times1.6\times10^{-19}}{1.675\times10^{-27}}$

or $v^2 = 0.0625 \times 10^8$; $v = 0.25 \times 10^4$ m s^{-1}

$\therefore$ Time taken $= \dfrac{\text{distance}}{\text{velocity}} = \dfrac{10}{0.25\times10^4}$

or $t = 4 \times 10^{-3}$ s

$\therefore$ Fraction that decays $= \dfrac{N}{N_0} = (1 - e^{-\lambda t})$

$$= 1 - \left\{e^{-\left(\frac{0.693}{700}\times 4\times10^{-3}\right)}\right\} = 3.9\times10^{-6}$$

8. **(b) :** Here,

$c_m(t) = 30\sin(300\pi t) + 10(\cos(200\pi t) - \cos(400\pi t))$

Compare this equation with standard equation of amplitude modulated wave,

$$c_m(t) = A_c\sin\omega_c t - \frac{\mu A_c}{2}\cos(\omega_c+\omega_m)t + \frac{\mu A_c}{2}\cos(\omega_c-\omega_m)t$$

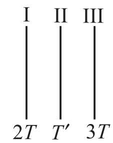
$A_c = 30$ V, $\omega_c = 300\pi \Rightarrow 2\pi\upsilon_c = 300\pi \Rightarrow \upsilon_c = 150$ Hz

$\omega_c - \omega_m = 200\pi \Rightarrow \upsilon_c - \upsilon_m = 100$ Hz

$\therefore\ \upsilon_m = 150 - 100 = 50$ Hz

$\dfrac{\mu A_c}{2} = 10,\ A_c = 30 \quad \therefore\ \mu = \dfrac{10}{15} = \dfrac{2}{3}$

9. **(c) :** Let T' be the temperature of the middle plate (II) and A be area of each plate.



Under steady state, the rate of energy received by the middle plate is equal to rate of energy emitted by it.

i.e., $\sigma A(3T)^4 - \sigma A(T')^4 = \sigma A(T')^4 - \sigma A(2T)^4$

or $\sigma A[(3T)^4 - (T')^4] = \sigma A[(T')^4 - (2T)^4]$

or $(3T)^4 - (T')^4 = (T')^4 - (2T)^4$

or $2(T')^4 = T^4(3^4 + 2^4) = T^4(81 + 16) = 97\,T^4$

or $T'^4 = \dfrac{97}{2}T^4$ or $T' = \left(\dfrac{97}{2}\right)^{1/4}T$

10. **(a) :** As rms value of current,

$I_{\text{rms}} = V_{\text{rms}}/Z$

So, net impedance across LCR circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(100)^2 + (\omega L - X_C)^2}$$

$$= \sqrt{(100)^2 + \left[\left(100\times\pi\times\frac{1}{\pi}\right) - (X_C)\right]^2}$$

$$\left(\frac{220}{2.2}\right)^2 = (100)^2 + ((100) - (X_C))^2 \Rightarrow X_C = 100\,\Omega$$

As, $\tan\theta = \dfrac{X_C}{R} = \dfrac{100}{100} = 1$

$\Rightarrow\ \theta = \tan^{-1}(1)$ or $\theta = 45°$

Power factor, $\cos\theta = \dfrac{1}{\sqrt{2}}$

11. **(d) :** From Einstein's photoelectric equation, we get

$$\frac{hc}{\lambda} = \frac{1}{2}mv^2 + h\upsilon_0 \qquad \text{...(i)}$$

and $\dfrac{hc}{3\lambda/4} = \dfrac{1}{2}mv'^2 + h\upsilon_0$

or $\dfrac{1}{2}mv'^2 = \dfrac{4}{3}\left(\dfrac{hc}{\lambda}\right) - h\upsilon_0 = \dfrac{4}{3}\left(h\upsilon_0 + \dfrac{1}{2}mv^2\right) - h\upsilon_0$ (Using (i))

$$= \frac{1}{3}h\upsilon_0 + \frac{4}{3}\left(\frac{1}{2}mv^2\right)$$

or $\dfrac{1}{2}mv'^2 > \dfrac{4}{3}\left(\dfrac{1}{2}mv^2\right)$ or $v'^2 > \dfrac{4}{3}v^2$

or $v' > v\left(\dfrac{4}{3}\right)^{1/2}$

12. (a) : Let the projectile be fired vertically upward from the earth's surface with velocity v and it reaches a maximum height h. By the law of conservation of mechanical energy,

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

or $$\frac{1}{2}mv^2 = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right] = \frac{GMmh}{(R)(R+h)}$$

or $$\frac{1}{2}mv^2 = \frac{mghR}{(R+h)} \qquad \left(\because\ g = \frac{GM}{R^2}\right)$$

As per question

$v = kv_e = k\sqrt{2gR}$ and $h = r - R$ $\quad (\because\ v_e = \sqrt{2gR})$

where r is the distance from the centre of the earth.

$$\therefore \quad \frac{1}{2}mk^2 2gR = \frac{mg(r-R)R}{r}$$

or $k^2 = \frac{r-R}{r}$ or $(1-k^2)r = R$ or $r = \frac{R}{1-k^2}$

13. (a) : Given : $\phi = at^2 + bt$

The magnitude of induced emf is

$$\varepsilon = \frac{d\phi}{dt} = \frac{d}{dt}(at^2 + bt) = 2at + b$$

Current flowing, $I = \frac{|\varepsilon|}{R} = \frac{2at+b}{R}$

$$\text{Average emf} = \frac{\int_0^\tau \varepsilon dt}{\int_0^\tau dt} = \frac{\int_0^\tau (2at+b)dt}{\tau} = a\tau + b$$

Total charge flowing,

$$q = \int_0^\tau I dt = \int_0^\tau \frac{(2at+b)}{R}dt = \frac{a\tau^2 + b\tau}{R}$$

14. (d) : Comparing the equation, $y = px - qx^2$ with the equation of projectile motion

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$ we get, $\tan\theta = p$ and

$$\frac{g}{2u^2\cos^2\theta} = q \quad \text{or} \quad u\cos\theta = \sqrt{\frac{g}{2q}}$$

The time of flight of the projectile is

$$T = \frac{2u\sin\theta}{g} = \frac{2u\sin\theta\cos\theta}{g\cos\theta} = \frac{2u\cos\theta\tan\theta}{g}$$

$$= \frac{2}{g}(u\cos\theta)(\tan\theta) = \frac{2}{g}\left(\sqrt{\frac{g}{2q}}\right)(p) = p\sqrt{\frac{2}{qg}}$$

15. (d) : Pitch of screw gauge $= \frac{1}{2}$ mm = 0.5 mm

Least count of the screw gauge

$$= \frac{0.5}{50}\text{mm} = 0.01 \text{ mm}$$

Zero error = –0.03 mm

Zero correction = + 0.03 mm

Main scale reading = 3 mm

Circular scale reading = 35

Observed diameter of the wire

= 3 mm + 35 × (0.01) mm = 3.35 mm

Corrected diameter of the wire = (3.35 + 0.03) mm

= 3.38 mm

16. (d) : Here, l = 85 cm = 0.85 m, v = 340 m s^{-1}

Pipe is closed from one end so it behaves as a closed organ pipe.

Frequencies in the closed organ pipe is given by,

$$\upsilon = \frac{(2n-1)v}{4l} \quad \text{where, } n = 1, 2, 3, 4, \ldots$$

According to question, υ < 1250 Hz

$$\left(\frac{2n-1}{4l}\right)v < 1250 \Rightarrow \frac{(2n-1)\times 340}{4\times 0.85} < 1250$$

$\Rightarrow$ $(2n - 1) < 12.5$

Possible value of n = 1, 2, 3, 4, 5, 6

So, number of possible natural frequencies lie below 1250 Hz is 6.

17. (d) : Potential at ∞, $V_\infty = 0$.

Potential at the surface of the sphere,

$$V_s = k\frac{Q}{R}. \qquad \left(\text{where } k = \frac{1}{4\pi\varepsilon_0}\right)$$

Potential at the centre of the sphere,

$$V_c = \frac{3}{2}k\frac{Q}{R}.$$

Let m and $-q$ be the mass and the charge of the particle respectively.

Let v_0 = speed of the particle at the centre of the sphere.

$$\frac{1}{2}mv^2 = -q[V_\infty - V_s] = qk\frac{Q}{R} \qquad \ldots\text{(i)}$$

$$\frac{1}{2}mv_0^2 = -q[V_\infty - V_c] = q\cdot\frac{3}{2}k\frac{Q}{R} \qquad \ldots\text{(ii)}$$

Dividing (ii) by (i),

$$\frac{v_0^2}{v^2} = \frac{3}{2} = 1.5 \quad \text{or} \quad v_0 = \sqrt{1.5}v$$

18. (b) : Applying Bohr model to the given system,

$$\frac{mv^2}{r_n}=\frac{k}{r_n} \quad ...(i)$$

and $mvr_n=\frac{nh}{2\pi}$ or $v=\frac{nh}{2\pi mr_n}$

Put in (i),

$$\frac{m}{r_n}\times\frac{n^2h^2}{4\pi^2m^2r_n^2}=\frac{k}{r_n}$$

$$r_n^2=\frac{n^2h^2}{4\pi^2 mk} \quad ...(ii)$$

$\therefore\quad r_n^2\propto n^2$ or $r_n\propto n$

K.E. of the electron, $T_n=\frac{1}{2}mv^2$

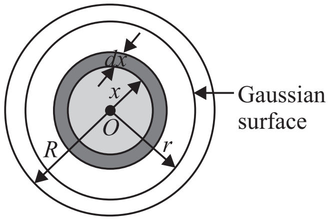
$$=\frac{1}{2}m\frac{n^2h^2}{4\pi^2m^2r_n^2}=\frac{n^2h^2}{8\pi^2mr_n^2}$$

Using (ii), we get

$$T_n=\frac{n^2h^2\,4\pi^2mk}{8\pi^2mn^2h^2}=\frac{k}{2}$$

$\therefore\quad T_n$ is independent of n.

19. (c) : Consider a thin spherical shell of radius x and thickness dx as shown in the figure.



Volume of the shell,

$dV = 4\pi x^2 dx$

Let us draw a Gaussian surface of radius $r(r < R)$ as shown in the figure above.

Total charge enclosed inside the Gaussian surface is

$$Q_{in}=\int_0^r \rho dV=\int_0^r \rho_0\left(\frac{5}{4}-\frac{x}{R}\right)4\pi x^2dx$$

$$=4\pi\rho_0\int_0^r\left(\frac{5}{4}x^2-\frac{x^3}{R}\right)dx=\pi\rho_0\left[\frac{5}{3}r^3-\frac{r^4}{R}\right]$$

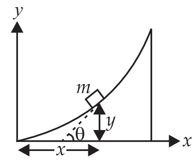
According to Gauss's law,

$$E4\pi r^2=\frac{Q_{in}}{\varepsilon_0}=\frac{\pi\rho_0}{\varepsilon_0}\left[\frac{5}{3}r^3-\frac{r^4}{R}\right];$$

$$\therefore\ E=\frac{\rho_0 r}{4\varepsilon_0}\left[\frac{5}{3}-\frac{r}{R}\right]$$

20. (b) : Block is under limiting friction, so

$\mu = \tan\theta \quad ...(i)$



Equation of the surface, $y=\frac{x^3}{6}$

Slope, $\frac{dy}{dx}=\frac{x^2}{2} \quad ...(ii)$

From eqns (i) and (ii),2 we get $\mu=\frac{x^2}{2}$

Here, $\mu = 0.5$ $\therefore 0.5=\frac{x^2}{2}$ or $x^2=1$ or $x=1$

So, $\quad y=\frac{1}{6}$

21. (1.06) : Here $qE = qvB\sin\phi$ or $v=\frac{E}{B\sin\phi}$

$v' = v\cos\phi$ = velocity along the field

$v'' = v\sin\phi$ = velocity perpendicular to field

By the dynamics of circular motion

$qv''B=\frac{mv''^2}{r}$ or $qB = m\omega$ or $T=\frac{2\pi m}{qB}$

$\therefore\quad p=T\times v\cos\phi=\frac{2\pi m}{qB}v\cos\phi$

$$=\frac{2\pi m}{qB}\left(\frac{E}{B\sin\phi}\right)\cos\phi$$

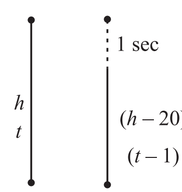
or $\quad p=\frac{2\pi mE}{qB^2}\cot\phi$

$$=\frac{2\pi\times1.67\times10^{-27}\times4.5\times10^4}{1.6\times10^{-19}\times40^2\times10^{-6}}\cot60^\circ$$

≈ 1.06 m

22. (31.2) : For first ball, $h=\frac{1}{2}gt^2 \quad ...(i)$

For second ball, $(h-20)=\frac{1}{2}g(t-1)^2 \quad ...(ii)$



$h-20=\frac{1}{2}g(t-1)^2$

or $\frac{gt^2}{2}-20=\frac{g}{2}(t^2-2t+1)$ (Using (i))

or $\frac{gt^2}{2}-20=\frac{gt^2}{2}-gt+\frac{g}{2}$

or $gt=\frac{g}{2}+20$ or $10t=\frac{10}{2}+20=25$

or $t=\frac{25}{10}=2.5 \quad ...(iii)$

$\because\ h=\frac{1}{2}gt^2\ \therefore\ h=\frac{1}{2}\times10\times(2.5)^2$

or $\quad h=\frac{10\times6.25}{2}$ or $h = 31.25$ m $= 31.2$ m

23. (0.5) : $\dfrac{\Delta V}{V} = \dfrac{\Delta(\pi r^2 L)}{\pi r^2 L} = \dfrac{r^2\Delta L + 2rL\Delta r}{r^2 L}$

or $\dfrac{\Delta V}{V} = \dfrac{\Delta L}{L} + 2\dfrac{\Delta r}{r}$; But $\Delta V = 0$ (given)

$\therefore \quad 0 = \dfrac{\Delta L}{L} + 2\dfrac{\Delta r}{r}$ or $\dfrac{\Delta L}{L} = -2\dfrac{\Delta r}{r}$

Poisson's ratio, $\sigma = -\dfrac{\Delta r/r}{\Delta L/L} = \dfrac{\Delta r/r}{2\Delta r/r} = \dfrac{1}{2} = 0.5$

24. (11.0) : $\beta = \dfrac{I_C}{I_B}$

$I_B = \dfrac{I_C}{\beta} = \dfrac{2.5}{200} = 0.0125$ mA

Applying Kirchhoff's law to base emitter loop,

$V_{CE} = V_C - I_C R_C$
$= 20 - (2.5 \times 10^{-3}) \times (5 \times 10^3) = 7.5$ V

$V_{BE} = V_B - I_B R_B$
$= 20 - (0.0125 \times 10^{-3}) \times (120 \times 10^3) = 18.5$ V

$\therefore \quad V_{BC} = (V_{BE} - V_{CE}) = (18.5 - 7.5) = 11$ V

25. (4.0) : On immersing the apparatus in a liquid of $\mu = 1.5$, wavelength $\lambda' = \dfrac{\lambda}{1.5} = \dfrac{\lambda}{3/2}$

As fringe width $\propto$ wavelength,

$\therefore$ fringe width $\beta' = \dfrac{\beta}{3/2}$

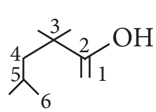
The distance between central maximum and tenth maximum is 3 cm in vacuum. When immersed in liquid, this distance would reduce to $\dfrac{3}{3/2}\text{ cm} = 2\text{ cm}$. As the position of central maximum does not change, therefore, y-coordinate of central maximum will remain 2 cm and that of tenth maximum would become 4 cm.

26. (d) : Among the given oxoacids of phosphorus, pyrophosphoric acid is a tetrabasic acid.

27. (b)

28. (c) : O_3 does not decolourise $KMnO_4$.

29. (c) :



3,3,5-Trimethylhex-1-en-2-ol

30. (a) : $\overset{+7}{KMnO_4} \rightarrow K_2\overset{+6}{MnO_4}$

Equivalent weight of $KMnO_4$

$= \dfrac{\text{Molecular weight}}{\text{Change in oxidation number}} = M$

31. (a) : The formation of CH_3OCH_3 may be represented as

$$2C_{(s)} + 3H_{2(g)} + \frac{1}{2}O_{2(g)} \longrightarrow CH_3{-}O{-}CH_{3(g)};\ \Delta H = ?$$

It involves vaporisation of 2 gram atoms of solid carbon and breaking of 3 moles of H – H bonds and $\dfrac{1}{2}$ mole of O=O bond resulting in the formation of 6 C—H bonds and 2 C—O bonds. Thus, the heat of formation of $CH_3{-}O{-}CH_3$ is given by

$$\Delta H = (2 \times 125 + 3 \times 103 + \frac{1}{2} \times 177) - (6 \times 87 + 2 \times 70)$$

$= -14.5$ kcal

32. (a) : $\begin{cases} ZnCO_3 \xrightarrow{\text{Calcination}} ZnO \\ ZnO + C \xrightarrow[\text{Reduction}]{\text{Carbon}} Zn + CO \end{cases}$

$\begin{cases} CaCO_3 \xrightarrow[\text{Calcination}]{\Delta} CaO \\ CaO \xrightarrow[\text{Salt electrolysis}]{\text{Molted}} \mathbf{Ca} \end{cases}$

$\begin{cases} Cu_2S + \frac{3}{2}O_2 \xrightarrow{\text{Roasting}} Cu_2O + SO_2 \\ 2Cu_2O + Cu_2S \xrightarrow[\text{Reduction}]{\text{Self}} 6Cu + SO_2 \end{cases}$

33. (b) : $\underset{(X)}{CaCO_3} \xrightleftharpoons{\text{Heat}} CaO + CO_2$

$CaO + H_2O \longrightarrow Ca(OH)_2$

$\underset{(Y)}{Ca(OH)_2} + CO_2 + H_2O \longrightarrow \underset{(Z)}{Ca(HCO_3)_2}$

$\underset{(Z)}{Ca(HCO_3)_2} \xrightarrow{\text{boil}} \underset{(X)}{CaCO_3} + CO_2 + H_2O$

34. (b) : $Hg^{2+} + 2KI \longrightarrow \underset{\text{(scarlet red)}}{HgI_2\downarrow} + 2K^+$

$HgI_2 + 2KI \longrightarrow \underset{\text{(Nessler's reagent)}}{K_2HgI_4}$

$Hg^{2+} + Co^{2+} + 4SCN^- \longrightarrow \underset{\text{(deep blue crystalline)}}{Co[Hg(SCN)_4]\downarrow}$

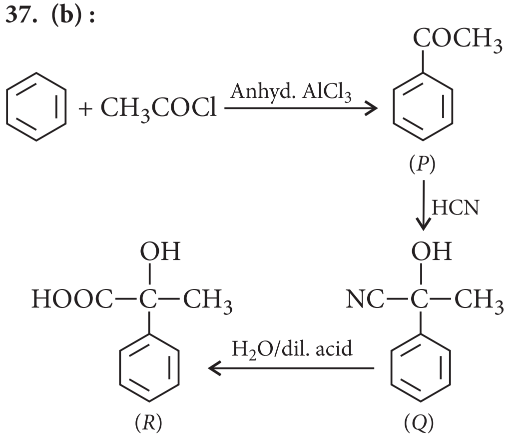
35. (d) : Hyperconjugation is the delocalisation of sigma electrons with π-bond. In carbocation, hyperconjugation is possible only when α-hydrogen is present, so in carbocations (a), (c) and (d) hyperconjugation is possible while (b) is stabilised by the interaction of the cyclopropyl bonding orbitals with the vacant p-orbital of carbon.

36. (b) :

$$\underset{A(C_2H_3N)}{CH_3C{\equiv}N} \xrightarrow{\text{Reduction}} \underset{\text{Ethyl amine }(B)}{CH_3CH_2NH_2} \xrightarrow{HNO_2} \underset{\text{Ethyl alcohol}}{CH_3CH_2OH}$$

$$\underset{\text{Ethyl amine }(B)}{CH_3CH_2NH_2} \xrightarrow{CHCl_3/\text{Alc. KOH}} \underset{\text{Ethyl isocyanide }(C)}{CH_3CH_2N{\overset{\rightarrow}{\equiv}}C}$$

37. (b) :



38. (a)

39. (c) : $\log k = -\dfrac{E_a}{2.303RT} + \log A$ (straight line)

Slope $= -\dfrac{E_a}{2.303R}$

First order reaction,

$k = \dfrac{2.303}{t}\log\dfrac{C}{C-x}$ (straight line)

Slope $= -\dfrac{k}{2.303}$

$t_{1/2} \propto a^{1-n}$; $n = 3$, $t_{1/2} \propto \dfrac{1}{a^2}$

$t_{1/2} = \dfrac{k'}{a^2} \Rightarrow t_{1/2} \cdot a^2 = k'$

k' is a proportionality constant.

Units of k for the first order reaction are independent of concentration units.

40. (c) : $K_4[Fe(CN)_6] \rightleftharpoons 4K^+ + [Fe(CN)_6]^{4-}$

Potassium hexacyanoferrate(II) gives a total of 5 ions in aqueous solution thus, it has the highest molar conductivity whereas other complexes will give lesser number of ions.

41. (b)

42. (a) : The polymer is glyptal and its monomers are ethylene glycol (CH_2OH-CH_2OH) and phthalic acid ($HOOC-C_6H_4-COOH$).

43. (a) : $2Ag_{(s)} \longrightarrow 2Ag^+\ (0.1\text{ M}) + 2e^-$

$2e^- + Zn^{2+}\ (0.1\text{ M}) \longrightarrow Zn_{(s)}$

$2Ag_{(s)} + Zn^{2+}\ (0.1\text{ M}) \longrightarrow 2Ag^+(0.1\text{ M}) + Zn_{(s)}$

$\therefore\quad E_{cell} = E^\circ_{cell} - \dfrac{0.0591}{2}\log_{10}\dfrac{[Ag^+]^2}{[Zn^{2+}]}$

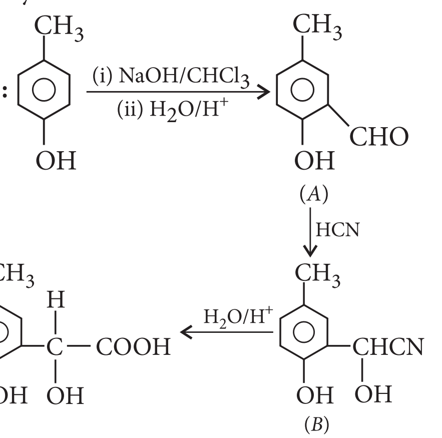
$\therefore\quad E_{cell} = -1.562 - \dfrac{0.0591}{2}\log_{10}\dfrac{(0.1)^2}{0.1}$

$= -1.562 - 0.03\log_{10}10^{-1}$

$\Rightarrow E_{cell} = -1.562 + 0.03 = -1.532\text{ V}$

44. (d) : Aspirin a weak acid is unionised in acidic medium due to common ion effect and completely ionised in alkaline medium.

45. (c) :



46. (2.57) : One unit cell in NaCl lattice has 4 NaCl formula units.

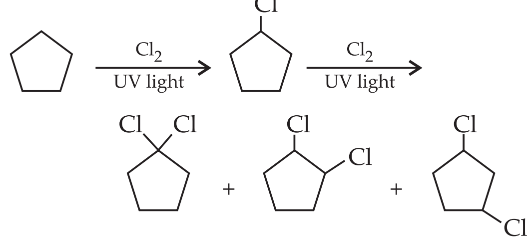
4×58.5 g NaCl $= 6.02 \times 10^{23}$ unit cells

(1 mole unit cells)

1 g NaCl $= \dfrac{6.02\times10^{23}}{4\times58.5} = 2.57 \times 10^{21}$ unit cells

47. (8) : Gold number of starch sol is the amount in milligrams required to prevent coagulation of 10 mL of gold sol when 1 mL of 10% NaCl solution is present. This amount is 0.008 g or 8 mg thus, gold number of starch is 8.

48. (5) : Since the hydrocarbon (X) gives only a single monochloride, it implies that all the C—H bonds must be of the same type. So, the given hydrocarbon should be cyclopentane.



49. **(16.89) :** $ClO_3^- \longrightarrow \overset{0}{Cl_2}$;

In $ClO_3^- = x - 6 = -1$ or $x = +5$

$$\text{Eq. mass of } ClO_3^- = \frac{\text{Mol. mass of } ClO_3^-}{\text{Change in oxidation number}}$$

$$= \frac{84.45}{5} = 16.89$$

50. **(4) :** Partial charge $= \dfrac{\text{Dipole moment}}{\text{Bond distance}}$

$$= \frac{1.2 \times 10^{-18}\text{ esu cm}}{1.0 \times 10^{-8}\text{ cm}} = 1.2 \times 10^{-10}\text{ esu}$$

The fraction of an electronic charge is

$$\frac{1.2 \times 10^{-10}\text{ esu}}{4.8 \times 10^{-10}\text{ esu}} = \frac{1}{4} \quad \therefore \quad \text{Value of } x = 4$$

51. **(c) :** The given function is

$$f(x) = \sin^{-1}\left(2x^2 + 3x + 1\right)$$

$f(x)$ will be defined when $-1 \le 2x^2 + 3x + 1 \le 1$

$$\Rightarrow -\frac{1}{2} \le x^2 + \frac{3}{2}x + \frac{1}{2} \le \frac{1}{2}$$

$$\Rightarrow -\frac{1}{2} \le (x)^2 + 2x\frac{3}{4} + \left(\frac{3}{4}\right)^2 + \frac{1}{2} - \frac{9}{16} \le \frac{1}{2}$$

$$\Rightarrow -\frac{1}{2} \le \left(x + \frac{3}{4}\right)^2 - \frac{1}{16} \le \frac{1}{2}$$

$$\Rightarrow -\frac{1}{2} + \frac{1}{16} \le \left(x + \frac{3}{4}\right)^2 \le \frac{1}{2} + \frac{1}{16}$$

$$\Rightarrow -\frac{7}{16} \le \left(x + \frac{3}{4}\right)^2 \le \frac{9}{16} \Rightarrow 0 \le \left(x + \frac{3}{4}\right)^2 \le \frac{9}{16}$$

$$\Rightarrow -\frac{3}{4} \le \left(x + \frac{3}{4}\right) \le \frac{3}{4} \Rightarrow -\frac{3}{2} \le x \le 0$$

52. **(b) :** For adj A, interchange the diagonal elements and change the sign of counter diagonal elements.

$$\text{We have } A^{-1} = \frac{1}{\cos^2\theta + \sin^2\theta}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = A^T$$

53. **(a) :** Let $I = \displaystyle\int_0^{\pi} e^{\sin^2 x}\cos^3 x\,dx$

$$= \int_0^{\pi/2} [e^{\sin^2 x}\cos^3 x + e^{\sin^2(\pi - x)}\cos^3(\pi - x)]\,dx$$

$$\left[\because \int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a - x)dx\right]$$

$$= \int_0^{\pi/2}\left[e^{\sin^2 x}\cos^3 x - e^{\sin^2 x}\cos^3 x\right]dx = 0$$

54. **(d) :** $\lim\limits_{x \to \infty} (x - 1 - ax - b) = 2$

$\Rightarrow \lim\limits_{x \to \infty} [(1 - a)x - 1 - b] = 2$

$\Rightarrow a = 1, b = -3 \Rightarrow a - b = 4.$

55. **(d) :** The equation of the line passing through $(3, b, 1)$ and $(5, 1, a)$ is $\dfrac{x-5}{2} = \dfrac{y-1}{1-b} = \dfrac{z-a}{a-1} = \mu\text{(say)}$

The line crosses the yz plane where $x = 0$, *i.e.*,

$$-5 = 2\mu \quad \therefore \quad \mu = -\frac{5}{2}$$

$$\text{Again, } y = \mu(1 - b) + 1 = \frac{17}{2}$$

$$\Rightarrow -\frac{5}{2}(1 - b) + 1 = \frac{17}{2}$$

$$\Rightarrow -\frac{5}{2}(1 - \ ) = \frac{15}{2} \Rightarrow (1 - b) = -3 \quad \therefore \quad b = 4$$

$$\text{Again, } z = \mu(a - 1) + a = -\frac{13}{2}$$

$$\Rightarrow -\frac{5}{2}(a - 1) + a = -\frac{13}{2} \Rightarrow -\frac{3}{2}a + \frac{5}{2} = -\frac{13}{2}$$

$$\Rightarrow -\frac{3}{2}a = -9 \Rightarrow a = 6$$

56. **(c) :** We have, $T_1 = {}^nC_0 = 1$ ...(i)

$T_2 = {}^nC_1 ax = 6x$ ...(ii)

$T_3 = {}^nC_2 (ax)^2 = 16x^2$ ...(iii)

From (ii), $\dfrac{n!}{(n-1)!}a = 6 \Rightarrow na = 6$ ...(iv)

From (iii), $\dfrac{n(n-1)}{2}a^2 = 16$ ...(v)

Solving (iv) and (v), we have $a = \dfrac{2}{3}$ and $n = 9$.

57. **(d) :** $\sin 36° \sin 72° \sin 108° \sin 144°$

$$= \sin^2 36° \sin^2 72° = \frac{1}{4}\{(2\sin^2 36°)(2\sin^2 72°)\}$$

$$= \frac{1}{4}\{(1 - \cos 72°)(1 - \cos 144°)\}$$

$$= \frac{1}{4}\{(1 - \sin 18°)(1 + \cos 36°)\}$$

$$= \frac{1}{4}\left[\left(1-\frac{\sqrt{5}-1}{4}\right)\left(1+\frac{\sqrt{5}+1}{4}\right)\right] = \frac{20}{16}\times\frac{1}{4} = \frac{5}{16}$$

58. (a) : Given, $f(x) = \dfrac{x}{1+x^2}$

$$\Rightarrow \quad f'(x) = \frac{(1+x^2)(1)-(x)(2x)}{(1+x^2)^2}$$

$$= \frac{1+x^2-2x^2}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$

For decreasing function $f'(x) < 0$, $\dfrac{1-x^2}{(1+x^2)^2} < 0$

$\Rightarrow \quad 1 - x^2 < 0$ and $(1 + x^2)^2 > 0$ ...(i)

So, function decreases in the interval

$(-\infty, -1] \cup [1, \infty)$

59. (b) : Possibility of getting a score 9 are (5, 4), (4, 5), (6, 3), (3, 6)

Probability of getting score 9 in a single throw

$= p = \dfrac{4}{36} = \dfrac{1}{9}$

$\therefore$ Required probability = probability of getting score 9 exactly twice

$$= {}^3C_2\left(\frac{1}{9}\right)^2 \times \left(\frac{8}{9}\right) = \frac{8}{243}.$$

60. (c) : Let α, β be the roots of

$x^2 - 2\sqrt{2}\,kx + 2e^{2\log k} - 1 = 0.$

$\therefore \quad \alpha\beta = 2e^{2\log k} - 1$ ...(i)

Product of roots $(\alpha\beta)$ is given 31

Hence from (i),

$2e^{2\log k} - 1 = 31$ or $2e^{\log k^2} - 1 = 31$

or $2k^2 = 32$ or $k^2 = 16$ or $k = \pm 4$.

As $\log(-4)$ is not defined so $k = 4$.

61. (d) : $16x^2 - 25y^2 + 400 = 0$

$\Rightarrow \quad \dfrac{y^2}{16} - \dfrac{x^2}{25} = 1 \Rightarrow a^2 = 16, b^2 = 25.$

$\therefore \quad$ Latus rectum $= \dfrac{2b^2}{a} = \dfrac{50}{4} = \dfrac{25}{2}.$

62. (c) : $I = \displaystyle\int_{-2}^{2}(x-[x])\,dx = \int_{-2}^{-1}(x+2)\,dx + \int_{-1}^{0}(x+1)\,dx$

$$+ \int_0^1 x\,dx + \int_1^2 (x-1)\,dx$$

$$= \left[\frac{x^2}{2}+2x\right]_{-2}^{-1} + \left[\frac{x^2}{2}+x\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2}-x\right]_1^2$$

$$= \frac{1}{2} - 2 - (2-4) + 0 - \left(\frac{1}{2}-1\right) + \frac{1}{2} + 2 - 2 - \left(\frac{1}{2}-1\right)$$

$= 2$

63. (d) : Word 'MATHEMATICS' has 2M, 2T, 2A, H, E, I, C, S. Therefore 4 letters can be chosen in the following ways.

Case I : 2 alike of one kind and 2 alike of second kind *i.e.*, ${}^3C_2 \Rightarrow$ No. of words $= {}^3C_2\,\dfrac{4!}{2!2!} = 18$

Case II : 2 alike of one kind and 2 different *i.e.*, ${}^3C_1 \times {}^7C_2$

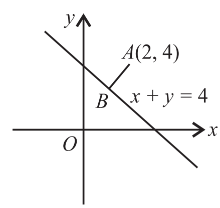
$\Rightarrow$ No. of words $= {}^3C_1 \times {}^7C_2 \times \dfrac{4!}{2!} = 756$

Case III : All are different *i.e.*, ${}^8C_4$

$\Rightarrow$ No. of words $= {}^8C_4 \times 4! = 1680.$

Hence total number of words are 2454.

64. (a) : From the figure let B is the foot of perpendicular so coordinates of B are $(h, 4 - h)$.



Slope of $BA = \dfrac{4-4+h}{2-h} = \dfrac{h}{2-h}$

Slope of given line = -1.

Since both the lines are perpendicular so

$\dfrac{h}{2-h}(-1) = (-1) \Rightarrow h = 1$

$\therefore$ Coordinates of foot of perpendicular are (1, 3).

65. (b) : $\tan 20° + 2\tan 50° - \tan 70°$

$$= \frac{\sin 20°}{\cos 20°} - \frac{\sin 70°}{\cos 70°} + 2\tan 50°$$

$$= \frac{\sin 20°\cos 70° - \cos 20°\sin 70°}{\cos 20°\cos 70°} + 2\tan 50°$$

$$= \frac{\sin(20°-70°)}{\frac{1}{2}[\cos(70°+20°)+\cos(70°-20°)]} + 2\tan 50°$$

$$= \frac{2\sin(-50°)}{\cos 90° + \cos 50°} + 2\tan 50°$$

$$= \frac{-2\sin 50°}{0 + \cos 50°} + 2\tan 50°$$

$= -2\tan 50° + 2\tan 50° = 0$

66. (d) : Given, a, b, c are in A.P. $\Rightarrow$ $2b = a + c$

So, $\dfrac{(a-c)^2}{(b^2-ac)} = \dfrac{(a-c)^2}{\left\{\left(\dfrac{a+c}{2}\right)^2 - ac\right\}}$

$= \dfrac{(a-c)^2 4}{[a^2+c^2+2ac-4ac]} = \dfrac{4(a-c)^2}{(a-c)^2} = 4$

67. (c) : $\sim p$: It is not that the earth is round

$\sim q$: It is not that $3 + 4 = 7$

$(\sim p) \vee (\sim q)$: It is not that the earth is round or it is not that $3 + 4 = 7$

68. (b) : Given data is 3, 10, 10, 4, 7, 10, 5

Here, $n = 7$

Mean $= \bar{x} = \dfrac{3+10+10+4+7+10+5}{7} = \dfrac{49}{7} = 7$

Mean deviation from mean $= \dfrac{\sum |x_i - \bar{x}|}{n}$

$= \dfrac{4+3+3+3+3+2}{7} = \dfrac{18}{7} = 2.57$

69. (b) : We have, $e^{\frac{dy}{dx}} = x+1$

$\Rightarrow \dfrac{dy}{dx} = \log_e(x+1)$

$\Rightarrow \int dy = \int \log_e(x+1)\,dx$

$\Rightarrow y = (x + 1)\log|x + 1| - (x + 1) + c$

When $y(0) = 3$ then, $3 = 0 - 1 + c \Rightarrow c = 4$

$\Rightarrow y = (x + 1)\log|x + 1| - x + 3$

70. (d) : $\lim\limits_{x\to 0}\left(\dfrac{1}{x} - \dfrac{2}{e^{2x}-1}\right) = \lim\limits_{x\to 0}\dfrac{e^{2x}-1-2x}{x(e^{2x}-1)}$

$= \lim\limits_{x\to 0}\dfrac{\left\{\left(1+2x+\dfrac{4x^2}{2!}+\dfrac{8x^3}{3!}+\ldots\right)-1-2x\right\}}{x\left\{\left(1+2x+\dfrac{4x^2}{2!}+\dfrac{8x^3}{3!}+\ldots\right)-1\right\}} = 1 = f(0)$

71. (1.414) : $(\sqrt{5} + \sqrt{3}i)^{33} = 2^{49}z$

Taking modulus on both sides

$2^{49}|z| = \left|\sqrt{5} + \sqrt{3}i\right|^{33}$

$\Rightarrow 2^{49}|z| = (\sqrt{5+3})^{33} = (\sqrt{8})^{33} = 2^{33}\,2^{33/2}$

or $|z| = \dfrac{2^{33}\,2^{33/2}}{2^{49}} = \dfrac{2^{33}\,2^{33/2}}{2^{33}\cdot 2^{16}} = 2^{\frac{33}{2}-16}$

$= 2^{1/2} = \sqrt{2} = 1.414$

72. (0) : $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-b & a(b-c) \end{vmatrix}$

[Applying $R_3 \to R_3 - R_2$ and $R_2 \to R_2 - R_1$]

$= 1 \cdot [a(b - c)(b - a) - c(a - b)(c - b)]$

$= (b - c)(b - a)(a - c) = (a - b)(b - c)(c - a)$

Again $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

So the given difference is 0.

73. (7) : The distance of the point '$\vec{a}$' from the plane $\vec{r}\cdot\vec{n} = q$ measured in the direction of the unit vector $\hat{b}$ is $= \dfrac{q - \vec{a}\cdot\vec{n}}{\hat{b}\cdot\vec{n}}$

Here $\vec{a} = \hat{i}+2\hat{j}+3\hat{k}$, $\vec{n} = \hat{i}+\hat{j}+\hat{k}$ and $q = 5$

Also, $\hat{b} = \dfrac{2\hat{i}+3\hat{j}-6\hat{k}}{\sqrt{(2)^2+(3)^2+(-6)^2}} = \dfrac{2\hat{i}+3\hat{j}-6\hat{k}}{7}$

$\therefore$ The required distance

$= \dfrac{5-(\hat{i}+2\hat{j}+3\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})}{\frac{1}{7}(2\hat{i}+3\hat{j}-6\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})} = \dfrac{5-(1+2+3)}{\frac{1}{7}(2+3-6)} = 7$

74. (0) : $\vec{a}\,.\,[(\vec{b}+\vec{c})\times(\vec{a}+\vec{b}+\vec{c})]$

$= \vec{a}\cdot[\vec{b}\times(\vec{a}+\vec{b}+\vec{c})+\vec{c}\times(\vec{a}+\vec{b}+\vec{c})]$

$= \vec{a}\cdot[\vec{b}\times\vec{a} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a} + \vec{c}\times\vec{b}]$

$= \vec{a}\cdot[\vec{b}\times\vec{a}] + \vec{a}\cdot[(\vec{c}\times\vec{a})] = 0 + 0 = 0$

75. (91) : $n = 1111\ldots1$(91 times)

$= 1 + 10 + 10^2 + \ldots + 10^{90}$

$= \dfrac{10^{91}-1}{10-1} = \dfrac{10^{91}-1}{10^7-1}\cdot\dfrac{10^7-1}{10-1}$

$= \dfrac{(10^7)^{13}-1}{10^7-1}\cdot\dfrac{10^7-1}{10-1}$

$= (10^{84} + 10^{77} + 10^{70} + \ldots + 10^7 + 1) \times (10^6 + 10^5 + 10^4 + \ldots + 10 + 1)$

= Product of two integers

$\therefore$ n is not a prime number

$\therefore$ $m = 91$