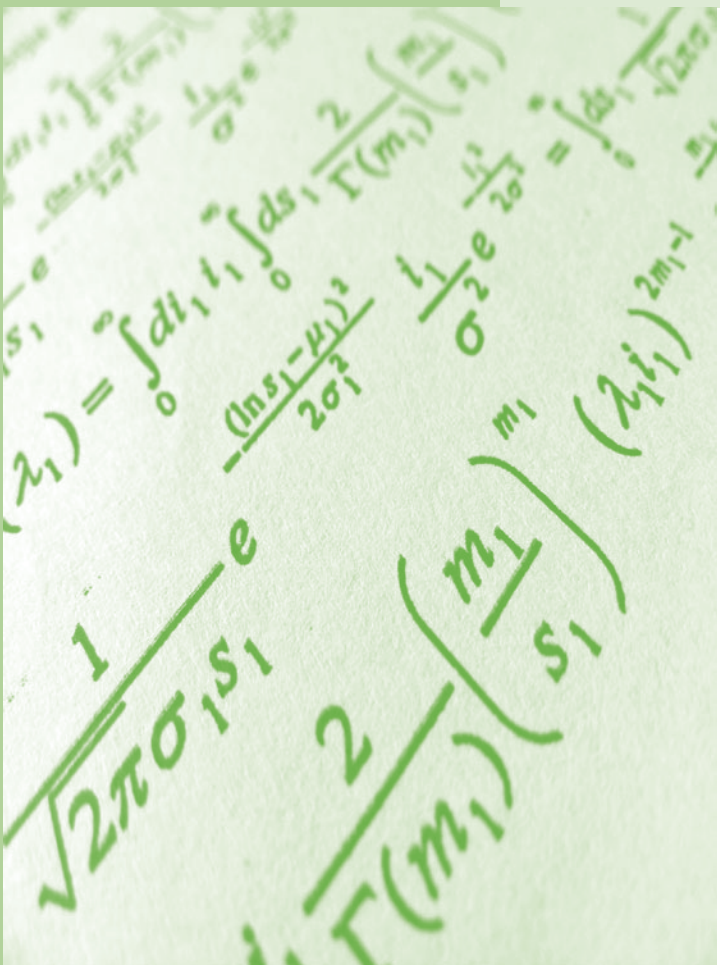
Chapter 4

Quadratic Equations and Inequalities



REMEMBER

Before beginning this chapter, you should be able to:

- Know the terms, such as quadratic expression, zero in a equation
- Understand reciprocal equation and maximum or minimum value of a quadratic equation

KEY IDEAS

After completing this chapter, you would be able to:

- Find the roots of a quadratic equation by factorization, using a formula, and graphical method
- Study nature and signs of the roots
- Construct a quadratic equation
- Solve word problems on quadratic equations

INTRODUCTION

Very often we come across many equations involving several powers of one variable. If the indices of all these powers are integers then the equation is called a polynomial equation. If the highest index of a polynomial equation in one variable is two, then it is a quadratic equation.

A quadratic equation is a second degree polynomial in x usually equated to zero. In other words, for an equation to be a quadratic, the coefficient of x^2 should not be zero and the coefficients of any higher power of x should be 0.

The general form of a quadratic equation is $ax^2 + bx + c = 0$, where $a \neq 0$ (and a, b, c are real). The following are some examples of quadratic equations.

1. $x^2 - 5x + 6 = 0$ (1)
2. $x^2 - x - 6 = 0$ (2)
3. $2x^2 + 3x - 2 = 0$ (3)
4. $2x^2 + x - 3 = 0$ (4)

ROOTS OF THE EQUATION

Just as a first degree equation in x has one value of x satisfying the equation, a quadratic equation in x has two values of x that satisfy the equation. The values of x that satisfy the equation are called the ROOTS of the equation. These roots may be real or complex.

The roots of the four quadratic equations given above are:

For Eq. (1), $x = 2$ and $x = 3$

For Eq. (2), $x = -2$ and $x = 3$

For Eq. (3), $x = \frac{1}{2}$ and $x = -2$

For Eq. (4), $x = 1$ and $x = \frac{-3}{2}$

In general, the roots of a quadratic equation can be found in two ways:

1. By factorizing the expression on the left hand side.
2. By using the standard formula.

   All the expressions may not be easy to factorize, whereas applying the formula is simple and straightforward.

Finding the Roots by Factorization

If the quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ is written in the form, $(x - \alpha)(x - \beta) = 0$, then the roots of the equation are α and β.

To find the roots of a quadratic equation, we should first express it in the form of $(x - \alpha)(x - \beta) = 0$, i.e., the left hand side, $ax^2 + bx + c$ of the quadratic equation $ax^2 + bx + c = 0$ should be factorized.

For this purpose, we should go through the following steps with the help of above equations.

Consider Equation (1)

Step 1: The equation is, $x^2 - 5x + 6 = 0$. Here $a = 1$, $b = -5$ and $c = 6$.

First write down b (the coefficient of x) as the sum of two quantities whose product is equal to ac. In this case, -5 has to be written as the sum of two quantities whose product is 6. We write -5 as $(-3) + (-2)$, because the product of (-3) and (-2) is equal to 6.

Step 2: Now, rewrite the equation. In this case, the given equation can be written as $x^2 - 3x - 2x + 6 = 0$.

Step 3: Consider the first two terms and rewrite them together after taking out their common factor. Similarly, the third and the fourth terms should be rewritten after taking out their common factor. In this process, we should ensure that what is left from the first and the second terms (after removing the common factor) is the same as that left from the third and fourth terms (after removing their common factor).

In this case, the equation can be rewritten as $x(x - 3) -2(x - 3) = 0$; now $(x - 3)$ is a common factor.

Step 4: If we take out $(x - 3)$ as the common factor, we can rewrite the given equation as $(x - 3)(x - 2) = 0$.

We know that if α and β are the roots of the given quadratic equation $(x - \alpha)(x - \beta) = 0$.

Hence, the roots of the given equation are 3 and 2.

Consider Equation (2)

The equation is, $x^2 - x - 6 = 0$. Here, the coefficient of x is -1 which can be rewritten as $(-3) + (+2)$, because the product of (-3) and 2 is -6, which is equal to 'ac' (1 multiplied by -6). Then, we can rewrite the equation as $(x - 3)(x + 2) = 0$ to get the roots as 3 and -2.

Consider Equation (3)

The equation is, $2x^2 + 3x - 2 = 0$. Here, the co-efficient of x is 3, which can be rewritten as $(+4) + (-1)$ so that their product is -4, which is the value of 'ac' (-2 multiplied by 2). Then, we can rewrite the equation as $(2x - 1)(x + 2) = 0$, obtaining the roots as $\frac{1}{2}$ and -2.

Consider Equation (4)

The equation is, $2x^2 + x - 3 = 0$. Here, the coefficient of x is 1, which can be rewritten as $(+3) + (-2)$ because their product is -6, which is equal to 'ac' (2 multiplied by -3). Then we can rewrite the given equation as $(x - 1)(2x + 3) = 0$ to get the roots as 1 and $\frac{-3}{2}$.

Finding the Roots by Using the Formula

For the quadratic equation $ax^2 + bx + c = 0$, we can use the standard formula given below to find out the roots of the equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Sum and Product of Roots of a Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$, let α and β be the roots, then

the sum of the roots, $(\alpha + \beta) = \frac{-b}{a}$

the product of the roots, $(\alpha\beta) = \frac{c}{a}$.

These two rules will be very helpful in solving problems on quadratic equation.

Nature of the Roots

We have already learnt that the roots of a quadratic equation with real coefficients can be real or complex. When the roots are real, they can be rational or irrational and, also, they can be equal or unequal.

Consider the expression $b^2 - 4ac$. Since $b^2 - 4ac$ determines the nature of the roots of the quadratic equation, it is called the DISCRIMINANT of the quadratic equation.

A quadratic equation has real roots only if $b^2 - 4ac \geq 0$.

If $b^2 - 4ac < 0$, then the roots of the quadratic equation are complex conjugates.

The following table gives us a clear idea about the nature of the roots of a quadratic equation, when a, b and c are all rational.

Condition	Nature of Roots
when $b^2 - 4ac < 0$	the roots are complex conjugates
when $b^2 - 4ac = 0$	the roots are rational and equal.
when $b^2 - 4ac > 0$ and a perfect square	the roots are rational and unequal.
when $b^2 - 4ac > 0$ and not a perfect square	the roots are irrational and unequal.

Notes

1. Whenever the roots of the quadratic equation are irrational, (a, b, c being rational), are of the form $a + \sqrt{b}$ and $a - \sqrt{b}$, i.e., whenever $a + \sqrt{b}$ is one root of a quadratic equation, $a - \sqrt{b}$ is the other root of the quadratic equation and vice-versa. In other words, if the roots of a quadratic equation are irrational, then they are conjugate to each other.
2. If the sum of the coefficients of a quadratic equation, say $ax^2 + bx + c = 0$, is zero, then its roots are 1 and $\frac{c}{a}$.

That is, if $a + b + c = 0$, then the roots of $ax^2 + bx + c = 0$ are 1 and $\frac{c}{a}$.

Signs of the Roots

We can comment on the signs of the roots, i.e., whether the roots are positive or negative, based on the sign of the sum of the roots and the product of the roots of the quadratic equation. The following table indicates the signs of the roots when the signs of the sum and the product of the roots are given.

Sign of Product of the Roots	Sign of Sum of the Roots	Sign of the Roots
+ve	+ve	Both the roots are positive.
+ve	−ve	Both the roots are negative.
−ve	+ve	One root is positive and the other negative. The numerically greater root is positive.
−ve	−ve	One root is positive and the other negative. The numerically greater root is negative.

CONSTRUCTING A QUADRATIC EQUATION

We can build a quadratic equation in the following cases:

1. When the roots of the quadratic equation are given.
2. When the sum of the roots and the product of the roots of the quadratic equation are given.

Case 1: If the roots of the quadratic equation are α and β, then its equation can be written as $(x - \alpha)(x - \beta) = 0$, i.e., $x^2 - x(\alpha + \beta) + \alpha\beta = 0$.

Case 2: If p is the sum of the roots of the quadratic equation and q is their product, then the equation can be written as $x^2 - px + q = 0$.

Constructing a New Quadratic Equation by Changing the Roots of a Given Quadratic Equation

If we are given a quadratic equation, we can build a new quadratic equation by changing the roots of this equation in the manner specified to us.

For example, consider the quadratic equation $ax^2 + bx + c = 0$ and let its roots be α and β respectively. Then, we can build new quadratic equations as per the following points:

1. A quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$, i.e., the roots are reciprocal to the roots of the given quadratic equation can be obtained by substituting $\left(\frac{1}{x}\right)$ for x in the given equation, which gives us $cx^2 + bx + a = 0$, i.e., we get the equation required by interchanging the coefficient of x^2 and the constant term.
2. A quadratic equation whose roots are $(\alpha + k)$ and $(\beta + k)$ can be obtained by substituting $(x - k)$ for x in the given equation.
3. A quadratic equation whose roots are $(\alpha - k)$ and $(\beta - k)$ can be obtained by substituting $(x + k)$ for x in the given equation.
4. A quadratic equation whose roots are $(k\alpha)$ and $(k\beta)$ can be obtained by substituting $\left(\frac{x}{k}\right)$ for x in the given equation.
5. A quadratic equation whose roots are $\left(\frac{\alpha}{k}\right)$ and $\left(\frac{\beta}{k}\right)$ can be obtained by substituting (kx) for x in the given equation.
6. A quadratic equation whose roots are $(-\alpha)$ and $(-\beta)$ can be obtained by replacing x by $(-x)$ in the given equation.

Finding the Roots of a Quadratic Equation by Graphical Method

First Method

First, let us learn how to draw the graph of $y = x^2$.

We assume certain real values for x, i.e., we substitute some values for x in $y = x^2$. We can find the corresponding values of y. We tabulate the values, as shown below.

x	5	4	3	2	1	0	−1	−2	−3	−4	−5
$y = x^2$	25	16	9	4	1	0	1	4	9	16	25

Plotting the points corresponding to the ordered pairs (5, 25), (4, 16), (3, 9), (2, 4), (1, 1), (0, 0) (−1, 1), (−2, 4), (−3, 9), (−4, 16) and (−5, 25) on the graph paper and joining them with a smooth curve we obtain the graph of $y = x^2$, as shown in the Fig. 4.1.

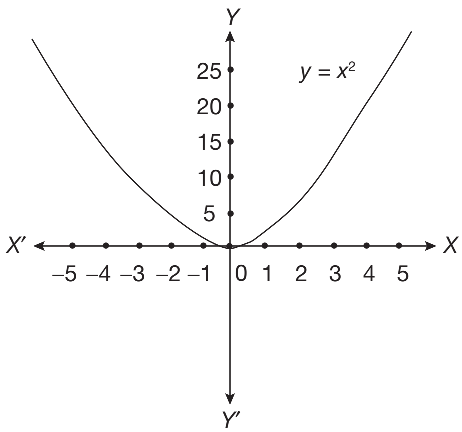
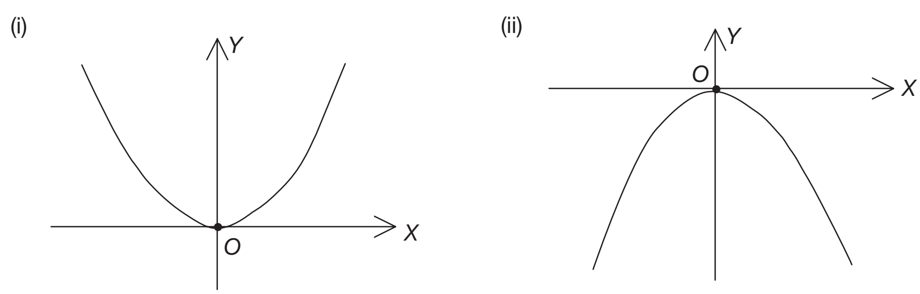


Figure 4.1

We observe the following about the graph of $y = x^2$.

1. It is a U shaped graph and it is called a parabola. The arms of the 'U' spread outwards.
2. For every value of x ($\neq 0$) we notice that y is always positive. Hence, the graph lies entirely in the first and second quadrants.
3. When $x = 0$, $y = 0 \quad \Rightarrow \quad y = x^2$ passes through origin.
4. The graph is symmetric about the Y-axis.
5. Using the graph of $y = x^2$, we can find the square of any real number as well as the square root of any non-negative real number.
   (i) for any given value of x, the corresponding value of y on the graph is its square and
   (ii) for any given value of y (≥ 0), the corresponding value of x on the graph is its square root.
6. The graph of $y = kx^2$, when $k > 0$ lies entirely in Q_1 and Q_2 and when $k < 0$ the graph lies entirely in Q_3 and Q_4 (see Fig. 4.2).



$y = kx^2$, $k > 0$ $\qquad$ $y = kx^2$, $k < 0$

Figure 4.2

The method of solving the quadratic equation of the form $px^2 + qx + r = 0$, whose roots are real is shown in the following example.

EXAMPLE 4.1

Solve $x^2 - 5x + 6 = 0$ using the graphical method.

SOLUTION

Let $y = x^2 - 5x + 6$

Prepare the following table by assuming different values for x.

x	0	1	2	3	−1	− 2	4	5
x^2	0	1	4	9	1	4	16	25
$5x$	0	5	10	15	−5	−10	20	25
$y = x^2 - 5x + 6$	6	2	0	0	12	20	2	6

Plot the points (0, 6), (1, 2), (2, 0), (3, 0), (−1, 12), (−2, 20), (4, 2) and (5, 6) on the graph and join the points with a smooth curve, as shown in the Fig. 4.3.

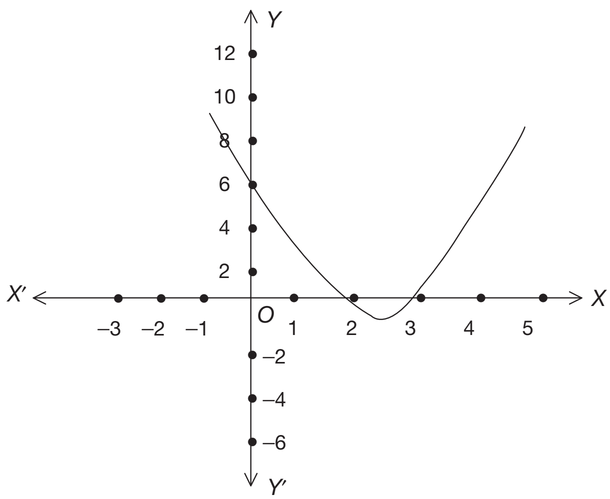


Figure 4.3

Here we notice that the given graph (parabola) intersects the X-axis at (2, 0) and (3, 0).

The roots of the given quadratic equation $x^2 - 5x + 6 = 0$ are $x = 2$ and $x = 3$.

∴ The roots of the given equation are the x-coordinates of the points of intersection of the curve with X-axis.

Notes

1. If the graph meets the X-axis at two distinct points, then the roots of the given equation are real and distinct (see Fig. 4.4).

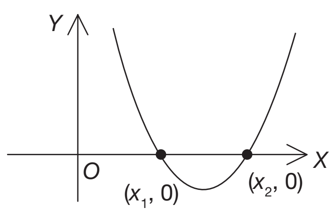


Figure 4.4

2. If the graph touches the X-axis at only one point, then the roots of the quadratic equation are real and equal (see Fig. 4.5).

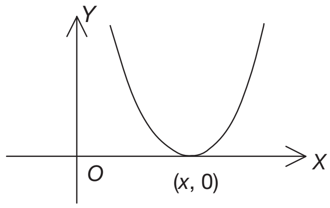


Figure 4.5

3. If the graph does not meet the X-axis, then the roots of the quadratic equation are not real, i.e., they are complex (see Fig. 4.6).

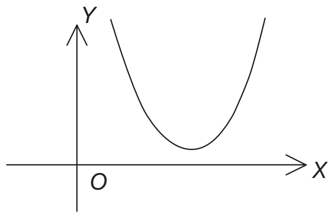


Figure 4.6

Second Method

We can also solve the quadratic equation $px^2 + qx + r = 0$ by considering the following equations:

$$y = px^2 \tag{1}$$

and
$$y = -qx - r \tag{2}$$

Clearly, $y = px^2$ is a parabola and $y = -qx - r$ is a straight line.

Step 1: Draw the graph of $y = px^2$ and $y = -qx - r$ on the same graph paper.

Step 2: Draw perpendiculars from the points of intersection of parabola and the straight line onto the X-axis. Let the points of intersection on the X-axis be $(x_1, 0)$ and $(x_2, 0)$.

Step 3: The x-coordinates of the points in Step (2), i.e., x_1 and x_2 are the two distinct roots of $px^2 + qx + r = 0$.

EXAMPLE 4.2

Solve $2x^2 - x - 3 = 0$.

SOLUTION

We know that the roots of $2x^2 - x - 3 = 0$ are the x-coordinates of the points of intersection of the parabola, $y = 2x^2$ and the straight line, $y = x + 3$.

(1) $y = 2x^2$

x	0	1	2	−1	−2
$y = 2x^2$	0	2	8	2	8

(2) $y = x + 3$

x	0	1	2	−1	−2	−3
$y = x + 3$	3	4	5	2	1	0

Draw the graph of $y = 2x^2$ and $y = x + 3$ (see Fig. 4.7).

Clearly, the perpendiculars drawn from the points of intersection of parabola and the line meet the X-axis at $\left(\frac{3}{2}, 0\right)$ and $(-1, 0)$.

$\therefore$ The roots of the given quadratic equation $2x^2 - x - 3 = 0$ are $\frac{3}{2}$ and -1.

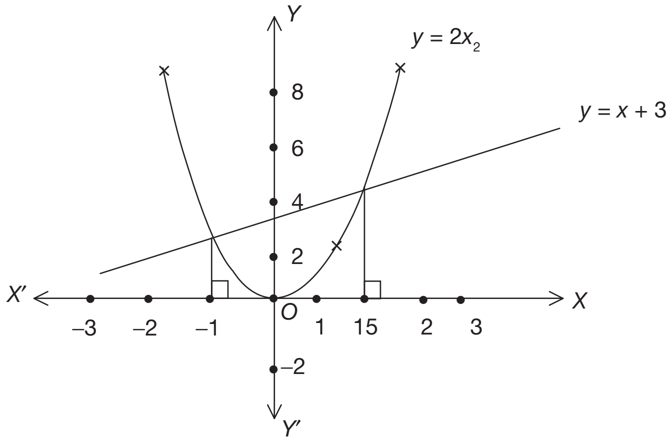


Figure 4.7

Notes

1. If the line meets the parabola at two points, then the roots of the quadratic equation are real and distinct.
2. If the line touches the parabola at only one point, then the quadratic equation has real and equal roots (see Fig. 4.8).

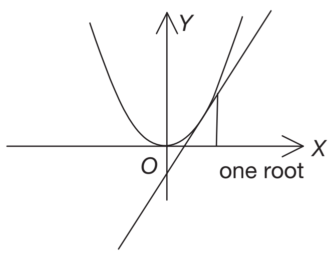


Figure 4.8

3. If the line does not meet the parabola, i.e., when the line and the parabola have no points in common, then the quadratic equation has no real roots. In this case, the roots of the quadratic equation are imaginary (see Fig. 4.9).

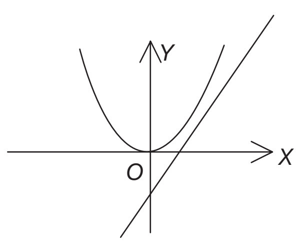


Figure 4.9

Equations of Higher Degree

The index of the highest power of x in the equation is called the degree of the equation. For example, if the highest power of x in the equation is x^3, then the degree of the equation is 3. An equation whose degree is 3 is called a cubic equation. A cubic equation will have three roots.

Note An equation whose degree is n will have n roots.

Maximum or Minimum Value of a Quadratic Expression

The quadratic expression $ax^2 + bx + c$ takes different values as x takes different values.

$$\begin{aligned} & ax^2 + bx + c \\ &= a\left(x^2 + \frac{b}{a}x\right) + c \\ &= a\left(x^2 + 2\left(\frac{b}{2a}\right)x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c \\ &= a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} \\ &= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a} \end{aligned}$$

For all the values of x, as x varies from $-\infty$ to $+\infty$, (i.e., when x is real), the quadratic expression $ax^2 + bx + c$

1. has a minimum value if $a > 0$ (i.e., a is positive). The minimum value of the quadratic expression is $\dfrac{(4ac - b^2)}{4a}$ and it occurs at $x = \dfrac{-b}{2a}$.
2. has a maximum value if $a < 0$ (i.e., a is negative). The maximum value of the quadratic expression is $\dfrac{(4ac - b^2)}{4a}$ and it occurs at $x = \dfrac{-b}{2a}$.

EXAMPLE 4.3

Find the roots of the equation $x^2 + 3x - 4 = 0$.

SOLUTION

$$\begin{aligned} & x^2 + 3x - 4 = 0 \\ \Rightarrow \quad & x^2 - x + 4x - 4 = 0 \\ \Rightarrow \quad & x(x - 1) + 4(x - 1) = 0 \\ \Rightarrow \quad & (x + 4)\,(x - 1) = 0. \end{aligned}$$

$\therefore\ x = -4$ or $x = 1$.

EXAMPLE 4.4

Find the roots of the equation $4x^2 - 13x + 10 = 0$.

SOLUTION

$$4x^2 - 13x + 10 = 0$$
$$\Rightarrow \quad 4x^2 - 8x - 5x + 10 = 0$$
$$\Rightarrow \quad 4x(x - 2) - 5(x - 2) = 0$$
$$\Rightarrow \quad (4x - 5)(x - 2) = 0.$$

$\therefore \; x = \dfrac{5}{4}$ or $x = 2$.

EXAMPLE 4.5

Find the roots of the equation $26x^2 - 43x + 15 = 0$.

SOLUTION

We have to write 43 as the sum of two parts whose product should be equal to $(26) \times (15)$.

$$26 \times 15 = 13 \times 30 \text{ and } 13 + 30 = 43.$$
$$\therefore \; 26x^2 - 43x + 15 = 0$$
$$\Rightarrow \quad 26x^2 - 13x - 30x + 15 = 0$$
$$\Rightarrow \quad (13x - 15)(2x - 1) = 0$$
$$\Rightarrow \quad x = \frac{15}{13} \text{ or } x = \frac{1}{2}.$$

We can also find the roots of the equation by using the formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{43 \pm \sqrt{(43)^2 - (1560)}}{52}$$
$$= \frac{43 \pm \sqrt{(1849) - (1560)}}{52}$$
$$= \frac{43 \pm \sqrt{289}}{52} \quad \Rightarrow \quad x = \frac{43 \pm 17}{52}$$
$$\Rightarrow \quad x = \frac{43 + 17}{52} \text{ or } \frac{43 - 17}{52} = \frac{60}{52} \text{ or } \frac{26}{52}$$
$$\therefore \quad x = \frac{15}{13} \text{ or } \frac{1}{2}.$$

EXAMPLE 4.6

Discuss the nature of the roots of the equation $4x^2 - 2x + 1 = 0$.

SOLUTION

Discriminant $= (-2)^2 - 4(4)(1) = 4 - 16 = -12 < 0$.

Since the discriminant is negative, the roots are imaginary.

EXAMPLE 4.7

If the sum of the roots of the equation $kx^2 - 3x + 9 = 0$ is $\frac{3}{11}$, then find the product of the roots of that equation.

SOLUTION

Sum of roots of the equation $= \frac{3}{k} = \frac{3}{11}$ (given)

$\therefore$ $k = 11$.

In the given equation, product of the roots $= \frac{9}{k}$.

As $k = 11$, product of the roots $= \frac{9}{11}$.

EXAMPLE 4.8

Form the quadratic equation whose roots are 2 and 7.

SOLUTION

Sum of the roots $= 2 + 7 = 9$

Product of the roots $= 2 \times 7 = 14$

We know that if p is the sum of the roots and q is the product of the roots of a quadratic equation, then its equation is $x^2 - px + q = 0$.

Hence, the required equation is $x^2 - 9x + 14 = 0$.

EXAMPLE 4.9

Form a quadratic equation with rational coefficients, one of whose roots is $3 + \sqrt{5}$.

SOLUTION

If $(3 + \sqrt{5})$ is one root, then the other root is $(3 - \sqrt{5})$.

Sum of the roots $= 6$

Product of the roots $= 4$

Thus, the required equation is $x^2 - 6x + 4 = 0$.

EXAMPLE 4.10

A person can buy 15 books less for ₹900 when the price of each book goes up by ₹3. Find the original price and the number of copies he could buy at the initial price.

SOLUTION

Let the number of books bought initially for ₹900 be 'x'. The original price of the each book was $\frac{900}{x}$. Now the price of the each book is increased by ₹3.

That is, the new price of each book is ₹$\left(\frac{900}{x}\right)+3$.

And the number of books bought is reduced by 15, i.e., $(x - 15)$.

Since the total amount spent is still ₹900, the product of the price of each book and the number of books are still 900.

$$\left[\left(\frac{900}{x}\right)+3\right](x-15) = 900$$

$$\Rightarrow \quad (900 + 3x)\ (x - 15) = 900x$$

$$\Rightarrow \quad 3x^2 + 855x - 13500 = 900x$$

$$\Rightarrow \quad 3x^2 - 45x - 13500 = 0$$

$$\Rightarrow \quad x^2 - 15x - 4500 = 0$$

$$\Rightarrow \quad x^2 - 75x + 60x - 4500 = 0$$

$$\Rightarrow \quad x(x - 75) + 60(x - 75) = 0$$

$$\Rightarrow \quad (x - 75)\ (x + 60) = 0$$

$$\Rightarrow \quad x = 75 \text{ or } -60.$$

Since x cannot be negative, $x = 75$.

Thus, the original price of the book $=\frac{900}{75}=$ ₹12.

EXAMPLE 4.11

If α and β are the roots of the equation $x^2 - 6x + 8 = 0$, then find the values of

(a) $\alpha^2 + \beta^2$

(b) $\frac{1}{\alpha}+\frac{1}{\beta}$

(c) $\alpha - \beta\ (\alpha > \beta)$

SOLUTION

From the given equation, we get $\alpha + \beta = 6$ and $\alpha\beta = 8$.

(a) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (6)^2 - 2(8) = 20$

(b) $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{6}{8}=\frac{3}{4}$

(c) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$$\Rightarrow \quad (\alpha - \beta) = \pm\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$$

$$= \sqrt{6^2 - 4(8)}$$

$$\Rightarrow \quad (\alpha - \beta) = \pm 2.$$

$$\therefore \quad \alpha - \beta = 2, \quad (\because \alpha > \beta).$$

EXAMPLE 4.12

Solve for x: $3^{x+1} + 3^{2x+1} = 270$.

SOLUTION

$$
\begin{aligned}
& 3^{x+1} + 3^{2x+1} = 270 \\
\Rightarrow\ & 3 \cdot 3^x + 3^{2x} \cdot 3 = 270 \\
\Rightarrow\ & 3^x + 3^{2x} = 90.
\end{aligned}
$$

Substituting $3^x = a$, we get,

$$
\begin{aligned}
& a + a^2 = 90 \\
\Rightarrow\ & a^2 + a - 90 = 0 \\
\Rightarrow\ & a^2 + 10a - 9a - 90 = 0 \\
\Rightarrow\ & (a + 10)(a - 9) = 0 \\
\Rightarrow\ & a = 9 \text{ or } a = -10.
\end{aligned}
$$

If $3^x = 9$, then $x = 2$.

If $3^x = -10$, which is not possible.

$\therefore\ x = 2$.

EXAMPLE 4.13

Solve $|x|^2 - 7|x| + 12 = 0$.

SOLUTION

Given equation is $|x|^2 - 7|x| + 12 = 0$

$$
\begin{aligned}
\Rightarrow\ & (|x| - 3)(|x| - 4) = 0 \\
\Rightarrow\ & |x| = 3 \text{ or } |x| = 4 \\
\Rightarrow\ & x = \pm 3 \text{ or } x = \pm 4.
\end{aligned}
$$

EXAMPLE 4.14

Solve $|x|^2 + 7|x| + 10 = 0$.

SOLUTION

Given equation is $|x|^2 + 7|x| + 10 = 0$

$$
\begin{aligned}
\Rightarrow\ & (|x| + 2)(|x| + 5) = 0 \\
\Rightarrow\ & |x| = -2 \text{ or } |x| = -5.
\end{aligned}
$$

But, the absolute value of any number can never be negative.

$\therefore$ No roots are possible for the given equation.

EXAMPLE 4.15

In writing a quadratic equation of the form $x^2 + bx + c = 0$, a student writes the coefficient of x incorrectly and finds the roots as -6 and 7. Another student makes a mistake in writing the constant term and finds the roots as 4 and 11. Find the correct quadratic equation.

(a) $x^2 + 15x - 42 = 0$ **(b)** $x^2 + x + 44 = 0$ **(c)** $x^2 - 15x - 42 = 0$ **(c)** $x^2 - x + 44 = 0$

HINTS

(i) Use sum of roots $= \frac{-b}{a}$, product of the roots $= \frac{c}{a}$.

(ii) Identify that the first student got the correct product and the second student got the correct sum.

EXAMPLE 4.16

A man bought 50 dozen fruits consisting of apples and bananas. An apple is cheaper than a banana. The number of dozens of apples he bought is equal to the cost per dozen of bananas in rupees and vice versa. If he had spent a total amount of ₹1050, find the number of dozens of apples and bananas he bought respectively.

(a) 12 and 38 **(b)** 14 and 36 **(c)** 15 and 35 **(d)** 18 and 32

HINTS

Form a quadratic equation in terms of apples or bananas.

EXAMPLE 4.17

If $3 \cdot 2^{2x+1} - 5 \cdot 2^{x+2} + 16 = 0$ and x is an integer, find the value of x.

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4

HINTS

(i) Let $2^x = P$.

(ii) Frame the quadratic equation in terms of P and solve it.

EXAMPLE 4.18

If $(x + 1)(x + 3)(x + 5)(x + 7) = 5760$, find the real values of x.

(a) $5, -13$ **(b)** $-5, 13$ **(c)** $-5, -13$ **(d)** $5, 13$

HINTS

(i) (a) (b) (c) (d) $= e$ can be written as (a d) (b c) $= e$.

(ii) Write the equation in the form of a quadratic equation.

WORKOUT

For what value of x: $-3x^2 + 5x - 12$ has maximum value?

(a) $-\frac{5}{3}$ **(b)** $\frac{5}{6}$ **(c)** $-\frac{5}{6}$ **(d)** $\frac{5}{3}$

$-3x^2 + 5x - 12$

QUADRATIC INEQUATIONS

Consider the quadratic equation $ax^2 + bx + c = 0$, $(a \neq 0)$, where a, b and c are real numbers.

The quadratic inequations related to $ax^2 + bx + c = 0$ are $ax^2 + bx + c < 0$ and $ax^2 + bx + c > 0$.

Assume that $a > 0$.

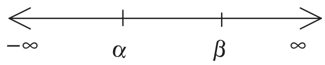
The following cases arise:

Case 1: If $b^2 - 4ac > 0$, then the equation $ax^2 + bx + c = 0$ has real and unequal roots.

Let α and $\beta(\alpha < \beta)$ be the roots.

Then,

$\therefore\ ax^2 + bx + c = a(x - \alpha)(x - \beta)$.



1. If $x < \alpha$, then $(x - \alpha) < 0$ and $(x - \beta) < 0$.

   $\therefore\ ax^2 + bx + c > 0$.
2. If $\alpha < x < \beta$, then $(x - \alpha) > 0$ and $(x - \beta) < 0$

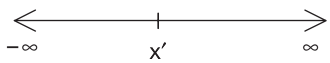
   $\therefore\ ax^2 + bx + c < 0$.
3. If $x > \beta$, then $x - \alpha > 0$ and $x - \beta > 0$.

   $\therefore\ ax^2 + bx + c > 0$.

Case 2: If $b^2 - 4ac = 0$, then $ax^2 + bx + c = 0$ has real and equal roots.

Let x' be the equal root.

$\Rightarrow\quad ax^2 + bx + c = a(x - x')(x - x')$



1. If $x < x'$. Then $x - x' < 0$.

   $\therefore\ ax^2 + bx + c > 0$.
2. If $x > x'$, then $x - x' > 0$.

   $\therefore\ ax^2 + bx + c > 0$.

Case 3: If $b^2 - 4ac < 0$, then $ax^2 + bx + c = 0$ has imaginary roots.

In this case, $ax^2 + bx + c > 0$, $\forall\ x \in R$.

The above concept can be summarised as:

1. If $\alpha < x < \beta$, then $(x - \alpha)(x - \beta) < 0$ and vice-versa.
2. If $x < \alpha$ and $x > \beta$ $(\alpha < \beta)$, then $(x - \alpha)(x - \beta) > 0$ and vice-versa.

Note If $a < 0$ and $b^2 - 4ac < 0$, then the solution for $ax^2 + bx + c > 0$ does not exist.

EXAMPLE 4.19

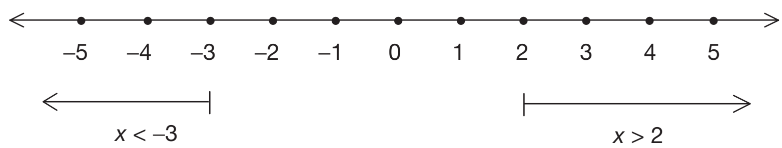
Solve the inequation $x^2 + x - 6 < 0$.

SOLUTION

Given inequation is $x^2 + x - 6 < 0$.

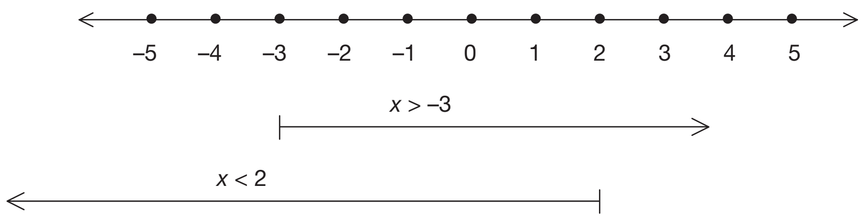
$$\Rightarrow \quad (x + 3)(x - 2) < 0$$
$$\Rightarrow \quad (x + 3) < 0,\ (x - 2) > 0 \text{ or } (x + 3) > 0,\ (x - 2) < 0$$
$$\Rightarrow \quad x < -3,\ x > 2 \text{ (Case 1) (or) } x > -3,\ x < 2 \text{ (Case 2)}$$

Case 1: $x < -3$ and $x > 2$



There exists no value of x so that $x < -3$ and $x > 2$ (as there is no overlap of the regions). Hence, in this case no value of x satisfies the given inequation.

Case 2: $x > -3$ and $x < 2$



All the points in the overlapping region, i.e., $-3 < x < 2$, satisfy the inequation. Hence, the solution set of the inequation. $x^2 + x - 6 < 0$ is $\{x/-3 < x < 2\}$ or $(-3, 2)$.

EXAMPLE 4.20

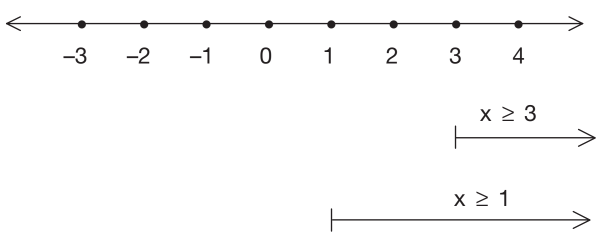
Solve for x: $x^2 - 4x + 3 \geq 0$.

SOLUTION

Given inequation is $x^2 - 4x + 3 \geq 0$.

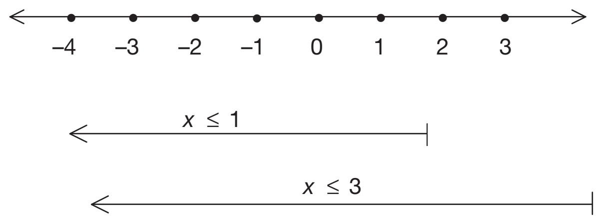
$$\Rightarrow \quad (x - 1)(x - 3) \geq 0$$
$$\Rightarrow \quad x - 1 \geq 0;\ x - 3 \geq 0 \text{ or } x - 1 \leq 0;\ x - 3 \leq 0$$
$$\Rightarrow \quad x \geq 1;\ x \geq 3 \text{ (Case 1) (or)}$$
$$x \leq 1;\ x \leq 3 \text{ (Case 2)}$$

Case 1: $x \geq 1$ and $x \geq 3$.



All the points in the overlapping region, i.e., $x \geq 3$, satisfy the given inequation.

Case 2: $x \leq 1$ and $x \leq 3$.



All the points in the overlapping region, i.e., $x \leq 1$, satisfy the given inequation.

Hence, the solution for the given inequation is $x \in (-\infty, 1] \cup [3, \infty)$.

EXAMPLE 4.21

Solve $x^2 + 6x + 13 > 0$.

SOLUTION

Given inequation is $x^2 + 6x + 13 > 0$.

Here, factorization is not possible.

Rewriting the given inequation we get,

$$(x^2 + 6x + 9) + 4 > 0$$

$$\Rightarrow \quad (x + 3)^2 + 4 > 0.$$

We know that $(x + 3)^2 \geq 0 \ \forall \ x \in R$,

$$(x + 3)^2 + 4 \geq 4 > 0 \ \forall \ x \in R.$$

$\therefore$ The required solution is the set of all real numbers, i.e., $(-\infty, \infty)$.

EXAMPLE 4.22

Solve $\dfrac{x^2 + 5x + 3}{x + 2} < x$.

SOLUTION

$$\frac{x^2 + 5x + 3}{x + 2} < x$$

$$\Rightarrow \quad \frac{x^2 + 5x + 3}{x + 2} - x < 0$$

$$\Rightarrow \quad \frac{x^2 + 5x + 3 - x^2 - 2x}{x + 2} < 0$$

$$\Rightarrow \quad \frac{3x + 3}{x + 2} < 0$$

$$\Rightarrow \quad \frac{x + 1}{x + 2} < 0. \qquad (1)$$

The solution of inequation (1) is same as $(x + 1)(x + 2) < 0$.

We know that $(x-\alpha)(x-\beta) < 0$

$$\Rightarrow \quad \alpha < x < \beta \qquad \text{where } (\alpha < \beta)$$

$\therefore$ $-2 < x < -1$.

Thus, the required solution is $-2 < x < -1$.

EXAMPLE 4.23

Solve $\dfrac{1}{x-1} < \dfrac{-2}{1-2x}$.

SOLUTION

$$\frac{1}{x-1} < \frac{-2}{1-2x}$$

$$\Rightarrow \quad \frac{1}{x-1} + \frac{2}{1-2x} < 0$$

$$\Rightarrow \quad \frac{1-2x+2x-2}{(x-1)(1-2x)} < 0$$

$$\Rightarrow \quad \frac{-1}{(x-1)(x-2x)} < 0$$

$$\Rightarrow \quad \frac{1}{(x-1)(1-2x)} > 0. \qquad (1)$$

Inequation (1) holds good if, $(x-1)(1-2x) > 0 \quad \Rightarrow \quad (x-1)(2x-1) < 0$.

We know that, $(x-\alpha)(x-\beta) < 0 \quad \Rightarrow \quad \alpha < x < \beta$, where $(\alpha < \beta)$.

$\therefore$ The solution of the given inequation is $\dfrac{1}{2} < x < 1$, i.e., $x \in \left(\dfrac{1}{2}, 1\right)$.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. For a quadratic equation $ax^2 + bx + c = 0$, if $a + b + c = 0$, then $x = 1$. (True/False)
2. If $a = c$, then the roots of $ax^2 + bx + c = 0$ are ______ to each other.
3. If $a^2 - b^2 > 0$, then $a > b$ or $a < -b$. (True/False)
4. If $b = 0$, then $ax^2 + bx + c = 0$ is called ______ quadratic equation.
5. If the sum of the roots of a quadratic equation is positive and the product of the roots is negative, then the numerically greater root is ______.
6. If we divide 8 into two parts such that their product is 15, then the parts are ______ and ______.
7. One side of a rectangle exceeds its other side by 2 cm. If its area is 24 cm^2, then the length and breadth are ______ and ______ respectively.
8. If the sum of a number and its reciprocal is 2, then the number is ______.
9. If we divide 4 into two parts such that the sum of their squares is 8, then the parts are ______ and ______.
10. If the product of two consecutive positive numbers is 20, then the numbers are ______ and ______.
11. The area of a right triangle is 32 sq. units. If its base is 4 times the altitude, then the altitude is ______.
12. The diagonal of a rectangle is 5 units. If one side is twice the other side, then the length is ______.
13. If α and β are the roots of the quadratic equation, such that $\alpha > \beta$, then for $(x - \alpha)(x - \beta) > 0$, the solution set of x is ______.
14. The positive number which is less than its square by 12 is ______.
15. If the product of two successive even numbers is 48, then the numbers are ______ and ______.
16. The point $(1, -6)$ satisfies the equation $y = x^2 - x - 6$. Is the statement true?
17. The line $y = 5x - 6$ meets the parabola $y = x^2$ in two points, where x-coordinates are ______ and ______.
18. The solution set of $x^2 - 4x + 4 < 0$ does not exist. (True/False)
19. If the product of Shiva's age 3 years ago and his age three years later is 16, then Shiva's present age is ______.
20. The solution set of $x^2 - 10x + 25 \geq 0$ exists. (True/False)
21. The sides of two squares are x cm and $(x + 2)$ cm. If the sum of their areas is 20 cm^2, then the sides of the squares are ______ and ______.
22. The solution set of $x^2 - 3x - 4 \leq 0$ is ______.
23. The solution set of $x^2 - 8x + 15 \geq 0$ is ______.
24. The solution set of $x^2 - 5x + 6 > 0$ is ______.
25. Find the positive number which is less than its square by 42.
26. If $mx^2 < nx$, where m and n are positive, then find the range of x.
27. The solution set of $x^2 - x - 6 < 0$ is ______.
28. Find the dimensions of a rectangular hall, if its length exceeds its breadth by 7 m and the area of the hall being 228 m^2.
29. If $(x + 1)^2 > mx$, and $x \in Z^+$, then the possible values of m are ________.
30. If $y = \sqrt{k\sqrt{k\sqrt{k\sqrt{k\ldots\infty}}}}$, where $k > 0$; find the value of y.

Short Answer Type Questions

31. The equation $(m + 1)x^2 + 2mx + 5x + m + 3 = 0$ has equal roots. Find the value of m.
32. The denominator of a fraction exceeds the numerator by 3. The sum of the fraction and its multiplicative inverse is $2\frac{9}{28}$. Find the fraction.
33. Find the value of q^2 in terms of p and r, so that the roots of the equation $px^2 + qx + r = 0$ are in the ratio of 3 : 4.
34. A number consists of two digits whose product is 18. If 27 is added to the number, the number

formed will have the digits in reverse order, when compared to the original number. Find the number.

35. Find the minimum value of $x^2 + 12x$.

36. If the sides of a right triangle are x, $3x + 3$, and $3x + 4$, then find the value of x.

37. Find the roots of the equation $\frac{1}{x} - \frac{1}{x-a} = \frac{1}{b} - \frac{1}{b-a}$, where $a \neq 0$.

38. Find the values of x which satisfy the inequation, $x - 5 < x^2 - 3x - 50$.

39. In a boys' hostel, there are as many boys in each room as the number of rooms. If the number of rooms is doubled and the number of boys in each room is reduced by 10, then the number of boys in the hostel becomes 1200. Find the number of rooms in the hostel.

40. If the equations $x^2 + 3x + 2 = 0$ and $x^2 + kx + 6 = 0$ have a common root, then find the values of k.

41. If $x^4 - 17x^2 + 16 = 0$, then find the sum of the squares of the roots.

42. If the quadratic equation $x^2 - mx - 4x + 1 = 0$ has real and distinct roots, then find the values of m.

43. $\sqrt{y+1} - \sqrt{y-1} = \sqrt{4y-1}$. Find the value of y.

44. Umesh and Varun are solving an equation of form $x^2 + bx + c = 0$. In doing so, Umesh commits a mistake in noting down the constant term and finds the roots as -3 and -12. And Varun commits a mistake in noting down the coefficient of x and finds the roots as -27 and -2. If so, find the roots of original equation.

45. Solve: $\frac{1}{4x+4} > \frac{2}{4x-2}$.

Essay Type Questions

46. If $-(4x + 27) < (x + 6)^2 < -4(6 + x)$, then find all the integral values of x.

47. Determine the values of x which satisfy the simultaneous inequations, $x^2 + 5x + 4 > 0$ and $-x^2 - x + 42 > 0$.

48. Solve $x^2 - 2x + 1 = 0$ graphically.

49. Draw the graph of $y = x^2 - x - 12$.

50. Solve $x^2 - x - 42 = 0$ graphically.

CONCEPT APPLICATION

Level 1

1. The roots of the equation $3x^2 - 2x + 3 = 0$ are
   (a) real and distinct.
   (b) real and equal.
   (c) imaginary.
   (d) irrational and distinct.

2. Find the sum and the product of the roots of the equation $\sqrt{3}x^2 + 27x + 5\sqrt{3} = 0$.
   (a) $-9\sqrt{3}, 5$ (b) $9\sqrt{3}, 5$
   (c) $6\sqrt{3}, -5$ (d) $6\sqrt{3}, 5$

3. If α and β are the roots of the equation $x^2 - 12x + 32 = 0$, then find the value of $\frac{\alpha^2 + \beta^2}{\alpha + \beta}$.
   (a) $\frac{-8}{3}$ (b) $\frac{8}{3}$
   (c) $\frac{-20}{3}$ (d) $\frac{20}{3}$

4. Find the maximum or minimum value of the quadratic expression, $x^2 - 3x + 5$ whichever exists.
   (a) The minimum value is $\frac{9}{10}$.
   (b) The minimum value is $\frac{11}{4}$.
   (c) The maximum value is $\frac{9}{10}$.
   (d) The maximum value is $\frac{11}{4}$.

5. Find the values of x which satisfy the equation $\sqrt{3x+7}-\sqrt{2x+3}=1$.

(a) 2, −2 (b) 4, 3

(c) 5, −1 (d) 3, −1

6. If one of the roots of a quadratic equation having rational coefficients is $\sqrt{7}-4$, then the quadratic equation is

(a) $x^2-2\sqrt{7}x-9=0$.

(b) $x^2-8x+9=0$.

(c) $x^2+8x+9=0$.

(d) $x^2-2\sqrt{7}x+9=0$.

7. If the quadratic equation $px^2+qx-r=0$ $(p\neq 0)$ is to be solved by the graphical method, then which of the following graphs have to be drawn?

(a) $y=x^2$, $y=r-qx$

(b) $y=px^2$, $y=qx-r$

(c) $y=x^2$, $qx+py-r=0$

(d) $y=x^2$, $qx-py=r$

8. If the roots of the equation $ax^2+bx+c=0$ are α and β, then the quadratic equation whose roots are $-\alpha$ and $-\beta$ is

(a) $ax^2-bx-c=0$

(b) $ax^2-bx+c=0$

(c) $ax^2+bx-c=0$

(d) $ax^2-bx+2c=0$

9. Find the sum and the product of the roots of the quadratic equation $-x^2-\frac{25}{3}x+25=0$.

(a) $\frac{25}{3}$, 25 (b) $\frac{-25}{3}$, 25

(c) $\frac{25}{3}$, −25 (d) $\frac{-25}{3}$, −25

10. For what value of k, if one root of the quadratic equation $9x^2-18x+k=0$ is double of the other?

(a) 36 (b) 9

(c) 12 (d) 8

11. The sum of a number and its square is greater than 6, then the number belongs to _______.

(a) $(-\infty, 2)\cup(3,\infty)$

(b) $(-\infty, -3)\cup(2,\infty)$

(c) $(2, 3)$

(d) $[2, 3]$

12. For which of the following intervals of x is $x^2>\frac{1}{x^2}$?

(a) $(-\infty, -1)\cup(1,\infty)$

(b) $(-\infty, -1)\cup(1,\infty)$

(c) $(-1, 1)$

(d) $[-1, 1]$

13. If x and y are two successive multiples of 2 and their product is less than 35, then find the range of x.

(a) $\{2, 4, 0\}$

(b) $\{-6, -4, -2, 2, 4, 6\}$

(c) $\{-6, -4, -2, 0, 2, 4\}$

(d) $\{-6, -4, -2, 0, 2, 4, 6\}$

14. If $x^2<n$, and $n\in(-\infty, 0)$, then x

(a) is any real number.

(b) is only positive number.

(c) has no value.

(d) is any negative number.

15. If α and β are the zeroes of the quadratic polynomial ax^2+bx+c such that x does not lie between α and β, then

(a) $a>0$ and $ax^2+bx+c<0$.

(b) $ax^2+bx+c<0$ and $a<0$.

(c) $a>0$ and $ax^2+bx+c>0$.

(d) Both (b) and (c)

16. The condition for the sum and the product of the roots of the quadratic equation $ax^2-bx+c=0$ to be equal, is

(a) $b+c=0$ (b) $b-c=0$

(c) $a+c=0$ (d) $a+b+c=0$

17. The quadratic equation having rational coefficients and one of the roots as $4+\sqrt{15}$, is

PRACTICE QUESTIONS

(a) $x^2 - 8x + 1 = 0$

(b) $x^2 + x - 8 = 0$

(c) $x^2 - x + 8 = 0$

(d) $x^2 + 8x + 8 = 0$

18. If α and β are the zeros of the quadratic polynomial $ax^2 + bx + c$ and x lies between α and β, then which of the following is true?

(a) If $a < 0$ then $ax^2 + bx + c > 0$.

(b) If $a > 0$ then $ax^2 + bx + c < 0$.

(c) If $a > 0$ then $ax^2 + bx + c > 0$.

(d) Both (a) and (b)

19. Find the nature of the roots of the equation $4x^2 - 2x - 1 = 0$.

(a) Real and equal

(b) Rational and unequal

(c) Irrational and unequal

(d) Imaginary

20. The solution of the inequation, $15x^2 - 31x + 14 < 0$ is given by

(a) $x \in \left(\frac{7}{5}, \infty\right)$ (b) $\frac{2}{3} < x < \frac{7}{5}$

(c) $x \in \left(\frac{7}{5}, \infty\right)$ (d) $x \in R$

21. If $mx^2 < nx$ such that m and n have opposite signs, then which of the following can be true?

(a) $x \in \left(\frac{n}{m}, \infty\right)$ (b) $x \in \left(-\infty, \frac{n}{m}\right)$

(c) $x \in \left(\frac{n}{m}, 0\right)$ (d) None of these

22. Find the range of values of x which satisfy the inequation, $(x + 1)^2 + (x - 1)^2 < 6$.

(a) $(-\sqrt{2}, \sqrt{2})$

(b) $(-1, 1)$

(c) $(-\infty, -2) \cup (2, \infty)$

(d) $(-\infty, -1) \cup (1, \infty)$

23. If the sum of the squares of three consecutive odd natural numbers is 155, then their product will be equal to

(a) 99 (b) 105

(c) 693 (d) 315

24. If $x^2 > 0$, then find the range of the values that x can take.

(a) $x = 0$ (b) $x \in R$

(c) $x \in (0, \infty)$ (d) $x \in R - \{0\}$

25. Find the range of the values of x which satisfy the inequation, $x^2 - 7x + 3 < 2x + 25$.

(a) $(-2, 11)$

(b) $(2, 11)$

(c) $(-\infty, -1) \cup (2, 11)$

(d) $(-8, -2) \cup [11, \infty)$

26. If A and B are the roots of the quadratic equation $x^2 - 12x + 27 = 0$, then $A^3 + B^3$ is ________.

(a) 27 (b) 729

(c) 756 (d) 64

27. By drawing which of the following graphs can the quadratic equation $4x^2 + 6x - 5 = 0$ be solved by graphical method?

(a) $y = x^2$, $3x - 2y - 5 = 0$

(b) $y = 4x^2$, $6x - 2y - 5 = 0$

(c) $y = x^2$, $6x - y - 5 = 0$

(d) $y = 2x^2$, $6x + 2y - 5 = 0$

28. If the quadratic equation $(a^2 - b^2)x^2 + (b^2 - c^2)x + (c^2 - a^2) = 0$ has equal roots, then which of the following is true?

(a) $b^2 + c^2 = a^2$ (b) $b^2 + c^2 = 2a^2$

(c) $b^2 - c^2 = 2a^2$ (d) $a^2 = b^2 + 2c^2$

29. Which of the following are the roots of the equation $|x|^2 + |x| - 6 = 0$?

(A) 2 (B) −2

(C) 3 (D) −3

(a) Both (A) and (B)

(b) Both (C) and (D)

(c) (A), (B), (C) and (D)

(d) None of these

30. What are the values of x which satisfy the equation, $\sqrt{5x-6} + \frac{1}{\sqrt{5x-6}} = \frac{10}{3}$?

(a) 3 (b) $4, \frac{11}{9}$

(c) $\frac{11}{9}$ (d) $3, \frac{11}{9}$

Level 2

31. If the roots of the quadratic equation $ax^2 + bx + c = 0$ are α and β, then the equation whose roots are α^2 and β^2 is

(a) $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$

(b) $a^2x^2 + b^2x + c^2 = 0$

(c) $a^2x^2 + (b^2 + 2ac)x + c^2 = 0$

(d) $a^2x^2 - (b^2 + 2ac)x + c^2 = 0$

32. If the roots of the equation $3ax^2 + 2bx + c = 0$ are in the ratio 2 : 3, then

(a) $8ac = 25b$

(b) $8ac = 9b^2$

(c) $8b^2 = 9ac$

(d) $8b^2 = 25ac$

33. Find the roots of the equation $l^2(m^2 - n^2)x^2 + m^2(n^2 - l^2)x + n^2(l^2 - m^2) = 0$.

(a) $1, \dfrac{n^2(l^2 - m^2)}{l^2(m^2 - n^2)}$ (b) $1, \dfrac{-m^2(l^2 - n^2)}{l^2(m^2 - n^2)}$

(c) $1, \dfrac{n^2(l^2 + m^2)}{l^2(m^2 - n^2)}$ (d) $1, \dfrac{-m^2(l^2 + n^2)}{l^2(m^2 - n^2)}$

34. Comment on the sign of the quadratic expression $x^2 - 5x + 6$ for all $x \in R$.

(a) $x^2 - 5x + 6 \geq 0$ when $2 \leq x \leq 3$ and $x^2 - 5x + 6 < 0$ when $x < 2$ or $x > 3$

(b) $x^2 - 5x + 6 \leq 0$ when $2 \leq x \leq 3$ and $x^2 - 5x + 6 > 0$ when $x < 2$ or $x > 3$

(c) $x^2 - 5x + 6 \leq 0$ when $-1 \leq x \leq 6$ and $x^2 - 5x + 6 > 0$ when $x < -1$ or $x > 6$

(d) $x^2 - 5x + 6 \geq 0$ when $-1 \leq x \leq 6$ and $x^2 - 5x + 6 < 0$ when $x < -1$ or $x > 6$

35. If $a - b$, $b - c$ are the roots of $ax^2 + bx + c = 0$, then find the value of $\dfrac{(a-b)(b-c)}{c-a}$.

(a) $\dfrac{b}{c}$ (b) $\dfrac{c}{b}$

(c) $\dfrac{ab}{c}$ (d) $\dfrac{bc}{a}$

36. The values of x for which $\dfrac{x+3}{x^2 - 3x - 54} \geq 0$ are ________.

(a) $(-6, -3) \cup (9, \infty)$

(b) $[-6, -3] \cup [9, \infty]$

(c) $(-6, -3) \cup (9, \infty)$

(d) $(-6, \infty)$

37. In a right triangle, the base is 3 units more than the height. If the area of the triangle is less than 20 sq. units, then the possible values of the base lie in the region ________.

(a) (4, 6) (b) (3, 8)

(c) (6, 8) (d) (5, 8)

38. The values of x for which $-2x - 4 \leq (x + 2)^2 \leq -2x - 1$ is satisfied are

(a) $[-5, -1]$

(b) $[-5, 0]$

(c) $[-5, -4] \cup [-2, -1]$

(d) $[-5, -4] \cup [-2, -1]$

39. If $\dfrac{x^2 + x - 12}{x^2 - 3x + 2} < 0$, then x lies in ________.

(a) $(-4, 3)$

(b) $(-4, 2)$

(c) $[-4, 1] \cup [2, 3]$

(d) $(-4, 1) \cup (2, 3)$

40. For all real values of x, $\dfrac{x^2 - \left(\dfrac{x}{2}\right) + 1}{x^2 + 1} - \dfrac{5}{4}$ is ______.

(a) equal to 1

(b) non-negative

(c) greater than $\dfrac{1}{4}$

(d) non-positive

41. If $x^2 - 4x + 3 > 0$ and $x^2 - 6x + 8 < 0$, then ______.

(a) $x > 3$ (b) $x < 4$

(c) $3 < x < 4$ (d) $1 < x < 2$

42. Find the product of the roots of $x^2 + 8x - 16 = 0$.

(a) 8 (b) −8

(c) 16 (d) −16

Level 3

43. If the roots of the equation $2x^2 + 7x + 4 = 0$ are in the ratio $p : q$, then find the value of $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}}$.

(a) $\pm\frac{7}{\sqrt{7}}$ (b) $\pm 7\sqrt{2}$

(c) $\pm\frac{7\sqrt{2}}{16}$ (d) $\pm\frac{7\sqrt{2}}{4}$

44. In a forest, a certain number of apes equal to the square of one-eighth of the total number of their group are playing and having great fun. The rest of them are twelve in number and are on an adjoining hill. The echo of their shrieks from the hills frightens them. They come and join the apes in the forest and play with enthusiasm. What is the total number of apes in the forest?

(a) 16 (b) 48

(c) 16 or 48 (d) 64

45. If the roots of the quadratic equation $x^2 - 2kx + 2k^2 - 4 = 0$ are real, then the range of the values of k is _________.

(a) $[-2, 2]$ (b) $[-\infty, -2] \cup [2, \infty]$

(c) $[0, 2]$ (d) None of these

46. Find the values of x for which the expression $x^2 - (\log_5 2 + \log_2 5)\, x + 1$ is always positive.

(a) $x > \log_2 5$ or $x < \log_5 2$

(b) $\log_5 2 < x < \log_2 5$

(c) $-\log_5 2 < x < \log_2 5$

(d) $x < -\log_5 2$ or $x > \log_2 5$

47. Find the values of x which satisfy the quadratic inequation $|x|^2 - 2\,|x| - 8 \leq 0$.

(a) $[-4, 4]$ (b) $[0, 4]$

(c) $[-4, 0]$ (d) $[-4, 2]$

48. The roots of the equation $x^2 - px + q = 0$ are consecutive integers. Find the discriminant of the equation.

(a) 1 (b) 2

(c) 3 (d) 4

49. Rohan and Sohan were attempting to solve the quadratic equation, $x^2 - ax + b = 0$. Rohan copied the coefficient of x wrongly and obtained the roots as 4 and 12. Sohan copied the constant term wrongly and obtained the roots as -19 and 3. Find the correct roots.

(a) -2 and -24 (b) 2 and 24

(c) 4 and 12 (d) -4 and -12

50. If $(x + 2)(x + 4)(x + 6)(x + 8) = 945$ and x is an integer, then find x.

(a) -1 or -11 (b) 1 or -11

(c) -1 or 11 (d) 1 or 11

51. The difference of the roots of $2y^2 - ky + 16 = 0$ is $\frac{1}{3}$. Find k.

(a) $\pm\frac{32}{3}$ (b) $\pm\frac{34}{3}$

(c) $\pm\frac{38}{3}$ (d) $\pm\frac{40}{3}$

52. Find the condition to be satisfied by the coefficients of the equation $px^2 + qx + r = 0$, so that the roots are in the ratio $3 : 4$.

(a) $12q^2 = 49pr$ (b) $12q^2 = -49pr$

(c) $49q^2 = 12\, pr$ (d) $49q^2 = -12pr$

53. If the roots of the equation $3x^2 + 9x + 2 = 0$ are in the ratio $m : n$, then find $\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}}$.

(a) $\frac{-3\sqrt{3}}{\sqrt{2}}$ (b) $\frac{3\sqrt{2}}{2}$

(c) $\frac{3\sqrt{3}}{\sqrt{2}}$ (d) $\frac{-3\sqrt{3}}{2}$

54. If $(p^2 - q^2)\, x^2 + (q^2 - r^2)\, x + r^2 - p^2 = 0$ and $(p^2 - q^2)\, y^2 + (r^2 - p^2)\, y + q^2 - r^2 = 0$ have a common root for all real values of p, q and r, then find the common root.

(a) -1 (b) 1

(c) 2 (d) -2

55. Which of the following are the roots of $|y|^2 - |y| - 12 = 0$?

(A) 4 (B) -4

(C) 3 (D) -3

(a) Both (A) and (B)

(b) Both (C) and (D)

(c) (A), (B), (C) and (D)

(d) None of these

56. Find the value of $\sqrt{30+\sqrt{30+\sqrt{30+\cdots\infty}}}$.

(a) 6 (b) −5

(c) Either (a) or (b) (d) Neither (a) nor (b)

57. If $y^2 + 6y - 3m = 0$ and $y^2 - 3y + m = 0$ have a common root, then find the possible values of m.

(a) $0, -\frac{27}{16}$ (b) $0, -\frac{81}{16}$

(c) $0, \frac{81}{16}$ (d) $0, \frac{27}{16}$

58. The students of a class contributed for a programme. Each student contributed the same amount. Had there been 15 more students in the class and each student had contributed ₹40 less, the total amount contributed would have increased from ₹3000 to ₹3200. Find the strength of the class.

(a) 25 (b) 15

(c) 10 (d) None of these

59. The graphs of $y = 2x^2$ and $y = ax + b$ intersect at two points (2, 8) and (6, 72). Find the quadratic equation in x whose roots are $a + 2$ and $\frac{b}{4} - 1$.

(a) $x^2 + 11x - 126 = 0$

(b) $x^2 - 11x + 126 = 0$

(c) $x^2 + 11x + 126 = 0$

(d) $x^2 - 11x - 126 = 0$

60. The equation $9y^2(m + 3) + 6(m - 3)y + (m + 3) = 0$, where m is real, has real roots. Which of the following is true?

(a) $m = 0$

(b) $m < 0$

(c) Either (a) or (b)

(d) Neither (a) nor (b)

61. Find the values of y which satisfy the quadratic inequalities below $y^2 + 5y + 4 \le 0$ and $y^2 - 2y - 15 \ge 0$.

(a) $-1 \le y \le 5$

(b) $-4 \le y \le -3$

(c) Either (a) or (b)

(d) Neither (a) nor (b)

62. If $\frac{y^2 + y - 6}{y^2 + y - 2} < 0$, then which of the following is true?

(a) $1 < y < 2$

(b) $-3 < y < -2$

(c) Either (a) or (b)

(d) Neither (a) nor (b)

63. The product of two consecutive even numbers exceeds twice their sum by more than 20. Which of the following is the range of values that the smaller of the numbers can take?

(a) $x > 4$ or $x < -6$

(b) $-6 < x < 4$

(c) $x > 6$ or $x < -4$

(d) $-4 < x < 6$

64. Which of the following statements about the sign of the quadratic expression $E = y^2 - 12y + 20$ is true?

(a) $E \le 0$ when $20 \le y \le 10$ and $E > 0$ when $y < 2$ or $y > 10$.

(b) $E \ge 0$ when $2 \le y \le 10$ and $E < 0$ when $y < 2$ or $y > 10$.

(c) $E \le 0$ when $-10 \le y \le -2$ and $E > 0$ when $y < -10$ or $y > -2$.

(d) $E \ge -2$ and $E < 0$ when $y < -10$ or $y > -2$.

65. If $|y|^2 - 4|y| - 60 \le 0$, then which of the following is the range of y?

(a) [−6, 6] (b) [0, 6]

(c) [0, 10] (d) [−10, 10]

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. True
2. reciprocal
3. True
4. pure
5. positive
6. 3, 5
7. 6 cm and 4 cm
8. 1
9. 2, 2
10. 4 and 5
11. 4 units
12. $2\sqrt{5}$
13. $x < \beta$ or $x > \alpha$
14. 4
15. 6 and 8
16. Yes
17. 3, 2
18. True
19. 5 years
20. True
21. 2, 4
22. $[-1, 4]$
23. $(-\infty, 3) \cup (5, \infty)$
24. $x \in (-\infty, 2) \cup (3, \infty)$
25. 7
26. $0 < x < \frac{n}{m}$
27. $-2 < x < 3$
28. Length = 19 m, breadth = 12 m
29. $(-\infty, 4)$
30. k

Short Answer Type Questions

31. $\frac{-13}{4}$
32. $\frac{4}{7}$
33. $q^2 = \frac{49pr}{12}$
34. 36
35. −36
36. $x = 7$
37. b and $(a - b)$
38. $\{x / -7 < x < 9\}$
39. $\{x / x < -12 \text{ or } x > -8\}$
40. 5 or 7
41. 34
42. $m \in (-\infty, -6) \cup (-2, \infty)$
43. There is no solution for the given equation.
44. −9, −6
45. $x \in \left(-\infty, \frac{-5}{2}\right) \cup \left(-1, \frac{1}{2}\right)$

Essay Type Questions

46. ϕ
47. $x \in (-7, -4) \cup (-1, 6)$
48. (1, 1)
50. $\{-6, 7\}$

CONCEPT APPLICATION

Level 1

1. (c)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (d)	**6.** (c)	**7.** (c)	**8.** (b)	**9.** (d)	**10.** (d)
11. (b)	**12.** (b)	**13.** (c)	**14.** (c)	**15.** (d)	**16.** (b)	**17.** (a)	**18.** (d)	**19.** (c)	**20.** (b)
21. (c)	**22.** (a)	**23.** (d)	**24.** (d)	**25.** (a)	**26.** (c)	**27.** (d)	**28.** (b)	**29.** (a)	**30.** (d)

Level 2

31. (a)	**32.** (d)	**33.** (a)	**34.** (b)	**35.** (b)	**36.** (a)	**37.** (b)	**38.** (d)	**39.** (d)	**40.** (d)
41. (c)	**42.** (d)	**47.** (b)	**46.** (d)	**47.** (c)					

Level 3

43. (a)	**44.** (a)	**45.** (a)	**48.** (a)	**49.** (d)	**50.** (b)	**51.** (b)	**52.** (a)	**53.** (a)	**54.** (b)
55. (a)	**56.** (a)	**57.** (d)	**58.** (a)	**59.** (d)	**60.** (c)	**61.** (b)	**62.** (c)	**63.** (c)	**64.** (a)
65. (d)									

CONCEPT APPLICATION

Level 1

1. Find the discriminant.
2. Use $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$.
3. Find the roots α and β or write $\alpha^2 + \beta^2$ as $(\alpha + \beta)^2 - 2\alpha\beta$ and $\alpha + \beta =$ sum of the roots and $\alpha\beta =$ product of the roots.
4. Use the formula and check the coefficient of x^2.
5. Square both sides.
6. If $a+\sqrt{b}$ is one root then $a-\sqrt{b}$ is other root.
7. Take the first degree expression and the constant on the other side.
8. Use $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.
9. Use $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$.
10. Simplify and solve by finding $\alpha + \beta$ and $\alpha\beta$. Where $\beta = 2\alpha$.
11. Let the number be x and frame the inequations.
12. Simplify and solve the inequation.
13. Assume the two successive multiples as n and $n + 2$.
14. x^2 is always positive and n is always negative.
16. Find the sum and the product of the roots then equate them.
17. If one of the roots is $a+\sqrt{b}$, this the other root is $a-\sqrt{b}$.
19. Use the formula of discriminant.
20. Factorise the LHS of the inequation.
21. Take nx on the left hand side and solve.
22. Simplify and solve the inequation.
23. Let the three consecutive odd natural numbers be $x - 2$, x and $x + 2$.
24. Observe the options.
25. Simplify and solve the inequation.
26. Find the roots or write $A^3 + B^3$ in terms of $A + B$ and AB.
27. Take $6x - 5$ on other side.
28. Observe the coefficient of each term and guess one root.
29. (i) Put $|x| = y$ and frame the equation.

    (ii) Solve for y.
30. Find the LCM and then square on both sides.

Level 2

31. Use $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.
32. If the roots of $ax^2 + bx + c = 0$ are in the ratio $m : n$, then $(m + n)^2\, a \cdot c = mnb^2$.
33. In the equation $ax^2 + bx + c = 0$ when $a + b + c = 0$, then the roots are 1 and $\frac{c}{a}$.
34. $(x - a)(x - b) \le 0$ when $a \le x \le b$.

    $(x - a)(x - b) \ge 0$ when $x \le a$ or $x \ge b$.
35. (i) $(a - b)\ (b - c) =$ product of the roots $= \frac{c}{a}$.

    (ii) $c - a = -(a - b + b - c) = -$(sum of the roots) $= \frac{b}{a}$.
36. (i) Express the quadratic equations given in the numerator and the denominator as the product of two linear factors.

    (ii) Form the various regions by using critical points.

    (iii) Identify the regions in which the inequality holds good.
37. (i) Let the height of the triangle be x. Hence base $= x + 3$.

    (ii) Form the quadratic inequation.
38. Form two different inequalities and find the common solution to them.
39. (i) Express the quadratic equations given in the numerator and the denominator as the product of two linear factors.

(ii) Form the various regions by using critical points.

(iii) Identify the regions in which the given inequality holds good.

40. Simplify the expression and identify whether it is non-positive or non-negative.

41. (i) If $(x - a)(x - b) > 0$, then $x \in (-\infty, a) \cup (b, \infty)$.

If $(x - a)(x - b) < 0$, then $x \in (a, b)$.

(ii) Take the common range of both the inequations.

42. $x^2 + 8x - 16 = 0$.

The product of the roots $= \dfrac{c}{a} = -16$.

Level 3

43. (i) Let the roots be pk and qk.

(ii) Use sum of roots $= \dfrac{-b}{a}$ and product of roots $= \dfrac{c}{a}$.

44. (i) Let the number of apes be equal to n.

(ii) The number of apes which are on the adjoining hills is $n - \dfrac{n^2}{64}$, which is equal to 12.

45. Discriminant ≥ 0.

46. (i) $x^2 - (\log_5 2 + \log_2 5)\, x + 1 > 0$.

(ii) $x^2 - (\log_5 2 + \log_2 5)\, x + 1$

$= (x - \log_5 2)\ (x - \log_2 5)$ and proceed.

47. (i) Put $|x| = y$.

(ii) Form a quadratic inequation in terms of y.

(iii) If $(x - a)\ (x - b) \leq 0$ then $x \in (a, b)$.

48. Let the roots be α and $\alpha + 1$.

$\alpha + (\alpha + 1) = p$, i.e., $2\alpha + 1 = p$.

$(\alpha)\ (\alpha + 1) = \dfrac{q}{1} = q$.

Discriminant $= p^2 - 4q = (2\alpha + 1)^2 - 4\ (\alpha)\ (\alpha + 1)$
$= 4\alpha^2 + 4\alpha + 1 - 4\alpha^2 - 4\alpha = 1$.

49. Rohan copied only the coefficient of x wrongly.

$\therefore$ He must have copied the constant term correctly.

$\therefore$ Correct product of the roots $= \dfrac{b}{1} = 4(12)$

$\Rightarrow \quad b = 48$

Sohan copied only the constant term wrongly.

$\therefore$ He must have copied the coefficient of x correctly.

$\therefore$ Correct sum of the roots $= a = -19 + 3 = -16$

Correct equation is $x^2 - (-16)x + 48 = 0$

$\Rightarrow \quad x^2 + 16x + 48 = 0$

$\Rightarrow \quad (x + 4)(x + 12) = 0$

$\Rightarrow \quad x = -4$ or -12.

$\therefore$ Correct roots are -4 and -12.

50. As x is an integer.

$\therefore$ $x + 2$, $x + 4$, $x + 6$ and $x + 8$ are four consecutive odd/even integers. Their product = 945, which is odd.

$\therefore$ $x + 2$, $x - 4$, $x + 6$ and $x + 8$ must be odd.

Factorising 945 as a product of 4 consecutive odd integers.

We have, $945 = 3(5)(7)(9) = (-3)(-5)(-7)(-9)$

$\therefore$ The smallest, i.e., $x + 2 = 3$ or -9.

$\therefore$ $x = 1$ or $x = -11$.

51. Let the roots of $2y^2 - ky + 16 = 0$ be α and β, where with $\alpha \geq \beta$. $\alpha + \beta = \dfrac{K}{2}$ and $\alpha\beta = 8$.

Difference of its roots > 0. $\alpha \neq \beta \therefore \alpha > \beta$

$\therefore$ Difference of its roots $= \alpha - \beta = \dfrac{1}{3}$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$$\left(\frac{1}{3}\right)^2 = \left(\frac{k}{2}\right)^2 - 4(8)$$

$$\left(\frac{k}{2}\right)^2 = \frac{289}{9} = \left(\frac{17}{3}\right)^2$$

$$\Rightarrow \quad \frac{k}{2} = \pm\frac{17}{3} \quad \Rightarrow \quad k = \pm\frac{34}{3}.$$

52. Let the common factor for the roots be y.

The roots are $3y$ and $4y$, $3y + 4y = \frac{-q}{p}$

$$\Rightarrow \quad y = \frac{-q}{7p}$$

$$(3y)(4y) = \frac{r}{p} \quad \Rightarrow \quad y^2 = \frac{r}{12p}$$

$$y^2 = \left(\frac{-q}{7p}\right)^2 = \frac{r}{12p} 12q^2 = 49pr.$$

53. Let the common factor for the roots be y.

The roots are my and ny.

$my + ny = -3 \quad \Rightarrow \quad (m+n)\, y = -3$

We can take m, n as positive and y as negative

$$(my)(ny) = \frac{2}{3}.$$

Now, $\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} = \frac{m+n}{\sqrt{mn}}$

Multiplying the numerator denominator by y, we have

$$\frac{m+n}{\sqrt{mn}} = \frac{y(m+n)}{y\sqrt{mn}} = \frac{y(m+n)}{\sqrt{(my)(ny)}}$$

$$= \frac{-3}{\sqrt{\frac{2}{3}}} = \frac{-3\sqrt{3}}{\sqrt{2}}.$$

54. $(p^2 - q^2)\, x^2 + (q^2 - r^2)\, x + (r^2 - p^2) = 0$ (1)

The sum of the coefficients $(p^2 - q^2) + (q^2 - r^2) + (r^2 - p^2) = 0$

$\therefore$ $x = 1$ is a root of Eq. (1)

$(p^2 - q^2)\, y^2 + (r^2 - p^2)\, y + q^2 - r^2 = 0$ (2)

$p^2 - q^2 + r^2 - p^2 + q^2 - r^2 = 0$

$\therefore$ $y = 1$ is a root of Eq. (2)

$\therefore$ 1 is the common root of Eqs. (1) and (2).

55. $|y|^2 - |y| - 12 = 0$

$(|y| - 4)\,(|y| + 3) = 0$

$|y| = 4$ or -3.

But $|y|$ must be non-negative.

$\therefore$ $|y| = 4$, i.e., $y = \pm 4$.

56. $x = \sqrt{30 + x}$

Squaring on both sides, we get $x^2 = 30 + x$

$x^2 - x - 30 = 0$

$(x - 6)(x + 5) = 0$

$x = 6$ or -5.

But x must be positive.

$\therefore$ $x = 6$.

57. Let the common root be c.

$c^2 + 6c - 3m = 0$ and $c^2 - 3c + m = 0$

That is, $3m = c^2 + 6c$ and $m = 3c - c^2$

$\therefore$ $3m = c^2 + 6c = 3\,(3c - c^2)$

$c^2 + 6c = 9c - 3c^2$

$4c^2 - 3c = 0$

$c = 0$ or $\frac{3}{4}$

If $c = 0$, $m = 0$

If $c = \frac{3}{4}$, $m = \frac{27}{16}$.

58. Let the strength of the class be x.

Let the amount each student contributed be ₹y.

$xy = 3000$ (1)

$(x + 15)(y - 40) = 3200$

That is, $xy + 15y - 40x - 600 = 3200$

From Eq. (1),

$$\Rightarrow \quad 15\left(\frac{3000}{x}\right) - 40x - 800 = 0$$

$\Rightarrow \quad x^2 + 20x - 1125 = 0$

$\Rightarrow \quad (x + 45)\,(x - 25) = 0$

$x = -45$ or 25.

But $x > 0 \quad \therefore$ $x = 25$.

59. At (2, 8) and (6, 72), $y = 2x^2 = ax + b$

$8 = 2a + b$ and $72 = 6a + b$.

Solving for a and b, $a = 16$ and $b = -24$.

The required equation is that whose roots are 18 and -7.

Sum of its roots = 11

Product of its roots = -126

$\therefore$ The Required equation is $x^2 - 11x - 126 = 0$.

60. Discriminant $(6(m-3))^2 - 4[9(m+3)(m+3)]$

$= 36[(m-3)^2 - (m+3)^2]$

$= 36((m^2 - 6m + 9 - (m^2 + 6m + 9))$

$= 36(-12m)$. This must be non-negative for the roots to be real.

$36(-12m) > 0$, i.e., $-2m \geq 0$, i.e., $m \leq 0$.

61. $y^2 + 5y + 4 \leq 0$

$\Rightarrow \quad (y+1)\,(y+4) \leq 0$ of $y + 1$ and $y + 4$

The expression with the smaller value is $y + 1$ and the one with the greater value is $y + 4$. If the product is negative, the smaller has to be negative and the greater is positive.

$\Rightarrow \quad -4 \leq y \leq 1 \qquad (1)$

Among $y - 5$ and $y + 3$, the smaller is $y - 5$, the greater, i.e., $y + 3$. If the product is positive either the smaller is positive or the greater is negative.

$y^2 - 2y - 15 \geq 0 \quad \Rightarrow \quad (y-5)(y+3) \geq 0$

$\Rightarrow \quad y - 5, 3 \geq 0$ or $y + 3 \leq 0$

$\Rightarrow \quad y \geq 5$

or $y \leq -3$. $\qquad (2)$

From Inequations (1) and (2)

$\Rightarrow \quad -4 \leq y \leq -3$.

62. $\dfrac{y^2 + y - 6}{y^2 + y - 2} < 0$

$\dfrac{(y+3)(y-2)}{(y+2)(y-1)} < 0$

$\dfrac{(y+3)(y-2)(y+2)(y-1)}{(y+2)^2(y-1)^2} < 0$

$(y+3)\,(y-2)\,(y+2)\,(y-1) < 0$,

i.e., $(y-2)\,(y-1)\,(y+2)\,(y+3) < 0$

×	✓	×	✓

$-\infty \qquad -3 \qquad -2 \qquad 1 \qquad 2$

$1 < y < 2$ or $-3 < y < -2$.

63. Let the smaller of the numbers be x. The greater number = $x + 2$.

$x(x+2) > [2(x + x + 2)] + 20$

$\Rightarrow \quad x^2 + 2x > 4x + 4 + 20$

$\Rightarrow \quad x^2 - 2x - 24 > 0 \; (x-6)(x+4) > 0$.

The smaller is positive as the greater is negative.

That is, $x > 6$ or $x < -4$.

64. $E = y^2 - 12y + 20 = (y-2)(y-10)$

We consider the following possibilities.

(1) $y < 2$. In this case, $y - 2 < 10$,

$y - 10 < 0$. $\therefore E > 0$

(2) $y = 2$ or 10. In this case $E = 0$

(3) $2 < y < 10$. In this case, $y - 2 > 0$ and

$y - 10 < 0$. $\therefore$ $E < 0$

(4) $y > 10$. In this case, $y - 2 > 0$,

$y - 10 > 0$. $\therefore$ $E > 0$

(2) and (3) $\Rightarrow$ $2 \leq y \leq 10$ $\Rightarrow$ $E \leq 0$. Only choice (1) is correct.

65. $|y|^2 - 4|y| - 60 \leq 0$

$\Rightarrow \quad x^2 - 4x - 60 \leq 0$

where $x = |y|$

$(x - 10)(\,x + 6) \leq 0$

$\Rightarrow \quad x < 10$ and $x \geq -6$

$\therefore -6 \leq |y| \leq 10$.

But $|y| \geq 0$.

$\therefore$ $0 \geq |y| \leq 10$

That is, $-10 \leq y \leq 10$.