Sample Question Paper - 48
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time : 2 Hr. **Max. Marks : 40**

General Instructions :

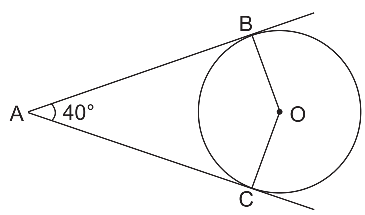
1. The question paper consists of 14 questions divided into three Sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

Section - A

1. The 4^{th} term of an A.P. is equal to 3 times the first term and 7^{th} term excess with the 3^{rd} term by 1. Find its n^{th} term.

2. If the quadratic equation $px^2 - 2\sqrt{5}\ px + 15 = 0$, has two equal roots then find the value of p.

3. In the given figure, AB and AC are tangents to the circle with centre O such that $\angle BAC = 40°$. Then find the $\angle BOC$.



4. The diameter of a sphere is 6 cm. It is melted and drawn in to a wire of diameter 2 mm. Find the length of the wire.

5. Consider the following distribution :

Marks obtained	Number of students
More than or equal to 0	63
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

Find the frequency of the class 30 – 40.

6. If the roots of the equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal, prove that $b + c = 2a$.

OR

Find the value of k for which the equation $x^2 + k(2x + k - 1) + 2 = 0$ has real and equal roots.

Section - B

7. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of mangoes	50 – 52	53 – 55	56 – 58	59 – 61	62 – 64
Number of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

OR

If the median of the following frequency distribution is 32·5. Find the values of f_1 and f_2.

Class	Frequency
0 – 10	f_1
10 – 20	5
20 – 30	9
30 – 40	12
40 – 50	f_2
50 – 60	3
60 – 70	2
Total	40

8. Draw two tangents to a circle of radius 3.5 cm, from a point P at a distance of 6.2 cm from its centre.

9. The arithmetic mean of the following frequency distribution is 53. Find the value of k.

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Frequency	12	15	32	k	13

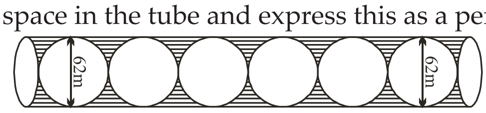
10. An aeroplane when flying at a height of H m from the ground passes vertically above another aeroplane at an instant when the angles of the elevation of the two planes from the point on the ground are θ and ϕ respectively. Prove that the vertical distance between the two planes is $\dfrac{H(\tan\theta - \tan\phi)}{\tan\theta}$.

Section - C

11. A solid cylinder of diameter 12 cm and height 15 cm is melted and recast into 12 toys in the shape of a right-circular cone mounted on a hemisphere. Find the radius of the hemisphere if the height of the cone is 3 times the radius.

OR

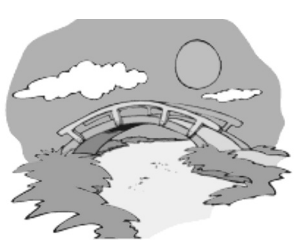
Six tennis balls of diameter 62 mm are placed in cylindrical tube. Find the volume of the six balls and the internal volume of unfilled space in the tube and express this as a percentage of the volume of the tube.

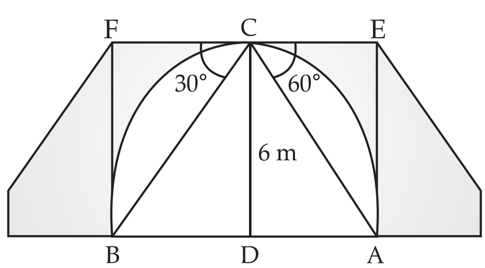


12. Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.

13. One day while sitting on the bridge across a river Aaradhya observes the angles of depression of the banks on opposite sides of the river to be 30° and 60° respectively as shown in the figure.

(Take $\sqrt{3}$ = 1.73)





Based on the above information, answer the following questions :

(i) If the bridge is at a height of 6 m, then find AD.

(ii) What is the width of the river ?

14. India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.

Based on the above information, answer the following questions :

(i) Find the production for first year.

(ii) In which year, the production is ₹ 29,200.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

Section - A

1.
$$t_4 = 3t_1$$

$\Rightarrow$
$$a + 3d = 3a$$

or
$$d = \frac{2}{3}a$$

$$t_7 = t_3 + 1$$
$$a + 6d = (a + 2d) + 1$$
$$\Rightarrow \quad 4d = 1$$
$$\Rightarrow \quad d = \frac{1}{4}$$
$$\therefore \quad a = \frac{3d}{2} = \frac{3}{8}$$
$$\therefore \quad t_n = a + (n-1)d$$
$$= \frac{3}{8} + (n-1).\frac{1}{4}$$
$$= \frac{1}{8}(2n+1)$$

2. The given quadratic equation is,
$$px^2 - 2\sqrt{5}\ px + 15 = 0$$
This equation is of the form
$$ax^2 + bx + c = 0$$
where, $a = p$, $b = -2\sqrt{5}\ p$, $c = 15$

We know , $\quad D = b^2 - 4ac$
$$= \left(-2\sqrt{5}p\right)^2 - 4 \times p \times 15$$
$$= 20p^2 - 60p$$
$$= 20p\ (p - 3)$$
For real and equal roots, we must have
$$D = 0 \Rightarrow 20p\ (p-3) = 0$$
$$\Rightarrow \quad p = 0, p = 3$$
$p = 0$, is not possible as whole equation will be zero.

Hence, 3 is the required value of p.

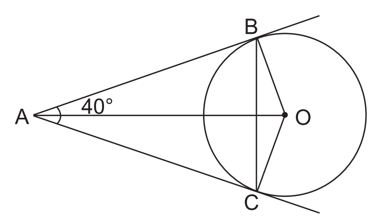
3. Given, $\quad AB = AC, OB = OC$ and $\angle A = 40°$

Join AO such that it is perpendicular to the chord BC and bisects $\angle A$.

Thus, $\quad \angle BAO = \angle OAC = 20°$

Also, as OB and OC are perpendicular to AB and AC respectively.

$\therefore \quad \angle ABO = \angle ACO = 90°$



Now, in ΔABO,
$$\angle BOA = 180° - (90° + 20°)$$
or $\quad \angle BOA = 180° - 110° = 70°$

Similarly in ΔACO,
$$\angle COA = 180° - (90° + 20°)$$
or $\quad \angle COA = 180° - 110° = 70°$

Hence, $\quad \angle BOC = \angle BOA + \angle COA$
$$= 70° + 70° = 140°.$$

4. $\quad$ Diameter of sphere = 6 cm
$$\therefore \quad \text{Radius } (r) = \frac{6}{2} = 3 \text{ cm}$$
$$\text{Volume} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (3)^3 \text{ cm}^3$$
$$= \frac{4}{3}\pi \times 3 \times 3 \times 3$$
$$= 36\pi \text{ cm}^3$$

$$\text{Diameter of wire} = 2 \text{ mm}$$

$$\therefore \quad \text{Radius } (r_2) = \frac{2}{2} = 1 \text{ mm} = \frac{1}{10} \text{ cm}$$

Let h be its length, then

$$\pi(r_2)^2 h = 36\pi$$

$$\Rightarrow \quad \pi \times \left(\frac{1}{10}\right)^2 h = 36\pi$$

$$\Rightarrow \quad \frac{\pi}{100} h = 36\pi$$

$$\Rightarrow \quad h = 36\pi \times \frac{100}{\pi} = 3600 \text{ cm}$$

$$\therefore \quad \text{Height or length of wire} = 3600 \text{ cm}$$

$$= \frac{3600}{100} = 36 \text{ m}$$

5.

Marks obtained	**Number of students**
0 – 10	(63 – 58) = 5
10 – 20	(58 – 55) = 3
20 – 30	(55 – 51) = 4
30 – 40	(51 – 48) = 3
40 – 50	(48 – 42) = 6
50 – 60	42 = 42

The frequency of the class interval 30 – 40 is 3.

6. Given, $(a-b)x^2 + (b-c)x + (c-a) = 0$

Comparing with $Ax^2 + Bx + C = 0$, we get

$$A = a - b, \ B = b - c \text{ and } C = c - a$$

Since, the roots are equal

$$\therefore \quad D = 0$$

$$\Rightarrow \quad B^2 - 4AC = 0$$

$$\Rightarrow \quad (b-c)^2 - 4(a-b)(c-a) = 0$$

$$\Rightarrow \quad b^2 + c^2 - 2bc - 4(ac - bc - a^2 + ab) = 0$$

$$\Rightarrow \quad b^2 + c^2 - 2bc - 4ac + 4bc + 4a^2 - 4ab = 0$$

$$\Rightarrow \quad b^2 + c^2 + 4a^2 + 2bc - 4ac - 4ab = 0$$

$$\Rightarrow \quad (b + c - 2a)^2 = 0$$

$$\Rightarrow \quad b + c - 2a = 0$$

$$\Rightarrow \quad b + c = 2a.$$

OR

Given equation is,

$$x^2 + k(2x + k - 1) + 2 = 0$$

$$\Rightarrow \quad x^2 + 2kx + k(k-1) + 2 = 0$$

Here $a = 1$, $b = 2k$ and $c = k(k-1) + 2$

For real and equal roots

$$b^2 - 4ac = 0$$

$$\Rightarrow \quad (2k)^2 - 4.1.\ (k\ (k-1) + 2) = 0$$

$$\Rightarrow \quad 4k^2 - 4\ (k^2 - k + 2) = 0$$

$$\Rightarrow \quad 4k^2 - 4k^2 + 4k - 8 = 0$$

$$\Rightarrow \quad 4k = 8$$

$$\Rightarrow \quad k = \frac{8}{4} = 2$$

Section - B

7.

C.I.	f_i	(x_i)	$d_i = x_i - 75$	$f_i d_i$
50 – 52	15	51	– 6	– 90
53 – 55	110	54	– 3	– 330
56 – 58	135	57 = A	0	0
59 – 61	115	60	3	345
62 – 64	25	63	6	150
	$\Sigma f_i = 400$			$\Sigma f_i d_i = 75$

Here, we have $\Sigma f_i d_i = 75$, A = 57

Now,
$$\bar{x} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$
$$= 57 + \frac{75}{400}$$
$$= 57 + 0.1875 = 57.1875$$

OR

Given, Median = 32·5

Class	Frequency	Cumulative Frequency
0 – 10	f_1	f_1
10 – 20	5	$f_1 + 5$
20 – 30	9	$f_1 + 14$
30 – 40	12	$f_1 + 26$
40 – 50	f_2	$f_1 + f_2 + 26$
50 – 60	3	$f_1 + f_2 + 29$
60 – 70	2	$f_1 + f_2 + 31$

Total frequency = 40

$\therefore$ $f_1 + f_2 + 31 = 40$

or $f_1 + f_2 = 9$ ...(i)

Also, $\frac{n}{2} = \frac{40}{2} = 20$

Median = 32·5 (Given)

which lies in the class interval (30 – 40)

$\therefore$ Median class = 30 – 40

$l = 30$

$f = 12$, $c.f = f_1 + 14$

$h = 10$

So,
$$\text{Median} = l + \left[\frac{\frac{n}{2} - c.f.}{f}\right] \times h$$

$$\Rightarrow \quad 32{\cdot}5 = 30 + \left[\frac{20 - (f_1 + 14)}{12}\right] \times 10$$

$$\Rightarrow \quad 32{\cdot}5 = 30 + \left(\frac{6 - f_1}{6}\right) \times 5$$

$\Rightarrow \quad 2 \cdot 5 = \frac{5}{6}(6 - f_1)$

$\Rightarrow \quad \frac{2.5 \times 6}{5} = 6 - f_1$

$\Rightarrow \quad 6 - f_1 = 3 \Rightarrow f_1 = 3$

From equation (i), we get

$$f_2 = 6$$

$\therefore \quad f_1 = 3, f_2 = 6.$

8. Steps of construction :

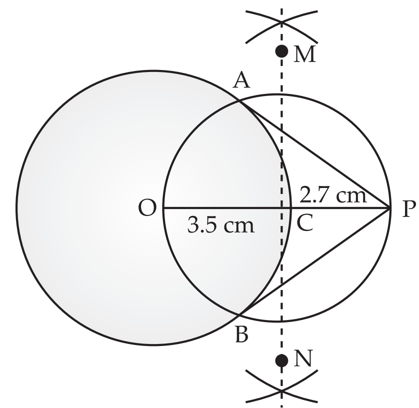
Step I : Draw a circle of radius 3.5 cm taking O as a centre.

Step II : Take a point P at distance of 6.2 cm from the centre.

Step III : Draw a bisector of OP which intersect OP at C.

Step IV : From C, of radius OC draw a circle which intersect the previous circle at A and B.

Step V : Join AP and BP.



$$OP = OC + CP$$
$$= 3.5 + 2.7 = 6.2 \text{ cm}$$

Hence AP and PB are the required tangents.

9. Given, Mean = 53

Class	Frequency (f_i)	Mid-value (x_i)	$f_i x_i$
0 – 20	12	10	120
20 – 40	15	30	450
40 – 60	32	50	1600
60 – 80	k	70	$70k$
80 – 100	13	90	1170
	$72 + k$		$3340 + 70k$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$53 = \frac{3340 + 70k}{72 + k}$$

$\Rightarrow \quad 53(72 + k) = 3340 + 70k$

$\Rightarrow \quad 3816 + 53k = 3340 + 70k$

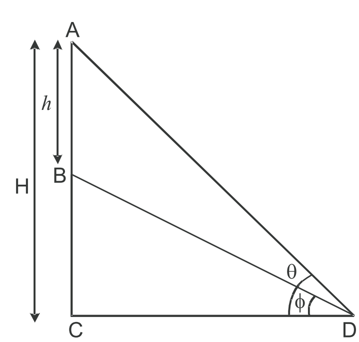
$\Rightarrow \quad k = 28$

10. Let AB be the distance between the two planes and D be the point of observation on the ground.

So $\quad AB = h$ m

Let BC be the height of the second plane from the ground and BC = $(H - h)$ m

Now, in ΔBDC,



$$\tan \phi = \frac{BC}{CD} = \frac{H - h}{CD}$$

$\Rightarrow \quad CD = (H - h) \cot \phi \qquad \ldots(i)$

and, in ΔADC

$$\tan \theta = \frac{AC}{CD} = \frac{H}{CD}$$

$\Rightarrow \quad CD = H \cot \theta \qquad \ldots(ii)$

From equations (i) and (ii), we have

$$(H - h)\cot\phi = H\cot\theta$$

$$\Rightarrow \quad H(\cot\phi - \cot\theta) = h\cot\phi$$

$$\Rightarrow \quad h = \frac{H(\cot\phi - \cot\theta)}{\cot\phi}$$

$$\Rightarrow \quad h = \frac{H\left(\frac{1}{\tan\phi} - \frac{1}{\tan\theta}\right)}{\frac{1}{\tan\phi}}$$

$$\Rightarrow \quad h = \frac{H(\tan\theta - \tan\phi)\tan\phi}{\tan\theta.\tan\phi}$$

$$\Rightarrow \quad h = \frac{H(\tan\theta - \tan\phi)}{\tan\theta}.$$

Hence Proved.

Section - C

11. Given, Diameter of the cylinder = 12 cm

$\therefore$ Radius = 6 cm

Height of the cylinder = 15 cm

Number of toys = 12

Let the radius of the hemisphere be r cm

Thus, Height of the cone = $3r$ cm

Now, Volume of cylinder $= \pi(6)^2\, 15\ \text{cm}^3$

$= (36)\,(15)\pi\ \text{cm}^3$

$= 540\pi\ \text{cm}^3$

Now, Total volume of 12 toys = 12 [Volume of cone + Volume of hemisphere]

Thus, Total volume of 12 toys $= 12\left[\frac{1}{3}\pi 3r(r)^2 + \frac{2}{3}\pi r^3\right]\text{cm}^3$

$= 4[3\pi r^3 + 2\pi r^3]\ \text{cm}^3$

$= 20\pi r^3\ \text{cm}^3$

Now, $20\pi r^3 = 540\pi$

$\Rightarrow \quad r^3 = 27$

$\Rightarrow \quad r^3 = (3)^3$

$\Rightarrow \quad r = 3$ cm

Thus, Radius of the hemisphere = 3 cm

OR

Diameter of the tennis balls = 62 mm.

$\because$ Radius of the balls and tube is half the diameter

$\therefore \quad r = \frac{1}{2} \times 62 = 31$ mm

Volume of one ball $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 31 \times 31 \times 31$

$= 124{,}838.476\ \text{mm}^3$

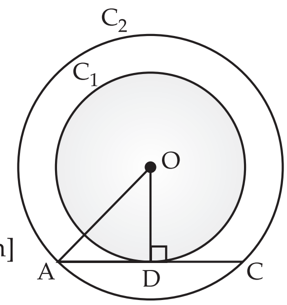
$= 124.838\ \text{cm}^3$

12. Let C_1 and C_2 be the two circles having same centre O.

And, AC is a chord which touches the circle C_1 at point D.

Join OD, So, $OD \perp AC$

[Since, perpendicular from centre bisects the chords]



$\therefore \quad AD = DC = 4 \text{ cm}$

Thus, in right angled ΔAOD,

$\therefore \quad OA^2 = AD^2 + DO^2$ [By Pythagoras theorem]

$\Rightarrow \quad DO^2 = 5^2 - 4^2 = 25 - 16 = 9$

$\Rightarrow \quad DO = 3 \text{ cm}$

Therefore, the radius of the inner circle, OD = 3 cm.

13. (i) Clearly, $\angle DAC = \angle ACE = 60°$

So, in ΔADC, we have

$$\tan 60° = \frac{CD}{AD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{6}{AD}$$

$$\Rightarrow \quad AD = \frac{6}{\sqrt{3}} \text{ m}$$

(ii) Clearly, $\angle FBC = \angle CBD = 30°$

In ΔCBD, $\tan 30° = \dfrac{CD}{BD}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{6}{BD}$$

$$\Rightarrow \quad BD = 6\sqrt{3} \text{ m}$$

$\therefore$ Width of the river = AB = AD + BD

$$= \frac{6}{\sqrt{3}} + 6\sqrt{3}$$

$$= 6\left(\frac{1}{\sqrt{3}} + \sqrt{3}\right)$$

$$= 6\left(\frac{4}{\sqrt{3}}\right) = \frac{24}{\sqrt{3}} \text{ m}$$

$$= 13.87 \text{ m}$$

14. (i) $a_6 = 16000$

and $a_9 = 22600$

Since, $a_n = a + (n-1)d$

$\Rightarrow \quad a_6 = a + 5d = 16000 \quad$ ...(i)

and $a_9 = a + 8d = 22600 \quad$ ...(ii)

Solving equations (i) and (ii), we get

$d = 2200$

and $a = 5000$

$\therefore$ It produced 5000 sets in 1st year

(ii) Given, $a_n = 29200$

$\Rightarrow \quad a + (n-1)d = 29200$

$\Rightarrow \quad 5000 + (n-1)2200 = 29200$

$\Rightarrow \quad (n-1)2200 = 24200$

$\Rightarrow \quad n = 12$