5. ARITHMETIC PROGRESSIONS

1. If a, b, c, d and e are in AP, then find the value of a - 4b + 6c – 4d + e

   [Sol. : a, b, c, d and e are in AP.

   Let k be its common difference.

   So, b = a + k, c = a + 2k, d = a + 3k and e = a + 4k.

   Now, a – 4b + 6c – 4d + e

   = a – 4(a + k) + 6(a + 2k) – 4(a + 3k) + a + 4k

   = a – 4a + 6a – 4a + a – 4k + 12k – 12k + 4k

   = 0 + 0 = 0**].**

2. Divide 16 into 4 parts which are in A.P. such that the product of the extremes is one less than sum of the means.

   [Sol. : Let the four parts of 16 which are in A.P. be a – 3d, a – d, a + d and a + 3d

   So, a – 3d + a – d + a + d + a + 3d = 16

   ⇒ 4a = 16 ⇒ a = 4.

   Thus, the four parts are 4 – 3d, 4 – d, 4 + d, 4 + 3d

   It is given that (4 – 3d)(4 + 3d) = (4 – d) + (4 + d) – 1

   ⇒ $16 - 9d^2 = 7$ ⇒ $9d^2 = 9$ ⇒ $d = \pm 1$.

   Taking d = 1, these parts are 1, 3, 5, 7.]

3. Find an AP whose sum of the first three terms is 21 and the sum of their squares is 155.

   [Ans : 5, 7, 9, ... (or) 9, 7, 5, ...]

4. If x,y,z are in A.P., then find the value of (x + y –z)(–x + y+ z).

   [Ans : $\frac{10zx - 3x^2 - 3z^2}{4}$]

5. Find the middle term of the A.P –11, –7,–3,.....45

   [Ans : 17]

6. In and orchard, there are 17 mango trees in the first row, 15 in the second, 13 in the third and so on. There are 3 mango trees in the last row. How many rows of trees are there in the orchard?

[Ans : 8 rows]

7. If third and ninth terms of an A.P are 4 and –8 respectively, then is there any term of this A.P which is zero?

[Hint : Yes, 5^{th} term of this A.P. is zero]

8. How many 3 digit numbers are divisible by 6 which always leave remainder 1?

[Ans : 150 numbers]

9. What is the 27^{th} positive odd number?

[Ans : 53]

10. Find the sum of all odd numbers between 10 and 200.

[Ans : 9975]

11. Find the sum of the first n terms of the sequence $< a_n >$, where $a_n = 5 - 6n$ and n is a natural number.

[Ans : $2n - 3n^2$]

12. Find $\left(4-\frac{1}{n}\right)+\left(7-\frac{2}{n}\right)+\left(10-\frac{3}{n}\right)+....$ upto n terms

[Sol. : $\left(4-\frac{1}{n}\right)+\left(7-\frac{2}{n}\right)+\left(10-\frac{3}{n}\right)+.......$ upto n terms

$$=(4+7+10+\text{....... upto n terms})-\left(\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\text{.........upto n terms}\right).$$

$$=\frac{n}{2}\{2\times 4+(n-1)\times 3\}-\frac{n}{2}\left\{2\times\frac{1}{n}+(n-1)\times\frac{1}{n}\right\}.$$

$$=\frac{n}{2}(8+3n-3)-\frac{n}{2}\left(\frac{2}{n}+1-\frac{1}{n}\right)$$

$$=(5+3n)\frac{n}{2}-\frac{n}{2}\left(\frac{1}{n}+1\right)$$

$$=\frac{n}{2}\left(5+3n-\frac{1}{n}-1\right)=\frac{n}{2}\left(4+3n-\frac{1}{n}\right)=\frac{1}{2}(3n^2+4n-1)]$$

13. Rahul purchased every year national saving certificates of value exceeding the last year purchase by ₹500. After 6 years, he finds the total value of the 500 certificates purchased by him as ₹13500. Find the value of the ceritificates purchased

i) In the first year

ii) In the 5th year.

[Ans : i) Rs. 10,000,

ii) Rs.3000]

14. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of houses following it. Find this value of x.

[Sol. : Here, we are given that

$\{1 + 2 + 3 + ...+(x - 1)\} = \{(x+ 1) + (x + 2) + (x + 3) + ...+49\}.$

$\Rightarrow \left(\frac{x-1}{2}\right)\{2\times1+(x-1-1)\times1\}=\left(\frac{49-x}{2}\right)\{2(x+1)+(49-x-1)\times1\}$

$\Rightarrow \quad (x-1)(2+x-2)=(49-x)(2x+2+49-x-1)$

$\Rightarrow \quad (x - 1)(x) = (49 - x)(50 + x)$

$\Rightarrow \quad x^2 - x = 2450 + 49x - 50x - x^2$

$\Rightarrow \quad 2x^2 = 2450 \quad \Rightarrow \quad x^2 = 1225.$

$\Rightarrow \quad x=\sqrt{1225}=35$.]

15. Priya is preparing for the Bicycle Marathon. Her racing bicycle has a device to calculate the number of kilometres she cycled. She decides to increase the distance she cycles everyday by a fixed number of kilometres.

i) On the first day Priya cycled 8 km. In 10 days she cycled a total of 170 km. How many kilometres did she cycle on the 3rd day?

ii) Priya plants to go on a cycle tour from Bangalore to Mangalore covering 425 km. She travels 20 km on day 1 and increases the distance covered each day by 5 km. In how many days will she reach her destination?

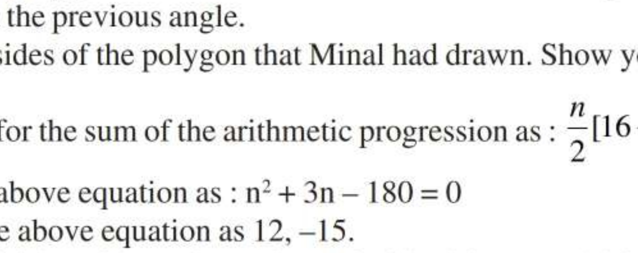
[Sol. : i) Applies the formula for the sum of n terms correctly and finds the value of the common difference d as 2.

Either applies the forula for the nth term or generates the pattern with a = 8 and d = 2 to find the distance cycled on the 3rd day as 12 km.

ii) Applies the formula for the sum of n terms of AP and represents the given scenario mathematically as $n^2 + 7n - 170 = 0$.

Solves the above quadratic equation and finds n as 10 days.]

16. The exterior angles marked in each of the polygons below are in arithmetic progression.



Minal drew one such polygon with n sides. The smallest exterior angle is 8° and each subsequent angle is 4° more than the previous angle.

Find the number of sides of the polygon that Minal had drawn. Show your steps.

[Ans : The equation for the sum of the arithmetic progression as : $\frac{n}{2}[16+4(n-1)]=360$

Simplifies the above equation as : $n^2 + 3n - 180 = 0$

The roots of the above equation as 12, –15.

The number of sides of the polygon that Minal had drawn as 12.]

17. Sana decided to start practicing for an upcoming marathon. She decided to gradually increase the duration. She ran for 10 mins on day 1 and increased the duration by 5 minutes every day.
From which day onwards will she be running for 2.5 hours or more? Show your work?

[Sol. : Converts 2.5 hours to minutes as $2.5 \times 60 = 150$ minutes.

Identifies that the increase in the running duration follows an arithmetic progression given by 10, 15, 20, 25, ...and assumes the day corresponding to 150 mins as n.

The value of n as follows :

$10 + (n - 1) \times 5 = 150$

$\Rightarrow \quad n = 29.$]

18. Two arithmetic progressions have the same first term. The common difference of one progression is 4 more than the other progression. 124th term of the first arithmetic progression is the same as 42nd term of the second.
Find one set of possible values of the common differences. Show your work.

[Sol. : Considers $d_1 = d_2 + 4$ or $d_2 = d_1 + 4$ where d_1 & d_2 are the common differences of the two arithmetic progressions.

Writes $a_{124} = b_{42}$

$\Rightarrow \quad a_1 + 123\, d_1 = b_1 + 41 d_2$

where a_1, a_{124}, b_1 & b_{42} are the 1st, 124th, 1st and 42nd terms of the two arithmetic progressions respectively.

Solves the above two equations and finds the value of d_1 as -2 and d_2 as -6 OR d_1 as 2 and d_2 as 6.

$a_{124} = b_{42}$

$\Rightarrow \quad a_1 + 123(4 + d_2) = b_1 + 41 d_2$

$\Rightarrow \quad 492 + 123\, d_2 = 41 d_2$

$\Rightarrow \quad d_2 = -6.$]

19. Huner said, "The value of the 20th term of ANY arithmetic progression is double that of the 10th term."
Is Huner's statement correct? Justify your answer

[Sol. : Huner statement is not correct.

Here Huner's statement is correct only when the first term of an arithmetic progression (AP), a and common difference, d are equal but not for any AP.]

20. The sum of the first two terms of an arithmetic progression is the same as the sum of the first seven terms of the same arithmetic progression.
Can such an arithmetic progression exist? Justify your answer.

[Sol. : An arithmetic progression having the sum of the first two terms same as the sum of the first seven terms can exist.

Justifies by writing that such an arithmetic progression exists if the sum of the first term and four times common difference is zero.

That is, $a + 4d = 0$, where a is the first term and d is the common difference.]