Sample Question Paper - 43
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time : 2 Hr. **Max. Marks : 40**

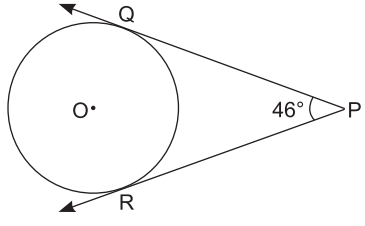
General Instructions :

1. The question paper consists of 14 questions divided into three Sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

Section - A

1. For what values of k, the roots of the equation $x^2 + 4x + k = 0$ are real?

2. In the given figure, PQ and PR are two tangents to a circle with centre O. If $\angle QPR = 46°$, then find $\angle QOR$.



OR

Prove that the line segment joining the points of contact of two parallel tangents of a circle passes through its centre.

3. From the following probability distribution, find the median class.

Cost of living index	1400-1550	1550-1700	1700-1850	1850-2000
Number of weeks	8	15	21	8

4. The first three terms of A.P. are $(3y - 1)$, $(3y + 5)$ and $(5y + 1)$. Then find y.

5. If k, $2k - 1$ and $2k + 1$ are three consecutive terms of an A.P., then find the value of k.

6. The diameter of the Moon is approximately one-fourth of the diameter of the Earth. What is the ratio (approximate) of their volumes ?

Section - B

7. Amit, standing on a horizontal plane, finds a bird flying at a distance of 200 m from him at an elevation of 30°. Deepak standing on the roof of a 50 m high building, finds the angle of elevation of the same bird to be 45°. Amit and Deepak are on opposite sides of the bird. Find the distance of the bird from Deepak.

8. The table below shows the distribution of marks obtained by students in an examination. Calculate the value of x, if the mean mark is 18.

Marks	5	10	15	20	25	30
No. of students	6	4	6	12	x	4

9. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm.

10. The following distribution shows the daily pocket allowance of children of a locality. If the mean pocket allowance is ₹ 18, find the missing frequency *f*.

Daily pocket allowance (in ₹)	11 – 13	13 – 15	15 – 17	17 – 19	19 – 21	21 – 23	23 – 25
Number of children	7	6	9	13	*f*	5	4

OR

To find out the concentration of SO_2 in the air (in parts per million *i.e.*, ppm), the data was collected for 30 localities in a certain city and is presented below :

Concentration of SO_2 (in ppm)	0.00 – 0.04	0.04 – 0.08	0.08 – 0.12	0.12 – 0.16	0.16 – 0.20	0.20 – 0.24
Frequency	4	9	9	2	4	2

Find the mean concentration of SO_2 in the air.

Section - C

11. The 16^{th} term of an A.P. is five times its third term. If its 10^{th} term is 41, then find the sum of its first fifteen terms.

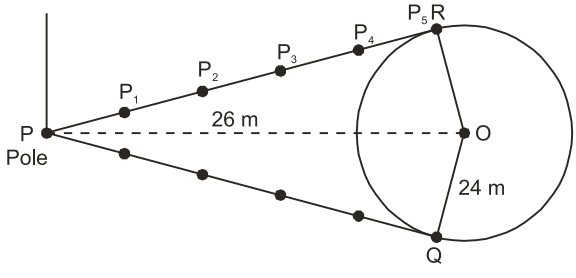
12. The shadow of a flagstaff is three times as long as the shadow of the flagstaff when the Sun rays meet the ground at an angle of 60°. Find the angle between the Sun rays and the ground at the time of longer shadow.

OR

From an aeroplane vertically above a straight horizontal plane, the angles of depression of two consecutive kilometer stones on the opposite sides of the aeroplane are found to be α and β. Show that the height of the aeroplane is $\frac{\tan\alpha.\tan\beta}{\tan\alpha+\tan\beta}$.

13. Read the following passage and answer the questions that follows :

There is a circular park of radius 24 m and a pole at a distance of 26 m from the centre of the park, as shown in the figure. From the pole, there are two paths PR and PQ, which are tangential to the park.



(i) Find the length of each path.

(ii) If five light poles are to be put along each tangential path at equal distances, find the distance between each consecutive light pole.

14. Vijay has rain water harvesting plant on his roof. After rain, all the water that is collected on the roof of 22 m × 20 m is drained into the cylindrical tank having diameter of base 2 m and height 3.5 m. It rained heavily last night and in the morning the tank is just full.

(i) How much water is collected in the tank (in litres)?

(ii) Find the rainfall in cm.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

Section – A

1. The given equation is $x^2 + 4x + k = 0$

On comparing the given equation with

$ax^2 + bx + c = 0$,

we get $a = 1$, $b = 4$ and $c = k$

For real roots, $D \geq 0$

or $b^2 - 4ac \geq 0$

$\Rightarrow$ $16 - 4k \geq 0$

or $k \leq 4$

$\therefore$ For $k \leq 4$, equation $x^2 + 4x + k$ will have real roots.

2. Join OP such that it bisects $\angle$QPR.

Thus, $\angle OPQ = \angle OPR = 23°$

Also, $\angle OQP = \angle ORP = 90°$ [Tangent is $\perp$ radius]

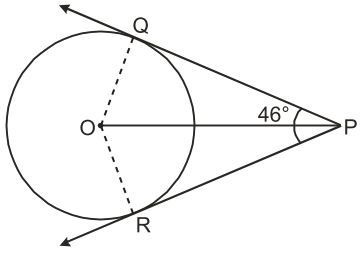
So, $\angle QOP = \angle ROP$

$= 180° - (90° + 23°)$

$= 180° - 113° = 67°$

Hence, $\angle QOR = \angle QOP + \angle ROP$

$= 67° + 67° = 134°$.



OR

Given, AC $\parallel$ DF

Let O be the centre of the circle. Join OB, OP, OE such that

$OB = OP = OE = r$

Also, let $OP \parallel BC \parallel ED$

Thus, $\angle CBO + \angle BOP = 180°$ [co-interior angles]

or $90° + \angle BOP = 180°$ [OB is perpendicular to AC]

or $\angle BOP = 90°$

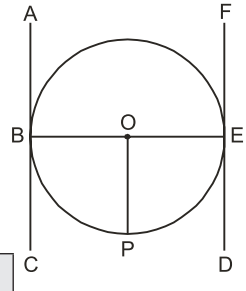
Similarly, $\angle POE = 90°$

Thus, $\angle BOE = \angle POE + \angle POB$

$= 90° + 90°$

$= 180°$

Hence, BOE is a striaght line passing through the centre O.



3.

Cost of living index	No. of weeks (f)	$c.f.$
1400 – 1550	8	8
1550 – 1700	15	23
1700 – 1850	21	44
1850 – 2000	8	52
	$\Sigma f = 52$	

Here, $N = 52$

$\Rightarrow$ $\frac{N}{2} = \frac{52}{2} = 26$

26 will lie in the class interval 1700 – 1850.

$\therefore$ Median class is 1700 – 1850.

4. The terms of an A.P. are

$(3y - 1)$, $(3y + 5)$ and $(5y + 1)$

Thus, $d = (3y + 5) - (3y - 1) = (5y + 1) - (3y + 5)$

$\Rightarrow$ $6 = 2y - 4$

$\Rightarrow$ $2y = 10$

$\Rightarrow$ $y = 5$.

5. The terms of an A.P. are

k, $(2k - 1)$ and $(2k + 1)$

Thus, $d = (2k - 1) - k = 2k + 1 - (2k - 1)$

$\Rightarrow$ $2k - 1 - k = 2k + 1 - 2k + 1$

$\Rightarrow$ $k - 1 = 2$

$\Rightarrow$ $k = 3$.

6. The diameter of Moon is approximately one-fourth of the diameter of Earth.

Let, Radius of Moon $= r$,

Then, Radius of Earth $= 4r$

$$\text{Required ratio} = \frac{\text{Volume of Moon}}{\text{Volume of Earth}}$$

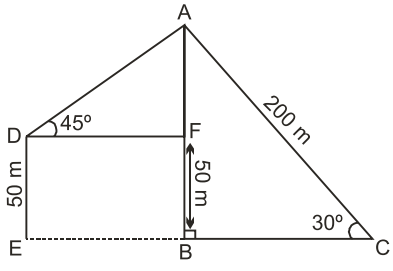
$$= \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi (4r)^3}$$

$$= \frac{r^3}{64r^3} = \frac{1}{64}$$

$= 1 : 64$

Section - B

7. Let Amit be at C point and bird is at A point, such that $\angle ACB = 30°$. AB is the height of bird from point B on ground and Deepak is at D point, DE is the building of height 50m.



Now, in right ΔABC, we have

$$\sin 30° = \frac{AB}{AC}$$

$\Rightarrow$ $\frac{1}{2} = \frac{AB}{200}$

$\Rightarrow$ $AB = 100$ m $(\because AB = AF + BF, 100 = AF + 50, AF = 50)$

In right ΔAFD, we have

$$\sin 45° = \frac{AF}{AD}$$

$\Rightarrow$ $\frac{1}{\sqrt{2}} = \frac{50}{AD}$

$\Rightarrow$ $AD = 50\sqrt{2}$ m

Hence, the distance of bird from Deepak is $50\sqrt{2}$ m.

8. Given, Mean = 18

Marks (x_i)	No. of students (f_i)	($f_i \times x_i$)
5	6	30
10	4	40
15	6	90
20	12	240
25	x	$25x$
30	4	120
Total	$32 + x$	$520 + 25x$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

or $\Sigma f_i \times \text{Mean} = \Sigma f_i x_i$

$\Rightarrow$ $(32 + x)18 = 520 + 25x$

or $520 + 25x = 576 + 18x$

$\Rightarrow$ $25x - 18x = 576 - 520$

$\Rightarrow$ $x = \frac{56}{7} = 8$

9. Steps of construction :

(1) Draw two concentric circles with centre O of radii 4 cm and 6 cm.

(2) Take a point P on the bigger circle of radius 6 cm.

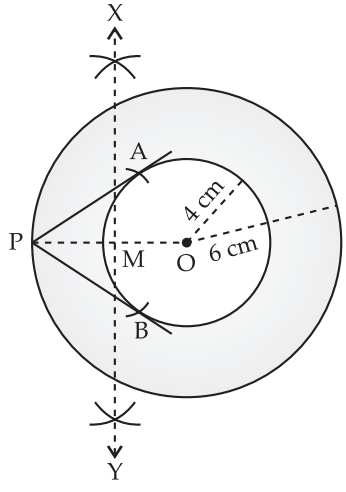
(3) Join OP with dotted line.

(4) Draw perpendicular bisector of OP which intersects OP at M.

(5) With P as centre and MP radius, mark two arcs on smaller circle of radius 4 cm at point A and B.

(6) Join PA and PB.

PA and PB are the required pair of tangents.



10. We have,

Daily pocket allowance (in ₹)	**Number of children f_i**	**Mid-value of x_i**	$f_i \times x_i$
11 – 13	7	12	84
13 – 15	6	14	84
15 – 17	9	16	144
17 – 19	13	18	234
19 – 21	f	20	$20f$
21 – 23	5	22	110
23 – 25	4	24	96
Total	$\Sigma f_i = 44 + f$		$\Sigma f_i x_i = 752 + 20f$

Now, $\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow$ $18 = \frac{752 + 20f}{44 + f}$

$\Rightarrow$ $18(44 + f) = 752 + 20f$

$\Rightarrow$ $18(44 + f = 752 + 20f$

$\Rightarrow$ $792 + 18f = 752 + 20f$

$\Rightarrow$ $2f = 40$

$\Rightarrow$ $f = 20.$

OR

Concentration of SO_2 (in ppm)	Frequency (f_i)	Class mark x_i	$d_i = x_i - 0.14$	$f_i d_i$
0.00 – 0.04	4	0.02	– 0.12	– 0.48
0.04 – 0.08	9	0.06	– 0.08	– 0.72
0.08 – 0.12	9	0.10	– 0.04	– 0.36
0.12 – 0.16	2	0.14 = A	0	0
0.16 – 0.20	4	0.18	0.04	0.16
0.20 – 0.24	2	0.22	0.08	0.16

Here, we have $\Sigma f_i = 30,\ \Sigma f_i d_i = -1.24$

and $A = 0.14$

$$\bar{x} = A + \frac{\Sigma f_i d_i}{\Sigma f_i} = 0.14 + \frac{-1.24}{30}$$

$$= 0.14 - 0.041 = 0.99$$

Section - C

11. Given that 16^{th} term of an A.P. is five time its third term.

$\therefore \quad a + (16-1)d = 5[a + (3-1)d]$

$\Rightarrow \quad a + 15d = 5[a + 2d]$

$\Rightarrow \quad a + 15d = 5a + 10d$

$\Rightarrow \quad 4a - 5d = 0$ ...(i)

Also given that,

$T_{10} = 41$

$\Rightarrow \quad a + (10-1)d = 41$

$\Rightarrow \quad a + 9d = 41$ ...(ii)

On multiplying equation (ii) by 4, we get

$4a + 36d = 164$ ...(iii)

Subtracting equation (iii) from (i), we get

$$\begin{array}{rr} 4a - 5d = & 0 \\ 4a + 36d = & 164 \\ - \quad - & - \\ \hline -41d = & -164 \end{array}$$

$\Rightarrow \quad d = 4$

On substituting the value of d in (i), we get

$4a - 5 \times 4 = 0$

$\Rightarrow \quad 4a = 20$

$\Rightarrow \quad a = 5$

Now, $S_{15} = \frac{15}{2}[2a + (15-1)d]$

$\Rightarrow \quad S_{15} = \frac{15}{2}(2 \times 5 + 14 \times 4)$

$$= \frac{15}{2} 2(5 + 14 \times 2)$$

$$= 15(5 + 28) = 15 \times 33$$

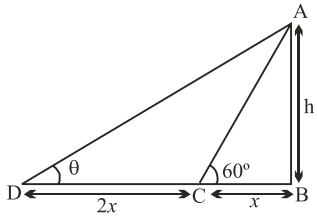
$\Rightarrow \quad S_{15} = 495$

Hence, sum of first fifteen terms is 495.

12. Let AB be the flagstaff and BC be the length of its shadow when the Sun rays meet the ground at an angle of 60°. Let θ be the angle between the Sun rays and the ground when the length of the shadow of the flagstaff is BD. Let h be the height of the flagstaff.

Let BC = x

Then, BD = $3x$ and CD = $2x$



In ΔACB, we have

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad h = \sqrt{3}x \qquad ...(i)$$

In ΔADB, we have

$$\tan \theta = \frac{AB}{BD}$$

$$\Rightarrow \quad \tan \theta = \frac{h}{3x}$$

$$\Rightarrow \quad \tan \theta = \frac{\sqrt{3}x}{3x} \qquad \text{[From (i)]}$$

$$\Rightarrow \quad \tan \theta = \frac{1}{\sqrt{3}}$$

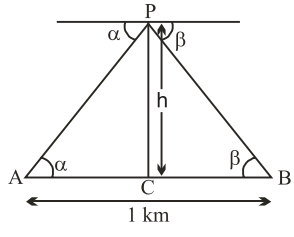
$$\Rightarrow \quad \tan \theta = \tan 30°$$

$$\Rightarrow \quad \theta = 30°$$

OR

Let P be the position of plane, A and B be the positions of two stones one kilometer apart. Angles of depression of stones A and B are α and β respectively. Let PC = h km.

In right-angled ΔACP, we have



$$\Rightarrow \quad \tan \alpha = \frac{PC}{AC} = \frac{h}{AC}$$

$$\Rightarrow \quad AC = \frac{h}{\tan \alpha} \qquad ...(i)$$

Now in right-angled ΔPCB, we have

$$\tan \beta = \frac{PC}{CB} = \frac{h}{CB}$$

$$\Rightarrow \quad CB = \frac{h}{\tan \beta} \qquad ...(ii)$$

Adding equations (i) and (ii), we get

$$AC + CB = \frac{h}{\tan \alpha} + \frac{h}{\tan \beta}$$

$$= h\left(\frac{\tan \beta + \tan \alpha}{\tan \alpha . \tan \beta}\right)$$

As it is given that

$$AC + CB = 1$$

$$\therefore \quad 1 = h\left(\frac{\tan \beta + \tan \alpha}{\tan \alpha . \tan \beta}\right)$$

$$\Rightarrow \quad h = \frac{\tan \alpha . \tan \beta}{\tan \alpha + \tan \beta}$$

13. (i) In right-angled triangle PRO

We have,

$$PR = \sqrt{PO^2 - OR^2}$$

$$= \sqrt{(26)^2 - (24)^2}$$

$$= \sqrt{676 - 576}$$

$$= \sqrt{100}$$

$$= 10 \text{ m.}$$

As PR and PQ are tangents from same external point so,

$$PR = PQ = 10 \text{ m.}$$

(ii) Let the distance between consecutive poles be x.

Hence, $5x = 10$

$\Rightarrow$ $x = 2$ m.

Each pole is at a distance of 2 m.

14. (i) We have,

$$\text{Radius of cylindrical tank, } r = \frac{2}{2} = 1 \text{ m}$$

$$\text{Height of cylindrical tank, } h = 3.5 \text{ m}$$

$$\text{Volume of cylindrical tank} = \pi r^2 h$$

$$= \frac{22}{7} \times (1)^2 \times 3.5$$

$$= 11 \text{ m}^3$$

Since, $1 \text{ m}^3 = 1000$ lit

$11 \text{ m}^3 = 11000$ litres.

11000 litres of water is collected in tank.

(ii) Let the rainfall be x m,

$\therefore$ Volume of water = Volume of cuboid

$= 22 \text{ m} \times 20 \text{ m} \times x \text{ m.}$

$= (22 \times 20 \times x) \text{ m}^3$

$= 440x \text{ m}^3$

Since, the tank is just full of water that drains out of the roof into the tank.

$\therefore$ Volume of water = Volume of the cylindrical vessel

$\Rightarrow$ $22 \times 20 \times x = 11$

$$x = \frac{11}{440}$$

$$= \frac{1}{40} \text{m}$$

$$= \frac{100}{40} \text{cm}$$

$$= 2.5 \text{ cm.}$$