UNIT TEST 3

Time Allowed : 1½ Hours **Max. Marks : 50**

Notes : *1. All questions are compulsory.*

2. Marks have been indicated against each question.

1. Differentiate $\sin^{-1}\left(x\sqrt{x}\right)$, $0 \le x \le 1$ w.r.t. x. **(1)**

2. Find the slope of the tangent to the curve :
$y = x^3 - 3x + 2$ at the point whose x-co-ordinate is 3. **(1)**

3. Prove that $\displaystyle\int_{-\pi/4}^{\pi/4} \sin^2 x\, dx = \frac{\pi}{4} - \frac{1}{2}$. **(2)**

4. Show that $(x^2 + xy)\, dy = (x^2 + y^2)\, dy$ is homogeneous. **(2)**

5. If $y = \tan^{-1}\left[\dfrac{\sqrt{1+x^2} - \sqrt{1-x^2}}{1+x^2+\sqrt{1-x}}\right]$, then prove that $\dfrac{dy}{dx}.\sqrt{1-x^4} = x$. **(2)**

6. A man 2 metres heigh walks at a uniform speed of 5 km/hr away from a lamp-post 6 metres high. Find the rate at which the length of his shadow increases. **(4)**

7. Find the equations of the normals to the curve $y = x^3 + 2x + 6$, which are parallel to the line $x + 14y + 4 = 0$. **(4)**

8. Evaluate : $\displaystyle\int x \tan^{-1} x\, dx$. **(4)**

9. Prove that $\displaystyle\int_0^{\pi} \frac{x \tan x}{\sec x \operatorname{cosec} x} dx = \frac{\pi^2}{4}$. **(4)**

10. Find the area of the smaller region bounded by the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ and the straight line $\dfrac{x}{3} + \dfrac{y}{2} = 1$. **(4)**

11. Solve $(1 + e^{x/y})\, dx + e^{x/y}\left(1 - \dfrac{x}{y}\right) dy = 0$. **(4)**

12. Prove that the semi-vertical angle of the right-circular cone of given volume and least covered surface area is $\cot^{-1}\sqrt{2}$. **(6)**

13. Evaluate $\displaystyle\int_1^4 f(x)\, dx$; when $f(x) = |x-1| + |x-2| + |x-3|$. **(6)**

14. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide that area of the square bounded by $x = 0$, $x = 4$ and $y = 0$ into three equal parts. **(6)**

Answers

1. $\dfrac{3}{2}\sqrt{\dfrac{x}{1-x^3}}$.

2. 24.

6. $\dfrac{5}{2}$ km/h.

7. $x + 14y - 254 = 0$, $x + 14y + 86 = 0$.

8. $(x^2+1)\tan^{-1}\dfrac{x}{2} - \dfrac{x}{2} + c$.

10. $\dfrac{3}{2}(\pi - 2)$ sq. units.

11. $y\, e^{x/y} + x = c$.

13. $\dfrac{19}{2}$.