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Electromagnetic Waves

IN THIS CHAPTER ....

In 1865, Maxwell predicted the existence of electromagnetic waves on the basis of his equations. He concluded that variation of electric and magnetic field vectors perpendicular to each other leads to the generation of electromagnetic disturbances in space. These disturbances possesses the properties of waves and can propagate in space without any medium. Thus, these waves are known as **electromagnetic waves**.

Displacement Current

It is a current which is produced due to the time rate of change of electric flux.

It is given by

$$i_D = \varepsilon_0 \frac{d\phi_E}{dt}$$

where, ε_0 is the absolute permittivity of free space and $d\phi_E/dt$ is rate of change of electric flux.

It is also known as **Maxwell's displacement current**.

So, total electric current through a parallel plate capacitor connected with a battery is given as $i = i_C + i_D = i_C + \varepsilon_0 \cdot \dfrac{d\phi_E}{dt}$

where, i_C is the **conduction current** which arises due to the flow of charges.

Important points related to displacement and conduction current are as follows

(i) The conduction and displacement current are individually discontinuous. But the currents together possess the property of continuity through any closed electric circuit.

(ii) Displacement current is precisely equal to the conduction current but they are present in different parts of the circuit.

(iii) The source of magnetic field is not just the conduction current but it is also due to the displacement current.

(iv) For a capacitor, inside the plates, $i_C = 0$ and $i = i_D$. But outside the plates, $i_D = 0$ and $i = i_C$.

__Example 1.__ The instantaneous displacement current of 1.0 A current in the space between the parallel plates of 1μF capacitor can be established by changing the potential difference of

(a) $10^5 Vs^{-1}$ (b) $10^6 Vs^{-1}$

(c) $10^{-6} Vs^{-1}$ (d) $10^7 Vs^{-1}$

Sol. (b) As, $i_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d}{dt}(EA) = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 A \frac{d}{dt}\left(\frac{V}{d}\right)$

or $$i_d = \frac{\varepsilon_0 A}{d} \times \frac{dV}{dt} = C\frac{dV}{dt}$$

or $$\frac{dV}{dt} = \frac{i_d}{C} = \frac{1.0}{10^{-6}} = 10^6 \text{ Vs}^{-1}$$

Thus, an instantaneous displacement current of 1.0 A can be set-up by changing the potential difference across the parallel plates of capacitor at the rate of 10^6 Vs^{-1}.

Maxwell's Equations

Maxwell gave the basic laws of electricity and magnetism in the form of four fundamental equations, which are known as Maxwell's equations.

These equations give complete description of all electromagnetic interactions.

These equations are as follows

(i) $\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$ (Gauss's law for electrostatics)

(ii) $\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$ (Gauss's law for magnetism)

(iii) $\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\phi_B}{dt}$

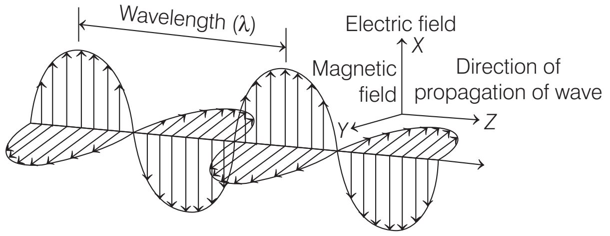
(Faraday's law of electromagnetic induction)

(iv) $\oint \mathbf{E} \cdot d\mathbf{l} = \mu_0\left(i_C + \varepsilon_0 \frac{d\phi_E}{dt}\right)$ (Ampere-Maxwell's law)

Electromagnetic Waves

Maxwell found that the **accelerated** or **oscillated** charges radiate electromagnetic waves. These charges produce an oscillating electric field in space, which produces an oscillating magnetic field, which in turn is a source of oscillating electric field and so on. Thus, oscillating electric and magnetic fields regenerate each other and electromagnetic waves propagates through the space.

Equation of electric field and magnetic field component of EM wave is given as



Here, $$E = E_x = E_0 \sin(\omega t - kZ)$$
$$B = B_y = B_0 \sin(\omega t - kZ)$$

where, propagation vector of wave vector, $k = \frac{2\pi}{\lambda}$ (λ is wavelength), angular frequency, $\omega = 2\pi\nu$ (ν is frequency), E_0 and B_0 are the amplitude of varying electric and magnetic field, respectively.

Characteristics of EM Waves

- These waves are self-sustaining oscillations of **E** and **B** in free space or vacuum.
- No material medium is required for their propagation.
- Electromagnetic waves could be diffracted, refracted and polarised.
- These waves are transverse in nature and travel in free space with the speed of light,

$$c \simeq 3 \times 10^8 \text{ m/s}$$

where, $$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{E_0}{B_0}$$

In isotropic medium, $$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}}$$

or $$v = \frac{1}{\sqrt{\mu_r \varepsilon_r}} \cdot \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{c}{n}$$

Here, $n = \sqrt{\mu_r \varepsilon_r}$ = refractive index of medium,

μ_r = relative permeability of medium

and ε_r = relative permittivity of medium.

- The average energy density (energy per unit volume) of electric field,

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 (E_0/\sqrt{2})^2 = \frac{1}{4}\varepsilon_0 E_0^2$$

The average energy density of magnetic field,

$$u_B = \frac{B^2}{2\mu_0} = \frac{(B_0/\sqrt{2})^2}{2\mu_0} = \frac{B_0^2}{4\mu_0}$$

$$\therefore \quad u_E = u_B \qquad \left(\because c = \frac{E_0}{B_0} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right)$$

Total average energy density,

$$u_{av} = u_E + u_B = 2u_E = 2u_B = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}\frac{B_0^2}{\mu_0}$$

The units of u_E and u_B are Jm^{-3}.

- Energy in electromagnetic waves is divided equally between electric and magnetic fields.
- The energy crossing per unit area per unit time, perpendicular to the direction of propagation of EM wave is called **intensity** of EM wave

i.e. Intensity, $$I = \frac{\text{Total EM energy}}{\text{Surface area} \times \text{Time}}$$
$$= u_{av} \times c = \frac{1}{2}\varepsilon_0 E_0^2 c = \frac{1}{2}\frac{B_0^2 c}{\mu_0} \frac{\text{W}}{\text{m}^2}$$

- If a portion of electromagnetic wave of energy U is propagating with speed c, then **linear momentum** of electromagnetic wave is given by

$$p = \frac{U}{c} \qquad \text{(complete absorption)}$$

If the wave is incident on a totally reflecting surface, then momentum delivered,

$$p = \frac{2U}{c}$$

- **Radiation pressure** exerted on the surface is defined as force per unit area (F/A). Thus,

$$p = \frac{F}{A} = \frac{1}{A}\left(\frac{dp}{dt}\right) = \frac{1}{A}\frac{d}{dt}\left(\frac{U}{c}\right) = \frac{1}{c}\frac{(dU/dt)}{A}$$

where, $\frac{dU/dt}{A}$ is the magnitude of the Poynting vector.

Thus, the radiation pressure p exerted on the perfectly absorbing surface, $p = \frac{S}{c}$.

If the surface is perfect reflector and incidence is normal, then $p = \frac{2S}{c}$.

- The rate of flow of energy crossing a unit area in an electromagnetic wave is described by the vector $\mathbf{S}$ called the **Poynting vector,** which is defined by the expression,

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$$

Its magnitude S is related to the rate at which energy is transported by a wave across a unit area at any instant.

or $$S = \frac{EB}{\mu_0}$$

The SI unit of poynting vector $\mathbf{S}$ is $Js^{-1}m^{-2}$ or Wm^{-2}.

Example 2. *If the magnetic field of a plane electromagnetic wave is given by*

$$B = 100 \times 10^{-6} \sin\left[2\pi \times 2 \times 10^{15}\left(t - \frac{x}{c}\right)\right],$$

then the maximum electric field associated with it is (Take, speed of light = 3×10^8 m/s) **[JEE Main 2019]**

(a) 6×10^4 N/C
(b) 4×10^4 N/C
(c) 3×10^4 N/C
(d) 4.5×10^4 N/C

Sol. (c) Given, instantaneous value of magnetic field,

$$B = 100 \times 10^{-6} \sin\left[2\pi \times 2 \times 10^{15}\left(t - \frac{x}{c}\right)\right]$$

and speed of light, $c = 3 \times 10^8$ ms^{-1}

For an electromagnetic wave, $E_{max} = B_{max} \times c$

where, E_{max} = maximum value of the electric field.

We get,

$$E_{max} = 100 \times 10^{-6} \times 3 \times 10^8 = 3 \times 10^4 \text{ N/C}$$

Example 3. *The electric field of a plane polarised electromagnetic wave in free space at time $t = 0$ is given by an expression*

$$\mathbf{E}(x, y) = 10\,\hat{\mathbf{j}}\cos(6x + 8z)$$

The magnetic field $\mathbf{B}(x, z, t)$ is given by (where, c is the velocity of light) **[JEE Main 2019]**

(a) $\frac{1}{c}(6\hat{\mathbf{k}} - 8\hat{\mathbf{i}})\cos[(6x + 8z + 10ct)]$
(b) $\frac{1}{c}(6\hat{\mathbf{k}} - 8\hat{\mathbf{i}})\cos[(6x + 8z - 10ct)]$
(c) $\frac{1}{c}(6\hat{\mathbf{k}} + 8\hat{\mathbf{i}})\cos[(6x - 8z + 10ct)]$
(d) $\frac{1}{c}(6\hat{\mathbf{k}} + 8\hat{\mathbf{i}})\cos[(6x + 8z - 10ct)]$

Sol. (b) Given, electric field is $E(x, y)$, *i.e.* electric field is in XY-plane which is given as

$$\mathbf{E} = 10\,\hat{\mathbf{j}}\cos(6x + 8z)$$

Since, the magnetic field given is $\mathbf{B}(x, z, t)$, this means $\mathbf{B}$ is in XZ-plane.

$\therefore$ Propagation of wave is in y-direction.

($\because$ for an electromagnetic wave, $\mathbf{E} \perp \mathbf{B} \perp$ propagation direction)

As, Poynting vector suggests that $\mathbf{E} \times \mathbf{B}$ is parallel to $(6\,\hat{\mathbf{i}} + 8\,\hat{\mathbf{k}})$.

Let $\hat{\mathbf{B}} = (x\,\hat{\mathbf{i}} + z\,\hat{\mathbf{k}})$, then

$$\hat{\mathbf{E}} \times \hat{\mathbf{B}} = \hat{\mathbf{j}} \times (x\,\hat{\mathbf{i}} + z\,\hat{\mathbf{k}}) = 6\,\hat{\mathbf{i}} + 8\,\hat{\mathbf{k}}$$

or $-x\,\hat{\mathbf{k}} + z\,\hat{\mathbf{i}} = 6\,\hat{\mathbf{i}} + 8\,\hat{\mathbf{k}}$ or $x = -8$ and $z = 6$

$$\therefore \quad \mathbf{B} = \frac{1}{c}(6\,\hat{\mathbf{k}} - 8\,\hat{\mathbf{i}})\cos(6x + 8z - 10ct) \qquad \left(\because \frac{|\mathbf{E}|}{|\mathbf{B}|} = c\right)$$

Example 4. *The electric fields of two plane electromagnetic plane waves in vacuum are given by*

$$\mathbf{E}_1 = E_0\,\hat{\mathbf{j}}\cos(\omega t - kx)$$

and

$$\mathbf{E}_2 = E_0\,\hat{\mathbf{k}}\cos(\omega t - ky)$$

At $t = 0$, a particle of charge q is at origin with a velocity $\mathbf{v} = 0.8c\,\hat{\mathbf{j}}$ (c is the speed of light in vacuum). The instantaneous force experienced by the particle is **[JEE Main 2020]**

(a) $E_0 q\,(0.8\,\hat{\mathbf{i}} - \hat{\mathbf{j}} + 0.4\,\hat{\mathbf{k}})$
(b) $E_0 q\,(0.4\,\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 0.8\hat{\mathbf{k}})$
(c) $E_0 q(-0.8\,\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$
(d) $E_0 q\,(0.8\,\hat{\mathbf{i}} + \hat{\mathbf{j}} + 0.2\,\hat{\mathbf{k}})$

Sol. (*d*) For first electromagnetic wave,

$$\mathbf{E}_1 = E_0\hat{\mathbf{j}}\cos(\omega t - kx)$$

So, magnetic field vector,

$$\mathbf{B}_1 = B_0\hat{\mathbf{k}}\cos(\omega t - kx) = \frac{E_0}{c}\hat{\mathbf{k}}\cos(\omega t - kx)$$

Also, for second electromagnetic wave,

$$\mathbf{E}_2 = E_0\hat{\mathbf{k}}\cos(\omega t - ky)$$

$$\Rightarrow \quad \mathbf{B}_2 = \frac{E_0}{c}\hat{\mathbf{i}}\cos(\omega t - ky)$$

Force on charged particle under influence of both electric and magnetic fields,

$$\mathbf{F} = q(\mathbf{v}\times\mathbf{B} + \mathbf{E}) \quad \text{...(i)}$$

Here, $\mathbf{v} = 0.8\,c\,\hat{\mathbf{j}}$

$$\because \quad \mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = \frac{\mathbf{E}_0}{c}\hat{\mathbf{k}}\cos(\omega t - kx) + \frac{E_0}{c}\hat{\mathbf{i}}\cos(\omega t - ky)$$

and at $t = 0$, at origin ($x = 0$, $y = 0$ and $z = 0$), we have

$$\mathbf{B} = \frac{E_0}{c}(\hat{\mathbf{i}} + \hat{\mathbf{k}})$$

and $\quad \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = E_0\hat{\mathbf{j}}\cos(\omega t - kx) + E_0\hat{\mathbf{k}}\cos(\omega t - ky)$

At $t = 0$ and at origin, $\mathbf{E} = E_0(\hat{\mathbf{j}} + \hat{\mathbf{k}})$

So, from Eq. (i), we get

$$\mathbf{F} = q\left[0.8\,c\,\hat{\mathbf{j}} \times \frac{E_0}{c}(\hat{\mathbf{i}} + \hat{\mathbf{k}}) + E_0(\hat{\mathbf{j}} + \hat{\mathbf{k}})\right]$$

$$= E_0 q\,[0.8\,\hat{\mathbf{j}} \times (\hat{\mathbf{i}} + \hat{\mathbf{k}}) + (\hat{\mathbf{j}} + \hat{\mathbf{k}})]$$

$$= E_0 q\,[0.8(-\hat{\mathbf{k}} + \hat{\mathbf{i}}) + (\hat{\mathbf{j}} + \hat{\mathbf{k}})]$$

$$= E_0 q\,(0.8\hat{\mathbf{i}} + \hat{\mathbf{j}} + 0.2\hat{\mathbf{k}})$$

Example 5. *A plane electromagnetic wave has frequency of 2.0×10^{10} Hz and its energy density is $1.02 \times 10^{-8}\ J/m^3$ in vacuum. The amplitude of the magnetic field of the wave is close to (Take, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \frac{N\text{-}m^2}{C^2}$ and speed of light $= 3 \times 10^8\ ms^{-1}$)* **[JEE Main 2020]**

(a) 190 nT (b) 160 nT

(c) 180 nT (d) 150 nT

Sol. (*b*) Energy density of magnetic field of an electromagnetic wave is given by

$$u_B = \frac{1}{2}\left(\frac{B_0^2}{\mu_0}\right)$$

Here, $\mu_0 = 4\pi \times 10^{-7}$ units and $u_B = 1.02 \times 10^{-8}\ \text{Jm}^{-3}$

So, amplitude of magnetic field,

$$B_0 = \sqrt{2\mu_0 u_B}$$

$$= \sqrt{2 \times 4\pi \times 10^{-7} \times 1.02 \times 10^{-8}}$$

$$= 16 \times 10^{-8}\ \text{T}$$

$$= 160\ \text{nT}$$

Example 6. *The mean intensity of radiation on the surface of the sun is about $10^8\ W/m^2$. The rms value of the corresponding magnetic field is closest to* **[JEE Main 2019]**

(a) 1 T (b) $10^2\ T$

(c) $10^{-4}\ T$ (d) $10^{-2}\ T$

Sol. (*c*) Mean radiation intensity is

$$I = \varepsilon_0 c E_{\text{rms}}^2 = \varepsilon_0 c (cB_{\text{rms}})^2 \qquad \left(\because \frac{E_{\text{rms}}}{B_{\text{rms}}} = c\right)$$

$$= \varepsilon_0 c^3 B_{\text{rms}}^2$$

$$\Rightarrow \quad B_{\text{rms}} = \sqrt{\frac{I}{\varepsilon_0 c^3}}$$

Substituting the given values, we get

$$= \sqrt{\left(\frac{10^8}{8.85 \times 10^{-12} \times (3 \times 10^8)^3}\right)}$$

$$= \sqrt{\left(\frac{10^8}{8.85 \times 27 \times 10^{12}}\right)}$$

$$\approx \sqrt{(10^{-8})} \approx 10^{-4}\ \text{T}$$

Example 7. *If the earth receives 2 cal min^{-1} cm^{-2} solar energy, then the amplitudes of electric field of radiation is (V/m)*

(a) 102.3 (b) 377

(c) 150 (d) 200

Sol. (a) From Poynting vector, we know that,

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} = EH\sin 90^\circ = EH$$

Given, $\quad S = 2\ \text{cal min}^{-1}\ \text{cm}^{-2}$

$$= \frac{2 \times 4.18 \times 10^4}{60}\ \text{Jm}^{-2}\ \text{s}^{-1}$$

As S represents energy flux per unit area per second, we have

$$EH = \frac{2 \times 4.18 \times 10^4}{60} = 1400$$

$$EH = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377$$

$$\Rightarrow \quad EH \times \frac{E}{H} = 1400 \times 377$$

$$\therefore \quad E = \sqrt{1400 \times 377} = 726.5\ \text{V/m}$$

Amplitude of electric field,

$$E_0 = E\sqrt{2} = 102.3\ \text{V/m}$$

Electromagnetic Spectrum

It is the orderly arrangement of the electromagnetic waves in increasing or decreasing wavelength λ or frequency ν. These waves includes gamma rays, X-rays, ultraviolet rays, visible light, infrared waves, microwaves and radio waves.

Tabular form of various electromagnetic waves of electromagnetic spectrum with their features and uses are as follows

Name of Wave	Frequency Range	Wavelength Range	Production	Detection	Uses
Radio waves	500 kHz to 1000 MHz	> 0.1 m	Rapid acceleration and decelerations of electrons in aerials.	Receiver's aerials	• These are used in AM (Amplitude Modulation) from 530 kHz to 1710 kHz and ground wave propagation. • These are used in TV waves ranging from 54 MHz to 890 MHz. • These are used in FM (Frequency Modulation) ranging from 88 MHz to 08 MHz.
Microwaves	1 GHz to 300 GHz	0.1 m to 1 mm	Klystron valve or magnetron valve.	Point contact diodes	• These are used in RADAR systems for aircraft navigation. • These are used in microwave oven for cooking purpose. • These are used in study of atomic and molecular structures.
Infrared waves (Heat waves)	3×10^{11} Hz to 4×10^{14} Hz	1 mm to 700 nm	Vibration of atoms and molecules	Thermopiles bolometer and infrared photographic film.	• These are used in physical therapy. • These are used in satellites for army purpose. • These are used in weather forecasting.
Light rays (Visible rays)	4×10^{14} Hz to 7×10^{14} Hz	700 nm to 400 nm	Electrons in atoms emit light, when they move from a higher energy level to a lower energy level.	Eye, photocells and photographic film.	• These are used by the optical organs of humans and animals for three primary purposes given below (i) To see things, avoid bumping into them and escape danger. (ii) To look for food. (iii) To find other living things with which to copulate, so as to prolong the species.
Ultraviolet rays	10^{14} Hz to 10^{16} Hz	400 nm to 1 nm	Inner shell electrons in atoms moving from higher energy level to a lower energy level.	Photocells and photographic film.	• These are used in burglar alarm. • These are used in checking mineral sample. • These are used to study molecular structure. • To kill germs in water purifiers. • Used in LASER eye surgery.
X-rays	3×10^{16} Hz to 3×10^{21} Hz	1 nm to 10^{-3} nm	X-ray tubes or inner shell electrons, bombarding metals by high energy electrons.	Photographic film, Geiger tubes and ionisation chamber.	• These are used in medicine to detect the fracture, diseased organs, stones in the body, etc. • These are used in engineering to detect fault, cracks in bridges and testing of welds. • These are used at metro stations to detect metals or explosive material.
Gamma (γ) rays	3×10^{18} Hz to 5×10^{22} Hz	$< 10^{-3}$ nm	Radioactive decay of the nucleus.	Photographic film and ionisation chamber.	• These are used to produce nuclear reactions. • These are used in radio therapy for the treatment of tumour and cancer. • These are used in food industry to kill pathogenic micro-organism.

Practice Exercise

ROUND I Topically Divided Problems

Displacement Current and Maxwell's Equations

1. The space between plates of parallel capacitor is filled with poorly conducting material. If an AC source is connected across the capacitor, current through material is
(a) only conduction current
(b) only displacement current
(c) displacement current as well as conduction current
(d) None of the above

2. The charge of a parallel plate capacitor is varying as $q = q_0 \sin 2\pi ft$. The plates are very large and close together (area = A, separation = d). Neglecting edge effects, the displacement current through the capacitor is
(a) $\frac{d}{A\varepsilon_0}$ (b) $\frac{d}{\varepsilon_0}\sin 2\pi ft$
(c) $2\pi f q_0 \cos 2\pi ft$ (d) $\frac{2\pi f q_0}{\varepsilon_0}\cos 2\pi ft$

3. A capacitor having a capacity of 2pF. Electric field across the capacitor is changing with a value of 10^{12} Vs^{-1}. The displacement current is
(a) 2 A (b) 4 A (c) 6 A (d) 10 A

4. A large parallel plate capacitor, whose plates have an area of $1\,\text{m}^2$ and are separated from each other by 1 mm, is being charged at a rate of $25\ \text{Vs}^{-1}$. If the dielectric between the plates has the dielectric constant 10, then the displacement current at this instant is
(a) 25 μA (b) 11 μA (c) 2.2 μA (d) 1.1 μA

5. A parallel plate capacitor consists of two circular plates with radius $R = 10$ cm separated by distance $d = 0.5$ mm. The capacitor is being charged at a uniform rate by applying a changing potential difference between the two plates. Calculate the displacement current for the capacitor. Assume that, the electric field is due to the displacement current only and rate at which the electric field between the plates changes is $5 \times 10^{13}\ \text{Vms}^{-1}$.
(a) 12.8 A (b) 12.6 A
(c) 13.9 A (d) 10.5 A

6. A parallel plate capacitor consists of two circular plates each of radius 2 cm, separated by a distance of 0.1 mm. If the potential difference across the plates is varying at the rate of $5 \times 10^6\ \text{Vs}^{-1}$, then the value of displacement current is
(a) 5.56 A (b) 5.56 mA
(c) 0.556 mA (d) 2.28 mA

7. Match the Column I with Column II and mark the correct option from the codes given below.

	Column I		Column II
A.	$\oint B\cdot dl = \mu_0 I$	1.	Gauss's law
B.	$\oint B\cdot dl = \mu_0\left(I + \epsilon_0 \frac{d\phi_E}{dt}\right)$	2.	Faraday's laws of electromagnetic induction
C.	$\oint E\cdot dl = -\frac{d}{dt}\oint_s B\cdot ds$	3.	Ampere-Maxwell's law
D.	$\oint E\cdot ds = \frac{1}{\varepsilon_0}\oint_s \rho\, ds$	4.	Ampere's circuital law

Codes

	A	B	C	D
(a)	1	2	3	4
(b)	4	3	2	1
(c)	4	2	1	3
(d)	3	1	2	4

Electromagnetic Waves and its Characteristics

8. Select the correct statement from the following. **[JEE Main 2013]**
(a) Electromagnetic waves cannot travel in vacuum.
(b) Electromagnetic waves are longitudinal waves.
(c) Electromagnetic waves are produced by charges moving with uniform velocity.
(d) Electromagnetic waves carry both energy and momentum as they propagate through space.

9. Which of the following is/are the property/properties of a monochromatic electromagnetic wave propagating in the free space?
I. Electric and magnetic fields will have a phase difference $\frac{\pi}{2}$.
II. The energy of the wave is distributed equally between electric and magnetic fields.
III. The pressure exerted by the wave is the product of its speed and energy density.

IV. The speed of the wave is equal to the ratio of the magnetic field to the electric field.

(a) I and III (b) Only II

(c) II and III (d) Only IV

10. If an electromagnetic wave is propagating in a medium with permittivity ε and permeabillity μ, then $\sqrt{\frac{\mu}{\varepsilon}}$ is the

(a) intrinsic impedance of the medium

(b) square of the refractive index of the medium

(c) refractive index of the medium

(d) energy density of the medium

11. A radio wave of frequency 90 MHz enters a ferrite rod. If $\varepsilon_r = 10^3$ and $\mu_r = 10$, then the velocity and wavelength of the wave in ferrite are

(a) $3\times10^8 \text{ms}^{-1}; 3.33\times10^{-2}\text{m}$

(b) $3\times10^6 \text{ms}^{-1}; 3.33\times10^{-2}\text{m}$

(c) $3\times10^8 \text{ms}^{-1}; 3.33\times10^{-1}\text{m}$

(d) $3\times10^7 \text{ms}^{-1}; 3.33\times10^{-3}\text{m}$

12. An electromagnetic wave of frequency $\nu = 3.0$ MHz passes from vacuum into a dielectric medium with permittivity $\varepsilon = 4.0$, then

(a) wavelength is doubled and the frequency remains unchanged

(b) wavelength is doubled and frequency becomes half

(c) wavelength is halved and frequency remains unchanged

(d) wavelength and frequency both remains unchanged

13. The amplitude of an electromagnetic wave in vacuum is doubled with no other changes made to the wave. As a result of this doubling of the amplitude, which of the following statement is true?

(a) The speed of wave propagation changes only.

(b) The frequency of the wave changes only.

(c) The wavelength of the wave changes only.

(d) None of the above

14. A plane electromagnetic wave is incident on a material surface. If the wave delivers momentum p and energy E, then

(a) $p=0, E=0$ (b) $p\neq0, E\neq0$

(c) $p\neq0, E=0$ (d) $p=0, E\neq0$

15. An electromagnetic wave going through vacuum is described by $E = E_0 \sin(kx-\omega t)$; $B = B_0 \sin(kx-\omega t)$. Which of the following equation is true?

(a) $E_0 k = B_0 \omega$ (b) $E_0 \omega = B_0 k$

(c) $E_0 B_0 = \omega k$ (d) None of these

16. An electromagnetic wave propagating along North has its electric field vector upwards. Its magnetic field vector point towards

(a) North (b) East (c) West (d) Downwards

17. In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2×10^{10} Hz and its amplitude is 54 V. Then, which of the following statement is correct?

(a) The amplitude of oscillating magnetic field will be 18×10^{-8} Wbm^{-2}.

(b) The amplitude of oscillating magnetic field will be 18×10^{-7} Wbm^{-2}.

(c) The wavelength of electromagnetic wave is 1.5 m.

(d) Both (a) and (c)

18. A plane electromagnetic wave propagating along y-direction can have the following pair of electric field (E) and magnetic field (B) components

[JEE Main 2021]

(a) E_y, B_y or E_z, B_z (b) E_y, B_x or E_x, B_y

(c) E_x, B_z or E_z, B_x (d) E_x, B_y or E_y, B_x

19. A plane electromagnetic wave of frequency 25 GHz is propagating in vacuum along the z-direction. At a particular point in space and time, the magnetic field is given by $\mathbf{B} = 5\times10^{-8}\,\hat{\mathbf{j}}$ T. The corresponding electric field $\mathbf{E}$ is (speed of light, $c = 3\times10^8$ ms^{-1})

[JEE Main 2020]

(a) $-1.66\times10^{-16}\hat{\mathbf{i}}$ V/m (b) $1.66\times10^{-16}\hat{\mathbf{i}}$ V/m

(c) $-15\,\hat{\mathbf{i}}$ V/m (d) $15\,\hat{\mathbf{i}}$ V/m

20. An electromagnetic wave in vacuum has the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$, which are mutually perpendicular to each other. The direction of polarisation is given by $\mathbf{X}$ and that of wave propagation by $\hat{\mathbf{k}}$, then

(a) $\mathbf{X} \parallel \mathbf{B}$ and $\hat{\mathbf{k}} \parallel \mathbf{B}\times\mathbf{E}$

(b) $\mathbf{X} \parallel \mathbf{E}$ and $\hat{\mathbf{k}} \parallel \mathbf{E}\times\mathbf{B}$

(c) $\mathbf{X} \parallel \mathbf{B}$ and $\hat{\mathbf{k}} \parallel \mathbf{E}\times\mathbf{B}$

(d) $\mathbf{X} \parallel \mathbf{E}$ and $\hat{\mathbf{k}} \parallel \mathbf{B}\times\mathbf{E}$

21. In a plane electromagnetic wave, the directions of electric field and magnetic field are represented by $\hat{\mathbf{k}}$ and $2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$, respectively. What is the unit vector along direction of propagation of the wave?

[JEE Main 2020]

(a) $\frac{1}{\sqrt{2}}(\hat{\mathbf{i}}+\hat{\mathbf{j}})$ (b) $\frac{1}{\sqrt{2}}(\hat{\mathbf{j}}+\hat{\mathbf{k}})$

(c) $\frac{1}{\sqrt{5}}(\hat{\mathbf{i}}+2\hat{\mathbf{j}})$ (d) $\frac{1}{\sqrt{5}}(2\hat{\mathbf{i}}+\hat{\mathbf{j}})$

22. The electromagnetic wave travelling along Z-axis is given as $\mathbf{E} = E_0 \cos(kz-\omega t)$. Choose the incorrect options from the following.

(a) The associated magnetic field is given as $\mathbf{B} = \frac{1}{c}\hat{\mathbf{k}}\times\mathbf{E}$

(b) The electromagnetic field can be written in terms of the associated magnetic field as $\mathbf{E} = c(\mathbf{B}\times\hat{\mathbf{k}})$

(c) $\hat{\mathbf{k}}\cdot\mathbf{E}=0, \hat{\mathbf{k}}\cdot\mathbf{B}=0$

(d) $\hat{\mathbf{k}}\times\mathbf{E}=0, \hat{\mathbf{k}}\times\mathbf{B}=0$

23. The electric field of a plane electromagnetic wave propagating along the x-direction in vacuum is $\mathbf{E} = E_0 \hat{\mathbf{j}} \cos(\omega t - kx)$. The magnetic field $\mathbf{B}$, at the moment $t = 0$ is **[JEE Main 2020]**

(a) $\mathbf{B} = E_0 \sqrt{\mu_0 \varepsilon_0} \cos(kx)\, \hat{\mathbf{j}}$

(b) $\mathbf{B} = \dfrac{E_0}{\sqrt{\mu_0 \varepsilon_0}} \cos(kx) \hat{\mathbf{j}}$

(c) $\mathbf{B} = \dfrac{E_0}{\sqrt{\mu_0 \varepsilon_0}} \cos(kx) \hat{\mathbf{k}}$

(d) $\mathbf{B} = E_0 \sqrt{\mu_0 \varepsilon_0} \cos(kx) \hat{\mathbf{k}}$

24. A plane electromagnetic waves travelling along the x-direction has a wavelength of 3 mm. The variation in the electric field occurs in the y-direction with an amplitude 66 Vm^{-1}. The equation for the electric and magnetic fields as a function of x and t are respectively

(a) $E_y = 33 \cos \pi \times 10^{11} \left(t - \dfrac{x}{c} \right)$ and

$B_z = 1.1 \times 10^{-7} \cos \pi \times 10^{11} \left(t - \dfrac{x}{c} \right)$

(b) $E_y = 11 \cos 2\pi \times 10^{11} \left(t - \dfrac{x}{c} \right)$ and

$B_y = 11 \times 10^{-7} \cos 2\pi \times 10^{11} \left(t - \dfrac{x}{c} \right)$

(c) $E_x = 33 \cos \pi \times 10^{11} \left(t - \dfrac{x}{c} \right)$ and

$B_y = 11 \times 10^{-7} \cos \pi \times 10^{11} \left(t - \dfrac{x}{c} \right)$

(d) $E_y = 66 \cos 2\pi \times 10^{11} \left(t - \dfrac{x}{c} \right)$ and

$B_z = 2.2 \times 10^{-7} \cos 2\pi \times 10^{11} \left(t - \dfrac{x}{c} \right)$

25. The magnetic field of a plane electromagnetic wave is $\mathbf{B} = 3 \times 10^{-8} \sin [200\pi(y + ct)] \hat{\mathbf{i}}$ T, where $c = 3 \times 10^8 \ \text{ms}^{-1}$ is the speed of light. The corresponding electric field is **[JEE Main 2020]**

(a) $\mathbf{E} = 9 \sin [200\pi(y + ct)]\, \hat{\mathbf{k}}$ V/m

(b) $\mathbf{E} = -10^{-6} \sin [200\pi(y + ct)]\, \hat{\mathbf{k}}$ V/m

(c) $\mathbf{E} = 3 \times 10^{-8} \sin [200\pi(y + ct)]\, \hat{\mathbf{k}}$ V/m

(d) $\mathbf{E} = -9 \sin [200\pi(y + ct)]\, \hat{\mathbf{k}}$ V/m

26. A linearly polarised electromagnetic wave given as $\mathbf{E}_r = -E_0 \hat{\mathbf{i}} \cos(kz - \omega t)$ is incident normally on a perfectly reflecting infinite wall at $z = a$. Assuming that the material of the wall is optically inactive, the reflected wave will be given as **[NCERT Exemplar]**

(a) $\mathbf{E}_r = -E_0 \hat{\mathbf{i}} \cos(kz + \omega t)$

(b) $\mathbf{E}_r = E_0 \hat{\mathbf{i}} \cos(kz + \omega t)$

(c) $\mathbf{E}_r = -E_0 \hat{\mathbf{i}} \cos(kz - \omega t)$

(d) $\mathbf{E}_r = -E_0 \hat{\mathbf{i}} \sin(kz - \omega t)$

27. For a plane electromagnetic wave, the magnetic field at a point x and time t is
$\mathbf{B}(x, t) = [1.2 \times 10^{-7} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \hat{\mathbf{k}}]$ T
The instantaneous electric field $\mathbf{E}$ corresponding to $\mathbf{B}$ is
(Given, speed of light, $c = 3 \times 10^8$ m/s) **[JEE Main 2020]**

(a) $\mathbf{E}(x, t) = [-36 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \hat{\mathbf{j}}]$ V/m

(b) $\mathbf{E}(x, t) = [36 \sin(1 \times 10^3 x + 0.5 \times 10^{11} t) \hat{\mathbf{j}}]$ V/m

(c) $\mathbf{E}(x, t) = [36 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \hat{\mathbf{k}}]$ V/m

(d) $\mathbf{E}(x, t) = [36 \sin(1 \times 10^3 x + 1.5 \times 10^{11} t) \hat{\mathbf{j}}]$ V/m

28. A plane electromagnetic wave is propagating along the direction $\dfrac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}$ with its polarisation along the direction $\hat{\mathbf{k}}$. The correct form of the magnetic field of the wave would be (here B_0 is an appropriate constant) **[JEE Main 2020]**

(a) $B_0 \hat{\mathbf{k}} \cos\left(\omega t - k \dfrac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}} \right)$

(b) $B_0 \dfrac{\hat{\mathbf{j}} - \hat{\mathbf{i}}}{\sqrt{2}} \cos\left(\omega t + k \dfrac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}} \right)$

(c) $B_0 \dfrac{\hat{\mathbf{i}} - \hat{\mathbf{j}}}{\sqrt{2}} \cos\left(\omega t - k \dfrac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}} \right)$

(d) $B_0 \dfrac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}} \cos\left(\omega t - k \dfrac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}} \right)$

29. An electromagnetic wave is represented by the electric field $\mathbf{E} = E_0 \, \hat{\mathbf{n}} \sin[\omega t + (6y - 8z)]$.
Taking unit vectors in x, y and z-directions to be $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$, the direction of propagation $\hat{\mathbf{s}}$, is **[JEE Main 2019]**

(a) $\hat{\mathbf{s}} = \dfrac{3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}}{5}$

(b) $\hat{\mathbf{s}} = \dfrac{-4\hat{\mathbf{k}} + 3\hat{\mathbf{j}}}{5}$

(c) $\hat{\mathbf{s}} = \left(\dfrac{-3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}}{5} \right)$

(d) $\hat{\mathbf{s}} = \dfrac{4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}}{5}$

30. A plane electromagnetic wave having a frequency $\nu = 23.9$ GHz propagates along the positive z-direction in free space. The peak value of the electric field is 60 V/m. Which among the following is the acceptable magnetic field component in the electromagnetic wave? **[JEE Main 2019]**

(a) $\mathbf{B} = 2 \times 10^7 \sin(0.5 \times 10^3 z + 1.5 \times 10^{11} t)\, \hat{\mathbf{i}}$

(b) $\mathbf{B} = 2 \times 10^{-7} \sin(0.5 \times 10^3 z - 1.5 \times 10^{11} t)\, \hat{\mathbf{i}}$

(c) $\mathbf{B} = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \hat{\mathbf{k}}$

(d) $\mathbf{B} = 2 \times 10^{-7} \sin(1.5 \times 10^2 x + 0.5 \times 10^{11} t)\, \hat{\mathbf{j}}$

31. The magnetic field of a plane electromagnetic wave is given by $\mathbf{B} = B_0[\cos(kz - \omega t)]\hat{\mathbf{i}} + B_1\cos(kz + \omega t)\hat{\mathbf{j}}$, where $B_0 = 3 \times 10^{-5}$ T and $B_1 = 2 \times 10^{-6}$ T. The rms value of the force experienced by a stationary charge $Q = 10^{-4}$ C at $z = 0$ is closest to **[JEE Main 2019]**

(a) 0.1 N (b) 3×10^{-2} N
(c) 0.6 N (d) 0.9 N

32. In free space, the energy of electromagnetic wave in electric field is U_E and in magnetic field is U_B. Then, **[JEE Main 2019]**

(a) $U_E = U_B$ (b) $U_E > U_B$
(c) $U_E < U_B$ (d) $U_E = \dfrac{U_B}{2}$

33. Assume that, a lamp radiates power P uniformly in all directions. What is the magnitude of electric field strength at a distance r from the lamp?

(a) $\dfrac{P}{\pi c \varepsilon_0 r^2}$ (b) $\dfrac{P}{2\pi c \varepsilon r^2}$
(c) $\sqrt{\dfrac{P}{2\pi \varepsilon_0 r^2 c}}$ (d) $\sqrt{\dfrac{P}{\pi \varepsilon_0 c r^2}}$

34. The amplitude of the electric field in a parallel beam of plane electromagnetic waves of intensity 53.1 Wm^{-2} is (Permittivity of free space $= 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$)

(a) 400 NC^{-1} (b) 50 NC^{-1}
(c) 100 NC^{-1} (d) 200 NC^{-1}

35. An earth orbiting statellite has solar energy collecting panel with total area $5\,m^2$. If solar radiations are perpendicular and completely absorbed, the average force associated with the radiation pressure is
(Solar constant = 1.4 kWm^{-2})

(a) 2.33×10^{-3} N
(b) 2.33×10^{-4} N
(c) 2.33×10^{-5} N
(d) 2.33×10^{-6} N

36. An electromagnetic wave with Poynting vector 6 Wm^{-2} is absorbed by a surface area $12\,m^2$. The force on the surface is

(a) 24×10^{-8} N
(b) 30×10^{-5} N
(c) 2.4×10^{-12} N
(d) 24×10^{8} N

37. A 27 mW laser beam has a cross-sectional area of 10 mm^2. The magnitude of the maximum electric field in this electromagnetic wave is given by (Take, permittivity of space, $\varepsilon_0 = 9 \times 10^{-12}$ SI units and speed of light, $c = 3 \times 10^8$ m / s) **[JEE Main 2019]**

(a) 1 kV/m (b) 0.7 kV/m
(c) 2 kV/m (d) 1.4 kV/m

38. The electric field intensity produced by the radiations coming from 100 W bulb at a 3 m distance is E. The electric field intensity produced by the radiations coming from 50 W bulb at the same distance is **[NCERT Exemplar]**

(a) $E/2$ (b) $2E$
(c) $E\sqrt{2}$ (d) $\sqrt{2}E$

39. A plane electromagnetic wave of wave intensity $6\,Wm^{-2}$ strikes a small mirror of area $30\,cm^2$, held perpendicular to the approaching wave. The momentum transferred (in $kg\text{-}ms^{-1}$) by the wave to the mirror each second will be

(a) 1.2×10^{-10} (b) 2.4×10^{-9}
(c) 3.6×10^{-8} (d) 4.8×10^{-7}

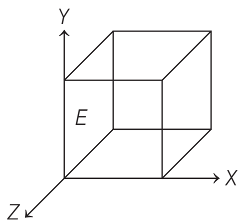
40. In a region of free space, the electric field at some instant of time is $\mathbf{E} = (80\hat{\mathbf{i}} + 32\hat{\mathbf{j}} - 64\hat{\mathbf{k}})$ V/m and the magnetic field is $\mathbf{B} = (0.2\,\hat{\mathbf{i}} + 0.08\,\hat{\mathbf{j}} + 0.29\,\hat{\mathbf{k}})\,\mu$T. The poynting vector for these fields is

(a) $-11.45\,\hat{\mathbf{i}} + 28.6\,\hat{\mathbf{j}}$
(b) $-28.6\,\hat{\mathbf{i}} + 11.45\,\hat{\mathbf{i}}$
(c) $28.6\,\hat{\mathbf{i}} - 11.45\,\hat{\mathbf{j}}$
(d) $11.45\,\hat{\mathbf{i}} - 28.6\,\hat{\mathbf{j}}$

41. A lamp emits monochromatic green light uniformly in all directions. The lamp is 3% efficient in converting electric power to electromagnetic waves and consume 100 W of power. The amplitude of the electric field associated with the electromagnetic radiation at a distance of 5 m from the lamp will be

(a) 2.68 Vm^{-1} (b) 3.15 Vm^{-1}
(c) 2.01 Vm^{-1} (d) 0

42. A cube of edge a has its edges parallel to X, Y and Z-axis of rectangular coordinate system. A uniform electric field $\mathbf{E}$ is parallel to Y-axis and a uniform magnetic field is $\mathbf{B}$ parallel to X-axis. The rate at which energy flows through each face of the cube is



(a) $\dfrac{a^2EB}{2\mu_0}$ parallel to XY-plane and zero in others
(b) $\dfrac{a^2EB}{\mu_0}$ parallel to XY-plane and zero in others
(c) $\dfrac{a^2EB}{2\mu_0}$ from all faces
(d) $\dfrac{a^2EB}{2\mu_0}$ parallel to yz faces and zero in others

Electromagnetic Spectrum

43. Radio waves diffract around building although light waves do not. The reason is that radio waves
(a) travel with speed target than c
(b) have much larger wavelength than light
(c) carry news
(d) are not electromagnetic waves

44. An electromagnetic wave is generated due to oscillation of a charged particle with a frequency 10^9 Hz.
(a) The wavelength of electromagnetic wave in vacuum is 60 cm
(b) This wave is radio wave
(c) This wave is microwave
(d) This wave is light wave

45. This question has Statement I and Statement II. Of the four choices given after the Statement, choose the one that best describes the two Statements. **[JEE Main 2015]**

I. Out of radio waves and microwaves, the radio waves undergo more diffraction.
II. Radio waves have greater frequency compared to microwaves.
(a) I is true, II is true and II is the correct explanation of I.
(b) I is false, II is true.
(c) I is true, II is false.
(d) I is true, II is true but II is not the correct explanation of I.

46. Choose the correct option relating wavelengths of different parts of electromagnetic wave spectrum. **[JEE Main 2020]**
(a) $\lambda_{\text{X-rays}} < \lambda_{\text{microwaves}} < \lambda_{\text{radio waves}} < \lambda_{\text{visible}}$
(b) $\lambda_{\text{visible}} > \lambda_{\text{X-rays}} > \lambda_{\text{radio waves}} > \lambda_{\text{microwaves}}$
(c) $\lambda_{\text{radio waves}} > \lambda_{\text{microwaves}} > \lambda_{\text{visible}} > \lambda_{\text{X-rays}}$
(d) $\lambda_{\text{visible}} < \lambda_{\text{microwaves}} < \lambda_{\text{radio waves}} < \lambda_{\text{X-rays}}$

47. Match Table I (Electromagnetic wave type) with Table II (Its association/application) and select the correct option from the codes given below the tables. **[JEE Main 2014]**

	Table I		Table II
A.	Infrared waves	1.	To treat muscular strain
B.	Radio waves	2.	For broadcasting
C.	X-rays	3.	To detect fracture of bones
D.	Ultraviolet rays	4.	Absorbed by the ozone layer of the atmosphere

Codes

	A	B	C	D		A	B	C	D
(a)	4	3	2	1	(b)	1	2	4	3
(c)	3	2	1	4	(d)	1	2	3	4

48. The correct sequence of the increasing wavelength of the given radiation sources is
(a) radioactive sources, X-ray tube, crystal oscillator, sodium vapour lamp
(b) radioactive source, X-ray tube, sodium vapour lamp, crystal oscillator
(c) X-ray tube, radioactive source, crystal oscillator, sodium vapour lamp
(d) X-ray tube, crystal oscillator, radioactive source, sodium vapour lamp

49. The correct match between the entries in table are : **[JEE Main 2020]**

	Table I (Radiation)		Table II (Wavelength)
A.	Microwaves	1.	100 m
B.	γ-rays	2.	10^{-15} m
C.	AM radio waves	3.	10^{-10} m
D.	X-rays	4.	10^{-3} m

Codes

	A	B	C	D		A	B	C	D
(a)	2	1	4	3	(b)	1	3	4	2
(c)	3	2	1	4	(d)	4	2	1	3

ROUND II Mixed Bag

Only One Correct Option

1. An EM wave radiates outwards from a dipole antenna with E_0 as the amplitude of its electric field vector. The electric field E_0 which transports significant energy from the source falls off as
(a) $\frac{1}{r^3}$ (b) $\frac{1}{r^2}$
(c) $1/r$ (d) remains constant

2. A sinusoidal voltage is applied directly across $8\,\mu$F capacitor. The frequency of the source is 3.00 kHz and the voltage amplitude is 30.0 V. Find the maximum value of displacement current between the plates of the capacitor.
(a) 42.5 A (b) $4.25\,\mu$A (c) 4.52 A (d) $4.52\,\mu$A

3. An EM wave of intensity I falls on a surface kept in vacuum and exerts radiation pressure p on it . Which of the following is false?
(a) Radiation pressure is I/c, if the wave is totally absorbed
(b) Radiation pressure is I/c, if the wave is totally reflected
(c) Radiation pressure is $2I/c$, if the wave is totally reflected
(d) Radiation pressure is in the range $I/c < p < 2I/c$ for the real surfaces

4. An EM wave from air enters a medium. The electric fields are $\mathbf{E}_1 = E_{01}\,\hat{\mathbf{x}} \cos\left[2\pi\nu\left(\frac{z}{c} - t\right)\right]$ in air and $\mathbf{E}_2 = E_{02}\,\hat{\mathbf{x}} \cos\left[k(2z - ct)\right]$ in medium, where the wave number k and frequency ν refer to their values in air. The medium is non-magnetic.

If ε_{r_1} and ε_{r_2} refer to relative permittivities of air and medium respectively, which of the following options is correct? **[JEE Main 2018]**

(a) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 4$ (b) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 2$ (c) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$ (d) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{2}$

5. An electromagnetic wave travels in vacuum along z-direction: $E = (E_1\hat{\mathrm{i}} + E_2\hat{\mathrm{j}})\cos(kz - \omega t)$. Choose the correct options from the following.

(a) The associated magnetic field is given as $\mathbf{B} = \frac{1}{c}(E_1\hat{\mathbf{i}} + E_2\hat{\mathbf{j}})\cos(kz - \omega t)$

(b) The associated magnetic field is given as $\mathbf{B} = \frac{1}{c}(E_1\hat{\mathbf{i}} - E_2\hat{\mathbf{j}})\cos(kz - \omega t)$

(c) The given electromagnetic field is circularly polarised

(d) None of the above

6. The electric field of a plane electromagnetic wave is given by $\mathbf{E} = E_0\hat{\mathbf{i}}\cos(kz)\cos(\omega t)$

The corresponding magnetic field $\mathbf{B}$ is then given by **[JEE Main 2019]**

(a) $\mathbf{B} = \frac{E_0}{c}\hat{\mathbf{j}}\sin(kz)\sin(\omega t)$

(b) $\mathbf{B} = \frac{E_0}{c}\hat{\mathbf{j}}\sin(kz)\cos(\omega t)$

(c) $\mathbf{B} = \frac{E_0}{c}\hat{\mathbf{k}}\sin(kz)\cos(\omega t)$

(d) $\mathbf{B} = \frac{E_0}{c}\hat{\mathbf{j}}\cos(kz)\sin(\omega t)$

7. The magnetic field of an electromagnetic wave is given by

$\mathbf{B} = 1.6 \times 10^{-6}\cos(2 \times 10^7 z + 6 \times 10^{15} t)(2\hat{\mathbf{i}} + \hat{\mathbf{j}})\ \mathrm{Wbm^{-2}}$

The associated electric field will be **[JEE Main 2019]**

(a) $\mathbf{E} = 4.8 \times 10^2\cos(2 \times 10^7 z - 6 \times 10^{15} t)(-2\hat{\mathbf{j}} + \hat{\mathbf{i}})\ \mathrm{Vm^{-1}}$

(b) $\mathbf{E} = 4.8 \times 10^2\cos(2 \times 10^7 z - 6 \times 10^{15} t)(2\hat{\mathbf{j}} + \hat{\mathbf{i}})\ \mathrm{Vm^{-1}}$

(c) $\mathbf{E} = 4.8 \times 10^2\cos(2 \times 10^7 z + 6 \times 10^{15} t)(\hat{\mathbf{i}} - 2\hat{\mathbf{j}})\ \mathrm{Vm^{-1}}$

(d) $\mathbf{E} = 4.8 \times 10^2\cos(2 \times 10^7 z + 6 \times 10^{15} t)(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\mathrm{Vm^{-1}}$

8. The electric field of a plane electromagnetic wave is given by $\mathbf{E} = E_0\,\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\cos(kz + \omega t)$

At $t = 0$, a positively charged particle is at the point $(x, y, z) = \left(0, 0, \frac{\pi}{k}\right)$.

If its instantaneous velocity at $(t = 0)$ is $v_0\hat{\mathbf{k}}$, the force acting on it due to the wave is **[JEE Main 2020]**

(a) zero (b) anti-parallel to $\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}$

(c) parallel to $\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}$ (d) parallel to $\hat{\mathbf{k}}$

9. The electric field of a plane electromagnetic wave is given by

$$\mathbf{E} = E_0(\hat{\mathbf{x}} + \hat{\mathbf{y}})\sin(kz - \omega t)$$

Its magnetic field will be given by **[JEE Main 2020]**

(a) $\frac{E_0}{c}(\hat{\mathbf{x}} + \hat{\mathbf{y}})\sin(kz - \omega t)$ (b) $\frac{E_0}{c}(-\hat{\mathbf{x}} + \hat{\mathbf{y}})\sin(kz - \omega t)$

(c) $\frac{E_0}{c}(\hat{\mathbf{x}} - \hat{\mathbf{y}})\sin(kz - \omega t)$ (d) $\frac{E_0}{c}(\hat{\mathbf{x}} - \hat{\mathbf{y}})\cos(kz - \omega t)$

10. An electromagnetic wave of intensity 50 $\mathrm{Wm^{-2}}$ enters in a medium of refractive index n without any loss. The ratio of the magnitudes of electric fields and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively, given by **[JEE Main 2019]**

(a) $\left(\frac{1}{\sqrt{n}}, \sqrt{n}\right)$ (b) $(\sqrt{n}, \sqrt{n})$

(c) $\left(\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)$ (d) $\left(\sqrt{n}, \frac{1}{\sqrt{n}}\right)$

11. An electron is constrained to move along the Y-axis with a speed of $0.1\,c$ (c is the speed of light) in the presence of electromagnetic wave, whose electric field is

$$\mathbf{E} = 30\hat{\mathbf{j}}\sin(1.5 \times 10^7 t - 5 \times 10^{-2} x)\ \mathrm{V/m}.$$

The maximum magnetic force experienced by the electron will be (Given, $c = 3 \times 10^8\ \mathrm{ms^{-1}}$ and electron charge $= 1.6 \times 10^{-19}$ C) **[JEE Main 2020]**

(a) 2.4×10^{-18} N (b) 4.8×10^{-19} N

(c) 1.6×10^{-19} N (d) 3.2×10^{-18} N

Numerical Value Questions

12. In a plane electromagnetic wave, the electric field oscillates with a frequency $2 \times 10^{10}\ \mathrm{s^{-1}}$ and amplitude 40 $\mathrm{Vm^{-1}}$, then the energy density due to electric field is $p \times 10^{-9}\ \mathrm{J/m^3}$. The value of p is ……… . ($\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{Fm^{-1}}$)

13. An electromagnetic wave of frequency 2 MHz propagates from vacuum to a non-magnetic medium of relative permittivity 9. Then, its wavelength decrease by .... m.

14. If 2.5×10^{-6} N average force is exerted by a light wave on a non-reflecting surface of 30 $\mathrm{cm^2}$ area during to minutes of time span, the energy flux of light just before it falls on the surface is ...... $\mathrm{W/cm^2}$.

(Round off to the nearest integer)

(Assume complete absorption and normal incidence conditions are there) **[JEE Main 2021]**

15. Suppose that, intensity of a laser is $\left(\frac{315}{\pi}\right)\ \mathrm{W/m^2}$.

The rms electric field (in V/m) associated with this source is close to the nearest integer is ....

(Take, $\varepsilon_0 = 8.86 \times 10^{-12}\ \mathrm{C^2N^{-1}m^{-2}}$ and $c = 3 \times 10^8\ \mathrm{ms^{-1}}$) **[JEE Main 2020]**

Answers

Round I

1. (c)	**2.** (c)	**3.** (a)	**4.** (c)	**5.** (c)	**6.** (c)	**7.** (b)	**8.** (d)	**9.** (b)	**10.** (a)
11. (b)	**12.** (c)	**13.** (d)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (a)	**18.** (c)	**19.** (d)	**20.** (b)
21. (a)	**22.** (d)	**23.** (d)	**24.** (d)	**25.** (d)	**26.** (b)	**27.** (a)	**28.** (c)	**29.** (c)	**30.** (b)
31. (c)	**32.** (a)	**33.** (c)	**34.** (d)	**35.** (c)	**36.** (a)	**37.** (d)	**38.** (a)	**39.** (a)	**40.** (d)
41. (a)	**42.** (b)	**43.** (b)	**44.** (b)	**45.** (c)	**46.** (c)	**47.** (d)	**48.** (b)	**49.** (d)	

Round II

1. (c)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (a)	**6.** (a)	**7.** (c)	**8.** (b)	**9.** (b)	**10.** (d)
11. (b)	**12.** 3.54	**13.** 100	**14.** 25	**15.** 194					

Solutions

1. Due to conductivity, conduction current is present and due to variation of electric field, displacement current is also present.

2. As, $i = \frac{dq}{dt} = \frac{d}{dt}(q_0 \sin 2\pi ft) = q_0 2\pi f \cos 2\pi ft$

3. As, $i = \frac{dQ}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt} = 2 \times 10^{-12} \times 10^{12} = 2$ A

4. As, $C = \frac{\varepsilon_0 KA}{d} = \frac{(8.85 \times 10^{-12}) \times 10 \times 1}{10^{-3}}$

$$= 8.85 \times 10^{-8} \text{ F}$$

Now, $i = \frac{dQ}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt} = 8.85 \times 10^{-8} \times 25$

$$= 2.2 \times 10^{-6} \text{A} = 2.2\,\mu\text{A}$$

5. Given, radius of plate, $R = 10$ cm $= 0.1$ m and cross-section area of a capacitor,

$$A = \pi R^2 = \pi (0.1)^2 = 3.14 \times 10^{-2} \text{ m}^2$$

Rate of change of electric field,

$$\frac{dE}{dt} = 5 \times 10^{13} \text{ Vms}^{-1}$$

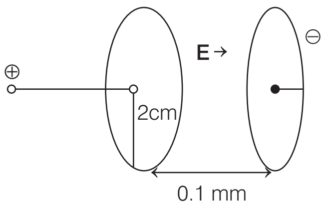
Thus, displacement current,

$$i_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 A \frac{dE}{dt} \qquad [\because \phi_E = EA]$$

$$= (8.85 \times 10^{-12})\,(3.14 \times 10^{-2})\,(5 \times 10^{13})$$

$\therefore \qquad i_d = 13.9$ A

6. According to the question, a parallel plate capacitor is shown in the figure below.



Given, radius = 2 cm, distance between plates, $d = 0.1$ mm and $\frac{dV}{dt} = 5 \times 10^6$ Vs^{-1}

Displacement current in the capacitor,

$$I_d = \varepsilon_0 \frac{d\phi}{dt} = \varepsilon_0 \frac{dE}{dt} A$$

$$I_d = \varepsilon_0 A \frac{dE}{dt}$$

$$\Rightarrow \qquad I_d = \varepsilon_0 \frac{A}{d}\frac{dV}{dt} \qquad \left(\because E = \frac{V}{d}\right)$$

Putting the given values, we get

$$\Rightarrow \qquad I_d = \frac{8.85 \times 10^{-12} \times \pi \times (4 \times 10^{-4}) \times 5 \times 10^6}{1 \times 10^{-4}} \text{ A}$$

$$\Rightarrow \qquad I_d = 556.28 \times 10^{-6} \text{ A} = 0.556 \text{ mA}$$

7. According to Gauss's law.

$$\oint \mathbf{E} \cdot \mathbf{dS} = \oint \frac{\rho}{\varepsilon_0} \cdot dS$$

$$\left[\because E = \frac{\rho}{\varepsilon_0}, \rho = \text{surface charge density}\right]$$

According to Faraday's law of electromagnetic induction,

$$\oint \mathbf{E} \cdot \mathbf{dl} = \frac{-d\phi}{dt} = \frac{-d}{dt}\oint \mathbf{B} \cdot \mathbf{dS}$$

$$\therefore \qquad \phi = \oint \mathbf{B} \cdot \mathbf{dS}$$

According to Ampere-Maxwell's law,

$$\oint \mathbf{B} \cdot \mathbf{d}l = \mu_0 \left(I + \varepsilon_0 \frac{d\phi_E}{dt}\right)$$

According to Ampere's circuital law,

$$\oint \mathbf{B} \cdot \mathbf{dl} = \mu_0 I$$

8. As electromagnetic wave contains both electric field and magnetic field. It carry both energy and momentum. Also, it can travel in vacuum and it is produced by oscillating charges.

9. The energy of electromagnetic wave is equally distributed between electric and magnetic fields. Some more properties of electromagnetic waves are as given below

(i) Electric and magnetic fields will have zero phase difference.

(ii) The pressure exerted by electromagnetic wave is the ratio of energy density to the speed of light.

(iii) The speed of wave is equal to the ratio of electric field to the magnetic field.

Hence, the correct option is (b).

10. As, $\sqrt{\dfrac{\mu}{\varepsilon}}$ = has the dimensions of resistance, hence it is called the intrinsic impedance of the medium.

11. As, $v_{\text{ferrite}} = \dfrac{c}{\sqrt{\mu_r \varepsilon_r}} = \dfrac{3 \times 10^8}{\sqrt{10 \times 10^3}} = 3 \times 10^6 \text{ ms}^{-1}$

and $\lambda_{\text{ferrite}} = \dfrac{v_{\text{ferrite}}}{\nu} = \dfrac{3 \times 10^6}{90 \times 10^6} = 3.33 \times 10^{-2}$ m

12. In vacuum, $\varepsilon_0 = 1$

In medium, $\varepsilon = 4$

$\therefore$ Refractive index, $\mu = \sqrt{\dfrac{\varepsilon}{\varepsilon_0}} = \sqrt{\dfrac{4}{1}} = 2$

Wavelength, $\lambda' = \dfrac{\lambda}{\mu} = \dfrac{\lambda}{2}$

and wave velocity, $v = \dfrac{c}{\mu} = \dfrac{c}{2}$ $\left(\because \mu = \dfrac{c}{v}\right)$

Hence, it is clear that wavelength and velocity will become half but frequency remains unchanged, when the wave is passing through any medium.

13. Velocity of an electromagnetic wave,
$c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ ms}^{-1}$, which is independent of amplitude of electromagnetic wave, its frequency and wavelength.

14. Electromagnetic wave carry momentum and hence, it can exert pressure on surfaces. Thus, they also transfer energy to the surface. To $p \neq 0$ and $E \neq 0$.

15. As, $\dfrac{E_0}{B_0} = c$

Also, $k = \dfrac{2\pi}{\lambda}$ and $\omega = 2\pi\nu$

Thus, we get $E_0 k = B_0 \omega$

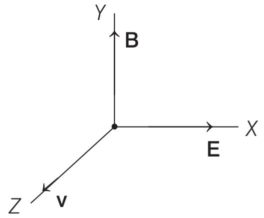
16. In electromagnetic wave, **E** and **B** are perpendicular to each other and to the direction of propagation, so the magnetic field vector points towards East.

17. As, $B_0 = \dfrac{E_0}{c} = \dfrac{54}{3 \times 10^8} = 18 \times 10^{-8}$ T or Wb m^{-2}

$\therefore \quad \lambda = \dfrac{c}{\nu} = \dfrac{3 \times 10^8}{2 \times 10^{10}} = 1.5 \times 10^{-2}$ m = 1.5 cm

18. The electromagnetic waves are produced due to sinusoidal variation of electric field vector and magnetic field vector perpendicular to each other as well as perpendicular to the direction of propagation of waves. Since, electromagnetic wave is propagating along y-direction, therefore **E** and **B** should be either along X-axis and Z-axis or along Z and X-axis, respectively.

19. Given, $\mathbf{B} = 5 \times 10^{-8} \hat{\mathbf{j}}$ and $\mathbf{v} = 3 \times 10^8 \hat{\mathbf{k}}$



Using $\mathbf{E} = \mathbf{B} \times \mathbf{v}$, we have

$$\mathbf{E} = (5 \times 10^{-8} \hat{\mathbf{j}}) \times (3 \times 10^8 \hat{\mathbf{k}}) = 15 \hat{\mathbf{i}} \text{ V/m}$$

20. In an electromagnetic wave, the direction of propagation of wave, electric field and magnetic field are mutually perpendicular to each other, *i.e.* the wave propagates perpendicular to E and B or along $\mathbf{E} \times \mathbf{B}$ i.e. $\hat{\mathbf{k}} \,||\, \mathbf{E} \times \mathbf{B}$. While polarisation of wave takes place parallel to electric field vector i.e. $\mathbf{X} \,||\, \mathbf{E}$.

21. Direction of propagation of an electromagnetic wave is given by $\mathbf{E} \times \mathbf{B}$.

$\therefore$ A unit vector in the direction of propagation

$$= \frac{\mathbf{E} \times \mathbf{B}}{|\mathbf{E} \times \mathbf{B}|} = \frac{\hat{\mathbf{k}} \times (2\hat{\mathbf{i}} - 2\hat{\mathbf{j}})}{|\mathbf{E} \times \mathbf{B}|}$$

$$\left[\begin{aligned} \because \mathbf{E} \times \mathbf{B} &= \hat{\mathbf{k}} \times (2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) \\ &= 2(\hat{\mathbf{k}} \times \hat{\mathbf{i}}) - 2(\hat{\mathbf{k}} \times \hat{\mathbf{j}}) \\ &= 2(\hat{\mathbf{j}}) - 2(-\hat{\mathbf{i}}) \\ &= 2\hat{\mathbf{j}} + 2\hat{\mathbf{i}} \\ \therefore |\mathbf{E} \times \mathbf{B}| &= \sqrt{2^2 + 2^2} = 2\sqrt{2} \end{aligned}\right]$$

$$= \frac{2\hat{\mathbf{j}} + 2\hat{\mathbf{i}}}{2\sqrt{2}} = \frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}$$

22. Suppose an electromagnetic wave is travelling along negative z-direction. Its electric field is given by
$\mathbf{E} = E_0 \cos(kz - \omega t)$

which is perpendicular to Z-axis. It acts along negative y-direction.

The associated magnetic field **B** in electromagnetic wave is along X-axis, *i.e.* along $\hat{\mathbf{k}} \times \mathbf{E}$.

As, $B_0 = \dfrac{E_0}{c}$

$\therefore \quad \mathbf{B} = \dfrac{1}{c}(\hat{\mathbf{k}} \times \mathbf{E})$

The associated electric field can be written in terms of magnetic field as $\mathbf{E} = c(\mathbf{B} \times \hat{\mathbf{k}})$.

23. Given, $\mathbf{E} = E_0 \hat{\mathbf{j}} \cos(\omega t - kx)$

This equation represents electric field vector of a wave which travels in positive x-direction, *i.e.* along $\hat{\mathbf{i}}$.

As, $\mathbf{E} \times \mathbf{B}$ = direction of wave propagation, we have

$$\hat{\mathbf{j}} \times \mathbf{B} = \hat{\mathbf{i}}$$

$\therefore$ $\mathbf{B}$ is along $\hat{\mathbf{k}}$.

Also, $\dfrac{E_0}{B_0} = c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

so $B_0 = E_0(\sqrt{\mu_0 \varepsilon_0})$

So, $\mathbf{B} = B_0 \hat{\mathbf{k}} \cos(\omega t - kx)$

or $\mathbf{B} = E_0(\sqrt{\mu_0 \varepsilon_0}) \hat{\mathbf{k}} \cdot \cos kx$

24. Here, $\lambda = 3\text{ mm} = 3 \times 10^{-3}\text{m}$, $E_0 = 66\text{ Vm}^{-1}$

$$\therefore \quad B_0 = \frac{E_0}{c} = \frac{66}{3 \times 10^8} = 2.2 \times 10^{-7}\text{ T}$$

As, electromagnetic wave is propagating along X-axis and electric field oscillation is along y-direction, the magnetic field oscillation is along z-direction using the relation for harmonic wave,

$$E_y = E_0 \cos \frac{2\pi}{\lambda}(ct - x)$$

$$E_y = E_0 \cos \frac{2\pi c}{\lambda}(t - x/c)$$

$$\therefore \quad E_y = 66 \cos \frac{2\pi \times 3 \times 10^8}{3 \times 10^{-3}}(t - x/c)$$

$$= 66 \cos 2\pi \times 10^{11}(t - x/c)$$

and $$B_z = B_0 \cos \frac{2\pi c}{\lambda}(t - x/c)$$

$$= 2.2 \times 10^{-7} \cos 2\pi \times 10^{11}(t - x/c)$$

25. We have the following facts

(i) An electromagnetic wave propagates in direction of vector $\mathbf{E} \times \mathbf{B}$.

(ii) Phase gives velocity of wave.

(iii) Electric field vector of electromagnetic wave is given by

$$\mathbf{E} = c\mathbf{B}$$

Here, $\mathbf{B} = B_0 \sin[200\pi(y + ct)]\hat{\mathbf{i}}$

$$= 3 \times 10^{-8} \sin[200\pi(y + ct)]\hat{\mathbf{i}}$$

$$\Rightarrow \quad E_0 = cB_0 = 3 \times 10^8 \times 3 \times 10^{-8} = 9\text{ V/m}$$

Phase = $y + ct$ = constant

$$\Rightarrow \quad \frac{dy}{dt} = -c$$

i.e. Wave propagates in negative y-direction.

$\Rightarrow$ $\mathbf{E} \times \mathbf{B}$ is parallel to $-\hat{\mathbf{j}}$.

$$\Rightarrow \quad \mathbf{E} \times \hat{\mathbf{i}} = \lambda(-\hat{\mathbf{j}})$$

So, $\mathbf{E} = -\hat{\mathbf{k}}$

Hence, electric field vector is given by

$$\mathbf{E} = E_0 \sin(ky + \omega t)(-\hat{\mathbf{k}})$$

$$= -9 \sin[200\pi(y + ct)]\hat{\mathbf{k}}\text{ V/m}$$

26. When a wave is reflected from denser medium, the reflected wave is without change in type of wave but with a change in phase by 180° or π radian. Therefore, for the reflected wave, we use $z = -z$, $\hat{\mathbf{i}} = -\hat{\mathbf{i}}$ and additional phase of π in the incident wave. The incident electromagnetic wave is $E_r = E_0 \hat{\mathbf{i}} \cos(kz - \omega t)$.

The reflected electromagnetic wave is

$$\mathbf{E}_r = E_0(-\hat{\mathbf{i}}) \cos[k(-z) - \omega t + \pi]$$

$$= -E_0 \hat{\mathbf{i}} \cos[-(kz + \omega t) + \pi]$$

$$= E_0 \hat{\mathbf{i}} \cos[-(kz + \omega t)] \quad [\because \cos(\theta + \pi) = -\cos\theta]$$

$$= E_0 \hat{\mathbf{i}} \cos(kz + \omega t) \quad [\because \cos(-\theta) = \cos\theta]$$

27. Given,

$\mathbf{B}(x, t) = [1.2 \times 10^{-7} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\hat{\mathbf{k}}]\text{T}$

Speed of light, $c = 3 \times 10^8$ m/s

Maximum value of electric field,

E_0 = (Maximum value of B_0) $\times$ (Speed of light)

$= B_0 \times c = 1.2 \times 10^{-7} \times 3 \times 10^8$

$= 36$ V/m

Since, wave is propagating in negative x-direction and magnetic field is in z-direction. So, electric field vector must be in negative y-direction as direction of wave propagation is in the direction of $\mathbf{E} \times \mathbf{B}$.

So, electric field at a point x and time t is given by

$\mathbf{E}(x, t) = -36 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\hat{\mathbf{j}}$ V/m

Here, phase of $\mathbf{E}$ will be same that of $\mathbf{B}$.

28. Given, direction of propagation of electromagnetic wave is along $\hat{\mathbf{i}} = \dfrac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}$, also direction of its electric field vector $\mathbf{E}$ (or direction of its polarisation) is $\hat{\mathbf{k}}$.

As, direction of propagation of wave is along $\mathbf{E} \times \mathbf{B}$, so direction of $\mathbf{B}$ is such that

$$\hat{\mathbf{k}} \times \hat{\mathbf{B}} = \frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}$$

$$\Rightarrow \quad \hat{\mathbf{B}} = \left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\right)\hat{\mathbf{k}}$$

$$= \frac{(\hat{\mathbf{i}} \times \hat{\mathbf{k}}) + (\hat{\mathbf{j}} \times \hat{\mathbf{k}})}{\sqrt{2}}$$

$$= \frac{(\hat{\mathbf{i}} - \hat{\mathbf{j}})}{\sqrt{2}}$$

So, correct expression of $\mathbf{B}$ is

$$\mathbf{B} = B_0 \hat{\mathbf{B}} \cos(\omega t - k\hat{\mathbf{i}})$$

$$= B_0\left(\frac{\hat{\mathbf{i}} - \hat{\mathbf{j}}}{\sqrt{2}}\right) \cdot \cos\left\{\omega t - k\left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\right)\right\}$$

29. Standard expression of electromagnetic wave is given by

$$\mathbf{E} = E_0\, \hat{\mathbf{n}}[\sin(\omega t - \mathbf{k} \cdot \hat{\mathbf{r}})] \quad \text{...(i)}$$

Here, $\mathbf{k}$ is the propagation vector.

Direction of propagation in this case is $\hat{\mathbf{k}}$.

Given expression of electromagnetic wave,

$$\mathbf{E} = E_0\,\hat{\mathbf{n}} \sin[\omega t + (6y - 8z)]$$

$$\Rightarrow \quad \mathbf{E} = E_0\,\hat{\mathbf{n}} \sin[\omega t - (8z - 6y)] \quad \ldots\text{(ii)}$$

Comparing Eq. (ii) with Eq. (i), we get

$$\mathbf{k}\cdot\hat{\mathbf{r}} = 8z - 6y \quad \ldots\text{(iii)}$$

Here, $\hat{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

and $\mathbf{k} = k_x\hat{\mathbf{i}} + k_y\hat{\mathbf{j}} + k_z\hat{\mathbf{k}}$

$$\therefore \quad \mathbf{k}\cdot\hat{\mathbf{r}} = xk_x + yk_y + zk_z \quad \ldots\text{(iv)}$$

From Eqs. (iii) and (iv), we get

$$xk_x = \text{zero} \Rightarrow k_x = 0$$

$$yk_y = -6y \Rightarrow k_y = -6$$

$$zk_z = 8z \Rightarrow k_z = 8$$

Hence, $\mathbf{k} = -6\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$

So, direction of propagation,

$$\hat{\mathbf{s}} = \hat{\mathbf{k}} = \frac{\mathbf{k}}{|\mathbf{k}|} = \frac{-6\hat{\mathbf{j}} + 8\hat{\mathbf{k}}}{\sqrt{6^2 + 8^2}}$$

$$= \frac{-6\hat{\mathbf{j}} + 8\hat{\mathbf{k}}}{10}$$

$$= \frac{-3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}}{5}$$

30. In an electromagnetic wave, magnetic field and electric field are perpendicular to each other and both are also perpendicular to the direction of propagation of wave.

Now, given direction of propagation is along z-direction. So, magnetic field is in either x or y-direction. Also, angular wave number for wave is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{c}$$

$$= \frac{2\pi \times 23.9 \times 10^9}{3 \times 10^8}$$

$$\approx 0.5 \times 10^3 \text{ m}^{-1}$$

and angular frequency ω for wave is

$$\omega = 2\pi\nu = 2\pi \times 23.9 \times 10^9 \text{ Hz}$$

$$= 1.5 \times 10^{11} \text{ Hz}$$

Magnitude of magnetic field is

$$B_0 = \frac{E_0}{c}$$

$$= \frac{60}{3 \times 10^8} = 2 \times 10^{-7}\,\text{T}$$

As the general equation of magnetic field of an electromagnetic wave propagating in + z- direction is given as

$$\mathbf{B} = B_0 \sin(kz - \omega t)\,\hat{\mathbf{i}} \text{ or } \hat{\mathbf{j}}$$

Thus, substituting the values of B_0, k and ω, we get

$$\Rightarrow \mathbf{B} = 2 \times 10^{-7} \sin(0.5 \times 10^3 z - 1.5 \times 10^{11} t)\,\hat{\mathbf{i}} \text{ or } \hat{\mathbf{j}}$$

31. Given, magnetic field of an electromagnetic wave is

$$\mathbf{B} = B_0[\cos(kz - \omega t)]\,\hat{\mathbf{i}} + B_1[\cos(kz + \omega t)]\,\hat{\mathbf{j}}$$

Here, $B_0 = 3 \times 10^{-5}$ T and $B_1 = 2 \times 10^{-6}$ T

Also, stationary charge, $Q = 10^{-4}$C at $z = 0$.

As charge is released from the rest at $z = 0$, in this condition.

Maximum electric field, $E_0 = cB_0$ and $E_1 = cB_1$

So, $E_0 = c \times 3 \times 10^{-5}$

and $E_1 = c \times 2 \times 10^{-6}$

Now,the direction of electric field of an electromagnetic wave is perpendicular to $\mathbf{B}$ and to the direction of propagation of wave $(\mathbf{E} \times \mathbf{B})$ which is $\hat{\mathbf{k}}$.

So, for E_0,

$$\mathbf{E}_0 \times \mathbf{B}_0 = \hat{\mathbf{k}}$$

$$\Rightarrow \quad \mathbf{E}_0 \times \hat{\mathbf{i}} = \hat{\mathbf{k}}$$

$$\Rightarrow \quad \mathbf{E}_0 = -\hat{\mathbf{j}}$$

Similarly,for E_1, $\mathbf{E}_1 \times \mathbf{B}_1 = \mathbf{k}$

$$\Rightarrow \quad \mathbf{E}_1 \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$$

$$\Rightarrow \quad \mathbf{E}_1 = \hat{\mathbf{i}}$$

$$\therefore \quad \mathbf{E}_0 = c \times 3 \times 10^{-5}(-\hat{\mathbf{j}})\,\text{NC}^{-1}$$

$$\mathbf{E}_1 = c \times 2 \times 10^{-6}(+\hat{\mathbf{i}})\,\text{NC}^{-1}$$

$\therefore$ Maximum force experienced by stationary charge is

$$\mathbf{F}_{\max} = Q\,\mathbf{E} = Q(\mathbf{E}_0 + \mathbf{E}_1)$$

$$= Q \times c\,[-3 \times 10^{-5}\,\hat{\mathbf{j}} + 2 \times 10^{-6}\hat{\mathbf{i}}]$$

$$\Rightarrow \quad |\mathbf{F}_{\max}| = 10^{-4} \times 3 \times 10^8 \times \sqrt{(3 \times 10^{-5})^2 + (2 \times 10^{-6})^2}$$

$$= 3 \times 10^4 \times 10^{-6}\sqrt{900 + 4}$$

$$= 3 \times 10^{-2} \times \sqrt{904} \approx 0.9\text{ N}$$

$\therefore$ rms value of experienced force is

$$F_{\text{rms}} = \frac{F_{\max}}{\sqrt{2}} = \frac{0.9}{\sqrt{2}} = 0.707 \times 0.9$$

$$= 0.6363\text{ N} \approx 0.6\text{ N}$$

32. Energy density of an electomagnetic wave in electric field,

$$U_E = \frac{1}{2}\varepsilon_0 \cdot E^2 \quad \ldots\text{(i)}$$

Energy density of an electromagnetic wave in magnetic field,

$$U_B = \frac{B^2}{2\mu_0} \quad \ldots\text{(ii)}$$

where, E = electric field,

B = magnetic field,

ε_0 = permittivity of free space

and μ_0 = magnetic permeability of free space.

From the theory of electromagnetic waves, the relation between μ_0 and ε_0 is

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \quad \ldots\text{(iii)}$$

where, c = velocity of light

$= 3 \times 10^8$ m/s

and $$\frac{E}{B} = c \qquad \text{...(iv)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{U_E}{U_B} = \frac{\frac{1}{2}\varepsilon_0 E^2}{\frac{1}{2}B^2 \times \frac{1}{\mu_0}} = \frac{\mu_0 \varepsilon_0 E^2}{B^2} \qquad \text{...(v)}$$

Using Eqs. (iii), (iv) and (v), we get

$$\frac{U_E}{U_B} = \frac{c^2}{c^2} = 1$$

Therefore, $U_E = U_B$

33. As, intensity, $I = \frac{P}{4\pi r^2}$

$= \text{Average energy density} \times \text{velocity}$

$= \frac{1}{2}\varepsilon_0 E_0^2 c$

$$\therefore \quad E_0 = \sqrt{\frac{2P}{4\pi\varepsilon_0 r^2 c}} = \sqrt{\frac{P}{2\pi\varepsilon_0 r^2 c}}$$

34. Given, intensity of electromagnetic wave ,

$I = 53.1 \text{ Wm}^{-2}$

Permittivity of free space,

$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$

Intensity of electromagnetic wave, $I = \frac{1}{2}\varepsilon_0 E_0^2 c$

where, E_0 = amplitude of the electric field.

$$E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}} = \sqrt{\frac{2 \times 53.1}{8.85 \times 10^{-12} \times 3 \times 10^8}}$$

$E_0 = 200 \text{ NC}^{-1}$

35. As, power $= I \times \text{area} = (1.4 \times 10^3) \times 5$

Force, $$F = \frac{\text{Power}}{c} = \frac{\text{Solar constant} \times \text{Area}}{\text{Speed of light}}$$

$$= \frac{1.4 \times 10^3 \times 5}{3 \times 10^8}$$

$= 2.33 \times 10^{-5}$ N

36. As, $F = PA$, but $P = \frac{1}{c}$

So, $$F = \frac{IA}{c} = \frac{6 \times 12}{3 \times 10^8} = 24 \times 10^{-8} \text{ N}$$

37. Given, power of laser beam,

$P = 27\text{mW} = 27 \times 10^{-3}$ W

Area of cross-section, A

$= 10 \text{ mm}^2 = 10 \times 10^{-6}\text{m}^2$

Permittivity of free space, $\varepsilon_0 = 9 \times 10^{-12}$ SI unit

Speed of light, $c = 3 \times 10^8$ m/s

Intensity of electromagnetic wave is given by the relation,

$$I = \frac{1}{2} n c \varepsilon_0 E^2$$

where, n is refractive index, for air $n = 1$.

$$\therefore \quad I = \frac{1}{2} c \cdot \varepsilon_0 E^2 \qquad \text{...(i)}$$

Also, $$I = \frac{P}{A} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{1}{2} c\varepsilon_0 E^2 = \frac{P}{A}$$

or $$E^2 = \frac{2P}{Ac\varepsilon_0}$$

or $$E = \sqrt{\frac{2 \times 27 \times 10^{-3}}{10 \times 10^{-6} \times 3 \times 10^8 \times 9 \times 10^{-12}}}$$

$\simeq 1.4 \times 10^3$ V/m

$= 1.4$ kV/m

38. Electric field intensity on a surface due to incident radiation is $E = \frac{U}{At} = \frac{P}{A}$, where $\frac{U}{t} = P$ = power

$\therefore$ $E \propto P$ (for the given area of the surface)

Hence, $$\frac{E'}{E} = \frac{P'}{P} = \frac{50}{100} = \frac{1}{2} \text{ or } E' = \frac{E}{2}$$

39. Momentum transferred in one second by electromagnetic wave to the mirror is

$$p = \frac{2S_{\text{av}}A}{c} = \frac{2 \times 6 \times (30 \times 10^{-4})}{3 \times 10^8}$$

$= 1.2 \times 10^{-10}$ kg-ms^{-1}

40. The poynting vector, $\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$

Substituting the given values, we get

$$\mathbf{S} = \frac{10^{-6}}{4\pi \times 10^{-7}} \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 80 & 32 & -64 \\ 0.2 & 0.08 & 0.29 \end{vmatrix} = 11.45\,\hat{\mathbf{i}} - 28.6\,\hat{\mathbf{j}}$$

41. Average intensity $= \frac{P}{4\pi R^2} = \frac{1}{2}\varepsilon_0 E_0^2 c$

or $$E_0 = \sqrt{\frac{P}{2\pi R^2 \varepsilon_0 c}}$$

$$= \sqrt{\frac{3}{2 \times 3.14 \times 25 \times 8.85 \times 10^{-12} \times 3 \times 10^8}}$$

$= 2.68 \text{ Vm}^{-1}$

42. Energy flowing per second per unit area from a face is $= \frac{1}{\mu_0}[\mathbf{E} \times \mathbf{B}]$. It will be in the negative z-direction. It shows that the energy will be flowing in faces parallel to XY-plane and is zero in all other faces. Total energy flowing per second from a face in XY-plane

$$= \frac{1}{\mu_0}(EB\sin 90^\circ)a^2 = \frac{EBa^2}{\mu_0}$$

43. Diffraction takes places when the wavelength of wave is comparable with the size of the obstacle in path. The wavelength of radio waves is greater than the wavelength of light waves. Therefore, radio waves are diffracted around building.

44. $\lambda = \frac{c}{f} = \frac{3\times 10^8}{10^9} = 3\times 10^{-1} = 0.3 \text{ m} = 30 \text{ cm}$

This wave is lying in radio wave region.

45. As, frequency of radio waves $= 3\times 10^8$ Hz

and frequency of microwaves $= 10^{10}$ Hz

$\therefore$ The frequency of radio waves is less than the frequency of microwaves.

Also, radio wave undergo more diffraction than microwaves.

46.

$$\lambda_{\text{X-rays}} = 0.01 \text{ Å to } 100 \text{ Å}$$

$$\lambda_{\text{visible}} = 3800 \text{ Å to } 7000 \text{ Å}$$

$$\lambda_{\text{microwaves}} = 10^7 \text{ Å to } 3\times 10^9 \text{ Å}$$

$$\lambda_{\text{radio waves}} = 10^7 \text{ Å to } 10^{15} \text{ Å}$$

So, it is clear from the values that

$\lambda_{\text{radio waves}} > \lambda_{\text{microwaves}} > \lambda_{\text{visible}} > \lambda_{\text{X-rays}}$

47. (i) Infrared waves are used to treat muscular strain.
(ii) Radio waves are used for broadcasting purposes.
(iii) X-rays are used to detect fracture of bones.
(iv) Ultraviolet rays are absorbed by the ozone layer of the atmosphere.

48. Radioactive source $\Rightarrow \gamma$-rays

X-ray tube $\Rightarrow$ X-rays

Sodium vapour lamp $\Rightarrow$ Visible light

Crystal oscillator $\Rightarrow$ Microwaves

$\therefore$ Radioactive source < X-ray tube < Sodium vapour lamp < Crystal oscillator

49. Energy is inversely proportional to wavelength, *i.e.*

$$E = \frac{hc}{\lambda}$$

$$\Rightarrow \qquad E \propto \frac{1}{\lambda}$$

Energies of given radiations have the following order

$$E_{\gamma\text{-rays}} > E_{\text{X-rays}} > E_{\text{microwaves}} > E_{\text{AM radio waves}}$$

$$\therefore \qquad \lambda_{\gamma\text{-rays}} < \lambda_{\text{X-rays}} < \lambda_{\text{microwaves}} < \lambda_{\text{AM radio waves}}$$

Thus, microwaves $\rightarrow 10^{-3}$ m

γ-rays $\rightarrow 10^{-15}$ m

AM radio waves $\rightarrow 100$ m

X-rays $\rightarrow 10^{-10}$ m

Hence, (A) $\rightarrow$ (4), (B) $\rightarrow$ (2), (C) $\rightarrow$ (1), (D) $\rightarrow$ (3).

Round II

1. From a diode antenna, the electromagnetic waves are radiated outwards. The amplitude of electric field vector (E_0) which transports significant energy from the source falls off intensity inversely as the distance (r) from the antenna, *i.e.* $E_0 \propto 1/r$.

2. Given, $C = 8\,\mu\text{F} = 8\times 10^{-6}$ F, $\nu = 3.00$ kHz

and $V_0 = 30.0$ V

Clearly, $\omega = 2\pi\nu = 2\pi \times (3.00\times 10^3 \text{ s}^{-1})$

$= 6\pi\times 10^3 \text{ s}^{-1}$

Voltage across the capacitor, $V = V_0 \sin \omega t$

$= (30.0) \sin (6\pi \times 10^3\ t)$

Displacement current,

$$i_d = i_c = \frac{dq}{dt} = \frac{d}{dt}(q) = \frac{d}{dt}(CV) = C\frac{dV}{dt}$$

$$= (8\times 10^{-6})\frac{d}{dt}[30.0 \sin (6\pi \times 10^3\ t)]$$

$$= (8\times 10^{-6})(30.0)\frac{d}{dt}[\sin (6\pi \times 10^3\ t)]$$

$$= (8\times 10^{-6})(30.0)(6\pi\times 10^3)\cos(6\pi\times 10^3\ t)$$

$$= 4.52 \cos (6\pi\times 10^3\ t) \text{A}$$

Hence, the displacement current varies sinusoidally with time and has a maximum value of 4.52 A.

3. Radiation pressure (p) is the force exerted by electromagnetic wave on unit area of the surface, *i.e.* rate of change of momentum per unit area of the surface.

Momentum per unit time per unit area

$$= \frac{\text{Intensity}}{\text{Speed of wave}} = \frac{I}{c}$$

Change in momentum per unit time per unit area $= \Delta I/c =$ radiation pressure (p), *i.e.* $p = \Delta I/c$.
Momentum of incident wave per unit time per unit area $= I/c$.

When wave is totally absorbed by the surface, the momentum of the reflected wave per unit time per unit area = 0.

Radiation pressure (p) = Change in momentum per unit time per unit area $= \frac{\Delta I}{c} = \frac{I}{c} - 0 = \frac{I}{c}$

When wave is totally reflected, then momentum of the reflected wave per unit time per unit area $= -I/c$.

Radiation pressure $(p) = \frac{I}{c} - \left(-\frac{I}{c}\right) = \frac{2I}{c}$

Here, p lies between I/c.

4. As electric field in air,

$$E_1 = E_{01}\,\hat{\mathbf{x}} \cos\left(\frac{2\pi\nu z}{c} - 2\pi\nu t\right)$$

$$\therefore \qquad \text{Speed in air} = \frac{2\pi\nu}{\left(\frac{2\pi\nu}{c}\right)} = c$$

Also, $$c = \frac{1}{\sqrt{\mu_0 \varepsilon_{r_1} \varepsilon_0}} \quad \ldots(\text{i})$$

In medium, $E_2 = E_{02}\,\hat{\mathbf{x}} \cos(2kz - kct)$

$\therefore$ Speed in medium $= \dfrac{kc}{2k} = \dfrac{c}{2}$

Also, $$\frac{c}{2} = \frac{1}{\sqrt{\mu_0 \varepsilon_{r_2} \varepsilon_0}} \quad \ldots(\text{ii})$$

As, medium is non-magnetic medium, $\mu_{\text{medium}} = \mu_{\text{air}}$

On dividing Eq. (i) by Eq. (ii), we have

$$2 = \sqrt{\frac{\varepsilon_{r_2}}{\varepsilon_{r_1}}} \Rightarrow \frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$$

5. In electromagnetic wave, the electric field vector is given as

$$\mathbf{E} = (E_1\hat{\mathbf{i}} + E_2\hat{\mathbf{j}})\cos(kz - \omega t)$$

In electromagnetic wave, the associated magnetic field vector,

$$\mathbf{B} = \frac{\mathbf{E}}{c} = \frac{(E_1\hat{\mathbf{i}} + E_2\hat{\mathbf{j}})}{c}\cos(kz - \omega t)$$

As, **E** and **B** are perpendicular to each other and the propagation of electromagnetic wave is perpendicular to **E** as well as **B**, so the given electromagnetic wave is plane polarised.

6. We know that, $\mathbf{E} \times \mathbf{B}$ represents direction of propagation of an electromagnetic wave

$$\Rightarrow \quad (\mathbf{E} \times \mathbf{B}) \,||\, v$$

$\therefore$ From the given electric field, we can state that direction of propagation is along Z-axis and direction of **E** is along X-axis.

Thus, from the above discussion, direction of **B** must be Y-axis.

From Maxwell's equation,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Here, $$\frac{\partial \mathbf{E}}{\partial Z} = -\frac{\partial \mathbf{B}}{\partial t} \quad \ldots(\text{i})$$

and $$B_0 = E_0 / c \quad \ldots(\text{ii})$$

Given, $\mathbf{E} = E_0\,\hat{\mathbf{i}}\cos kz \cos \omega t$

$$\Rightarrow \quad \frac{-\partial \mathbf{E}}{\partial Z} = kE_0 \sin kz \cos \omega t$$

$\therefore$ Using Eq. (i), we get

$$\frac{\partial \mathbf{B}}{\partial t} = kE_0 \sin kz \cos \omega t$$

Integrating both sides of the above equation w.r.t. t, we get

$$\mathbf{B} = \frac{k}{\omega} E_0 \sin kz \sin \omega t$$

$$= \frac{E_0}{c} \sin kz \sin \omega t$$

$$\Rightarrow \quad \mathbf{B} = \frac{E_0}{c} \sin(kz) \sin(\omega t)\,\hat{\mathbf{j}}$$

7. Given, $\mathbf{B} = 1.6 \times 10^{-6} \cos(2 \times 10^7 z + 6 \times 10^{15} t)$

$$(2\hat{\mathbf{i}} + \hat{\mathbf{j}})\ \text{Wbm}^{-2}$$

From the given equation, it can be said that the electromagnetic wave is propagating along negative z-direction, *i.e.* $-\hat{\mathbf{k}}$.

Equation of associated electric field will be

$$\mathbf{E} = (|\mathbf{B}_0| c)\cos(kz + \omega t) \cdot \hat{\mathbf{n}}$$

where, $\hat{\mathbf{n}}$ = a vector perpendicular to **B**.

So,

$$|\mathbf{E}_0| = |\mathbf{B}_0| \cdot c$$
$$= 1.6 \times 10^{-6} \times 3 \times 10^8 = 4.8 \times 10^2\ \text{V/m}$$

Since, we know that for an electromagnetic wave, **E** and **B** are mutually perpendicular to each other.

So, $$\mathbf{E} \cdot \mathbf{B} = 0$$

From the given options, when $\hat{\mathbf{n}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}}$

$$\mathbf{E} \cdot \mathbf{B} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}}) \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) = 0$$

Also, when $\hat{\mathbf{n}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

$$\mathbf{E} \cdot \mathbf{B} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}}) \cdot (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) = 0$$

But, we also know that the direction of propagation of electromagnetic wave is perpendicular to both **E** and **B**, *i.e.* it is in the direction of $\mathbf{E} \times \mathbf{B}$.

Again, when $\hat{\mathbf{n}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}}$

$$\mathbf{E} \times \mathbf{B} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}}) \times (\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) = -\hat{\mathbf{k}}$$

and when $\hat{\mathbf{n}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

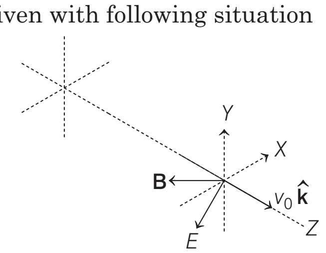
$$\mathbf{E} \times \mathbf{B} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}}) \times (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) = \hat{\mathbf{k}}$$

But, it is been given in the question that the direction of propagation of wave is in $-\hat{\mathbf{k}}$.

Thus, associated electric field will be

$$\mathbf{E} = 4.8 \times 10^2 \cos(2 \times 10^7 z + 6 \times 10^{15} t)(\hat{\mathbf{i}} - 2\hat{\mathbf{j}})\ \text{Vm}^{-1}$$

8. We are given with following situation



Given, $$\mathbf{E} = E_0\left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\right) \cdot \cos(kz + \omega t)$$

So, electromagnetic wave is propagating along Z-axis and its electric vector **E** is perpendicular to Z-axis; and is along angle bisector of IIIrd quadrant of XY-plane as shown in the figure. So, magnetic vector **B** is along angle bisector of IInd quadrant of XY-plane and is perpendicular to both **E** and **v** vectors.

Now, force due to electric field is along **E** vector and force due to magnetic field is obtained by Fleming's left hand rule.

Thus, direction of magnetic force is also along $\mathbf{E}$. So, force on charged particle will be along $\mathbf{E}$ or along $-\left(\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}\right)$ vector.

Hence, force on charged particle is anti-parallel to $\left(\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}\right)$ vector.

9. Electric field vector of the given EM wave is

$$\mathbf{E} = E_0\,(\hat{\mathbf{x}}+\hat{\mathbf{y}})\sin(kz-\omega t)$$

$\therefore$ Amplitude of electric field $= E_0$

and unit vector of electric field,

$$\hat{\mathbf{E}} = \frac{\hat{\mathbf{x}}+\hat{\mathbf{y}}}{\sqrt{1^2+1^2}} = \frac{\hat{\mathbf{x}}+\hat{\mathbf{y}}}{\sqrt{2}}$$

The coefficient of k in the given equation is z. So, wave travels along positive z-direction. So, unit vector into the direction of propagation of EM wave $\hat{\mathbf{c}} = \hat{\mathbf{z}}$.

Now, for magnetic field vector of the given EM wave, we can conclude

Amplitude of magnetic field, $B_0 = \frac{E_0}{c}$

and unit vector of magnetic field $= \hat{\mathbf{B}}$

So, the expected equation of magnetic field vector will be

$$\mathbf{B} = B_0(\hat{\mathbf{B}})\sin(kz-\omega t)$$

$$= \frac{E_0}{c}(\hat{\mathbf{B}})\sin(kz-\omega t) \quad \ldots\text{(i)}$$

Also, $\quad \hat{\mathbf{E}}\times\hat{\mathbf{B}} = \hat{\mathbf{c}} = \hat{\mathbf{z}} \quad \ldots\text{(ii)}$

Now, let's check the given alternatives

(a) $\mathbf{B} = \frac{E_0}{c}(\hat{\mathbf{x}}+\hat{\mathbf{y}})\sin(kz-\omega t)$

Here, $\quad \hat{\mathbf{B}} = \frac{\hat{\mathbf{x}}+\hat{\mathbf{y}}}{\sqrt{1^2+1^2}} = \frac{\hat{\mathbf{x}}+\hat{\mathbf{y}}}{\sqrt{2}}$

So, $\hat{\mathbf{E}}\times\hat{\mathbf{B}} = \left(\frac{\hat{\mathbf{x}}+\hat{\mathbf{y}}}{\sqrt{2}}\right)\times\left(\frac{\hat{\mathbf{x}}+\hat{\mathbf{y}}}{\sqrt{2}}\right)$

$$= \frac{1}{2}[(\hat{\mathbf{x}}\times\hat{\mathbf{x}})+(\hat{\mathbf{x}}\times\hat{\mathbf{y}})+(\hat{\mathbf{y}}\times\hat{\mathbf{x}})+(\hat{\mathbf{y}}\times\hat{\mathbf{y}})]$$

$$= \frac{1}{2}[(0)+(\hat{\mathbf{z}})+(-\hat{\mathbf{z}})+(0)]$$

$$= \frac{1}{2}[\hat{\mathbf{z}}-\hat{\mathbf{z}}] = 0$$

which is impossible for an EM wave.

So, this option is incorrect.

(b) $\mathbf{B} = \frac{E_0}{c}(-\hat{\mathbf{x}}+\hat{\mathbf{y}})\sin(kz-\omega t)$

Here, $\quad \hat{\mathbf{B}} = \frac{-\hat{\mathbf{x}}+\hat{\mathbf{y}}}{\sqrt{1^2+1^2}} = \frac{-\hat{\mathbf{x}}+\hat{\mathbf{y}}}{\sqrt{2}}$

So, $\hat{\mathbf{E}}\times\hat{\mathbf{B}} = \left(\frac{\hat{\mathbf{x}}+\hat{\mathbf{y}}}{\sqrt{2}}\right)\times\left(-\frac{\hat{\mathbf{x}}+\hat{\mathbf{y}}}{\sqrt{2}}\right)$

$$= \frac{1}{2}[-(\hat{\mathbf{x}}\times\hat{\mathbf{x}})+(\hat{\mathbf{x}}\times\hat{\mathbf{y}})-(\hat{\mathbf{y}}\times\hat{\mathbf{x}})+(\hat{\mathbf{y}}\times\hat{\mathbf{y}})]$$

$$= \frac{1}{2}[-(0)+(\hat{\mathbf{z}})-(-\hat{\mathbf{z}})+(0)]$$

$$= \frac{1}{2}[\hat{\mathbf{z}}+\hat{\mathbf{z}}] = \frac{1}{2}(2\,\hat{\mathbf{z}}) = \hat{\mathbf{z}}$$

which satisfies for Eq. (ii).

So, this option is correct.

(c) $\mathbf{B} = \frac{E_0}{c}(\hat{\mathbf{x}}-\hat{\mathbf{y}})\sin(kz-\omega t)$

Here, $\hat{\mathbf{B}} = \frac{\hat{\mathbf{x}}-\hat{\mathbf{y}}}{\sqrt{1^2+1^2}} = \frac{\hat{\mathbf{x}}-\hat{\mathbf{y}}}{\sqrt{2}}$

So, $\hat{\mathbf{E}}\times\hat{\mathbf{B}} = \left(\frac{\hat{\mathbf{x}}+\hat{\mathbf{y}}}{\sqrt{2}}\right)\times\left(\frac{\hat{\mathbf{x}}-\hat{\mathbf{y}}}{\sqrt{2}}\right)$

$$= \frac{1}{2}[(\hat{\mathbf{x}}\times\hat{\mathbf{x}})-(\hat{\mathbf{x}}\times\hat{\mathbf{y}})+(\hat{\mathbf{y}}\times\hat{\mathbf{x}})-(\hat{\mathbf{y}}\times\hat{\mathbf{y}})]$$

$$= \frac{1}{2}[(0)-(\hat{\mathbf{z}})+(-\hat{\mathbf{z}})-(0)]$$

$$= \frac{1}{2}[-\hat{\mathbf{z}}-\hat{\mathbf{z}}]$$

$$= \frac{1}{2}(-2\,\hat{\mathbf{z}}) = -\hat{\mathbf{z}}$$

which does not satisfy for Eq. (ii).

So, this option is incorrect.

(d) $\mathbf{B} = \frac{E_0}{c}(\hat{\mathbf{x}}-\hat{\mathbf{y}})\cos(kz-\omega t)$

This option is incorrect, as the function in Eq. (i) is a "sine" function, not a "cosine" function.

Hence, option (b) is correct.

10. In the free space, the speed of electromagnetic wave is given as

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{E_0}{B_0} \quad \ldots\text{(i)}$$

where, E_0 and B_0 are the amplitudes of varying electric and magnetic fields, respectively.

Now, when it enters in a medium of refractive index n, its speed is given as

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{K\varepsilon_0\mu}} = \frac{E}{B} \quad \ldots\text{(ii)}$$

where, K is dielectric strength of the medium.

Using Eqs. (i) and (ii), we get

$$\frac{v}{c} = \frac{1}{\sqrt{K}} \quad \ldots\text{(iii)}$$

($\because$ For a transparent medium, $\mu_0 \approx \mu$)

Also, refractive index of medium is n and is given as

$$\frac{c}{v} = n \quad \text{or} \quad \frac{v}{c} = \frac{1}{n} \quad \ldots\text{(iv)}$$

$\therefore$ From Eqs. (iii) and (iv), we get

$$n = \sqrt{K} \quad \text{or} \quad K = n^2 \quad \ldots\text{(v)}$$

The intensity of an EM wave is given as

$$I = \frac{1}{2}\varepsilon_0 E_0^2 c$$

and in the medium, it is given as $I' = \frac{1}{2} K\varepsilon_0 E^2 v$

It is given that, $I = I'$

$\Rightarrow \quad \frac{1}{2}\varepsilon_0 E_0^2 c = \frac{1}{2} K\varepsilon_0 E^2 v$

or $\quad \left(\frac{E_0}{E}\right)^2 = \frac{Kv}{c} \quad \ldots(\text{vi})$

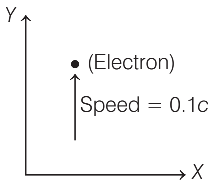
From Eqs. (iv), (v) and (vi), we get

$$\left(\frac{E_0}{E}\right)^2 = \frac{n^2}{n} = n \quad \text{or} \quad E_0 / E = \sqrt{n}$$

Similarly, $\frac{1}{2}\cdot\frac{B_0^2}{\mu_0} c = \frac{1}{2}\cdot\frac{B^2}{\mu_0} v$

$\Rightarrow \quad \frac{B_0}{B} = \frac{1}{\sqrt{n}}$

11. The moving electron is shown in the following figure.



In electromagnetic wave, $B_{\max} = \frac{E_{\max}}{c}$

We know that, magnetic force on a moving charge (electron) is given by

$$\mathbf{F} = q\mathbf{v}\times\mathbf{B}$$

Here, angle between $\mathbf{v}$ and $\mathbf{B}$ is 90°.

So, $\quad F_{\max} = qvB_{\max}\sin\theta$

$$= qvB_{\max}\sin 90^\circ = \frac{qvE_{\max}}{c}$$

Given, $\quad c = 3\times 10^8$ m/s, $v = 0.1\,c$

Charge on electron, $q = 1.6\times 10^{-19}$ C

Substituting these values in above equation, we get

$$F_{\max} = \frac{1.6\times 10^{-19}\times 0.1\times 3\times 10^8\times 30}{3\times 10^8}$$

$$= 4.8\times 10^{-19}\ \text{N}$$

12. Energy density due to electric field,

$$u = \frac{1}{2}\varepsilon_0 E^2$$

Here, $\quad E = \frac{E_0}{\sqrt{2}} = \frac{40}{\sqrt{2}}$

$\therefore \quad u = \frac{1}{2}\times 8.85\times 10^{-12}\times\frac{40\times 40}{2}$

$$= 3.54\times 10^{-9}\ \text{J/m}^3$$

$\therefore$ Hence, p is 3.54.

13. In vacuum, wavelength,

$$\lambda_0 = \frac{c}{f_0} = \frac{3\times 10^8}{2\times 10^6} = 1.5\times 10^2 = 150\ \text{m}$$

In medium, speed of wave,

$$v = \frac{c}{\sqrt{\varepsilon_r\mu_r}} \approx \frac{c}{\sqrt{\varepsilon_r}}$$

(As medium is non-magnetic)

$\therefore \quad v = \frac{c}{\sqrt{9}} = \frac{c}{3} = 1\times 10^8\,\text{ms}^{-1}$

Frequency remaining same, wavelength in medium is

$$\lambda = \frac{v}{f_0} = \frac{10^8}{2\times 10^6} = 50\ \text{m}$$

$\therefore$ Wavelength decreases by 100 m.

14. As, $F = pA = \frac{UA}{c}$

$\Rightarrow \quad U = \frac{Fc}{A}$

$$= \frac{2.5\times 10^{-6}\times 3\times 10^8}{30} = 25\ \text{W/cm}^2$$

15. Given, intensity, $I = \left(\frac{315}{\pi}\right)$ W/m^2

Absolute permittivity, $\varepsilon_0 = 8.86\times 10^{-12}\,\text{C}^2\text{N}^{-1}\text{m}^{-2}$

Speed of light, $c = 3\times 10^8$ m/s

As, intensity, $\quad I = \frac{1}{2}\varepsilon_0 E_0^2 c$

or $\quad E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}} = \sqrt{\frac{2\times\left(\frac{315}{\pi}\right)}{8.86\times 10^{-12}\times 3\times 10^8}}$

$$E_0 \approx 275\ \text{V/m}$$

$\therefore \quad E_{\text{rms}} = \frac{E_0}{\sqrt{2}} = \frac{275}{\sqrt{2}} \simeq 194\ \text{V/m}$