VERY SIMILAR
PRACTICE TEST 7

Hints and Explanations

1. **(a) :** Here, $u = 0,\ a = \dfrac{eE}{m}$

$\therefore \quad v = u + at = 0 + \dfrac{eE}{m}t$

de-Broglie wavelength,

$$\lambda = \frac{h}{mv} = \frac{h}{m(eEt/m)} = \frac{h}{eEt}$$

Rate of change of de-Broglie wavelength

$$\frac{d\lambda}{dt} = \frac{h}{eE}\left(-\frac{1}{t^2}\right) = \frac{-h}{eEt^2}$$

2. **(c) :** According to Bernoulli's theorem,

$$P + \frac{1}{2}\rho v^2 = \text{constant.}$$

Near the ends, the velocity of liquid is higher so that pressure is lower as a result the liquid rises at the sides to compensate for this drop of pressure.

i.e., $\rho gh = \dfrac{1}{2}\rho v^2 = \dfrac{1}{2}\rho r^2\omega^2$

Hence, $h = \dfrac{r^2\omega^2}{2g} = \dfrac{r^2(2\pi\upsilon)^2}{2g} = \dfrac{2\pi^2 r^2\upsilon^2}{g}$

$$= \frac{2 \times \pi^2 \times (0.05)^2 \times 2^2}{9.8}$$

$= 0.02$ m $= 2$ cm

3. **(c) :** Intensity $= \dfrac{1}{2}A^2\omega^2\rho v = \dfrac{1}{2}A^2(2\pi\upsilon)^2\rho v$

$= 2\pi^2\upsilon^2 A^2\rho v$

Also, intensity $= \dfrac{\text{power}}{4\pi r^2}$

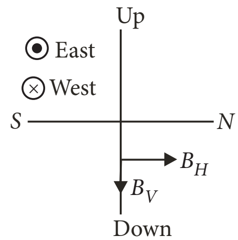
$\therefore \quad 2\pi^2\upsilon^2 A^2\rho v = \dfrac{\text{power}}{4\pi r^2}$

$$A = \frac{1}{2\pi r\upsilon}\sqrt{\frac{\text{power}}{2\pi\rho v}}$$

$$A = \frac{1}{2\pi(10)(1000)}\sqrt{\frac{10}{2\pi(1.29)340}}$$

$= 9.6 \times 10^{-7}$ m $= 0.96$ µm

4. **(a) :** Let us first decide the directions which can best represent the situation.



Here, $B_H = B\cos\delta$

$= 0.39 \times \cos 35°$

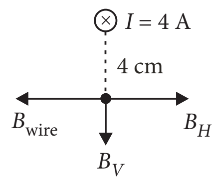
$B_H = 0.32$ G

and $B_V = B\sin\delta$

$= 0.39 \times \sin 35°$

$B_V = 0.22$ G

Telephone cable carry a total current of 4.0 A in direction east to west. We want resultant magnetic field 4.0 cm below.



Now, $B_{wire} = \dfrac{\mu_0}{4\pi}\dfrac{2I}{r}$

$= 10^{-7} \times \dfrac{2 \times 4}{4 \times 10^{-2}}$

$= 2 \times 10^{-5}$ T $= 0.2$ G

Net magnetic field,

$$B_{net} = \sqrt{(B_H - B_{wire})^2 + B_V^2}$$

$$B_{net} = \sqrt{(0.12)^2 + (0.22)^2} = 0.25\text{ G}$$

5. **(a) :** Gravitational potential energy of mass m at any point at a distance r from the centre of earth is

$$U = -\frac{GMm}{r}$$

At the surface of earth $r = R$,

$\therefore \quad U_s = -\dfrac{GMm}{R} = -mgR \qquad \left(\because\ g = \dfrac{GM}{R^2}\right)$

At the height $h = nR$ from the surface of earth

$r = R + h = R + nR = R(1 + n)$

$\therefore \quad U_h = -\dfrac{GMm}{R(1+n)} = -\dfrac{mgR}{(1+n)}$

Change in gravitational potential energy is

$\Delta U = U_h - U_s$

$= -\dfrac{mgR}{(1+n)} - (-mgR)$

$$= -\frac{mgR}{1+n} + mgR \quad = mgR\left(1 - \frac{1}{1+n}\right)$$

$$= mgR\left(\frac{n}{1+n}\right)$$

6. (b) : $\gamma_{\text{mixture}} = \dfrac{\dfrac{n_1\gamma_1}{\gamma_1 - 1} + \dfrac{n_2\gamma_2}{\gamma_2 - 1}}{\dfrac{n_1}{\gamma_1 - 1} + \dfrac{n_2}{\gamma_2 - 1}}$

Here, $n_1 = 3,\ \gamma_1 = \frac{5}{3}$, $n_2 = 1, \gamma_2 = \frac{7}{5}$

$$\therefore \quad \gamma_{\text{mixture}} = \frac{\dfrac{3\times\frac{5}{3}}{\left[\frac{5}{3}-1\right]} + \dfrac{1\times\frac{7}{5}}{\left[\frac{7}{5}-1\right]}}{\dfrac{3}{\left[\frac{5}{3}-1\right]} + \dfrac{1}{\left[\frac{7}{5}-1\right]}}$$

$$= \frac{\frac{15}{2} + \frac{7}{2}}{\frac{9}{2} + \frac{5}{2}} = \frac{22}{14} = \frac{11}{7}$$

7. (b) : Net moment of inertia of the system,
$I = I_1 + I_2 + I_3$
The moment of inertia of a shell about its diameter,

$$I_1 = \frac{2}{3}mr^2$$

The moment of inertia of a shell about its tangent is given by

$$I_2 = I_3 = I_1 + mr^2 = \frac{2}{3}mr^2 + mr^2 = \frac{5}{3}mr^2$$

$$\therefore \quad I = 2\times\frac{5}{3}mr^2 + \frac{2}{3}mr^2 = \frac{12mr^2}{3} = 4mr^2$$

8. (a) : Given, $\vec{r} = \{(2t)\hat{i} + (2t^2)\hat{j}\}\text{m}$
Comparing it with standard equation of position vector, $\vec{r} = x\hat{i} + y\hat{j}$, we get $x = 2t$ and $y = 2t^2$

$$\Rightarrow \quad v_x = \frac{dx}{dt} = 2 \text{ and } v_y = \frac{dy}{dt} = 4t$$

$$\therefore \quad \tan\theta = \frac{v_y}{v_x} = \frac{4t}{2} = 2t$$

Differentiating with respect to time we get,

$$(\sec^2\theta)\frac{d\theta}{dt} = 2$$

or $(1+\tan^2\theta)\dfrac{d\theta}{dt} = 2$ or $(1+4t^2)\dfrac{d\theta}{dt} = 2$

or $\dfrac{d\theta}{dt} = \dfrac{2}{1+4t^2}$

at $t = 2$ s, $\left(\dfrac{d\theta}{dt}\right) = \dfrac{2}{1+4(2)^2} = \dfrac{2}{17}$ rad s^{-1}

9. (b) : Induced emf, $E = -\dfrac{d\phi}{dt} = 2at - a\tau$,

Current through the loop, $I = \dfrac{E}{R} = \dfrac{2at - a\tau}{R}$ ...(i)
dQ = amount of heat generated in a time $dt = I^2Rdt$.
$\therefore$ Total amount of heat generated in that time is

$$Q = \int dQ = \int_0^\tau I^2Rdt = \int_0^\tau \frac{(2at - a\tau)^2 R\,dt}{R^2} \qquad \text{(Using (i))}$$

By solving integration, we get

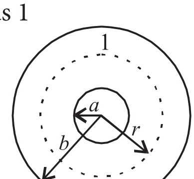
$$Q \approx \frac{a^2\tau^3}{3R}$$

10. (c) : As $F \propto S^{-1/3}$,
$\therefore$ Acceleration, $a \propto S^{-1/3}$

$$a = \frac{dv}{dt} = \frac{dv}{dS}\cdot\frac{dS}{dt} = \frac{dv}{dS}v \quad \textit{i.e.,}\ v\frac{dv}{dS} \propto S^{-1/3}$$

Integrating both sides, we get
$v^2 \propto S^{2/3}$ or $v \propto S^{1/3}$
As $P = Fv$
$\therefore P \propto S^{-1/3}\, S^{1/3}$ or $P \propto S^0$.
i.e., power is independent of S.

11. (d) : Given : Current density $J = Cr^2$
For a region $a < r < b$. The Amperian loop is a circle labelled as 1



According to Ampere's circuital law

$$= \oint \vec{B}.d\vec{l} = \mu_0 I$$

$$B(2\pi r) = \mu_0\int JdA = \mu_0\int_a^r Cr'^2(2\pi r'dr')$$

$$= \mu_0 C2\pi\int_a^r r'^3\,dr' = \mu_0 C2\pi\left[\frac{r'^4}{4}\right]_a^r$$

$$B2\pi r = \frac{\mu_0 C2\pi}{4}[r^4 - a^4]; \quad B = \frac{\mu_0 C}{4r}[r^4 - a^4]$$

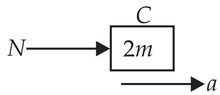
12. (b) : Total mass of the system = $2m + m + 2m = 5m$

$\therefore$ Horizontal acceleration of the system,

$$a = \frac{F}{5m} \quad \text{...(i)}$$

Let N be the normal reaction between blocks B and C.

From the free body diagram of C, as shown in figure.



$$N = 2ma = 2m \times \frac{F}{5m} = \frac{2}{5}F \text{ (Using (i))} \quad \text{...(ii)}$$

Block B will not slide (downwards), if force of friction; $f \geq m_B g$, *i.e.*, $\mu N \geq mg$

$$\mu\left(\frac{2}{5}F\right) \geq mg$$

$$F \geq \frac{5}{2\mu}mg$$

Hence, $F_{\min} = \frac{5}{2\mu}mg$

13. (b) : From the relation, $h = ut + \frac{1}{2}gt^2$

$h = \frac{1}{2}gt^2 \Rightarrow g = \frac{2h}{t^2}$ ($\because u = 0$, body initially at rest)

Taking natural logarithm on both sides, we get

$$\ln g = \ln h - 2\ln t$$

Differentiating, $\frac{\Delta g}{g} = \frac{\Delta h}{h} - 2\frac{\Delta t}{t}$

For maximum permissible error,

$$\text{or} \left(\frac{\Delta g}{g} \times 100\right)_{\max} = \left(\frac{\Delta h}{h} \times 100\right) + 2 \times \left(\frac{\Delta t}{t} \times 100\right)$$

According to problem

$$\frac{\Delta h}{h} \times 100 = e_1 \text{ and } \frac{\Delta t}{t} \times 100 = e_2$$

Therefore, $\left(\frac{\Delta g}{g} \times 100\right)_{\max} = e_1 + 2e_2$

14. (d) : As $U = U_0 e^{-\frac{2t}{RC}}$

$$\therefore \frac{U_0}{2} = U_0 e^{\frac{-2t_1}{RC}}$$

$$\Rightarrow \frac{2t_1}{RC} = \log_e 2 \quad \text{...(i)}$$

Also, $q = q_0 e^{-\frac{t}{RC}}$

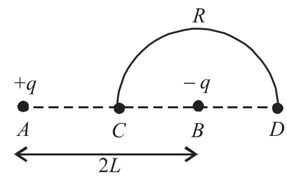
$$\therefore \frac{q_0}{4} = q_0 e^{-\frac{t_2}{RC}}$$

$$\Rightarrow \frac{t_2}{RC} = \log_e 4 = 2\log_e 2 \quad \text{...(ii)}$$

From equation (i) and (ii)

$$\frac{2t_1}{t_2} = \frac{\log_e 2}{2\log_e 2} \text{ or } \frac{t_1}{t_2} = \frac{1}{4}$$

15. (c) :



From figure, $AC = L$, $BC = L$, $BD = BC = L$

$AD = AB + BD = 2L + L = 3L$

Potential at C is given by

$$V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{AC} + \frac{(-q)}{BC}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{L} - \frac{q}{L}\right] = 0$$

Potential at D is given by

$$V_D = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{AD} + \frac{(-q)}{BD}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{3L} - \frac{q}{L}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{L}\left[\frac{1}{3} - 1\right] = \frac{-q}{6\pi\varepsilon_0 L}$$

Work done in moving charge $+Q$ along the semicircle CRD is given by

or $K_1 = \frac{5.5}{55}$ MeV

16. (b) : From the law of conservation of linear momentum, we get

$$p_1 = p_2$$

But $p = \sqrt{2mK}$ where K = kinetic energy

$$\therefore \sqrt{2(216m)K_1} = \sqrt{2(4m)K_2}$$

where K_1 is for nucleus and K_2 is for α particle

or $216K_1 = 4K_2$ or $K_2 = 54K_1$ .... (i)

Given : $K_1 + K_2 = 5.5$ MeV .... (ii)

From equations (i) and (ii), we get

$K_1 + 54K_1 = 5.5$ MeV

or $55K_1 = 5.5$ MeV

or $K_1 = \frac{5.5}{55}$ MeV

or $K_1 = \frac{1}{10}$ MeV

$\because \quad K_2 = 54K_1$ or $K_2 = \frac{54}{10}$ MeV

or $K_2 = 5.4$ MeV

$\therefore$ Kinetic energy of α particle $= 5.4$ MeV

17. (a) : $I_g = 16 \times 30\ \mu A = 480 \times 10^{-6}$ A

Let G be the resistance of galvanometer and R be the resistance connected in series to convert the galvanometer into voltmeter of range 0 to 3 V.

Then, $G + R = \frac{V}{I_g} = \frac{3}{480 \times 10^{-6}}$

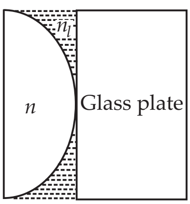
$= 6.25 \times 10^3 = 6.25$ kΩ

$\therefore$ A resistance R nearly 6 kΩ is to be used in series.

18. (d) : It is clear from given logic circuit, that output Y is low when both the inputs are high, otherwise it is high. Thus logic circuit is NAND gate.

A	B	Y
1	1	0
0	0	1
0	1	1
1	0	1

19. (c) :



According to lens maker's formula

The focal length of plano convex lens is

$$\frac{1}{f} = (n-1)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$$

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R}\right) = \frac{1}{2R}$$

or $R = \frac{f}{2}$ ...(i)

The focal length of liquid lens is

$$\frac{1}{f_l} = (n_l - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$$

$$\frac{1}{f_l} = -\frac{(n_l - 1)}{R}$$

$$\frac{1}{f_l} = -\frac{2(n_l - 1)}{f} \quad \text{(Using (i))}$$

Effective focal length of the combination is

$$\frac{1}{2f} = \frac{1}{f} + \frac{1}{f_l}$$

$$\frac{1}{2f} = \frac{1}{f} - \frac{2(n_l - 1)}{f}$$

$$n_l = \frac{5}{4} = 1.25$$

20. (c) : The transmitted power in AM wave is

$$P_t = \left(1 + \frac{\mu^2}{2}\right)P_c$$

where μ is the modulation index and P_c is the power in carrier wave.

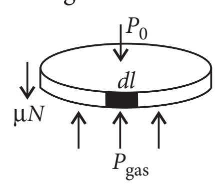
When $\mu = 0$, $P_t = P_c$

and when $\mu = 1$, $P_t = \left(1 + \frac{1}{2}\right)P_c = \frac{3}{2}P_c$

$\therefore$ The percentage increased in transmitted power

$$= \frac{\frac{3}{2}P_c - P_c}{P_c} \times 100\% = \frac{1}{2} \times 100\% = 50\%$$

21. (6.28) : Since the volume of the gas is constant



$$\frac{P_i}{T_i} = \frac{P_f}{T_f}$$

$$P_f = P_i\left(\frac{T_f}{T_i}\right) = 2P_i = 2P_0$$

$$(\because \quad T_f = 2T_i)$$

At equilibrium, $P_0 \times A + \mu N = 2P_0 A$

or $N = \frac{P_0 A}{\mu}$

N is the total normal force exerted by the tube on the cork, hence contact force per unit length is

$$\frac{dN}{dl} = \frac{N}{2\pi r} = \frac{P_0 A}{2\pi \mu r} = \frac{P_0 A}{6.28\ \mu r}$$

$n = 6.28$

22. (8.0) : Amplitude of the damped oscillation at time t is given by

$A = A_0 e^{-bt/2m}$

When $t = 1$ minute, $A = A_0/2$, then

$$\frac{A_0}{2} = A_0 e^{-b \times 1/2m}$$

or $e^{b/2m} = 2$ ...(i)

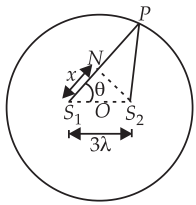
When $t = 3$ minute, $A = \frac{A_0}{x}$, then

$$\frac{A_0}{x} = A_0 e^{-b\times 3/2m}$$

or $\quad x = e^{3b/2m} = \left(e^{b/2m}\right)^3 = 2^3 = 8.0 \quad$ (Using (i))

23. (12.0) : As it is clear from figure, path difference between light waves from S_1 and S_2 is

$x = S_1N = S_1S_2\cos\theta = 3\lambda\cos\theta$



For constructive interference

$x = 3\lambda\cos\theta = n\lambda$

$$\cos\theta = \frac{n}{3}$$

Putting $n = 0, 1, 2, 3$

$$\theta = 90^\circ, \theta = \cos^{-1}\frac{1}{3}, \theta = \cos^{-1}\frac{2}{3}; \theta = 0^\circ$$

These are the positions of one quadrant. They repeat in all the four quadrants. Total number of distinct bright fringes = 12.0

24. (105) : Let the successive loops formed be p and $(p + 1)$ for frequencies 315 Hz and 420 Hz.

$$\therefore\ \upsilon = \frac{p}{2L}\sqrt{\frac{T}{\mu}} = \frac{pv}{2L}$$

$$\therefore\ \frac{pv}{2L} = 315\text{ Hz and } \frac{(p+1)v}{2L} = 420\text{ Hz}$$

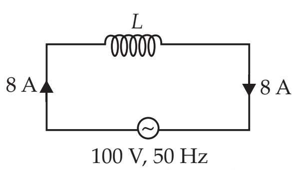
or $\quad \frac{(p+1)v}{2L} - \frac{pv}{2L} = 420\text{ Hz} - 315\text{ Hz}$

or $\quad \frac{v}{2L} = 105\text{ Hz} \Rightarrow \frac{1\times v}{2L} = 105\text{ Hz}$

$p = 1$ for fundamental mode of vibration of string.

$\therefore$ Lowest resonant frequency = 105 Hz.

25. (10.6) : An ideal choke has zero ohmic resistance.

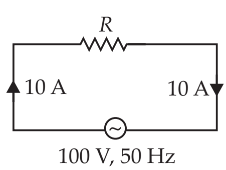


The inductive reactance of a choke is

$X_L = 2\pi\upsilon L$

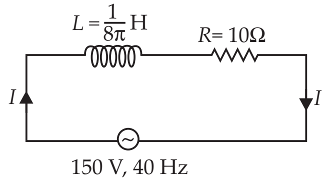
$$\therefore\ \frac{100}{8} = 2\pi\times 50\times L$$

or $\quad L = \frac{1}{8\pi}\text{ H}$



The resistance of the resistor is

$$R = \frac{100}{10} = 10\ \Omega$$



Now, the inductive reactance of the choke is

$$X'_L = 2\pi\upsilon' L = 2\pi\times 40\times\frac{1}{8\pi} = 10\ \Omega$$

The impedance of the circuit is

$$Z = \sqrt{R^2 + X_L'^2} = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2}\ \Omega$$

Current in the circuit is

$$I = \frac{V}{Z} = \frac{150}{10\sqrt{2}} = 10.6\text{ A}$$

26. (c) : Molar ratio $A : \text{O} = \frac{50}{12} : \frac{50}{16} = 4.16 : 3.12$

$= 1.33 : 1 = 4 : 3$

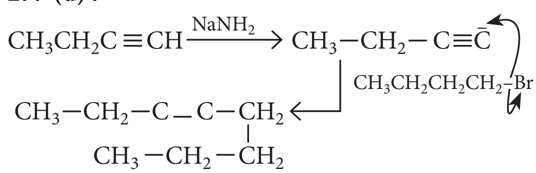
E.F. = A_4O_3

E.F.W. = (12 × 4) + (16 × 3) = 96

$$n = \frac{\text{M.F.W}}{\text{E.F.W}} = \frac{288}{96} = 3$$

Molecular formula = $(A_4O_3)_3 = A_{12}O_9$

27. (d) :



28. (a) : $\lambda^\circ_{HCOOH} = \lambda^\circ_{H^+} + \lambda^\circ_{HCOO^-}$

$= 349.6 + 54.6$

$= 404.2\text{ S cm}^2\text{ mol}^{-1}$

$$\alpha = \frac{\Lambda_m}{\Lambda_m^\circ} = \frac{46.1}{404.2} = 0.114 \text{ or } 11.4\%$$

$$K_a = \frac{C\alpha^2}{1-\alpha} = \frac{0.025\times(0.114)^2}{1-0.114}$$

$$= \frac{0.025\times 0.114\times 0.114}{0.886}$$

$= 3.67\times 10^{-4}$

29. (c)

30. (c) : Silicates are very inert and are not easily hydrolysed (statement 2 is wrong). Each oxygen atom in SiO_2 is shared by 2 silicon atoms (not four). So statement 4 is wrong.

31. (c) : A solution can be colloidal only above CMC. A solution with concentration lower than CMC will be true solution.

32. (d) **33. (b)** **34. (a)**

35. (a) : Compound (a) is enantiomer of compound (I) because the configuration of two groups, *i.e.*, CH_3 and C_2H_5 in them is reversed at the chiral carbon.

36. (d) : Electronegativity decreases down the group as the size of the atom increases.

37. (d) : $\underset{(A)}{CH_2}=\underset{(B)}{C}=\underset{(C)}{\overset{CH_3}{\overset{|}{C}}}-CH=\underset{(A)}{CH_2} \xrightarrow[Zn]{O_3/H_2O}$

$\underset{(A)}{2HCHO} + \underset{(B)}{CO_2} + \underset{(C)}{CH_3COCHO}$

38. (b) : $CH_{4(g)} + 2H_2S_{(g)} \overset{K}{\rightleftharpoons} CS_{2(g)} + 4H_{2(g)}$

$$K = \frac{[CS_2][H_2]^4}{[CH_4][H_2S]^2} \quad \text{...(i)}$$

$$[CS_2] = \left(\frac{3}{10}\right); [H_2] = \left(\frac{3}{10}\right);$$

$$[CH_4] = \left(\frac{2}{10}\right); [H_2S] = \left(\frac{4}{10}\right)$$

Substituting all the concentration values in equation (i), we get

$$K = \frac{\left(\frac{3}{10}\right)\left(\frac{3}{10}\right)^4}{\left(\frac{2}{10}\right)\left(\frac{4}{10}\right)^2} = \frac{3^5}{10^5} \times \frac{10^3}{2 \times 4^2} = \frac{243}{3200} \Rightarrow 0.076$$

Therefore, $K > 2.5 \times 10^{-3}$ (K_c). Hence, the reaction will proceed in backward direction.

39. (d) : There is no change in volume from $A \to B$.

40. (c) : H_2O and NH_3 have abnormally high boiling points because of their tendency to form hydrogen bonds. NH_3 has higher boiling point than phosphine and the boiling point increases down the group because of increase in size. Hence, the order of boiling point will be

$H_2O > SbH_3 > NH_3 > AsH_3 > PH_3$

41. (d) : Vapour pressure of all liquids are different at their freezing points.

42. (c) : Five-membered rings are named as furanose while six-membered rings are named as pyranose rings.

43. (d) : $E_n = -\frac{13.6\,Z^2}{n^2}$ eV atom^{-1}

(P) For He^+, $Z = 2$; for ground state of He^+, $n = 1$
Hence, $E_1 = -13.6 \times 2^2 = -54.4$ eV atom^{-1}
But $E_1 = P.E. + K.E.$

$$-54.4 \text{ eV atom}^{-1} = P.E. - \frac{P.E.}{2} = \frac{P.E.}{2}$$

$\Rightarrow$ $P.E. = -108.8$ eV atom^{-1}

(Q) $K.E. = -\frac{P.E.}{2} = -\frac{-(-108.8)}{2} = +54.4$ eV atom^{-1}

(R) $E_1 = E_{total} = -54.4$ eV atom^{-1}
(S) For ionization of 1st excited state of He^+, $n_1 = 2, n_2 = \infty$

Hence, $I.E. = \Delta E = 13.6 \times Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$$= 13.6 \times 2^2\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right] = +13.6 \text{ eV atom}^{-1}$$

44. (b) : 10 mL of 1 M $KMnO_4$ oxidises 10 mL of 5 M $FeSO_4$ in acidic medium.

$2KMnO_4 + 8H_2SO_4 + 10FeSO_4 \longrightarrow K_2SO_4 + 2MnSO_4 + 5Fe_2(SO_4)_3 + 8H_2O$

45. (d) : Cupellation can be used in such a case.

46. (2) : Phenelzine and equanil are tranquilisers and morphine is narcotic analgesic and bithionol is an antiseptic. Alitame is used as an artificial sweetening agent. Novestrol is an antifertility drug. Dimetapp and seldane are antihistamines.

47. (4) : BF_3, $AlCl_3$ are electron deficient molecules while in PCl_5 and SF_6 molecules, P and S possess more than 8 electrons in their valence shells.

48. (4) : $\underline{P}O_4^{3-} \rightarrow x + 4(-2) = -3, \Rightarrow x = +5$
$H_2\underline{S}_2O_8 \rightarrow 2(+1) + 2x + 2(-1) + 6(-2) = 0$
$\Rightarrow$ $x = +6$ (two oxygen atoms with peroxide linkage).
$H_2\underline{S}O_5 \rightarrow 2(+1) + x + 2(-1) + 3(-2) = 0$
$\Rightarrow$ $x = +6$ (two oxygen atoms with peroxide linkage)
$\underline{O}F_2 \rightarrow x + 2(-1) = 0, \Rightarrow x = +2$
$\underline{Cr}_2O_7^{2-} \rightarrow 2x + 7(-2) = -2, \Rightarrow x = +6$
$\underline{Cr}O_5 \rightarrow x + 4(-1) + (-2) = 0,$
$\Rightarrow$ $x = +6$
(four oxygen atoms with peroxide linkage).

49. (2) : Both aliphatic and aromatic secondary amines undergo Liebermann's nitroso reaction. Thus, this test is used to separate secondary amines from primary and tertiary amines.

50. (15.05) : The compound $Fe_{0.93}O_{1.00}$ is non-stoichiometric where electrical neutrality is achieved by converting appropriate Fe^{2+} ions into Fe^{3+} ions.

There are 7 Fe^{2+} ions missing out of the expected 100 Fe^{2+} ions. The missing 2 × 7 positive charge is compensated by the presence of Fe^{3+} ions. Replacement of one Fe^{2+} ion by Fe^{3+} ion increases one positive charge.
Thus, 14 positive charges are compensated by the presence of 14 Fe^{3+} ions out of a total of 93 Fe ions.
Hence, % of Fe^{3+} ions present = $\frac{14}{93} \times 100 = 15.05$

51. (a) : Given, $f(x)=\frac{1}{\sqrt{|x|-x}}$
Clearly, $|x| - x > 0$
$\Rightarrow$ $|x| > x \Rightarrow x$ is negative
$\therefore$ The domain of the function $f(x)$ is $(-\infty, 0)$

52. (c) : $(A + B)(A - B) = A^2 - B^2$
$\Rightarrow$ $AA + BA - AB - BB = A^2 - B^2$
$\Rightarrow$ $A^2 + BA - AB - B^2 = A^2 - B^2$
$\Rightarrow$ $BA - AB = 0 \Rightarrow BA = AB$
$\therefore$ $(ABA^{-1})^2 = (BAA^{-1})^2 = (BI)^2 = B^2$

53. (a) : Given, $k\int_0^1 x\cdot f(3x)dx = \int_0^3 t\cdot f(t)dt$ ...(i)

Let $I = k\int_0^1 x\cdot f(3x)dx$
Put $3x = t \Rightarrow dx = dt/3$.
So, $x = 0 \Rightarrow t = 0; x = 1 \Rightarrow t = 3$

$\therefore$ $I = k\int_0^3 \frac{1}{3}\frac{t}{3}f(t)\,dt$

$= \frac{k}{9}\int_0^3 t\cdot f(t)\,dt = \int_0^3 t\cdot f(t)dt$ [Using (i)]

$\Rightarrow$ $\frac{k}{9} = 1 \Rightarrow k = 9$

54. (a)

55. (b) : We have,
$\frac{S_n}{S'_n} = \frac{(n/2)[2a+(n-1)d]}{(n/2)[2a'+(n-1)d']} = \frac{3n+8}{7n+15}$, where S_n and S'_n are sum of n terms of given two A.P.

or $\frac{a+\left(\frac{n-1}{2}\right)d}{a'+\left(\frac{n-1}{2}\right)d'} = \frac{3n+8}{7n+15}$ ...(i)

Choosing $(n - 1)/2 = 11$ or $n = 23$ in (i), we get
$$\frac{T_{12}}{T'_{12}} = \frac{a+11d}{a'+11d'} = \frac{7}{16}$$

56. (c) : The distance of the point C from the line $\vec{r} = \vec{a} + s\vec{b}$ is $\frac{|(\vec{c}-\vec{a})\times\vec{b}|}{|\vec{b}|}$

Here, $\vec{a} = \hat{i} - 2\hat{j} - \hat{k}, \vec{c} = 4\hat{i} + 2\hat{j} + 5\hat{k}$ and
$\vec{b} = 6\hat{i} + 3\hat{j} + 2\hat{k}$

$$\therefore \quad (\vec{c}-\vec{a})\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 6 \\ 6 & 3 & 2 \end{vmatrix}$$

$= -10\hat{i} + 30\hat{j} - 15\hat{k} = 5(-2\hat{i} + 6\hat{j} \quad 3\hat{k})$

Hence, $d = \frac{|(\vec{c}-\vec{a})\times\vec{b}|}{|\vec{b}|} = \frac{5\times 7}{7} = 5.$

57. (b) : Clearly,
$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + ... + {}^nC_nx^n$

$\left(1+\frac{1}{x}\right)^n = {}^nC_0 + {}^nC_1\frac{1}{x} + {}^nC_2\frac{1}{x^2} + ... + {}^nC_n\left(\frac{1}{x}\right)^n$

Now, required coefficient of $\frac{1}{x}$ is given by

${}^nC_0\,{}^nC_1 + {}^nC_1\,{}^nC_2 + ... + {}^nC_{n-1}\,{}^nC_n = \frac{(2n)!}{(n-1)!(n+1)!}$

58. (a) : Clearly, the lines will be concurrent if
$$\begin{vmatrix} 1 & -2 & 3 \\ k & 3 & 1 \\ 4 & -k & 2 \end{vmatrix} = 0$$
$\Rightarrow$ $1(6 + k) + 2(2k - 4) + 3(-k^2 - 12) = 0$
or $3k^2 - 5k + 38 = 0.$
No real roots since the discriminant is negative.

59. (c) : Mean square deviation about the value $x = 0$ is

$\frac{1}{N}\sum_{i=1}^{n} f_i x_i^2 = \frac{1}{2^n}\sum_{r=0}^{n} r^2\,{}^nC_r$ [where, $N = \sum f$]

$= \frac{1}{2^n}n\cdot\sum_{r=1}^{n} r\cdot{}^{n-1}C_{r-1} = \frac{1}{2^n}n\sum_{r=1}^{n}[(r-1)+1]\,{}^{n-1}C_{r-1}$

$= \frac{1}{2^n}\cdot n\left\{\sum_{r=1}^{n}(r-1)\,{}^{n-1}C_{r-1} + \sum_{r=1}^{n}{}^{n-1}C_{r-1}\right\}$

$$= \frac{1}{2^n} \cdot n\left\{(n-1)\sum_{r=2}^{n} {}^{n-2}C_{r-2} + 2^{n-1}\right\}$$

$$= \frac{1}{2^n} n\left\{(n-1)\, 2^{n-2} + 2^{n-1}\right\}$$

$$= \frac{n(n+1)2^{n-2}}{2^n} = \frac{n(n+1)}{4}$$

60. (c) : Given, $\cos\alpha = \dfrac{2\cos\beta - 1}{2 - \cos\beta}$

$$\Rightarrow \quad \frac{\dfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}}{} = \frac{2\left(1-\tan^2\frac{\beta}{2}\right)-\left(1+\tan^2\frac{\beta}{2}\right)}{2\left(1+\tan^2\frac{\beta}{2}\right)-\left(1-\tan^2\frac{\beta}{2}\right)}$$

$$\Rightarrow \quad \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{1-3\tan^2\frac{\beta}{2}}{1+3\tan^2\frac{\beta}{2}}$$

Applying componendo and dividendo, we get

$$\tan^2\frac{\alpha}{2} = 3\tan^2\frac{\beta}{2} \quad \Rightarrow \quad \tan\frac{\alpha}{2}\cdot\cot\frac{\beta}{2} = \sqrt{3}$$

61. (d) : Given limit $= \lim\limits_{x\to 0} \dfrac{(1+x)\ln(1+x) - x}{x^2}$

$$= \lim_{x\to 0} \frac{\ln(1+x)}{2x} \quad \text{(By L' Hospital's Rule)}$$

$$= \lim_{x\to 0} \frac{1}{2}\ln(1+x)^{\frac{1}{x}} = \frac{1}{2}$$

62. (c) : We have,

$$z = (1+i\sqrt{3})^{100} = 2^{100}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{100}$$

$$= 2^{100}\left(\cos\frac{100\pi}{3} + i\sin\frac{100\pi}{3}\right)$$

$$= 2^{100}\left(-\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right) = 2^{100}\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)$$

Now, $\dfrac{2\,\mathrm{Re}(z)}{\sqrt{3}\,\mathrm{Im}(z)} = \dfrac{2\times 2^{100}\cdot(-1/2)}{\sqrt{3}\cdot 2^{100}\cdot(-\sqrt{3}/2)} = \dfrac{2}{3}$

63. (a) : Since $\int e^x(f(x) + f'(x))dx = e^x f(x) + C$

$$\therefore \int e^x\left(\operatorname{cosec}^{-1}x + \frac{(-1)}{x\sqrt{x^2-1}}\right)dx = e^x \operatorname{cosec}^{-1}x + C$$

64. (b) : Clearly,

$$\Delta = \frac{1}{abc}\begin{vmatrix} a^3+ax & a^2b & a^2c \\ ab^2 & b^3+bx & b^2c \\ c^2a & c^2b & c^3+cx \end{vmatrix}$$

$$= \begin{vmatrix} a^2+x & a^2 & a^2 \\ b^2 & b^2+x & b^2 \\ c^2 & c^2 & c^2+x \end{vmatrix}$$

$$= (a^2+b^2+c^2+x)\times\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+x & b^2 \\ c^2 & c^2 & c^2+x \end{vmatrix}$$

(Applying $R_1 \to R_1 + R_2 + R_3$)

$$= (a^2+b^2+c^2+x)\begin{vmatrix} 1 & 0 & 0 \\ b^2 & x & 0 \\ c^2 & 0 & x \end{vmatrix} \quad \begin{pmatrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{pmatrix}$$

$$= x^2(a^2 + b^2 + c^2 + x)$$

Hence Δ is divisible by x^2 as well as by x.

65. (c) : Converse of given statement is
If Ram will get a mobile then he secures 100 marks in Math.

66. (b) : The given differential equation is
$ydx + (x + x^2y)dy = 0$

$\Rightarrow ydx + xdy + x^2ydy = 0 \Rightarrow d(xy) = -x^2ydy$

$$\Rightarrow \frac{d(xy)}{(xy)^2} = -\frac{dy}{y} \quad \Rightarrow \int\frac{d(xy)}{(xy)^2} = -\int\frac{dy}{y}$$

$\Rightarrow -\dfrac{1}{xy} = -\log_e|y| + c,$ which is the required solution of the given differential equation.

67. (d) : Equation of tangent at $(t^2, 2t)$ is

$x = ty - t^2$ ...(i)

Equation of normal to ellipse at $(\sqrt{5}\cos\alpha, 2\sin\alpha)$

is $\sqrt{5}x\sec\alpha - 2y\operatorname{cosec}\alpha = 1$ ...(ii)

Now, (i) = (ii)

$$\Rightarrow \sqrt{5}\sec\alpha = \frac{2\operatorname{cosec}\alpha}{t} = -\frac{1}{t^2}$$

$\Rightarrow \cos\alpha = -\sqrt{5}t^2$ and $\sin\alpha = -2t$

$\Rightarrow \cos^2\alpha + \sin^2\alpha = 5t^4 + 4t^2 = 1$

$$\Rightarrow t^2 = \frac{1}{5}.$$

Also, $\dfrac{\sin\alpha}{\cos\alpha} = \dfrac{-2t}{-\sqrt{5}t^2} = \dfrac{2}{\sqrt{5}} \times \dfrac{1}{t} = \dfrac{2}{\sqrt{5}} \times (\pm\sqrt{5})$

$\therefore$ $\tan\alpha = \pm 2 \Rightarrow \tan\alpha = -2, \tan\alpha = 2$

68. (d): Given, $f(x) = ax + \dfrac{b}{x} \Rightarrow f'(x) = a - \dfrac{b}{x^2}$

For minima, $f'(x) = 0 \Rightarrow a = \dfrac{b}{x^2} \Rightarrow x = \sqrt{\dfrac{b}{a}}$

$$f''(x) = \frac{2b}{x^3} \Rightarrow f''\left(\sqrt{\frac{b}{a}}\right) = \frac{2b}{\left(\frac{b}{a}\right)^{3/2}} > 0$$

Hence $f(x)$ is minimum at $x = \sqrt{\dfrac{b}{a}}$ and its minimum value is $a \times \sqrt{\dfrac{b}{a}} + \dfrac{b}{\left(\sqrt{\frac{b}{a}}\right)} = \sqrt{ab} + \sqrt{ab} = 2\sqrt{ab}$

69. (b) : Clearly x should lie in $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ and y in $[0, \pi]$ in order to get the integer value of $\sin^{-1}(\sin x) + \cos^{-1}(\cos y)$.

$\Rightarrow$ $x = 1$ and $y = 1, 2, 3$

$\therefore$ Required probability $= \dfrac{3}{16}$.

70. (b) : The 4 odd digits 1, 3, 3, 1 can be arranged in the 4 odd places in $\dfrac{4!}{2!2!} = 6$ ways and 3 even digits 2, 4, 2 can be arranged in the three even places in $\dfrac{3!}{2!} = 3$ ways. Hence the required number of ways $= 6 \times 3 = 18$.

71. (7) : Here $\angle C = 90°$, $\angle A + \angle B = 90°$

$c^2 = a^2 + b^2$ and $2b = a + c$ (or $2a = b + c$)

Since $c = 2b - a$ and $c^2 = a^2 + b^2$

$\Rightarrow$ $(2b - a)^2 = a^2 + b^2$ or $\dfrac{b}{a} = \dfrac{4}{3}$

$\Rightarrow$ $\dfrac{\sin B}{\sin A} = \dfrac{4}{3}$ or $\dfrac{\sin B + \sin A}{\sin B - \sin A} = \dfrac{7}{1}$

or $\cot\dfrac{B - A}{2} = \dfrac{7}{1}$ $\Rightarrow$ $\cos\dfrac{A - B}{2} = \dfrac{7}{5\sqrt{2}}$

Also $5(\sin A + \sin B) = 5\sqrt{2}\cos\left(\dfrac{A - B}{2}\right) = 7$

72. (1) : Since, a, b, c are roots of given equation, therefore, $a + b + c = -1$, $ab + bc + ca = -333$ and $abc = 1002$

Now, as we know that

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$\therefore$ $a^3 + b^3 + c^3 = (a + b + c)((a + b + c)^2 - 3(ab + bc + ca)) + 3abc$

$= (-1)(1 - 3(-333)) + 3 \times 1002$

$= 3006 - 1000 = 2006 = P$

73. (34) : We know, $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$

or $|\vec{a}|^2|\vec{b}|^2 = |\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2$

$= (|\vec{a} \times \vec{b}|)^2 + (|\vec{a} \cdot \vec{b}|)^2 = (5)^2 + (3)^2 = 25 + 9 = 34$

74. (4.5) : The given lines will intersect, if shortest distance between them is zero, *i.e.*,

$$\begin{vmatrix} 3-1 & k+1 & 0-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $2(3 - 8) - (k + 1)(2 - 4) - (4 - 3) = 0$

$\Rightarrow$ $k = \dfrac{9}{2} = 4.5$

75. (4) : We have, $\sin\theta(1 + \sin^2\theta) = 1 - \sin^2\theta$

$\Rightarrow$ $\sin\theta(2 - \cos^2\theta) = \cos^2\theta$

Squaring both sides, we get

$\sin^2\theta(2 - \cos^2\theta)^2 = \cos^4\theta$

$\Rightarrow$ $(1 - \cos^2\theta)(4 - 4\cos^2\theta + \cos^4\theta) = \cos^4\theta$

$\Rightarrow$ $-\cos^6\theta + 5\cos^4\theta - 8\cos^2\theta + 4 = \cos^4\theta$

$\therefore$ $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$