

## 11. AREAS RELATED TO CIRCLES

1. A bicycle wheel makes 5000 revolutions in moving 10 km. Find the radius of the wheel.

[ Ans : radius =  $\frac{7}{22}m$  ]

2. An athlete runs on a circular track of radius 49 m and covers a distance 3080 m along its boundary. How many rounds has he taken to cover this distance.

[ Ans : Required number of rounds = 10 ]

3. The diameters of front and rear wheels of a tractor are 21 cm and 84 cm respectively. Find the number of revolutions that the rear wheel makes to cover the distance which front wheel covers in 20 seconds at a speed of 252 km/h.

[ Sol : Radii of the wheels are  $\frac{21}{2}cm$  and  $\frac{84}{2} = 42$  cm respectively.

Distance travelled by front wheel in 20 seconds at a speed of 252 km/h

$$\begin{aligned} &= \frac{25200000 \times 20cm}{60 \times 60} \\ &= \frac{840000}{6}cm = 140000cm. \end{aligned}$$

Distance travelled by rear wheel in one revolution =  $2\pi r = 2 \times \frac{22}{7} \times 42cm = 264cm$

So, number of revolutions to cover 140000 cm.

$$= \frac{140000}{264} = \frac{17500}{33} = 530\frac{10}{33} = 530 \text{ approx.}]$$

4. The area of two concentric circles are 9856 m<sup>2</sup> and 5544 m<sup>2</sup>. Find the width of the path between them  $\left( Use \pi = \frac{22}{7} \right)$

[Ans : Width of the path = 14 m]

5. The cost of fencing a circular field at the rate of ₹30 per metre is ₹6600. The field is to be ploughed at the rate of ₹1.50 per square metre. Find the cost of ploughing the field.  $\left(Use \pi = \frac{22}{7}\right)$

**[Ans :** Cost of fencing = ₹5775]

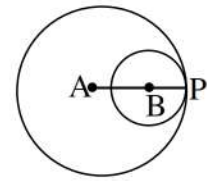
6. Two circles touch each other internally. The sum of their areas is  $116\pi$  sq. cm and distance between their centres is 6 cm. Find the radii of the circles.

**[Sol. :** Let the radii of the two circles be  $r_1$  and  $r_2$ .

So, we have :

$$\pi r_1^2 + \pi r_2^2 = 116\pi$$

$$\Rightarrow r_1^2 + r_2^2 = 116 \quad \dots\dots\dots(1)$$



Also, we have :

$$AB = AP - BP$$

$$\Rightarrow 6 = r_1 - r_2 \quad \dots\dots\dots(2)$$

$$\Rightarrow r_1^2 + r_2^2 - 2r_1r_2 = 36$$

$$\Rightarrow 2r_1r_2 = 116 - 36 \quad [From (1)]$$

$$\Rightarrow 2r_1r_2 = 80.$$

So, from (1),

$$r_1^2 + r_2^2 + 2r_1r_2 = 116 + 80 = 196.$$

$$\Rightarrow r_1 + r_2 = 14 \quad \dots\dots\dots(3)$$

Solving (2) and (3), we get

$$r_1 = 10 \text{ cm and } r_2 = 4 \text{ cm.}$$

Thus, radii of the circles are 10 cm and 4 cm.]

7. A truck has front wheels of radius 0.7 m and its rear wheels have radius 1.4 m. If the rear wheel takes 500 revolutions to travel a certain distance, how many revolutions must the front wheel had taken to cover the same distance ? Also, find the difference in their areas.  $\left(Use \pi = \frac{22}{7}\right)$

**[Sol. :** Distance covered by rear wheel in one revolution =  $2\pi r = 2\pi \times 1.4m$

So, distance covered by rear wheel in 500 revolutions =  $2 \times \pi \times 1.4 \times 500m$

Now, distance covered by front wheel in one revolution =  $2\pi \times (0.7)m$

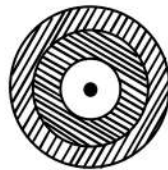
So, number of revolutions by front wheel to cover  $2\pi \times 1.4 \times 500m$

$$= \frac{2\pi \times 1.4 \times 500}{2 \times \pi \times 0.7} = 2 \times 500 = 1000 ]$$

8. A circular park has a path all around it for walking. If the difference between the radius of the park and that of the circle including the path is 35 m and area of the path is  $15400 \text{ m}^2$ , find the sum of the outer and inner radii.  $\left( \text{Use } \pi = \frac{22}{7} \right)$

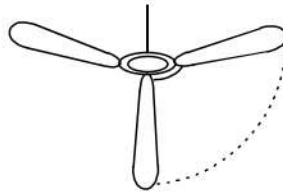
[Ans : 140 m]

9. An archery target has three regions formed by three concentric circles as shown in the figure. If the diameters of the concentric circles are in the ratio 1 : 2 : 3, then find the ratio of the areas and perimeters of three regions.



[Ans : 1 : 3 : 5 and 1 : 2 : 3]

10. A ceiling fan has three wings as shown in the figure. Find the length of arc described between two consecutive wings, where length of each wing is 0.98 m.



[Ans : Length of arc = 2.05 m]

11. With the vertices A, B and C of an equilateral triangle ABC as centres, arcs are drawn with radii 7cm each. If the side of the triangle is 40 cm, then find the area of the region of the triangle not included in the sectors. (Use  $\pi = 3.14$ )

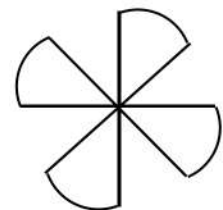
[Sol : Area of the triangle not included in the sectors =  $(400\sqrt{3} - 76.93) \text{ cm}^2$ ]

12. An exhaust fan has four blades of same size with radius of each blade 20 cm as shown in the figure. Find the area of each blade, if they all are equally spaced.

[Sol. : Area of each blade =  $\frac{D}{360^\circ} \times \pi \times 20^2 \text{ cm}^2$

$$= \frac{45^\circ}{360^\circ} \times 3.14 \times 400 \text{ cm}^2 \quad \left[ D = \frac{360^\circ}{8} = 45^\circ \right]$$

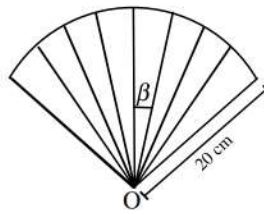
$$= \frac{3.14 \times 400}{8} \text{ cm}^2 = 3.14 \times 50 \text{ cm}^2 = 157 \text{ cm}^2 ]$$



13. A circular disc of radius 6 cm is divided into three sectors with central angles  $120^\circ$ ,  $150^\circ$  and  $90^\circ$ . What part of the whole area is the area of the sector with central angle of  $120^\circ$  ? Also, find the ratio of the areas of the three sectors.

[Ans :  $\frac{1}{3}, 4 : 5 : 3$ ]

14. The figure below is a part of a circle with centre O. Its area is  $\frac{1250\pi}{9} \text{ cm}^2$  and the 10 sectors are identical.



Find the value of  $\beta$ , in degrees. Show your steps.

[Sol :  $\beta = 12.5^\circ$ ]

15. Avikant bought a pair of glasses with wiper blades. He was curious to know the area being cleaned by each of the wiper blades. With the help of a ruler and a protractor, he found the length of each blade as 3 cm and the angle swept as  $60^\circ$ .

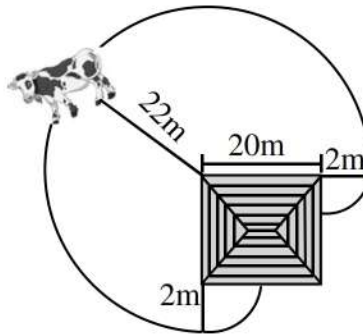


- Find the area that each wiper cleans in one swipe, in terms of  $\pi$ .
- If the diameter of each circular glass is 5 cm, what percent of the area of the glass will be cleaned by the blade in one swipe?

Show your work.

[Sol : i)  $1.5\pi$       ii)  $\frac{25\pi}{4} \text{ cm}^2$ ]

16. A cow is tied at one of the corners of a square shed. The length of the rope is 22 m. The cow can only eat the grass outside the shed as shown below.



What is the area that the cow can graze on? Show your steps.

(Note : Give the answer in terms of  $\pi$  )

[Ans : Total area = (3 quarters sector with radius 22 m) + (2  $\times$  one - quarter sector with radius 2 m)

$$\begin{aligned} \text{Total area} &= \left( \frac{3}{4} \times \pi \times 22^2 \right) + \left( 2 \times \frac{1}{4} \times \pi \times 2^2 \right) \\ &= \Rightarrow \text{Total area} = 363\pi + 2\pi = 365\pi m^2 \end{aligned}$$

17. A circle has radius 5 cm. Three chords of lengths 6 cm, 8 cm and 10 cm are drawn. Which chord subtends the largest angle on its minor arc?

[Sol : The chord of length 6 cm subtends the largest angle on its minor arc.]

18. Ramit drew two circles of different radii. Each of them had an arc that subtended an equal angle at the centre.

He said, "Both arcs are of the same length".

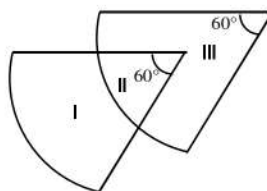
i) Is Ramit right?

ii) If both radii and angles subtended by the two arcs are different, can the arc lengths be the same? Give valid reasons

[Sol. : i) Writes that Ramit is not right and gives a reason. For example, two arcs are not of the same length as arc length is directly proportional to the radius of the circle which is different.

ii) Writes that the arc lengths can be the same and gives a reason. for example, if the product of the measure of the angle and the radius is the same for both circles, then arc lengths will be the same.]

19. Shown below are two overlapping sectors of a circle. The radii of the sectors are 6 cm and 8 cm. The figure is divided into three regions - I, II and III.



Find the difference in the areas of regions I and III. Show your work.

(Note : Take  $\pi = \frac{22}{7}$  )

[Sol : The area of the 8 cm sector as  $\frac{60^\circ}{360^\circ} \times \pi \times 8^2 = \frac{64\pi}{6} \text{ cm}^2$  .

The area of the 6 cm sector as  $\frac{60^\circ}{360^\circ} \times \pi \times 6^2 = \frac{36\pi}{6} \text{ cm}^2$

The difference in the areas of regions I and III as area of (I + II) – area of

$$\text{(II + III)} = \frac{28}{6} \times \frac{22}{7} = \frac{44}{3} \text{ cm}^2 ]$$