COMPETENCY BASED QUESTIONS

1. REAL NUMBERS

1. Show that the numbers 143 and 187 are not co-prime.

   [Hint : To show that HCF $(143,187) \neq 1$ **]**

2. Pens are sold in pack of 8 and notepads are sold in pack of 12. Find the least number of pack of each type that one should buy so that there are equal number of pens and notepads.

   [Hint : LCM (8, 12) = 24

   Least number of packs of pens = 3

   Least number of packs of note pads = 2 **]**

3. A boy with a collection of marbles realises that if he makes a group of 5 or 6 marbles at a time, there are always 2 marbles left. Can you explain why the boy cannot have prime number of marbles ?

   [Hint : The LCM of 5 and $6 = 5 \times 6 = 30$

   So, the number of marbles that the boy can have $= 30n + 2$

   So, $30n + 2 = 2(15n + 1)$

   Which has a factor 2 other than 1 and the numer itself. So, it can not be a prime number**]**

4. A rectangular courtyard is 18 m 72 cm long and 13 m 20 cm broad. It is to be paved with square tiles of the same size. Find the least possible number of such tiles.

   [Hint : Need to calculate HCF(1872, 1320) i.e, 24

   $$\text{Required number of least tiles} = \frac{1872 \times 1320}{24 \times 24} = 4290$$ **]**

5. If two positive integers x and y are expressible in terms of primes as $x = p^2 q^3$ and $y = p^3 q$, what can you say about their LCM and HCF. Is LCM a multiple of HCF ? Explain.

   [Sol. : LCM $= p^3q^3$, HCF $= p^2q$

   Yes, LCM is a multiple of HCF.

   Alert check : In general, LCM is a multiple of HCF (or) HCF is a factor of LCM**]**

6. Dhudnath has two vessels containing 720 mL and 405 mL of milk respectively. Milk from these containers is poured into glasses of equal capacity to their brim. Find the minimum number of glasses that can be filled.

   [Sol. : HCF (720, 405) = 45

   $$\text{Required number} = \frac{720}{45} + \frac{405}{45}$$

   $$= 16 + 9 = 25$$ **]**

7. Two brands of chocolates are available in pack of 24 and 15 respectively. If I need to buy an equal number of chocolates of both kinds, what is the least number of boxes of each kind I would need to buy ?

[Ans : 5 number of boxes in pack of 24 chocolates and 8 number of boxes in pack of 15 chocolates]

8. Find the largest number which divides 70 and 125 leaving remainders 5 and 8 respectively.

(Answer is 13)

9. Can the number 6^n, n being a natural number, end with the digit 5? Give reasons.

(Ans : No)

10. A sweet shopkeeper prepares 396 gulab jamuns and 342 ras-gullas. He packs them in containers. Each container consists of either gulab jamuns or ras-gullas have but have equal number of pieces. Find the number of pieces he should put in each box so that number of boxes are least.

[Hint : Need to calculate HCF (396, 342) = 18]

11. Find two numbers which on multiplication with $\sqrt{180}$ gives a rational number. Are these numbers rational or irrational ?

[Sol. : $\sqrt{180} = \sqrt{2\times2\times3\times3\times5}$

$= 2\times3\times\sqrt{5}$

So, the required numbers may be $\sqrt{5}$ and $2\sqrt{5}$. These numbers are irrational]

12. Prove that the sum of a rational number and irrational number is always irrational.

[Sol. : Let the rational number be x and the irrational number be y.

Their sum = x + y.

Let us assume that x + y is not irrational, i.e., it is rational.

So, it must be of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Thus, $x + y = \frac{p}{q}$

$\Rightarrow \quad y = x - \frac{p}{q}$

Now, $x - \frac{p}{q}$ is a rational number as sum or difference of two rational numbers is rational number.

$\Rightarrow$ y is a rational number.

But it is a contradiction to the fact that y is an irrational number.

Hence, our assumption is wrong.

Therefore, the sum x + y is an irrational number.

13. Is square root of every non-square number always irrational ? Find the smallest natural number which divides 2205 to make its square root a rational number.

[Sol. : Yes, square root of every non - square number is an irrational number.

Now, we have : $2205 = 5 \times 21 \times 21$

If we divide 2205 by 5, then it will give a perfect square 21×21, whose square root will be 21 and 21 is a rational number.

Thus, required least natural number is 5.]

14. The sum of LCM and HCF of two numbers is 7380. If the LCM of these numbers is 7340 more than their HCF, find the product of the two numbers.

[Sol. : Let LCM = x and HCF = y.

Then, $x + y = 7380$ .....(1)

Also, $x = y + 7340$

$\Rightarrow \quad x - y = 7340$ .....(2)

Adding (1) and (2), we get

$x + y = 7380$

$x - y = 7340$

$\Rightarrow \quad 2x = 14720$

$$\Rightarrow \quad x = \frac{14720}{2} = 7360$$

From eqn. (1),

$7360 + y = 7380$

$\Rightarrow \quad y = 7380 - 7360 = 20.$

So, product of the two numbers

$= HCF \times LCM$

$= 20 \times 7360 = 147200.$]

15. Find the greatest number of six digits exactly divisible by 18, 24 and 36.

[Hint : Find the LCM (18, 24, 36), we get 72.

Divide the greatest 6 - digit number, 999999 by 72,

we get remainder as 63

Now, the required number is $999999 - 63 = 9, 99, 936$]

16. Jenny and Sally bought a special 360 days joint membership of a tennis club. Jenny will use the club every alternate day and sally will use the club every third day. They both use the club on the first day. How many days will neither person use the club in the 360 days ?

[Hint : Jenny will use the club in 180 days and Sally will use the club in 120 days

Common days = HCF (180, 120) = 60

Either person use the club = $180 + 120 - 60 = 240$

Neither person use the club = $360 - 240 = 120$]

17. Write the HCF and LCM of the smallest odd composite number and the smallest odd prime number. If an odd number p divides q^2, then will it divide q^3 also ? Explain.

[Hint : HCF = 3, LCM = 9, yes**]**

18. A dining hall has a length of 8.25 m, breadth of 6.75 m, and height of 4.50 m. What is the length of the longest unmarked ruler that can exactly measure the three dimensions of the hall? Show your steps and give valid reasons.

[Answer : 75 cm (or) 0.75 m**]**

19. The prime factorisation of a prime number is the number itself. How many factors and prime factors does the square of a prime number have?

[Answer : For the square of a prime number :

number of factors = 3

number of prime factors = 1**]**

20. If the prime factorisation of a natural number, n is $2^3 \times 3^2 \times 5^2 \times 7$, then write the number of consecutive zeroes in n.

[Sol. : Consider $n = 2^3 \times 3^2 \times 5^2 \times 7$

$= 2 \times 3^2 \times (2 \times 5)^2 \times 7$

$= 2 \times 3^2 \times 7 \times (10)^2$

$\therefore$ There are 2 consecutive zeroes.**]**