08

Centre of Mass

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- Motion of Centre of Mass
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Centre of mass of a system of particles is the point that behaves as whole mass of the system is concentrated on it and all the external forces are acting on it.

For rigid bodies, centre of mass is independent of the state of the body, *i.e.* whether it is in rest or in accelerated motion, centre of mass will remain same.

Position of Centre of Mass

For different particles system, the position of centre of mass is as given below,

(i) **Two Particles System** If a system consists two particles of masses m_1, m_2 and respective position vectors $\mathbf{r}_1, \mathbf{r}_2$, then the position of centre of mass is given by

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2}$$

If $m_1 = m_2 = m$(say), then $\mathbf{r}_{CM} = \dfrac{\mathbf{r}_1 + \mathbf{r}_2}{2}$.

(ii) **System of *n* Particles** If a system consists of n particles, of masses m_1, $m_2, \dots m_n$ with $\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_n$ as their respective position vectors, at a given instant of time, then the position vector of CM, *i.e.* $\mathbf{r}_{CM}$ of the system at that instant is given by

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + \dots m_n\mathbf{r}_n}{m_1 + m_2 + \dots + m_n} = \frac{\sum_{i=1}^{n} m_i\mathbf{r}_i}{\sum_{i=1}^{n} m_i} = \frac{\sum_{i=1}^{n} m_i\mathbf{r}_i}{M}$$

In terms of coordinates, $\mathbf{x}_{CM} = \dfrac{\sum_{i=1}^{n} m_i\mathbf{x}_i}{M}$

Further, $\mathbf{y}_{CM} = \dfrac{\sum_{i=1}^{n} m_i\mathbf{y}_i}{M}$

$$\mathbf{z}_{CM} = \frac{\sum_{i=1}^{n} m_i\mathbf{z}_i}{M}$$

Note The centre of mass of a body may lie within or outside the body. It is not at all necessary that some mass has to be present at the centre of mass.

Example 1. *The position vectors of three particles of masses $m_1 = 1$ kg, $m_2 = 2$ kg and $m_3 = 3$ kg are $\mathbf{r}_1 = (\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \hat{\mathbf{k}})$ m, $\mathbf{r}_2 = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$ m and $\mathbf{r}_3 = (2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$ m respectively. Find position vector of their centre of mass.*

(a) $\frac{1}{3}(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}})$
(b) $\frac{1}{2}(3\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})$
(c) $\frac{1}{3}(3\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})$
(d) $\frac{1}{2}(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}})$

Sol. (b) The position vector of CM of the three parts will be given by

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3}{m_1 + m_2 + m_3}$$

Substituting the values, we get

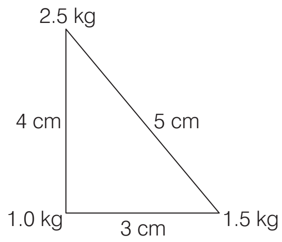
$$\mathbf{r}_{CM} = \frac{1(\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \hat{\mathbf{k}}) + 2(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + 3(2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}})}{1 + 2 + 3}$$

$$= \frac{9\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 3\hat{\mathbf{k}}}{6}$$

$$\therefore \quad \mathbf{r}_{CM} = \frac{1}{2}(3\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) \text{ m}$$

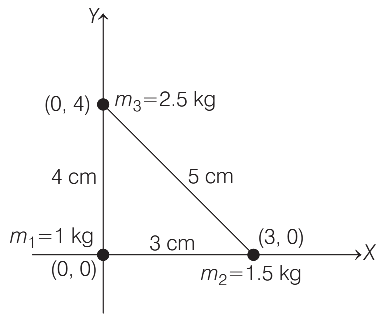
Example 2. *Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The centre of mass of the system is at a point*

[JEE Main 2020]



(a) 2.0 *cm right and* 0.9 *cm above* 1 *kg mass*
(b) 0.6 *cm right and* 2.0 *cm above* 1 *kg mass*
(c) 1.5 *cm right and* 1.2 *cm above* 1 *kg mass*
(d) 0.9 *cm right and* 2.0 *cm above* 1 *kg mass*

Sol. (d) We choose origin as shown in the figure.



Using $x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$, we have

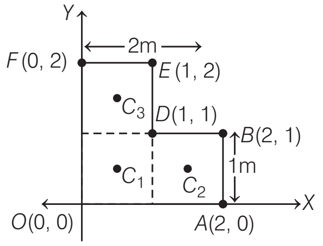
$$x_{CM} = \frac{1 \times 0 + 1.5 \times 3 + 2.5 \times 0}{1 + 1.5 + 2.5} = 0.9 \text{ cm}$$

Similarly,

$$y_{CM} = \frac{1 \times 0 + 1.5 \times 0 + 2.5 \times 4}{1 + 1.5 + 2.5} = 2.0 \text{ cm}$$

So, centre of mass (CM) is 0.9 cm right and 2.0 cm above 1 kg mass.

Example 3. *The centre of mass of a uniform L-shaped lamina (a thin flat plate) with dimensions as shown is (g mass of lamina is 3 kg)*



(a) $\left(\frac{6}{5}, \frac{5}{6}\right)$
(b) $\left(\frac{5}{6}, \frac{5}{6}\right)$
(c) $\left(\frac{5}{6}, \frac{6}{5}\right)$
(d) $\left(\frac{7}{5}, \frac{6}{5}\right)$

Sol. (b) Taking the *L*-shape to consist of 3 squares each of length 1m. The mass of each square is 1kg, since the lamina is uniform. The centre of mass C_1, C_2 and C_3 of the squares are, by symmetry their geometric centres and have coordinates $\left(\frac{1}{2}, \frac{1}{2}\right)$, $\left(\frac{3}{2}, \frac{1}{2}\right)$, $\left(\frac{1}{2}, \frac{3}{2}\right)$ respectively. We take the masses of the squares to be concentrated at these points. The centre of mass of the whole *L* shape (x, y) is the centre of mass of these mass points.

Hence,
$$X = \frac{\left[1\left(\frac{1}{2}\right) + 1\left(\frac{3}{2}\right) + 1\left(\frac{1}{2}\right)\right] \text{kg-m}}{(1 + 1 + 1) \text{ kg}} = \frac{5}{6} \text{ m}$$

and
$$Y = \frac{\left[1\left(\frac{1}{2}\right) + 1\left(\frac{1}{2}\right) + 1\left(\frac{3}{2}\right)\right] \text{kg-m}}{(1 + 1 + 1) \text{ kg}} = \frac{5}{6} \text{ m}$$

Centre of Mass of Rigid Continuous Bodies

A body is said to be a rigid body when it has perfectly definite shape and volume.

The distance between all points of particles of such a body do not change, while applying any force on it.

Since, a real rigid body contains so many particles (atoms) that we can treat it as a continuous distribution of matter. The particles then become differential mass elements dm, and the coordinates of the centre of mass are defined as

$$x_{CM} = \frac{1}{M}\int x\, dm$$

$$y_{CM} = \frac{1}{M}\int y\, dm$$

$$z_{CM} = \frac{1}{M}\int z\, dm$$

For uniform objects which have uniform density or mass per unit volume, we have

$$\rho = \frac{dm}{dV} = \frac{M}{V}$$

where dV is the volume occupied by a mass element dm and V is the total volume of the object. Thus, we find that

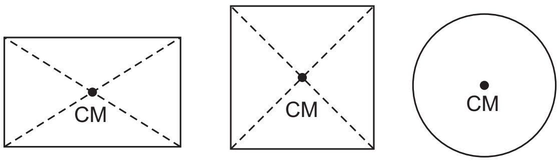
$$x_{\mathrm{CM}} = \frac{1}{V}\int x\,dV$$

$$y_{\mathrm{CM}} = \frac{1}{V}\int y\,dV$$

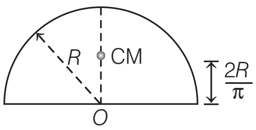
$$z_{\mathrm{CM}} = \frac{1}{V}\int z\,dV$$

Centre of Mass of Different Rigid Bodies

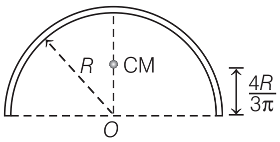
- For a *uniform rectangular, square* or *circular plate*, CM lies at its centre.



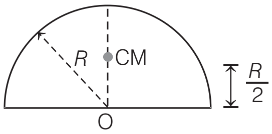
- Centre of mass of a **uniform semicircular** ring lies at a distance of $h = \frac{2R}{\pi}$ from its centre, on the axis of symmetry, where R is the radius of the ring.



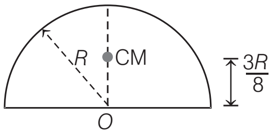
- For a **uniform semicircular disc** of radius R, CM lies at a distance of $h = \frac{4R}{3\pi}$ from the centre on the axis of symmetry as shown in the following figure.



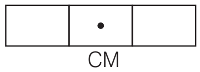
- Centre of mass of a **hemispherical shell** of radius R lies at a distance of $h = R/2$ from its centre on the axis of symmetry as shown in the following figure.



- For a **solid hemisphere** of radius R, CM lies at a distance of $h = \frac{3R}{8}$ from its centre on the axis of symmetry.



- Centre of mass of a uniform rod is located at its mid-point.

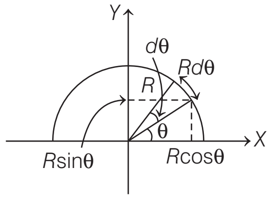


Example 4. *Find the centre of mass of a uniform semicircular ring of radius R and mass M.*

(a) $0, 2\pi R$ (b) $0, \frac{2R}{\pi}$ (c) $0, \frac{3R}{\pi}$ (d) $0, \pi R$

Sol. (*b*) Consider the centre of the ring as origin. Let a differential element of length dl of the ring whose radius vector makes an angle θ with the X-axis. If the angle subtended by the length dl is $d\theta$ at the centre, then $dl = R\,d\theta$.

Let λ be the mass per unit length.



Then, mass of this element is $dm = \lambda R\,d\theta$

$$X_{\mathrm{CM}} = \frac{1}{m}\int_0^{\pi} (R\cos\theta)\lambda R\,d\theta = 0$$

$$\Rightarrow \quad Y_{\mathrm{CM}} = \frac{1}{m}\int_0^{\pi} (R\sin\theta)\cdot\lambda R\,d\theta$$

$$\text{or} \quad = \frac{\lambda R^2}{m}\int_0^{\pi}\sin\theta\,d\theta = \frac{\lambda R^2}{\lambda\pi R}[-\cos\theta]_0^{\pi}$$

$$\Rightarrow \quad Y_{\mathrm{CM}} = \frac{2R}{\pi}$$

$$\text{As,} \quad m = \int_0^{\pi}\lambda R\,d\theta = 2\pi R$$

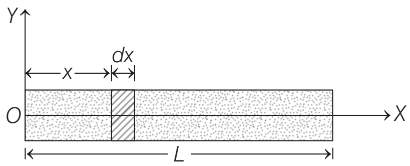
Example 5. *A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$, where a and b are constants and $0 \le x \le L$. The value of x for the centre of mass of the rod is at* ***[JEE Main 2020]***

(a) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$ (b) $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$

(c) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$ (d) $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$

Sol. (*a*) For a continuous mass distribution,

$$x_{\mathrm{CM}} = \frac{\int x\,dm}{\int dm}$$



Here, mass of an elemental length dx of rod,

$$dm = \rho dx = \left[a + b\left(\frac{x}{L}\right)^2\right]dx$$

So, $$x_{CM} = \frac{\int_0^L x\left(a + \frac{bx^2}{L^2}\right)dx}{\int_0^L \left(a + \frac{bx^2}{L^2}\right)dx}$$

$$= \frac{\int_0^L \left(ax + \frac{b\,x^3}{L^2}\right)dx}{\int_0^L \left(a + \frac{b\,x^2}{L^2}\right)dx}$$

$$= \frac{\left[\left(\frac{ax^2}{2} + \frac{b}{L^2}\cdot\frac{x^4}{4}\right)\right]_0^L}{\left[\left(ax + \frac{bx^3}{3L^2}\right)\right]_0^L}$$

$$= \frac{\frac{aL^2}{2} + \frac{bL^2}{4}}{aL + \frac{bL}{3}}$$

$$= \left(\frac{\frac{a}{2} + \frac{b}{4}}{a + \frac{b}{3}}\right)L$$

So, $$x_{CM} = \frac{3}{4}\left(\frac{2a + b}{3a + b}\right)L$$

Position of Centre of Mass after Removal of a Part from a Rigid Body

If some mass or area is removed from a rigid body, then the position of centre of mass of the remaining portion is obtained from the following formulae.

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 - m_2\mathbf{r}_2}{m_1 - m_2}$$

or $$\mathbf{r}_{CM} = \frac{A_1\mathbf{r}_1 - A_2\mathbf{r}_2}{A_1 - A_2}$$

In terms of coordinates,

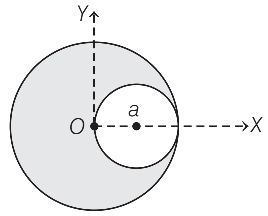
$$x_{CM} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2} \text{ or } x_{CM} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2}$$

$\therefore$ $$y_{CM} = \frac{m_1y_1 - m_2y_2}{m_1 - m_2} \text{ or } y_{CM} = \frac{A_1y_1 - A_2y_2}{A_1 - A_2}$$

and $$z_{CM} = \frac{m_1z_1 - m_2z_2}{m_1 - m_2} \text{ or } z_{CM} = \frac{A_1z_1 - A_2z_2}{A_1 - A_2}$$

Here, $m_1, A_1, \mathbf{r}_1, x_1, y_1$ and z_1 are the values for the mass of the whole body before the mass has been removed while $m_2, A_2, \mathbf{r}_2, x_2, y_2$ and z_2 are the values for the mass which has been removed.

Example 6. *Find the position of centre of mass of the uniform lamina shown in figure is*



(a) $\left(-\frac{a}{6}, 0\right)$ (b) (0, 0) (c) $\left(\frac{a}{6}, 0\right)$ (d) $\left(\frac{a}{6}, -\frac{a}{6}\right)$

Sol. (a) Here, A_1 = area of complete circle $= \pi a^2$

and A_2 = area of small circle $= \pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{4}$

(x_1, y_1) = coordinates of centre of mass of large circle = (0, 0)

and (x_2, y_2) = coordinates of centre of mass of small circle $= \left(\frac{a}{2}, 0\right)$.

Using $$x_{CM} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2}$$

We get, $$x_{CM} = \frac{\frac{-\pi a^2}{4}\left(\frac{a}{2}\right)}{\pi a^2 - \frac{\pi a^2}{4}} = \frac{-\left(\frac{1}{8}\right)}{\left(\frac{3}{4}\right)} a = -\frac{a}{6}$$

and $y_{CM} = 0$ as y_1 and y_2 both are zero. Therefore, the coordinates of centre of mass of the lamina shown in figure are $\left(-\frac{a}{6}, 0\right)$.

Motion of Centre of Mass

Consider the motion of a system of n particles of individual masses $m_1, m_2, \ldots, m_n$ and total mass M. It is assumed that no mass enters or leaves the system during its motion, so that M remains constant. Then, velocity of centre of mass is

$$\mathbf{v}_{CM} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + \ldots + m_n\mathbf{v}_n}{m_1 + m_2 + \ldots + m_n}$$

or $$\mathbf{v}_{CM} = \sum_{i=1}^{n} \frac{m_i\mathbf{v}_i}{M}$$

Acceleration of the centre of mass is

$$\mathbf{a}_{CM} = \frac{m_1\mathbf{a}_1 + m_2\mathbf{a}_2 + \ldots + m_n\mathbf{a}_n}{M}$$

or $$\mathbf{a}_{CM} = \sum_{i=1}^{n} \frac{m_i\mathbf{a}_i}{M}$$

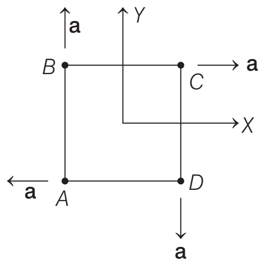
Further, in accordance with Newton's second law of motion, $\mathbf{F} = m\mathbf{a}$.

We can write, force on centre of mass is

$$\mathbf{F}_{CM} = \sum_{i=1}^{n} F_i$$

If total net force acting on a system of particles is zero, then $\mathbf{a}_{CM} = 0$. Hence, in the absence of any net external force acting on a system, the centre of mass of the system is either at rest or in uniform motion along a given straight line.

Example 7. *Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown in the figure. The acceleration of the centre of mass of the particles (in ms^{-2}) is* ***[JEE Main 2019]***

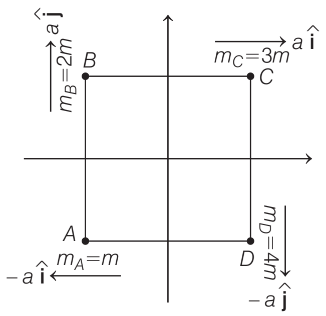


(a) $\frac{a}{5}(\hat{\mathbf{i}} - \hat{\mathbf{j}})$ (b) $a(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

(c) zero (d) $\frac{a}{5}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

Sol. (a) For a system of discrete masses, acceleration of centre of mass (CM) is given by

$$a_{CM} = \frac{m_A a_A + m_B a_B + m_C a_C + m_D a_D}{m_A + m_B + m_C + m_D}$$



where, $m_A = m, m_B = 2m, m_C = 3m$ and $m_D = 4m$,

$|a_A| = |a_B| = |a_C| = |a_D| = a$ (according to the question)

$$a_{CM} = \frac{-ma\hat{i} + 2ma\hat{\mathbf{j}} + 3ma\hat{i} - 4ma\hat{j}}{m + 2m + 3m + 4m}$$

$$= \frac{2a\hat{i} - 2a\hat{j}}{10}$$

$$= \frac{a}{5} \cdot (\hat{i} - \hat{j}) \text{ ms}^{-2}$$

Note Centre of mass is not the geometric centre.

Collision

The physical interaction of two or more bodies in which equal and opposite forces act upon each other causing the exchange of energy and momentum is called collision.

Types of Collision

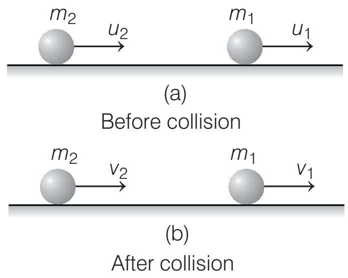
Collision between two bodies may be classified in two ways

(i) **Elastic Collision** A collision is said to be elastic, if total linear momentum and kinetic energy remains conserved before and after collision.

(ii) **Inelastic Collision** A collision is said to be inelastic, if only linear momentum remains conserved and not the kinetic energy, the collision is said to be perfectly inelastic, if approaching particles permanently stick to each other and move with a common velocity.

Elastic Collision in One Dimension

Let the two balls of mass m_1 and m_2, collide each other elastically with velocities u_1 and u_2 in the directions shown as in Fig. (a). Their velocities become v_1 and v_2 after the collision along the same line.



(a)
Before collision

(b)
After collision

Since, in a perfectly elastic collision, total energy and total linear momentum of colliding particles remains conserved, then

Relative velocity of approach = Relative velocity of separation, *i.e.* $u_1 - u_2 = v_2 - v_1$

$$\Rightarrow \quad v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \quad \ldots\text{(i)}$$

$$\text{and} \quad v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 \quad \ldots\text{(ii)}$$

Special Cases of Elastic Collision in One Dimension

- If $m_1 = m_2$, then from Eqs. (i) and (ii), we get

$$v_1 = u_2 \text{ and } v_2 = u_1$$

i.e. when two particles of equal mass collide elastically and the collision is head on, they exchange their velocities.

- If $m_1 >> m_2$ and $u_1 = 0$, then

$$\frac{m_2}{m_1} \approx 0$$

Now, we get

$$v_1 \approx 0 \text{ and } v_2 \approx -u_2$$

i.e. The particle of mass m_1 remains at rest while the particle of mass m_2 bounces back with same velocity.

- If $m_2 >> m_1$ and $u_1 = 0$

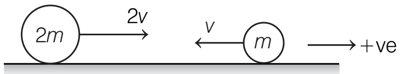
Now, we get, $v_1 \approx 2u_2$ and $v_2 \approx u_2$

i.e. The mass m_1 (lighter particle) moves with velocity $2u_2$, while the velocity of mass m_2 (heavier particle) remains same.

Example 8. *Two particles of mass m and 2m moving in opposite directions collide elastically with velocities v and 2v. Find their velocities after collision.*

(a) 0, 3v (b) 3v, 0 (c) 3v, 3v (d) 0, 0

Sol. (b) Here, $u_1 = -v, u_2 = 2v, m_1 = m$ and $m_2 = 2m$



Substituting these values in Eqs. (i) and (ii), we get

$$v_1 = \left(\frac{m-2m}{m+2m}\right)(-v) + \left(\frac{4m}{m+2m}\right)(2v)$$

or $$v_1 = \frac{v}{3} + \frac{8v}{3} = 3v$$

and $$v_2 = \left(\frac{2m-m}{m+2m}\right)(2v) + \left(\frac{2m}{m+2m}\right)(-v)$$

or $$v_2 = \frac{2}{3}v - \frac{2}{3}v = 0$$

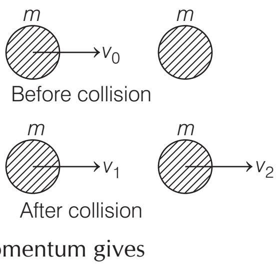
i.e. The second particle (of mass 2m) comes to a rest while the first (of mass m) moves with velocity 3v in the direction shown in figure.

Example 9. *In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles after collision, is* **[JEE Main 2018]**

(a) $\frac{v_0}{4}$ (b) $\sqrt{2}\,v_0$ (c) $\frac{v_0}{2}$ (d) $\frac{v_0}{\sqrt{2}}$

Sol. (b) Final kinetic energy is 50% more than initial kinetic energy

$$\Rightarrow \quad \frac{1}{2}mv_2^2 + \frac{1}{2}mv_1^2 = \frac{150}{100} \times \frac{1}{2}mv_0^2 \quad \ldots\text{(i)}$$



Conservation of momentum gives

$$mv_0 = mv_1 + mv_2$$

$$v_0 = v_2 + v_1 \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we have

$$v_1^2 + v_2^2 + 2v_1v_2 = v_0^2$$

$$\Rightarrow \quad 2v_1v_2 = \frac{-v_0^2}{2}$$

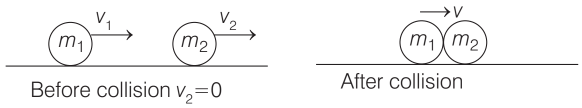
$$\therefore \quad (v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2 = 2v_0^2$$

or $$v_{\text{rel}} = \sqrt{2}v_0$$

Inelastic Collision in One Dimension

In an inelastic collision, the total linear momentum as well as total energy remain conserved but total kinetic energy after collision is not equal to kinetic energy before collision.

For inelastic collision,



Common speed, $v = \dfrac{m_1 v_1}{m_1 + m_2}$

and loss of kinetic energy, $\Delta K = \dfrac{m_1 m_2 (v_1 - v_2)^2}{2(m_1 + m_2)}$

Here, $v_2 = 0$

$$\therefore \quad \Delta K = \frac{m_1 m_2 v_1^2}{2(m_1 + m_2)}$$

which is a positive quantity. Therefore, kinetic energy is lost mainly in the form of light, sound and heat.

Example 10. *A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is*

(a) 0.16 J (b) 1.00 J
(c) 0.67 J (d) 0.34 J

Sol. (c) From law of conservation of momentum, we have

$$m_1v_1 + m_2v_2 = (m_1 + m_2)\,v$$

Given, $m_1 = 0.50$ kg, $v_1 = 2$ ms^{-1}, $m_2 = 1$ kg, $v_2 = 0$ [at rest]

$$0.5 \times 2 + 1 \times 0 = 1.5 \times v \quad \text{[assumed that 2 nd body is at rest]}$$

$$\Rightarrow \quad v = \frac{2}{3}$$

$$\therefore \quad \Delta K = K_f - K_i$$

$$= \frac{1.5 \times \left(\frac{2}{3}\right)^2}{2} - (0.5) \times \frac{2^2}{2} = -\frac{2}{3}\text{ J} = -0.67\text{ J}$$

So, energy lost is 0.67 J.

Newton's Law of Restitution

According to this law, the ratio of relative velocity of separation after collision to the relative velocity of approach before collision remains constant,

$$e = \frac{v_2 - v_1}{u_2 - u_1}$$

where, e is called the **coefficient of restitution** and it is constant for two particular objects.

Following are few important points related to coefficient of restitution.

- In general, $0 \le e \le 1$
- $e = 0$, for completely inelastic collision, as both the objects stick together and $e = 1$, for an elastic collision.
- If $0 < e < 1$, the collision is said to be partially elastic.

- For head-on collision, the final velocities of the colliding bodies can be written as

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right) u_1 + \frac{(1+e)m_2}{(m_1+m_2)} u_2$$

and $$v_2 = \frac{(1+e)m_1}{(m_1+m_2)} u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right) u_2$$

Putting, $e = 1$, we will get formulae of v_1 and v_2 for an elastic collision.

Similarly, putting $e = 0$, we get formulae for inelastic collision.

- The loss in KE during an inelastic collision can be given as,

$$\Delta K = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(1 - e^2)(u_1 - u_2)^2$$

However, if the target is massive (*i.e.* $m_2 >> m_1$) and $u_2 = 0$, then the lighter body loses all its kinetic energy.

Example 11. *A ball of mass m moving with a speed v makes a head-on collision with an identical ball at rest. The kinetic energy after collision of the balls is three-fourth the original kinetic energy. The coefficient of restitution (e) is*

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

Sol. (c) From the law of conservation of momentum,

$$mv = mv_1 + mv_2$$

or $$v = v_1 + v_2 \quad \text{...(i)}$$

which gives, $$e = \frac{v_2 - v_1}{v - 0}$$

Before collision

After collision

or $$v_2 - v_1 = ev \quad \text{...(ii)}$$

Adding Eqs. (i) and (ii), we get

$$v_2 = \frac{v + ev}{2}$$

and $$v_1 = \frac{(1-e)v}{2}$$

$$\frac{3}{4}\frac{mv^2}{2} = \frac{m}{2}\left[\frac{v^2(1+e)^2}{4} + \frac{(1-e)^2 v^2}{4}\right]$$

$$\Rightarrow \quad 3 = (1+e)^2 + (1-e)^2 = 2(1+e^2)$$

$$\Rightarrow \quad e^2 = \frac{1}{2}$$

or $$e = \frac{1}{\sqrt{2}}$$

Rebounding of a Ball on Collision with the Floor

- Speed of the ball after the nth rebound

$$v_n = e^n v_0 = e^n \sqrt{2gh_0}$$

- Height covered by the ball after the nth rebound

$$h_n = e^{2n} h_0$$

- Total distance (vertical) covered by the ball before it stops bouncing

$$H = h_0 + 2h_1 + 2h_2 + 2h_3 + \dots = h_0\left(\frac{1+e^2}{1-e^2}\right)$$

- Total time taken by the ball before it stops bouncing

$$T = t_0 + t_1 + t_2 + t_3 + \dots$$

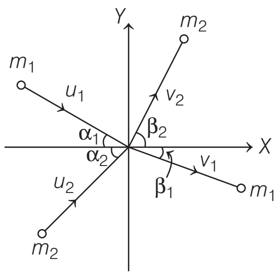
$$= \sqrt{\frac{2h_0}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + \dots = \sqrt{\frac{2h_0}{g}}\left(\frac{1+e}{1-e}\right)$$

Two Dimensional or Oblique Collision

If the initial and final velocities of colliding bodies do not lie along the same line, then the collision is called two dimensional or oblique collision.

In horizontal direction,

$$m_1 u_1 \cos\alpha_1 + m_2 u_2 \cos\alpha_2 = m_1 v_1 \cos\beta_1 + m_2 v_2 \cos\beta_2$$

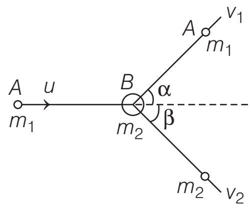


In vertical direction,

$$m_1 u_1 \sin\alpha_1 - m_2 u_2 \sin\alpha_2 = m_1 v_1 \sin\beta_1 - m_2 v_2 \sin\beta_2$$

If $m_1 = m_2$ and $\alpha_1 + \alpha_2 = 90°$, then $\beta_1 + \beta_2 = 90°$.

If a particle A of mass m_1 moving along X-axis with a speed u and makes an elastic collision with another stationary body B of mass m_2. Then,



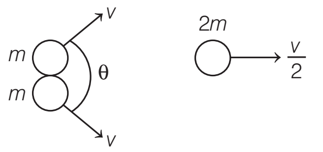
$$m_1 v_1 \sin\alpha = m_2 v_2 \sin\beta$$

Example 12. *After perfectly inelastic collision between two identical particles moving with same speed in different directions, the speed of the particles becomes half the initial speed. Find the angle between the two before collision.*

(a) 120° (b) 140° (c) 90° (d) 100°

Sol. (*a*) Let θ be the desired angle. Linear momentum of the system will remain conserved. Hence,

$$p^2 = p_1^2 + p_2^2 + 2p_1p_2 \cos\theta$$



or $\quad \left\{2m\left(\frac{v}{2}\right)\right\}^2 = (mv)^2 + (mv)^2 + 2(mv)(mv)\cos\theta$

or $\quad 1 = 1 + 1 + 2\cos\theta$

or $\quad \cos\theta = -\frac{1}{2}$

$\therefore \quad \theta = 120°$

Example 13. *A particle of mass m is moving with speed 2v and collides with a mass 2m moving with speed v in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass m, which move at angle 45° with respect to the original direction. The speed of each of the moving particle will be*

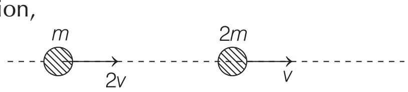
[JEE Main 2019]

(*a*) $\sqrt{2}\,v$ $\qquad$ (*b*) $\frac{v}{\sqrt{2}}$

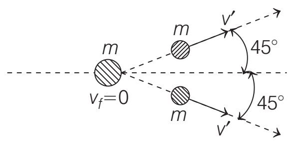
(*c*) $\frac{v}{(2\sqrt{2})}$ $\qquad$ (*d*) $2\sqrt{2}\,v$

Sol. (*d*) According to the questions,

Initial condition,



Final condition,



As we know that, in collision, linear momentum is conserved in both *x* and *y* directions separately.

So, $\quad (p_x)_{\text{initial}} = (p_x)_{\text{final}}$

$m(2v) + 2m(v) = 0 + mv'\cos 45° + mv'\cos 45°$

$\Rightarrow \quad 4mv = \frac{2m}{\sqrt{2}}v'$

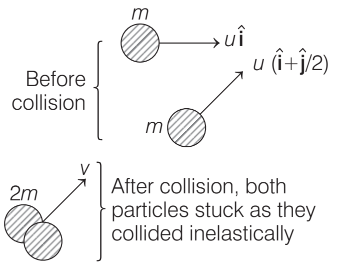
$\Rightarrow \quad v' = 2\sqrt{2}v$

Example 14. *Two particles of equal mass m have respective initial velocities $u\hat{\mathbf{i}}$ and $u\left(\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{2}\right)$. They collide completely inelastically. The energy lost in the process is*

[JEE Main 2020]

(*a*) $\frac{3}{4}mu^2$ $\qquad$ (*b*) $\sqrt{\frac{2}{3}}mu^2$ $\qquad$ (*c*) $\frac{1}{3}mu^2$ $\qquad$ (*d*) $\frac{1}{8}mu^2$

Sol. (*d*) Collision between particles are as shown in the figure.



From momentum conservation, we have

$$m(u\hat{\mathbf{i}}) + mu\left(\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{2}\right) = 2m\mathbf{v}$$

$$\Rightarrow \quad \mathbf{v} = \frac{u}{2}\hat{\mathbf{i}} + u\left(\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{4}\right) = \frac{3}{4}u\hat{\mathbf{i}} + \frac{1}{4}u\hat{\mathbf{j}}$$

Initial kinetic energy of particles,

$$K_1 = \frac{1}{2}mu^2 + \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2}mu^2 + \frac{1}{4}mu^2 = \frac{3}{4}mu^2$$

Final kinetic energy of combined particles,

$$K_2 = \frac{1}{2}(2m)v^2$$

$$= \frac{1}{2} \times 2m \times \left(\sqrt{\left(\frac{3}{4}u\right)^2 + \left(\frac{1}{4}u\right)^2}\right)^2$$

$$= \frac{5}{8}mu^2$$

Change in kinetic energy or energy lost

$$= K_1 - K_2$$

$$= \frac{3}{4}mu^2 - \frac{5}{8}mu^2$$

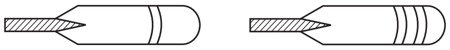
$$= \frac{1}{8}mu^2$$

Practice Exercise

ROUND I Topically Divided Problems

Position of Centre of Mass

1. A cricket bat is cut at the location of its centre of mass as shown. Then,



(a) the two pieces will have the same mass
(b) the bottom piece will have larger mass
(c) the handle piece will have larger mass
(d) mass of handle piece is double the mass of bottom piece

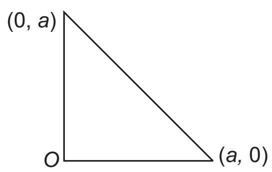
2. Which of the following does the centre of mass lie outside the body? **[NCERT Exemplar]**

(a) A pencil (b) A shotput
(c) A dice (d) A bangle

3. The centre of mass of two particles with masses 4 kg and 2 kg located at (1,0,1) and (2,2,0) respectively has coordinates.

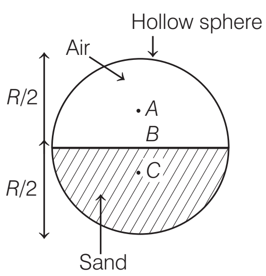
(a) (1/3, 2/3, 2/3) (b) (4/3, 1/3, 1/3)
(c) (2/3, 1/3, 1/3) (d) (4/3, 2/3, 2/3)

4. Three rods of the same mass are placed as shown in figure. What will be the coordinates of centre of mass of the system?



(a) $\left(\frac{a}{2}, \frac{a}{2}\right)$ (b) $\left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$ (c) $(\sqrt{2}\,a, \sqrt{2}\,a)$ (d) $\left(\frac{a}{3}, \frac{a}{3}\right)$

5. Which of the following points is the likely position of the centre of mass of the system shown in figure? **[NCERT Exemplar]**



(a) A (b) B (c) C (d) D

6. Four particles of mass 1 kg, 2 kg, 3 kg and 4 kg are placed at the corners A, B, C and D respectively of a square $ABCD$ of edge X-axis and edge AD is taken along Y-axis, the coordinates of centre of mass in SI unit is

(a) (1, 1) m
(b) (5, 7) m
(c) (0.5, 0.7) m
(d) None of the above

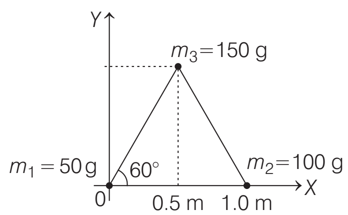
7. In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 Å (1 Å = 10^{-10} m). Find the approximate location of the centre of mass of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus. **[NCERT]**

(a) $\mathbf{r}_{CM}$ = 1.24 Å
(b) $\mathbf{r}_{CM}$ = 2.24 Å
(c) $\mathbf{r}_{CM}$ = 0.24 Å
(d) $\mathbf{r}_{CM}$ = 3.24 Å

8. The centre of mass of three particles of masses 1 kg, 2 kg and 3 kg is at (2, 2, 2). The position of the fourth mass of 4 kg to be placed in the system as that the new centre of mass is at (0, 0, 0) is **[EAMCET]**

(a) $(-3, -3, -3)$ (b) $(-3, 3, -3)$
(c) $(2, 3, -3)$ (d) $(2, -2, 3)$

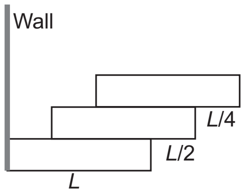
9. Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be **[JEE Main 2019]**



(a) $\left(\frac{\sqrt{3}}{4}\text{ m}, \frac{5}{12}\text{ m}\right)$ (b) $\left(\frac{7}{12}\text{ m}, \frac{\sqrt{3}}{8}\text{ m}\right)$

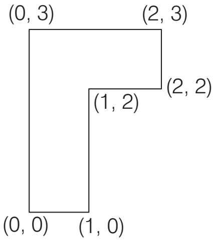
(c) $\left(\frac{7}{12}\text{ m}, \frac{\sqrt{3}}{4}\text{ m}\right)$ (d) $\left(\frac{\sqrt{3}}{8}\text{ m}, \frac{7}{12}\text{ m}\right)$

10. Three bricks each of length L and mass M are arranged from the wall as shown. The distance of the centre of mass of the system from the wall is



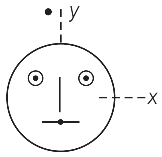
(a) $\frac{L}{4}$ (b) $\frac{L}{2}$ (c) $\frac{3}{2}L$ (d) $\frac{11}{12}L$

11. The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4 kg. (The coordinates of the same are shown in figure) are **[JEE Main 2020]**



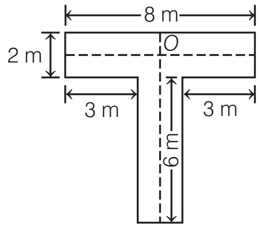
(a) (1.25 m, 1.50 m) (b) (1 m, 1.75 m)
(c) (0.75 m, 0.75 m) (d) (0.75 m, 1.75 m)

12. Look at the drawing given in the figure, which has been drawn with ink of uniform line thickness. The mass of ink used to draw each of the two inner circles and each of the two line segments is m. The mass of the ink used to draw the outer circle is $6m$. The coordinates of the centres of the different parts are outer circle (0, 0) left inner circle $(-a, a)$, right inner circle (a, a), vertical line (0, 0) and horizontal line $(0, -a)$. The y-coordinate of the centre of mass of the ink in this drawing is



(a) $\frac{a}{10}$ (b) $\frac{a}{8}$ (c) $\frac{a}{12}$ (d) $\frac{a}{3}$

13. The distance of the centre of mass of the T-shaped plate from O is



(a) 7 m (b) 2.7 m (c) 4 m (d) 1 m

14. A circular hole of radius 1 cm is cut-off from a disc of radius 6 cm. The centre of hole is 3 cm from the centre of the disc. The position of centre of mass of the remaining disc from the centre of disc is

(a) $-\frac{3}{35}$ cm (b) $\frac{1}{35}$ cm
(c) $\frac{3}{10}$ cm (d) None of these

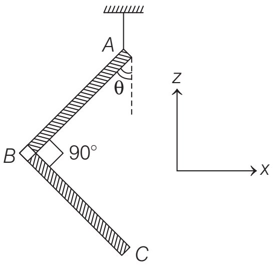
15. The density of a non-uniform rod of length 1m is given by $\rho(x) = a(1 + bx^2)$ where a and b are constants and $0 \le x \le 1$. The centre of mass of the rod will be at **[NCERT Exemplar]**

(a) $\frac{3(2+b)}{4(3+b)}$ (b) $\frac{4(2+b)}{3(3+b)}$
(c) $\frac{3(2+b)}{4(2+b)}$ (d) $\frac{4(3+b)}{3(2+b)}$

16. A non-uniform thin rod of length L is placed along X-axis as such its one of ends is at the origin. The linear mass density of rod is $\lambda = \lambda_0 x$. The distance of centre of mass of rod from the origin is

(a) $\frac{L}{2}$ (b) $\frac{2L}{3}$
(c) $\frac{L}{4}$ (d) $\frac{L}{5}$

17. An L-shaped object made of thin rods of uniform mass density is suspended with a string as shown in figure. If $AB = BC$ and the angle is made by AB with downward vertical is θ, then **[JEE Main 2019]**



(a) $\tan\theta = \frac{2}{\sqrt{3}}$ (b) $\tan\theta = \frac{1}{2\sqrt{3}}$
(c) $\tan\theta = \frac{1}{2}$ (d) $\tan\theta = \frac{1}{3}$

Motion of Centre of Mass

18. An isolated particle of mass m is moving in a horizontal plane (x-y), along the X-axis, at a certain height above the ground. It suddenly explodes into two fragments of masses $m/4$ and $3m/4$. An instant later, the smaller fragment is at $y = +15$ cm. The larger fragment at this instant is at

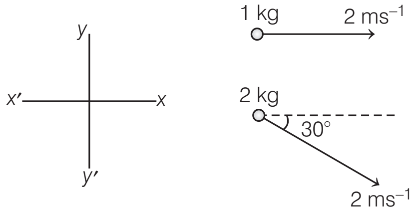
(a) $y = -5$ cm (b) $y = +20$ m
(c) $y = +5$ cm (d) $y = -20$ cm

19. Two identical particles move towards each other with velocity $2v$ and v, respectively. The velocity of centre of mass is

(a) v (b) $\frac{v}{3}$

(c) $\frac{v}{2}$ (d) zero

20. Find the velocity of centre of the system shown in the figure.



(a) $\left(\frac{2+2\sqrt{3}}{3}\right)\hat{\mathbf{i}}-\frac{2}{3}\hat{\mathbf{j}}$ (b) $4\,\hat{\mathbf{i}}$

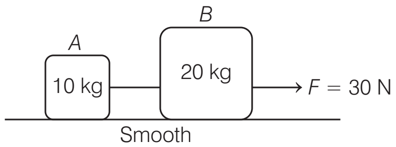
(c) $\left(\frac{2-2\sqrt{3}}{3}\right)\hat{\mathbf{i}}-\frac{2}{3}\hat{\mathbf{j}}$ (d) None of these

21. Consider a two particle system with particles having masses m_1 and m_2. If the first particles is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved, so as to keep the centre of mass at the same position?

(a) $\frac{m_2}{m_1}d$ (b) $\frac{m_1}{m_1+m_2}d$

(c) $\frac{m_1}{m_2}d$ (d) d

22. Two blocks A and B are connected by a massless string (shown in figure). A force of 30 N is applied on block B. The distance travelled by centre of mass in 2 s starting from rest is



(a) 1 m (b) 2 m

(c) 3 m (d) None of these

23. A particle of mass m moving with a velocity $(3\,\hat{\mathbf{i}}+2\,\hat{\mathbf{j}})\,\text{ms}^{-1}$ collides with a stationary body of mass M and finally moves with a velocity $(-2\,\hat{\mathbf{i}}+\hat{\mathbf{j}})\,\text{ms}^{-1}$. If $\frac{m}{M}=\frac{1}{13}$, then

(a) the impulse received by M is $m\,(5\,\hat{\mathbf{i}}+\hat{\mathbf{j}})$

(b) the velocity of the M is $\frac{1}{13}(5\,\hat{\mathbf{i}}+\hat{\mathbf{j}})$

(c) the coefficient of restitution $\frac{11}{17}$

(d) All of the above are correct

Collision in One Dimension

24. Two equal masses m_1 and m_2 moving along the same straight line with velocities $+3$ m/s and -5 m/s respectively collide elastically. Their velocities after the collision will be respectively

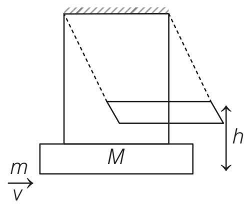
(a) $+4$ m/s for both

(b) -3 m/s and $+5$ m/s

(c) -4 m/s and $+4$ m/s

(d) -5 m/s and $+3$ m/s

25. A large block of wood of mass $M=5.99$ kg is hanging from two long massless cords. A bullet of mass $m=10$ g is fired into the block and gets embedded in it. The (block + bullet) then swing upwards, their centre of mass rising a vertical distance $h=9.8$ cm before the (block + bullet) pendulum comes momentarily rest at the end of its arc. The speed of the bullet just before collision is (Take, $g=9.8\ \text{ms}^{-2}$) **[JEE Main 2021]**



(a) 841.4 m/s (b) 811.4 m/s

(c) 831.4 m/s (d) 821.4 m/s

26. A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one-fourth of its original speed. What is the mass of the second body? **[JEE Main 2019]**

(a) 1.5 kg (b) 1.2 kg

(c) 1.8 kg (d) 1.0 kg

27. A small block of mass M moves with velocity $5\ \text{ms}^{-1}$ towards an another block of same mass M placed at a distance of 2 m on a rough horizontal surface. Coefficient of friction between the blocks and ground is 0.25. Collision between the two blocks is elastic, the separation between the blocks, when both of them come to rest, is ($g=10\ \text{ms}^{-2}$)

(a) 3 m (b) 4 m

(c) 2 m (d) 1.5 m

28. A body of mass m_1 moving with an unknown velocity of $v_1\hat{\mathbf{i}}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{\mathbf{i}}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{\mathbf{i}}$ and $v_4\hat{\mathbf{i}}$, respectively. If $m_2=0.5m_1$ and $v_3=0.5\,v_1$, then v_1 is **[JEE Main 2019]**

(a) v_4+v_2 (b) $v_4-\frac{v_2}{4}$

(c) $v_4-\frac{v_2}{2}$ (d) v_4-v_2

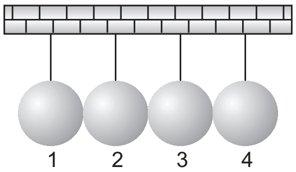
29. An α-particle of mass m suffers one-dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing 64% of its initial kinetic energy. The mass of the nucleus is **[JEE Main 2019]**

(a) 1.5 m (b) 4 m
(c) 3.5 m (d) 2 m

30. A thick uniform bar lies on a frictionless horizontal surface and is free to move in any way on the surface. Its mass is 0.16 kg and length is 1.7 m. Two particles each of mass 0.08 kg are moving on the same surface and towards the bar in the direction perpendicular to the bar, one with a velocity of 10 ms^{-1} and other with velocity 6 ms^{-1}. If collision between particles and bar is completely inelastic, both particles strike with the bar simultaneously. The velocity of centre of mass after collision is

(a) 2 ms^{-1} (b) 4 ms^{-1}
(c) 10 ms^{-1} (d) 167 ms^{-1}

31. In the given figure four, identical spheres of equal mass m are suspended by wires of equal length l_0, so that all spheres are almost touching to one other. If the sphere 1 is released from the horizontal position and all collisions are elastic, the velocity of sphere 4 just after collision is



(a) $\sqrt{2\,gl_0}$ (b) $\sqrt{3\,gl_0}$
(c) $\sqrt{gl_0}$ (d) $\frac{\sqrt{g\,l_0}}{2}$

32. A body of mass 1 kg falls freely from a height of 100 m on a platform of mass 3 kg which is mounted on a spring having spring constant $k = 1.25 \times 10^6$ N/m. The body sticks to the platform and the spring's maximum compression is found to be x. Given that $g = 10\,ms^{-2}$, the value of x will be close to **[JEE Main 2019]**

(a) 8 cm (b) 4 cm
(c) 40 cm (d) 80 cm

33. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son, so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is **[JEE Main 2019]**

(a) 0.28 ms^{-1} (b) 0.20 ms^{-1}
(c) 0.47 ms^{-1} (d) 0.14 ms^{-1}

34. A block of mass 1.9 kg is at rest at the edge of a table of height 1 m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 m/s in the horizontal direction just before the collision, then the kinetic energy just before the combined system strikes the floor, is (Take, $g = 10\,m/s^2$ and assume there is no rotational motion and loss of energy after the collision is negligible) **[JEE Main 2020]**

(a) 20 J (b) 19 J (c) 21 J (d) 23 J

35. In an elastic head on collision between two particles,

(a) velocity of separation is equal to the velocity of approach
(b) velocity of the target is always more than the velocity of the projectile
(c) the maximum velocity of the target is double to that of the projectile
(d) maximum transfer of kinetic energy occurs when masses of both projectile and target are equal

36. A sphere of mass m moving with a constant velocity v hits another stationary sphere of same mass. If e is the coefficient of restitution, then the ratio of velocity of two spheres after collision will be

(a) $\frac{1-e}{1+e}$ (b) $\frac{1+e}{1-e}$ (c) $\frac{e+1}{e-1}$ (d) $\frac{e-1}{e+1}t^2$

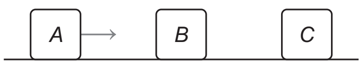
37. A particle of mass m collides with another stationary particle of mass M. If the particle m stops just after collision, the coefficient of restitution for collision is equal to

(a) 1 (b) $\frac{m}{M}$ (c) $\frac{M-m}{M+m}$ (d) $\frac{m}{M+m}$

38. In a one dimensional collision between two identical particles A and B, where B is stationary and A has momentum p before impact. During impact B gives an impulse J to A. Then coefficient of restitution between the two is

(a) $\frac{2J}{p}-1$ (b) $\frac{2J}{p}+1$
(c) $\frac{J}{p}+1$ (d) $\frac{J}{p}-1$

39. Three identical blocks A, B and C are placed on horizontal frictionless surface. The blocks A and C are at rest. But A is approaching towards B with a speed 10 ms^{-1}. The coefficient of restitution for all collisions is 0.5. The speed of the block C just after collision is



(a) 11.25 ms^{-1} (b) 6 ms^{-1}
(c) 8 ms^{-1} (d) 10 ms^{-1}

40. A bullet of mass m hits a target of mass M hanging by a string and gets embedded in it. If the block rises to a height h as a result of this collision, the velocity of the bullet before collision is

(a) $v = \sqrt{2gh}$

(b) $v = \sqrt{2gh}\left(1 + \frac{m}{M}\right)$

(c) $v = \left(1 + \frac{M}{m}\right)\sqrt{2gh}$

(d) $v = \sqrt{2gh}\left(1 - \frac{m}{M}\right)$

41. If a ball is dropped from rest, its bounces from the floor. The coefficient of restitution is 0.5 and the speed just before the first bounce is 5 ms^{-1}. The total time taken by the ball to come to rest is

(a) 2 s

(b) 1 s

(c) 0.5 s

(d) 0.25 s

42. A tennis ball bounces down flight of stairs striking each step in turn and rebounding to the height of the step above. The coefficient of restitution has a value

(a) 1/2 (b) 1

(c) $1/\sqrt{2}$ (d) $1/2\sqrt{2}$

43. Two bodies A and B of definite shape (dimensions of bodies are not ignored). A is moving with speed of 10 ms^{-1} and B is in rest, collides elastically. The

(a) body A comes to rest and B moves with speed of 10 ms^{-1}

(b) they may move perpendicular to each other

(c) A and B may come to rest

(d) they must move perpendicular to each other

Collision in Two Dimensions

44. A ball moving with a certain velocity hits another identical ball at rest. If the plane is frictionless and collision is elastic, the angle between the directions in which the balls move after collision, will be

(a) 30° (b) 60°

(c) 90° (d) 120°

45. A body at rest breaks up into 3 parts. If 2 parts having equal masses fly off perpendicularly each after with a velocity of 12 m/s, then the velocity of the third part which has 3 times mass of each part is

(a) $4\sqrt{2}$ m/s at an angle of 45° from each body

(b) $24\sqrt{2}$ m/s at an angle of 135° from each body

(c) $6\sqrt{2}$ m/s at 135° from each body

(d) $4\sqrt{2}$ m/s at 135° from each body

46. A particle of mass m with an initial velocity $u\hat{\mathbf{i}}$ collides perfectly elastically with a mass $3m$ at rest. It moves with a velocity $v\,\hat{\mathbf{j}}$ after collision, then v is given by **[JEE Main 2020]**

(a) $v = \sqrt{\frac{2}{3}}u$ (b) $v = \frac{1}{\sqrt{6}}u$ (c) $v = \frac{u}{\sqrt{2}}$ (d) $v = \frac{u}{\sqrt{3}}$

47. A smooth steel ball strikes a fixed smooth steel plate at an angle θ with the vertical. If the coefficient of restitution is e, the angle at which the rebounce will take place is

(a) θ (b) $\tan^{-1}\left(\frac{\tan\theta}{e}\right)$

(c) $e\tan\theta$ (d) $\tan^{-1}\left(\frac{e}{\tan\theta}\right)$

48. Particle A of mass m_1 moving with velocity $(\sqrt{3}\,\hat{\mathbf{i}} + \hat{\mathbf{j}})\ ms^{-1}$ collides with another particle B of mass m_2 which is at rest initially. Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be the velocities of particles A and B after collision, respectively. If $m_1 = 2m_2$ and after collision $\mathbf{v}_1 = (\hat{\mathbf{i}} + \sqrt{3}\,\hat{\mathbf{j}})\ ms^{-1}$, then the angle between $\mathbf{v}_1$ and $\mathbf{v}_2$ is **[JEE Main 2020]**

(a) 15° (b) 60° (c) −45° (d) 105°

49. A mass m moves with a velocity v and collides inelastically with another identical mass. After collision, the 1st mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the second mass after collision.

(a) v (b) $\sqrt{3}\,v$ (c) $\frac{2}{\sqrt{3}}v$ (d) $\frac{v}{\sqrt{3}}$

50. A particle of mass m moving in the x-direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y-direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to **[JEE Main 2015]**

(a) 44% (b) 50% (c) 56% (d) 62%

51. A particle of mass m is projected with a speed u from the ground at an angle $\theta = \frac{\pi}{3}$ w.r.t. horizontal (X-axis). When it has reached its maximum height, it collides completely inelastically with another particle of the same mass and velocity $u\hat{\mathbf{i}}$. The horizontal distance covered by the combined mass before reaching the ground is **[JEE Main 2020]**

(a) $\frac{3\sqrt{3}}{8}\frac{u^2}{g}$ (b) $\frac{3\sqrt{2}}{4}\frac{u^2}{g}$

(c) $\frac{5}{8}\frac{u^2}{g}$ (d) $2\sqrt{2}\frac{u^2}{g}$

ROUND II Mixed Bag

Only One Option is Correct

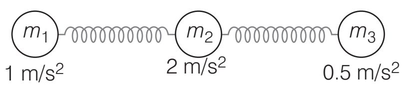
1. A 10 kg object collides with stationary 5 kg object and after collision they stick together and move forward with velocity 4 ms^{-1}. What is the velocity with which the 10 kg object hit the second one?
(a) 4 ms^{-1} (b) 6 ms^{-1}
(c) 10 ms^{-1} (d) 12 ms^{-1}

2. A bullet of mass M hits a block of mass M'. The energy transfer is maximum, when
(a) $M' = M$ (b) $M' = 2M$
(c) $M' << M$ (d) $M' >> M$

3. Two bodies having masses m_1 and m_2 and velocities $\mathbf{u}_1$ and $\mathbf{u}_2$ collide and form a composite system of $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = 0 (m_1 \neq m_2)$. The velocity of the composite system is
(a) zero (b) $\mathbf{u}_1 + \mathbf{u}_2$
(c) $\mathbf{u}_1 - \mathbf{u}_2$ (d) $\dfrac{\mathbf{u}_1 + \mathbf{u}_2}{2}$

4. A gas molecule of mass m strikes the wall of the container with a speed v at an angle θ with the normal to the wall at the point of collision. The impulse of the gas molecule has a magnitude
(a) $3mv$ (b) $2mv\cos\theta$
(c) mv (d) zero

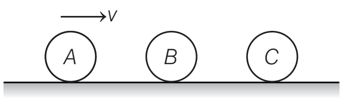
5. A system of three particles having masses $m_1 = 1$ kg, $m_2 = 2$ kg and $m_3 = 4$ kg respectively is connected by two light springs. The acceleration of the three particles at any instant are 1 ms^{-2}, 2 ms^{-2} and 0.5 ms^{-2} respectively directed as shown in the figure. The net external force acting on the system is



(a) 1 N (b) 7 N (c) 3 N (d) 6 N

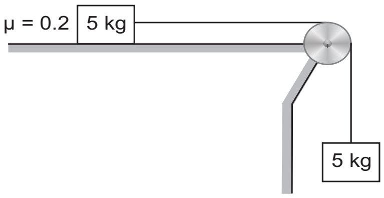
6. A loaded spring gun of mass M fires a shot of mass m with a velocity v at an angle of elevation θ. The gun was initially at rest on a horizontal frictionless surface. After firing, the centre of mass of gun-shot system
(a) moves with a velocity $\dfrac{mv}{M}$
(b) moves with a velocity $\dfrac{mv}{M\cos\theta}$ in the horizontal direction
(c) remains at rest
(d) moves with velocity $\dfrac{(M-m)v}{(M+m)}$ in the horizontal direction

7. Three identical balls A, B and C are lying on a horizontal frictionless table as shown in figure. If ball A is imparted a velocity v towards B and C and the collisions are perfectly elastic, then finally



(a) ball A comes to rest and balls B and C roll out with speed $v/2$ each
(b) balls A and B are at rest and ball C rolls out with speed v
(c) all the three balls roll out with speed $v/3$ each
(d) all the three balls come to rest

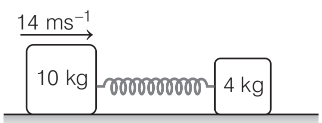
8. In the figure shown below, the magnitude of acceleration of centre of mass of the system is (Take, $g = 10 ms^{-2}$)



(a) 4 ms^{-2} (b) 10 ms^{-2}
(c) $2\sqrt{2}$ ms^{-2} (d) 5 ms^{-2}

9. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward with a velocity 100 ms^{-1} from the ground. The bullet gets embedded in the wood. Then, the maximum height to which the combined system reaches above the top of the building before falling below is (Take, $g = 10 ms^{-2}$) **[JEE Main 2019]**
(a) 20 m (b) 30 m
(c) 10 m (d) 40 m

10. Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulsive force gives a velocity of 14 ms^{-1} to the heavier block in the direction of the lighter block. The velocity of centre of mass of the system at that moment is

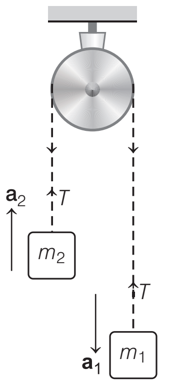


(a) 30 ms^{-1} (b) 20 ms^{-1}
(c) 10 ms^{-1} (d) 5 ms^{-1}

11. A particle is projected with 200 ms^{-1}, at an angle of 60º. At the highest point, it explodes into three particles of equal masses. One goes vertically upward with velocity 100 ms^{-1}, the second particle goes vertically downward with the same velocity as the first. Then, what is the velocity of the third particle?
(a) 120 ms^{-1} with 60º angle
(b) 200 ms^{-1} with 30º angle
(c) 50 ms^{-1} vertically upwards
(d) 300 ms^{-1} horizontally

12. An object of mass m_1 collides with another object of mass m_2, which is at rest. After the collision the objects move with equal speeds in opposite direction. The ratio of the masses $m_2 : m_1$ is **[JEE Main 2021]**
(a) 3 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 1

13. Two bodies of masses m_1 and m_2 ($m_1 > m_2$) respectively are tied to the ends of a massless string, which passes over a light and frictionless pulley. The masses are initially at rest and then released. Then acceleration of the centre of mass of the system is



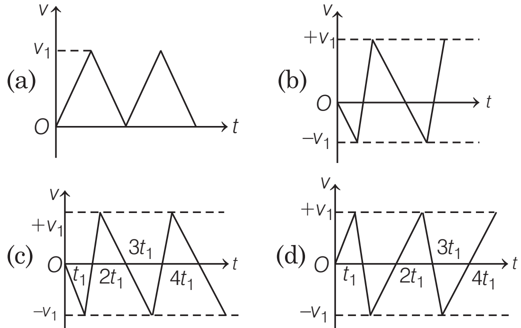
(a) $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$ (b) $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$
(c) g (d) zero

14. Three identical sphere lie at rest along a line on a smooth horizontal surface. The separation between any two adjacent spheres is L. The first sphere is moved with a velocity u towards the second sphere at time $t = 0$. The coefficient of restitution for collision between any two blocks is 1/3. Then choose the correct statement.
(a) The third sphere will start moving at $t = \frac{5L}{2u}$
(b) The third sphere will start moving at $t = \frac{4L}{u}$
(c) The centre of mass of the system will have a final speed $u/3$.
(d) The centre of mass of the system will have a final speed u

15. Two carts on horizontal straight rails are pushed apart by an explosion of a powder charge Q placed between the carts. Suppose the coefficient of friction between carts and rails are identical. If the 200 kg cart travels a distance of 36 m and stops, the distance covered by the cart weighing 300 kg is
(a) 32 m (b) 24 m (c) 16 m (d) 12 m

16. The masses of five balls at rest and lying at equal distances in a straight line are in geometrical progression with ratio 2 and their coefficients of restitution are each 2/3. If the first ball be started towards the second with velocity u, then the velocity communicated to 5th ball is
(a) $\frac{5}{9}u$ (b) $\left(\frac{5}{9}\right)^2 u$ (c) $\left(\frac{5}{9}\right)^3 u$ (d) $\left(\frac{5}{9}\right)^4 u$

17. Consider a rubber ball freely falling from a height $h = 4.9$ m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time and the height as function of time will be



18. A ball strikes a horizontal floor at an angle θ = 45°. The coefficient of restitution between the ball and the floor is $e = 1/2$. The fraction of its kinetic energy lost in collision is
(a) 5/8 (b) 3/8
(c) 3/4 (d) 1/4

19. A ball falls freely from a height of 45 m. When the ball is at a height of 25m, it explodes into two equal pieces. One of them moves horizontally with a speed of 10 ms^{-1}. The distance between the two pieces when both strike the ground is
(a) 10 m (b) 20 m
(c) 15 m (d) 30 m

20. A set on n identical cubical blocks lies at rest parallel to each other along a line on a smooth horizontal surface. The separation between the near surface of any two adjacent block is L. The block at one end is given a speed v towards the next one at time $t = 0$. All collisions are completely inelastic, then

(a) the last block starts moving at $t = \frac{(n-1)}{v} L$

(b) the last block starts moving at $t = \frac{n(n-1)L}{2v}$

(c) the centre of mass of the system will have a final speed v

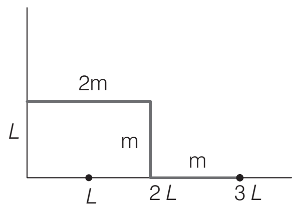
(d) the centre of mass of the system will have a final speed zero

21. A block C of mass m is moving with velocity v_0 and collides elastically with block A of mass m and connected to another block B of mass $2m$ through spring constant k. What is k? If x_0 is compression of spring when velocity of A and B is same.

(a) $\frac{mv_0^2}{x_0^2}$ (b) $\frac{mv_0^2}{2x_0^2}$

(c) $\frac{3}{2}\frac{mv_0^2}{x_0^2}$ (d) $\frac{2}{3}\frac{mv_0^2}{x_0^2}$

22. The position vector of the centre of mass $\mathbf{r}_{CM}$ of an asymmetric uniform bar of negligible area of cross-section as shown in figure is **[JEE Main 2019]**



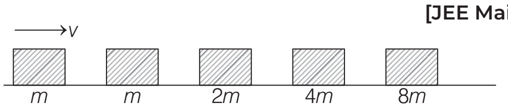
(a) $\mathbf{r} = \frac{13}{8}L\,\hat{\mathbf{x}} + \frac{5}{8}L\,\hat{\mathbf{y}}$

(b) $\mathbf{r} = \frac{11}{8}L\,\hat{\mathbf{x}} + \frac{3}{8}L\,\hat{\mathbf{y}}$

(c) $\mathbf{r} = \frac{3}{8}L\,\hat{\mathbf{x}} + \frac{11}{8}L\,\hat{\mathbf{y}}$

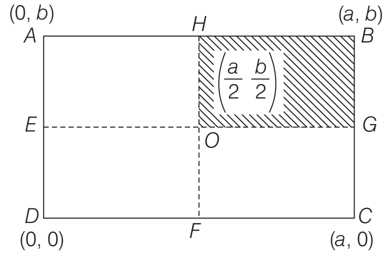
(d) $\mathbf{r} = \frac{5}{8}L\,\hat{\mathbf{x}} + \frac{13}{8}L\,\hat{\mathbf{y}}$

23. Blocks of masses m, $2m$, $4m$ and $8m$ are arranged in a line on a frictionless floor. Another block of mass m, moving with speed v along the same line (see figure) collides with mass m in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time, the last block of mass $8m$ starts moving, the total energy loss is p% of the original energy. Value of p is close to **[JEE Main 2020]**



(a) 77 (b) 87 (c) 94 (d) 37

24. A uniform rectangular thin sheet $ABCD$ of mass M has length a and breadth b, as shown in the figure. If the shaded portion $HBGO$ is cut-off, the coordinates of the centre of mass of the remaining portion will be **[JEE Main 2019]**



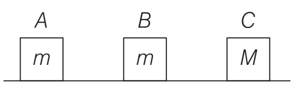
(a) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (b) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

(c) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (d) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

25. A particle of mass m is dropped from a height h above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely inelastically, then the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is **[JEE Main 2020]**

(a) $\sqrt{\frac{1}{2}}$ (b) $\frac{1}{2}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\sqrt{\frac{3}{4}}$

26. Three blocks A, B and C are lying on a smooth horizontal surface as shown in the figure. A and B have equal masses m while C has mass M. Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of $\frac{M}{m}$? **[JEE Main 2019]**



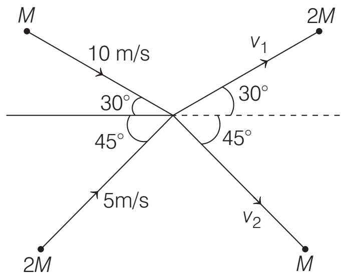
(a) 4 (b) 2 (c) 3 (d) 5

27. A simple pendulum is made of a string of length l and a bob of mass m, is released from a small angle θ_0. It strikes a block of mass M, kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. Then, M is given by **[JEE Main 2019]**

(a) $m\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$ (b) $\frac{m}{2}\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$

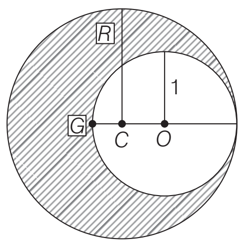
(c) $m\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$ (d) $\frac{m}{2}\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$

28. Two particles of masses M and $2M$, moving as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speed v_1 and v_2 are nearly **[JEE Main 2019]**



(a) 6.5 m/s and 3.2 m/s (b) 3.2 m/s and 6.3 m/s
(c) 3.2 m/s and 12.6 m/s (d) 6.5 m/s and 6.3 m/s

29. As shown in figure, when a spherical cavity (centred at O) of radius 1 is cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, *i.e.* on the surface of the cavity. R can be determined by the equation



[JEE Main 2020]

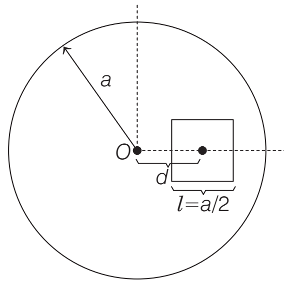
(a) $(R^2 + R + 1)(2 - R) = 1$
(b) $(R^2 + R - 1)(2 - R) = 1$
(c) $(R^2 - R - 1)(2 - R) = 1$
(d) $(R^2 - R + 1)(2 - R) = 1$

Numerical Value Questions

30. The centre of mass of a solid hemisphere of radius 8 cm is x cm from the centre of the flat surface. Then, value of x is .... **[JEE Main 2020]**

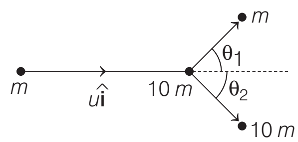
31. A body A, of mass $m = 0.1$ kg has an initial velocity of $3\hat{\mathbf{i}}$ ms^{-1}. It collides elastically with another body, B of the same mass which has an initial velocity of $5\hat{\mathbf{j}}$ ms^{-1}. After collision, A moves with a velocity $\mathbf{v} = 4(\hat{\mathbf{i}} + \hat{\mathbf{j}})$. The energy of B after collision is written as $\frac{x}{10}$ J. The value of x is .... **[JEE Main 2020]**

32. A square shaped hole of side $l = \frac{a}{2}$ is curved out at a distance $d = \frac{a}{2}$ from the centre O of a uniform circular disc of radius a. If the distance of the centre of mass of the remaining portion from O is $-\frac{a}{X}$, value of X (to the nearest integer) is .... **[JEE Main 2020]**

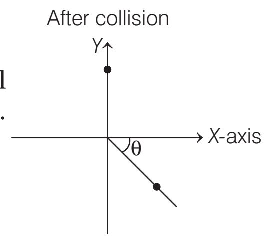


33. Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is half of their initial speed. The angle between the initial velocities of the two bodies (in degree) is .... **[JEE Main 2020]**

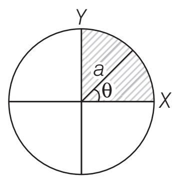
34. A particle of mass m is moving along the X-axis with initial velocity $u\hat{\text{i}}$. It collides elastically with a particle of mass 10 m at rest and then moves with half its initial kinetic energy (see figure). If $\sin\theta_1 = \sqrt{n}\sin\theta_2$, then value of n is .... **[JEE Main 2020]**



35. A ball of mass 10 kg moving with a velocity $10\sqrt{3}$ ms^{-1} along X-axis, hits another ball of mass 20 kg which is at rest. After collision, the first ball comes to rest and the second one disintegrates into two equal pieces. One of the pieces starts moving along Y-axis at a speed of 10 m/s. The second piece starts moving at a speed of 20 m/s at an angle θ (degree) with respect to the X-axis. The configuration of pieces after collision is shown in the figure. The value of θ to the nearest integer is ..... **[JEE Main 2021]**



36. The disc of mass M with uniform surface mass density σ is shown in the figure. The centre of mass of the quarter disc (the shaded area) is at the position $\frac{x}{3}\frac{a}{\pi}, \frac{x}{3}\frac{a}{\pi}$ where x is ....
(Round off to the nearest integer)
(Here, a is an area as shown in the figure.)
[JEE Main 2021]



Answers

Round I

1. (b)	**2.** (d)	**3.** (d)	**4.** (d)	**5.** (c)	**6.** (c)	**7.** (a)	**8.** (a)	**9.** (c)	**10.** (d)
11. (d)	**12.** (a)	**13.** (b)	**14.** (a)	**15.** (a)	**16.** (b)	**17.** (d)	**18.** (a)	**19.** (c)	**20.** (a)
21. (c)	**22.** (b)	**23.** (d)	**24.** (d)	**25.** (c)	**26.** (b)	**27.** (a)	**28.** (d)	**29.** (b)	**30.** (b)
31. (a)	**32.** (*)	**33.** (b)	**34.** (c)	**35.** (d)	**36.** (a)	**37.** (b)	**38.** (a)	**39.** (a)	**40.** (c)
41. (c)	**42.** (c)	**43.** (b)	**44.** (c)	**45.** (d)	**46.** (c)	**47.** (b)	**48.** (d)	**49.** (c)	**50.** (c)
51. (a)									

Round II

1. (b)	**2.** (a)	**3.** (a)	**4.** (b)	**5.** (c)	**6.** (c)	**7.** (b)	**8.** (c)	**9.** (d)	**10.** (c)
11. (d)	**12.** (a)	**13.** (a)	**14.** (a)	**15.** (c)	**16.** (d)	**17.** (c)	**18.** (b)	**19.** (b)	**20.** (b)
21. (d)	**22.** (a)	**23.** (c)	**24.** (b)	**25.** (c)	**26.** (a)	**27.** (a)	**28.** (d)	**29.** (a)	**30.** 3
31. 1	**32.** 23	**33.** 120	**34.** 10	**35.** 30	**36.** 4				

Solutions

Round I

1. Centre of mass is closer to massive part of the body, therefore the bottom piece of bat has larger mass.

2. Centre of mass of a bangle lies at the centre of the bangle, which is outside the body.

3. Here, $m_1 = 4$ kg, $m_2 = 2$ kg

$$x_1 = 1, y_1 = 0, z_1 = 1, x_2 = 2, y_2 = 2, z_2 = 0$$

The coordinates of centre are

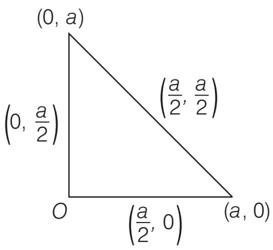
$$x = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{4\times1 + 2\times2}{4+2} = \frac{4}{3}$$

$$y = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{4\times0 + 2\times2}{4+2} = \frac{2}{3}$$

$$z = \frac{m_1z_1 + m_2z_2}{m_1 + m_2} = \frac{4\times1 + 2\times0}{4+2} = \frac{2}{3}$$

4. As shown in figure, centre of mass of respective rods are at their respective mid-points.

Hence centre of mass of the system has coordinates (X_{CM}, Y_{CM}), then

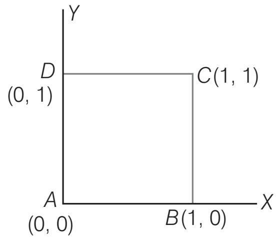


$$X_{CM} = \frac{m\times\frac{a}{2} + m\times\frac{a}{2} + m\times0}{3m} = \frac{a}{3}$$

$$Y_{CM} = \frac{m\times0 + m\times\frac{a}{2} + m\times\frac{a}{2}}{3m} = \frac{a}{3}$$

5. The position of centre of mass of the system shown in figure is likely to be at C. This is because lower part of the sphere containing sand is heavier than upper part of the sphere containing air.

6. $X_{CM} = \dfrac{m_AX_A + m_BX_B + m_CX_C + m_DX_D}{m_A + m_B + m_C + m_D}$



$$= \frac{1\times0 + 2\times1 + 3\times1 + 4\times0}{1+2+3+4}$$

$$= \frac{2+3}{10} = \frac{1}{2} = 0.5 \text{ m}$$

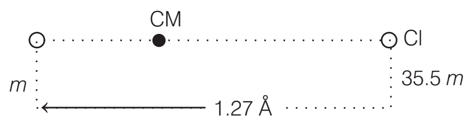
Similarly, $Y_{CM} = \dfrac{m_AY_A + m_BY_B + m_CY_C + m_DY_D}{m_A + m_B + m_C + m_D}$

$$= \frac{1\times0 + 2\times0 + 3\times1 + 4\times1}{1+2+3+4}$$

$$= \frac{7}{10} = 0.7 \text{ m}$$

7. Let mass of hydrogen atom $= m$

$\therefore$ Mass of chlorine atom $= 35.5m$



Let hydrogen atom be at origin, *i.e.* position vector of it, $r_1 = 0$.

$\therefore$ Position vector of chlorine atom, $r_2 = 1.27 \times 10^{-10}$ m

Position vector to centre of mass is given by

$$r_{CM} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

$$= \frac{m \times 0 + 35.5\, m \times 1.27 \times 10^{-10}}{m + 35.5\, m}$$

$$= \frac{35.5 \times 1.27 \times 10^{-10}}{36.5}$$

$$= 1.235 \times 10^{-10} \text{ m} = 1.24 \text{ Å}$$

8. $m_1 = 1$ kg, $m_2 = 2$ kg, $m_3 = 3$ kg

Position of centre of mass (2, 2, 2).

$$m_4 = 4 \text{ kg}$$

New position of centre of mass (0, 0 ,0).

For initial position,

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$2 = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{1 + 2 + 3}$$

$$m_1 x_1 + m_2 x_2 + m_3 x_3 = 12$$

Similarly, $m_1 y_1 + m_2 y_2 + m_3 y_3 = 12$

and $m_1 z_1 + m_2 z_2 + m_3 z_3 = 12$

For new position,

$$x'_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4}{m_1 + m_2 + m_3 + m_4}$$

$$0 = \frac{12 + 4 \times x_4}{1 + 2 + 3 + 4}$$

$\Rightarrow \quad 4x_4 = -12 \quad \Rightarrow \quad x_4 = -3$

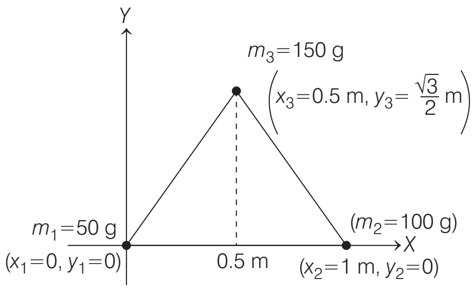
Similarly, $y_4 = -3 \quad \Rightarrow \quad z_4 = -3$

$\therefore$ Position of fourth mass $(-3, -3, -3)$.

9. The height of equilateral Δ is

$$h = y_3 = \sqrt{(1)^2 - (0.5)^2} = \sqrt{3}/2 \text{ m}$$

Thus, coordinates of three masses are (0, 0), (1, 0) and $\left(0.5, \frac{\sqrt{3}}{2}\right)$.



Using, $X_{CM} = \dfrac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$,

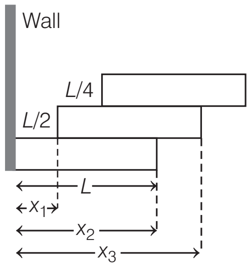
$$= \frac{50 \times 0 + 100 \times 1 + 150 \times 0.5}{50 + 100 + 150}$$

$$= \frac{175}{300} = \frac{7}{12} \text{ m}$$

Similarly, $Y_{CM} = \dfrac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$

$$= \frac{50 \times 0 + 100 \times 0 + 150 \times \frac{\sqrt{3}}{2}}{50 + 100 + 150} = \frac{\sqrt{3}}{4} \text{ m}$$

10. From figure,



$$x_1 = \frac{L}{2},\ x_2 = \frac{L}{2} + \frac{L}{2} = L$$

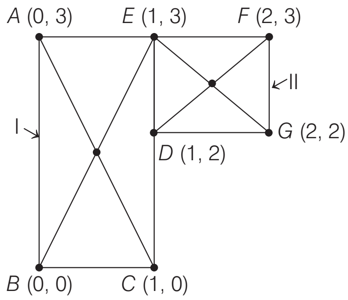
$$x_3 = \frac{L}{2} + \frac{L}{4} + \frac{L}{2} = \frac{5L}{4}$$

$$\therefore \quad x_{CM} = \frac{M_1 x_1 + M_2 x_2 + M_3 x_3}{M_1 + M_2 + M_3}$$

$$= \frac{M \times \frac{L}{2} + M \times L + M \times \frac{5L}{4}}{M + M + M}$$

$$= \frac{\frac{11}{4} ML}{3M} = \frac{11}{12} L$$

11. Given lamina consists of 2 parts I and II as shown in the figure.



As mass of uniform lamina is 4 kg, mass of part I is $m_1 = 3$ kg and mass of part II is $m_2 = 1$ kg.

These masses can be assumed to be concentrated at geometrical centres of sections I and II.

So, $m_1 = 3$ kg has coordinates $x_1 = 0.5$ m, $y_1 = 1.5$ m

and $m_2 = 1$ kg has coordinates $x_2 = 1.5$ m, $y_2 = 2.5$ m

Now, we use formula of centre of mass (CM) to find X_{CM} and Y_{CM}.

So, $\quad X_{CM} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

$$= \frac{(3 \times 0.5) + (1 \times 1.5)}{4} = 0.75 \text{ m}$$

and $\quad Y_{CM} = \dfrac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$

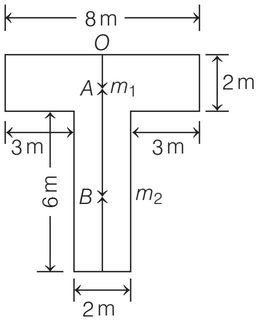
$$= \frac{(3 \times 1.5) + (1 \times 2.5)}{4} = 1.75 \text{ m}$$

12. $y_{CM} = \dfrac{m_1 y_1 + m_2 y_2 + m_3 y_3 + m_4 y_4 + m_5 y_5}{m_1 + m_2 + m_3 + m_4 + m_5}$

$$= \frac{(6m)(0) + m(a) + m(a) + m(0) + m(-a)}{6m + m + m + m + m} = \frac{a}{10}$$

13. Coordinate of centre of mass is given by

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$



Taking parts A and B as two bodies of same system.

$$m_1 = l \times b \times \sigma = 8 \times 2 \times \sigma = 16\sigma$$

$$m_2 = l \times b \times \sigma = 6 \times 2 \times \sigma = 12\sigma$$

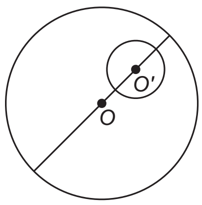
Choosing O as origin,

$$x_1 = 1 \text{ m}, \quad x_2 = 2 + 3 = 5 \text{ m}$$

$$\therefore \quad X_{CM} = \frac{16\sigma \times 1 + 12\sigma \times 5}{16\sigma + 12\sigma} = \frac{19}{7}$$

$$= 2.7 \text{ m from } O$$

14. For the calculation of the position of centre of mass, cut-off mass is taken as negative. The mass of disc is



$$m_1 = \pi r_1^2 \sigma$$

$$= \pi (6)^2 \sigma = 36\pi\sigma$$

where σ is surface mass density.

The mass of cutting portion is

$$m_2 = \pi (1)^2 \sigma = \pi\sigma$$

$$x_{CM} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}$$

Taking origin at the centre of disc,

$$x_1 = 0, x_2 = 3 \text{ cm}$$

$$x_{CM} = \frac{36\pi\sigma \times 0 - \pi\sigma \times 3}{36\pi\sigma - \pi\sigma}$$

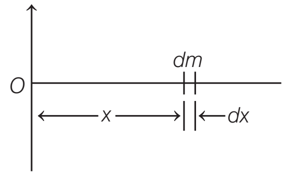
$$= \frac{-3\pi\sigma}{35\pi\sigma} = -\frac{3}{35} \text{ cm}$$

15. Here, $\rho(x) = a(1 + bx^2)$

When $b \to 0$, $(x) = a =$ constant, *i.e.* density of rod of length 1 m is constant. In that event, centre of mass of rod would lie at 0.5 m , (*i.e.* at the centre of rod.)

When we try $b \to 0$ in all the four given options, we find choice (a) alone given $x = \dfrac{3(2 + b)}{4(3 + b)} = \dfrac{6}{12} = 0.5$

16. The mass of considered element is

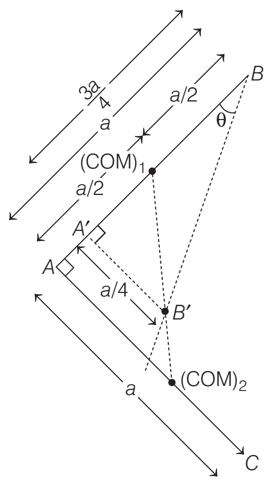


$$dm = \lambda\, dx = \lambda_0 x\, dx$$

$$\therefore \quad x_{CM} = \frac{\int_0^1 x\, dm}{\int dm} = \frac{\int_0^1 x(\lambda_0 x\, dx)}{\int_0^1 \lambda_0 x\, dx}$$

$$= \frac{\lambda_0 \left[\frac{x^3}{3}\right]_0^L}{\lambda_0 \left[\frac{x^2}{2}\right]_0^L} = \frac{\lambda_0 \frac{L^3}{3}}{\lambda_0 \frac{L^2}{2}} = \frac{2}{3} L$$

17. The given system of rods can be drawn using geometry as,



where, $(COM)_1$ and $(COM)_2$ are the centre of mass of both rods AB and AC, respectively.

So, in $\Delta A'BB'$,

$$\tan\theta = \frac{A'B'}{A'B} = \frac{\frac{a}{4}}{\frac{3a}{4}} = \frac{1}{3} \quad \text{or} \quad \tan\theta = \frac{1}{3}$$

18. Since there is no external force acting on the particle, hence

$$y_{CM} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = 0,$$

Hence, $\left(\dfrac{m}{4}\right) \times (15) + \left(\dfrac{3m}{4}\right)(y_2) = 0$

$\Rightarrow \quad y_2 = -5 \text{ cm}$

19. Let mass of each body be m.

$$\Rightarrow \qquad \mathbf{v}_{CM} = \frac{m \times 2v - mv}{m + m} = \frac{v}{2}$$

20. Here, $m_1 = 1$ kg, $\mathbf{v}_1 = 2\,\hat{\mathbf{i}}$

$$m_2 = 2 \text{ kg}, \ \mathbf{v}_2 = 2\cos 30^\circ\,\hat{\mathbf{i}} - 2\sin 30^\circ\,\hat{\mathbf{j}}$$

$$\mathbf{v}_{CM} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2}$$

$$= \frac{1 \times 2\,\hat{\mathbf{i}} + 2\,(2\cos 30^\circ\,\hat{\mathbf{i}} - 2\sin 30^\circ\,\hat{\mathbf{j}})}{1 + 2}$$

$$= \frac{2\,\hat{\mathbf{i}} + 2\sqrt{3}\,\hat{\mathbf{i}} - 2\,\hat{\mathbf{j}}}{3} = \left(\frac{2 + 2\sqrt{3}}{3}\right)\hat{\mathbf{i}} - \frac{2}{3}\,\hat{\mathbf{j}}$$

21. To keep applied centre of mass at the same position, velocity of centre of mass is zero, so

$$\frac{m_1\mathbf{v}_1 + m_2\,\mathbf{v}_2}{m_1 + m_2} = 0$$

where, $\mathbf{v}_1$ and $\mathbf{v}_2$ are velocities of particles 1 and 2 respectively.

$$\Rightarrow \qquad m_1\frac{d\mathbf{r}_1}{dt} + m_2\frac{d\mathbf{r}_2}{dt} = 0 \quad \left[\because v_1 = \frac{d\mathbf{r}_1}{dt} \text{ and } v_2 = \frac{d\mathbf{r}_2}{dt}\right]$$

$$\Rightarrow \qquad m\,d\mathbf{r}_1 + m_2\,d\mathbf{r}_2 = 0$$

Let 2nd particle has been displaced by distance x.

$$\Rightarrow \qquad m_1\,(d) + m_2\,(x) = 0 \quad \Rightarrow \quad x = -\frac{m_1 d}{m_2}$$

(negative sign shows that both the particles have to move in opposite directions.)

22. The acceleration of centre of mass is

$$a_{CM} = \frac{F}{m_A + m_B} = \frac{30}{10 + 20} = 1 \text{ ms}^{-2}$$

$$\therefore \qquad s = \frac{1}{2}a_{CM}\,t^2 = \frac{1}{2} \times 1 \times 2^2 = 2 \text{ m}$$

23. (a) Impulse received by m

$$\mathbf{J} = m\,(\mathbf{v}_f - \mathbf{v}_i)$$
$$= m\,(-2\,\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\,\hat{\mathbf{i}} - 2\,\hat{\mathbf{j}})$$
$$= m\,(-5\,\hat{\mathbf{i}} - \hat{\mathbf{j}})$$

and impulse received by M

$$= -\mathbf{J} = m\,(5\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

(b) $mv = m\,(5\,\hat{\mathbf{i}} + \hat{\mathbf{j}})$

or $\qquad v = \dfrac{m}{M}(5\,\hat{\mathbf{i}} + \hat{\mathbf{j}}) = \dfrac{1}{13}(5\,\hat{\mathbf{i}} + \hat{\mathbf{j}})$

(c) $e =$ (relative velocity of separation/relative velocity of approach) in the direction of $-\hat{\mathbf{j}} = 11/17$

24.

As $m_1 = m_2$, therefore after elastic collision, velocities of masses get interchanged.

i.e. velocity of mass, $m_1 = -5$ m/s

and velocity of mass, $m_2 = +3$ m/s

25. From energy conservation

$$\left[\begin{array}{l}\text{after bullet gets embedded till the} \\ \text{system comes momentarily at rest}\end{array}\right]$$

$$(M + m)\,g\,h = \frac{1}{2}(M + m)v_1^2$$

(where, v_1 is velocity after collision)

$$\therefore \qquad v_1 = \sqrt{2gh}$$

Applying momentum conservation, (just before and just after collision)

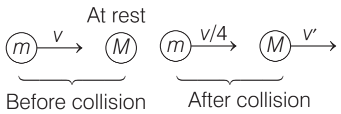
$$mv = (M + m)v_1$$

$$v = \left(\frac{M + m}{m}\right)v_1$$

$$= \frac{6}{10 \times 10^{-3}} \times \sqrt{2 \times 9.8 \times 9.8 \times 10^{-2}}$$

$$\approx 831.55 \text{ m/s}$$

26. Given situation is as shown



Using momentum conservation law for the given system,

$$(\text{Total momentum})_{\text{before collision}} = (\text{Total momentum})_{\text{after collision}}$$

$$\Rightarrow \qquad m(v) + M\,(0) = m\left(\frac{v}{4}\right) + M(v') \qquad \ldots\text{(i)}$$

$$\because \qquad e = 1 \text{ and}$$

we know that, $\qquad e = -\dfrac{v_2 - v_1}{u_2 - u_1}$

$$\Rightarrow \qquad 1 = -\frac{v' - v/4}{0 - v}$$

$$\Rightarrow \qquad v = v' - v/4$$

or $\qquad v' = 5v/4 \qquad \ldots\text{(ii)}$

Using value from Eq. (ii) into Eq. (i), we get

$$mv = \frac{mv}{4} + M\left(\frac{5v}{4}\right)$$

$$M = \frac{3}{5}m = \frac{3}{5} \times 2 = 1.2 \text{ kg}$$

27. Retardation due to friction

$$a = \mu g = (0.25)\,(10) = 2.5 \text{ ms}^{-2}$$

Collision is elastic, *i.e.* after collision, first block comes to rest and the second block acquires the velocity of first block or we can understand it is this manner that second block is permanently at rest while only the first block moves. Distance travelled by it will be,

$$s = \frac{v^2}{2\,a} = \frac{(5)^2}{(2)\,(2.5)} = 5 \text{ m}$$

$\therefore$ Final separation will be $(s - 2) = 3$ m

28. Given, $m_2 = 0.5 m_1$

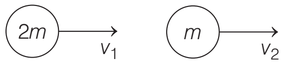
$\Rightarrow \qquad m_1 = 2m_2$

Let $\qquad m_2 = m$, then, $m_1 = 2m$

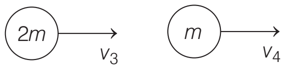
Also, $\qquad v_3 = 0.5 v_1$

Given situation of collinear collision is as shown below.

Before collision,



After collision,



$\therefore$ According to the conservation of linear momentum,
Initial momentum = Final momentum

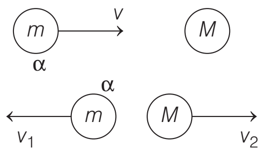
$$m_1 v_1 \hat{\mathbf{i}} + m_2 v_2 \hat{\mathbf{i}} = m_1 v_3 \hat{\mathbf{i}} + m_2 v_4 \hat{\mathbf{i}}$$

$$\Rightarrow \quad 2mv_1 \hat{\mathbf{i}} + mv_2 \hat{\mathbf{i}} = 2m(0.5 v_1)\hat{\mathbf{i}} + mv_4 \hat{\mathbf{i}}$$

$$\Rightarrow \quad v_4 = v_1 + v_2$$

$$\Rightarrow \quad v_1 = v_4 - v_2$$

29. We have following collision, where mass of α particle $= m$ and mass of nucleus $= M$



Let α-particle rebounds with velocity v_1, then
Given; final energy of $\alpha = 36\%$ of initial energy

$$\Rightarrow \quad \frac{1}{2} m v_1^2 = 0.36 \times \frac{1}{2} m v^2$$

$$\Rightarrow \quad v_1 = 0.6\, v \qquad \ldots\text{(i)}$$

As unknown nucleus gained 64% of energy of α, we have

$$\frac{1}{2} M v_2^2 = 0.64 \times \frac{1}{2} m v^2$$

$$\Rightarrow \quad v_2 = \sqrt{\frac{m}{M}} \times 0.8\, v \qquad \ldots\text{(ii)}$$

From momentum conservation, we have

$$mv = Mv_2 - mv_1$$

Substituting values of v_1 and v_2 from Eqs. (i) and (ii), we have

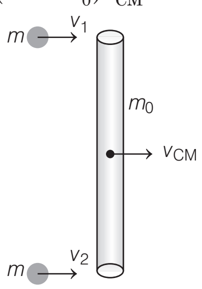
$$mv = M\sqrt{\frac{m}{M}} \times 0.8\, v - m \times 0.6\, v$$

$$\Rightarrow \quad M = 4m$$

30. Here, $m = 0.08$ kg, $\quad m_0 = 0.16$ kg

According to conservation principle of momentum,

$$mv_1 + mv_2 = (2m + m_0)\, v_{CM}$$



$$\therefore \quad v_{CM} = \frac{mv_1 + mv_2}{2m + m_0}$$

$$= \frac{0.08 \times 16}{0.16 + 0.16} = \frac{1.28}{1.32} = \frac{128}{32} = 4 \text{ ms}^{-1}$$

31. When the sphere 1 is released from horizontal position, from energy conservation, potential energy at height l_0 = kinetic energy at bottom

or $\qquad mgl_0 = \frac{1}{2} mv^2 \quad$ or $\quad v = \sqrt{2\, gl_0}$

Since, all collisions are elastic, so velocity of sphere 1 is transferred to sphere 2, then from 2 to 3 and finally from 3 to 4. Hence, just after collision, the sphere 4 attains a velocity to $\sqrt{2\, gl_0}$.

32. Initial compression of the spring,

$$mg = k\left(\frac{x_0}{100}\right) \qquad (x_0 \text{ in cm})$$

$$\Rightarrow \quad x_0 = \frac{3 \times 10 \times 100}{1.25 \times 10^6} = \frac{3}{1250}$$

which is very small and can be neglected.

Applying conservation of momentum before and after the collision, *i.e.* momentum before collision = momentum after collision.

$$m \times \sqrt{2gh} = (m + M)\, v$$

($\because$ Velocity of the block just before the collision is $v^2 - 0^2 = 2gh \quad$ or $\quad v = \sqrt{2gh}$)

After substituting the given values, we get

$$1 \times \sqrt{2 \times 10 \times 100} = 4v$$

or $\qquad 4v = 20\sqrt{5}$

So, $\qquad v = 5\sqrt{5}$ m/s

Let this be the maximum velocity, then for the given system, using

$$\frac{1}{2} mv^2 = \frac{1}{2} kx^2$$

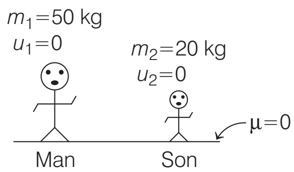
$$\therefore \quad \frac{1}{2} \times 4 \times 125 = \frac{1}{2} \times 1.25 \times 10^6 \times \left(\frac{x}{100}\right)^2$$

$$\Rightarrow \quad 4 = 10^4 \times \frac{x^2}{10^4}$$

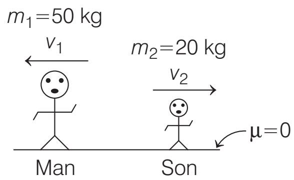
or $\qquad x = 2$ cm

$\therefore$ No option given is correct.

33. The given situation can be shown as below



Before collision



After collision

Using momentum conservation law,

(Total momentum)$_{\text{before collision}}$

$= $ (Total momentum)$_{\text{after collision}}$

$$(m_1 \times 0) + (m_2 \times 0) = m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2$$

$$0 = m_1(-v_1)\hat{\mathbf{i}} + m_2 v_2 \hat{\mathbf{i}}$$

$$\Rightarrow \quad m_1 v_1 = m_2 v_2$$

$$\Rightarrow \quad 50 v_1 = 20 v_2$$

$$\Rightarrow \quad v_2 = 2.5 v_1 \qquad \ldots\text{(i)}$$

Again, relative velocity $= 0.70$ m/s

But from figure, relative velocity $= v_1 + v_2$

$$\therefore \quad v_1 + v_2 = 0.7 \qquad \ldots\text{(ii)}$$

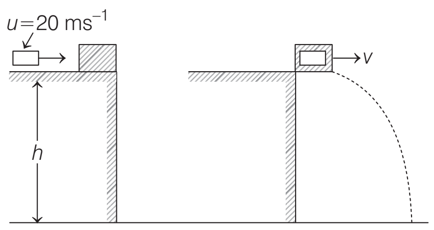
From Eqs. (i) and (ii), we get

$$v_1 + 2.5 v_1 = 0.7$$

$$\Rightarrow \quad v_1 (3.5) = 0.7$$

$$v_1 = \frac{0.7}{3.5} = 0.20 \text{ m/s}$$

34. When the bullet undergoes an inelastic collision with block, a part of KE of bullet is lost.



When bullet + block system falls from height h, its total energy (kinetic + potential) becomes kinetic energy, so kinetic energy of bullet + block system at bottom just before collision is equal to total energy just after collision.

Now, by law of conservation of momentum, we have

$$mu = (m + M)v$$

$$\Rightarrow \quad v = \frac{mu}{m+M} = \frac{0.1 \times 20}{(0.1 + 1.9)} = 1 \text{ ms}^{-1}$$

Total energy of bullet and block just after collision

$$= \text{KE} + \text{PE} = \frac{1}{2}(m + M)v^2 + (m + M)gh$$

$$= \frac{1}{2} \times 2 \times 1^2 + 2 \times 10 \times 1$$

$$= 1 + 20 = 21 \text{ J}$$

35. For elastic collision $e = 1$ and velocity of separation is equal to velocity of approach.

The velocity of the target may be more, equal or less than that of projectile depending on their masses.

The maximum velocity of target is double to that of projectile, when projectile is extremely massive as compared to the target.

Maximum kinetic energy is transferred from projectile to target when their masses are exactly equal.

36. Given, $m_1 = m_2 = m$, $u_1 = v$ and $u_2 = 0$

$$v_1 = \frac{v}{2}(1 - e)$$

$$v_2 = \frac{v}{2}(1 + e)$$

$$\therefore \quad \frac{v_1}{v_2} = \left(\frac{1-e}{1+e}\right)$$

37. As net horizontal force acting on the system is zero, hence momentum must remain conserved.

Hence, $\quad mu + 0 = 0 + Mv_2$

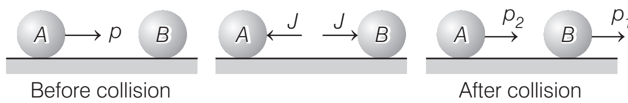
$$\Rightarrow \quad v_2 = \frac{mu}{M}$$

As per definition,

$$e = \frac{(v_2 - v_1)}{(u_2 - u_1)} = \frac{v_2 - 0}{0 - u}$$

$$= \frac{v_2}{u} = \frac{\frac{mu}{M}}{u} = \frac{m}{M}$$

38. Let p_1 and p_2 be the momenta of A and B after collision.



Then applying impulse = change in linear momentum for the two particles

For B $\quad J = p_1$

For A $\quad J = p - p_2$

or $\quad p_2 = p - J$

Coefficient of restitution, $e = \dfrac{p_1 - p_2}{p}$

$$= \frac{p_1 - p + J}{p}$$

$$= \frac{J - p + J}{p}$$

$$= \frac{2J}{p} - 1$$

39. For collision between block A and B,

$$e = \frac{v_B - v_A}{u_A - u_B}$$

$$= \frac{v_B - v_A}{10 - 0} = \frac{v_B - v_A}{10}$$

$$\therefore \quad v_B - v_A = 10e = 10 \times 0.5 = 5 \qquad \ldots\text{(i)}$$

From principle of momentum conservation,

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

or $\quad m \times 10 + 0 = mv_A + mv_B$

$$\therefore \quad v_A + v_B = 10 \qquad \ldots\text{(ii)}$$

Adding Eqs. (i) and (ii), we get

$$v_B = 7.5 \text{ ms}^{-1} \qquad \ldots\text{(iii)}$$

Similarly, for collision between B and C,

$$v_C - v_B = 7.5e = 7.5 \times 0.5 = 3.75$$

$$\therefore \quad v_C - v_B = 3.75 \text{ ms}^{-1} \qquad \ldots\text{(iv)}$$

Adding Eqs. (iii) and (iv), we get

$$v_C = 11.25 \text{ ms}^{-1}$$

40. If initial velocity of bullet be v, then after collision combined velocity of bullet and target is

$$v' = \frac{mv}{(M+m)}$$

and $$h = \frac{v'^2}{2g} \text{ or } v' = \sqrt{2gh}$$

$\therefore$ $$\frac{mv}{(M+m)} = \sqrt{2gh}$$

$\Rightarrow$ $$v = \left(\frac{M+m}{m}\right)\cdot\sqrt{2gh} = \left(1+\frac{M}{m}\right)\sqrt{2gh}$$

41. Acceleration, $g = \dfrac{v - v_0}{t}$

$\therefore$ $$v = 0$$

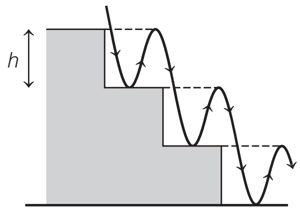
Speed before first bounce

$$v_0 = -5 \text{ ms}^{-1}$$

$\therefore$ $$t = \frac{v_B - v_A}{g}$$

$$= \frac{0-(-5)}{10} = \frac{5}{10} = 0.5 \text{ s}$$

42. As shown in the following figure, ball is falling from height $2h$ and rebounding to a height h only. It means that velocity of ball just before collision.



$$u = \sqrt{\frac{2(2h)}{g}} = \sqrt{\frac{4h}{g}}$$

and velocity just after collision.

$$v = -\sqrt{\frac{2h}{g}}$$

$\therefore$ $$e = \frac{-v}{u} = \frac{\sqrt{\frac{2h}{g}}}{\sqrt{\frac{4h}{g}}} = \frac{1}{\sqrt{2}}$$

43. (a) This is only possible when collision is head-on elastic.

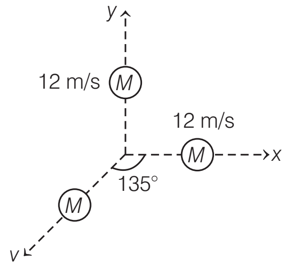
(b) When collision is oblique elastic, then in this case, both bodies move perpendicular to each other after collision.

(c) Since, in elastic collision, kinetic energy of system remains constant so, this is not possible.

(d) The same reason as (b).

44. This is an example of elastic oblique collision. When a moving body collides obliquely with another identical body at rest, then during elastic collision, the angle of divergence will be 90°.

45. The momentum of third part will be equal and opposite of the resultant of momentum of rest two equal parts.



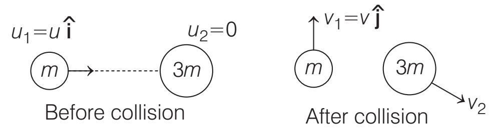
Let v be the velocity of third part.

By the conservation of linear momentum,

$$3m \times v = m \times 12\sqrt{2}$$

$\Rightarrow$ $$v = 4\sqrt{2} \text{ m/s}$$

46. As collision is elastic as shown below, both momentum and KE are conserved.



Momentum conservation gives,

$$mu\hat{\mathbf{i}} = mv\hat{\mathbf{j}} + 3m\mathbf{v}_2$$

$\Rightarrow$ $$\mathbf{v}_2 = \frac{1}{3}(u\hat{\mathbf{i}} - v\hat{\mathbf{j}})$$

$\Rightarrow$ $$|\mathbf{v}_2| = \sqrt{\frac{u^2+v^2}{9}}$$

or $$v_2^2 = (u^2+v^2)/9 \qquad \text{...(i)}$$

Kinetic energy conservation gives,

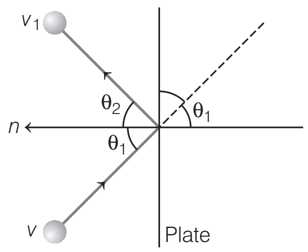
$\Rightarrow$ $$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}3mv_2^2$$

$\Rightarrow$ $$u^2 = v^2 + 3v_2^2 \qquad \text{...(ii)}$$

Substituting value of v_2 from Eq (i) into Eq (ii), we get

$$v = \frac{u}{\sqrt{2}}$$

47. Since, no force is present along the surface, so momentum conservation principle for ball is applicable along the surface of plate.



$$mv\sin\theta_1 = mv_1\sin\theta_2$$

or $$v\sin\theta_1 = v_1\sin\theta_2$$

$$e = \frac{v_1\cos\theta_2}{v\cos\theta_1} = \frac{v_1\cos\theta_2}{v\cos\theta}$$

$\therefore \quad v_1 = \cos\theta_2 = ev\cos\theta$

$$\therefore \quad \frac{v_1 \sin\theta_2}{v_1\cos\theta_2} = \frac{v\sin\theta}{ev\cos\theta} = \frac{\tan\theta}{e}$$

$$\therefore \quad \tan\theta = \frac{\tan\theta}{e}$$

$$\therefore \quad \theta_2 = \tan^{-1}\left(\frac{\tan\theta}{e}\right)$$

48. Given that, $\mathbf{u}_1 = (\sqrt{3}\,\hat{\mathbf{i}} + \hat{\mathbf{j}})$ m/s, $\mathbf{u}_2 = 0$

$\mathbf{v}_1 = (\hat{\mathbf{i}} + \sqrt{3}\,\hat{\mathbf{j}})$ m/s and $m_1 = 2m_2$

Using conservation of linear momentum,

$$\mathbf{p}_i = \mathbf{p}_f$$

$$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$$

$$2m_2(\sqrt{3}\,\hat{\mathbf{i}} + \hat{\mathbf{j}}) + m_2(0) = 2m_2(\hat{\mathbf{i}} + \sqrt{3}\,\hat{\mathbf{j}}) + m_2\mathbf{v}_2$$

$$\mathbf{v}_2 = 2(\sqrt{3}\,\hat{\mathbf{i}} + \hat{\mathbf{j}}) - 2(\hat{\mathbf{i}} + \sqrt{3}\,\hat{\mathbf{j}})$$

$$= 2(\sqrt{3} - 1)(\hat{\mathbf{i}} - \hat{\mathbf{j}}) \text{ m/s}$$

Let the angle between $\mathbf{v}_1$ and $\mathbf{v}_2$ be θ, then

$$\cos\theta = \frac{\mathbf{v}_1 \cdot \mathbf{v}_2}{v_1 v_2}$$

$$= \frac{(\hat{\mathbf{i}} + \sqrt{3}\,\hat{\mathbf{j}})\cdot 2(\sqrt{3}-1)(\hat{\mathbf{i}} - \hat{\mathbf{j}})}{2\cdot 2\sqrt{2}(\sqrt{3}-1)}$$

$$(\because v_1 = 2 \text{ m/s}, v_2 = 2\sqrt{2}(\sqrt{3}-1) \text{ m/s})$$

$$= \frac{2(\sqrt{3}-1) - 2\sqrt{3}(\sqrt{3}-1)}{4\sqrt{2}(\sqrt{3}-1)}$$

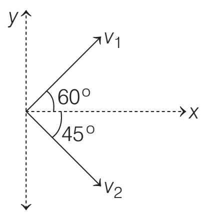
$$= \frac{2(\sqrt{3}-1)(1-\sqrt{3})}{4\sqrt{2}(\sqrt{3}-1)}$$

$$\cos\theta = \frac{1-\sqrt{3}}{2\sqrt{2}} = -0.259$$

$$\Rightarrow \quad \theta = 105^\circ$$

Alternate solution

Directly observing the direction of $\mathbf{v}_1$ and $\mathbf{v}_2$.



$$\theta_1 = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 60^\circ$$

$$\Rightarrow \quad \theta_2 = \tan^{-1}(-1) = 45^\circ$$

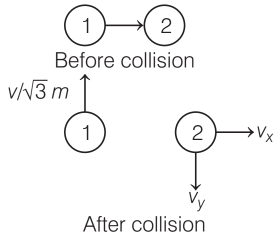
$$\therefore \quad \theta = \theta_1 + \theta_2 = 60^\circ + 45^\circ = 105^\circ$$

49. In x-direction, apply conservation of momentum, we get

$$mu_1 + 0 = mv_x$$

$$\Rightarrow \quad mv = mv_x$$

$$\Rightarrow \quad v_x = v$$



In y-direction, apply conservation of momentum, we get

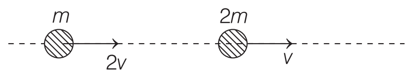
$$0 + 0 = m\left(\frac{v}{\sqrt{3}}\right) - mv_y$$

$$\Rightarrow \quad v_y = \frac{v}{\sqrt{3}}$$

Velocity of second mass after collision,

$$v' = \sqrt{\left(\frac{v}{\sqrt{3}}\right)^2 + v^2} = \sqrt{\frac{4}{3}v^2} \quad \text{or} \quad v' = \frac{2}{\sqrt{3}}v$$

50. Consider the movement of two particles as shown below.



Conserving linear momentum in x-direction

$$(p_i)x = (p_f)x$$

or $\quad 2mv = (2m + m)\,v_x$

or $\quad v_x = \dfrac{2}{3}v$

Conserving linear momentum in y-direction

$$(p_i)y = (p_f)y$$

or $\quad 2mv = (2m + m)\,v_y$

or $\quad v_y = \dfrac{2}{3}v$

Initial kinetic energy of the two particles system is

$$E_i = \frac{1}{2}m\,(2v)^2 + \frac{1}{2}(2m)\,(v)^2$$

$$= \frac{1}{2}\times 4mv^2 + \frac{1}{2}\times 2mv^2$$

$$= 2mv^2 + mv^2 = 3mv^2$$

Final energy of the combined two particles system is

$$E_f = \frac{1}{2}(3m)\,(v_x^2 + v_y^2)$$

$$= \frac{1}{2}(3m)\left[\frac{4v^2}{9} + \frac{4v^2}{9}\right]$$

$$= \frac{3m}{2}\left[\frac{8v^2}{9}\right] = \frac{4mv^2}{3}$$

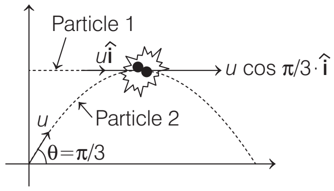
Loss in the energy $\Delta E = E_i - E_f$

$$= mv^2\left[3 - \frac{4}{3}\right] = \frac{5}{3}mv^2$$

Percentage loss in the energy during the collision

$$\frac{\Delta E}{E_i}\times 100 = \frac{(5/3)\,mv^2}{3mv^2}\times 100 = \frac{5}{9}\times 100 \simeq 56\%$$

51. Collision is as shown in the figure.



Velocity of the particle projected from origin at its topmost point,

$$\mathbf{u}_2 = u \cos \frac{\pi}{3} \cdot \hat{\mathbf{i}} = \frac{u}{2} \hat{\mathbf{i}}$$

By conservation of momentum (velocity of combined mass after collision (v)), we have

$$mu\hat{\mathbf{i}} + m \frac{u}{2} \hat{\mathbf{i}} = 2m\mathbf{v}$$

$$\Rightarrow \quad \mathbf{v} = \frac{3}{4} u\hat{\mathbf{i}}$$

Time of fall of combined mass from h_{max},

$$t = \frac{u \sin \theta}{g} = \frac{u \sin \frac{\pi}{3}}{g} = \frac{\sqrt{3}}{2} \frac{u}{g}$$

During this time, combined particle keeps on moving with a horizontal speed of $|\mathbf{v}| = \frac{3}{4} u$.

So, horizontal distance covered by combined mass before reaching the ground,

$$R = \text{speed} \times \text{time} = \frac{3}{4} u \times \frac{\sqrt{3}}{2} \frac{u}{g} = \frac{3\sqrt{3}}{8} \cdot \frac{u^2}{g}$$

Round II

1. As, $m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$

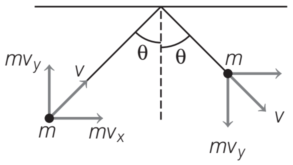
$$\Rightarrow \quad 10 \times u_1 + 5 \times 0 = (10 + 5) \times 4$$

$$\Rightarrow \quad u_1 = \frac{15 \times 4}{10} = 6 \text{ ms}^{-1}$$

2. If $M = M'$, then bullet will transfer whole of its velocity (and consequently 100% of its KE) to block and will itself come to rest as per theory of collision.

3. Since net momentum of the composite system is zero, hence resultant velocity of the composite system should also be zero.

4. In the following figure, it can be seen that the component of momentum along X-axis (parallel to the wall of container) remains unchanged even after the collision.



$\therefore$ Impulse = change in momentum of gas molecule along Y-axis, *i.e.* in a direction normal to the wall $= 2mv\cos\theta$

5. $\because \quad \mathbf{a}_{CM} = \dfrac{\mathbf{F}_{eq}}{(m_1 + m_2 + m_3)} = \dfrac{m_1\mathbf{a}_1 + m_2\mathbf{a}_2 + m_3\mathbf{a}_3}{(m_1 + m_2 + m_3)}$

$$\therefore \quad \mathbf{F}_{eq} = m_1\mathbf{a}_1 + m_2\mathbf{a}_2 + m_3\mathbf{a}_3$$
$$= 1 \times 1 + 2 \times 2 + 4 \times (-0.5) = 1 + 4 - 2 = 3 \text{ N}$$

6. Since gun-shot system is an isolated closed system, its centre of mass must remain at rest.

7. When two identical balls collide head-on elastically, they exchange their velocities. Hence when A collides with B, A transfers its whole velocity to B. When B collides with C, B transfers its whole velocity to C. Hence, finally A and B will be at rest and only C will be moving forward with a speed v.

8. $a_{\text{system}} = \dfrac{5g - 5\mu g}{5 + 5} = 4 \text{ m/s}^2$

$$a_{CM} = \frac{m_1\mathbf{a}_1 + m_2\mathbf{a}_2}{m_1 + m_2} = \frac{5(4\hat{\mathbf{i}}) + 5(4\hat{\mathbf{j}})}{10}$$
$$= \frac{5\sqrt{4^2 + 4^2}}{10} = 2\sqrt{2} \text{ m/s}^2$$

9. Velocity of bullet is very high compared to velocity of wooden block so, in order to calculate time for collision, we take relative velocity nearly equal to velocity of bullet.

So, time taken for particles to collide is

$$t = \frac{d}{v_{rel}} = \frac{100}{100} = 1 \text{ s}$$

Speed of block just before collision is,

$$v_1 = gt = 10 \times 1 = 10 \text{ ms}^{-1}$$

Speed of bullet just before collision is

$$v_2 = u - gt = 100 - 10 \times 1 = 90 \text{ ms}^{-1}$$

Let v = velocity of bullet + block system, then by conservation of linear momentum, we get

$$-(0.03 \times 10) + (0.02 \times 90) = (0.05)\, v$$

$$\Rightarrow \quad v = 30 \text{ ms}^{-1}$$

Now, maximum height reached by bullet and block is

$$h = \frac{v^2}{2g}$$

$$\Rightarrow \quad h = \frac{30 \times 30}{2 \times 10}$$

$$\Rightarrow \quad h = 45 \text{ m}$$

$\therefore$ Height covered by the system from point of collision = 45 m

Now, distance covered by bullet before collision in 1 s.

$$= 100 \times 1 - \frac{1}{2} \times 10 \times 1^2 = 95 \text{ m}$$

Distance of point of collision from the top of the building

$$= 100 - 95 = 5 \text{ m}$$

$\therefore$ Maximum height to which the combined system reaches above the top of the building before falling below $= 45 - 5 = 40$ m.

10. At the time of applying the impulsive force on block of 10 kg pushes the spring forward but 4 kg mass is at rest. Hence,

$$v_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{10 \times 14 + 4 \times 0}{10 + 4}$$
$$= \frac{140}{14} = 10 \text{ ms}^{-1}$$

11. At the highest point momentum of particle before explosion

$$\mathbf{p} = mv \cos 60^\circ$$
$$= m \times 200 \times \frac{1}{2} = 100\, m \text{ horizontally.}$$

Now, as there is no external force during explosion, hence

$$\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = \text{constant}$$

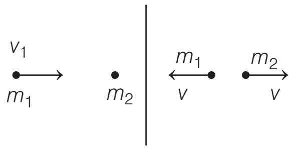
However, since velocities of two fragments, of masses $m/3$ each, are 100 ms^{-1} downward and 100 ms^{-1} upward.

Hence, $\quad \mathbf{p}_1 = -\mathbf{p}_2$

or $\quad \mathbf{p}_1 + \mathbf{p}_2 = 0$

$$\mathbf{p}_3 = \frac{m}{3} \cdot \mathbf{v}_3 = \mathbf{p} = 100\, m \text{ horizontally}$$
$$v_3 = 300 \text{ ms}^{-1} \text{ horizontally}$$

12.



$$m_1 v_1 = -m_1 v + m_2 v$$
$$v_1 = -v + \frac{m_2}{m_1} v$$
$$\frac{(v_1 + v)}{v} = \frac{m_2}{m_1}$$
$$e = \frac{2v}{v_1} = 1$$
$$v = \frac{v_1}{2}$$
$$\frac{v_1 + v_1/2}{v_1/2} = \frac{m_2}{m_1} \Rightarrow 3 = \frac{m_2}{m_1}$$

Hence, the ratio of masses $m_2 : m_1$ is 3 : 1.

13. In the pulley arrangement, $|\mathbf{a}_1| = |\mathbf{a}_2| = a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$

but $\mathbf{a}_1$ is in downward direction and $\mathbf{a}_2$ in the upward direction, *i.e.* $\mathbf{a}_2 = -\mathbf{a}_1$.

$\therefore$ Acceleration of centre of mass

$$\mathbf{a}_{CM} = \frac{m_1 \mathbf{a}_1 + m_2 \mathbf{a}_2}{m_1 + m_2}$$
$$= \frac{m_1 \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g - m_2 \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g}{(m_1 + m_2)}$$
$$= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$$

14. First sphere will take a time t_1 to start motion in second sphere on colliding with it, where $t_1 = \frac{L}{u}$.

Now speed of second sphere will be

$$v_2 = \frac{u}{2}(1 + e) = \frac{2}{3} u \qquad \left(\because e = \frac{1}{3}\right)$$

Hence, time taken by second sphere to start motion in third sphere $t_2 = \frac{L}{2/3\, u} = \frac{3\, L}{2\, u}$.

$\therefore$ Total time $\quad t = t_1 + t_2 = \frac{L}{u} + \frac{3\, L}{2\, u} = \frac{5\, L}{2\, u}$

15. Consider the two cart system as a single system. Due to explosion of power, total momentum of system remains unchanged, *i.e.* $\mathbf{p}_1 + \mathbf{p}_2 = 0$ or $m_1 v_1 = m_2 v_2$, hence

$$\frac{v_1}{v_2} = \frac{m_2}{m_1}$$

As coefficient of friction between carts and rails are identical, hence $a_1 = a_2$ and at the time of stopping, final velocity of cart is zero. Using equation $v^2 - u^2 = 2\, as$, we have

$$\frac{s_1}{s_2} = \frac{v_1^2}{v_2^2} = \frac{m_2^2}{m_1^2}$$

$\Rightarrow$
$$s_2 = \frac{s_1 m_1^2}{m_2^2} = \frac{36 \times (200)^2}{(300)^2} = 16 \text{ m}$$

16. We know that velocity of 2nd ball after collision is given by

$$v_2 = \frac{u_1 (1 + e) m_1}{(m_1 + m_2)} + u_2 \frac{(m_2 - m_1 e)}{(m_1 + m_2)}$$

In present problem $u_2 = 0, m_2 = 2\, m_1$ and $e = 2/3$, hence

$$v_2 = \frac{u\left(1 + \frac{2}{3}\right) m_1}{(m_1 + 2\, m_1)} = \frac{5}{9} u$$

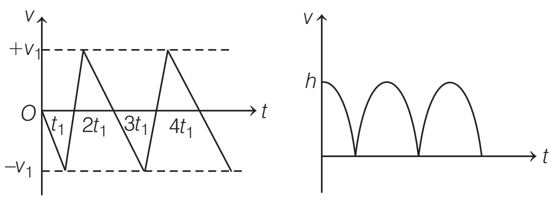
As four exactly similar type of collisions are taking place successively, hence velocity communicated to fifth ball

$$v_5 = \left(\frac{5}{9}\right)^4 u$$

17. $h = \frac{1}{2} gt^2$ (parabolic)

$v = -gt$ and after the collision $v = gt$ (straight line).

Collision is perfectly elastic, then ball reaches to same height again and again with same velocity.



18. Let ball strikes at a speed u, the $K_1 = \frac{1}{2} mu^2$.

Due to collision, tangential component of velocity remains unchanged at $u \sin 45^\circ$, but the normal component of velocity change to $u \sin 45^\circ = \frac{1}{2} u \cos 45^\circ$

$\therefore$ Final velocity of ball after collision

$$v = \sqrt{(u \sin 45^\circ) + \left(\frac{1}{2} u \cos 45^\circ\right)^2}$$

$$= \sqrt{\left(\frac{u}{\sqrt{2}}\right)^2 + \left(\frac{u}{2\sqrt{2}}\right)^2} = \sqrt{\frac{5}{8}}\, u$$

Hence, final kinetic energy

$$K_2 = \frac{1}{2} mv^2 = \frac{5}{16} mu^2$$

$\therefore$ Fractional loss in KE

$$= \frac{K_1 - K_2}{K_1}$$

$$= \frac{\frac{1}{2} mu^2 - \frac{5}{16} mu^2}{\frac{1}{2} mu^2} = \frac{3}{8}$$

19. Let at the time of explosion velocity of one piece of mass $m/2$ is $(10\,\hat{\mathbf{i}})$. If velocity of other be $\mathbf{v}_2$, then from conservation law of momentum (since there is no force in horizontal direction), horizontal component of $\mathbf{v}_2$, must be $-10\,\hat{\mathbf{i}}$.

$\therefore$ Relative velocity of two parts in horizontal direction $= 20\ \text{ms}^{-1}$.

Time taken by ball to fall through 45 m,

$$= 20 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 45}{10}} = 3\,\text{s}$$

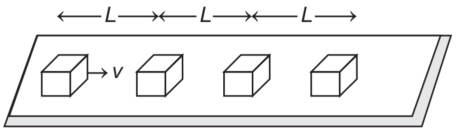
and time taken by ball to fall through first 20 m,

$$t' = \sqrt{\frac{2h'}{g}} = \sqrt{\frac{2 \times 20}{10}} = 2\,\text{s}$$

Hence time taken by ball pieces to fall from 25 m height to ground $= t - t' = 3 - 2 = 1\,\text{s}$.

$\therefore$ Horizontal distance between the two pieces at the time of striking on ground $= 20 \times 1 = 20$ m.

20.



Since, collision is perfectly inelastic, so all the block will stick together one by one and move in a form of combined mass.

Time required to cover distance (d) by first block $= \dfrac{L}{v}$.

Now first and second block will stick together and move with $v/2$ velocity (by applying conservation of momentum) and combined system will take $\dfrac{L}{v/2} = \dfrac{2L}{v}$ to reach upto block third.

Now, these three blocks will move with velocity $v/3$ and combined system will take time $\dfrac{L}{v/3} = \dfrac{3L}{v}$ to reach upto the fourth block.

So, total time $\dfrac{L}{v} + \dfrac{2L}{v} + \dfrac{3L}{v} + \ldots \dfrac{(n-1)L}{v} = \dfrac{n(n-1)L}{2v}$

Final velocity of the centre of mass of the system will be v/n.

21. Using law of conservation of linear momentum, we have

$$mv_0 = mv + 2mv$$

$$\Rightarrow \quad v = \frac{v_0}{3}$$

Using conservation of energy, we have

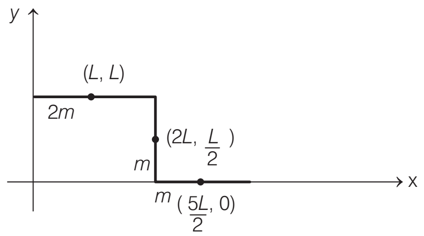
$$\frac{1}{2} mv_0^2 = \frac{1}{2} x_0^2 + \frac{1}{2}(3m)v^2$$

where, $x_0 =$ compression in the spring.

$$\therefore \quad mv_0^2 = kx_0^2 + (3m)\frac{v_0^2}{9}$$

$$\Rightarrow \quad k = \frac{2mv_0^2}{3x_0^2}$$

22. For given system of rods, masses and coordinates of centre of rods are as shown in the following figure.



So, $X_{\text{COM}} = \left(\dfrac{2mL + m2L + m\dfrac{5L}{2}}{4m}\right) = \dfrac{13}{8} L$

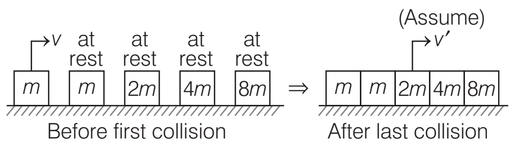
and $Y_{\text{COM}} = \dfrac{2mL + m \times \dfrac{L}{2} + m \times 0}{4m} = \dfrac{5L}{8}$

So, position vector of COM is

$$\mathbf{r}_{\text{COM}} = X_{\text{COM}}\hat{\mathbf{x}} + Y_{\text{COM}}\hat{\mathbf{y}}$$

$$= \frac{13}{8} L\hat{\mathbf{x}} + \frac{5}{8} L\hat{\mathbf{y}}$$

23.



Since, all the collisions are perfectly inelastic, so after the final collision, all blocks will be moving together. Let their final velocity be v'.

By law of conservation of linear momentum,

$$(\mathbf{p}_{\text{sys}})_i = (\mathbf{p}_{\text{sys}})_f$$

$$\Rightarrow \quad mv + m(0) + 2m(0) + 4m(0) + 8m(0) = (m + m + 2m + 4m + 8m)v'$$

$$\Rightarrow \quad mv = 16mv'$$

$$\Rightarrow \quad v' = \frac{v}{16} \qquad \ldots\text{(i)}$$

Now, initial kinetic energy of system,

$$(K_{\text{sys}})_i = \frac{1}{2}mv^2 + 0 + 0 + 0 + 0 = \frac{1}{2}mv^2$$

And final kinetic energy of system,

$$(K_{\text{sys}})_f = \frac{1}{2}(m + m + 2m + 4m + 8m)(v')^2$$

$$= \frac{1}{2} \times 16m \times \left(\frac{v}{16}\right)^2$$

$$= \frac{1}{2} \times 16m \times \frac{v^2}{256}$$

$$= \frac{1}{32}mv^2$$

Loss in kinetic energy,

$$(\Delta K_{\text{sys}})_{\text{loss}} = (K_{\text{sys}})_i - (K_{\text{sys}})_f$$

$$= \frac{1}{2}mv^2 - \frac{1}{32}mv^2$$

$$= \frac{15}{32}mv^2$$

% loss in kinetic energy,

$$\%(\Delta K_{\text{sys}})_{\text{loss}} = \frac{(\Delta K_{\text{sys}})_{\text{loss}}}{(K_{\text{sys}})_i} \times 100\%$$

$$= \frac{\frac{15}{32}mv^2}{\frac{1}{2}mv^2} \times 100\%$$

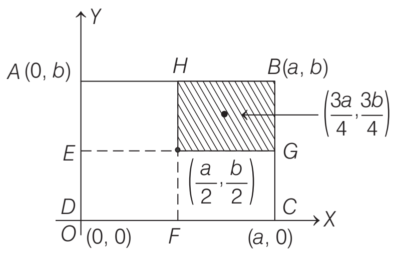
$$= \frac{15}{16} \times 100\%$$

$$= 93.75\%$$

Given that, $\% (\Delta K_{\text{sys}})_{\text{loss}} = p\%$

so, $p = 93.75 \simeq 94$

24. The given rectangular thin sheet $ABCD$ can be drawn as shown in the figure below,



Here,

Area of complete lamina, $A_1 = ab$

Area of shaded part of lamina $= \frac{a}{2} \times \frac{b}{2} = \frac{ab}{4}$

(x_1, y_1) = coordinates of centre of mass of complete lamina $= \left(\frac{a}{2}, \frac{b}{2}\right)$

(x_2, y_2) = coordinates of centre of mass of shaded part of lamina $= \left(\frac{3a}{4}, \frac{3b}{4}\right)$

$\therefore$ Using formula for centre of mass, we have

$$X_{\text{CM}} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2}$$

$$= \frac{ab\left(\frac{a}{2}\right) - \frac{ab}{4}\left(\frac{3a}{4}\right)}{ab - \frac{ab}{4}}$$

$$= \frac{\frac{8a^2b - 3a^2b}{16}}{\frac{3ab}{4}} = \frac{5a}{12}$$

Similarly, $$Y_{\text{CM}} = \frac{A_1y_1 - A_2y_2}{A_1 - A_2}$$

$$= \frac{ab\left(\frac{b}{2}\right) - \frac{ab}{4}\left(\frac{3b}{4}\right)}{ab - \frac{ab}{4}} = \frac{5b}{12}$$

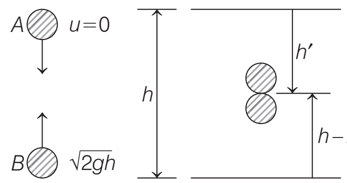
25. Let particles collide at some distance h' from top at time t_0. Then,

$$h' = \frac{1}{2}gt_0^2 \quad \text{... (i)}$$

(for particle A)

and $$h - h' = \sqrt{2gh} \cdot t_0 - \frac{1}{2}gt_0^2 \quad \text{.... (ii)}$$

(for particle B)



From these equations, particles meet after time t_0 given by

$$t_0 = \frac{h}{\sqrt{2gh}} = \sqrt{\frac{h}{2g}}$$

Velocities of particles A and B at instant of collision are $v_A = gt_0$ and $v_B = \sqrt{2gh} - gt_0$.

Hence,

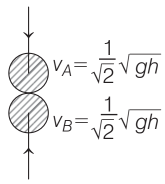
$$v_A = g \times \sqrt{\frac{h}{2g}} = \sqrt{\frac{1}{2}gh} = \frac{1}{\sqrt{2}}\sqrt{gh}$$

and $$v_B = \sqrt{2gh} - g\sqrt{\frac{h}{2g}}$$

$$= \left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)\sqrt{gh} = \frac{1}{\sqrt{2}} \times \sqrt{gh}$$

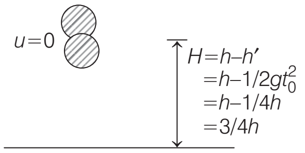
So, particles collide as shown in the figure.

From momentum conservation, we can see that particles stuck, $p_{\text{initial}} = p_{\text{final}}$.



This means the combined system of particles comes to rest ($v_{\text{combined mass}} = 0$) instantaneously.

Now, we have to calculate time of fall of combined mass.



Combined mass starts with $u = 0$

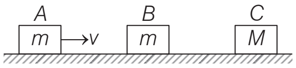
and its height above earth's surface is $H = \frac{3}{4}h$.

So, time taken by combined mass to reach ground is given by

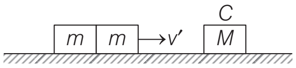
$$H = ut + \frac{1}{2}at^2$$

$$\Rightarrow \qquad \frac{3}{4}h = \frac{1}{2}g \times t^2 \Rightarrow \sqrt{\frac{3}{2}}\sqrt{\frac{h}{g}} = t$$

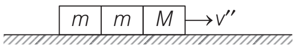
26. Initially, block A is moving with velocity v as shown in the figure below,



Now, A collides with B such that they collide inelastically. Thus, the combined mass (say) move with the velocity v as shown below.



Then, this combined system is collided inelastically again with the block C. So, now the velocity of system be v'' as shown below.



Thus, according to the principle of conservation of momentum,

initial momentum of the system = final momentum of the system

$$\Rightarrow \qquad m\,v = (2m + M)\,v''$$

$$\text{or} \qquad v'' = \left(\frac{mv}{2m + M}\right) \qquad \text{... (i)}$$

Initial kinetic energy of the system,

$$(\text{KE})_i = \frac{1}{2}mv^2 \qquad \text{...(ii)}$$

Final kinetic energy of the system, $(\text{KE})_f$

$$= \frac{1}{2}(2m + M)(v'')^2 = \frac{1}{2}(2m + M)\left(\frac{mv}{2m + M}\right)^2$$

[$\because$ using Eq. (i)]

$$= \frac{1}{2} \cdot \frac{v^2 m^2}{(2m + M)} \qquad \text{...(iii)}$$

Dividing Eq. (iii) and Eq. (ii), we get

$$\frac{(\text{KE})_f}{(\text{KE})_i} = \frac{\frac{\frac{1}{2}m^2 v^2}{(2m + M)}}{\frac{1}{2}mv^2} = \frac{m}{2m + M} \qquad \text{... (iv)}$$

It is given that $\frac{5}{6}$th of $(\text{KE})_i$ is lost in this process.

$$\Rightarrow \qquad (\text{KE})_f = \frac{1}{6}(\text{KE})_i$$

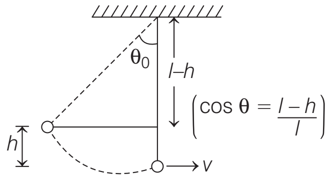
$$\Rightarrow \qquad \frac{(\text{KE})_f}{(\text{KE})_i} = \frac{1}{6} \qquad \text{... (v)}$$

Comparing Eq. (iv) and Eq. (v), we get

$$\frac{m}{2m + M} = \frac{1}{6} \Rightarrow 6m = 2m + M$$

$$4m = M \quad \Rightarrow \frac{M}{m} = 4$$

27. Initially, when pendulum is released from angle θ_0 as shown in the figure below,

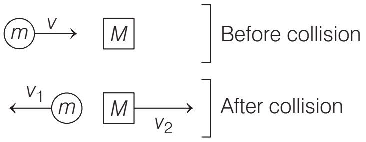


We have, $\quad mgh = \frac{1}{2}mv^2$

Here, $\qquad h = l - l\cos\theta_0$

So, $\qquad v = \sqrt{2gl\,(1 - \cos\theta_0)} \qquad \text{...(i)}$

With velocity v, bob of pendulum collides with block. After collision, let v_1 and v_2 are final velocities of masses m and M respectively as shown.



Then, if pendulum is deflected back upto angle θ_1, then

$$v_1 = \sqrt{2gl\,(1 - \cos\theta_1)} \qquad \text{...(ii)}$$

Using definition of coefficient of restitution to get

$$e = \frac{|\text{velocity of separation}|}{|\text{velocity of approach}|}$$

$$1 = \frac{v_2 - (-v_1)}{v - 0} \Rightarrow v = v_2 + v_1 \qquad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii), we get

$$\Rightarrow \quad \sqrt{2gl(1 - \cos\theta_0)} = v_2 + \sqrt{2gl(1 - \cos\theta_1)}$$

$$\Rightarrow \qquad v_2 = \sqrt{2gl}\,(\sqrt{1 - \cos\theta_0} - \sqrt{1 - \cos\theta_1}) \qquad \text{...(iv)}$$

According to the law of conservation of momentum,

initial momentum of the system = final momentum of the system

$$\Rightarrow \qquad mv = Mv_2 - mv_1$$

$$Mv_2 = m\sqrt{2gl}\,(\sqrt{1 - \cos\theta_0} + \sqrt{1 - \cos\theta_1}) \qquad \text{...(v)}$$

Dividing Eq. (v) by Eq. (iv), we get

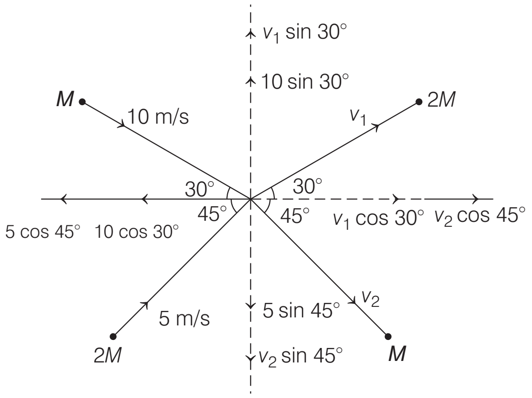
$$\Rightarrow \qquad \frac{M}{m} = \frac{\sqrt{1 - \cos\theta_0} + \sqrt{1 - \cos\theta_1}}{\sqrt{1 - \cos\theta_0} - \sqrt{1 - \cos\theta_1}}$$

$$\frac{M}{m} = \frac{\sin\left(\frac{\theta_0}{2}\right) + \sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_0}{2}\right) - \sin\left(\frac{\theta_1}{2}\right)}$$

For small θ_0, we have

$$\frac{M}{m}=\frac{\frac{\theta_0}{2}+\frac{\theta_1}{2}}{\frac{\theta_0}{2}-\frac{\theta_1}{2}} \quad \text{or} \quad M=m\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$$

28. The given condition can be drawn as shown below.



Applying linear momentum conservation law in x-direction, we get

Initial momentum = Final momentum

$$(M\times 10\cos 30^\circ)+(2M\times 5\cos 45^\circ) = (M\times v_2\cos 45^\circ)+(2M\times v_1\cos 30^\circ)$$

$$\Rightarrow \quad 5\sqrt{3}+5\sqrt{2}=\frac{v_2}{\sqrt{2}}+v_1\sqrt{3} \quad \dots \text{(i)}$$

Similarly, applying linear momentum conservation law in y-direction, we get

$$(M\times 10\sin 30^\circ)-(2M\times 5\sin 45^\circ) = (M\times v_2\sin 45^\circ)-(2M\times v_1\sin 30^\circ)$$

$$\Rightarrow \quad 5-5\sqrt{2}=\frac{v_2}{\sqrt{2}}-v_1 \quad \dots \text{(ii)}$$

Subtracting Eq. (ii) from Eq. (i), we get

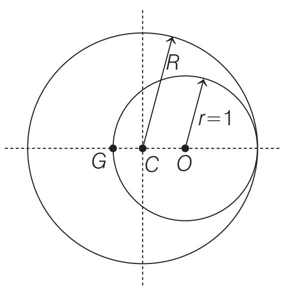
$$(5\sqrt{3}+5\sqrt{2})-(5-5\sqrt{2}) = \left(\frac{v_2}{\sqrt{2}}+v_1\sqrt{3}\right)-\left(\frac{v_2}{\sqrt{2}}-v_1\right)$$

$$\Rightarrow \quad v_1 = 6.516\,\text{m/s} \approx 6.5 \text{ m/s} \quad \dots \text{(iii)}$$

Substituting the value from Eq. (iii) in Eq. (i), we get

$$\Rightarrow \quad v_2 \approx 6.3 \text{ m/s}$$

29. If we combine the remaining and removed spherical parts at their initial relative positions, we get complete sphere as shown in the figure.



So, for complete sphere,

$$X_{\text{CM}}=\frac{m_{\text{remaining}}\times x_{\text{remaining}}+m_{\text{removed}}\times x_{\text{removed}}}{m_{\text{remaining}}+m_{\text{removed}}}$$

As, X_{CM} is at C which is taken at origin of our chosen reference axis, so $X_{\text{CM}}=0$.

$$\Rightarrow \quad m_{\text{remaining}}\times x_{\text{remaining}}+m_{\text{removed}}\times x_{\text{removed}}=0 \quad \dots \text{(i)}$$

Here, $m_{\text{remaining}}=\left(\frac{4}{3}\pi R^3-\frac{4}{3}\pi 1^3\right)\rho$

where, ρ = density of sphere,

$$x_{\text{remaining}}=2-R,$$

$$M_{\text{removed}}=-\left(\frac{4}{3}\pi 1^3\right)\rho.$$

Here, mass removed is negative

and $\quad x_{\text{removed}}=R-1$.

So, from Eq. (i), we get

$$\left(\frac{4}{3}\pi R^3-\frac{4}{3}\pi\right)\rho\times(2-R)=\left(\frac{4}{3}\pi\right)\rho\times(R-1)$$

$$\Rightarrow \quad (R^3-1)(2-R)=R-1$$

$$\Rightarrow \quad (R^2+R+1)(2-R)=1$$

30. The distance of centre of mass of a solid hemisphere from the centre of the flat surface is given by

$$d=\frac{3R}{8}$$

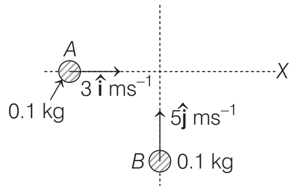
Here, $\quad R=8$ cm

$\therefore \quad d=\dfrac{3\times 8}{8}=3$ cm

Given, $\quad d=x$ cm

$\therefore \quad x=3$

31. Given situation is shown in the figure.



Total initial momentum, $\mathbf{p}_i=m_Av_A+m_Bv_B$

$$=0.1\times 3\hat{\mathbf{i}}+0.1\times 5\hat{\mathbf{j}}$$

$$=0.3\,\hat{\mathbf{i}}+0.5\,\hat{\mathbf{j}}\,(\text{kg-ms}^{-1})$$

Final velocity of A, $\quad v_A=4\,(\hat{\mathbf{i}}+\hat{\mathbf{j}})$

Let final velocity of B be v_B.

Then, final momentum after collision,

$$\mathbf{p}_f=m_Av_A+m_Bv_B$$

$$=0.1\times 4(\hat{\mathbf{i}}+\hat{\mathbf{j}})+0.1\times v_B$$

Now, by conservation of momentum, we have

$$\mathbf{p}_i=\mathbf{p}_f$$

$$\Rightarrow \quad 0.3\hat{\mathbf{i}}+0.5\hat{\mathbf{j}}=0.4\hat{\mathbf{i}}+0.4\hat{\mathbf{j}}+0.1v_B$$

$$\Rightarrow \quad v_B=-\hat{\mathbf{i}}+\hat{\mathbf{j}}$$

Kinetic energy of B after collision will be

$$K=\frac{1}{2}mv^2=\frac{1}{2}\times 0.1\times\left|(-\hat{\mathbf{i}}+\hat{\mathbf{j}})\right|^2$$

$$=\frac{1}{2}\times 0.1\times 2=0.1\text{ J}$$

It is given, energy of B after collision is $\dfrac{x}{10}$.

So, $\dfrac{x}{10}=0.1$ or $x=1$.

32. Centre of mass of remaining portion is given by

$$X_{\text{CM}} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}$$

Here note that, removed mass (m_2) is treated as a negative mass.

Let x = mass per unit area of uniform disc.

Then, m_1 = mass of complete disc $= x \cdot \pi a^2$

and m_2 = mass of square portion $= x \cdot \frac{a^2}{4}$.

Also, x_1 = x-coordinate of centre of mass of $m_1 = 0$

and x_2 = x-coordinate of centre of mass of $m_2 = d$.

$$\Rightarrow \quad X_{\text{CM}} = \frac{x\pi a^2 \times 0 - x\frac{a^2}{4} \times d}{x\pi a^2 - x\frac{a^2}{4}} = \frac{-a}{2(4\pi - 1)} \quad \text{...(i)}$$

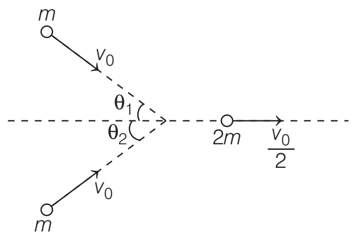
$$\left(\text{Since, } d = \frac{a}{2}\right)$$

Given, $X_{\text{CM}} = -\frac{a}{X}$ ...(ii)

Comparing the Eqs. (i) and (ii), we get

$\therefore \quad X = 2(4\pi - 1) = 23.12$ or $X \approx 23$.

33. Let initial velocities of two bodies are making angle θ_1 and θ_2 with horizontal direction as shown in figure.



Initial momentum, $\mathbf{p}_i = \mathbf{p}_1 + \mathbf{p}_2$

$= \{mv_0 \cos\theta_1 \hat{\mathbf{i}} + mv_0 \sin\theta_1 \hat{\mathbf{j}}\} + \{mv_0 \cos\theta_2 \hat{\mathbf{i}} + mv_0 \sin\theta_2 - \hat{\mathbf{j}}\}$

$= mv_0(\cos\theta_1 + \cos\theta_2)\hat{\mathbf{i}} + mv_0(\sin\theta_1 - \sin\theta_2)\hat{\mathbf{j}}$

Final momentum, $\mathbf{p}_f = (2m)\left(\frac{v_0}{2}\right)\hat{\mathbf{i}}$

$$\Rightarrow \quad \mathbf{p}_f = mv_0 \hat{\mathbf{i}}$$

In collision momentum remains conserved, so, applying momentum conservation,

$$\mathbf{p}_f = \mathbf{p}_i$$

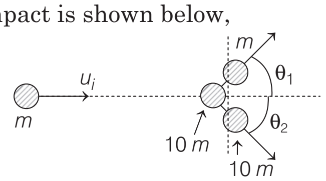
or $\quad mv_0 \hat{\mathbf{i}} = mv_0(\cos\theta_1 + \cos\theta_2)\hat{\mathbf{i}} + mv_0(\sin\theta_1 - \sin\theta_2)\hat{\mathbf{j}}$

$\Rightarrow \quad \sin\theta_1 - \sin\theta_2 = 0 \Rightarrow \theta_1 = \theta_2$

and $\quad mv_0 = 2mv_0 \cos\theta$ or $\cos\theta = \frac{1}{2}$ or $\theta = 60°$

But angle between initial velocities is $\theta_1 + \theta_2$ which is equal to $60° + 60° = 120°$.

34. Given, impact is shown below,



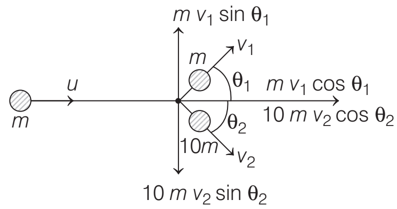
For particle 1, final KE is equals to half of its initial value,

$$K_f = \frac{1}{2} K_i$$

$$\Rightarrow \quad \frac{1}{2} m_1 v_1^2 = \left(\frac{1}{2} m_1 u^2\right) \times \frac{1}{2}$$

Final velocity of m_1 will be, $v_1 = \frac{u}{\sqrt{2}}$

Momentum in y-direction is conserved,



i.e. $\quad 10mv_2 \sin\theta_2 = mv_1 \sin\theta_1$

Here, $v_1 = \frac{u}{\sqrt{2}}$ and $\sin\theta_1 = \sqrt{n}\sin\theta_2$

So, we have

$$10mv_2 \cdot \sin\theta_2 = \frac{mu}{\sqrt{2}} \cdot \sqrt{n}\sin\theta_2$$

$$\Rightarrow \quad v_2 = \frac{u\sqrt{n}}{10\sqrt{2}} \quad \text{...(i)}$$

Also collision is elastic, so KE is conserved.

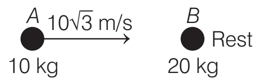
$$\Rightarrow \quad \frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}(10m)v_2^2$$

Substituting values of v_1 and v_2, we have

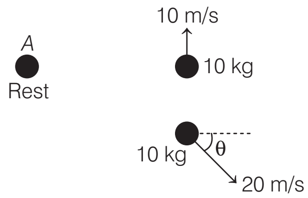
$$u^2 = \left(\frac{u}{\sqrt{2}}\right)^2 + 10\left(\frac{u^2 n}{100 \times 2}\right)$$

$$\Rightarrow \quad \frac{u^2}{2} = \frac{u^2 n}{10 \times 2} \Rightarrow n = 10$$

35. Before collision



After collision



From conservation of momentum along X-axis,

$$\text{p}_i = \text{p}_f$$

$$10 \times 10\sqrt{3} = 200\cos\theta$$

$$\cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30°$$

36. Centre of mass of the quarter disc is at $\frac{4a}{3\pi}, \frac{4a}{3\pi}$

According to the centre of mass of the quarter disc (the shaded area) is at $\frac{x}{3}\frac{a}{\pi} \cdot \frac{x}{3} \cdot \frac{a}{\pi}$, So, $x = 4$