UNIT TEST 2

Time Allowed : 1½ Hours **Max. Marks : 50**

Notes : **1.** *All questions are compulsory.*

2. *Marks have been indicated against each question.*

1. For a 2 × 2 matrix, $A = [a_{ij}]$, whose elements are given by :

$$a_{ij} = \frac{i}{j},$$

write the value of a_{12}. **(1)**

2. If A is a square matrix such that $A^2 = A$, then write the value of $(I + A)^2 - 3A$. **(1)**

3. Find the value of 'x' if $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$. **(2)**

4. If $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 5 & 3 & 8 \end{vmatrix}$, write the minor of the element a_{22}. **(2)**

5. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$.

Use this result to find A^4. **(2)**

6. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, prove that :

$$A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}, n \in \mathbf{N}.$$ **(4)**

7. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then verify that $A'A = I$. **(4)**

8. Find the inverse of the following by using elementary transformations :

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}.$$ **(4)**

9. If x, y, z are different and

$$\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0, \text{ show that } 1 + xyz = 0.$$ **(4)**

10. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find $(A')^{-1}$. **(4)**

11. Solve the following system of equations :

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2 ;\ x, y, z \neq 0.$$ **(4)**

12. Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that :

$$(aI + bA)^n = a^n I + n\, a^{n-1} bA,$$

where I is the identity matrix of order 2 and $n \in \mathbf{N}$. **(6)**

13. Prove that : $\begin{bmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{bmatrix} = 2abc\,(a+b+c)^3$. **(6)**

14. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 3 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

to solve the system of equations :

$$x - y + 2z = 1,\ 2y - 3z = 1,\ 3x - 2y + 4z = 2.$$ **(6)**

Answers

1. $\frac{1}{2}$. **2.** I . **3.** $\pm 2\sqrt{2}$. **4.** -7.

5. $\begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$. **8.** $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$. **10.** $\begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$.

11. $x = 2, y = 3, z = 5$. **14.** $AB = 8I$; $x = 3, y = -2, z = -1$.