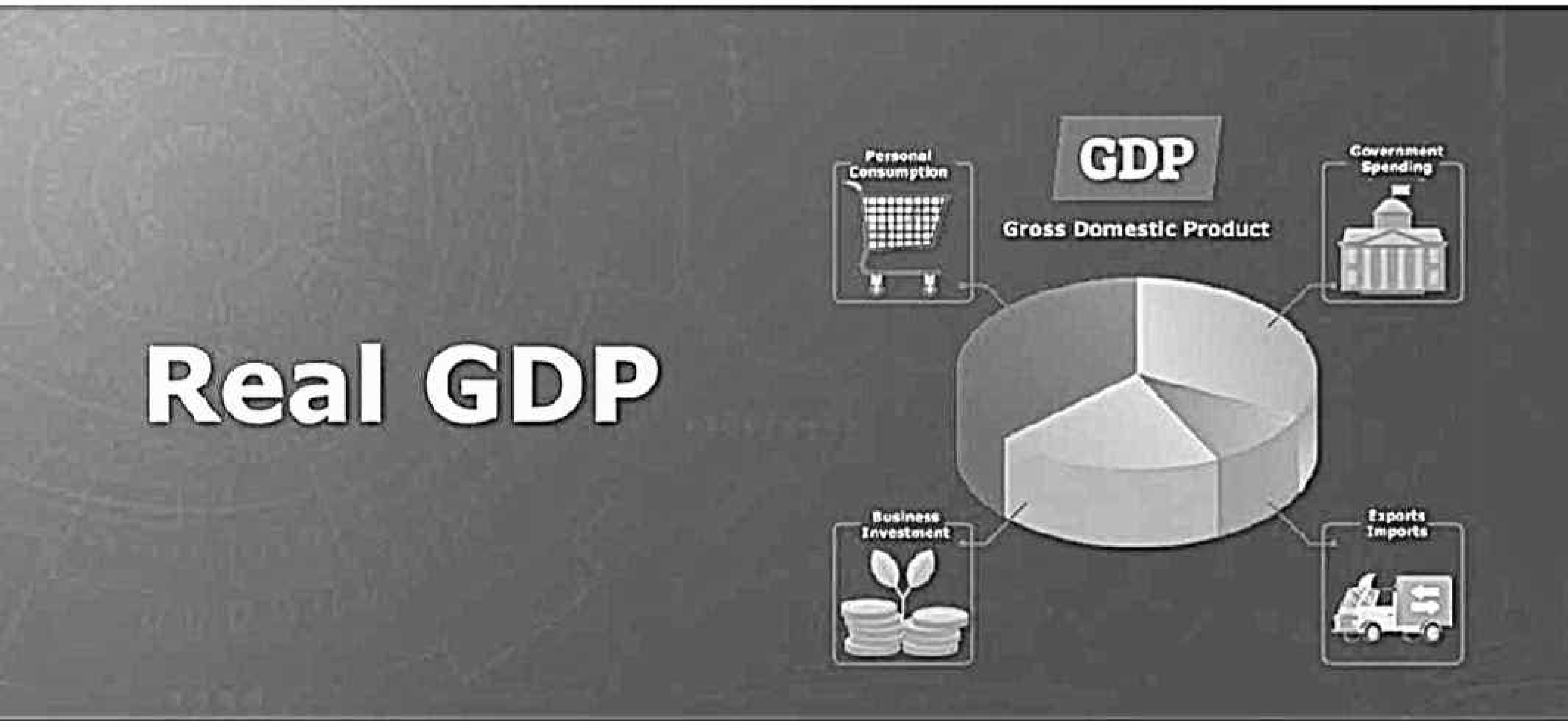
4 Determinants



> Determinants are very useful in Science, Engineering and Economics. For instance, they can be used to determine the level of short-run real gross domestic product (real GDP) in a country's economy. Experts use determinants to find combinations of interest rates and output (GDP) such that the money market is in equilibrium.

Topic Notes

- *Determinants: Its Properties and Area of a Triangle*
- *Minors and Cofactors*
- *Applications of Determinants and Matrices*

DETERMINANTS: ITS PROPERTIES AND AREA OF A TRIANGLE

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TOPIC 1

DETERMINANT

Now, we know that a system of algebraic equations can be expressed in the form of matrices. In order to determine whether this system of algebraic equations has a unique solution or not, a number, which is inferred from the equations is calculated. This number is called a determinant. Determinants have wide applications in the fields of engineering, science, economics, social sciences, etc. In this chapter, we will study till third order determinants.

Let A be any square matrix. We can associate a unique expression or number (real or complex)with this square matrix, called determinant of A. It is denoted by det (A) or |A|.

⚠ Caution

- *|A| is read as 'determinant of A' and not 'modulus of A'.*
- *Only square matrices have determinants. Non-square matrices do not have determinants.*

TOPIC 2

DETERMINANT OF A SQUARE MATRIX OF DIFFERENT ORDERS

Determinant of a Square Matrix of Order 1

Let A = $[a_{11}]$ be a square matrix of order 1. Then we define $|A| = a_{11}$

Determinant of a Square Matrix of Order 2

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a square matrix of order 2.

Then we define |A| as

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

Example 1.1: **Evaluate the following determinants:**

(A) $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$

(B) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

(C) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$ [NCERT]

Ans. (A) $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} = 2(-1) - 4(-5) = -2 + 20 = 18$

(B) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = \cos\theta.\cos\theta - (-\sin\theta)\sin\theta$

$$= \cos^2\theta + \sin^2\theta = 1$$

(C) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

$$= (x^2 - x + 1)(x + 1) - (x - 1)(x + 1)$$
$$= (x + 1)(x^2 - x + 1 - x + 1)$$
$$= (x + 1)(x^2 - 2x + 2), \text{ or } x^3 - x^2 + 2$$

Example 1.2: A = $\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, **then show that** |2A| = 4|A|. [NCERT]

Ans. Here, $2A = 2\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$

So, $|2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix}$

$$= (2)(4) - (4)(8)$$
$$= 8 - 32 = -24$$

And, $4|A| = 4[(1)(2) - (2)(4)]$

$$= 4(2 - 8)$$
$$= -24$$

Thus, $|2A| = 4|A|$

Example 1.3: If $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$, then find the value of x. [NCERT]

Ans. Given: $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$.

$$\therefore (2)(5) - (3)(4) = (x)(5) - (3)(2x)$$
$$\Rightarrow 10 - 12 = 5x - 6x$$
$$\Rightarrow x = 2$$

Determinant of a Square Matrix of Order 3

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be a square matrix of order 3. Then we define |A| as

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11}(-1)^{1+1}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{12}(-1)^{1+2}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}(-1)^{1+3}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{12}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

⚠ Caution

- *The above expansion of |A| is the expansion along the first row. We can also find the expansion of |A| along any row or column. The value of |A| remains the same.*

Example 1.4: Evaluate $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$ [NCERT]

Ans. Expanding $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$ along first column, we have

$$\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 3\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} - 1\begin{vmatrix} -4 & 5 \\ 3 & 1 \end{vmatrix} + 2\begin{vmatrix} -4 & 5 \\ 1 & -2 \end{vmatrix}$$

$$= 3(1 + 6) - 1(-4 - 15) + 2(8 - 5)$$
$$= 21 + 19 + 6$$
$$= 46$$

Important

- *For any square matrix of order n, |A'| = |A|.*
- *For any square matrix of order n, $|kA| = k^n|A|$.*
- *For any square matrices A and B of the same order, |AB| = |A||B|.*
- *A square matrix A is said to be singular, if |A| = 0.*

TOPIC 3

AREA OF A TRIANGLE USING DETERMINANTS

In earlier classes, we have studied that, area (A) of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$A = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| \quad ...(i)$$

Now we write the expression (i) in the form of a determinant as

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \quad ...(ii)$$

⚠ Caution

- *Since area of a triangle is a positive quantity, we always take the absolute value of the determinant in (ii).*

Example 1.5: Find the area of a triangle whose vertices are (2, 7), (1, 1) and (10, 8). **[NCERT]**

Ans. The area (A) of the triangle with given vertices is given by

$$A = \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

Expanding along R_1, we have

$$A = \frac{1}{2}|2(1 - 8) - 7(1 - 10) + 1(8 - 10)|$$

$$= \frac{1}{2}|-14 + 63 - 2|$$

$$= \frac{1}{2}|47|, \text{ or } \frac{47}{2} \text{ sq. units}$$

Condition of Collinearity

Three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are said to be collinear, if they lie on the same straight line. In other words, we can say that if the area of a triangle formed by vertices A, B and C is zero, then points are collinear. Therefore, if

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0,$$

i.e., $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$,

Then, $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear.

Example 1.6: Show that the points A(*a*, *b* + *c*), B(*b*, *c* + *a*) and C(*c*, *a* + *b*) are collinear. **[NCERT]**

Ans. The given three points A(*a*, *b* + *c*), B(*b*, *c* + *a*) and C(*c*, *a* + *b*) will be collinear, if the value of the determinant $\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$ is zero.

Let $\Delta = \begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$.

Then, $\Delta = a(c + a - a - b) - (b + c)(b - c) + 1(ba + b^2 - c^2 - ca)$
$= a(c - b) - (b^2 - c^2) + ba + b^2 - c^2 - ca$
$= 0$

Thus, the points A(a, $b + c$), B(b, $c + a$) and C(c, $a + b$) are collinear.

Equation of a Straight Line Passing Through Two Given Points A(x_1, y_1) and B(x_2, y_2) in a Plane

Let A(x_1, y_1) and B(x_2, y_2) be given two points. Let P(x, y) be any general point on the line passing through A and B, then the three points A, B and P must be collinear, and hence

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

Thus, the equation of a line passing through two given points A(x_1, y_1) and B(x_2, y_2) is given by

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

Important

The coordinates of the general point P(x, y) can be placed in 2nd or 3rd row, the equation of the line will remain the same.

Example 1.7: Find the equation of a line joining A(1, 2) and B(3, 6), using determinants. [NCERT]

Ans. Let P(x, y) be any general point on the line passing through A and B, then the three points A, B and P must be collinear, and hence

$$\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$$

$\Rightarrow x(2 - 6) - y(1 - 3) + 1(6 - 6) = 0$

$\Rightarrow \quad -4x + 2y = 0$

Thus, the equation of the line is $y = 2x$.

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. Value of k, for which $A = \begin{bmatrix} k & 8 \\ 4 & 2k \end{bmatrix}$ is a singular matrix, is:

(a) 4 (b) – 4

(c) ±4 (d) 0

[CBSE Term-1 SQP 2021]

Ans. (c) ±4

As A is singular matrix

$\Rightarrow \quad |A| = 0$

$\Rightarrow \quad 2k^2 - 32 = 0 \Rightarrow k = \pm 4$

[CBSE Marking Scheme Term-1 SQP 2021]

Explanation: $\because$ A is a singular matrix

$\therefore \quad |A| = 0$

$\Rightarrow \quad \begin{vmatrix} k & 8 \\ 4 & 2k \end{vmatrix} = 0$

$\Rightarrow \quad 2k^2 - 32 = 0$

$\Rightarrow \quad k^2 = 16$

$\Rightarrow \quad k = \pm 4$

2. If $A = \begin{bmatrix} 6x & 8 \\ 3 & 2 \end{bmatrix}$ is singular matrix, then the value of x is:

(a) 3 (b) –2

(c) 0 (d) 2

[Delhi Gov. 2022]

Ans. (d) 2

Explanation: $A = \begin{bmatrix} 6x & 8 \\ 3 & 2 \end{bmatrix}$

$\Rightarrow \quad |A| = 12x - 24 = 0$

$\Rightarrow \quad x = 2.$

3. Given that A is a non-singular matrix of order 3 such that $A^2 = 2A$, then value of |2A| is:

(a) 4 (b) 8

(c) 64 (d) 16

[CBSE Term-1 SQP 2021]

Ans. (c) 64

$A^2 = 2A$

$\Rightarrow |A^2| = |2A|$

$\Rightarrow |A^2| = 2^3|A|$ as $|kA| = k^n|A|$ for a matix of order n

$\Rightarrow$ either $|A| = 0$ or $|A| = 8$

But A is non-singular matrix

$\therefore |A| = 8^2 = 64$

[CBSE Marking Scheme Term-1 SQP 2021]

Explanation:

$$A^2 = 2A$$

$\Rightarrow \quad |A^2| = |2A|$

$\Rightarrow \quad |A^2| = 2^3|A|$

$\therefore |kA| = k^n |A|$ for a matrix of order n

$\Rightarrow \quad |A^2| - 8|A| = 0$

$\Rightarrow \quad |A|(|A|-8) = 0$

$\Rightarrow$ either $|A| = 0$ or $|A| = 8$

But A is non-singular matrix

$\therefore \quad |A| = 8$

Now, $|2A| = 2^3|A|$

$= 8 \times 8$

$= 64$

4. The equation $\begin{vmatrix} x & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$ is satisfied by:

(a) $x = 1$ (b) $x = 2$

(c) $x = 3$ (d) $x = 4$

5. If for matrix $A = \begin{bmatrix} \alpha & -2 \\ -2 & \alpha \end{bmatrix}$, $|A^3| = 125$, then the value of α is:

(a) ± 3 (b) -3

(c) ± 1 (d) 1

Ans. *(a)* ± 3

Explanation: We have,

$$|A| = \begin{vmatrix} \alpha & -2 \\ -2 & \alpha \end{vmatrix} = \alpha^2 - 4$$

We know $|A^n| = |A|^n$

$\therefore \quad |A^3| = |A|^3$

$\because \quad |A^3| = 125$ [Given]

$\Rightarrow \quad |A|^3 = (5)^3$

$\Rightarrow \quad |A| = 5$

$\Rightarrow \quad \alpha^2 - 4 = 5$

$\Rightarrow \quad \alpha^2 = 9$

$\Rightarrow \quad \alpha = \pm 3.$

6. Let $A = \begin{bmatrix} 1 & \sin\alpha & 1 \\ -\sin\alpha & 1 & \sin\alpha \\ -1 & -\sin\alpha & 1 \end{bmatrix}$, where $0 \le \alpha \le 2\pi$, then:

(a) $|A| = 0$ (b) $|A| \in (2, \infty)$

(c) $|A| \in (2, 4)$ (d) $|A| \in [2, 4]$

[CBSE Term-1 SQP 2021]

7. If A is a skew-symmetric matrix of order 3, then the value of $|A|$ is:

(a) 3 (b) 0

(c) 9 (d) 27 [CBSE 2020]

Ans. *(b) 0*

Explanation: By definition of skew-symmetric matrix, $A^T = -A$

$\Rightarrow \quad |A^T| = |-A|$

$\Rightarrow \quad |A| = (-1)^3|A| \quad [\because |A^T| = |A|]$

$\Rightarrow \quad |A| = -|A|$

$\Rightarrow \quad 2|A| = 0$

$\Rightarrow \quad |A| = 0$

8. If one root of the equation $\begin{vmatrix} 7 & 6 & x \\ 2 & x & 2 \\ x & 3 & 7 \end{vmatrix} = 0$ is $x = -9$, then the other two roots are:

(a) 2, 6 (b) 3, 6

(c) 2, 7 (d) 3, 7

9. If A is a 3×3 matrix such that $|A| = 8$, then $|3A|$ equals:

(a) 8 (b) 24

(c) 72 (d) 216 [CBSE 2020]

10. If A is a non-singular square matrix of order 3 such that $|A| = 3$, then value of $|2A^T|$ is:

(a) 3 (b) 6

(c) 12 (d) 24

[Delhi Gov. 2022]

Ans. *(d) 24*

Explanation:

We know $|KA| = K^n|A|$

Where n in order of matrix A

Here, $n = 3$

$\therefore \quad |2A^T| = 2^3|A^T|$

$= 8|A| \quad [\because |A^T| = |A|]$

$= 8 \times 3 = 24$

11. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then

(a) $x = 3, y = 1$ (b) $x = 1, y = 3$

(c) $x = 0, y = 3$ (d) $x = 0, y = 0$

Ans. *(d)* $x = 0, y = 0$

Explanation: Here, $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}$

$= 6i(-3 + 3) + 3i(4i + 20) + 1(12 - 60i)$

$= -12 + 60i + 12 - 60i = 0$

Thus, $x = 0, y = 0$

12. If $f(x) = \begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$, then

(a) $f(a) = 0$ (b) $f(b) = 0$

(c) $f(0) = 0$ (d) $f(1) = 0$

[NCERT Exemplar]

13. If T_p, T_q, T_r are the p^{th}, q^{th} and r^{th} terms of an A.P., then $\begin{vmatrix} T_p & T_q & T_r \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} =$

(a) $p + q + r$ (b) 0

(c) -1 (d) 1

Ans. *(b) 0*

Explanation: Let a be the first term and d be the common difference of an A.P. Then,

$$T_p = a + (p-1)d$$
$$T_q = a + (q-1)d$$
$$T_r = a + (r-1)d \quad ...(i)$$

Expanding the given determinant along first row, we have

$$\begin{vmatrix} T_p & T_q & T_r \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = T_p(q-r) - T_q(p-r) + T_r(p-q)$$

$$= \{a + (p-1)d\}(q-r) - \{a + (q-1)d\}(p-r) + \{a + (r-1)d\}(p-q)$$

$$= (aq - ar - ap + ar + ap - aq) + \{(p-1)(q-r) - (q-1)(p-r) + (r-1)(p-q)\}d$$

$$= 0 + \{(pq - q + r - pr) - (pq - p + r - qr) + (pr - p - qr + q)\}d$$

$$= (pq - q + r - pr - pq - p - r + qr + pr - p + q - qr)d$$

$$= 0$$

14. The value of the determinant $\begin{vmatrix} 1 & x & y+z \\ 1 & y & z+x \\ 1 & z & x+y \end{vmatrix}$ is:

(a) $x + y + z$ (b) 0

(c) $(-y)(x-y)(x-y)$ (d) $1 + x + y + z$

Ans. *(b) 0*

Explanation: Here, $\Delta = \begin{vmatrix} 1 & x & y+z \\ 1 & y & z+x \\ 1 & z & x+y \end{vmatrix}$

Expanding along C_1, we get

$$\Delta = 1\,[y(x+y) - (z+x)z] - 1\,[x(x+y) - (y+z)z] + 1\,[x(z+x) - (y+z)y]$$

$$= xy + y^2 - z^2 - xz - x^2 - xy + yz + z^2 + xz + x^2 - y^2 - y^2$$

$$= 0$$

15. The area of a triangle with vertices (-3, 0), (3, 0) and (0, k) is 9 sq. units. The value of k is ($k > 0$):

(a) 3 (b) 6

(c) 9 (d) 12

[Delhi Gov. 2022]

Ans. *(a) 3*

Explanation: We have, $\Delta = 9$ sq. units

$$\Rightarrow \quad \frac{1}{2}\begin{bmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{bmatrix} = 9$$

$$\frac{1}{2}|-3(0-k) - 0 + 1(3k - 0| = 9$$

$$\Rightarrow \quad |6k| = 18$$

$$\Rightarrow \quad 6k = \pm 18$$

$$\Rightarrow \quad k = \pm 3$$

But, $k > 0$

$$\therefore \quad k = 3$$

16. Three points P $(2x, x+3)$, Q $(0, x)$ and R $(x+3, x+6)$ are collinear, then x is:

(a) 0 (b) 2

(c) 3 (d) 1

[CBSE Term-1 2021]

Ans. *(d) 1*

Explanation: For collinearity of three points

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} 2x & x+3 & 1 \\ 0 & x & 1 \\ x+3 & x+6 & 1 \end{vmatrix} = 0$$

Using $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\begin{vmatrix} 2x & x+3 & 1 \\ -2x & -3 & 8 \\ 3-x & 3 & \end{vmatrix} = 0$$

Expanding along C_1, we get

$$\Rightarrow 2x[x - (x+6)] - 0 + (x+3)[(x+3) - x] = 0$$

$$\Rightarrow \quad 2x(-6) + (x+3)3 = 0$$

$$\Rightarrow \quad -12x + 3x + 9 = 0$$

$$\Rightarrow \quad -9x = -9$$

$$\Rightarrow \quad x = 1.$$

17. **While eating a slice of pizza with her friends, Shefali noted that it is shaped like a triangle. She questioned her friends about computing the area of the pizza slice using determinants, given the vertices of the slice.**

If area of a triangle is 35 sq. units with vertices (2, –6), (5, 4) and (k, 4), then k is:

(a) 12 (b) –2

(c) –12, –2 (d) 12, –2 [NCERT]

Ans. *(d) 12, –2*

Explanation: Area of the triangle with given vertices is $\frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$

Expanding this determinant along 3rd row, we have

$\frac{1}{2}$ {k(– 6 – 4) – 4(2 – 5) + 1(8 + 30)},

i.e., $-5k + 25$

Equating it to ± 35, we have

$$-5k + 25 = \pm 35$$

$\Rightarrow$ $k = -2, 12$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

18. **Evaluate $\begin{vmatrix} \tan\theta & \sec\theta \\ \sec\theta & \tan\theta \end{vmatrix}$.**

19. **If P is a 4 × 4 matrix such that |P| ≠ 0 and |2P| = k|P|, find the value of k.**

Ans. We have $|2P| = k|P|$

We know that, if A is a square matrix of order n, then $|kA| = k^n|A|$.

$\therefore$ $|2P| = 2^4|P|$

$\Rightarrow$ $2^4|P| = k|P|$

$\Rightarrow$ $k = 2^4$

$= 16$

20. **If A is a square matrix satisfying A′A = I, write the value of |A|.** **[CBSE 2019]**

21. **Evaluate $\begin{vmatrix} \cos 20° & \sin 20° \\ \sin 70° & \cos 70° \end{vmatrix}$.**

22. **If A = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$, find the value of |AB|.**

23. **If $\begin{vmatrix} x & 3 \\ 1 & 2 \end{vmatrix} = \begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix}$, then find the value of x.**

Ans. We have,

$$\begin{vmatrix} x & 3 \\ 1 & 2 \end{vmatrix} = \begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix}$$

$$2x - 3 = 0 + 2$$

$\Rightarrow$ $2x = 5$

$\Rightarrow$ $x = \frac{5}{2}$

24. **If $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$, then find the value of x.** **[CBSE 2013]**

25. **If $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$, write the value of x.** **[CBSE 2016]**

Ans. Given: $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$

On expanding along C_1, we get

$x(-x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta)$

$+ \cos\theta(-\sin\theta + x\cos\theta) = 8$

$\Rightarrow -x^3 - x + x(\sin^2\theta + \cos^2\theta) - \sin\theta\cos\theta$

$+ \cos\theta\sin\theta = 8$

$\Rightarrow -x^3 - x + x(1) = 8$

$\Rightarrow x^3 = -8$

$\therefore x = -2$ is only the real value of x.

26. **Find the area of the triangle whose vertices are A(4, 5), B(–1, 3) and (2, –1).**

27. **Find the equation of line joining points (3, 2) and (–1, 4) by using determinant.**

Ans. The equation of a line passing through points (x_1, y_1) and (x_2, y_2) is:

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

We have, $(x_1, y_1) = (3, 2)$

and $(x_2, y_2) = (-1, 4)$

$$\therefore \quad \begin{vmatrix} x & y & 1 \\ 3 & 2 & 1 \\ -1 & 4 & 1 \end{vmatrix} = 0$$

On expanding along R_1, we get

$x(2-4) - y(3+1) + 1(12+2) = 0$

$\Rightarrow \quad -2x - 4y + 14 = 0$

$\Rightarrow \quad 2x + 4y - 14 = 0$

28. Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$.

Ans. Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$

On expanding along first row, we get

$$\Delta = 1\begin{vmatrix} 1+\sin\theta & 1 \\ 1 & 1+\cos\theta \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 1+\cos\theta \end{vmatrix} + 1\begin{vmatrix} 1 & 1+\sin\theta \\ 1 & 1 \end{vmatrix}$$

$= [(1+\sin\theta)(1+\cos\theta) - 1] - [1 + \cos\theta - 1] + [1 - 1 - \sin\theta]$

$= 1 + \cos\theta + \sin\theta + \sin\theta\cos\theta - 1 - \cos\theta - \sin\theta$

$= \sin\theta\cos\theta$

$= \left(\frac{1}{2}\right)(2\sin\theta\cos\theta) = \frac{1}{2}\sin 2\theta$

We know that, maximum value of $\sin 2\theta$ is 1.

$\therefore \quad \Delta_{max} = \frac{1}{2} \times 1 = \frac{1}{2}.$

29. Find the value of λ, if the points, $(\lambda - 1, -1)$, $(3\lambda - 1, 2)$ and $(2\lambda + 1, 2\lambda)$ are collinear.

Ans. We have, given points are collinear.

$$\therefore \quad \begin{vmatrix} \lambda-1 & -1 & 1 \\ 3\lambda-1 & 2 & 1 \\ 2\lambda+1 & 2\lambda & 1 \end{vmatrix} = 0$$

On expanding along C_3, we get

$1[(3\lambda - 1)(2\lambda) - 2(2\lambda + 1) - 1[(\lambda - 1)(2\lambda) - (-1)(2\lambda + 1) + 1[\lambda - 1)2 - (-1)(3\lambda - 1)]$

$\Rightarrow (6\lambda^2 - 2\lambda - 4\lambda - 2) - (2\lambda^2 - 2\lambda + 2\lambda + 1) + (2\lambda - 2 + 3\lambda - 1) = 0$

$4\lambda^2 - \lambda - 6 = 0$

$$\therefore \quad \lambda = \frac{+1 \pm \sqrt{(-1)^2 + 4 \times 4 \times 6}}{2 \times 4}$$

$$= \frac{1 \pm \sqrt{1 + 96}}{8}$$

$$= \frac{1 \pm \sqrt{97}}{8}$$

30. If [.] denotes the greatest integer less than or equal to the real number under consideration, and $1 \le x < 2$, $0 \le y < 1$ and $-1 \le z < 0$, find the value of the determinant.

Ans.
$$\Delta = \begin{vmatrix} [x] & [y]+2 & [z] \\ [x] & [y] & [z]+2 \\ [x]+2 & [y] & [z] \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0+2 & -1 \\ 1 & 0 & -1+2 \\ 1+2 & 0 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 3 & 0 & -1 \end{vmatrix}$$

On expanding along C_2, we get

$\Delta = 2(-1-3) + 0 + 0$

$= -8$

31. If $A = \begin{bmatrix} p & 2 \\ 2 & p \end{bmatrix}$ and $|A^3| = 125$, then find the value of p. **[CBSE 2019]**

Ans. We have, $A = \begin{bmatrix} p & 2 \\ 2 & p \end{bmatrix}$

$\therefore \quad |A| = \begin{vmatrix} p & 2 \\ 2 & p \end{vmatrix}$

$= p^2 - 4$

Also, we have $|A^3| = 125$

$\Rightarrow \quad |A|^3 = 125$

$\Rightarrow \quad (p^2 - 4)^3 = 5^3$

On taking cube roots, we get

$p^2 - 4 = 5$

$\Rightarrow \quad p^2 = 9$

$\Rightarrow \quad p = \pm 3$

32. If area of a triangle is 35 square units with vertices (2, –6), (5, 4) and (k, 4), find the value of k.

Ans. We have,

area of triangle = 35 sq. units

$$\therefore \quad \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} = \pm 35$$

$$\Rightarrow \quad \begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} = \pm 70$$

On expanding along C_1, we get

$2(4 - 4) - 5(-6 - 4) + k(-6 - 4) = \pm 70$

$\Rightarrow \quad 0 + 50 - 10k = \pm 70$

$\Rightarrow \quad 10k = 50 \mp 70$

When we take positive sign,

$10k = 50 + 70$

$\Rightarrow \quad 10k = 120$

$\Rightarrow \quad k = 12$

When we take negative sign,

$10k = -20$

$\Rightarrow \quad k = -2$

Hence, values of k are 12 and –2.

33. Show that the points (a + 5, a – 4), (a – 2, a + 3) and (a, a) do not lie on a straight line for any value of a. [NCERT Exemplar]

Ans. Given, points are $(a + 5, a - 4)$, $(a - 2, a + 3)$ and (a, a).

If the points do not form a straight line, then they will form a triangle.

$$\therefore \quad \text{Area, } \Delta = \frac{1}{2}\begin{vmatrix} a+5 & a-4 & 1 \\ a-2 & a+3 & 1 \\ a & a & 1 \end{vmatrix}$$

$= \frac{1}{2}$ $[(a + 5) [(a + 3) - a] - (a - 4)$

$\{(a - 2) - a\} + 1 \{(a - 2) a - (a + 3)a\}$

$= \frac{1}{2}[3a + 15 + 2a - 8 - 5a]$

$= \frac{7}{2} \neq 0$

$\therefore \quad \Delta \neq 0$

Also, Δ is independent of a.

Hence, the given points neither form a straight line nor froms a triangle.

34. Find the value of λ, if area of triangle is 4 sq. units, whose vertices are A(–2, 0), B(0, 4) and C(0, λ).

Ans. Area of triangle having vertices A(–2, 0), B(0, 4) and C(0, λ) is

$$\frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & \lambda & 1 \end{vmatrix} = \pm 4$$

On expanding along R_1, we get

$\frac{1}{2}[-2(4 - \lambda) + 0 + 1(0)] = \pm 4$

$\Rightarrow \quad -8 + 2\lambda = \pm 8$

Taking '–' sign,

$-8 + 2\lambda = -8$

$\Rightarrow \quad 2\lambda = 0$

$\lambda = 0$

Taking '+' sign

$-8 + 2\lambda = 8$

$\Rightarrow \quad 2\lambda = 16$

$\Rightarrow \quad \lambda = 8$

Hence, values of λ are 0 and 8.

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

35. If A + B + C = 0, then prove that:

$$\begin{vmatrix} 1 & \cos C & \cos B \\ \cos C & 1 & \cos A \\ \cos B & \cos A & 1 \end{vmatrix} = 0$$ **[NCERT Exemplar]**

Ans. $$\text{L.H.S.} = \begin{vmatrix} 1 & \cos C & \cos B \\ \cos C & 1 & \cos A \\ \cos B & \cos A & 1 \end{vmatrix}$$

Expanding along R_1

$= 1(1 - \cos^2 A) - \cos C(\cos C - \cos A \cos B)$
$+ \cos B(\cos C.\cos A - \cos B)$

$= \sin^2 A - \cos^2 C + \cos A \cos B \cos C$
$+ \cos A \cos B \cos C - \cos^2 B$

$= \sin^2 A - \cos^2 B - \cos^2 C + 2 \cos A.\cos B \cos C$

$= -\cos(A + B)\cos(A - B) + \cos C(2 \cos A \cos B$
$- \cos C)$

$[\because \cos^2 B - \sin^2 A = \cos(A + B).\cos(A - B)]$

$= -\cos(-C)\cos(A-B) + \cos C\,(2\cos A.\cos B - \cos C)$

$[\because A + B + C = 0]$

$= -\cos C\,(\cos A\cos B + \sin A\sin B) + \cos C\,(2\cos A\cos B - \cos C)$

$[\because \cos(-C) = \cos C]$

$= \cos C\,[-\cos A\cos B - \sin A\sin B + 2\cos A\cos B - \cos C]$

$= \cos C\,[\cos(A+B) - \cos C]$

$= \cos C\,[\cos(-C) - \cos C]$

$= \cos C\,[\cos C - \cos C]$

$= 0 =$ R.H.S.

Hence, proved.

⚠ Caution

- *Don't forget the formulae of expansion in trigonometry, studied in previous class.*

MINORS AND COFACTORS 2

TOPIC 1

MINOR

Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its i^{th} row and j^{th} column in which element a_{ij} lies. The minor of a_{ij} is denoted by M_{ij}.

Illustration: To find the minor of element (–2) in the determinant $\Delta = \begin{vmatrix} 2 & 5 & -1 \\ 5 & 0 & -2 \\ 1 & -1 & 0 \end{vmatrix}$, we need to delete 2^{nd} row and 3^{rd} column in which (–2) lies.

Thus, $M_{23} = \begin{vmatrix} 2 & 5 \\ 1 & -1 \end{vmatrix}$, or $M_{23} = (2)(-1) - (1)(5) = -7$

Similarly, to find the minor of element (1) in the determinant $\Delta = \begin{vmatrix} 2 & 5 & -1 \\ 5 & 0 & -2 \\ 1 & -1 & 0 \end{vmatrix}$, we need to delete 3^{rd} row and 1^{st} column in which (1) lies.

Thus, $M_{31} = \begin{vmatrix} 5 & -1 \\ 0 & -2 \end{vmatrix}$ or $M_{31} = (5)(-2) - (0)(-1) = -10$

TOPIC 2

COFACTOR

Cofactor of an element a_{ij}, denoted by A_{ij}, is defined by

$$A_{ij} = (-1)^{i+j} M_{ij}$$

where, M_{ij} is the minor of a_{ij}.

Illustration: The cofactor of element (– 2) in the determinant $\Delta = \begin{vmatrix} 2 & 5 & -1 \\ 5 & 0 & -2 \\ 1 & -1 & 0 \end{vmatrix}$ is given by

$$A_{23} = (-1)^{2+3} M_{23}$$

i.e., $A_{23} = -\begin{vmatrix} 2 & 5 \\ 1 & -1 \end{vmatrix} = -(-7) = 7$

Similarly, the cofactor of element (1) in the determinant $\Delta = \begin{vmatrix} 2 & 5 & -1 \\ 5 & 0 & -2 \\ 1 & -1 & 0 \end{vmatrix}$, is given by

$$A_{31} = (-1)^{3+1} M_{31}$$

i.e., $A_{23} = \begin{vmatrix} 5 & -1 \\ 0 & -2 \end{vmatrix} = -10$

Important Results

(1) Let A_{ij} denote the cofactor of a_{ij} in $A = [a_{ij}]$. Then,
$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$ (Expanding along first row)
$= a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$ (Expanding along second row)
$= a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33}$ (Expanding along third row)
$= a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$ (Expanding along first column)
$= a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32}$ (Expanding along second column)
$= a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$ (Expanding along third column)

(2) If elements of a row (or column) are multiplied with cofactors of any other row (or column), then their sum is zero.

Consider $A = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$.

Then, $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23}$

$= (2)\left\{-\begin{vmatrix} -3 & 5 \\ 5 & -7 \end{vmatrix}\right\} + (-3)\left\{\begin{vmatrix} 2 & 5 \\ 1 & -7 \end{vmatrix}\right\} + (5)\left\{-\begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix}\right\}$

$= (2)(4) + (-3)(-19) + (5)(-13)$

$= 8 + 57 - 65$

$= 0$

Example 2.1: Write minors and cofactors of the elements of $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$ [NCERT]

Ans. We have

$$M_{11} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10 + 1 = 11;$$

and $A_{11} = (-1)^{1+1} M_{11} = 11$

$$M_{12} = \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6 + 0 = 6;$$

and $A_{12} = (-1)^{1+2} M_{12} = -6$

$$M_{13} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3;$$

and $A_{13} = (-1)^{1+3} M_{13} = 3$

$$M_{21} = \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = 0 - 4 = -4;$$

and $A_{21} = (-1)^{2+1} M_{21} = 4$

$$M_{22} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2;$$

and $A_{22} = (-1)^{2+2} M_{22} = 2$

$$M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1;$$

and $A_{23} = (-1)^{2+3} M_{23} = -1$

$$M_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = 0 - 20 = -20;$$

and $A_{31} = (-1)^{3+1} M_{31} = -20$

$$M_{32} = \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -1 - 12 = -13;$$

and $A_{32} = (-1)^{3+2} M_{32} = 13$

$$M_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5 - 0 = 5;$$

and $A_{33} = (-1)^{3+3} M_{31} = 5$

TOPIC 3

ADJOINT OF A MATRIX

Let $A = [a_{ij}]$ be a square matrix and A_{ij} be the cofactor of the element a_{ij}. Then the matrix $[A_{ij}]$ is called the cofactor matrix of the matrix A.

The transpose of the matrix $[A_{ij}]$ is called the adjoint of the matrix A.

Illustration: Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$. Then, its cofactor matrix is $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$ and is written as cof A.

So, its adjoint matrix is $\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$ and is written as *adj* A.

Important

➥ *It is quite easy to write the adjoint matrix of a 2 × 2 matrix.*
Let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Then, the adj A is obtained by interchanging a and d, and by changing signs of b and c.

Thus, $$adj\, A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Noteworthy Results

(1) $A(\text{adj } A) = (\text{adj } A)A = |A|I_n$
(2) $|\text{adj } A| = |A|^{n-1}$
(3) $|A\,(\text{adj } A)| = |A|^n$
(4) $|\text{adj }(\text{adj } A)| = |A|^{(n-1)^2}$
(5) $A(\text{adj } A) = |A|^{n-2}A$
(6) $\text{adj }(kA) = k^{n-1}(\text{adj } A)$
(7) $\text{adj }(A') = (\text{adj } A)'$
(8) $\text{adj }(AB) = (\text{adj } A)(\text{adj } B)$

Example 2.2: Find the adjoint of the matrix $A = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{vmatrix}$. [NCERT]

Ans. Let A_{ij} be the cofactor of the element a_{ij} in $A = [a_{ij}] = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{vmatrix}$.

Then, $$A_{11} = (-1)^{1+1}\begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3;$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 5 \\ -2 & 1 \end{vmatrix} = -12;$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = 6;$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = 1;$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 5;$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = 2;$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -11;$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = -1;$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 5;$$

$$\text{Thus, adj A} = \begin{bmatrix} 3 & -12 & 6 \\ 1 & 5 & 2 \\ -11 & -1 & 5 \end{bmatrix}' = \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$$

TOPIC 4

INVERSE OF A MATRIX

A square matrix A is invertible if it is non-singular, *i.e.*, if $|A| \neq 0$.

The inverse (A^{-1}) of a non-singular matrix A is defined as:

$$A^{-1} = \frac{1}{|A|}(\text{adj A})$$

Important

- *Since* $AA^{-1} = I,$

$\Rightarrow \quad |AA^{-1}| = 1$

$\Rightarrow \quad |A||A^{-1}| = 1$

Thus, $\quad |A^{-1}| = \dfrac{1}{|A|}$

Example 2.3: Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$. **[NCERT]**

Ans. Here, $|A| = 1(8 - 6) + 1(0 + 9) + 2(0 - 6)$

$= 2 + 9 - 12 = -1 \neq 0$

So, A is invertible, as A is non-singular.

Let A_{ij} be the cofactor of the element a_{ij} in $A = [a_{ij}] = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$. Then,

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 2 & -3 \\ -2 & 4 \end{vmatrix} = 2;$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 0 & -3 \\ 3 & 4 \end{vmatrix} = -9;$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 0 & 2 \\ 3 & -2 \end{vmatrix} = -6;$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} -1 & 2 \\ -2 & 4 \end{vmatrix} = 0;$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -2;$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} = -1;$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix} = -1;$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 2 \\ 0 & -3 \end{vmatrix} = 3;$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} = 2$$

$$\Rightarrow \text{adj A} = \begin{bmatrix} 2 & -9 & -6 \\ 0 & -2 & -1 \\ -1 & 3 & 2 \end{bmatrix}' = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

Now, $A^{-1} = \dfrac{1}{|A|}(\text{adj A})$

$$= \frac{1}{(-1)}\begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

Example 2.4: Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ **and** $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$

Verify that $(AB)^{-1} = B^{-1}A^{-1}$. **[NCERT]**

Ans. Here, $AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$

$$= \begin{bmatrix} 18+49 & 24+63 \\ 12+35 & 16+45 \end{bmatrix}$$

$$= \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$$

$$|AB| = 67 \times 61 - 87 \times 47$$
$$= 4087 - 4089$$
$$= -2 \neq 0$$

So, AB is invertible, as AB is non-singular.

Thus, $(AB)^{-1} = \frac{1}{(-2)} \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$ ...(i)

Further, $|A| = 3 \times 5 - 2 \times 7 = 15 - 14 = 1 \neq 0$

So, A is invertible, as A is non-singular.

Thus, $(A)^{-1} = \frac{1}{1} \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$ ...(ii)

Also, $|B| = 6 \times 9 - 7 \times 8 = 54 - 56 = -2 \neq 0$

So, B is invertible, as B is non-singular.

Thus, $(B)^{-1} = \frac{1}{(-2)} \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$ ...(iii)

From (ii) and (iii), we have

$$(B)^{-1}(A)^{-1} = \frac{1}{(-2)} \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix} \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$= \frac{1}{(-2)} \begin{bmatrix} 45+16 & -63-24 \\ -35-12 & 49+18 \end{bmatrix}$$

$$= \frac{1}{(-2)} \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$$

Comparing it with (i), we get

$$(AB)^{-1} = B^{-1}A^{-1}$$

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. Given that $A = [a_{ij}]$ is a square matrix of order 3×3 and $|A| = -7$, then the value of $\sum_{i=1}^{3} a_{i2}A_{i2}$, where A_{ij} denotes the cofactor of element a_{ij} is:

(a) 7 **(b) − 7**

(c) 0 **(d) 49**

[CBSE Term-1 SQP 2021]

Ans. *(b)* -7

$$|A| = -7$$

$$\therefore \sum_{i=1}^{3} a_{i2}A_{i2} = a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32}$$

$$|A| = -7$$

[CBSE Marking Scheme Term-1 SQP 2021]

Explanation:

$$\sum_{i=1}^{3} a_{i2}A_{i2} = a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32}$$

= Determinant of matrix A expanded along C_2

$= |A|$

$= -7$

2. The cofactor of an element −5 in the determinant $\begin{vmatrix} -1 & 2 & 3 \\ 4 & -7 & -1 \\ -2 & -5 & 6 \end{vmatrix}$ is:

(a) 11 **(b) −12**

(c) − 11 **(d) 12**

Ans. *(a) 11*

Explanation: The cofactor of an element − 5 in the given determinant is

$$(-1)^{3+2} \begin{vmatrix} -1 & 3 \\ 4 & -1 \end{vmatrix} = -(1-12) = 11$$

3. Given that A is a square matrix of order 3 and $|A| = -4$, then $|adj\ A|$ is equal to:

(a) −4 **(b) 4**

(c) −16 **(d) 16**

[CBSE Term-1 SQP 2021]

4. If A is an invertible matrix of order 2. Then, $\det(A^{-1})$ is equal to:

(a) det (A) **(b) $\frac{1}{\det(A)}$**

(c) 1 **(d) 0** **[NCERT]**

Ans. *(b)* $\frac{1}{det(A)}$

Explanation: A being an invertible matrix,

$\therefore \quad AA^{-1} = I$

$\Rightarrow \quad |AA^{-1}| = 1$

or $\quad |A||A^{-1}| = 1$

$\Rightarrow \quad \det(A^{-1}) = \dfrac{1}{\det(A)}$

5. For any 2 × 2 matrix, if A (adj A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then |A| is equal to:

(a) 20 (b) 100
(c) 10 (d) 0

Ans. *(c) 10*

Explanation: Given, A (adj A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$.

$\therefore \quad |A(\text{adj } A)| = 100$

$\Rightarrow \quad |A|^2 = 100$

$\{\because |A(\text{adj } A)| = |A|^n$, for a matrix A of order $n\}$

$\Rightarrow \quad |A| = 10$

6. If $A^{-1} = \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$, then |adj A| =

(a) $\frac{1}{9}$ (b) $\frac{1}{81}$
(c) – 9 (d) – 81

[Delhi Gov. 2021]

Ans. *(b)* $\frac{1}{81}$

Explanation: We have,

$|A^{-1}| = 3(1 - 4) + 0$ [Expanding along c_1]

$= -9$

We know, $|A^{-1}| = \dfrac{1}{|A|}$

or, $|A| = \dfrac{1}{|A^{-1}|} = -\dfrac{1}{9}$

Also, we know that

$|\text{adj. } A| = |A|^{n-1}$

Where n is order of matrix A

$= |A|^{3-1}$

$= |A|^2 = \left(-\frac{1}{9}\right)^2$

$= \dfrac{1}{81}$

7. If A = $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then the value of |adj A| is:

(a) a^{27} (b) a^9
(c) a^6 (d) a^2

[DIKSHA]

8. If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then:

(a) $a = 1, b = 1$
(b) $a = \cos 2\theta, b = \sin 2\theta$
(c) $a = \sin 2\theta, b = \cos 2\theta$
(d) $a = 1, b = -1$

Ans. *(b)* $a = \cos 2\theta$, $b = \sin 2\theta$

Explanation: Given:

$$\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix},$$

we have

$$\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \times \frac{1}{1+\tan^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \cos^2\theta\begin{bmatrix} 1-\tan^2\theta & -2\tan\theta \\ 2\tan\theta & 1-\tan^2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \cos^2\theta\begin{bmatrix} \frac{\cos 2\theta}{\cos^2\theta} & -\frac{\sin 2\theta}{\cos^2\theta} \\ \frac{\sin 2\theta}{\cos^2\theta} & \frac{\cos 2\theta}{\cos^2\theta} \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$\Rightarrow a = \cos 2\theta, b = \sin 2\theta.$

9. If A = $\begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, then det [adj(adj A)] is:

(a) 14^4 (b) 14^3
(c) 14^2 (d) 14

10. If A is a singular matrix, then adj A is:

(a) non-singular (b) singular

(c) symmetric (d) not defined

Ans. *(b) singular*

Explanation: As A is a singular matrix, $|A| = 0$

Further, we know that $|\text{adj } A| = |A|^{n-1}$

$\Rightarrow \quad |\text{adj } A| = 0 \qquad \{\because |A| = 0\}$

Thus, adj A is a singular matrix.

11. If A is any square matrix of order 3 × 3 such that |adj A| = 256, then the sum of all possible values of |A| is:

(a) 256 (b) 16

(c) – 16 (d) 0 [Delhi Gov. 2022]

Ans. *(d) 0*

Explanation: We know,

$$|\text{adj } A| = |A|^{n-1}$$

Where, n is order of matrix A.

Here, $n = 3$

$\therefore \quad |\text{adj } A| = |A|^2$

$\Rightarrow \quad |A|^2 = 256 = (\pm 16)^2$

$\Rightarrow \quad |A| = \pm 16$

$\therefore \quad$ Required sum $= 16 + (-16) = 0$

12. Ⓐ**If** $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$, **then the cofactor A_{21} is**

(a) $hc - fg$ (b) $hc + fg$

(c) $fg - hc$ (d) $-(hc + fg)$

13. If $A^2 - A + 1 = 0$, then the inverse of A is

(a) A^{-2} (b) $I - A$

(c) 0 (d) A

Ans. *(b) $I - A$*

Explanation: Given $A^2 - A + I = 0$

$\therefore \quad A^{-1}(A^2 - A + I) = A^{-1}0$

$A^{-1}A^2 - A^{-1}A + A^{-1}I = 0$

$(A^{-1}A)A - I + A^{-1} = 0$

$\Rightarrow \quad IA - I + A^{-1} = 0 \; [\because A^{-1}A = I, A^{-1}I = A^{-1}]$

$\Rightarrow \quad A - I + A^{-1} = 0$

or $\quad A^{-1} = I - A$

14. If A and B are invertible matrices, then which of the following is not true?

(a) $\text{adj } A = |A|.A^{-1}$

(b) $|A^{-1}| = (|A|)^{-1}$

(c) $(AB)^{-1} = B^{-1}.A^{-1}$

(d) $(A + B)^{-1} = B^{-1} + A^{-1}$

Ans. *(d) $(A + B)^{-1} = B^{-1} + A^{-1}$*

Explanation: For instance, if $A = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$

Then, $\quad A + B = \begin{bmatrix} 1 & 5 \\ 3 & 3 \end{bmatrix}$

$$\Rightarrow \quad (A + B)^{-1} = \frac{-1}{12}\begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix} \qquad \text{...(i)}$$

Further,

$$B^{-1} + A^{-1} = \frac{-1}{2}\begin{bmatrix} 3 & -2 \\ -1 & 0 \end{bmatrix} + \frac{-1}{6}\begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{3}{2} & \frac{3}{2} \\ \frac{5}{6} & -\frac{1}{6} \end{bmatrix} \qquad \text{...(ii)}$$

From (i) and (ii), we have

$(A + B)^{-1} \neq B^{-1} + A^{-1}$

15. If the value of a third order determinant is 12, then the value of the determinant formed by replacing each element by its cofactor will be:

(a) 12 (b) 144

(c) – 12 (d) 13

Ans. *(b) 144*

Explanation: Let $\Delta = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be a determinant of third order.

We know that $|A'| = |A|$.

$\therefore \quad |\text{adj } A| = |\text{cof } A| \qquad \text{...(i)}$

Further, $\quad |\text{adj } A| = |A|^{n-1} \qquad \text{...(ii)}$

From (i) and (ii), we have

$$|\text{cof } A| = |A|^{n-1} = |A|^2 = 12^2 = 144$$

16. Ⓐ **If x, y and z are non-zero real numbers, then the inverse of the matrix $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is:**

(a) $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(b) $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(c) $\frac{1}{xyz}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

(d) $\frac{1}{xyz}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

17. For matrix $A=\begin{bmatrix} 2 & 5 \\ -11 & 7 \end{bmatrix}$, (adj A)' is equal to:

(a) $\begin{bmatrix} -2 & -5 \\ 11 & -7 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & 5 \\ 11 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix}$

[CBSE Term-1 SQP 2021]

Ans. *(c)* $\begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$

$$\text{adj } A = \begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix}$$

$$\Rightarrow \quad (\text{adj } A) = \begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$$

[CBSE Marking Scheme Term-1 SQP 2021]

Explanation: Cofactors of matrix A are :

$$A_{11} = 7, A_{12} = 11; A_{21} = -5, A_{22} = 2$$

We know, $\text{adj. } A = \begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}^T$

$$\Rightarrow \quad [\text{adj. } A]^T = \left[\begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}^T\right]^T$$

$$\Rightarrow \quad [\text{adj. } A]^T = \begin{bmatrix} 7 & 11 \\ 5 & 2 \end{bmatrix}$$

18. For $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then $14A^{-1}$ is given by :

(a) $14\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$

(c) $2\begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix}$ (d) $2\begin{bmatrix} -3 & -1 \\ 1 & -2 \end{bmatrix}$

[CBSE Term-1 SQP 2021]

Ans. *(b)* $\begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$

$$|A| = 7, \text{adj } A = \begin{bmatrix} 2 & -2 \\ 1 & 3 \end{bmatrix}$$

$$\therefore \quad 14A^{-1} = \begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$$

[CBSE Marking Scheme Term-1 SQP 2021]

Explanation: we have

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\therefore \quad |A| = 6 - (-1) = 7$$

$\therefore$ A^{-1} exist.

$$\therefore \quad \text{adj. } A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}\text{adj.} A$$

$$= \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

So, $$14A^{-1} = 14 \times \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$= 2\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$$

VERY SHORT ANSWER Type Questions (VSA)

[**1** mark]

19. Find the cofactors of all the elements of $\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$. **[CBSE 2020]**

Ans. Let $$\Delta = \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$$

Cofactor of 1 is 3

Cofactor of – 2 is – 4

Cofactor of 4 is 2

Cofactor of 3 is 1

20. If $A = \begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$, then write the cofactor of the element a_{21} of its 2nd row. **[CBSE 2015]**

Ans. We have $A = \begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$

Now, cofactors of $a_{21} = (-1)^{2+1} \begin{vmatrix} 6 & -3 \\ -7 & 3 \end{vmatrix}$

$$= -1(18 - 21)$$
$$= 3$$

21. Find the minor and cofactor of element -7 in the determinant $\Delta = \begin{vmatrix} 1 & 2 & 4 \\ 2 & -8 & 3 \\ 2 & -7 & 4 \end{vmatrix}$.

22. In the interval $\frac{\pi}{2} < x < \pi$, find the value of x for which the matrix $\begin{bmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{bmatrix}$ is singular. **[CBSE 2015]**

23. If for any 2×2 square matrix A, A(adj A) $= \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $|A|$. **[CBSE 2017]**

Ans. We have, $A(\text{adj } A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

$\therefore \quad |A (\text{adj } A)| = \begin{vmatrix} 8 & 0 \\ 0 & 8 \end{vmatrix}$

$\Rightarrow \quad |A|\,|\text{adj } (A)| = 64 - 0$

$\Rightarrow \quad |A|\,|A|^{2-1} = 64$

$\Rightarrow \quad |A|^2 = 8^2$

$\Rightarrow \quad |A| = \pm 8$

24. If A is a square matrix of order 3 such that $|\text{adj } A| = 64$, then find $|A|$. **[CBSE 2013]**

25. If $A = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix}$, then find $|\text{adj } (A)|$.

26. Find the adjoint of a matrix $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$.

Ans. We have, $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$

The cofactors of elements of the determinant are

$$A_{11} = +(4) = +4$$
$$A_{12} = -(3) = -3$$
$$A_{21} = -(-2) = 2$$
$$A_{22} = +(1) = 1$$

$\because \quad \text{adj } (A) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}^T$

$$= \begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}^T = \begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$$

27. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, then write A^{-1} in terms of A. **[CBSE 2011]**

Ans. We have

$$A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$$

So, $\quad \text{adj } A = \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$

and $\quad |A| = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix}$

$$= -4 - 15 = -19$$

$\therefore \quad A^{-1} = \frac{1}{|A|} \text{adj}(A)$

$$= \frac{1}{-19} \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$$

$$= \frac{1}{19} \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$$

$$= \frac{1}{19} A$$

28. Rishika and Anwesha were very bright and sincere students and always did very well in examinations. Soon after their second class tests results in Mathematics and English were announced, they wrote a matrix to compare their performance in the two class tests held in the two subjects.

$$\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

Find the minors and cofactors of matrix $P = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$.

Ans. Minors of matrix P are

$M_{11} = -1, M_{12} = 2, M_{21} = 3, M_{22} = 1$

and cofactors of matrix P are

$A_{11} = (-1)^{1+1} M_{11} = +(-1) = -1$

$A_{12} = (-1)^{1+2} M_{12} = -(2) = -2$

$A_{21} = (-1)^{2+1} M_{21} = -(3) = -3$

$A_{22} = (-1)^{2+2} M_{22} = +(1) = 1$

SHORT ANSWER Type-I Questions (SA-I)

[**2** marks]

29. Find the minors and cofactors of the diagonal elements of the determinant $\begin{vmatrix} 3 & -1 & 3 \\ 4 & 2 & 2 \\ 1 & 3 & 1 \end{vmatrix}$.

Ans. Let $\det(A) = \begin{vmatrix} 3 & -1 & 3 \\ 4 & 2 & 2 \\ 1 & 3 & 1 \end{vmatrix}$

The diagonal elements of the given determinant are $a_{11} = 3, a_{22} = 2$ and $a_{33} = 1$.

Thus, the minors of the diagonal elements of the determinant A are

$$M_{11} = \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} = 2 - 6 = -4$$

$$M_{22} = \begin{vmatrix} 3 & 3 \\ 1 & 1 \end{vmatrix} = 3 - 3 = 0$$

$$M_{33} = \begin{vmatrix} 3 & -1 \\ 4 & 2 \end{vmatrix} = 6 - (-4) = 10$$

The cofactors of the diagonal elements of the determinant A are

$C_{11} = (-1)^{1+1} M_{11} = +(-4) = -4$

$C_{22} = (-1)^{2+2} M_{12} = +(0) = 0$

$C_{33} = (-1)^{3+3} M_{13} = +(10) = 10$

30. Using cofactors of the elements of third row, evaluate $\Delta = \begin{vmatrix} 3 & 4 & 5 \\ -1 & 2 & 3 \\ 0 & 1 & 4 \end{vmatrix}$.

Ans. We have, $\Delta = \begin{vmatrix} 3 & 4 & 5 \\ -1 & 2 & 3 \\ 0 & 1 & 4 \end{vmatrix}$

Using cofactor of the elements of third row, we have

$$\Delta = a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33}$$

$$= 0\begin{vmatrix} 4 & 5 \\ 2 & 3 \end{vmatrix} - 1\begin{vmatrix} 3 & 5 \\ -1 & 3 \end{vmatrix} + 4\begin{vmatrix} 3 & 4 \\ -1 & 2 \end{vmatrix}$$

$$= 0(12 - 10) - 1(9 + 5) + 4(6 + 4)$$

$$= 0 - 14 + 40 = 26$$

31. Find the inverse of the matrix $P = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.

32. Using cofactors of elements of second row, evaluate $\Delta = \begin{vmatrix} 1 & u & v+w \\ 1 & v & u+w \\ 1 & w & u+v \end{vmatrix}$.

33. Find |adj (adj A)|, if $A = \begin{bmatrix} 4 & -1 \\ 3 & 2 \end{bmatrix}$.

Ans. We have $A = \begin{bmatrix} 4 & -1 \\ 3 & 2 \end{bmatrix}$

$\therefore$ $\text{adj } A = \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}$

$\Rightarrow$ $\text{adj (adj } A) = \begin{bmatrix} 4 & -1 \\ 3 & 2 \end{bmatrix}$

$\Rightarrow$ $|\text{adj (adj } A)| = \begin{vmatrix} 4 & -1 \\ 3 & 2 \end{vmatrix}$

$= 8 + 3 = 11$

34. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ **be such that** $A^{-1} = kA$, **then find the value of** k. **[CBSE 2018]**

35. Find the adjoint of a matrix $A = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 1 \\ 5 & 2 & 1 \end{bmatrix}$.

Ans. We have $A = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 1 \\ 5 & 2 & 1 \end{bmatrix}$

Now cofactors of all elements of determinant A are

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} = +(2-2) = 0$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 4 & 1 \\ 5 & 1 \end{vmatrix} = -(4-5) = 1$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 4 & 2 \\ 5 & 2 \end{vmatrix} = +(8-10) = -2$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} = -(-1-4) = 5$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 3 & 2 \\ 5 & 1 \end{vmatrix} = +(3-10) = -7$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 3 & -1 \\ 5 & 2 \end{vmatrix} = -(6+5) = -11$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} = +(-1-4) = -5$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} = -(3-8) = 5$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 3 & -1 \\ 4 & 2 \end{vmatrix} = +(6+4) = 10$$

$$\because \text{ adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$\therefore \text{ adj } A = \begin{bmatrix} 0 & 1 & -2 \\ 5 & -7 & -11 \\ -5 & 5 & 10 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 5 & -5 \\ 1 & -7 & 5 \\ -2 & -11 & 10 \end{bmatrix}$$

36. **If B is a matrix of order 2 × 2, then prove that $(B^3)^{-1} = (B^{-1})^3$.**

Ans. We know, $(B^2)^{-1} = (BB)^{-1}$

$$= B^{-1}B^{-1}$$

$$= (B^{-1})^2$$

$$\therefore \quad (B^3)^{-1} = (BB^2)^{-1}$$

$$= (B^2)^{-1}B^{-1}$$

$$= (B^{-1})^2B^{-1}$$

$$= (B^{-1})^{2+1}$$

$$= (B^{-1})^3$$

Hence proved.

37. **Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$.** **[CBSE 2018]**

Ans. We have, $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

So, $|A| = \begin{vmatrix} 2 & -3 \\ -4 & 7 \end{vmatrix}$

$$= 14 - 12$$

$$= 2$$

So A^{-1} exist

Now, $\text{adj }(A) = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$$\therefore \quad A^{-1} = \frac{\text{adj }(A)}{|A|}$$

$$= \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} \quad \text{...(i)}$$

Hence, A^{-1} is $\frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

Now, we have to show that

$$2A^{-1} = 9I - A$$

Taking R.H.S.

$$= 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 9-2 & 3 \\ 4 & 9-7 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$= 2A^{-1} =$ L.H.S.[From eq. (i)]

38. Ⓐ **If $A = \begin{bmatrix} 4 & -1 \\ 3 & 2 \end{bmatrix}$, then prove that $|A^{-1}| = |A|^{-1}$.**

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

39. Consider a non-singular matrix $A = \begin{bmatrix} 4 & 2 \\ -1 & 2 \end{bmatrix}$. Verify that $(A^T)^{-1} = (A^{-1})^T$.

Ans. We have $A = \begin{bmatrix} 4 & 2 \\ -1 & 2 \end{bmatrix}$

So, $\text{adj } A = \begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}$

and $|A| = \begin{vmatrix} 4 & 2 \\ -1 & 2 \end{vmatrix}$

$= 8 + 2 = 10$

$\therefore \quad A^{-1} = \frac{1}{|A|} \text{adj}(A)$

$= \frac{1}{10}\begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}$

Now, R.H.S. $= (A^{-1})^T$

$= \frac{1}{10}\begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}^T$ ...(i)

$= \frac{1}{10}\begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}$

$\because \quad A^T = \begin{bmatrix} 4 & 2 \\ -1 & 2 \end{bmatrix}^T = \begin{bmatrix} 4 & -1 \\ 2 & 2 \end{bmatrix}$

$\therefore \quad \text{adj}(A^T) = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}$

and, $|A^T| = \begin{vmatrix} 4 & 2 \\ -1 & 2 \end{vmatrix} = 8 + 2 = 10$

So, L.H.S $= (A^T)^{-1}$

$= \frac{1}{|A^T|}(\text{adj}(A^T))$

$= \frac{1}{10}\begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}$ ...(ii)

From (i) and (ii),

L.H.S. = R.H.S.

Hence, verified

40. Find the adjoint of the matrix $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that $A(\text{adj } A) = |A|\, I_3$. [CBSE 2015]

Ans. We have $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

Now, cofactors of elements of determinant A are

$A_{11} = (-1)^{1+1}\begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} = (1-4) = -3$

$A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix} = -(2+4) = -6$

$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} = (-4-2) = -6$

$A_{21} = (-1)^{2+1}\begin{vmatrix} -2 & -2 \\ -2 & 1 \end{vmatrix} = -(-2-4) = 6$

$A_{22} = (-1)^{2+2}\begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = (-1+4) = 3$

$A_{23} = (-1)^{2+3}\begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -(2+4) = -6$

$A_{31} = (-1)^{3+1}\begin{vmatrix} -2 & -2 \\ 1 & -2 \end{vmatrix} = (4+2) = 6$

$A_{32} = (-1)^{3+2}\begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -(2+4) = -6$

$A_{33} = (-1)^{3+3}\begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = (-1+4) = 3$

$\therefore \quad \text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$

$= \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}^T$

$= \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$

Also, $|A| = \begin{vmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{vmatrix}$

$= -1(1-4) + 2(2+4) - 2(-4-2)$

$= -1(-3) + 2(6) - 2(-6)$

$= 3 + 12 + 12 = 27$

Now,

$$A\,(\text{adj } A) = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}\begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3+12+12 & -6-6+12 & -6+12-6 \\ -6-6+12 & 12+3+12 & 12-6-6 \\ -6+12-6 & 12-6-6 & 12+12+3 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{bmatrix} = 27\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= 27I_3 = |A|I_3$

Hence, proved.

41. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, **then verify the following results**

(A) $|\text{adj}\,(\text{adj } A)| = |A|$

(B) $\text{adj}\,(A^T) = (\text{adj } A)^T$.

42. **Suppose matrix P and Q are defined as $P = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} 8 & 3 \\ 2 & 1 \end{bmatrix}$. Verify the result adj (PQ) = adj (Q) adj (P).**

43. Find matrix P of order 2 × 2 of the following equation, by using inverse method $P\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$.

Ans. Let $Q = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$

Then, $PQ = 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 6I$ ...(i)

Now $|Q| = \begin{vmatrix} 1 & -2 \\ 1 & 4 \end{vmatrix}$

$= 4 + 2 = 6$

Here $|Q| \neq 0$, so its inverse exist.

$\because$ $\text{adj } Q = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

$\therefore$ $Q^{-1} = \frac{1}{|Q|}\text{adj}(Q)$

$= \frac{1}{6}\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

On post multiplying Q^{-1} in eq. (i) on both sides, we get

$\Rightarrow$ $(PQ)Q^{-1} = (6I)Q^{-1}$

$\Rightarrow$ $P(QQ^{-1}) = 6(IQ^{-1})$

$\Rightarrow$ $PI = 6IQ^{-1}$

$\Rightarrow$ $P = 6Q^{-1}$

$P = 6 \times \frac{1}{6}\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

44. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, **then find $(A^T)^{-1}$.**

[CBSE 2015]

45. Suppose $A = \begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$, then verify that $(AB)^{-1} = B^{-1}A^{-1}$.

Ans. Given matrices are $A = \begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$.

Now, $AB = \begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$

$= \begin{bmatrix} 8-5 & 4+20 \\ 6-2 & 3+8 \end{bmatrix}$

Also, $= \begin{bmatrix} 3 & 24 \\ 4 & 11 \end{bmatrix}$

$\because$ $|AB| = \begin{vmatrix} 3 & 24 \\ 4 & 11 \end{vmatrix}$

$= 33 - 96 = -63$

$\therefore$ $(AB)^{-1} = \frac{1}{|AB|}\text{adj}(AB)$

$= \frac{1}{-63}\begin{bmatrix} 11 & -24 \\ -4 & 3 \end{bmatrix}$ ...(i)

$\because$ $|A| = \begin{vmatrix} 4 & 5 \\ 3 & 2 \end{vmatrix}$

$= 8 - 15 = -7$

and $|B| = \begin{vmatrix} 2 & 1 \\ -1 & 4 \end{vmatrix}$

$= 8 + 1 = 9$

$\therefore$ $A^{-1} = \frac{1}{|A|}\text{adj}(A)$

$= \frac{1}{-7}\begin{bmatrix} 2 & -5 \\ -3 & 4 \end{bmatrix}$

and $B^{-1} = \frac{1}{|B|} adj(B)$

$$= \frac{1}{9}\begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}$$

Now, $B^{-1}A^{-1} = \frac{1}{9 \times (-7)}\begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -5 \\ -3 & 4 \end{bmatrix}$

$$= -\frac{1}{63}\begin{bmatrix} 8+3 & -20-4 \\ 2-6 & -5+8 \end{bmatrix}$$

$$= -\frac{1}{63}\begin{bmatrix} 11 & -24 \\ -4 & 3 \end{bmatrix} \quad ...(ii)$$

From eqs. (i) and (ii), we conclude that

$(AB)^{-1} = B^{-1}A^{-1}$.

Hence proved.

46. If matrix $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ such that $A^2 + 8I = xA$, find the values of x. Also find A^{-1}.

Ans. We have $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$

$$\therefore \quad A^2 = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9+7 & 3+5 \\ 21+35 & 7+25 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix}$$

We have

$$A^2 + 8I = xA \quad ...(i)$$

$$\therefore \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix} + \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} = \begin{bmatrix} 3x & x \\ 7x & 5x \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 24 & 8 \\ 56 & 40 \end{bmatrix} = \begin{bmatrix} 3x & x \\ 7x & 5x \end{bmatrix}$$

On comparing element a_{12} both sides, we get

$$x = 8$$

Put $x = 8$ in Eq. (i), we get

$$A^2 + 8I = 8A \quad ...(ii)$$

On post multipying eq. (ii) by A^{-1}, we get

$$A^2A^{-1} + 8IA^{-1} = 8AA^{-1}$$

$$\Rightarrow A(AA^{-1}) + 8A^{-1} = 8I \quad [\because AA^{-1} = I]$$

$$\Rightarrow AI + 8A^{-1} = 8I$$

$$\Rightarrow 8A^{-1} = 8I - A$$

$$= \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 8-3 & 0-1 \\ 0-7 & 8-5 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{8}\begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$$

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

47. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find adj A and verify that A(adj A) = (adj A)A = |A| I_3.

Ans. Given: $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now, cofactors of determinant A are

$$A_{11} = +\begin{vmatrix} \cos\alpha & 0 \\ 0 & 1 \end{vmatrix} = \cos\alpha$$

$$A_{12} = -\begin{vmatrix} \sin\alpha & 0 \\ 0 & 1 \end{vmatrix} = -\sin\alpha$$

$$A_{13} = +\begin{vmatrix} \sin\alpha & \cos\alpha \\ 0 & 0 \end{vmatrix} = 0$$

$$A_{21} = -\begin{vmatrix} -\sin\alpha & 0 \\ 0 & 1 \end{vmatrix} = \sin\alpha$$

$$A_{22} = +\begin{vmatrix} \cos\alpha & 0 \\ 0 & 1 \end{vmatrix} = \cos\alpha$$

$$A_{23} = -\begin{vmatrix} \cos\alpha & -\sin\alpha \\ 0 & 0 \end{vmatrix} = 0$$

$$A_{31} = +\begin{vmatrix} -\sin\alpha & 0 \\ \cos\alpha & 0 \end{vmatrix} = 0$$

$$A_{32} = -\begin{vmatrix} \cos\alpha & 0 \\ \sin\alpha & 0 \end{vmatrix} = 0$$

$$A_{33} = +\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix} = \cos^2\alpha + \sin^2\alpha = 1$$

$\therefore$ adj (A) = $\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

$$= \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, A(adj A) = $\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\alpha+\sin^2\alpha+0 & \cos\alpha\sin\alpha-\sin\alpha\cos\alpha+0 & 0+0+0 \\ \sin\alpha\cos\alpha-\cos\alpha\sin\alpha+0 & \sin^2\alpha+\cos^2\alpha+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{...(i)}$$

Also,

adj (A)A = $\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\alpha+\sin^2\alpha+0 & -\cos\alpha\sin\alpha+\sin\alpha\cos\alpha+0 & 0+0+0 \\ -\sin\alpha\cos\alpha+\cos\alpha\sin\alpha+0 & \sin^2\alpha+\cos^2\alpha+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{...(ii)}$$

And,

$$|A| = \begin{vmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding along C_3, we get

$|A| = 1 (\cos^2\alpha + \sin^2\alpha)$

$= 1$

and $|A|I_3 = 1\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{...(iii)}$$

From eqs. (i), (ii) and (iii), we conclude that

A(adj A) = (adj A)A = $|A|\ I_3$

Hence, verified

48. **Find A^{-1} if A = $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and show that**

$A^{-1} = \dfrac{A^2 - 3I}{2}$ [NCERT Exemplar]

49. If P = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, prove that $P^2 - 4P - 5I = O$.

Hence, find A^{-1}.

Ans. We have, P = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

So, $P^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

Now, L.H.S. = $P^2 - 4P - 5I$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9-4-5 & 8-8-0 & 8-8-0 \\ 8-8-0 & 9-4-5 & 8-8-0 \\ 8-8-0 & 8-8-0 & 9-4-5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

= 0 = R.H.S.

Hence proved.

Now, $|P| = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$

$= 1(1 - 4) - 2(2 - 4) + 2(4 - 2)$

$= -3 + 4 + 4 = 5$

Here $|P| \neq 0$, so its inverse exists.

Consider $P^2 - 4P - 5I = 0$

On pre-multiplying both sides by P^{-1}, we get

$P^{-1}P^2 - 4P^{-1}P - 5P^{-1}I = P^{-1}O$

$\Rightarrow \quad (P^{-1}P)P - 4I - 5P^{-1} = O \qquad [\because PP^{-1} = I]$

$\Rightarrow \quad IP - 4I - 5P^{-1} = O$

$\Rightarrow \quad P^{-1} = \frac{1}{5}(P - 4I)$

$$= \frac{1}{5}\left(\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}\right)$$

$$= \frac{1}{5}\begin{bmatrix} 1-4 & 2 & 2 \\ 2 & 1-4 & 2 \\ 2 & 2 & 1-4 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

50. **Show that for the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$,**

$A^3 - 6A^2 + 5A + 11I = O$. Hence, find A^{-1}.

[CBSE 2019]

51. Given $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}, B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$,

compute $(AB)^{-1}$.

Ans. Given, $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 5(3-4) - 0 + 4(4-3)$$

$= -5 + 4 = -1$

Here $|A| \neq 0$, so A^{-1} exists.

Now cofactors of elements of determinant A are

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = (3-4) = -1$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} = -(2-2) = 0$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = (4-3) = 1$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 0 & 4 \\ 2 & 1 \end{vmatrix} = -(0-8) = 8$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 5 & 4 \\ 1 & 1 \end{vmatrix} = (5-4) = 1$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 5 & 0 \\ 1 & 2 \end{vmatrix} = -(10-0) = -10$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 0 & 4 \\ 3 & 2 \end{vmatrix} = (0-12) = -12$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 5 & 4 \\ 2 & 2 \end{vmatrix} = -(10-8) = -2$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 5 & 0 \\ 2 & 3 \end{vmatrix} = (15-0) = 15$$

$$\therefore \quad \text{adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 8 & -12 \\ 0 & 1 & -2 \\ 1 & -10 & 15 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{-1}\begin{bmatrix} -1 & 8 & -12 \\ 0 & 1 & -2 \\ 1 & -10 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$$

By using property of inverse matrix,

$(AB)^{-1} = B^{-1}A^{-1}$

$$= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0-3 & -8-3+30 & 12+6-45 \\ 1+0-3 & -8-4+30 & 12+8-45 \\ 1+0-4 & -8-3+40 & 12+6-60 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix}$$

APPLICATIONS OF DETERMINANTS AND MATRICES

3

TOPIC 1

CONSISTENT/INCONSISTENT SYSTEM OF EQUATIONS

In this section, we shall discuss applications of determinants and matrices for solving a system of linear equations in two or three variables and for checking the consistency of the system of linear equations.

A system of linear equations is said to be consistent if its solution (one or more) exits, while a system of linear equations is said to be inconsistent, if its solution does not exist.

⚠ Caution

➥ *Here, we shall restrict ourselves to the system of linear equations having unique solutions only.*

TOPIC 2

SOLUTION OF A SYSTEM OF LINEAR EQUATIONS, USING INVERSE OF A MATRIX

Consider a system of linear equations

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

This system of linear equations can be expressed by a single matrix equation as

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

or, $AX = B$

where, $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$, , $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

{Here, A is called the coeffcient matrix.}

Case I: If $|A| \neq 0$, *i.e.*, if A is a non-singular matrix, then A^{-1} exists.

From AX = B, we have

$$A^{-1}(AX) = A^{-1}B \text{ \{Pre-multiplying by } A^{-1}\}$$

$\Rightarrow$ $(A^{-1}A)X = A^{-1}B$

$\Rightarrow$ $IX = A^{-1}B$

$\Rightarrow$ $X = A^{-1}B$

Thus, the system AX = B has a unique solution given by $X = A^{-1}B$.

⚠ Caution

➥ *This matrix equation provides unique solution for the given system of linear equations as inverse of a matrix is unique. This method of solving system of linear equations is known as Matrix Method.*

Example 3.1: **Solve the following system of linear equations, using matrix method.**

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

[NCERT]

Ans. The given system of linear equations can be expressed by a single matrix equation as

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

or, $AX = B$

where $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$.

Here, $|A| = 2(-4 + 4) + 3(-6 + 4) + 5(3 - 2)$
$= -1 \neq 0$,

i.e., A is a non-singular matrix, and hence A^{-1} exists.

From AX = B, we have

$$X = A^{-1}B \quad ...(i)$$

Cofactors of all elements of determinant A are:

$$A_{11} = (-1)^{1+1}M_{11} = \begin{vmatrix} 2 & -4 \\ 1 & -2 \end{vmatrix}$$
$$= -4 + 4 = 0$$

$$A_{12} = (-1)^{1+2}M_{12} = -\begin{vmatrix} 3 & -4 \\ 1 & -2 \end{vmatrix}$$

$$= 6 - 4 = 2$$

$$A_{13} = (-1)^{1+3}M_{13} = \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix}$$

$$= 3 - 2 = 1$$

$$A_{21} = (-1)^{2+1}M_{21} = -\begin{vmatrix} -3 & 5 \\ 1 & -2 \end{vmatrix}$$

$$= -6 + 5 = -1$$

$$A_{22} = (-1)^{2+2}M_{22} = \begin{vmatrix} 2 & 5 \\ 1 & -2 \end{vmatrix}$$

$$= -4 - 5 = -9$$

$$A_{23} = (-1)^{2+3}M_{23} = -\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix}$$

$$= -3 - 2 = -5$$

$$A_{31} = (-1)^{3+1}M_{31} = \begin{vmatrix} -3 & 5 \\ 2 & -4 \end{vmatrix}$$

$$= 12 - 10 = 2$$

$$A_{32} = (-1)^{3+2}M_{32} = -\begin{vmatrix} 2 & 5 \\ 3 & -4 \end{vmatrix}$$

$$= 8 + 15 = 23$$

$$A_{33} = (-1)^{3+3}M_{33} = \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix}$$

$$= 4 + 9 = 13$$

This gives $\text{adj } A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$; and thus,

$$A^{-1} = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}, \text{ or } \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

From (i), we have

$$X = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \quad -5 \quad +6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

or, $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Thus, $x = 1$, $y = 2$ and $z = 3$.

Case II: If $|A| = 0$, *i.e.*, if A is a singular matrix, then A^{-1} does not exist.

In this case, we calculate (adj A)B.

(1) If (adj A)B ≠ O, then solution does not exist and given system of equations is inconsistent.

(2) If (adj A)B = O, then either no solution exist or infinitely many solutions exist, and accordingly, the given system of equations will be inconsistent or consistent, respectively.

Example 3.2: Examine the consistency of the following system of linear equations.

(A) $x + 2y = 2$, $2x + 3y = 3$ **(B)** $x + 3y = 5$, $2x + 6y = 8$ **[NCERT]**

Ans. (A) The given system of equations can be written in matrix form as,

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

or $AX = B$

where, $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1 \neq 0$

Hence, given system of equations is consistent.

(B) The given systems of equations can be written in matrix form as,

$$\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

or $AX = B$

where, $A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix} = 6 - 6 = 0$

So, we have to calculate (adj A)B.

Now, $(\text{adj } A) = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

So, $(\text{adj } A)B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 8 \end{bmatrix}$

$$= \begin{bmatrix} 30 - 24 \\ -10 + 8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq O$$

Hence, the given system of equations is inconsistent.

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. The system of linear equations:

$x + y + z = 2;\ 2x + y - z = 3;\ 3x + 2y + kz = 4$

has a unique solution if

(a) $k \neq 0$ (b) $-1 < k < 1$

(c) $-2 < k < 2$ (d) $k = 0$

Ans. *(a)* $k \neq 0$

Explanation: The system of equation will have a unique solution if $\Delta = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{bmatrix} \neq 0$

$\Rightarrow 1(k + 2) - 1(2k + 3) + 1(4 - 3) \neq 0$

$\Rightarrow k \neq 0$

2. The system of linear equations:

$x + y + z = 5;\ x + 2y + 3z = 9;\ x + 3y + \lambda z = m$

has a unique solution if

(a) $\lambda \neq 5, m = 13$ (b) $\lambda \neq 5$

(c) $\lambda = 5, m \neq 13$ (d) $m \neq 13$

3. The matrix equation for the problem "The sum of three numbers x, y and z is 6. If we multiply z by 3 and add y to it, we get 11. By adding x and z, we get $2y$" is:

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$

Ans. *(d)* $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$

Explanation: The system of linear equations, thus, formed is

$x + y + z = 6;\ y + 3z = 11;\ x - 2y + z = 0$

4. The system of linear equations

$$5x + ky = 5$$
$$3x + 3y = 5;$$

Will be consistent if

(a) $k \neq -3$ (b) $k = -5$

(c) $k = 5$ (d) $k \neq 5$

[CBSE Term-1 2021]

Ans. *(d)* $k \neq -5$

Explanation:

Here $A = \begin{bmatrix} 5 & k \\ 3 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$

For consistent system, $|A| \neq 0$

$\therefore \quad |A| = \begin{vmatrix} 5 & k \\ 3 & 3 \end{vmatrix} \neq 0$

$\Rightarrow \quad 15 - 3k \neq 0$

$\Rightarrow \quad -3k \neq -15$

$\Rightarrow \quad k \neq 5.$

5. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, then:

(a) $A^{-1} = B$ (b) $A^{-1} = 6B$

(c) $B^{-1} = B$ (d) $B^{-1} = \frac{1}{6}A$

[CBSE Term-1 SQP 2021]

6. The system of linear equations $kx + y + z = 1$, $x + ky + z = 1$ and $x + y + kz = 1$ has a unique solution under which one of the following conditions?

(a) $k \neq 1$ and $k \neq 2$

(b) $k \neq 1$ and $k \neq -2$

(c) $k \neq -2$ and $k \neq 2$

(d) None of these

Ans. *(b)* $k \neq 1$ and $k \neq -2$

Explanation: Given system of linear equations can be written in matrix form as,

$$\begin{bmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Consider $A = \begin{bmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

The condition for unique solution is $|A| \neq 0$

$$\therefore \quad \begin{bmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{bmatrix} \neq 0$$

On expanding along R_1, we get

$k(k^2 - 1) - 1(k - 1) + 1(1 - k) \neq 0$

$\Rightarrow k(k - 1)(k + 1) - 2(k - 1) \neq 0$

$\Rightarrow (k - 1)^2(k + 2) \neq 0$

$\Rightarrow k \neq 1$ and $k \neq -2$

7. The system of equations $x + 2y = 2$ and $2x + 3y = 3$ is

(a) Consistent **(b) Inconsistent**

(c) Both (a) and (b) **(d) None of these**

Ans. *(a) Consistent*

Explanation: Given system of equations can be written in matrix form as

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Consider $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \end{bmatrix}$

Then $AX = B$

Now, $|A| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}$

$= 3 - 4 = -1 \neq 0$

Hence, given system of equations is consistents.

8. The existence of the unique solution of the following system of equations, depends on

$$x + y + z = \lambda$$
$$5x - y + \mu z = 10$$
$$2x + 3y - z = 6$$

(a) μ only **(b) λ only**

(c) Both λ and μ **(d) Neither λ nor μ**

Ans. *(a) μ only*

Explanation: Given system of equations can be written in matrix form as

$$\begin{bmatrix} 1 & 1 & 1 \\ 5 & -1 & \mu \\ 2 & 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \lambda \\ 10 \\ 6 \end{bmatrix}$$

Here $A = \begin{bmatrix} 1 & 1 & 1 \\ 5 & -1 & \mu \\ 2 & 3 & -1 \end{bmatrix}$

For the existence of the unique solution, $|A| \neq 0$, so the existence of unique solution of given system of equations depends on $|A|$ only, *i.e.*, on μ only.

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

9. *The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for cooperation and helping others and some others (say z) for supervising the workers to keep the colony neat and clean.*

(I) The sum of all awardees is 12.

(II) Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33.

(III) The sum of the number of awardees for honesty and supervision is twice the number of awardees for cooperation and helping others.

(A) The group of equations for the statements (I), (II) and (III) respectively are:

(a) $x + y + z = 12; 3x + 2(y + z) = 33; x + z = 2y$

(b) $x + y + z = 12; 3x + 2(y + z) = 33; y + z = 2x$

(c) $x + y + z = 12; 3z + 2(x + y) = 33; x + z = 2y$

(d) $x + y + z = 12; 2x + 3(y + z) = 33; x + z = 2y$

(B) The above group of equations in matrix form AX = B, can be written as:

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & 1 \\ -3 & 2 & 1 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 33 \\ 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 & 1 \\ -3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 33 \\ 12 \end{bmatrix}$

(C) **|A| is equal to:**

(a) 3 (b) 2

(c) 12 (d) – 14

(D) **adj (A) is:**

(a) $\begin{bmatrix} 4 & -3 & -1 \\ 4 & 0 & -4 \\ 4 & -3 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & -1 \\ -2 & 0 & 2 \\ 4 & -1 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 3 & -1 \\ 1 & 0 & 3 \\ -3 & 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$

(E) **The number of awardees of each category is**

(a) $x = 3; y = 5; z = 4$

(b) $x = 3; y = 4; z = 5$

(c) $x = 5; y = 4; z = 3$

(d) $x = 4; y = 5; z = 3$

Ans. (A) *(d)* $x + y + z = 12;\ 2x + 3(y + z) = 33;$ $x + z = 2y$

(D) *(d)* $\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$

Explanation: From part (B), we have

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}$$

So, cofactors of all elements of determinant of A are:

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 3 & 3 \\ -2 & 1 \end{vmatrix} = 9;$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = 1;$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -7;$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} = -3$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0;$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = 3$$

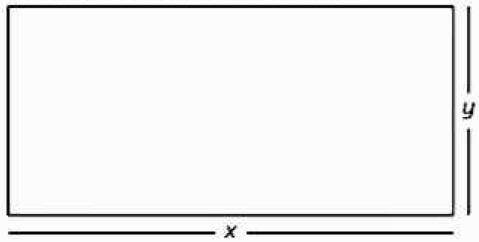
$$A_{31} = (-1)^{3+1}\begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix} = 0;$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = -1;$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1$$

$$\text{So, adj (A)} = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

10. *Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m².*



(A) Form the equations of given situation in matrix form.

(B) Using matrix method, find the area of rectangular field.

Ans. (A) According to the question

$$(x - 50)(y + 50) = xy$$
$$\Rightarrow \quad xy + 50x - 50y - 2500 = xy$$
$$\text{or} \quad x - y = 50 \quad ...(i)$$
$$\text{Also} \quad (x - 10)(y - 20) = xy - 5300$$
$$\Rightarrow \quad xy - 20x - 10y + 200 = xy - 5300$$
$$\Rightarrow \quad -20x - 10y = -5500$$
$$\text{or} \quad 2x + y = 550 \quad ...(ii)$$

Thus, system of equation (i) and (ii) in matrix from is

$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$$

(B) From part (A), we have

$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 50 \\ 550 \end{bmatrix}$$

$$= \frac{1}{1+2}\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 50 \\ 550 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 50+550 \\ -100+550 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix}600\\450\end{bmatrix} = \begin{bmatrix}200\\150\end{bmatrix}$$

∴ $x = 200, y = 150$

∴ Area of field $= xy$

$= 200 \times 150$

$= 30{,}000 \text{ m}^2$

11. *Three friends, Ravi, Sanjay and Ramesh went to Delhi to visit Qutab-Minar. They are carrying a number of visiting cards but they do not want to reveal the actual number of it. They start playing with each other in the garden nearby the monument. Total number of visiting cards they have is 18. Twice the number of visiting cards Ramesh has when added to Ravi's number of cards gives 21. On adding Sanjay's and Ramesh's number of visiting cards to thrice the Ravi's number of cards, they give 36. Assume the number of cards Ravi, Sanjay and Ramesh have are x, y and z respectively.*

(A) Write the given information in matrix form as AX = B.

(B) Find the number of visiting cards with Ravi, Sanjay and Ramesh respectively.

Ans. (A) According to the question,

$x + y + z = 18$ ...(i)

Also $x + 2z = 21$

or, $x + 0y + 2z = 21$ ....(ii)

And $3x + y + z = 36$ ....(iii)

The system of equation (i), (ii), and (iii) in matrix form is:

$$\begin{bmatrix}1 & 1 & 1\\1 & 0 & 2\\3 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}18\\21\\36\end{bmatrix}$$

$$AX = B$$

Where $A = \begin{bmatrix}1 & 1 & 1\\1 & 0 & 2\\3 & 1 & 1\end{bmatrix}$, $X = \begin{bmatrix}x\\y\\z\end{bmatrix}$,

$$B = \begin{bmatrix}18\\21\\36\end{bmatrix}$$

12. *A Class XII teacher, after teaching the topic of "matrices", tries to assess the performance of her students over this topic.*

(A) For a square matrix $A = [a_{ij}]$ of order 3, adjoint of A is

(a) $[A_{ij}]$, where A_{ij} denotes the cofactor of a_{ij} in A

(b) $[M_{ij}]$, where M_{ij} denotes the minor of a_{ij} in A

(c) $[A_{ij}]^T$, where A_{ij} denotes the cofactor of a_{ij} in A

(d) $[M_{ij}]^T$, where M_{ij} denotes the minor of a_{ij} in A

(B) Let A_{ij} denotes the cofactor of a_{ij} in $A = [a_{ij}]$. Then, |A| is given by

(a) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$

(b) $a_{31}A_{13} + a_{32}A_{23} + a_{33}A_{33}$

(c) $a_{21}A_{13} + a_{22}A_{23} + a_{23}A_{33}$

(d) $a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33}$

(C) Let A_{ij} and M_{ij} respectively denote the cofactor and minor of a_{ij} in $A = [a_{ij}]$. Then,

(a) $A_{ij} = -M_{ij}$ (b) $A_{ij} = (-1)^{i+j} M_{ij}$

(c) $A_{ij} = (-1)^{i-j} M_{ij}$ (d) $A_{ij} = (-1)^{i \times j} M_{ij}$

(D) For a square matrix A of order n, which of the following is true?

(a) $|\text{adj } A| = |A|^{n-1}$

(b) $|\text{adj } A| = |A|^{n}$

(c) $|\text{adj } A| = |A|^{n+1}$

(d) $|\text{adj } A| = |A|^{1+n}$

(E) A square matrix A is singular, if

(a) $|A| = 0$ (b) $|A(\text{adj } A)| = 0$

(c) $|A| \neq 0$ (b) $|A(\text{adj } A)| \neq 0$

Ans. (A) *(c) $[A_{ij}]^T$, where A_{ij} denotes the cofactor of a_{ij} in A*

(D) *(a) $|\text{adj } A| = |A|^{n-1}$*

13. *A typist charges ₹ 145 for typing 10 English and 3 Hindi pages, while charges for 3 English and 10 Hindi pages are ₹ 180. However, typist charged ₹ 2 per page from a poor student Shyam for 5 Hindi pages and ₹ 1 per page for 4 English pages.*

Let the charges for typing one English and one Hindi page be ₹ x and ₹ y respectively.

(A) The pair of linear equations formed with the given situation is:

(a) $5x + 4y = 65; 4x + 5y = 61$

(b) $10x + 3y = 145; 3x + 10y = 180$

(c) $3x + 10y = 145; 10x + 3y = 180$

(d) $4x + 5y = 61; 5x + 4y = 65$

(B) The amount charged by the typist from Shyam, is:

(a) ₹ 3 (b) ₹ 4

(c) ₹ 10 (d) ₹ 14

(C) On converting the pair of linear equations, formed in part (A), in the matrix form AX = B, the matrix A is:

(a) $\begin{bmatrix}5 & 4\\4 & 5\end{bmatrix}$ (b) $\begin{bmatrix}3 & 10\\10 & 3\end{bmatrix}$

(c) $\begin{bmatrix}4 & 5\\5 & 4\end{bmatrix}$ (d) $\begin{bmatrix}10 & 3\\3 & 10\end{bmatrix}$

(D) The values of x and y respectively are:

(a) 10, 15 (b) 15, 10

(c) 9, 5 (d) 5, 9

(E) How much less was charged from the poor student?

(a) ₹ 91 (b) ₹ 62

(c) ₹ 101 (b) ₹ 84

Ans. (A) *(b)* $10x + 3y = 145; 3x + 10y = 180$

(D) *(a)* 10, 15

Explanation: From part (C), we have,

$$AX = B$$

$$\Rightarrow \quad X = A^{-1} B$$

$$\because \quad |A| = \begin{bmatrix} 10 & 3 \\ 3 & 10 \end{bmatrix}$$

$$= 100 - 9 = 91$$

$$\text{and} \quad \text{adj } A = \begin{bmatrix} 10 & -3 \\ -3 & 10 \end{bmatrix}$$

$$\text{So} \quad X = A^{-1} B$$

$$= \left[\frac{1}{|A|} \text{adj } A\right] B$$

$$= \frac{1}{91}\begin{bmatrix} 10 & -3 \\ -3 & 10 \end{bmatrix}\begin{bmatrix} 145 \\ 180 \end{bmatrix}$$

$$= \frac{1}{91}\begin{bmatrix} 1450-540 \\ -435+1800 \end{bmatrix}$$

$$= \frac{1}{91}\begin{bmatrix} 910 \\ 1365 \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \end{bmatrix}$$

$$\therefore \quad x = 10, y = 15$$

14. *On her birthday, Seema decided to donate some money to children of orphanage home. If there are 8 children less, everyone will get ₹ 10 more. However, if there are 16 children more, everyone will get ₹ 10 less. Let the number of children be x and the amount distributed by Seem a to each child be ₹ y.*

(A) The total amount of money distributed by Seema is

(a) ₹ $(x + y)$ (b) ₹ $(y - x)$

(c) ₹ $(x \times y)$ (b) ₹ $(x - y)$

(B) The system of equations formed, from the given conditions, are

(a) $5x - 4y = 40$ and $8y - 5x = 80$

(b) $5x + 4y = 40$ and $8y + 5x = 80$

(c) $5x + 4y = 40$ and $- 8y + 5x = 80$

(d) $5x - 4y = 40$ and $5x - 8y = 80$

(C) The system of equations in matrix form is:

(a) $\begin{bmatrix} 5 & -5 \\ -4 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 80 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & -4 \\ -5 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 80 \end{bmatrix}$

(c) $\begin{bmatrix} -5 & 8 \\ 5 & -4 \end{bmatrix}[x \quad y] = [40 \quad 80]$

(d) $\begin{bmatrix} 5 & -8 \\ 5 & -4 \end{bmatrix}[x \quad y] = [80 \quad 40]$

(D) The adjoint of the matrix $\begin{bmatrix} 5 & -4 \\ -5 & 8 \end{bmatrix}$ is

(a) $\begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & -8 \\ -5 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & -4 \\ -5 & 8 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 8 \\ 5 & 4 \end{bmatrix}$

(E) The values of 'x' and 'y' respectively are:

(a) $x = 32, y = 35$ (b) $x = 30, y = 32$

(c) $x = 35, y = 32$ (d) $x = 32, y = 30$

Ans. (B) *(d)* $5x - 4y = 40$ and $5x - 8y = 80$

Explanation: According to the question,

$(x - 8)(y + 10) = xy$ and $(x + 16)(y - 10) = xy$

or $10x - 8y = 80$ and $16y - 10x = 160$

or $5x - 4y = 40$ and $8y - 5x = 80$

(C) *(b)* $\begin{bmatrix} 5 & -4 \\ -5 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 80 \end{bmatrix}$

15. *A trust having a fund of ₹ 30000 wish to invest into two different types of bonds. The first bond pays 5% interest per annum which will be given to an orphanage and the second bond pays 7% interest per annum which will be given to 'Cancer Aid Society' and NGO. The trust obtain an annual total interest of ₹ 1800.*

(A) What is the adjoint of the matrix $\begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix}$?

(B) If the amount invested in two bonds are ₹ x and ₹ y, respectively then find the values of x and y using matrix method.

Ans. (B) According to the question

$$x + y = 30,000 \quad ...(i)$$

And 5% of x + 7% of y = 1800

or $$5x + 7y = 180000 \quad ....(ii)$$

System of equation (i) and (ii) in matrix from is

$$\begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 30000 \\ 180000 \end{bmatrix}$$

Or $$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix}^{-1}\begin{bmatrix} 30000 \\ 180000 \end{bmatrix}$$

So, $$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 7 & -1 \\ -5 & 1 \end{bmatrix}\begin{bmatrix} 30000 \\ 180000 \end{bmatrix}$$

$$= \begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$$

$$x = 15000, y = 15000$$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

16. If $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix}$, **determine the values of x, y and z.**

Ans. $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x+0+0 \\ 0+y+0 \\ 0+0+z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix}$$

$\Rightarrow x = 2, y = -3$ and $z = 0$.

17. Solve $\begin{bmatrix} 3 & -4 \\ 9 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$ **for x and y.**

Ans. $$\begin{bmatrix} 3 & -4 \\ 9 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 9 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 10 \\ 2 \end{bmatrix}$$

$$= \frac{1}{6+36}\begin{bmatrix} 2 & 4 \\ -9 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 2 \end{bmatrix}$$

$$= \frac{1}{42}\begin{bmatrix} 20+8 \\ -90+6 \end{bmatrix}$$

$$\Rightarrow x = \frac{28}{42} \text{ and } y = -\frac{84}{42}$$

$$\Rightarrow x = \frac{2}{3} \text{ and } y = -2$$

18. If $A = \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$ **and AX = B, then find the values of x and y.**

Ans. We have, $$A = \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}$$

$$\therefore \quad |A| = 6 - 16 = -10 \neq 0$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}\text{adj}(A)$$

$$= \frac{1}{-10}\begin{bmatrix} 3 & -4 \\ -4 & 2 \end{bmatrix}$$

Now, $AX = B$ [Given]

$$\Rightarrow \quad X = A^{-1}B$$

$$= -\frac{1}{10}\begin{bmatrix} 3 & -4 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 8 \\ 11 \end{bmatrix}$$

$$= -\frac{1}{10}\begin{bmatrix} 24-44 \\ -32+22 \end{bmatrix}$$

$$= -\frac{1}{10}\begin{bmatrix} -20 \\ -10 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Hence, $x = 2$ and $y = 1$.

19. For what values of k, the system of linear equations

$$x + y + z = 2$$
$$2x + y - z = 3$$
$$3x + 2y + kz = 4$$

has a unique solution? **[CBSE 2016]**

Ans. Given system of equations can be written in matrix form as

$$\begin{bmatrix}1 & 1 & 1\\2 & 1 & -1\\3 & 2 & k\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\3\\4\end{bmatrix} \quad ...(i)$$

Let $A = \begin{bmatrix}1 & 1 & 1\\2 & 1 & -1\\3 & 2 & k\end{bmatrix}$, $B = \begin{bmatrix}2\\3\\4\end{bmatrix}$ and $X = \begin{bmatrix}x\\y\\z\end{bmatrix}$

Then eq. (i) becomes

$$AX = B$$

The condition for unique solution is $|A| \neq 0$

$$\therefore \quad |A| = \begin{vmatrix}1 & 1 & 1\\2 & 1 & -1\\3 & 2 & k\end{vmatrix} \neq 0$$

$\Rightarrow 1(k+2) - 1(2k+3) + 1(4-3) \neq 0$

$\Rightarrow k + 2 - 2k - 3 + 1 \neq 0$

$\Rightarrow -k \neq 0 \Rightarrow k \neq 0$

Hence, for unique solution of the given system of equations, k should be non-zero real number.

SHORT ANSWER Type-I Questions (SA-I)

[**2** marks]

20. Show that the following system of equations is in consistent

$$6x + 4y = 3$$
$$9x + 6y = 3$$

Ans. Given system of equations can be written in matrix form as

$$\begin{bmatrix}6 & 4\\9 & 6\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}3\\3\end{bmatrix}$$

Consider $A = \begin{bmatrix}6 & 4\\9 & 6\end{bmatrix}$, $B = \begin{bmatrix}3\\3\end{bmatrix}$ and $X = \begin{bmatrix}x\\y\end{bmatrix}$

Then it becomes

$$AX = B$$

Now $\quad |A| = \begin{vmatrix}6 & 4\\9 & 6\end{vmatrix}$

$$= 36 - 36 = 0$$

and $\quad (\text{adj } A)\, B = \begin{bmatrix}6 & -4\\-9 & 6\end{bmatrix}\begin{bmatrix}3\\3\end{bmatrix}$

$$= \begin{bmatrix}18-12\\-27+18\end{bmatrix} = \begin{bmatrix}6\\-11\end{bmatrix}$$

Here, $|A| = 0$ and $(\text{adj } A)B \neq 0$, so the given system of questions is inconsistent.

21. Ⓐ**Use matrix method to examine the following system of equations for consistency or inconsistency.**

$4x - 2y = 3,\ 6x - 3y = 5$

22. Solve the system of equations by matrix method.

$5x + 2y = 3$ and $3x + 2y = 5$

Ans. Given system of equations can be written in matrix form is

$$\begin{bmatrix}5 & 2\\3 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}3\\5\end{bmatrix}$$

Consider $A = \begin{bmatrix}5 & 2\\3 & 2\end{bmatrix}$, $B = \begin{bmatrix}3\\5\end{bmatrix}$ and $X = \begin{bmatrix}x\\y\end{bmatrix}$

Then it becomes

$$AX = B$$

or, $\quad X = A^{-1}B$

Now, $\quad |A| = \begin{vmatrix}5 & 2\\3 & 2\end{vmatrix}$

$$= 10 - 6 = 4$$

Here $|A| \neq 0$, so A^{-1} exists.

$$\because \quad A^{-1} = \frac{1}{|A|}\text{adj}(A)$$

$$= \frac{1}{4}\begin{bmatrix}2 & -2\\-3 & 5\end{bmatrix}$$

Now, $\quad X = A^{-1}B$

$$= \frac{1}{4}\begin{bmatrix}2 & -2\\-3 & 5\end{bmatrix}\begin{bmatrix}3\\5\end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix}6-10\\-9+25\end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix}-4\\16\end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}-1\\4\end{bmatrix}$$

Hence, $x = -1$ and $y = 4$.

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

23. Solve the following system of equations by using matrix method

$5x - 7y - 2 = 0$ and $7x - 5y - 3 = 0$

Ans. We have,

$5x - 7y = 2$ and $7x - 5y = 3$

Given system of equations can be written in matrix form as

$$\begin{bmatrix} 5 & -7 \\ 7 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Consider, $A = \begin{bmatrix} 5 & -7 \\ 7 & -5 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \end{bmatrix}$

Then it becomes

$$AX = B$$

or $$X = A^{-1}B$$

Now $$|A| = \begin{vmatrix} 5 & -7 \\ 7 & -5 \end{vmatrix}$$

$$= -25 + 49 = 24$$

Here, $|A| \neq 0$, so A^{-1} exists.

Now, cofactors of all elements of determinant A are

$$A_{11} = +(-5) = -5,$$
$$A_{12} = -(7) = -7,$$
$$A_{21} = -(-7) = 7$$

and $$A_{22} = +(5) = 5$$

$\because$ $$\text{adj}(A) = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} -5 & 7 \\ -7 & 5 \end{bmatrix}$$

$\therefore$ $$A^{-1} = \frac{1}{|A|}\text{adj}(A)$$

$$= \frac{1}{24}\begin{bmatrix} -5 & 7 \\ -7 & 5 \end{bmatrix}$$

Now, $$X = A^{-1}B$$

$$= \frac{1}{24}\begin{bmatrix} -5 & 7 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$= \frac{1}{24}\begin{bmatrix} -10+21 \\ -14+15 \end{bmatrix}$$

$$= \frac{1}{24}\begin{bmatrix} 11 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{11}{24} \\ \frac{1}{24} \end{bmatrix}$$

Hence, $x = \frac{11}{24}$ and $y = \frac{1}{24}$

24. Use matrix method to solve the following system of equations

$x - 2y = 4$ and $3x - 5y = 7$

25. Solve the following system of linear equations

$$x - y + 2z = 7$$
$$3x + 4y - 5z = -5$$

and $$2x - y + 3z = 12$$ **[CBSE 2012]**

26. Using matrix method, solve the system of equations $3x + 2y - 2z = 3$, $x + 2y + 3z = 6$, $2x - y + z = 2$. [NCERT Exemplar]

27. The monthly incomes of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5 : 7. If each saves ₹ 15,000 per month, then find their monthly incomes using matrices.

Ans. Let the monthly income of Aryan be ₹ $3x$ and that of Babban be ₹ $4x$.

Also, let monthly expenditure of Aryan be ₹ $5y$ and that of Babban be ₹ $7y$.

According to given problem, we have

$$3x - 5y = ₹\,15000$$
$$4x - 7y = ₹\,15000$$

These equations can be written in matrix form as

$$\begin{bmatrix} 3 & -5 \\ 4 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$$

Consider, $A = \begin{bmatrix} 3 & -5 \\ 4 & -7 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$,

$B = \begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$, then it becomes

$$AX = B$$

Now $$|A| = \begin{vmatrix} 3 & -5 \\ 4 & -7 \end{vmatrix} = -21 + 20 = -1$$

Here $|A| \neq 0$, so A^{-1} exists. Thus, by system of linear equations, it has a unique solution and it is given by $X = A^{-1}B$.

Now, $\text{adj}(A) = \begin{bmatrix} -7 & 5 \\ -4 & 3 \end{bmatrix}$

$\therefore$ $$A^{-1} = \frac{\text{adj}(A)}{|A|} = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}$$

Now, $X = A^{-1}B = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 30000 \\ 15000 \end{bmatrix}$

$\Rightarrow x = 30000$ and $y = 15000$

Hence, monthly income of Aryan

$= 3 \times 30000$

$=$ ₹ 90,000

and Monthly income of Babban

$= 4 \times 30000 =$ ₹ 1,20,000

LONG ANSWER Type Questions (LA)

[**4** & **5** marks]

28. If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}.

Using A^{-1}, solve the system of linear equations $x - 2y = 10$, $2x - y - z = 8$, $-2y + z = 7$.

[NCERT Exemplar]

Ans. We have, $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$

$\therefore \quad |A| = 1(-1-2) - 2(-2-0) + 0$

$= -3 + 4$

$= 1 \neq 0$

$\therefore A^{-1}$ exist.

Now, cofactors of A are

$A_{11} = (-1)^{1+1}(-1-2)$
$= -3$

$A_{12} = (-1)^{1+2}(-2-0)$
$= 2$

$A_{13} = (-1)^{1+3}(2-0)$
$= 2$

$A_{21} = (-1)^{2+1}(2-0)$
$= -2$

$A_{22} = (-1)^{2+2}(1-0)$
$= 1$

$A_{23} = (-1)^{2+3}(-1-0)$
$= 1$

$A_{31} = (-1)^{3+1}(-4+0)$
$= -4$

$A_{32} = (-1)^{3+2}(-2-0)$
$= 2$

$A_{33} = (-1)^{3+3}(-1+4)$
$= 3$

$\therefore \quad \text{adj. } A = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}^T$

$= \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

$\therefore \quad A^{-1} = \frac{1}{|A|} \text{adj. } A$

$= \frac{1}{1}\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

$\Rightarrow \quad A^{-1} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

Now, the system of equations is:

$x - 2y = 10$
$2x - y - z = 8$
$-2y + z = 7$

which is of the form $CX = D$.

where, $C = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $D = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

Here, $C = A^T$

Now, $CX = D$

$\Rightarrow \quad C^{-1}CX = C^{-1}D$

$\Rightarrow \quad IX = C^{-1}D$

$\Rightarrow \quad X = [A^T]^{-1}D$

$\Rightarrow \quad X = [A^{-1}]^T D \quad [\because (A^T)^{-1} = (A^{-1})^T]$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -30+16+14 \\ -20+8+7 \\ -40+16+21 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$

Hence, $x = 0$, $y = -5$, $z = -3$.

⚠ **Caution**

➥ *Compare the given system of equations with the given matrix, so it becomes easy to solve them.*

29. Determine the product of $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, and then use this to solve the system of equations

$$x - y + z = 4,$$
$$x - 2y - 2z = 9$$
and $$2x + y + 3z = 1$$ **[CBSE 2017]**

Ans. Let $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

Then,

$$AB = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow AB = 8I$

On post-multiplying both sides by B^{-1}, we get

$$ABB^{-1} = 8IB^{-1}$$
$$AI = 8B^{-1}$$

$$\Rightarrow \quad B^{-1} = \frac{1}{8}A$$

$$= \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

Now, we have to find the solution of given system of equations.

Here, we write the given system of equations in matrix form as

$$BX = C \quad \text{...(i)}$$

where $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and

$C = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$

On pre-multiplying Eq. (i) by B^{-1}, we get

$$B^{-1}BX = B^{-1}C$$
$$IX = B^{-1}C$$
$$\Rightarrow \quad X = B^{-1}C$$

$$\Rightarrow \quad X = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$X = \frac{1}{8}\begin{bmatrix} -16 & +36 & +4 \\ -28 & +9 & +3 \\ 20 & -27 & -1 \end{bmatrix}$$

$$X = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

On comparing the corresponding elements, we get

$x = 3, y = -2, z = -1$

30. Given $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, find BA and use this to solve the system of equations $x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 7$. **[NCERT Exemplar]**

31. Using matrix method, solve the following system of equation $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$, $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$, $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$, $x, y, z \neq 0$.

[NCERT]

Ans. We have system of equations as

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{2} = 1, \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

Given equations written in matrix form as

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}\begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

Consider $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$, $X = \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix}$

and $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

Then it becomes

$$AX = B$$

Now $|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$

$= 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$

$= 150 + 330 + 720 = 1200$

Here $|A| \neq 0$, so A^{-1} exists. Thus by system of equations it has a unique solution and it is $X = A^{-1}B$.

Cofactors of elements of A are

$A_{11} = 75$, $A_{12} = 110$, $A_{13} = 72$, $A_{21} = 150$, $A_{22} = -100$, $A_{23} = 0$, $A_{31} = 75$, $A_{32} = 30$, $A_{33} = -24$.

$\therefore$ adj $A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

$= \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

We know,

$$A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$= \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/5 \end{bmatrix}$$

$\therefore \quad \frac{1}{x} = \frac{1}{2} \Rightarrow x = 2$

$\frac{1}{y} = \frac{1}{3} \Rightarrow y = 3$

$\frac{1}{z} = \frac{1}{5} \Rightarrow z = 5$

32. **If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find A^{-1} and use it to solve the following systems of the equations:**

$$x + 2y - 3z = 6$$
$$3x + 2y - 2z = 3$$
$$3x + 2y - z = 5$$

[CBSE 2020]

33. A shopkeeper has 3 varieties of pens 'A', 'B' and 'C'. Meena purchased 1 pen of each variety for a total of ₹ 21. Jeevan purchased 4 pens of 'A' variety, 3 pens of 'B' variety and 3 pens of 'C' variety for ₹ 60, while Shikha purchased 6 pens of 'A' variety 2 pens of 'B' variety and 3 pens of 'C' variety for ₹ 70. Using matrix method, find the cost of each variety of pen. [CBSE 2016]

Ans. Let one pen of variety 'A' costs ₹ x, one pen of variety 'B' costs ₹ y and one pen of variety 'C' costs ₹ z.

According to given problem

$$x + y + z = ₹\,21$$
$$4x + 3y + 2z = ₹\,60$$
and $$6x + 2y + 3z = ₹\,70$$

The given system of equations can be written in matric form:

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$$

Consider $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$.

Then, it becomes

$$AX = B$$

Now, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{vmatrix}$

$= 1(9 - 4) - 1(12 - 12) + 1(8 - 18) = -5$

Here $|A| \neq 0$, so A^{-1} exists. Thus, by system of equations it has a unique solution and it is given by $X = A^{-1}B$.

Now, cofactors of a_{11} elements of A are

$A_{11} = 5$, $A_{12} = 0$, $A_{13} = -10$, $A_{21} = -1$, $A_{22} = -3$, $A_{23} = 4$, $A_{31} = -1$, $A_{32} = 2$, $A_{33} = -1$

$\because \quad$ adj $A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

$= \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$

$\therefore \quad A^{-1} = \frac{1}{|A|}(\text{adj } A)$

$$= \frac{1}{(-5)}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$

Consider $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{(-5)}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}\begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{(-5)}\begin{bmatrix} -25 \\ -40 \\ -40 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix} \Rightarrow x = 5, y = 8, z = 8$$

Hence, cost of 1 pen of variety 'A' is ₹ 5

Cost of 1 pen of variety 'B' is ₹ 8

Cost 1 pen of variety 'C' is ₹ 8

34. Ⓐ**An amount of ₹ 5000 is put into three investments at the rate of interest of 6%, 7% and 8% per annum respectively. The total annual interest is ₹ 358. If the combined income from the first two investments is ₹ 70 more than the income from the third, find the amount of each investment by matrix method.**

TOPPER'S CORNER

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

1. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, find the value of x.

Ans.

$12x + 14 = 32 - 42$

$12x + 14 = -10$

[CBSE Topper 2014]

2. If for any 2 × 2 square matrix A, A(adj A) = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of |A|.

Ans.

$A(\text{adj} A) = |A| I_n$

$A(\text{adj} A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

$|A| \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 8 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$|A| = 8$

[CBSE Topper 2017]

3. If A is a skew-symmetric matrix of order 3, then prove that det A = 0.

Ans.

A is skew symmetric

$A = -A^T$

$|A| = (-1)^3 |A^T|$

$|A| = -|A|$ $\quad [\because |A| = |A^T|]$

$2|A| = 0$

$|A| = 0$

det $A = 0$

[CBSE Topper 2016]

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

4. Using properties of determinants, prove that

$$\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix} = 9\,(3xyz + xy + yz + zx)$$

Ans.

$\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix}$

$C_1 \to C_1 - C_2$

$$\begin{vmatrix} 0 & 1 & 1+3x \\ 3y & 1 & 1 \\ -3x & 1+3x & 1 \end{vmatrix}$$

taking 3 common from C_1

$$3\begin{vmatrix} 0 & 1 & 1+3x \\ y & 1 & 1 \\ -x & 1+3x & 1 \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$$3\begin{vmatrix} 0 & 1 & 1+3x \\ y & 0 & -3x \\ -x & 1+3x & 1 \end{vmatrix}$$

Expanding along R_1

$3[-1(y - 3xz) + (1+3x)(y+3xy)]$

$3[-y + 3xz + y + 3xy + 3xy + 9xyz]$

$3[9xyz + 3xz + 3xy + 3xy)$

$9(3xyz + xy + yz + xz)$ Ans.

[CBSE Topper 2018]

5. Using the properties of determinants, solve the following for x.

$$\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$$

Ans.

$$\Delta = \begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\Delta = \begin{vmatrix} 3x+7 & 3x+7 & 3x+7 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix}$$

$$\Delta = 3x+7\begin{vmatrix} 1 & 1 & 1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix}$$

$C_1 \to C_1 - C_2 \quad C_2 \to C_2 - C_3$

$$\Delta = (3x+7)\begin{vmatrix} 0 & 0 & 1 \\ 7 & -3 & x+2 \\ -3 & -4 & x+6 \end{vmatrix}$$

expanding by R_1

$\Delta = (3x+7)[-28 - 9]$

$\Delta = 0$

$\Rightarrow 3x+7 = 0$

$x = -\frac{7}{3}$

[CBSE Topper 2015]

LONG ANSWER Type Questions **(LA)**

[**4** & **5** marks]

6. Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school A wants to award ₹ x each, ₹ y each and ₹ z each for the three reswpective values to 3, 2 and 1 students respectively with a total award money of ₹ 1,600. School B wants to spend ₹ 2,300 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value is ₹ 900, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for award.

Ans. Let The amounts awarded for sincerity, truthfulness and helpfulness are Rs x, Rs y and Rs z respectively. [awarded by schools A and B]

The equations are :

$$3x + 2y + z = 1600$$
$$4x + y + 3z = 2300$$
$$x + y + z = 900$$

$$\underbrace{\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}_{3\times3}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times1}}_{X} = \underbrace{\begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}_{3\times1}}_{B}$$

$\therefore X = A^{-1}B$

$$|A| = 3(1-3) - 2(4-3) + 1(4-1)$$
$$= 3(-2) - 2 + 3$$
$$= -6 - 2 + 3$$
$$= -8 + 3$$
$$= -5$$

$|A| \neq 0 \quad \therefore A^{-1}$ exists.

$\therefore X = A^{-1}B$

$$A^{-1} = \frac{adj(A)}{|A|}$$

Let A_{ij} represent the cofactors of elements in A.

$A_{11} = -2$

$A_{12} = -1$

$A_{13} = 3$

$A_{21} = -1$ $\quad \therefore adj(A) =$ (transpose of co-factor matrix)

$A_{22} = 2$

$A_{23} = -1$

$A_{31} = 5$

$A_{32} = -5$ $\quad \therefore adj(A) =$ (co-factor matrix)'

$A_{33} = -5$

7. $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$

Ans. TP $\Delta : \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ac \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$

Proof $\quad C_1 \to C_1 - 2C_3$

$\Delta = \begin{vmatrix} b^2+c^2 & a^2 & bc \\ c^2+a^2 & b^2 & ac \\ a^2+b^2 & c^2 & ab \end{vmatrix}$

$C_1 \to C_1 + C_2$

Now taking $a^2+b^2+c^2$ common from C_1 :-

$\Delta = (a^2+b^2+c^2) \begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ac \\ 1 & c^2 & ab \end{vmatrix}$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$\Delta = (a^2+b^2+c^2) \begin{vmatrix} 1 & a^2 & bc \\ 0 & b^2-a^2 & ac-bc \\ 0 & c^2-a^2 & ab-bc \end{vmatrix}$

Taking $(a-b)$ common from R_2 & $(c-a)$ common from C_3

$\Delta = (a^2+b^2+c^2)(a-b)(c-a) \begin{vmatrix} 1 & a^2 & bc \\ 0 & -(a+b) & c \\ 0 & a+c & -b \end{vmatrix}$

Expanding along C_1

$\Delta = (a^2+b^2+c^2)(a-b)(c-a)\left(-(a+b)\cdot b + (a+c)c\right)$

$= (a^2+b^2+c^2)(a-b)(c-a)(b^2 - c^2 + bc - ba)$

$= (a^2+b^2+c^2)(a-b)(b-c)(c-a)(a+b+c)$ H.P.

As $(b-c)(a+b+c) = ab + b^2 + bc - ac - bc - c^2$
$= b^2 - c^2 + ab - ac$

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