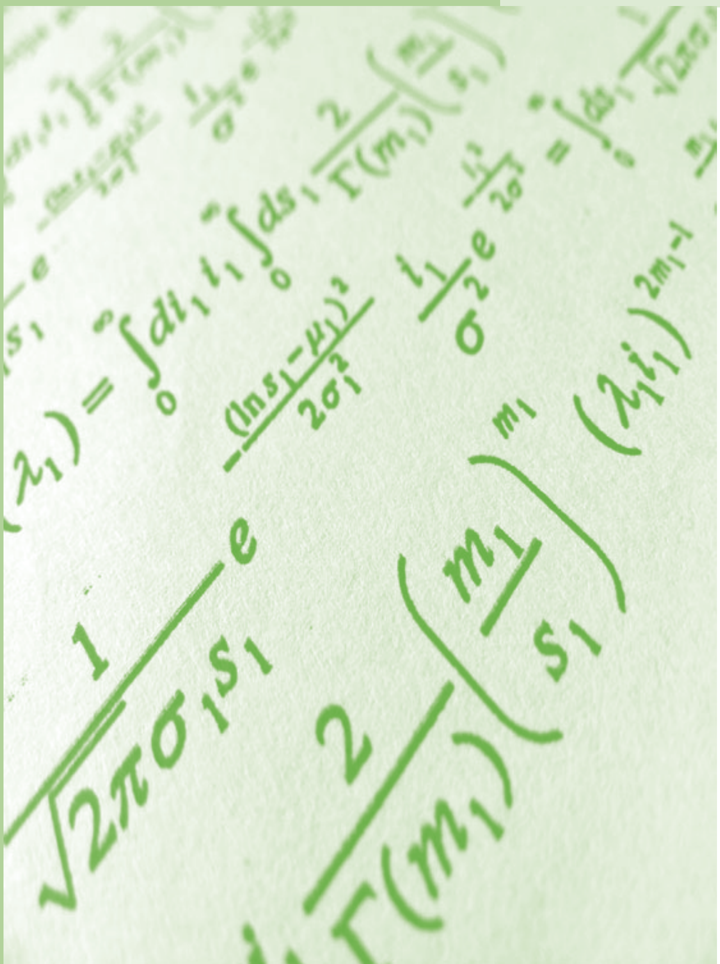
Chapter 9

Limits



REMEMBER

Before beginning this chapter, you should be able to:

- State the term function
- Apply basic operations on functions

KEY IDEAS

After completing this chapter, you would be able to:

- Identify limit of a function
- Understand meaning of symbols used in limits
- Study evaluation of limits by different methods
- Obtain limits of a function using a formula

INTRODUCTION

Limit of a Function

Let $y = f(x)$ be a function of x and let 'a' be any real number.

We first understand what a 'limit' is. A limit is the value, a function approaches, as the independent variable of the function (usually 'x') gets nearer and nearer to a particular value. In other words, when x is very close to a certain number, say a, what is $f(x)$ very close to? It may be equal to $f(a)$ but may be different. It may exist even when $f(a)$ is not defined.

Meaning of '$x \to a$'

Let x be a variable and 'a' be a constant. If x assumes values nearer and nearer to 'a', then we say that 'x tends to a' or 'x approaches a' and is written as '$x \longrightarrow a$'. By $x \longrightarrow a$, we mean that $x \neq a$ and x may approach 'a' from left or right, which is explained in the example given below.

What is the limit of the function $f(x) = x^2$ as x approaches 3? The expression 'the limit as x approaches to 3' is written as: $\lim\limits_{x \to 3}$. Let us check out some values of $\lim\limits_{x \to 3}$, as x increases and gets closer to 3, without ever exactly getting three.

When $x = 2.9$, $f(x) = 8.4100$

When $x = 2.99$, $f(x) = 8.9401$

When $x = 2.999$, $f(x) = 8.9940$

When $x = 2.9999$, $f(x) = 8.9994$

As x increases and approaches 3, $f(x)$ gets closer and closer to 9 and since x tends to 3 from the left this is called the 'left-hand limit' and is written as $\lim\limits_{x \to 3^-}$.

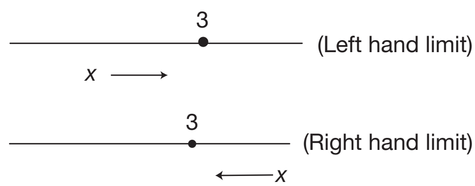
Now, let us see what happens when x is greater than 3.

When $x = 3.1$, $f(x) = 9.6100$

When $x = 3.01$, $f(x) = 9.0601$

When $x = 3.001$, $f(x) = 9.0060$

When $x = 3.0001$, $f(x) = 9.0006$



As x decreases and approaches 3, $f(x)$ still approaches 9. This is called the 'right-hand limit' and is written as $\lim\limits_{x \to 3^+}$.

As $\lim\limits_{x \to 3^-} f(x) \lim\limits_{x \to 3^+} f(x) = 9$, we write that $\lim\limits_{x \to 3} x^2 = 9$.

Meaning of the Symbol: $\lim\limits_{x \to a} f(x) = l$

Let $f(x)$ be a function of x where x takes values closer and closer to 'a' ($\neq a$), and let $f(x)$ assume values nearer and nearer to l.

We say, $f(x)$ tends to the limit 'l' as x tends to 'a'.

The following are some simple algebraic rules of limits.

1. $\lim\limits_{x \to a} k\, f(x) = k \lim\limits_{x \to a} f(x)$

2. $\lim\limits_{x \to a} [f(x) \pm g(x)] = \lim\limits_{x \to a} f(x) \pm \lim\limits_{x \to a} g(x)$

3. $\lim_{x\to a}[f(x)\cdot g(x)] = \lim_{x\to a} f(x)\cdot \lim_{x\to a} g(x)$

4. $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ (where $\lim_{x\to a} g(x) \neq 0$)

Notes

1. If the left hand limit of a function is not equal to the right hand limit of the function, then the limit does not exist.
2. A limit equal to infinity does not imply that the limit does not exist.

Example: The limit of a chord of a circle passing through a fixed point Q and a variable point P as P approaches Q is the tangent to the circle at P.

If PQ is chord of a circle, when P approaches Q along the circle, then the chord becomes the tangent to the circle at P.

Problems: While evaluating the limits we use the following methods.

1. Method of substitution
2. Method of factorization
3. Method of rationalization
4. Using the formula $\lim_{x\to a}\left[\dfrac{x^n - a^n}{x - a}\right] = na^{n-1}$

Method of Substitution

In this method we directly substitute the value of x in the given function to obtain the limit value.

Examples:

1. $\lim_{x\to 2}[x^2 + 5x + 6] = (2)^2 + 5(2) + 6 = 20.$

2. $\lim_{x\to 3}\left[\dfrac{x^2 + 4x + 1}{x + 5}\right] = \dfrac{(3)^2 + 4(3) + 1}{3 + 5} = \dfrac{22}{8} = \dfrac{11}{4}.$

Indeterminate Form If $f(x) = \dfrac{x^2 - 9}{x - 3}$, then $f(3) = \dfrac{3^2 - 9}{3 - 3} = \dfrac{0}{0}$, which is not defined. By substituting $x = 3$, the function assumes a form whose value can’t be determined this form is called an indeterminate form.

Method of Factorization

If $\dfrac{f(x)}{g(x)}$ assumes an indeterminate form when $x = a$, then there exists a common factor for $f(x)$ and $g(x)$.

We remove the common factors, and use the substitution method to find the limit.

EXAMPLE 9.1

Evaluate $\lim_{x\to 2}\left[\frac{x^2-5x+6}{x^2-3x+2}\right]$.

SOLUTION

When $x = 2$, $\frac{4-10+6}{4-6+2} = \frac{0}{0}$ an indeterminate form.

Now, $x^2 - 5x + 6 = (x - 3)(x - 2)$

$x^2 - 3x + 2 = (x - 1)(x - 2)$

$$\lim_{x\to 2}\left[\frac{x^2-5x+6}{x^2-3x+2}\right] = \lim_{x\to 2}\left[\frac{(x-3)(x-2)}{(x-1)(x-2)}\right]$$

$$\lim_{x\to 2}\left[\frac{x-3}{x-1}\right] = \left[\frac{2-3}{2-1}\right] = -1.$$

Method of Rationalization

If $\frac{f(x)}{g(x)}$ assumes an indeterminate form when $x = a$ and $f(x)$ or $g(x)$ are irrational functions, then we rationalize $f(x)$, $g(x)$ and cancel the common factor, then use the substitution method to find the limit.

EXAMPLE 9.2

Evaluate $\lim_{x\to 0}\left[\frac{x}{1-\sqrt{1-x}}\right]$.

SOLUTION

When $x = 0$, the given function assumes the form $\frac{0}{1-\sqrt{1-0}} = \frac{0}{0}$, an indeterminate form.

Since $g(x)$, the denominator is an irrational form, we multiply the numerator and denominator with the rationalizing factor of $g(x)$.

$$\lim_{x\to 0}\left[\frac{x}{1-\sqrt{1-x}}\right] = \lim_{x\to 0}\frac{x}{\left(1-\sqrt{1-x}\right)}\times\frac{\left(1+\sqrt{1-x}\right)}{\left(1+\sqrt{1-x}\right)}$$

$$= \lim_{x\to 0}\frac{x\left(1+\sqrt{1-x}\right)}{1-1+x}$$

$$= \lim_{x\to 0}\frac{x\left(1+\sqrt{1-x}\right)}{x}$$

$$= \lim_{x\to 0}\left(1+\sqrt{1-x}\right) = 1+\sqrt{1} = 2.$$

EXAMPLE 9.3

Evaluate $\lim_{x\to3}\frac{\left(3-\sqrt{6+x}\right)}{x-3}$.

SOLUTION

When $x=3$, $\frac{\left(3-\sqrt{6+3}\right)}{3-3}=\frac{0}{0}$, an indeterminate form. As $f(x)$ is irrational, we multiply the numerator and denominator with the rationalizing factor of $f(x)$.

$$\lim_{x\to3}\frac{\left(3-\sqrt{6+x}\right)}{x-3}=\lim_{x\to3}\frac{\left(3-\sqrt{6+x}\right)}{x-3}\times\frac{\left(3+\sqrt{6+x}\right)}{\left(3+\sqrt{6+x}\right)}=\lim_{x\to3}\frac{9-(6+x)}{(x-3)\left(3+\sqrt{6+x}\right)}$$

$$=\lim_{x\to3}\frac{3-x}{(x-3)\left(3+\sqrt{6+x}\right)}=\lim_{x\to3}\frac{-1}{\left(3+\sqrt{6+x}\right)}=\frac{-1}{\left(3+\sqrt{6+3}\right)}$$

$$=\frac{-1}{(3+3)}=-\frac{1}{6}.$$

EXAMPLE 9.4

Evaluate $\lim_{x\to0}\frac{\sqrt{4+x}-\sqrt{4-x}}{\sqrt{9+x}-\sqrt{9-x}}$.

SOLUTION

When $x=0$, $\frac{\sqrt{4}-\sqrt{4}}{\sqrt{9}-\sqrt{9}}=\frac{0}{0}$, an indeterminate form.

Here $f(x)$ and $g(x)$ both are irrational functions. We multiply both the numerator and the denominator with their rationalizing factors.

$$\lim_{x\to0}\frac{\sqrt{4+x}-\sqrt{4-x}}{\sqrt{9+x}-\sqrt{9-x}}\times\frac{\sqrt{4+x}+\sqrt{4-x}}{\sqrt{9+x}+\sqrt{9-x}}\times\frac{\sqrt{9+x}-\sqrt{9-x}}{\sqrt{4+x}+\sqrt{4-x}}$$

$$\lim_{x\to0}\frac{(4+x)-(4-x)}{(9+x)-(9-x)}\times\frac{\sqrt{9+x}+\sqrt{9-x}}{\sqrt{4+x}+\sqrt{4-x}}=\lim_{x\to0}\frac{2x}{2x}\times\frac{\sqrt{9+x}+\sqrt{9-x}}{\sqrt{4+x}+\sqrt{4-x}}$$

$$=\lim_{x\to0}\frac{\sqrt{9+x}+\sqrt{9-x}}{\sqrt{4+x}+\sqrt{4-x}}=\frac{\sqrt{9}+\sqrt{9}}{\sqrt{4}+\sqrt{4}}=\frac{2\sqrt{9}}{2\sqrt{4}}=\frac{3}{2}.$$

Using the Formula

1. $\lim_{x\to a}\left[\frac{x^n-a^n}{x-a}\right]=na^{n-1}$ (where 'n' is a any rational number).

Proof:

Case 1: n is a positive integer.

$$\lim_{x\to a}\left[\frac{x^n-a^n}{x-a}\right]=\lim_{x\to a}\left[\frac{(x-a)(x^{n-1}+x^{n-2}a+x^{n-3}a^2+\cdots+a^{n-1})}{x-a}\right]$$

$$=\lim_{x\to a}[x^{n-1}+x^{n-2}a+x^{n-3}a^2+\cdots+a^{n-1}]$$

$$=a^{n-1}+a^{n-2}\ a+a^{n-3}\ a^2+\cdots+a^{n-1}(n\text{ terms})$$

$$=a^{n-1}+a^{n-1}+a^{n-1}+\cdots+a^{n-1}(n\text{ terms})=n\cdot a^{n-1}.$$

Case 2: n is negative integer.

Let $n=-m$ (m is positive integer)

$$\lim_{x\to a}\left[\frac{x^n-a^n}{x-a}\right]=\lim_{x\to a}\left[\frac{x^{-m}-a^{-m}}{x-a}\right]$$

$$=\lim_{x\to a}\frac{\frac{1}{x^m}-\frac{1}{a^m}}{x-a}=\lim_{x\to a}\frac{\frac{a^m-x^m}{x^ma^m}}{x-a}=\lim_{x\to a}\frac{-(x^m-a^m)}{(x-a)x^ma^m}=-\lim_{x\to a}\frac{x^m-a^m}{x-a}\times\lim_{x\to a}\frac{1}{x^ma^m}$$

$$=-ma^{m-1}\frac{1}{a^ma^m}\ (\because\ m\text{ is positive interget using case (1)})$$

$$=-m\frac{a^{m-1}}{a^{2m}}=-ma^{m-1-2m}=(-m)a^{-m-1}$$

$$\Rightarrow\quad \lim_{x\to a}\left[\frac{x^n-a^n}{x-a}\right]=na^{n-1}.\quad(\because\ n=-m)$$

Case 3: n is a rational number.

Let $n=\frac{p}{q}$ where p, q are integers and $q\neq 0$.

$$\lim_{x\to a}\left[\frac{x^n-a^n}{x-a}\right]=\lim_{x\to a}\left[\frac{x^{\frac{p}{q}}-a^{\frac{p}{q}}}{x-a}\right]=\lim_{x\to a}\left[\frac{\left(x^{\frac{1}{q}}\right)^p-\left(a^{\frac{1}{q}}\right)^p}{x-a}\right]$$

Let $x^{\frac{1}{q}}=y$ then $x=y^q$ and $a^{\frac{1}{q}}=b$ then $a=b^q$.

$x\to a\quad\Rightarrow\quad y^q\to b^q\quad\Rightarrow\quad y\to b.$

$$\lim_{x\to a}\left[\frac{(x^{\frac{1}{q}})^p-(a^{\frac{1}{q}})^p}{x-a}\right]=\lim_{x\to a}\left[\frac{y^p-b^p}{y^q-b^q}\right]$$

$$=\lim_{x\to b}\frac{\frac{y^p-b^p}{y-b}}{\frac{y^q-b^q}{y-b}}=\frac{\lim\limits_{x\to b}\frac{y^p-b^p}{y-b}}{\lim\limits_{x\to b}\frac{y^q-b^q}{y-b}}=\frac{p\cdot b^{p-1}}{q\cdot b^{q-1}}\quad(\because\ p,q\text{ are integers}).$$

$$=\left(\frac{p}{q}\right)b^{p-1-(q-1)}=\left(\frac{p}{q}\right)b^{p-q}=\left(\frac{p}{q}\right)b^{q\left(\frac{p}{q}-1\right)}=\left(\frac{p}{q}\right)(b^q)^{\left(\frac{p}{q}-1\right)}$$

$$\lim_{x\to a}\left[\frac{x^n-a^n}{x-a}\right]=na^{n-1}.\quad\left(\because\ n=\frac{p}{q};\,b^q=a\right)$$

Hence when n is rational number,

$$\lim_{x\to a}\left[\frac{x^n - a^n}{x - a}\right] = na^{n-1}.$$

Note $\lim_{x\to a}\left[\frac{x^m - a^m}{x^n - a^n}\right] = \frac{m}{n} a^{m-n}.$

EXAMPLE 9.5

Evaluate $\lim_{x\to 4}\left[\frac{x^4 - 256}{x - 4}\right]$.

SOLUTION

$$\lim_{x\to 4}\left[\frac{x^4 - 256}{x - 4}\right]$$

$$= \lim_{x\to 4}\left[\frac{x^4 - 4^4}{x - 4}\right]$$

$$= 4 \times 4^{4-1} = 4 \times 4^3 = 256.$$

EXAMPLE 9.6

Evaluate $\lim_{x\to 3}\left[\frac{x^5 - 243}{x^2 - 9}\right]$.

SOLUTION

$$\lim_{x\to 3}\left[\frac{x^5 - 243}{x^2 - 9}\right] = \lim_{x\to 3}\left[\frac{x^5 - 3^5}{x^2 - 3^2}\right]$$

$$= \frac{5}{2} 3^{5-2} \left(\because \lim_{x\to a}\left[\frac{x^m - a^m}{x^n - a^n}\right] = \frac{m}{n} a^{m-n}\right) = \frac{5}{2} \cdot 3^3 = \frac{135}{2}.$$

EXAMPLE 9.7

If $\lim_{x\to 2}\left[\frac{x^n - 2^n}{x - 2}\right] = 32$, then find the value of n.

SOLUTION

$$\lim_{x\to 2}\left[\frac{x^n - 2^n}{x - 2}\right] = n \cdot 2^{n-1}$$

$$32 = n \cdot 2^{n-1}$$

$$4 \times 8 = n \cdot 2^{n-1}$$

$$4 \times 2^3 = n \cdot 2^{n-1}$$

$$4 \times 2^{4-1} = n \times 2^{n-1}.$$

$\therefore n = 4.$

Limits as x Tends to Infinity

We know, when x tends to infinity $\frac{1}{x}$ tends to 0. While evaluating limits at infinity put $x = \frac{1}{y}$.

EXAMPLE 9.8

Evaluate $\lim_{x \to \infty} \frac{2x+3}{x-5}$.

SOLUTION

Put $x = \frac{1}{y}$; if $x \to \infty$, $y \to 0$

$$\therefore \quad \lim_{x \to \infty} \frac{2x+3}{x-5} \quad \Rightarrow \quad \lim_{y \to 0} \frac{\frac{2}{y}+3}{\frac{1}{y}-5} = \lim_{y \to 0} \frac{2+3y}{1-5y} = \frac{2+0}{1-0} = 2.$$

EXAMPLE 9.9

Evaluate $\lim_{x \to \infty} \frac{3x^2+4x+5}{4x^2+7}$.

SOLUTION

Put $x = \frac{1}{y}$, when $x \to \infty$, $y \to 0$

$$\lim_{x \to \infty} \frac{3x^2+4x+5}{4x^2+7} = \lim_{y \to 0} \frac{3 \cdot \frac{1}{y^2} + 4 \cdot \frac{1}{y} + 5}{4 \cdot \frac{1}{y^2} + 7}$$

$$= \lim_{y \to 0} \frac{3+4y+5y^2}{4+7y^2} = \frac{3+0+0}{4+7(0)} = \frac{3}{4}.$$

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. When the number of sides of a polygon tends to infinity, it approaches _______.
2. The scientists who put calculus in mathematical form are _______ and _______.
3. The radius of circle tends to zero then it approaches a _______.
4. _______ helps in finding the areas of the curves.
5. Evaluate $\lim_{x\to 2}(2x-2)$.
6. The limiting position of a secant is _______.
7. $\lim_{x\to a} x^n + ax^{n-1} + a^2x^{n-2} + \cdots + a^n =$ _______.
8. $\lim_{x\to 3} \log(4x-11) =$ _______.
9. A circle is inscribed in a polygon. As the number of sides increases, the difference in areas of circle and polygon _______.
10. $\lim_{x\to 0} \dfrac{x^2+8x}{x} =$ _______.
11. $\lim_{x\to\infty} \dfrac{n(n+1)}{n^2} =$ _______.
12. $\lim_{x\to\infty} \dfrac{1}{n} =$ _______.
13. $\lim_{x\to 1} \dfrac{\sqrt[5]{x}-1}{\sqrt[4]{x}-1} =$ _______.
14. $\lim_{x\to -a} \dfrac{x^n+a^n}{x+a}$ (where n is an odd natural number) $=$ _______.
15. $\lim_{x\to 0^-} \dfrac{|x|}{x} =$ _______.
16. $\lim_{x\to 3} \sqrt{9-x^2} =$ _______.
17. Evaluate: $\lim_{x\to 3} \dfrac{|x-3|}{x-3}$.
18. Evaluate: $\lim_{x\to a} \left[\dfrac{x^n-a^n}{x^m-a^m}\right]$.
19. $\lim_{x\to 5} \dfrac{\sqrt{x}-\sqrt{5}}{x-5} =$ _______.
20. If $\lim_{x\to 0} \left[\dfrac{2x^2+3x+b}{x^2+4x+3}\right] = 2$, then the value of b is _______.
21. What is the value of $\lim_{x\to 1}(x^3+1)(x^2-2x+4)$?
22. Evaluate: $\lim_{x\to 3} \left[\dfrac{x^3-27}{x-3}\right]$.
23. Evaluate: $\lim_{x\to\infty} \left[\dfrac{x^2+x+6}{x+1}\right]$.
24. Evaluate: $\lim_{x\to\infty} \left[\dfrac{4x^n+1}{5x^{n+1}+5}\right]$.
25. Evaluate: $\lim_{x\to a} \dfrac{x^{\frac{1}{4}}-a^{\frac{1}{4}}}{x^4-a^4}$.
26. Evaluate: $\lim_{x\to -2} \dfrac{x^7+128}{x+2}$.
27. Evaluate: $\lim_{x\to 1} \left[\dfrac{x^2+3x+2}{x^2-5x+3}\right]$.
28. Evaluate: $\lim_{x\to 5} \sqrt{25-x^2}$.
29. In finding $\lim_{x\to a} f(x)$, we replace x by $\dfrac{1}{n}$ then the limit becomes _______.
30. $\lim_{x\to\infty} \dfrac{7x-3}{8x-10} =$ _______.

Short Answer Type Questions

31. Evaluate: $\lim_{x\to\infty} \dfrac{11|x|+7}{8|x|-9}$.
32. Evaluate: $\lim_{x\to 2} \left[\dfrac{2x^2-9x+10}{5x^2-5x-10}\right]$.
33. Evaluate: $\lim_{x\to 3} \dfrac{x^5-243}{x-3}$.
34. Evaluate: $\lim_{x\to\infty} \dfrac{x^5+3x^4-4x^3-3x^2+2x+1}{2x^5+4x^2-9x+16}$.

35. Evaluate: $\lim\limits_{x\to a}\dfrac{x^{14}-a^{14}}{x^{-7}-a^{-7}}$.

36. Evaluate: $\lim\limits_{x\to 2}\dfrac{x-2}{\sqrt{x+2}-2}$.

37. Evaluate: $\lim\limits_{x\to 0}\dfrac{\sqrt{5+x}-\sqrt{5-x}}{\sqrt{10+x}-\sqrt{10-x}}$.

38. Evaluate: $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x+x^2+x^3}-1}{x}$.

39. $\lim\limits_{x\to\infty}\dfrac{x^n+a^n}{x^n-a^n}=$ ______.

40. Evaluate: $\lim\limits_{x\to 1}\left[\dfrac{x^4-2x^3-x^2+2x}{x-1}\right]$.

41. Evaluate: $\lim\limits_{x\to a}\dfrac{\sqrt{x+a}-\sqrt{2a}}{x-a}$.

42. Evaluate: $\lim\limits_{n\to\infty}\left(\sum\limits_{r=0}^{n}\dfrac{1}{2^r}\right)$.

43. If $\lim\limits_{x\to -3}\dfrac{x^k+3^k}{x+3}=405$, where k is an odd natural number then, $k=$ ______.

44. $\lim\limits_{x\to 4^-}\dfrac{|x-4|}{x-4}=$ ______.

45. $\lim\limits_{x\to 3}\dfrac{\log(2x-3)-\log(3x+2)}{\log(2x+1)}=$ ______.

Essay Type Questions

46. Evaluate: $\lim\limits_{x\to\infty}\dfrac{n(1+4+9+16+\cdots+n^2)}{n^4+8n^3}$.

47. Evaluate: $\lim\limits_{x\to\infty}\dfrac{n^2(1+2+3+4+\cdots+n)}{n^4+4n^2}$.

48. $\lim\limits_{x\to 1}\dfrac{\sqrt{x+1}-\sqrt{5x-3}}{\sqrt{2x+3}-\sqrt{4x+1}}=$ ______.

49. $\lim\limits_{n\to\infty}\dfrac{1+3+5+7+\ldots n \text{ terms}}{2+4+6+8+\ldots n \text{ terms}}=$ ______.

50. $\lim\limits_{x\to 3}\left[\dfrac{2}{x-3}+\dfrac{2}{x^2-7x+12}\right]=$ ______.

CONCEPT APPLICATION

Level 1

1. Evaluate: $\lim\limits_{x\to 3}(4x^2+3)=$

(a) 36 (b) 39
(c) 40 (d) None of these

2. $\lim\limits_{x\to\infty}\dfrac{2x+4}{x-2}=$

(a) 1 (b) 0
(c) 2 (d) 6

3. $\lim\limits_{x\to 5^+}\dfrac{|x-5|}{x-5}=$

(a) 1 (b) −1
(c) 0 (d) Cannot be determined

4. Evaluate $\lim\limits_{x\to 1}\dfrac{2x^2+4x+4}{2x-1}$.

(a) 1 (b) 10
(c) 20 (d) 5

5. Evaluate: $\lim\limits_{x\to 20}\dfrac{\sqrt{x+5}+5}{\sqrt{x+5}-5}$.

(a) 1 (b) 2
(c) 4 (d) ∞

6. $\lim\limits_{x\to 0}\dfrac{\sqrt{8-3x}+\sqrt{8+4x}}{\sqrt{2-3x}}=$

(a) 5 (b) 3
(c) 2 (d) 4

7. $\lim\limits_{x\to\infty}\dfrac{(x-1)(2x-1)}{(x-4)(x-7)}=$

(a) 0 (b) 1
(c) 2 (d) −1

8. $\lim\limits_{x\to 2}\dfrac{x^{-8}\ \frac{1}{256}}{x-2}=$

(a) $-\frac{1}{32}$ (b) $-\frac{1}{128}$

(c) $-\frac{1}{256}$ (d) $-\frac{1}{64}$

9. $\lim_{x\to 2} \frac{x^2+2x-8}{2x^2-3x-2} =$

(a) $\frac{1}{5}$ (b) $\frac{6}{5}$

(c) $\frac{3}{2}$ (d) $-\frac{1}{6}$

10. $\lim_{x\to 4^-} \sqrt{16-x^2}$ is a/an

(a) complex number.

(b) real number.

(c) natural number.

(d) integer.

11. $\lim_{x\to 0} \log\left(\frac{2x+1}{5x+4}\right) =$

(a) $-2\log 2$ (b) $\log 4$

(c) $-\log\left(\frac{1}{2}\right)$ (d) $-3\log 2$

12. $\lim_{x\to 3} \frac{\sqrt{x+3}+\sqrt{x+6}}{\sqrt{x+1}-2} =$

(a) 1 (b) 7

(c) 14 (d) None of these

13. $\lim_{x\to 5} \frac{\sqrt{x+20}-\sqrt{3x+10}}{5-x} =$

(a) $-\frac{2}{5}$ (b) $\frac{2}{5}$

(c) $\frac{1}{5}$ (d) $\frac{-1}{5}$

14. $\lim_{x\to -3} \frac{x^2-2x-15}{x^2+2x-3} =$

(a) 1 (b) 2

(c) -3 (d) -4

15. $\lim_{x\to \infty} \frac{(x-5)(x+7)}{(x+2)(5x+1)} =$

(a) $-\frac{1}{5}$ (b) 25

(c) 5 (d) $\frac{1}{5}$

16. Evaluate, $\lim_{x\to 2} \frac{f(x)-f(2)}{x-2}$, where $f(x) = x^2 - 4x$.

(a) -1 (b) 2

(c) 0 (d) 4

17. Evaluate $\lim_{x\to 9} \frac{2x-7\sqrt{x}+3}{3x-11\sqrt{x}+6}$.

(a) $\frac{3}{4}$ (b) $\frac{5}{3}$

(c) $\frac{5}{7}$ (d) $\frac{3}{7}$

18. $\lim_{x\to b} \frac{\sqrt{x}-\sqrt{b}}{x-b} =$

(a) $\frac{1}{2b}$ (b) $\frac{\sqrt{2}}{b}$

(c) $\frac{1}{2\sqrt{b}}$ (d) None of these

19. $\lim_{x\to 3} \frac{x-3}{\sqrt{x+6}-\sqrt{2x+3}} =$

(a) -5 (b) -6

(c) 9 (d) 6

20. $\lim_{x\to -2} \frac{x+2}{\sqrt{2x+8}-\sqrt{2-x}} =$

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) 1 (d) $\frac{4}{3}$

21. Evaluate $\lim_{x\to 1} \left[\frac{\log x^3 + \log\left(\frac{1}{x^2}\right) + \log 2}{\log(x^3+3)}\right]$.

(a) $\frac{3}{2}$ (b) 1

(c) $\frac{1}{2}$ (d) 2

22. $\lim_{x\to 3} \frac{x^{-5}-\frac{1}{243}}{x-3}$?

(a) $-\frac{5}{729}$ (b) $-\frac{5}{243}$

(c) $\frac{5}{81}$ (d) None of these

23. $\lim_{x\to\infty} \frac{4x-3}{(2x+3)} =$

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) 2

24. Evaluate: $\lim_{d\to 3} \frac{d^3-27}{d-3}$

(a) 3 (b) 9

(c) 27 (d) None of these

25. $\lim_{x\to 3^-} \sqrt{9-x^2}$ is a /an

(a) natural number.

(b) real number.

(c) imaginary number.

(d) integer.

26. Evaluate $\lim_{x\to 4} \frac{3x-8\sqrt{x+4}}{5x-9\sqrt{x}-2}$.

(a) $\frac{1}{5}$ (b) $\frac{4}{11}$

(c) $\frac{3}{10}$ (d) 5

27. For some real number k, the value of $\lim_{x\to -k} \frac{x^5+k^5}{x+k}$ can be

(a) 50 (b) 60

(c) 70 (d) 80

28. Evaluate $\lim_{x\to 1} \frac{f(x)-f(1)}{x-1}$, where $f(x) = x^2 - 2x$.

(a) −1 (b) 0

(c) 1 (d) 2

29. Evaluate $\lim_{x\to a} \frac{x^{\frac{1}{5}}-a^{\frac{1}{5}}}{x^{\frac{-4}{5}}-a^{\frac{-4}{5}}}$.

(a) $\frac{-a}{4}$ (b) $\frac{-1}{4a}$

(c) $\frac{1}{4a}$ (d) $\frac{a}{4}$

30. $\lim_{x\to\infty} \frac{6x^4+7x^3+2x+1}{x^4+1} =$

(a) 1 (b) 2

(c) 3 (d) 6

Level 2

31. Evaluate $\lim_{x\to -2} \frac{2x^3-x^2-13x-6}{3x^2+x-10}$.

(a) $-\frac{5}{7}$ (b) $-\frac{17}{5}$

(c) $-\frac{15}{11}$ (d) $-\frac{10}{3}$

32. Evaluate $\lim_{x\to -1} \frac{\log x^2 - \log\left(\frac{1}{x^4}\right) + \log 3}{\log\left(\frac{x^3}{-3}\right)}$.

(a) 1 (b) 0

(c) −1 (d) Does not exist

33. $\lim_{x\to\infty} \left\{\frac{3x}{\sqrt{x^2+5x-6}+2x}\right\} =$

(a) −1 (b) 1

(c) 0 (d) ∞

34. $\lim_{x\to\infty} \frac{(4x+5)(2x-1)}{(27x^2+1)} =$

(a) $\frac{4}{27}$ (b) $\frac{8}{27}$

(c) $\frac{2}{27}$ (d) $\frac{6}{27}$

35. $\lim_{x\to 1} \frac{x^3-6x^2+11x-6}{x^2-5x+4} =$

(a) $\frac{2}{3}$ (b) $-\frac{5}{3}$

(c) $\frac{5}{3}$ (d) $\frac{-2}{3}$

36. If $\lim_{x\to a} \frac{x^5-a^5}{x-a} = 5$, then find the sum of the possible real values of a.

(a) 0 (b) 2

(c) 3 (d) Cannot be determined

37. $\lim_{x\to 0} \dfrac{x}{\sqrt{3+x}-\sqrt{3-x}} =$

(a) $\sqrt{3}$ (b) $\sqrt{-3}$

(c) $2\sqrt{3}$ (d) $-2\sqrt{3}$

38. $\lim_{x\to 1} \dfrac{4-\sqrt{15+x}}{1-x} =$

(a) $\dfrac{1}{6}$ (b) $\dfrac{1}{8}$

(c) $\dfrac{1}{10}$ (d) $\dfrac{1}{12}$

39. $\lim_{x\to 0} \dfrac{\sqrt{1+x+x^2+x^3}-1}{x} =$

(a) 1 (b) 2

(c) $\dfrac{1}{2}$ (d) $\dfrac{1}{4}$

40. $\lim_{x\to 0} \dfrac{\sqrt[4]{x+16}-2}{x} =$

(a) $\dfrac{1}{16}$ (b) $\dfrac{1}{8}$

(c) $\dfrac{1}{32}$ (d) $\dfrac{1}{20}$

41. $\lim_{x\to \infty} \dfrac{(x+2)^{10}+(x+4)^{10}+\cdots+(x+20)^{10}}{x^{10}+1} =$

(a) 12 (b) 20

(c) 40 (d) 10

42. $\lim_{x\to \infty} \dfrac{2+6+10+14+\ldots 2n \text{ terms}}{2+4+6+8+\ldots n \text{ terms}} =$

(a) 4 (b) 16

(c) 8 (d) 10

43. Evaluate: $\lim_{x\to -a} \dfrac{1}{x}\left[\dfrac{1}{x+a}+\dfrac{b+a}{x^2-bx+ax-ab}\right]$

(a) $\dfrac{1}{a}$ (b) $\dfrac{1}{a+b}$

(c) $\dfrac{1}{a(a+b)}$ (d) $\dfrac{1}{a(a-b)}$

44. $\lim_{x\to 5} \dfrac{\sqrt{2x-3}-\sqrt{3x-8}}{\sqrt{2x-1}-\sqrt{3x-6}} =$

(a) $\dfrac{2\sqrt{7}}{3}$ (b) $\dfrac{3}{2\sqrt{7}}$

(c) $\dfrac{4}{3}\sqrt{7}$ (d) $\dfrac{3}{\sqrt{7}}$

45. If $\lim_{x\to a} \dfrac{x^7-a^7}{x-a} = 7$, then find the number of possible real values of a.

(a) 0 (b) 1

(c) 2 (d) Cannot be determined

Level 3

46. $\lim_{n\to \infty} \dfrac{1+4+9+16+\cdots+n^2}{4n^3+1} =$

(a) $\dfrac{1}{12}$ (b) $\dfrac{1}{24}$

(c) $\dfrac{1}{8}$ (d) $\dfrac{1}{16}$

47. $\lim_{x\to 0} \dfrac{\sqrt[5]{x+32}-2}{x} =$

(a) $\dfrac{1}{16}$ (b) $\dfrac{1}{45}$

(c) $\dfrac{1}{80}$ (d) $\dfrac{1}{100}$

48. $\lim_{x\to 2} \dfrac{\sqrt{x^3-2x^2+2x-3}-1}{x-2} =$

(a) 4 (b) 8

(c) 6 (d) 3

49. Evaluate: $\lim_{x\to \infty} \dfrac{(1^2+2^2+3^2+\cdots+n^2)}{n^2(1+3+5+7+\cdots+n \text{ terms})}$.

(a) 0 (b) 1

(c) ∞ (d) Does not exist

50. Evaluate: $\lim_{x\to 0} \dfrac{\sqrt{6+x}-\sqrt{6-x}}{\sqrt{8+x}-\sqrt{8-x}}$.

(a) $\sqrt{3}$ (b) $\dfrac{-5\sqrt{3}}{3}$

(c) $\dfrac{4\sqrt{3}}{3}$ (d) $\dfrac{2\sqrt{3}}{3}$

51. Evaluate: $\lim_{\theta \to 0} \cot\theta - \operatorname{cosec}\theta$.

(a) 0 (b) 2

(c) 4 (d) None of these

52. $\lim_{x\to\infty} \dfrac{(x+1)^{100} + (x+2)^{100} + \cdots + (x+50)^{100}}{x^{100} + x^{99} + \cdots + x + 1}$ = ______.

(a) 100 (b) 1

(c) 21 (d) 50

53. $\lim_{x\to 3^-} \dfrac{|2x-6|}{2x-6}$ is ______.

(a) 1 (b) −1

(c) 0 (d) ∞

54. If $\lim_{x\to -2} \dfrac{x^p + 2^p}{x+2} = 80$ (where p is an odd number), then p can be _____.

(a) 3 (b) 5

(c) 7 (d) 9

55. If $\lim_{x\to m} \dfrac{x^3 - m^3}{x - m} = 3$, then find the number of possible values of m.

(a) 0 (b) 2

(c) 1 (d) 3

56. $\lim_{x\to 3} \dfrac{\sqrt{5x+1} - \sqrt{7x-5}}{\sqrt{7x+4} - \sqrt{5x+10}}$ = ________.

(a) 0 (b) $-\dfrac{5}{4}$

(c) $\dfrac{5}{4}$ (d) ∞

57. $\lim_{x\to a} (x-a)\left(\dfrac{1}{x-a} - \dfrac{1}{x^2 - (a+b)x + ab}\right)$ = ________.

(a) $\dfrac{a-b-1}{a-b}$ (b) 0

(c) 1 (d) $\dfrac{a+b}{a-b}$

58. $\lim_{x\to \frac{2}{5}} \dfrac{1}{|5x-2|}$ = ________.

(a) 0 (b) ∞

(c) 1 (d) Does not exist

59. $\lim_{x\to\infty} \dfrac{5x^2}{\sqrt{25x^4 + 13x^3 + 14x^2 + 17x + 6}}$ is ________.

(a) 1 (b) $\dfrac{1}{5}$

(c) $\dfrac{5}{14}$ (d) None of these

60. $\lim_{x\to -1} \dfrac{x^5 + 1}{x+1}$ = ________.

(a) 1 (b) −5

(c) 5 (d) None of these

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. circle
2. Newton and Leibnitz
3. point
4. integration
5. 2
6. tangent
7. na^n
8. 0
9. decreases.
10. 8
11. 1
12. 0
13. $\frac{4}{5}$
14. $n(-a)^{n-1}$
15. -1
16. does not exist
17. Limit does not exist
18. $\frac{n}{m} a^{n-m}$
19. $\frac{1}{2\sqrt{5}}$
20. 6
21. 6.
22. 27
23. ∞
24. 0
25. $\frac{1}{16} \cdot a^{\frac{-15}{4}}$
26. 448
27. -6
28. Limit does not exist
29. $\lim_{\frac{1}{n}\to a} f\left(\frac{1}{n}\right)$
30. $\frac{7}{8}$

Short Answer Type Questions

31. $\frac{11}{8}$
32. $\frac{-1}{15}$
33. 405
34. $\frac{1}{2}$
35. $-2a^{21}$.
36. 4
37. $\sqrt{2}$
38. $\frac{1}{2}$
39. 1
40. -2
41. $\frac{1}{2\sqrt{2a}}$
42. 2
43. -1
44. Cannot be determined
45. $\log_7\left(\frac{6}{11}\right)$

Essay Type Questions

46. $\frac{1}{3}$
47. $\frac{1}{2}$
48. $\sqrt{10}$
49. 1
50. 0

CONCEPT APPLICATION

Level 1

1. (b)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (d)	**6.** (d)	**7.** (c)	**8.** (d)	**9.** (b)	**10.** (b)
11. (a)	**12.** (d)	**13.** (c)	**14.** (b)	**15.** (d)	**16.** (c)	**17.** (c)	**18.** (c)	**19.** (b)	**20.** (d)
21. (c)	**22.** (a)	**23.** (d)	**24.** (c)	**25.** (b)	**26.** (b)	**27.** (d)	**28.** (b)	**29.** (a)	**30.** (d)

Level 2

31. (c)	**32.** (c)	**33.** (b)	**34.** (b)	**35.** (d)	**36.** (a)	**37.** (a)	**38.** (b)	**39.** (c)	**40.** (c)
41. (d)	**42.** (c)	**43.** (c)	**44.** (d)	**45.** (c)					

Level 3

46. (a)	**47.** (c)	**48.** (d)	**49.** (a)	**50.** (d)	**51.** (a)	**52.** (d)	**53.** (b)	**54.** (b)	**55.** (b)
56. (b)	**57.** (a)	**58.** (b)	**59.** (a)	**60.** (c)					

CONCEPT APPLICATION

Level 1

1. Substitute $x = 3$ in the given function.
2. Divide both numerator and denominator by x then use if $x \to \infty$ then $\frac{1}{x} \to 0$.
3. When $x > 5, |x - 5| = x - 5$.
4. Substitute $x = 1$ in the given expression.
5. Substitute $x = 20$.
6. Substitute $x = 0$.
7. Divide both numerator and denominator by x^2. Now substitute, $\frac{1}{x} = 0$.
8. Use the formula, $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$.
9. Factorise the numerator and denominator and remove common factor. Now substitute $x = 2$.
10. Apply the concept of left hand limit.
11. Substitute $x = 0$.
12. Rationalize the denominator.
13. Rationalize the numerator and cancel common factor. Then substitute, $x = 5$.
14. Factorize the numerator and denominator and cancel the common factor and then substitute $x = -3$.
15. Divide both the numerator and the denominator by x^2. Then substitute, $\frac{1}{x} = 0$.
16. Substitute then factorize numerator $f(x)$, then cancel the common factor in both the numerator and the denominator, then substitute $x = 2$.
17. Factorize numerator and denominator, eliminate the common factor then substitute $x = 9$.
18. Use the formula, $\lim_{x \to a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}$.
19. Rationalize the denominator and remove the common factor. Now substitute $x = 3$.
20. Rationalize the denominator and cancel the common factor and then substitute $x = -2$.
21. Put $x = 1$ in the given limit.
22. Use the formula, $\lim_{x \to a} \frac{x^n - a^n}{x - a}$ by converting the given limit into the form.
23. Divide both the numerator and the denominator by x and apply the concept hat, as $x \to \infty$, $\frac{1}{x} \to 0$.
24. Use the formula, $\lim_{x \to a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}$.
25. Apply the concept of left hand limit.
26. Factorize the numerator and denominator and cancel the common factor which is in both numerator and denominator then substitute $x = 4$.
27. Use the formula, $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$.
28. Calculate $f(1)$ substitute $f(x)$ and $f(1)$, factorise numerator, then cancel the common factor and substitute $x = 1$.
29. Use the formula, $\lim_{x \to a} \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n} a^{m-n}$.
30. (i) The highest power of x is to be taken common from numerator and denominator.

    (ii) Put $\frac{1}{x}, \frac{1}{x^2} \ldots = 0$ as $x \to \infty \frac{1}{x}, \frac{1}{x^2} \to 0$.

Level 2

31. Factorize the numerator and the denominator and cancel the common factor then substitute $x = -2$.
32. Put $x = -1$ in the given limit.
33. Numerator and denominator are of the same degree.

    $\therefore$ Limiting value of the given function $= \frac{\text{Coefficient of } x \text{ in numerator}}{\text{Coefficient of } x \text{ in denominator}}$.

34. Divide both the numerator and the denominator by x^2 and then use when

$$x \to \infty \quad \Rightarrow \quad \frac{1}{x} \to 0.$$

35. Factorize the numerator and the denominator, and cancel the common factor, then substitute $x = 1$.

36. (i) $\lim_{x \to a} \frac{x^m - a^m}{x - a} = ma^{m-1}$.

(ii) Equating ma^{m-1} with 5 obtain the value of a.

(iii) Add all the possible real values of a.

37. Rationalize the denominator and cancel common factor and then substitute $x = 0$.

38. Rationalize the numerator and cancel the common factor and then substitute $x = 1$.

39. Rationalize the numerator and cancel the common factor x, then substitute $x = 0$.

40. Use the formula, $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$.

41. Take common the highest power of x and cancel it, then substitute $\frac{1}{x} = 0$ as $x \to \infty$, $\frac{1}{x} \to 0$.

42. Find the sum of $2n$ terms in numerator and sum of n terms in denominator and take common the highest power of n in both the numerator and the denominator and then substitute $\frac{1}{n} = 0$ as $n \to \infty$, $\frac{1}{n} \to 0$.

43. Simplify and then factorise the numerator. Cancel the common factor and then substitute $x = -a$.

44. Rationalize the numerator and denominator and cancel the common factor and then substitute $x = 5$.

45. (i) Use the formula, $\lim_{x \to a} \frac{x^m - a^m}{n^n - a^n} = \frac{m}{n} a^{m-n}$.

(ii) $7a^6 = 7$.

(iii) Now, find the number of possible real values of a.

Level 3

46. Write formula for sum of the squares of first n natural numbers, i.e., $\frac{n(n+1)(2n+1)}{6}$.

Divide the numerator and the denominator by n^3.

Then substitute $\frac{1}{n} = 0$ as $n \to \infty$, $\frac{1}{n} = 0$.

47. Use the formula, $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$.

48. (i) Rationalize the numerator and find its factors

(ii) Cancel the common factor of numerator and denominator and substitute the value of limit.

49. Substitute the formulae for sum of the squares of n natural numbers and sum of n natural number and sum of n odd natural numbers and simplify.

50. Rationalize both numerator and denominator.

51. $\lim_{\theta \to 0} \cot\theta - \operatorname{cosec}\theta$

$= \infty - \infty$, i.e., undetermined form.

$\cot\theta - \operatorname{cosec}\theta$

$$= \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta} = \frac{\cos\theta - 1}{\sin\theta}$$

$$= \frac{\cos^2\theta - 1}{\sin\theta(\cos\theta + 1)}$$

$$= \frac{-\sin\theta}{\sin\theta(\cos\theta + 1)} = \frac{-\sin\theta}{\cos\theta + 1}$$

$\therefore \quad \lim_{\theta \to 0} \cot\theta - \operatorname{cosec}\theta$

$$= \lim_{\theta \to 0} \frac{-\sin\theta}{\cos\theta + 1} = \frac{-\sin 0}{\cos 0 + 1} = \frac{-0}{1+1} = 0.$$

52. $\lim_{x \to \infty} \frac{(x+1)^{100} + (x+2)^{100} + \cdots + (x+50)^{100}}{x^{100} + x^{99} + \cdots + x + 1}$

$$\lim_{\frac{1}{x} \to \infty} \frac{x^{100}\left(\left(1 + \frac{1}{x}\right)^{100} + \left(1 + \frac{2}{x}\right)^{100} + \cdots + \left(1 + \frac{50}{x}\right)^{100}\right)}{x^{100}\left(1 + \frac{1}{x} + \cdots + \frac{1}{x^{99}} + \frac{1}{x^{100}}\right)}$$

$$= \frac{1 + 1 + \cdots + 1(50 \text{ terms})}{1} = 50.$$

53. $\lim_{x \to 3^-} \frac{|2x - 6|}{2x - 6}$

$$= \lim_{x \to 3^-} \frac{-(2x-6)}{(2x-6)} (\text{for } x < 3, 2x - 6 < 0) = -1.$$

54. $\lim\limits_{x\to -2} \dfrac{x^p + 2^p}{x+2} = 80$

$$\Rightarrow \quad \lim_{x\to -2} \frac{x^p - (-2)^p}{x-(-2)} = 80$$

$$\Rightarrow \quad p(-2)^{p-1} = 5(-2)^{5-1} \quad \therefore\ p = 5.$$

55. $\lim\limits_{x\to m} \dfrac{x^3 - m^3}{x-m} = 3$

$$\Rightarrow \quad 3m^2 = 3 \quad \Rightarrow \quad m^2 = 1 \quad \Rightarrow \quad m = \pm 1$$

$\therefore$ There are two values for m.

56. $\lim\limits_{x\to 3} \dfrac{\sqrt{5x+1} - \sqrt{7x-5}}{\sqrt{7x+4} - \sqrt{5x+10}} = \dfrac{0}{0}$

That is, undetermined form.
Multiply both the numerator and the denominator with $\left(\sqrt{5x+1} + \sqrt{7x-5}\right)\left(\sqrt{7x+4} + \sqrt{5x+10}\right)$

$$\Rightarrow \quad \lim_{x\to 3} \frac{[5x+1-(7x-5)]\left(\sqrt{7x+4} + \sqrt{5x+10}\right)}{(7x+4-5x-10)\left(\sqrt{5x+1} + \sqrt{7x-5}\right)}$$

$$\Rightarrow \quad \lim_{x\to 3} \frac{(-2x+6)\left(\sqrt{7x+4} + \sqrt{5x+10}\right)}{(2x-6)\left(\sqrt{5x+1} + \sqrt{7x-5}\right)}$$

$$= \lim_{x\to 3} \frac{-\left(\sqrt{7x+4} + \sqrt{5x+10}\right)}{\left(\sqrt{5x+1} + \sqrt{7x-5}\right)}$$

$$= \frac{-\left(\sqrt{25} + \sqrt{25}\right)}{\left(\sqrt{16} + \sqrt{16}\right)} = \frac{-10}{8} = \frac{-5}{4}.$$

57. $\lim\limits_{x\to a} (x-a)\left(\dfrac{1}{x-a} - \dfrac{1}{x^2-(a+b)x+ab}\right)$

$$= \lim_{x\to a} (x-a)\left(\frac{1}{x-a} - \frac{1}{(x-a)(x-b)}\right)$$

$$= \lim_{x\to a} (x-a)\left(\frac{x-b-1}{(x-a)(x-b)}\right)$$

$$\lim_{x\to a} (x-a)\left(\frac{x-b-1}{x-b}\right) = \frac{a-b-1}{a-b}.$$

58. $\lim\limits_{x\to \frac{2}{5}} \dfrac{1}{|5x-2|}$

As $|5x-2| \geq 0$

$\lim\limits_{x\to \frac{2}{5}} \dfrac{1}{|5x-2|}$ is exist

$$\therefore \quad \lim_{x\to \frac{2}{5}} \frac{1}{|5x-2|} = \frac{1}{0} = \infty.$$

59. $\lim\limits_{x\to\infty} \dfrac{5x^2}{\sqrt{25x^4 + 13x^3 + 14x^2 + 17x + 6}}$

$$= \lim_{x\to\infty} \frac{5x^2}{x^2\sqrt{25 + 13\left(\frac{1}{x}\right) + 14\left(\frac{1}{x^2}\right) + 17\left(\frac{1}{x^3}\right) + 6\left(\frac{1}{x^4}\right)}}$$

As $x \to \infty$, $\dfrac{1}{x} \to 0$

$$= \lim_{\frac{1}{x}\to 0} \frac{5}{\sqrt{25 + 13\left(\frac{1}{x}\right) + 14\left(\frac{1}{x}\right)^2 + 17\left(\frac{1}{x}\right)^3 + 6\left(\frac{1}{x}\right)^4}}$$

$$= \frac{5}{\sqrt{25+10}} = \frac{5}{5} = 1.$$

60. $\lim\limits_{x\to -1} \dfrac{x^5+1}{x+1}$

$$\lim_{x\to -1} \frac{x^5 - (-1)^5}{x-(-1)} = 5(-1)^{5-1} = 5(-1)^4 = 5.$$