12 LINEAR PROGRAMMING

Objectives

After studying the material of this chapter, you should be able to :

- Understand the concept of constraints and their graphs.
- Understand the definition of Linear Programming.
- Understand various definitions viz. Feasible Region, Feasible Solution; etc.
- Understand Corner Point Method and its application.

The mathematical experience of the student is incomplete if he never had the opportunity to solve a problem invented by himself

—G. Polya

INTRODUCTION

The subject namely Linear Programming had its birth during the Second World War (1939 – 45), arising mainly out of the necessity to plan the war activities so that the damages on the enemy camps are large at minimum cost and loss.

But in later years, the subject was found increasingly useful for economic planning of the activities of the industrial and commercial organisations.

The persons who developed and enriched this subject are the Russian Mathematician **L. Kantorovich,** the American Economists **F.L. Hitchcock** and **G.B. Dantzig. Kantorovich** and **Kooperman** were awarded nobel prize in the year 1974 in Economics for their poineering work in Linear Programming.

L. Kantorovich

F.L. Hitchcock

G.B. Dantzig

In Linear Programming Problems (LPP) we are faced with the problem of either a maximization or a minimization for getting an optimum schedule. In real life, the LPP are so complicated that computers are required for solving them.

Chapter at Glance

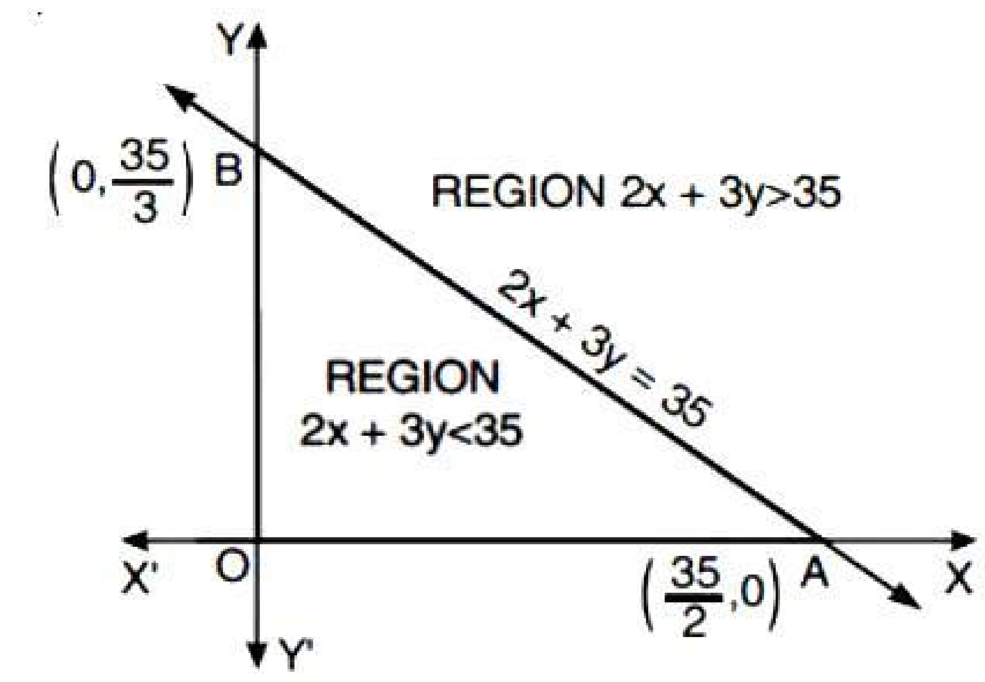
SUB CHAPTER 12.1 Constraints

12.1. LINEAR CONSTRAINTS

Let us consider an example in which we have a sum of ₹ 350 and we want to purchase items (I), costing ₹ 20 each, and items (II), costing ₹ 30 each. We can make different choices and we have the following results :

Items (I)	Items (II)	Money Left	
17	×	₹ 10	[$\because$ $350 - 17 \times 20 = 10$]
×	11	₹ 20	[$\because$ $350 - 11 \times 30 = 20$]
5	8	₹ 10	[$\because$ $350 - (5 \times 20 + 8 \times 30) = 10$]
7	7	×	[$\because$ $350 - (7 \times 20 + 7 \times 30) = 0$]

Obviously, we cannot purchase 8 items (I) and 7 items (II) because the total cost will be = 8 × 20 + 7 × 30 = ₹ 370, which is more than ₹ 350, which we can not spend.



Let us denote that 'x' be the number of items (I) and 'y' that of items (II) to be purchased.

Clearly 'x' and 'y' can assume only positive integral values. Thus, the above problem becomes :

Find positive integers x and y so that $20x + 30y \leq 350$ *i.e.* $2x + 3y \leq 35$.

The solution of the above problem is the set of pairs (x, y) satisfying $2x + 3y \leq 35$.

Draw the st. line $2x + 3y = 35$.

This meets x-axis $(y = 0)$ at A $\left(\frac{35}{2}, 0\right)$ and y-axis $(x = 0)$ at B $\left(0, \frac{35}{3}\right)$. The region is as shown in the figure. The solution of the above problem is the set of those points (x, y) whose co-ordinates are integers (can be zero) and lying in the region OAB (shaded) including the st. line AB.

We discuss different cases :

When we purchase only items (I).

Here the solution will be on OA.

When we purchase only items (II).

Here the solution will be on OB.

In order to utilise the whole money, we can have : $x = 1$, $y = 11$; $x = 7$, $y = 7$; $x = 10$, $y = 5$; etc.

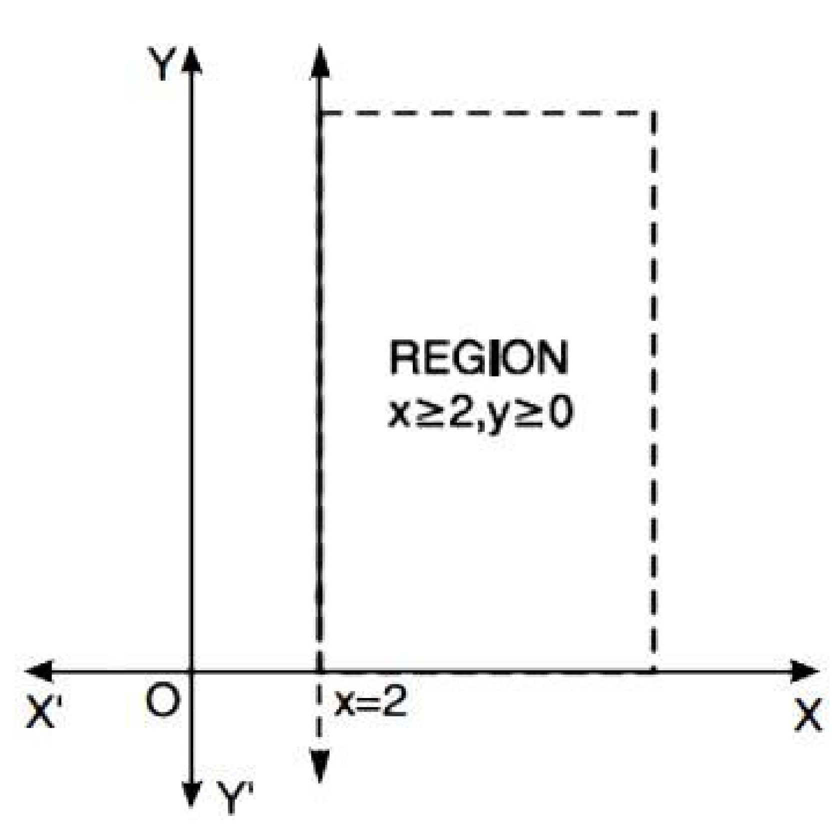
[∵ *In each case* $2x + 3y = 35$]

Here the solution will be on the line segment [AB].

When we purchase minimum 2 items (I) and 3 items (II).

Then $x \geq 2$, $y \geq 3$; $2x + 3y \leq 35$.

The points (x, y) for which y is non-negative and $x \geq 2$ and for which x is non-negative and $y \geq 3$, which are as shown below :



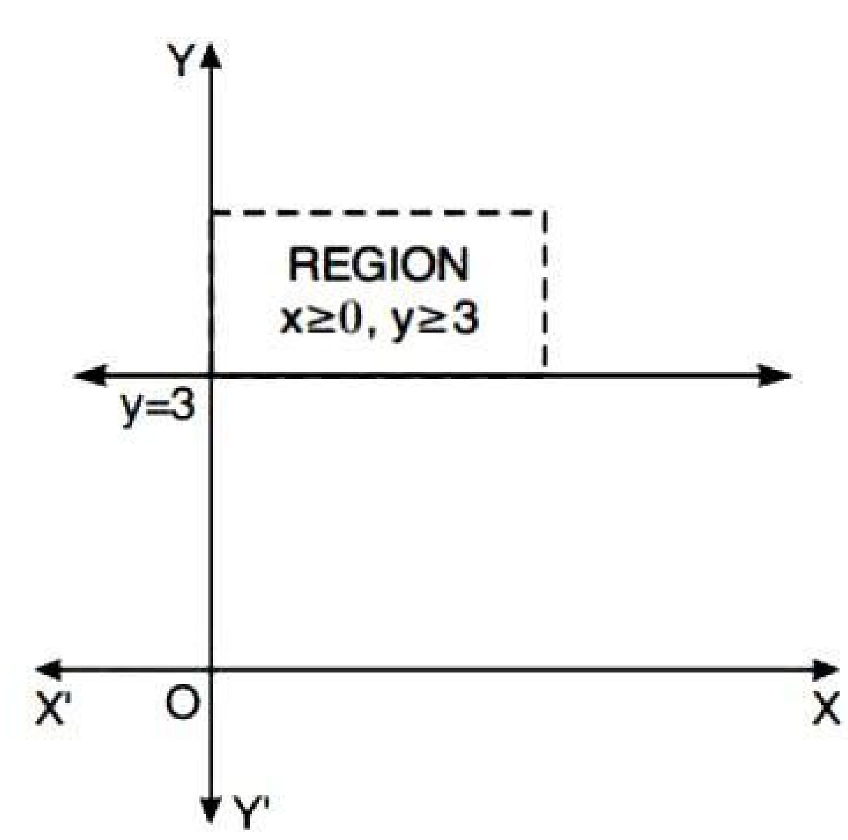


Fig.

The points (x, y) satisfying $x \geq 2$, $y \geq 3$, $2x + 3y \leq 35$ lie in the shaded region as below :

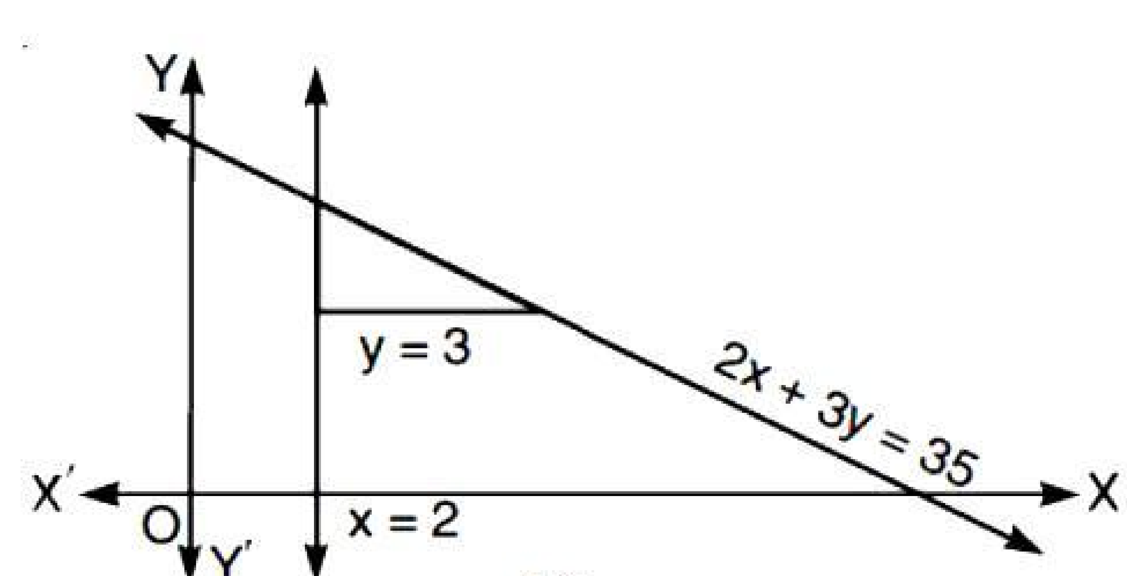


Fig.

The conditions $x \geq 2$, $y \geq 3$, $2x + 3y \leq 35$ are called **constraints** because they restrict us for the selection of x, y. Such constraints, as used above, are called **linear constraints** because they are expressed as linear functions.

ILLUSTRATIVE EXAMPLES

Example 1. Draw the graph of the following LPP :

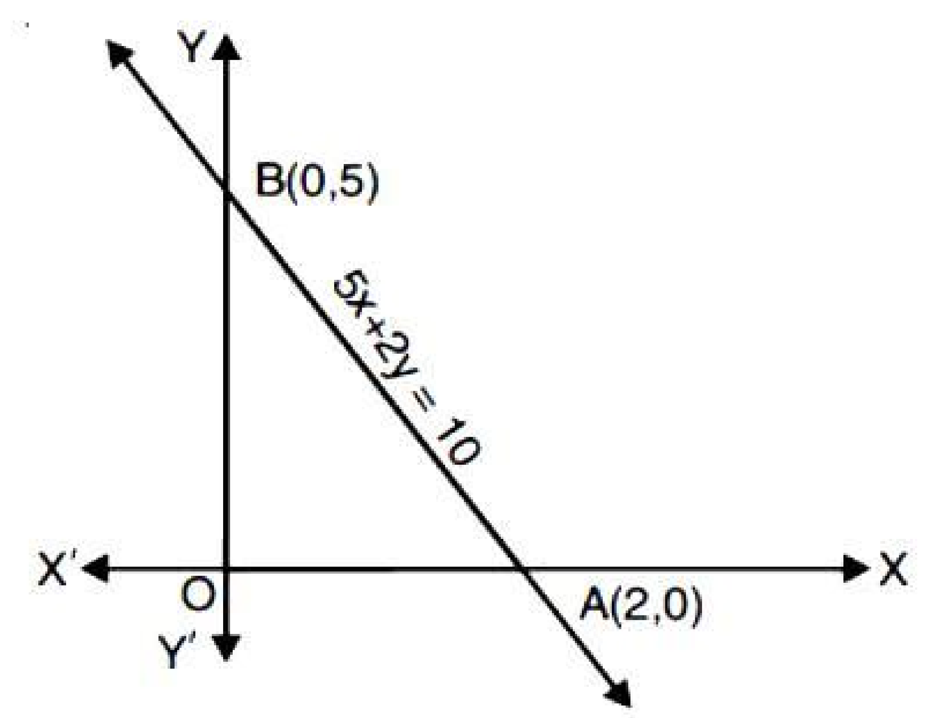
$5x + 2y \leq 10, x \geq 0, y \geq 0$. *(H.B. 2010)*

Solution. Draw the line AB : $5x + 2y = 10$ ...(1),

which meets x-axis at A (2, 0) and y-axis at B (0, 5).

Also $x = 0$ is y-axis and $y = 0$ is x-axis.

Hence, the graph of the given L.P.P. is as shown (shaded) :



Example 2. Solve the system of linear inequations :
$x + 2y \leq 10$; $2x + y \leq 8$. *(J. & K. B. 2010)*

Solution. Draw the st. lines $x + 2y = 10$ and $2x + y = 8$. These lines meet at E (2, 4).

Hence, the solution of the given linear inequations is shown as shaded in the following figure.

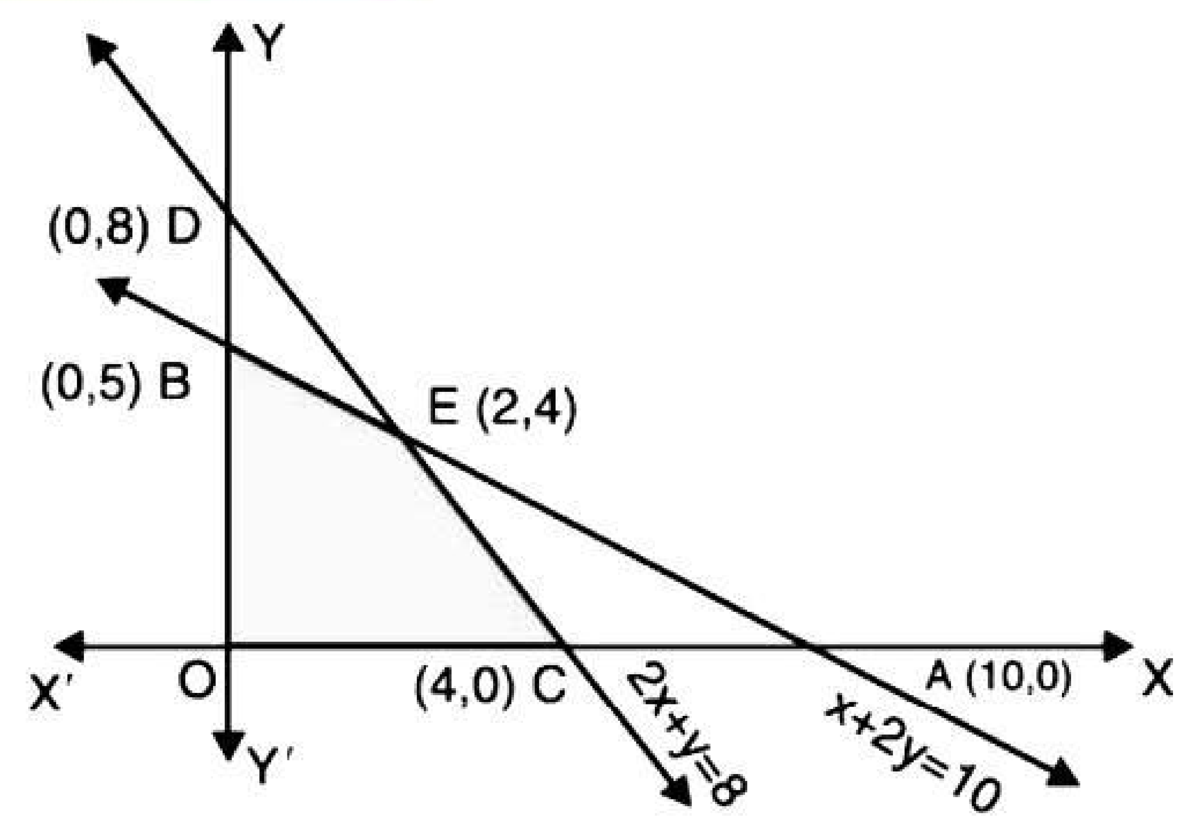


Fig.

Example 3. Find the linear constraints for which the shaded area in the figure below is the solution set :

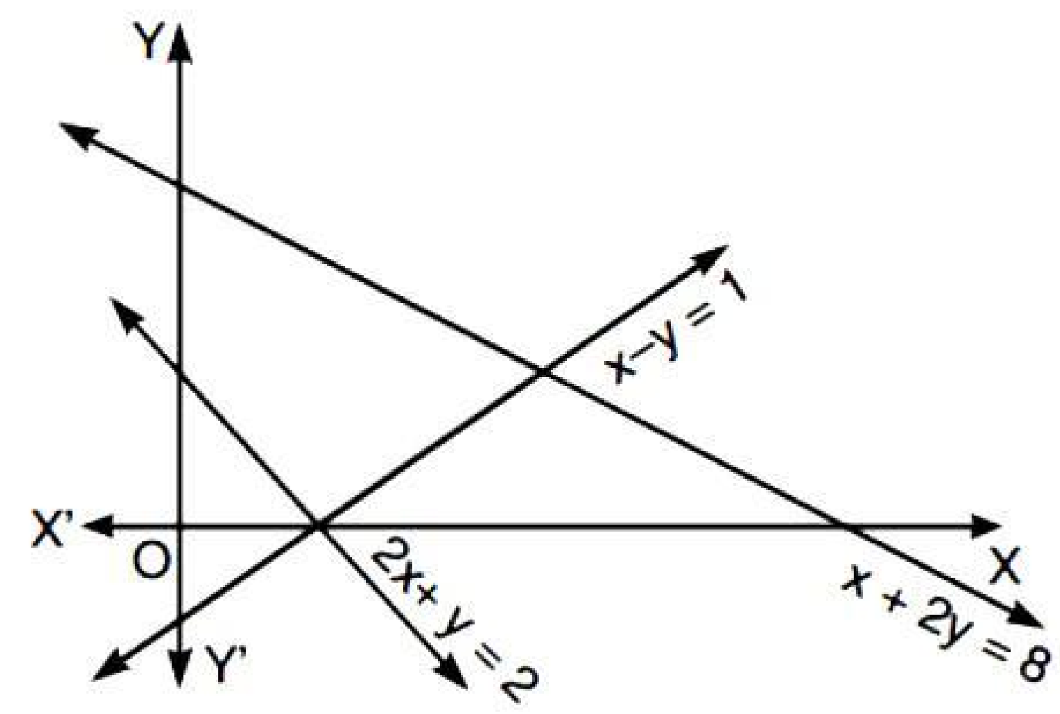


Fig.

Solution.

From the above shaded portion, the linear constraints are : $2x + y \geq 2$, $x - y \leq 1$, $x + 2y \leq 8$, $x \geq 0$, $y \geq 0$.

EXERCISE 12 (a)

Fast Track Answer Type Questions

FTATQ

1. (*a*) Draw the feasible region of inequation :

$x + y \leq 4$; $x \geq 0$, $y \geq 0$. *(Kashmir B. 2017)*

(*b*) Draw the graph of the following LPP :

$3x + y \leq 17$; $x, y \geq 0$. *(H.B. 2010)*

2. Find the solution set of the system of linear constraints : $x + y \leq 6$; $x \geq 1$ and $y < 1$ by graph.

Short Answer Type Questions

SATQ

Draw the diagrams of the solution sets of the following (3 – 6) linear constraints :

3. $x + 2y \leq 8$, $3x + 2y \leq 12$; $x, y \geq 0$. *(Kashmir B. 2017)*

4. $3x + 4y \leq 60$, $x + 3y \leq 30$; $x, y \geq 0$.

5. $3x + 2y \leq 14$, $3x + y \leq 9$, $x, y \geq 0$.

6. $x + y \leq 5$, $4x + y \geq 4$, $x + 5y \geq 5$, $x \leq 4$, $y \leq 3$.

7. Verify that the solution set of the following constraints is empty :

$3x + 4y \geq 12$, $x + 2y \leq 3$, $x \geq 0$, $y \geq 1$.

8. Verify that the solution set of the following constraints : is not empty and is unbounded :

$x - 2y \geq 0$,

$2x - y \leq -2$.

9. Find the linear constraints for which the shaded area in the following figure is the solution set :

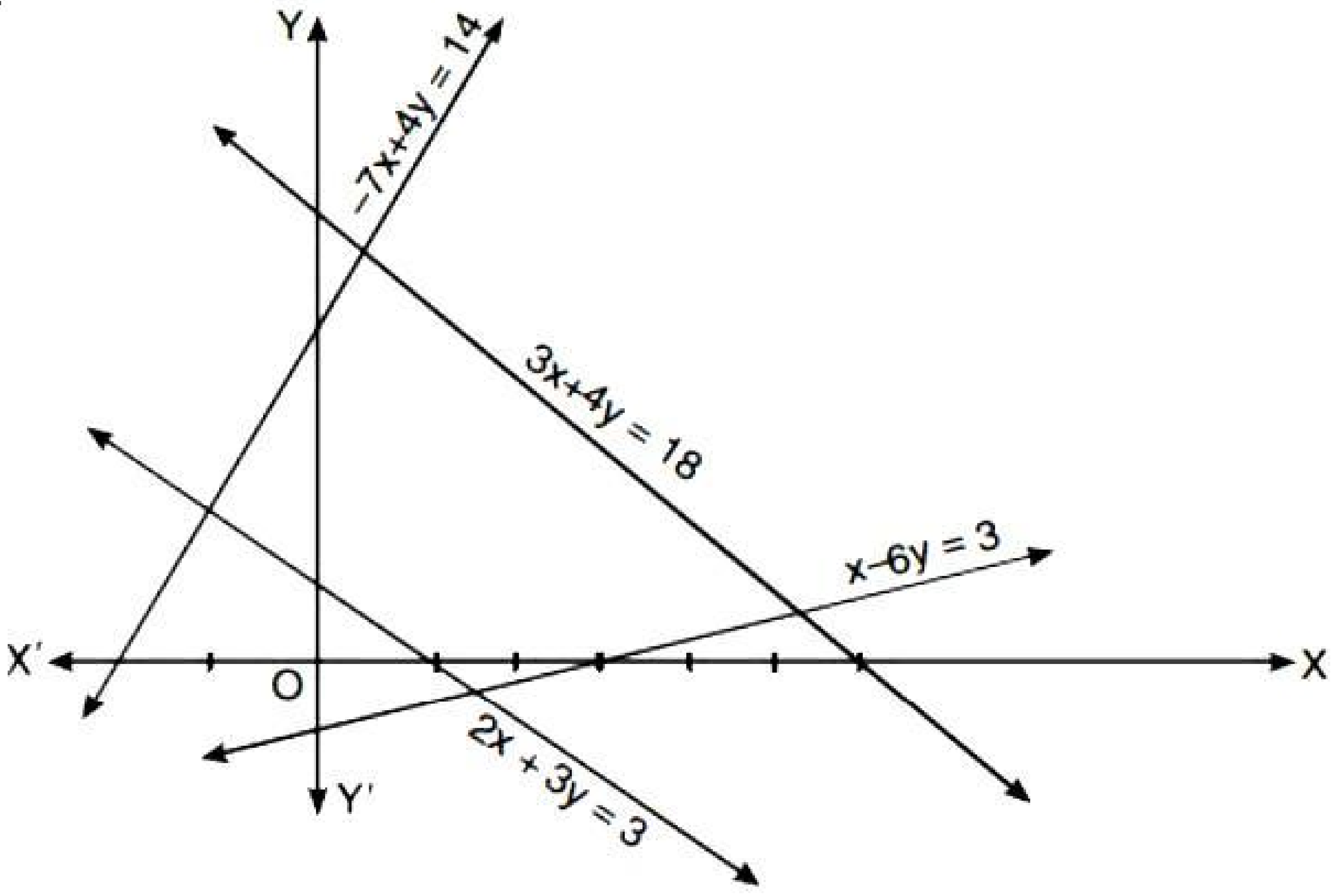


Fig.

10. Find the linear constraints for which the shaded area as shown in the figure is a solution set.

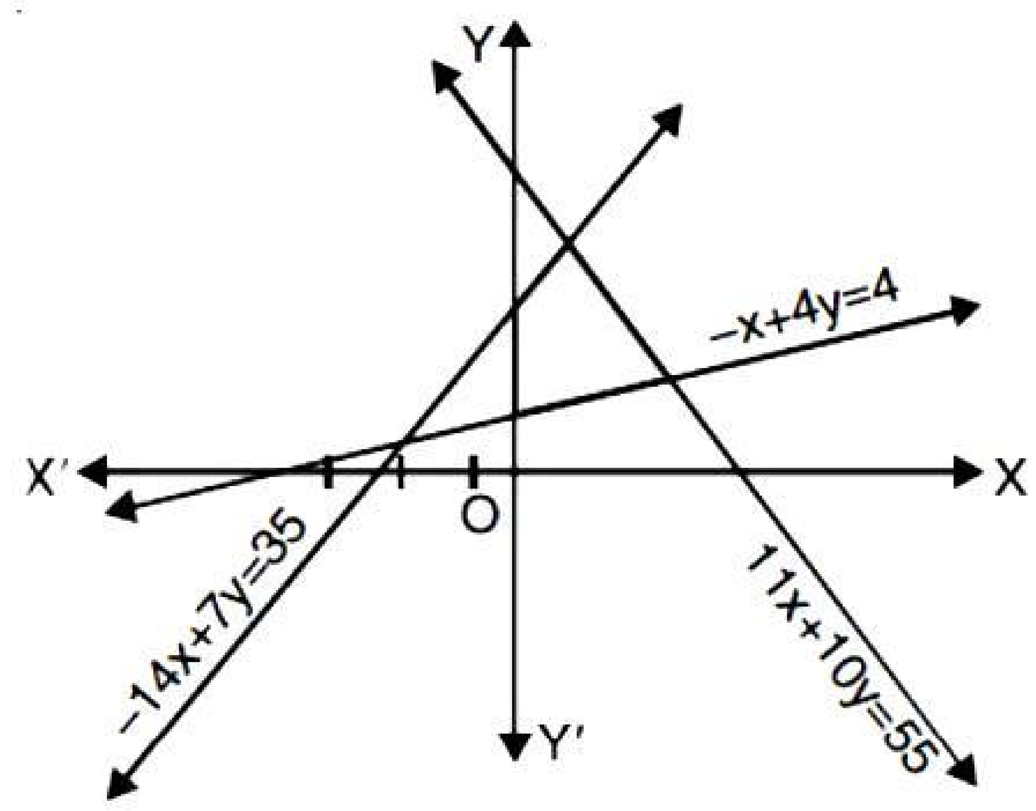


Fig.

Answers

9. $2x + 3y \geq 3$, $x - 6y \leq 3$, $-7x + 4y \leq 14$, $3x + 4y \leq 18$, $x \geq 0$, $y \geq 0$.

10. $-x + 4y \geq 4$, $11x + 10y \leq 55$, $-14x + 7y \geq 35$, $x \geq 0$, $y \geq 0$.

SUB CHAPTER 12.2 Linear Programming

Definition

Linear programming is a tool, which is used in decision making in business for obtaining maximum and minimum values of quantities subject to certain constraints.

12.2. MATHEMATICAL FORMULATION

Let the linear function Z be defined by (*i*) $Z = c_1x_1 + c_2x_2 + ... + c_nx_n$, where *c*'s are constants.

Let a_{ij} be (*mn*) constants and b_j be a set of *m* constants such that :

(*ii*) $a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \quad (\leq, =, \geq)\ b_1$

$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \quad (\leq, =, \geq)\ b_2$

....

....

$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \quad (\leq, =, \geq)\ b_m$

and let (*iii*) $x_1 \geq 0, x_2 \geq 0, ..., x_n \geq 0$.

The problem of determining the values of $x_1, x_2, ..., x_n$, which maximizes (or minimizes) Z and satisfying above (*ii*) is called **General Linear Programming Problem (L.P.P.).**

Note : In (*ii*), there may be any sign ≤, =, or ≥.

12.3. SOME DEFINITIONS

Before proceeding further, we define some terms :

(*a*) Objective Function. The linear function $Z = ax + by$, where *a*, *b* are constants, which is to be maximized or minimized is called a **linear objective function** of **LPP.** ***(Tripura B. 2016; Kashmir B. 2016, 11; Jammu B. 2015)***

The variables *x* and *y* are called **decision variables.**

(*b*) Constraints. The linear inequalities or inequations or restrictions on the variables of a linear programming problem are called **constraints** of **LPP.**

The conditions $x \geq 0$, $y \geq 0$ are called **non-negative restrictions** of **LPP.**

(*c*) Optimisation Problem. A problem, in which it is required to maximize or minimize a linear function (say of two variables *x* and *y*) subject to certain constraints, determined by a set of linear inequalities, is called the **optimisation problem.**

(d) Solution. The values of x, y, which satisfy the constraints of LPP is called the **solution** of the **LPP.**

(e) Feasible Solution. Any solution to LPP, which satisfies the non-negative restrictions of the problem, is called **a feasible solution** to **LPP.** *(Tripura B. 2016; Jammu B. 2015)*

(f) Optimum Solution. Any feasible solution, which optimizes (minimizes or maximizes) the objective function of LPP is called an **optimum solution** of the general **LPP.** *(Jammu B. 2018; Kashmir B. 2016)*

12.4. CONVEX SET (POLYGON)

In a problem of linear programming, the set of feasible solutions is a *polygon* in the positive quadrant (*i.e.*, a closed figure bounded by straight lines).

If we say that the set is a *'convex set'*, it means that if we take any two points in the set, the line joining them also lies in the set. Thus, the set of feasible solutions is a *convex polygon.*

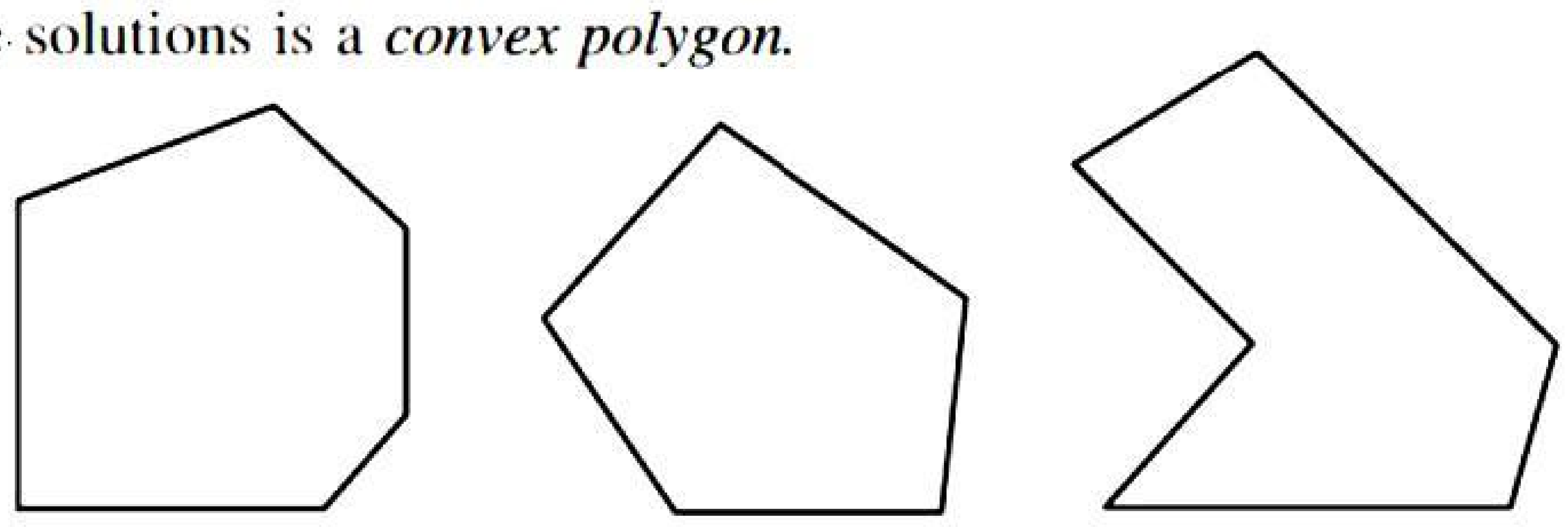


Fig.

In the above figures, (*i*) and (*ii*) are convex and (*iii*) is not convex. After determining the convex polygon, we select a point or points in it which will make the objective function maximum or minimum. It may be noted with care that we do not have to consider all the points inside the polygon because the objective function is linear. Thus, we consider the values of the objective function at the vertices of the set of feasible solutions. The largest (smallest) of these values is the maximum (minimum) value of the objective function.

12.5. GRAPHICAL METHOD

Let us consider an example :

Example : A furniture dealer deals in only two items tables and chairs. He has ₹ 50,000·00 to invest and a space to store at most 60 pieces. A table costs him ₹ 2,500·00 and a chair ₹ 500·00. He can sell a table at a profit of ₹ 250·00 and a chair at a profit of ₹ 75·00. Assuming that he can sell all the items that he buys, how should he invest his money in order that he may maximize his profit ? *(Karnataka B. 2014)*

Solution. We formulate the problem mathematically.

Max. possible investment = ₹ 50,000·00, Max. storage space = 60 pieces of furniture.

	Cost	*Profit*
Table :	₹ 2,500·00	₹ 250·00
Chair :	₹ 500·00	₹ 75·00

Let 'x' and 'y' be the number of tables and chairs respectively. Then we have the following constraints :

$x \geq 0$ ...(1) $\qquad y \geq 0$ ...(2)

$2500x + 500y \leq 50000$ *i.e.* $5x + y \leq 100$ ...(3)

and $x + y \leq 60$ ...(4)

Let Z be the profit, then $Z = 250x + 75y$ ...(5)

We are to **maximize 'Z'** subject to constraints (1) – (4).

Let us graph the constraints given in (1) – (4).

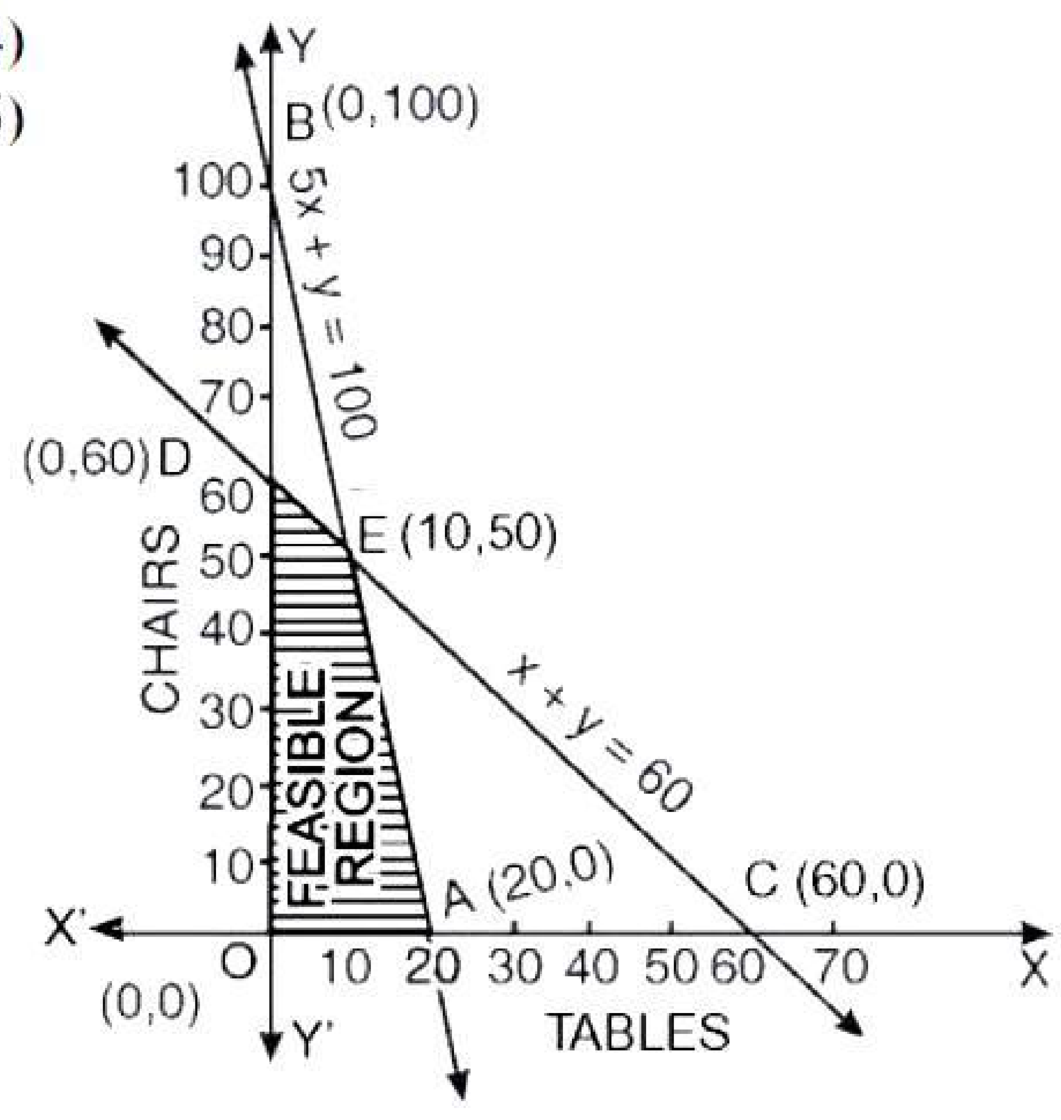


Fig.

The graph of this system (shaded region) consists of the points, which are common to all half-planes, determined by the inequations (1) – (4).

Each point in this region is called a **Feasible choice.**

The region is called the **Feasible region.** Every point of this region is called the **Feasible solution.**

Thus we have :

(I) Feasible Region. The common region, which is determined by all the constraints including non-negative constraints $x, y \geq 0$ of a LPP is called feasible region (or **solution region**). In the above figure, OAED (shaded) is the feasible region of the problem. *(Karnataka B. 2017; Jammu B. 2015)*

The region, other than feasible one, is called an **Infeasible region.**

(II) Feasible Solution. The points, which lie on the boundary and within the feasible region, represent the **feasible solution** of the constraints.

In the above figure, every part in the boundary and within the feasible region OAED, represents the feasible solution of the problem.

Any part outside the feasible solution is called **Infeasible solution**.

In the above figure, (25, 40) is an infeasible solution of the problem.

(III) Optimal (Feasible) Solution.

Any point in the feasible region, which gives the optimal value (maximum or minimum) of the objective function, is called **optimal solution.**

In the above figure, every point in the feasible region OAED satisfies all the constraints as given in (1) – (4).

These points are **infinitely many**.

To find the exact point, we use the following theorems (the proofs of which are beyond the scope of the book).

Theorem I. Let R be the feasible region (convex polygon) for LPP and $Z = ax + by$, the objective function.

When Z has an optimal value (max. or min.) when x, y are subject to constraints, this optimal value will occur at a corner point* (vertex).

Theorem II. Let R be the feasible region for LPP and

$Z = ax + by$, the objective function.

If R is bounded**, then the objective function Z has both maximum and minimum values on R and each occurs at a corner point* (vertex).

KEY POINT

When R is unbounded, max. (or min.) value of objective function may not exist. In case it exists, it occurs at a corner point of R.

In the above example, the corner points (vertices) of the bounded feasible region are :

O (0, 0), A (20, 0), E (10, 50) and D (0, 60).

Thus we have :

Vertex of Feasible Region	Corresponding value of Z (in ₹)
O : (0, 0)	0
A : (20, 0)	5000
E : (10, 50)	**6250 (Maximum)**
D : (0, 60)	4500

It is observed that maximum profit is ₹ 6,250 when the dealer buys 10 tables and 50 chairs.

12.6. CORNER POINT METHOD

In order to solve a Linear Programming Problem we use *Corner Point Method,* which is as follows :

*Corner point of a feasible region, is a point in the region, which is the intersection of the boundary lines.

A feasible region is **bounded if it can be enclosed within a circle, otherwise it is called unbounded.

CORNER POINT METHOD

Step I. Obtain the feasible region of LPP and determine its corner points (vertices).
Step II. Evaluate the objective function Z = $ax + by$ at each corner point (vertex).
Let M and m be the largest and smallest values at these points.
Step III. (*a*) When the feasible region is bounded,
then M and m are the maximum and minimum values of Z.
(*b*) When the feasible region is unbounded, then
(*i*) M is the maximum value of Z if the open half-plane determined by $ax + by > M$ has no common point with the feasible region; otherwise Z has no maximum value.
(*ii*) m is the minimum value of Z if the open half-plane determined by $ax + by < m$ has no common point with the feasible region; otherwise Z has no minimum value.

Frequently Asked Questions

FAQs

Example 1. A small firm manufactures neclaces and bracelets. The total number of neclaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a neclace. The maximum number of hours available per day is 16. If the profit on a neclace is ₹ 100 and that on a bracelet is ₹ 300. Formulate an LPP for finding how many of each should be produced daily to maximize the profit ?

It is being given that at least one of each must be produced. *(C.B.S.E. 2017)*

Solution. Let 'x' neclaces and 'y' bracelets be manufactured per day.

Then LPP problem is :

Maximize : $Z = 100x + 300y$

Subject to the constraints :

$x + y \le 24$,

$(1)(x) + \frac{1}{2}y \le 16$, *i.e.* $2x + y \le 32$

and $x \ge 1$ and $y \ge 1$ *i.e.* $x - 1 \ge 0$ and $y - 1 \ge 0$.

Example 2. Old hens can be bought for ₹ 2·00 each and young ones at ₹ 5.00 each. The old hens lay 3 eggs per week and the young hens 5 eggs per week, each egg being worth 30 paise. A hen costs ₹ 1.00 per week to feed. A man has only ₹ 80 to spend for hens. Formulate the problem for maximum profit per week, assuming that he cannot house more than 20 hens. *(H.B. 2010)*

Solution. Let 'x' be the number of old hens and 'y' the number of young hens.

$$\text{Profit} = (3x + 5y)\frac{30}{100} - (x + y)(1)$$

$$= \frac{9x}{10} + \frac{3}{2}y - x - y = \frac{y}{2} - \frac{x}{10} = \frac{5y - x}{10}.$$

∴ LPP problem is :

Maximize $Z = \frac{5y - x}{10}$ subject to :

$x \ge 0, y \ge 0, x + y \le 20$ and $2x + 5y \le 80$.

Example 3. Maximize Z = 5x + 3y
subject to the constraints :
3x + 5y ≤ 15, 5x + 2y ≤ 10, x ≥ 0, y ≥ 0.

(N.C.E.R.T.; H.B. 2015, 14, 13; H.P.B. 2017, 13 S, 12, 10 ; Kerala B. 2013; Jammu B. 2012)

Solution. The system of constraints is :

$3x + 5y \le 15$ ...(1)

$5x + 2y \le 10$ ...(2)

and $x \ge 0, y \ge 0$ ...(3)

The shaded region in the following figure is the feasible region determined by the system of constraints (1) – (3) :

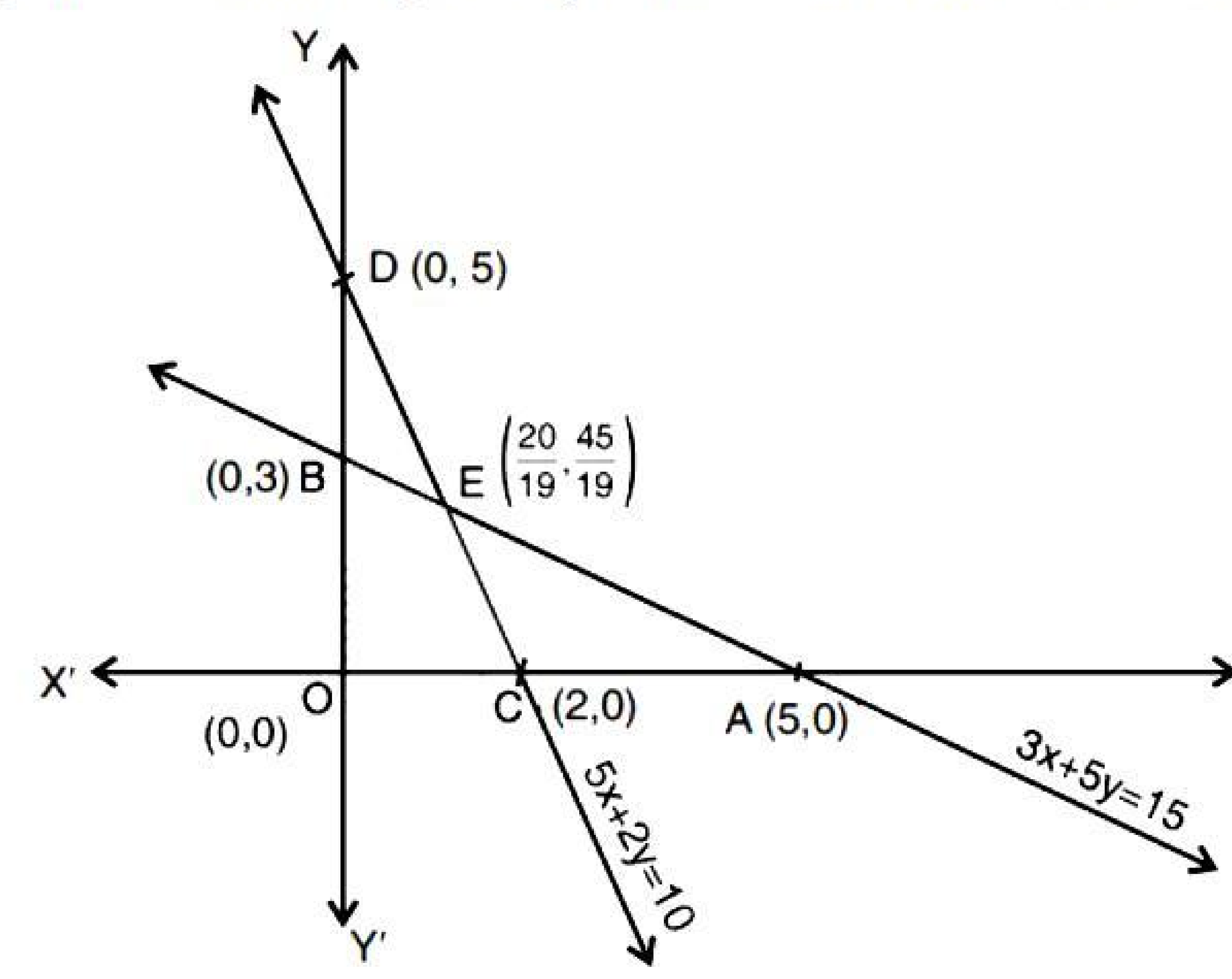


Fig.

It is observed that the feasible region OCEB is bounded. Thus we use ***Corner Point Method*** to determine the maximum value of Z, where :

$Z = 5x + 3y$ ...(4)

The co-ordinates of O, C, E and B are (0, 0), (2, 0), $\left(\frac{20}{19}, \frac{45}{19}\right)$ (*Solving 3x + 5y = 15 and 5x + 2y = 10*) and (0, 3) respectively.

We evaluate Z at each corner point :

Corner Point	Corresponding Value of Z
O : (0, 0)	0
C : (2, 0)	10
E : $\left(\frac{20}{19}, \frac{45}{19}\right)$	$\frac{235}{19}$ **(Maximum)**
B : (0, 3)	9

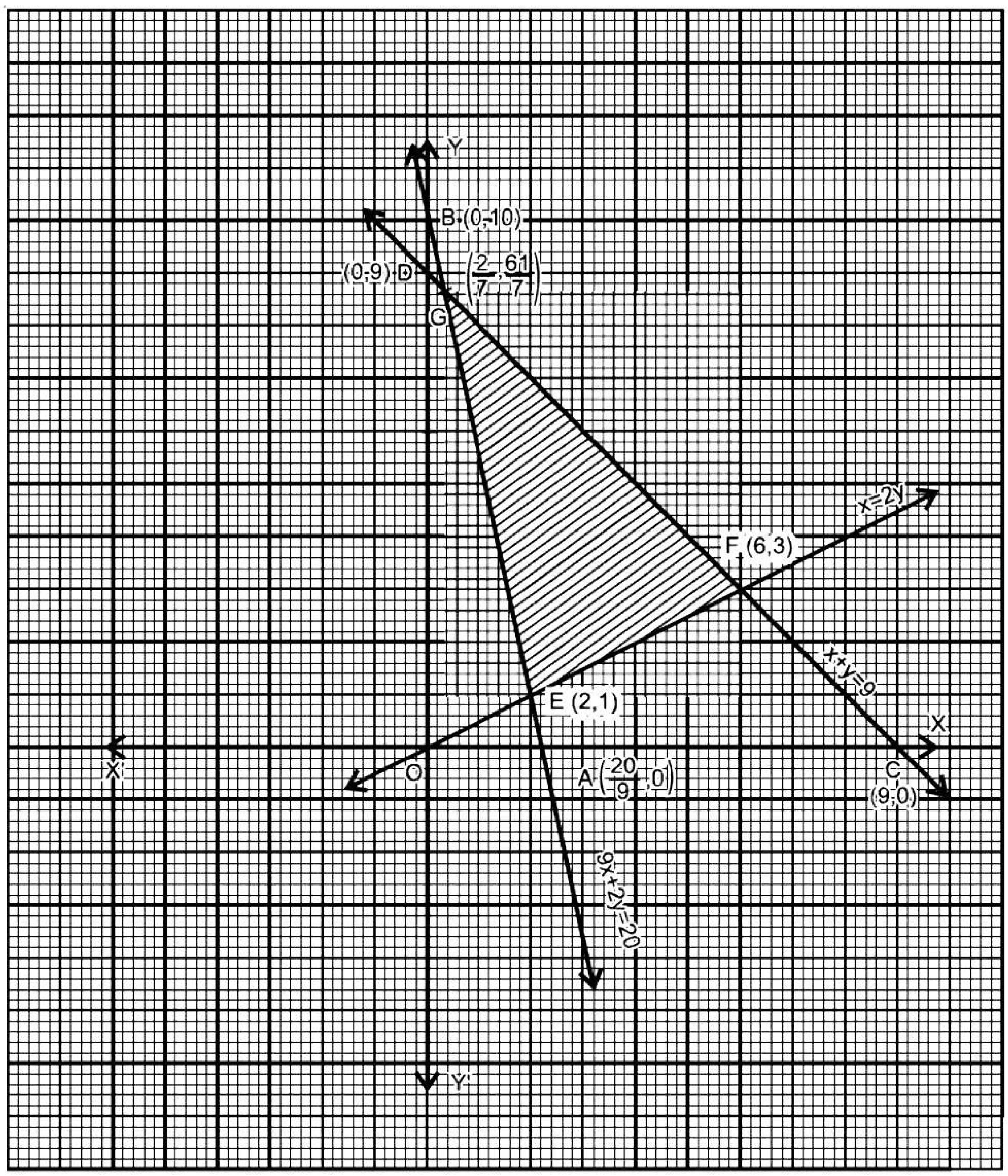
Hence, $Z_{max} = \frac{235}{19}$ at the point $\left(\frac{20}{19}, \frac{45}{19}\right)$.

Example 4. Graphically maximise Z = 9x + 10y subject to constraints :
9x + 2y ≥ 20, x – 2y ≥ 0, x + y ≤ 9, x ≥ 0, y ≥ 0. *(P.B. 2016)*

Solution. The system of constraints is :

$$9x + 2y \geq 20 \quad ...(1)$$
$$x - 2y \geq 0 \quad ...(2)$$
$$x + y \leq 9 \quad ....(3)$$
and $$x \geq 0, y \geq 0 \quad ...(4)$$

The shaded region in the following figure is the feasible region determined by the system of constraints (1) – (4).



It is observed that the feasible region EFG is bounded.
Thus we use ***Corner Point Method*** to determine the maximum value of Z, where $Z = 9x + 10y$ ....(5)

The co-ordinates of E, F and G are (2, 1), (6, 3) and $\left(\frac{2}{7}, \frac{61}{7}\right)$ respectively.

[*Solving $9x + 2y = 20$ and $x - 2y = 0$, we get E (2, 1); etc.*]

We evaluate Z at each corner point :

Corner Point	**Corresponding value of Z**
E : (2, 1)	28
F : (6, 3)	84
G : $\left(\frac{\mathbf{2}}{\mathbf{7}}, \frac{\mathbf{61}}{\mathbf{7}}\right)$	$\frac{\mathbf{628}}{\mathbf{7}}$ **(Maximum)**

Hence, $Z_{max} = \frac{628}{7}$ at the point $\left(\frac{2}{7}, \frac{6}{7}\right)$.

Example 5. Solve the following LPP graphically :

Maximise : Z = 4x + y

Subject to following constraints :

$$\begin{aligned} x + y &\le 50 \\ 3x + y &\le 90 \\ x &\ge 10 \\ \text{and} \quad x, y &\ge 0. \end{aligned}$$

(C.B.S.E. 2017)

Solution. The system of constraints is :

$x + y \le 50$ ....(1)

$3x + y \le 90$ ....(2)

$x \ge 10$ ....(3)

and $x \ge 0;\ y \ge 0$ ....(4)

The shaded region in the following figure is the feasible region determined by the system of constraints (1) – (4) :

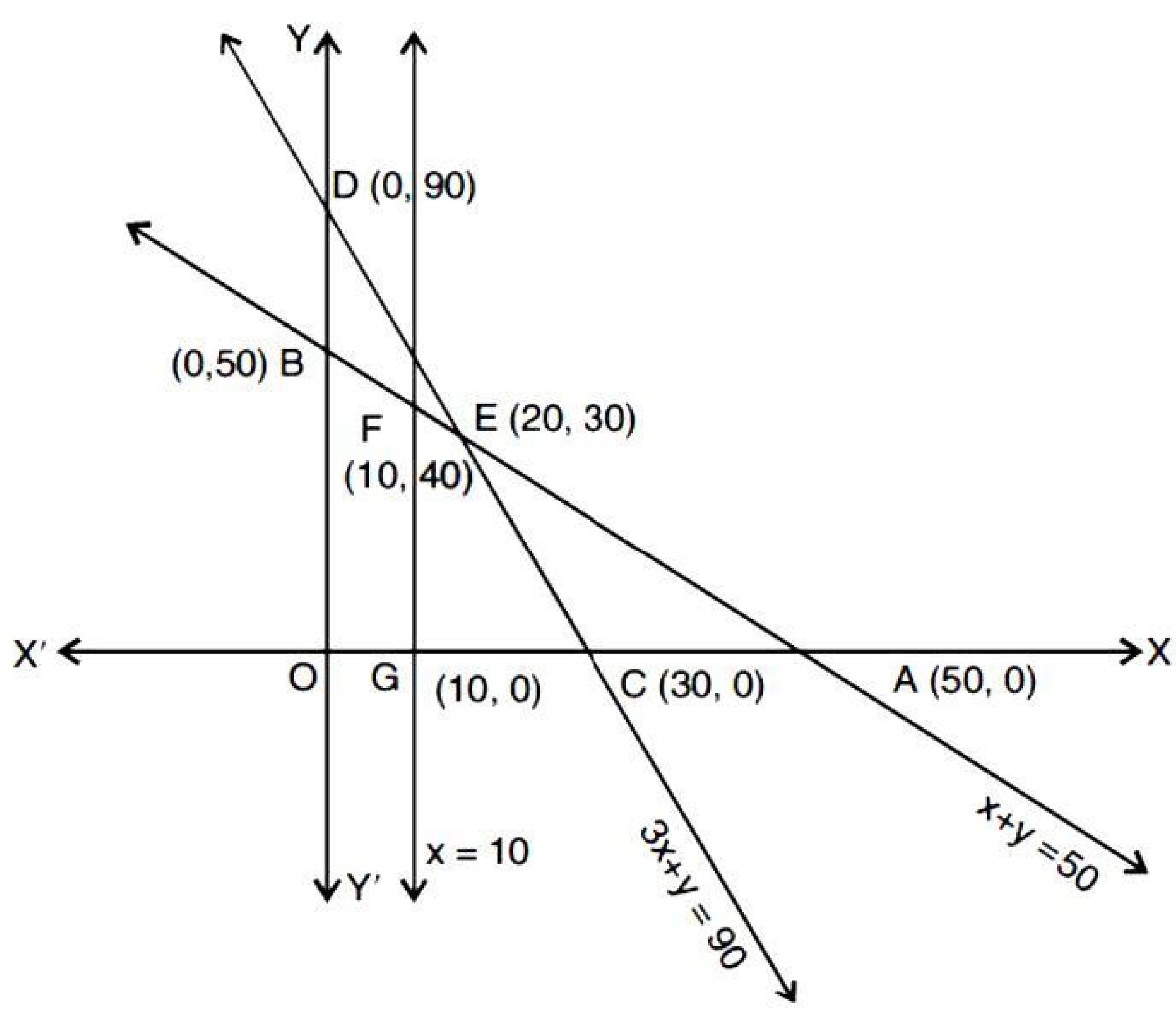


Fig.

It is observed that the feasible region GCEF is bounded.
Thus we use ***Corner Point Method*** to determine the maximum value of Z, where $Z = 4x + y$ ....(5)

The co-ordinates of G, C, E and F are (10, 0), (30, 0),

(20, 30) [*Solving* $x + y = 5$ *and* $3x + y = 90$] and

(10, 40) [*Solving* $x + y = 50$ *and* $x = 10$] respectively

We evaluate Z at each corner point :

Corner Point	Corresponding Value of Z
G : (10, 0)	40
C : (30, 0)	**120 (Maximum)**
E : (20, 30)	110
F : (10, 40)	80

Hence, $Z_{max} = 120$ at the point (30, 0).

Example 6. Minimize Z = 3x + 2y

subject to the constraints :

x + y ≥ 8, 3x + 5y ≤ 15, x ≥ 0, y ≥ 0. (*N.C.E.R.T.*)

Solution. The system of constraints is :

$x + y \geq 8$ ...(1)

$3x + 5y \leq 15$ ...(2)

and $x, y \geq 0$ ...(3)

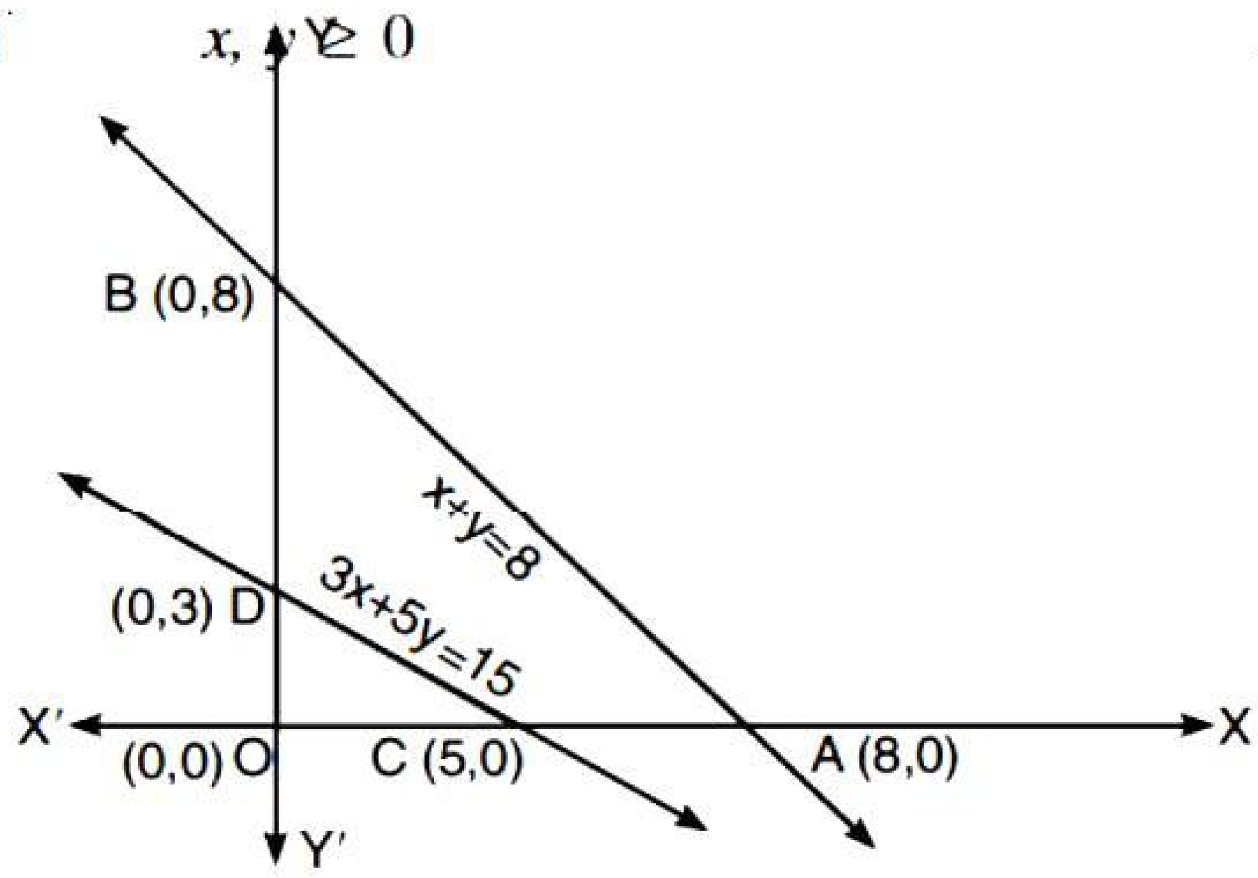


Fig.

It is observed that there is no point, which satisfies all (1) – (3) simultaneously.

Thus there is no feasible region.

Hence, there is no feasible solution.

Example 7. Maximise and Minimise :

Z = 4x + 3y – 7

subject to the constraints :

x + y ≤ 10, x + y ≥ 3, x ≤ 8, y ≤ 9, x, y ≥ 0.

(*P.B. 2018*)

Solution. The given system of constraints is :

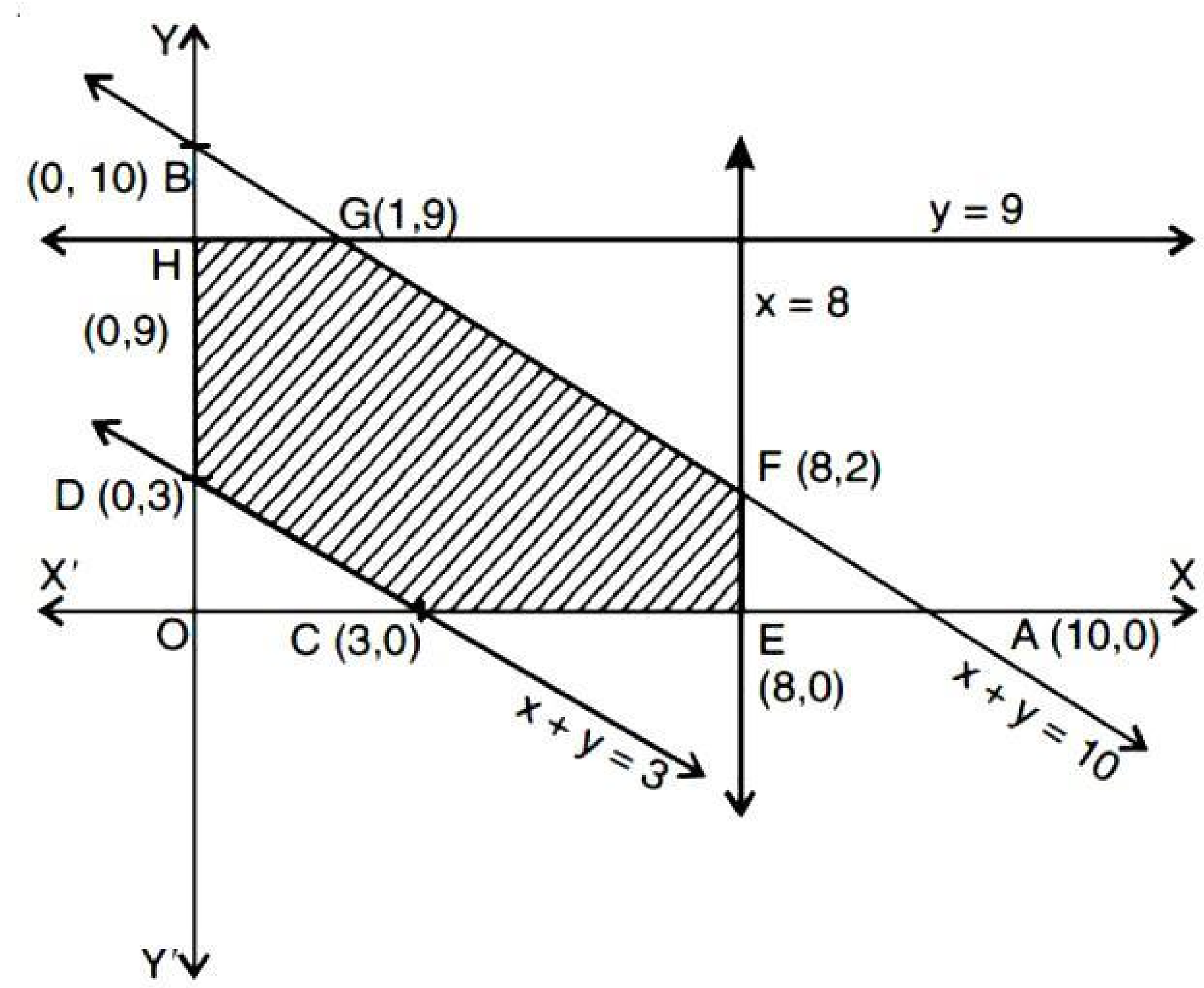
$x + y \leq 10$ ...(1)

$x + y \geq 3$ ...(2)

$x \leq 8$ ...(3)

$y \leq 9$ ...(4)

$x, y \geq 0$ ...(5)



The shaded region in the above figure is the feasible region determined by the system of constraints (1) – (5). It is observed that the feasible region DCEFGH is bounded. Thus we use ***Corner Point Method*** to determine the maximum and minimum values of Z, where :

$$Z = 4x + 3y - 7 \qquad ...(6)$$

The co-ordinates of D, C, E, F, G and H are respectively (0, 3), (3, 0), (8, 0), (8, 2),

[*Solving* $x = 8$ *and* $x + y = 10$]

(1, 9) [*Solving* $y = 9$ *and* $x + y = 10$]

and (0, 9).

We evaluate Z at each corner point.

Corner Point	Corresponding Value of Z
D : (0, 3)	**2 (Minimum)**
C : (3, 0)	5
E : (8, 0)	25
F : (8, 2)	**31 (Maximum)**
G : (1, 9)	24
H : (0, 9)	20

Hence, $Z_{max} = 31$ at (8, 2) and $Z_{min} = 2$ at (0, 3).

Example 8. Determine graphically the minimum value of the objective function :

$$Z = -50x + 20y$$

subject to the constraints :

$2x - y \geq -5,\ 3x + y \geq 3,\ 2x - 3y \leq 12,\ x, y \geq 0.$

(N.C.E.R.T.)

Solution. The system of constraints is :

$2x - y \geq -5$ ...(1)

$3x + y \geq 3$ ...(2)

$2x - 3y \leq 12$ ...(3)

and $x, y \geq 0$ ...(4)

The shaded region in the following figure is the feasible region determined by the system of constraints (1) – (4).

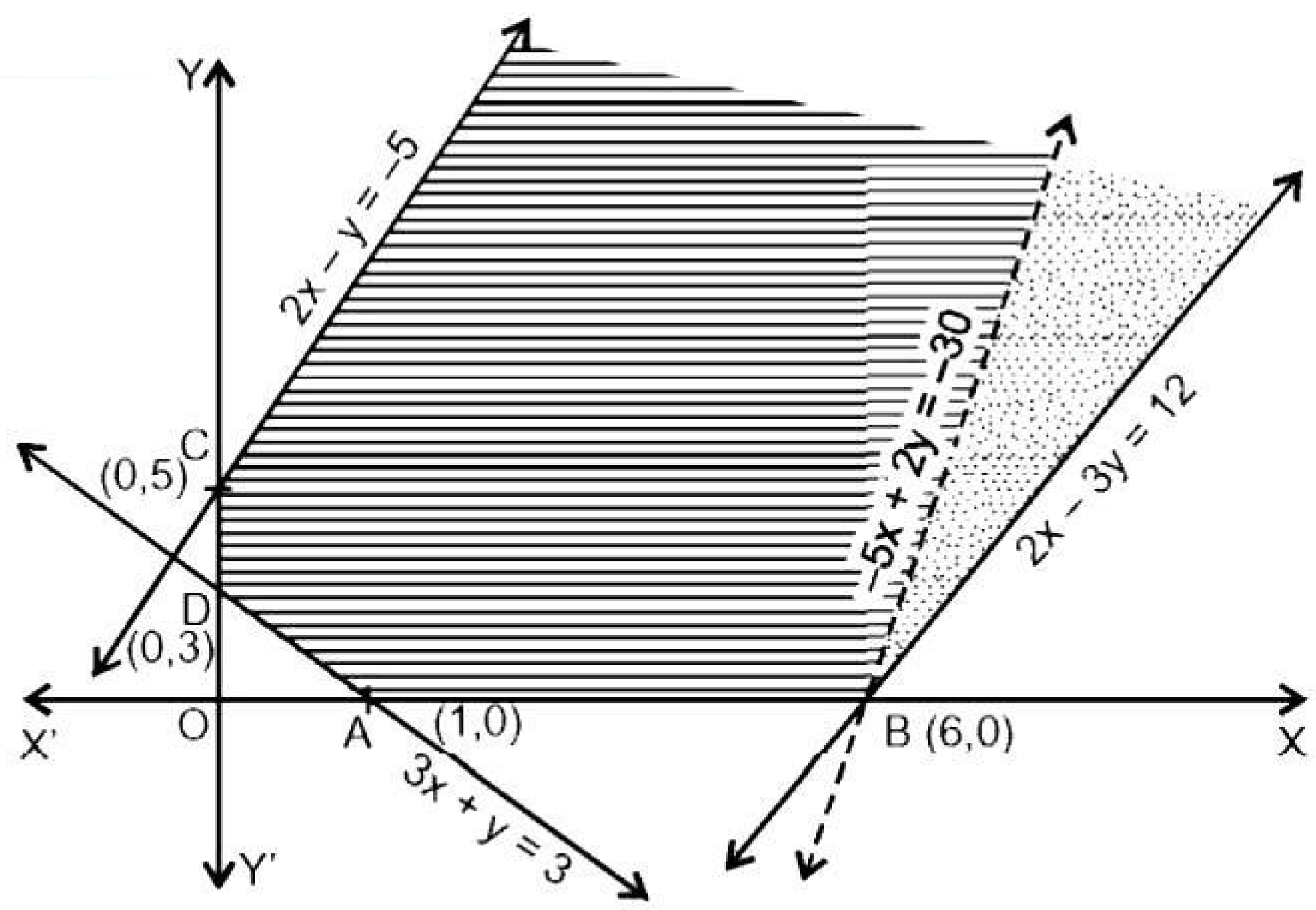


Fig.

It is observed that the feasible region is unbounded.

We evaluate $Z = -50x + 20y$ at the corner points :

A (1, 0), B (6, 0), C (0, 5) and D (0, 3) :

Corner Point	Corresponding Value of Z
A : (1, 0)	– 50
B : (6, 0)	**– 300 (Minimum)**
C : (0, 5)	100
D : (0, 3)	60

From the table, we observe that – 300 is the minimum value of Z.

But the feasible region is unbounded.

∴ – 300 may or may not be the minimum value of Z.

For this, we draw the graph of the inequality.

$-50x + 20y < -300$

i.e. $-5x + 2y < -30.$

Since the remaining half-plane has common points with the feasible region,

∴ $Z = -50x + 20y$ has no minimum value.

Example 9. Minimize and Maximize Z = 5x + 2y, subject to the following constraints :

$x - 2y \leq 2,\ 3x + 2y \leq 12,\ -3x + 2y \leq 3,\ x \geq 0,\ y \geq 0.$

(A.I.C.B.S.E. 2015)

Solution. The given system of constraints is :

$x - 2y \leq 2$ ...(1)

$3x + 2y \leq 12$ ...(2)

$-3x + 2y \leq 3$ ...(3)

and $x \geq 0,\ y \geq 0$ ...(4)

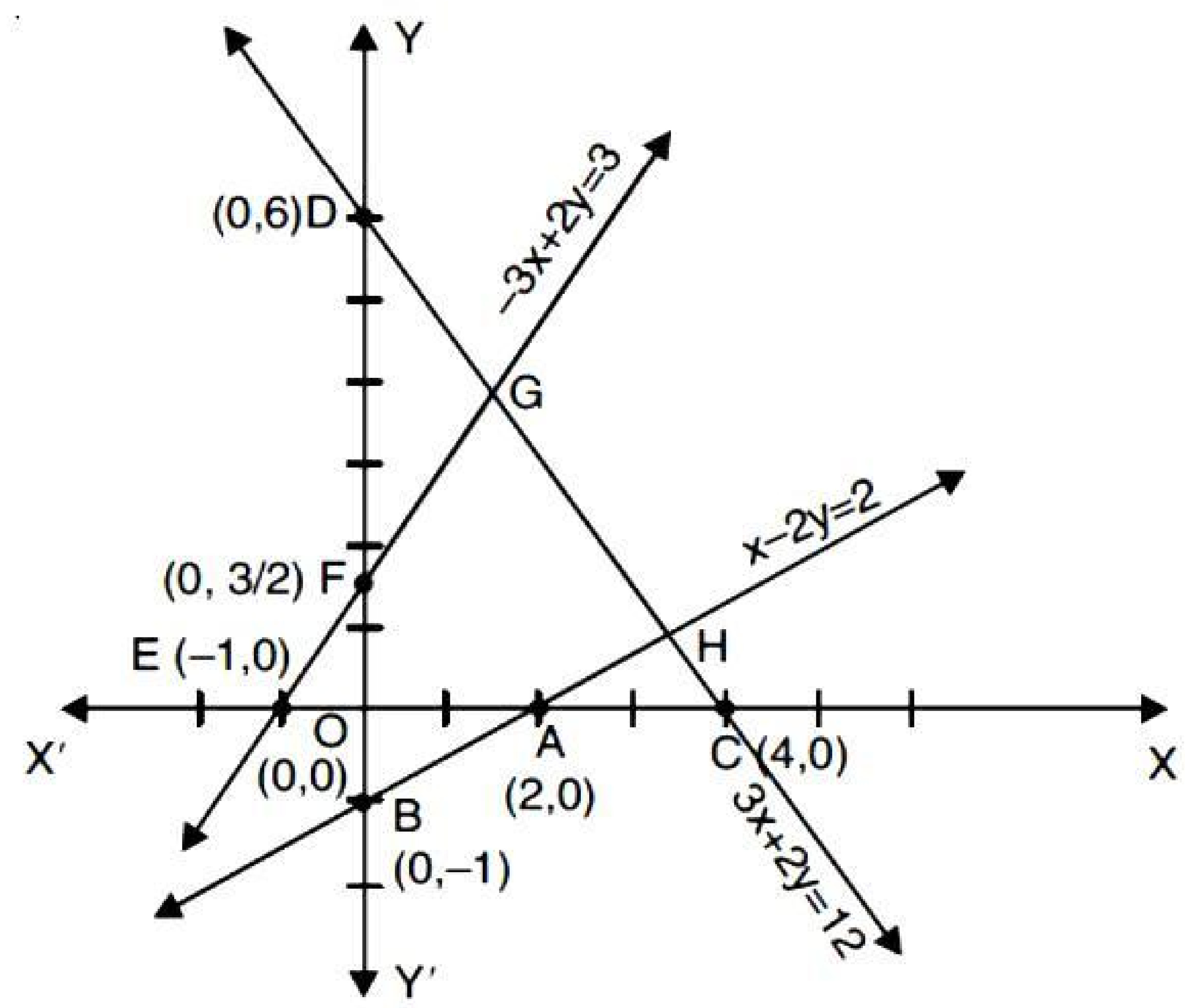


Fig.

The shaded region in the above figure is the feasible region determined by the system of constraints (1) – (4). It is observed that the feasible region OAHGF is bounded. Thus we use ***Corner Point Method*** to determine the maximum and minimum value of Z, where

$Z = 5x + 2y$ ...(5)

The co-ordinates of O, A, H, G and F are :

$(0, 0), (2, 0), \left(\frac{7}{2}, \frac{3}{4}\right), \left(\frac{3}{2}, \frac{15}{4}\right)$ and $\left(0, \frac{3}{2}\right)$ respectively.

[Solving $x - 2y = 2$ and $3x + 2y = 12$ for H and $-3x + 2y = 3$ and $3x + 2y = 12$ for G]

We evaluate Z at each corrner point :

Corner Point	Corresponding value of Z
O : (0, 0)	**0 (Minimum)**
A : (2, 0)	10
H : $\left(\frac{7}{2}, \frac{3}{4}\right)$	**19 (Maximum)**
G : $\left(\frac{3}{2}, \frac{15}{4}\right)$	15
F : $\left(0, \frac{3}{2}\right)$	3

Hence, $Z_{max} = 19$ at $\left(\frac{7}{2}, \frac{3}{4}\right)$ and $Z_{min} = 0$ at (0, 0).

EXERCISE 12 (b)

Fast Track Answer Type Questions

FTATQ

1. Two tailors, A and B earn ₹ 300 and ₹ 400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers per day while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulates this as an LPP. *(A.I.C.B.S.E. 2017)*

2. A dietician wishes to mix two types of food in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food 'I' contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C while Food 'II' contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₹ 50 per kg to purchase Food 'I' and ₹ 70 per kg to purchase Food 'II'. Formulate the problem for minimum cost of such a mixture. ***(H.B. 2010)***

3. A firm can produce three types of cloth say 'A', 'B' and 'C'. Three kinds of wool are required for it, say red wool, green wool and blue wool. One unit length of type 'A' cloth needs 2 yards of red wool and 3 yards of blue wool, one unit length of type 'B' cloth needs 3 yards of red wool, 2 yards of green wool and 2 yards of blue wool, one unit length of type 'C' cloth needs 5 yards of green wool and 4 yards of blue wool. The firm has only a stock of 8 yards of red wool, 10 yards of green wool and 15 yards of blue wool. It is assumed that the income obtained from one unit of type 'A' cloth is ₹ 3, for type 'B' cloth is ₹ 5 and for type 'C' cloth is ₹ 4. Formulate the problem as a LPP so as to maximize the profit of the firm by using the available materials.

4. A furniture firm manufactures chairs and tables, each requiring the use of three machines 'A', 'B' and 'C'. Producing of one chair requires 2 hours on machine 'A', 1 hour on machine 'B' and 1 hour on machine 'C'. Each table requires 1 hour each on machines 'A' and 'B' and 3 hours on machine 'C'. The profit realized by selling one chair is ₹ 30 while for a table is ₹ 60. The total time available per week on machine 'A' is 70 hours, on machine 'B' is 40 hours and on machine 'C' is 90 hours. Find the mathematical formulation so as to find the number of chairs and tables that should be made per week so as to maximize the profit.

5. (*a*) (*i*) Maximize $Z = x + 2y$ subject to :
   $x + y \geq 5,\ x \geq 0,\ y \geq 0$ ***(Jammu B. 2014)***
   (*ii*) Maximize $Z = 2x + 3y$ subject to :
   $x + 2y \leq 6,\ x \geq 4,\ y \geq 0$ ***(Jammu B. 2013)***
   (*iii*) Maximize $Z = 4x + y$ subject to :
   $x + y \leq 50;\ x, y \geq 0$ ***(Kashmir B. 2013)***
   (*iv*) Maximize $Z = x + 2y$ subject to :
   $2x + y \leq 6,\ x, y \geq 0$ ***(Kashmir B. 2015)***
   (*v*) Maximize $Z = 5x + 3y$ subject to :
   $3x + 5y \leq 10,\ x, y \geq 0$ ***(Jammu B. 2017)***
   (*vi*) Maximize $Z = 3y + 5x$ subject to :
   $3x + 5y \leq 15,\ x, y \geq 0$ ***(Jammu B. 2017)***
   (*b*) (*i*) Minimize $Z = 3x + 2y$ subject to :
   $x + y \geq 8,\ x, y \geq 0.$ ***(Jammu B. 2015W)***
   (*ii*) Minimize $Z = 3x + 9y$ subject to :
   $x + 3y \leq 60,\ x \leq y$ and $x, y \geq 0.$ ***(Jammu B. 2017)***

Short Answer Type Questions

SATQ

Solve the following Linear Programming Problems graphically.

Maximize (6 – 16) :

	OBJECTIVE FUNCTION	CONSTRAINTS	
6.	$Z = 6x + 8y$	$x + y \leq 6,\ 3x + y \geq 6,\ x - y \geq 0,\ x \geq 0,\ y \geq 0.$	***(P.B. 2017)***
7.	$Z = 4x + y$	$x + y \leq 50,\ 3x + y \leq 90,\ x \geq 0,\ y \geq 0.$	***(N.C.E.R.T. ; C.B.S.E. 2017; H.P.B. 2016, 14, 11; Jammu B. 2016; Uttarakhand B. 2015; H.B. 2014, 13 ; P.B. 2012 ; Kashmir 2011)***
8.	$Z = 3x + 2y$	$x + 2y \leq 10,\ 3x + y \leq 15,\ x \geq 0,\ y \geq 0.$	***(N.C.E.R.T.; H.P.B. 2018, 17, 16, 14, 13 S; Jammu B. 2015 W, 15; H.B. 2014; P.B. 2013, 12 ; Uttarakhand B. 2013)***
9.	$Z = 13x + 3y$	$x + y \leq 6,\ 3x + 2y \leq 15,\ x \geq 0,\ y \geq 0.$	***(P.B. 2011)***
10.	$Z = 3x + 5y$	$x + y \geq 2,\ x + 3y \geq 3,\ x \geq 0,\ y \geq 0.$	***(Rajasthan B. 2012)***
11.	$Z = 4x + 7y$	$x + 2y \leq 20,\ x + y \leq 15,\ x \geq 0,\ y \geq 0.$	***(P.B. 2012)***
12.	$Z = -3x + 4y$	$x + 2y \leq 8,\ 3x + 2y \leq 12,\ x \geq 0,\ y \geq 0.$	***(Jammu B. 2013)***
13.	$Z = 6x + 11y$	$2x + y \leq 104,\ x + 2y \leq 76,\ x, y \geq 0.$	***(H.B. 2017, 11)***
14.	$Z = 34x + 45y$	$x + y \leq 300,\ 2x + 3y \leq 70,\ x, y \geq 0.$	***(A.I.C.B.S.E. 2017)***
15.	$Z = 7x + 4y$	$2x + y \leq 10,\ x + 2y \leq 12,\ x \geq 0,\ y \geq 0.$	***(P.B. 2015)***

16. (*i*)	$Z = 20x + 10y$	$x + 2y \le 28, 3x + y \le 24, x \ge 2, x, y \ge 0.$	*(C.B.S.E. 2017)*
(*ii*)	$Z = 7x + 10y$	$4x + 6y \le 240, 6x + 3y \le 240, x \ge 10, x \ge 0, y \ge 0.$	*(A.I.C.B.S.E. 2017)*

Minimize (17 – 25) :

	OBJECTIVE FUNCTION	CONSTRAINTS	
17.	$Z = x + 2y$	$2x + y \ge 3, x + 2y \ge 6, x, y \ge 0.$	*(H.P.B. 2018)*
18.	$Z = 200x + 500y$	$x + 2y \ge 10, 3x + 4y \le 24, x \ge 0, y \ge 0.$	*(N.C.E.R.T.; Kashmir B. 2017, 11; H.P.B. 2017; H.B. 2016;)*
19.	$Z = -3x + 4y$	$x + 2y \le 8, 3x + 2y \le 12, x \ge 0, y \ge 0.$	*(N.C.E.R.T.; H.P.B. Model Paper 2018; H.B. 2018; Kerala B. 2018; H.P.B. 2018, 15, 12, 10 S; P.B. 2014 S; Jammu B. 2014, 12)*
20.	$Z = 3x + 5y$	$x + 3y \ge 3, x + y \ge 2, x \ge 0, y \ge 0.$	*(N.C.E.R.T.; H.B. 2015)*
21.	$Z = 2x + 3y$	$x \ge 0, y \ge 0, 1 \le x + 2y \le 10.$	*(N.C.E.R.T.; H.P.B. 2015,12)*
22.	$Z = 3x + 9y$	$x + 3y \le 60, x + y \ge 10, x \le y, x \ge 0, y \ge 0.$	*(H.B. 2017; H.P.B. 2015; Kerala B. 2014)*
23.	$Z = 5x + 10y$	$x + y \ge 60, x + 2y \le 120, x - 2y \ge 0, x, y \ge 0.$	*(C.B.S.E. 2017; Jammu B. 2017; Bihar B. 2014)*
24.	$Z = 2x + 5y$	$2x + 4y \le 8, 3x + y \le 6, x + y \le 4, x \ge 0, y \ge 0.$	*(C.B.S.E. 2015)*
25.	$Z = 200x + 500y$	$x + 2y \ge 10, 3x + 4y \le 24, x \ge 0, x \ge 0, y \ge 0.$	*(Kashmir B. 2016)*

26. Maximize, if possible :

(*i*) $Z = 3x + 2y$ subject to the constraints :
$x - y \le 1, x + y \ge 3, x \ge 0, y \ge 0$

(*ii*) $Z = 3x + 4y$ subject to the constraints :
$x - y \le -1, -x + y \le 0, x \ge 0, y \ge 0.$

27. Maximize :
$Z = -x + 2y$, subject to the constraints :
$x \ge 3, x + y \ge 5, x + 2y \ge 6, y \ge 0.$ *(N.C.E.R.T.)*

28. Maximize :
$Z = x + y$, subject to the constraints :
$x - y \le -1, -x + y \le 0, x \ge 0, y \ge 0.$ *(N.C.E.R.T.)*

29. Verify that the following problem has no feasible solution :
Maximize $Z = 4x_1 + 2x_2$, subject to the constraints :
$2x_1 + 3x_2 \le 18, x_1 + x_2 \ge 12, x_1, x_2 \ge 0.$

Long Answer Type Questions

LATQ

Minimize and Maximize (30-33) :

	OBJECTIVE FUNCTION	CONSTRAINTS	
30.	$Z = 3x + 9y$	$x + 3y \le 60, x + y \ge 10, x \le y, x \ge 0, y \ge 0.$	*(N.C.E.R.T.; Jammu B. 2018; Assam B. 2017; Kashmir B. 2011)*
31.	$Z = 5x + 10y$	$x + 2y \le 120, x + y \ge 60, x - 2y \ge 0, x \ge 0, y \ge 0.$	*(N.C.E.R.T.; P.B. 2014; Kashmir B. 2013; H.P.B. 2010 S)*
32.	$Z = 15x + 30y$	$x + y \le 8, 2x + y \ge 8, x - 2y, x, y \ge 0.$	*(P.B. 2018)*
33.	$Z = x + 2y$	$x + 2y \ge 100, 2x - y \le 0, 2x + y \le 200, x, y \ge 0.$	*(N.C.E.R.T.; A.I.C.B.S.E. 2017; Jammu B. 2015; Assam B. 2015; Karnataka B. 2014; Kashmir B. 2012 S)*
34. (*i*)	$Z = 3x + 2y$	$x + 3y \le 60, x + y \ge 10, x \le y, x, y \ge 0$	*(Assam B. 2016)*
(*ii*)	$Z = 800x + 1200y$	$3x + 4y \le 80, x + 3y \le 30, x \ge 0, y \ge 0.$	*(Assam B. 2018)*

35. Consider the following LPP :
Maximize $Z = 3x + 2y$ subject to the constraints :
$x + 2y \le 10, 3x + y \le 15, x, y \ge 0.$
(*a*) Draw the feasible region.
(*b*) Find the corner points of the feasible region.
(*c*) Find the maximum value of Z. *(Kerala B. 2016)*

Answers

1. Minimize Z = $300x + 400y$ subject to : $3x + 5y \geq 30$, $x + y \geq 8$, $x, y \geq 0$.
2. Minimize Z = $50x + 70y$ subject to : $2x + y \geq 8$, $x + 2y \geq 10$, $x \geq 0$, $y \geq 0$.
3. Maximize P = $3x + 5y + 4z$ subject to : $2x + 3y \leq 8$, $2y + 5z \leq 10$, $3x + 2y + 4z \leq 15$, $x \geq 0$, $y \geq 0$, $z \geq 0$.
4. Maximize Z = $30x + 60y$ subject to : $2x + y \leq 70$, $x + y \leq 40$, $x + 3y \leq 90$, $x, y \geq 0$.
5. (*a*) (*i*) 10 at (0, 5) (*ii*) 12 at (6, 0) (*iii*) 200 at (50,0) (*iv*) 12 at (10, 6) (*v*) 25 at (5, 0) (*vi*) 25 at (5, 0)
   (*b*) (*i*) 16 at (0, 8) (*ii*) 180 and (15, 15).
6. 42 at (0, 6).
7. 120 at (30, 0).
8. 18 at (4, 3).
9. 65 at (5, 0),
10. 7 at $\left(\frac{3}{2}, \frac{1}{2}\right)$.
11. 75 at (10, 5).
12. 16 at (0, 4).
13. 440 at (44, 16).
14. 1190 at (35, 0).
15. $\frac{112}{3}$ at $\left(\frac{8}{3}, \frac{14}{3}\right)$.
16. (*i*) 200 at (4, 12) (*ii*) 410 at (30, 20).
17. 6 on the line segment joining (0, 3) and (6, 6).
18. 2300 at (4, 3).
19. – 12 at (4, 0).
20. 7 at $\left(\frac{3}{2}, \frac{1}{2}\right)$.
21. $\frac{3}{2}$ at $\left(0, \frac{1}{2}\right)$.
22. 60 at (5, 5).
23. 400 at (40, 20).
24. 10 at (0, 2).
25. 2300 at (4, 3).
26. (*i*) Not finite (*ii*) No max. value.

27. – 29. No max. value.

30. Min = 60 at (5, 5); Max = 180 at all points of the line segment joining the points (15, 15) and (0, 20).
31. Min = 300 at (60, 0); Max = 600 at all points of line segment joining the points (120, 0) and (60, 30).
32. Min = 80 at $\left(\frac{16}{5}, \frac{8}{5}\right)$; Max = 240 at (0, 8).
33. Min = 100 at all points on the line segment joining the points (0, 50) and (20, 40).
    Max = 400 at (0, 200).
34. (*i*) Min = 20 at (0, 10); Max = 180 at (60, 0)
    (*ii*) Min = 0 at (0, 0); Max = 216 at (24, 2).
35. (*b*) (0, 0), (5, 0), (4, 3) and (0, 5)
    (*c*) 18 at (4, 3).

12.7. TYPES OF LINEAR PROGRAMMING PROBLEMS (LPP)

We list below a few important linear programming problems :

(*a*) Diet Problems. Here we obtain the amount of different kinds of constituents and nutrients, which are to be included in the diet in order to minimize the cost of the desired diet so that it contains certain minimum amount of each constituent and nutrient.

(*b*) Transportation Problems. Here we obtain a transportation schedule so as to find the cheapest way of transferring a product from factories (plants) which are situated at different places to different markets.

(*c*) Manufacturing Problems. Here we obtain a number of units of different products, which are to be produced and sold by a firm, when each product needs machine hours, fixed manpower, labour hour per unit of product, warehouse space per unit of output; etc. so as to yield maximum profit.

Now we shall solve some of these types of linear programming problems (LPP).

Frequently Asked Questions

FAQs

Example 1. A dealer in rural area wishes to purchase a number of sewing machines. He has only ₹ 5,760·00 to invest and has a space for at most 20 items. An electronic sewing machine costs him ₹ 360·00 and a manually operated sewing machine ₹ 240·00. He can sell electronic sewing machine at a profit of ₹ 22·00 and a manually operated sewing machine at a profit of ₹ 18·00. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit. Make it a linear programming problem and solve it graphically.

(C.B.S.E. 2014)

Solution. Let 'x' be the number of electronic operated machines and 'y' that of manually operated machines be purchased.

Then the LPP problem is as follows :

Maximize : $Z = 22x + 18y$

Subject to : $x + y \leq 20$ ....(1)

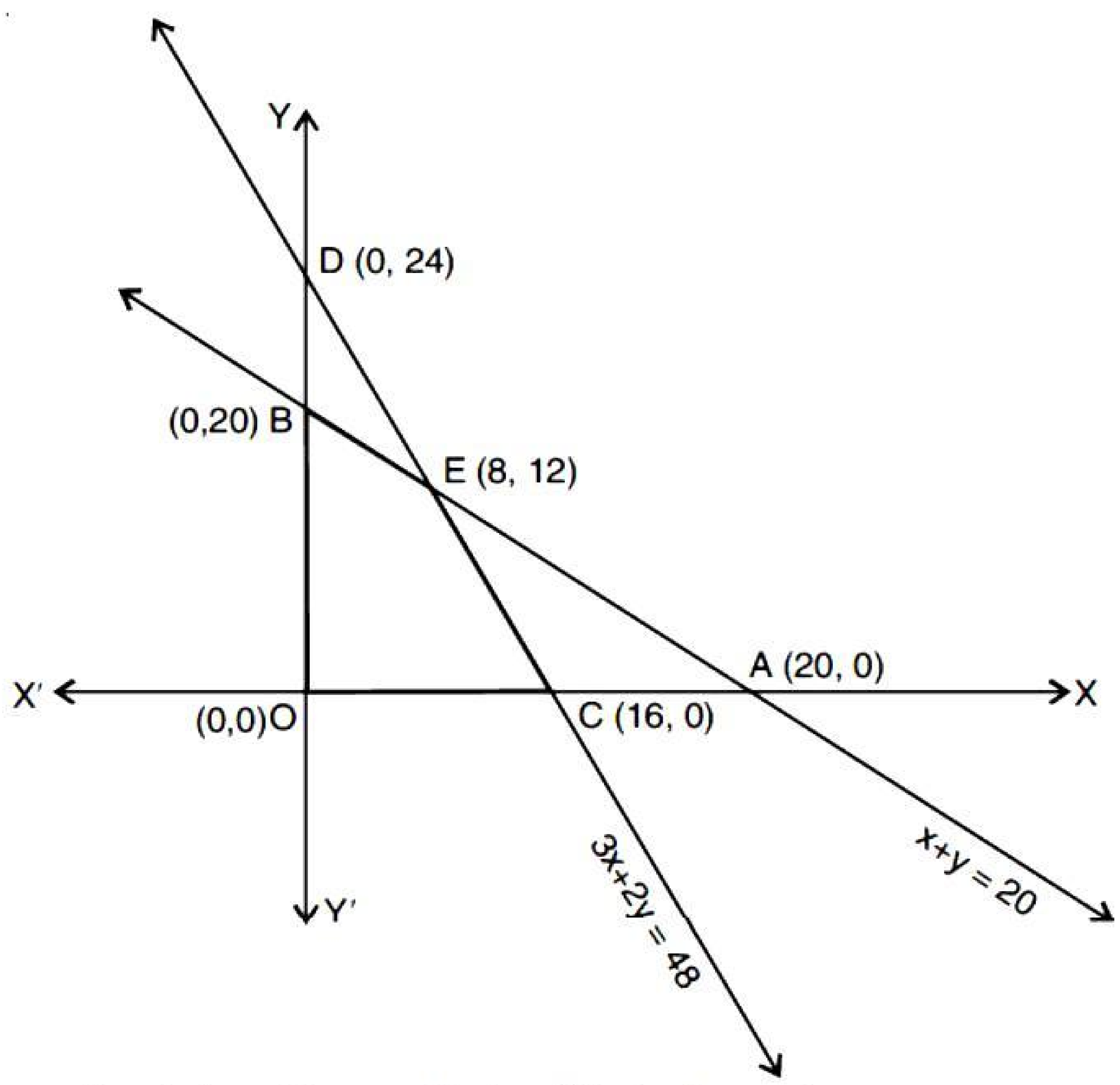
$360x + 240y \leq 5760$

i.e. $3x + 2y \leq 48$ ...(2)

and $x \geq 0, y \geq 0$ ...(3)

The shaded region of the figure represents the feasible region OCEB, which is bounded, where O is (0, 0), C is (16, 0), B is (0, 20) and E is (8, 12).

[$\because$ *Solving* $x + y = 20$ *and* $3x + 2y = 48$; $x = 8, y = 12$]



Applying ***Corner Point Method,*** we have :

Corner Point	Z = 22x + 18y
O : (0, 0)	0
C : (16, 0)	352
E : (8, 12)	**392 (Maximum)**
B : (0, 20)	360

Thus, Z is maximum at E (8, 12).

Hence, the dealer should invest in 8 electronic and 12 manually operated machines.

Example 2. If a man rides his motor cycle at 25 km/hr., he has to spend ₹ 2 per km on petrol, if he rides at a faster speed of 40 km/hr., the petrol cost increases to ₹ 5 per km. He has ₹ 100 to spend on petrol and wishes to find maximum distance he can travel within one hour. Express this as a linear programming problem and then solve it graphically.

(Meghalaya B. 2017; Type : Nagaland B. 2016)

Solution. Let the man travel 'x' km at 25 km/hr. and 'y' km at 40 km/hr.

Clearly $x \geq 0$ ...(1) and $y \geq 0$ ...(2)

Since the total money is ₹ 100,

$\therefore$ $2x + 5y \leq 100$ ...(3)

Since the total time is 1 hour,

$\therefore$ $\frac{x}{25} + \frac{y}{40} \leq 1$ *i.e.* $8x + 5y \leq 200$ ...(4)

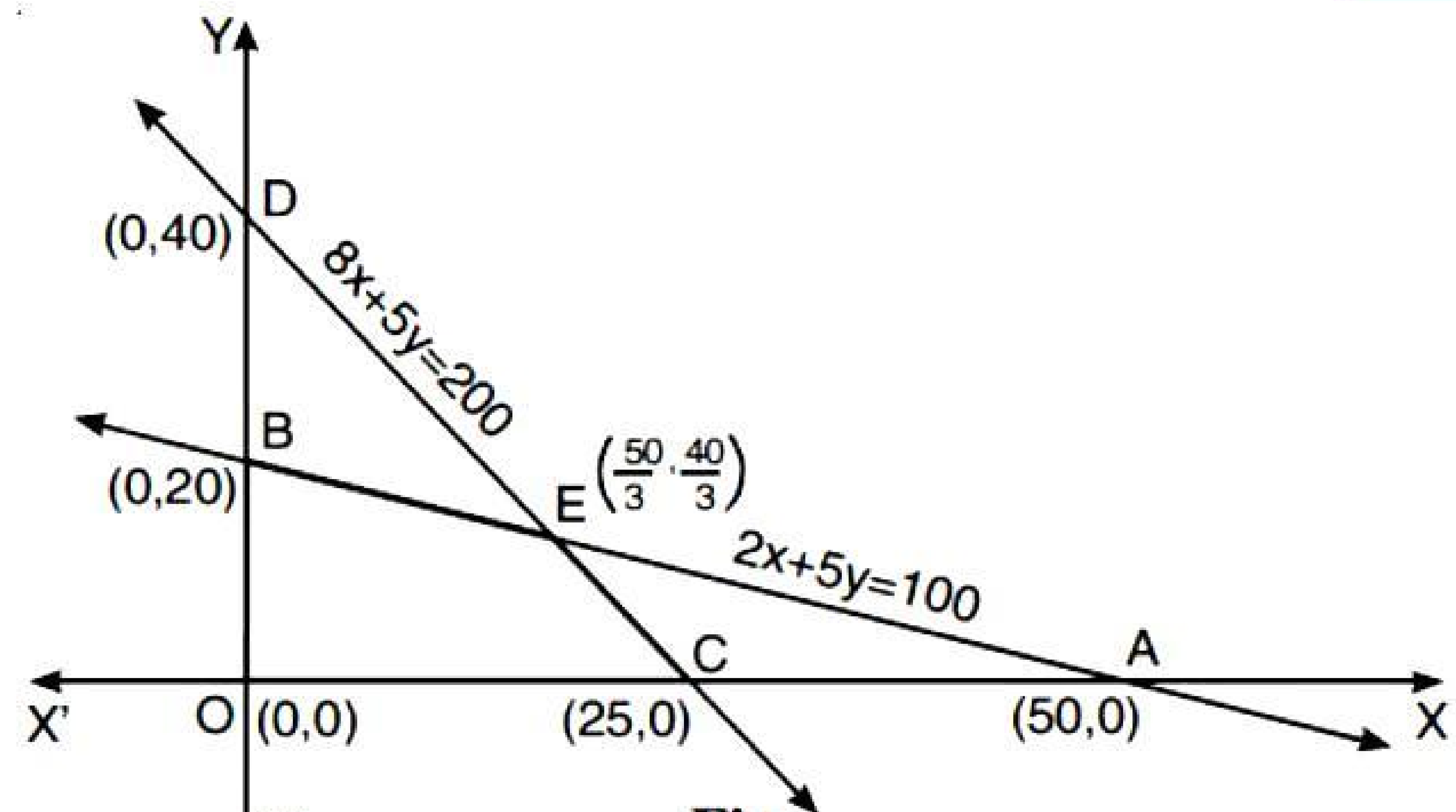


Fig.

The objective function or the distance D, in one hour is :

$D = x + y$ ...(5)

Draw the lines :

$x = 0, y = 0, 2x + 5y = 100$

and $8x + 5y = 200$.

The feasible region (shown shaded) OCEB is bounded, where O is (0, 0), C is (25, 0), B is (0, 20) and E is $\left(\frac{50}{3}, \frac{40}{3}\right)$.

[$\because$ *Solving* $2x + 5y = 100$ *and* $8x + 5y = 200$; $x = \frac{50}{3}, y = \frac{40}{3}$]

Applying ***Corner Point Method,*** we have :

Corner Point	D = x + y
O : (0, 0)	0
C : (25, 0)	25
E : $\left(\frac{50}{3}, \frac{40}{3}\right)$	**30 (Maximum)**
B : (0, 20)	20

Hence, maximum distance travelled = 30 km.

Example 3. (Diet Problem) Every gram of wheat provides 0·1 g of protein and 0·25 g of carbohydrates. The corresponding values for rice are 0·05 g and 0·5 g respectively. Wheat costs ₹ 4 per/kg and rice ₹ 6 per/kg. The minimum daily requirements of protein and carbohydrates for an average child are 50 g and 200 g respectively. In what quantities should wheat and rice be mixed in the daily diet so as to provide the maximum daily requirements of protein and carbohydrates at minimum cost ? Frame an L.P.P. and solve it graphically.

Solution. Let quantity of wheat = x grams.

Quantity of rice = y grams.

Cost of wheat ₹ 4 per/kg and rice ₹ 6 per/kg.

Therefore, $Z = \frac{4x}{1000} + \frac{6y}{1000}$.

The minimum daily requirement of protein = 50 gm and wheat provides 0·1 gm and rice 0·05 gm protein.

Therefore $0{\cdot}1\ x + 0{\cdot}05\ y \geq 50$.

The minimum daily requirements of carbohydrates = 200 gm.

Carbohydrates in wheat = 0·25 gm and in rice = 0·5 gm.

Therefore, $0{\cdot}25x + 0{\cdot}5y \geq 200$.

Thus the linear programming problem is as follows :

Minimise : $Z = \dfrac{4x}{1000} + \dfrac{6y}{1000}$

subject to the constraints :

$0{\cdot}1x + 0{\cdot}05y \geq 50$

$0{\cdot}25x + 0{\cdot}5y \geq 200$

$x, y \geq 0.$

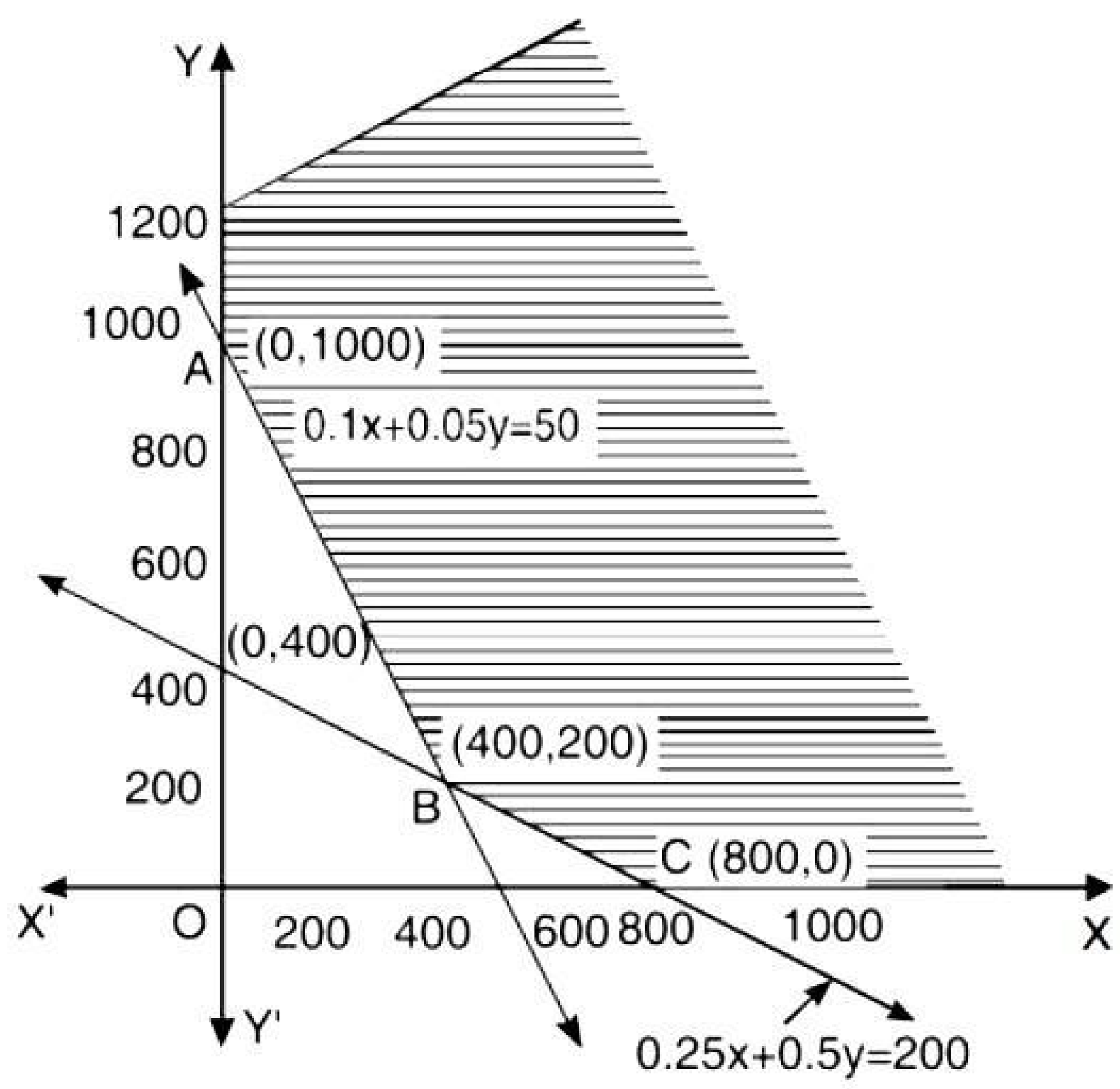


Fig.

The graph of the constraints can be easily obtained. As $x \geq 0$ and $y \geq 0$, the graph has been drawn in the first quadrant only.

The coordinates of A are (0, 1000), B are (400, 200) and C are (800, 0).

The feasible region is not bounded.

Applying ***Corner Point Method,*** we have :

Corner Point	$Z = \dfrac{4x}{1000} + \dfrac{6y}{1000}$
A : (0, 1000)	$0 + \dfrac{6}{1000} \times 1000 = ₹\ 6$
B : (400, 200)	$\dfrac{4 \times 400}{1000} + \dfrac{6 \times 200}{1000}$ **= ₹ 2·80 (Minimum)**
C : (800, 0)	$\dfrac{4 \times 800}{1000} + \dfrac{6 \times 0}{1000} = ₹\ 3{\cdot}20$

Hence, minimum cost is ₹ 2·80 when

$x = 400$ g and $y = 200$ g.

Example 4. (Manufacturing Problem) A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is atmost 24. It takes one hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is ₹ 300 and that on a chain is ₹ 190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an L.P.P. and solve it graphically.

(Meghalaya B. 2018; C.B.S.E. 2010 ; P.B. 2010)

Solution. Let 'x' and 'y' be the number of gold rings and chains respectively.

We have :

$$x \geq 0 \quad ....(1) \quad y \geq 0 \qquad ....(2)$$

$$x + y \leq 24 \quad ....(3) \quad x + \frac{y}{2} \leq 16 \qquad ....(4)$$

The objective function, or the profit, Z is :

$$Z = 300x + 190y \qquad ....(5)$$

We have to maximize Z subject to (1) – (4).

For solution set, we draw the lines :

$$x = 0, y = 0, x + y = 24, 2x + y = 32.$$

The lines $x + y = 24$ and $2x + y = 32$ meet at E (8, 16).

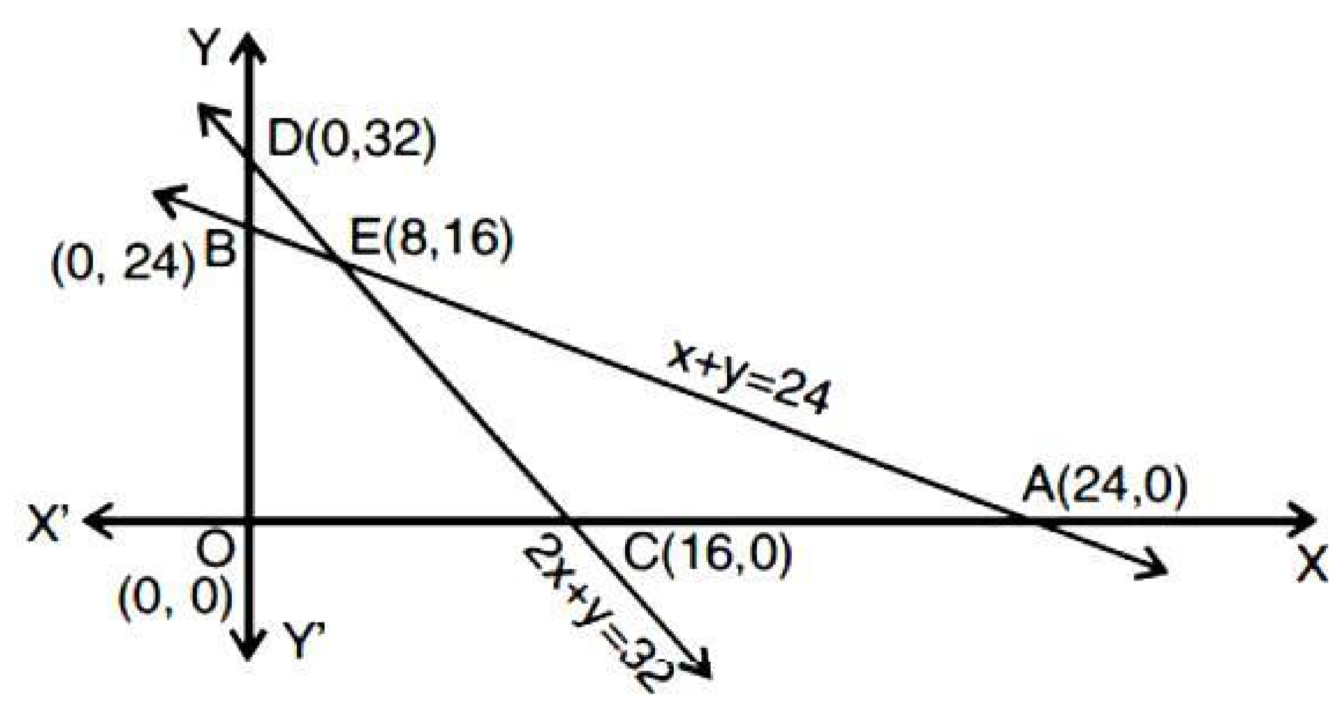


Fig.

The shaded portion represents the feasible region, which is bounded.

Applying ***Corner Point Method,*** we have :

Corner point	**Z = 300x + 190y**
O : (0, 0)	0
C : (16, 0)	4800
E : (8, 16)	**5440 (Maximum)**
B : (0, 24)	4560

Hence, the maximum profit is ₹ 5,440 and it is obtained when 8 gold rings and 16 chains are manufactured.

Example 5. A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours per day. Each unit of product A requires 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on second machine. Each unit of product A is sold at ₹ 7 profit and that of B at a profit of ₹ 4. Find the production level per day for maximum profit graphically. *(C.B.S.E. 2016)*

Solution. Let the manufacturer produce 'x' units of product A and 'y' units of product B.

Let the total profit, $Z = 7x + 4y$ ...(1)

Then LPP is:

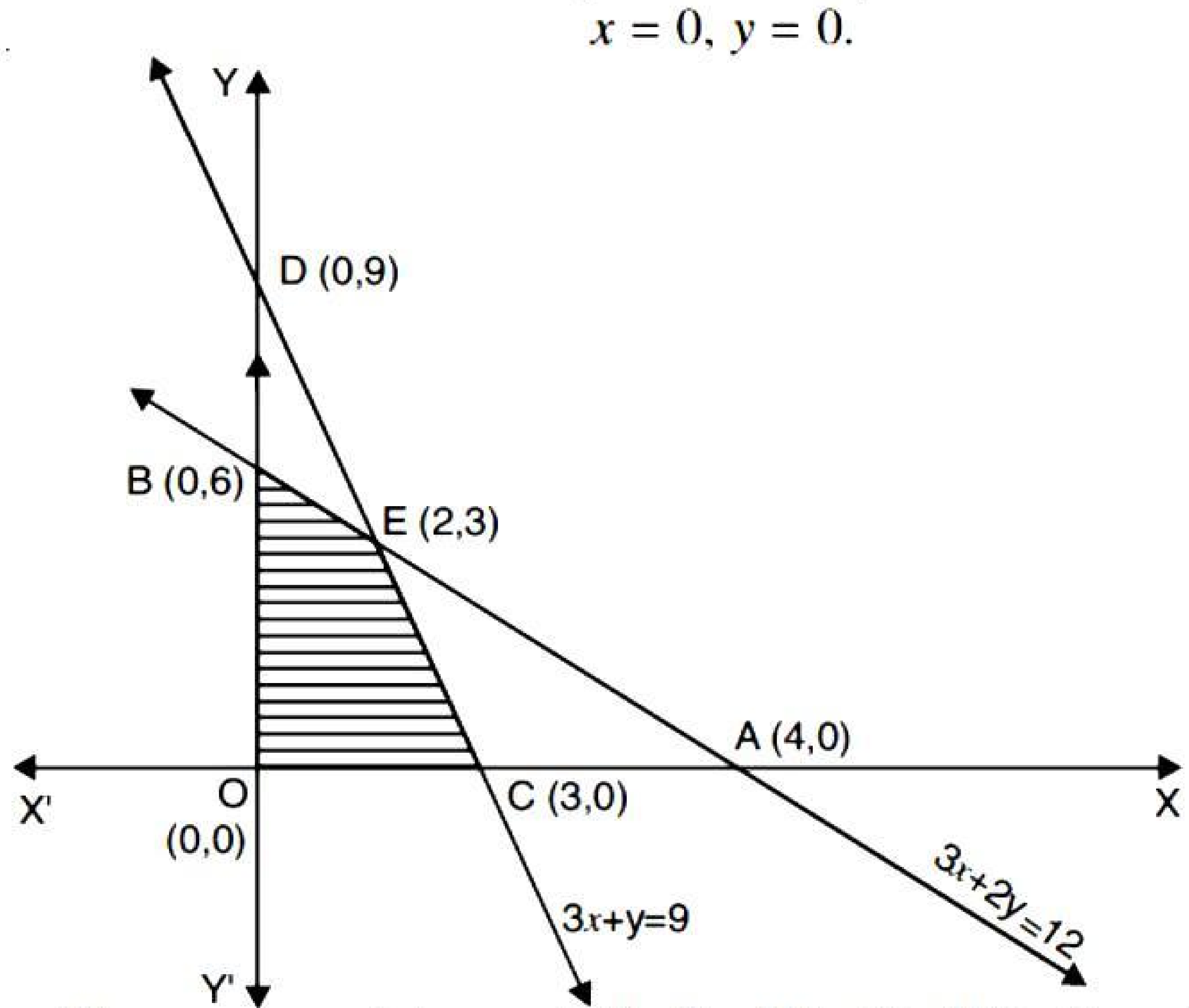
Maximize $Z = 7x + 4y$ subject to constraints:

$$3x + 2y \le 12 \quad ...(2)$$

$$3x + y \le 9 \quad ...(3)$$

and $$x \ge 0, y \ge 0 \quad ...(4)$$

Draw the st-lines $3x + 2y = 12$, $3x + y = 9$, $x = 0$, $y = 0$.



The corner points are O(0, 0), C(3, 0), B(0, 6) and E(2, 3) [Solving $3x + y = 9$, $3x + 2y = 12$]. The shaded region represents the feasible region, which is bounded.

Applying ***Corner Point Method*** :

Corner Point	$Z = 7x + 4y$
O : (0, 0)	0
C : (3, 0)	21
E : (2, 3)	**26 (Maximum)**
B : (0, 6)	24

For maximum profit, the manufacturer should manufacture 2 unit of product A and 3 units of product B.

Example 6. Two tailors A and B earn ₹ 150 and ₹ 200 per day respectively. A can stitch 6 shirts and 4 pants while B can stitch 10 shirts and 4 pants per day. How many days shall each work if it is desired to produce (at least) 60 shirts and 32 pants at a minimum labour cost ? Make it an LPP and solve the problem graphically. ***(Type : Mizoram B. 2016; C.B.S.E. 2009 C)***

Solution. Let the tailor A work for 'x' days and B for 'y' days.

We have :	*Shirts*	*Pants*
Tailor (A)	6	4
Tailor (B)	10	4

Thus we have the following constraints :

$$x \ge 0 \quad ...(1)$$

$$y \ge 0 \quad ...(2)$$

$$6x + 10y \ge 60$$

i.e. $$3x + 5y \ge 30 \quad ...(3)$$

$$4x + 4y \ge 32$$

i.e. $$x + y \ge 8 \quad ...(4)$$

The objective function, or the cost, Z is :

$$Z = 150x + 200y \quad ...(5)$$

For the solution set, we draw the lines :

$x = 0$, $y = 0$, $3x + 5y = 30$, $x + y = 8$.

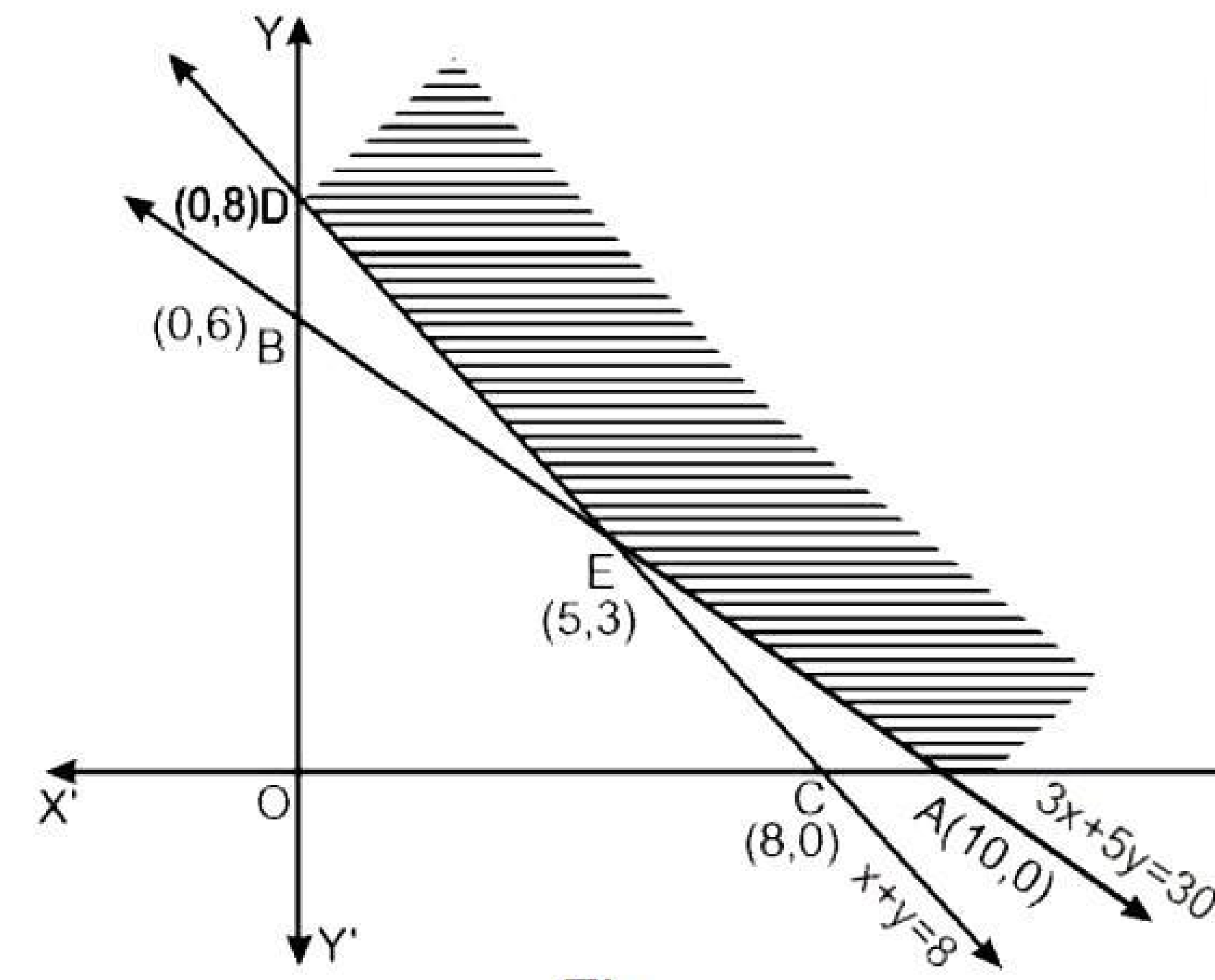


Fig.

The feasible region (shaded) is unbounded.

Let us evaluate Z at the corner points :

A (10, 0), D (0, 8) and E (5, 3).

[*Solving $x + y = 8$, $3x + 5y = 30$; $x = 5$, $y = 3$*]

Applying ***Corner Point Method,*** we have :

Corner point	Z = 150x + 200y
A : (10, 0)	1500
E : (5, 3)	**1350 (Minimum)**
D : (0, 8)	1600

Hence, the tailor A should work for 5 days and B for 3 days for the minimum cost of ₹ 1,350.

Example 7. (Allocation Problem) A farmer has a supply of chemical fertilizer of type I which contains 10% nitrogen and 6% phosphoric acid and type II fertilizer which contains 5% nitrogen and 10% phosphoric acid. After testing the soil condition of a field, it is found that at least 14 kg of nitrogen and 14 kg of phosphoric acid are required for a good crop. The fertilizer type I costs ₹ 2·00 per kg and the type II ₹ 3·00 per kg. How many kilograms of each fertilizer should be used to meet the requirement and the cost be minimum ?

Solution. Let 'x' kg of Type I and 'y' kg of type II be required.

Thus the problem is :

Minimize : $Z = 2x + 3y$ subject to :

$$x \ge 0,\ y \ge 0,\ \frac{10x}{100} + \frac{5y}{100} \ge 14 \text{ and } \frac{6x}{100} + \frac{10y}{100} \ge 14$$

i.e. $x \ge 0$, $y \ge 0$, $2x + y \ge 280$ and $3x + 5y \ge 700$.

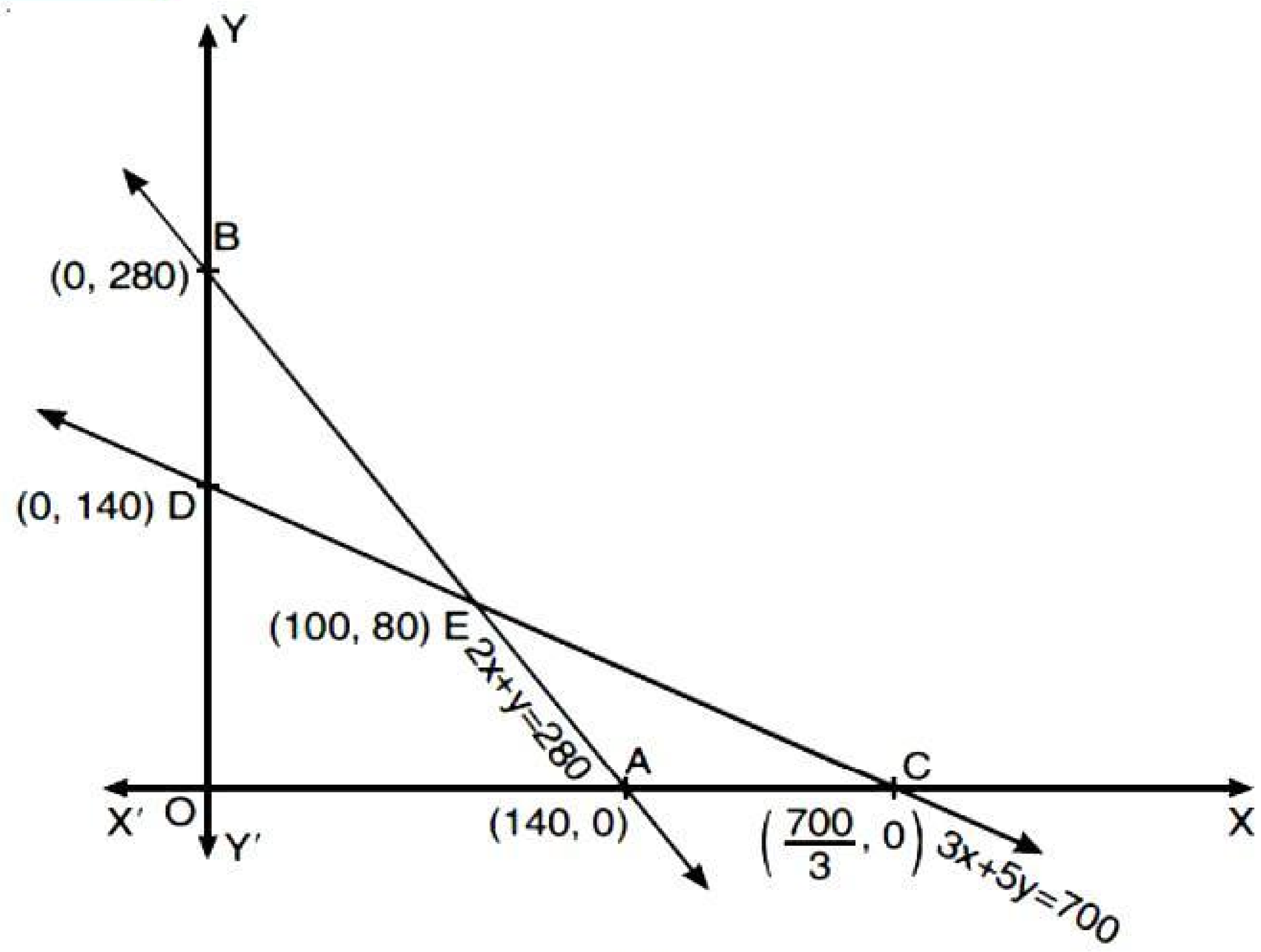


Fig.

The shaded portion represents the feasible region and is unbounded.

The feasible region (shaded) is unbounded.

Let us evaluate Z at the corner points :

$$C\left(\frac{700}{3}, 0\right), B\ (0, 280) \text{ and } E\ (100, 80).$$

[*Solving* $2x + y = 280$, $3x + 5y = 700$; $x = 100$, $y = 80$]

Applying ***Corner Point Method,*** we have :

Corner point	Z = 2x + 3y
C : $\left(\frac{700}{3}, 0\right)$	$\frac{1400}{3}$
E : (100, 80)	**440 (Minimum)**
B : (0, 280)	840

Hence, the least cost is ₹ 440 and it is obtained when 100 kg of type I and 80 kg of type II are used.

Example 8. A manufacturer makes two types of toys A and B. Three machines are needed for this purpose and the time (in minutes) required for each toy on the machines is given below :

Types of Toys	Machines		
	I	II	III
A	20	10	10
B	10	20	30

The machines I, II and III are available for a maximum of 3 hours, 2 hours and 2 hours 30 minutes respectively. The profit on each toy of type A is ₹ 50 and that of type B is ₹ 60. Formulate the above problem as a LPP and solve graphically to maximize profit.

(C.B.S.E. Sample Paper 2019)

Solution. Let 'x' and 'y' be the number of toys of type A and type B respectively.

We have : $x \geq 0$ ...(1) $y \geq 0$ ...(2)

$20x + 10y \leq 180$ ...(3)

$10x + 2y \leq 120$ ...(4)

and $10x + 30y \leq 150$ ...(5)

The objective function, or the profit, Z is

$Z = 50x + 60y$ ...(6)

We have to maximize Z subject to (1) – (5).

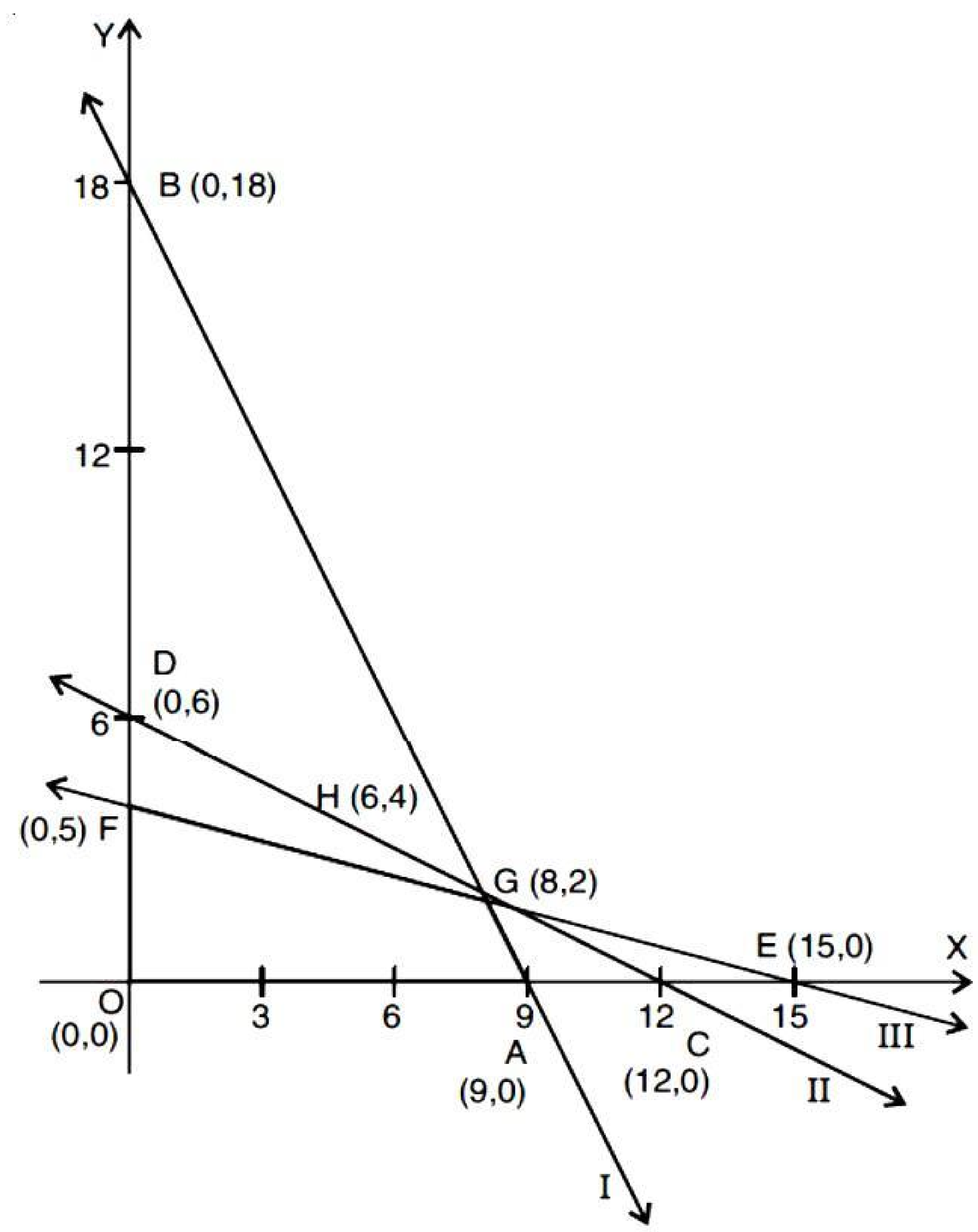
For solution set, we solve the lines :

$x = 0, y = 0$

$20x + 10y = 180$ *i.e.* $2x + y = 18$ ...(I)

$10x + 20y = 120$ *i.e.* $x + 2y = 12$ ...(II)

and $10x + 30y = 150$ *i.e.* $x + 3y = 15$ ...(III)



The corner points are O(0, 0), A(9, 0),

G(8, 2) [*Solve* $2x + y = 18$ *and* $x + 2y = 12$]

H(6, 3) [*Solve* $x + 2y = 12$ *and* $x + 3y = 15$]

and F(0, 5).

The shaded region represents the feasible region, which is bounded.

Applying ***Corner Point Method*** **:**

Cover Point	Z = 50x + 60y
O : (0, 0)	0
A : (9, 0)	450
G : (8, 2)	**520 (Maximum)**
H : (6, 0)	480
F : (0, 5)	300

Hence, maximum profit is ₹ 520 when 8 toys of type A and 2 toys of type B are sold.

Example 9. (Manufacturing Problem). A manufacturer considers that men and women workers are equally efficient and so he pays them at the same rate. He has 30 and 17 units of workers (male and female) and capital respectively, which he uses to produce two types of goods A and B. To produce one unit of A, 2 workers and 3 units of capital are required while 3 workers and 1 unit of capital are required to produce one unit of B. If A and B are priced at ₹ 100 and ₹ 120 per unit respectively, how should he use his resources to maximise the total revenue ? Form the above as an LPP and solve graphically. *(A.I.C.B.S.E. 2013)*

Solution. Let 'x' units of goods A and 'y' units of goods B be produced.

Then LPP is as below :

Maximise : $Z = 100x + 120y$ ...(1)

subject to : $2x + 3y \leq 30$ ...(2)

$3x + y \leq 17$ ...(3)

and $x \geq 0, y \geq 0$ ...(4)

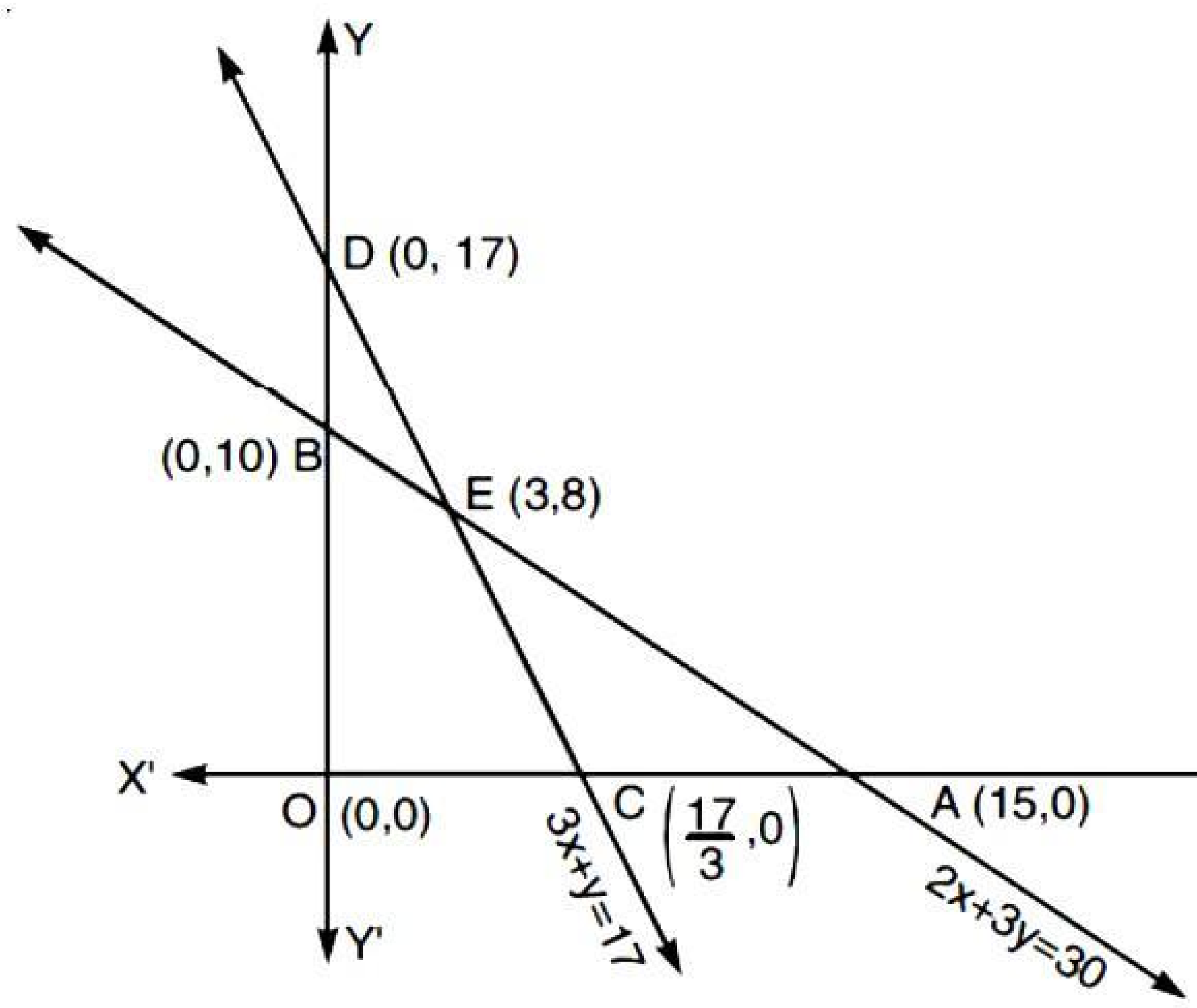


Fig.

The shaded portion represents the feasible region OCEB, which is bounded.

Its vertices are O(0, 0), $C\left(\frac{17}{3},0\right)$, B (0, 10) and E (3, 8).

[*Solving $2x + 3y = 30$ and $3x + y = 17$; $x = 3$, $y = 8$*]

Applying ***Corner Point Method,*** we have :

Corner point	Z = 100x + 120y
O : (0, 0)	0
C : $\left(\frac{17}{3},0\right)$	$\frac{1700}{3}$
E : (3, 8)	**1260 (Maximum)**
B : (0, 10)	1200

Hence, the maximum total revenue is obtained when 3 units of goods A and 8 units of goods B be produced.

Example 10. A cooperative society of farmers has 50 hectares of land to grow crops A and B. The profits from crops A and B per hectare are estimated as ₹ 10,500 and ₹ 9,000 respectively. To control weeds, a liquid pesticide has to be used for crops A and B at the rate of 20 litres and 10 litres per hectare, respectively. Further not more than 800 litres of pesticide should be used in order to protect fish and wildlife using a pond which collects drainage from this land. Keeping in mind that the protection of fish and other wildlife is more important than earning profit, how much land should be allocated to each crop so as to maximize the total profit ? Form an LPP from the above and solve it graphically.

(C.B.S.E. 2013)

Solution. Let 'x' and 'y' hectares of land be allocated to crops A and B respectively. Then the problem is :

Maximize : $Z = 10{,}500x + 9{,}000y$ subject to :

$x \geq 0, y \geq 0, x + y \leq 50, 20x + 10y \leq 800.$

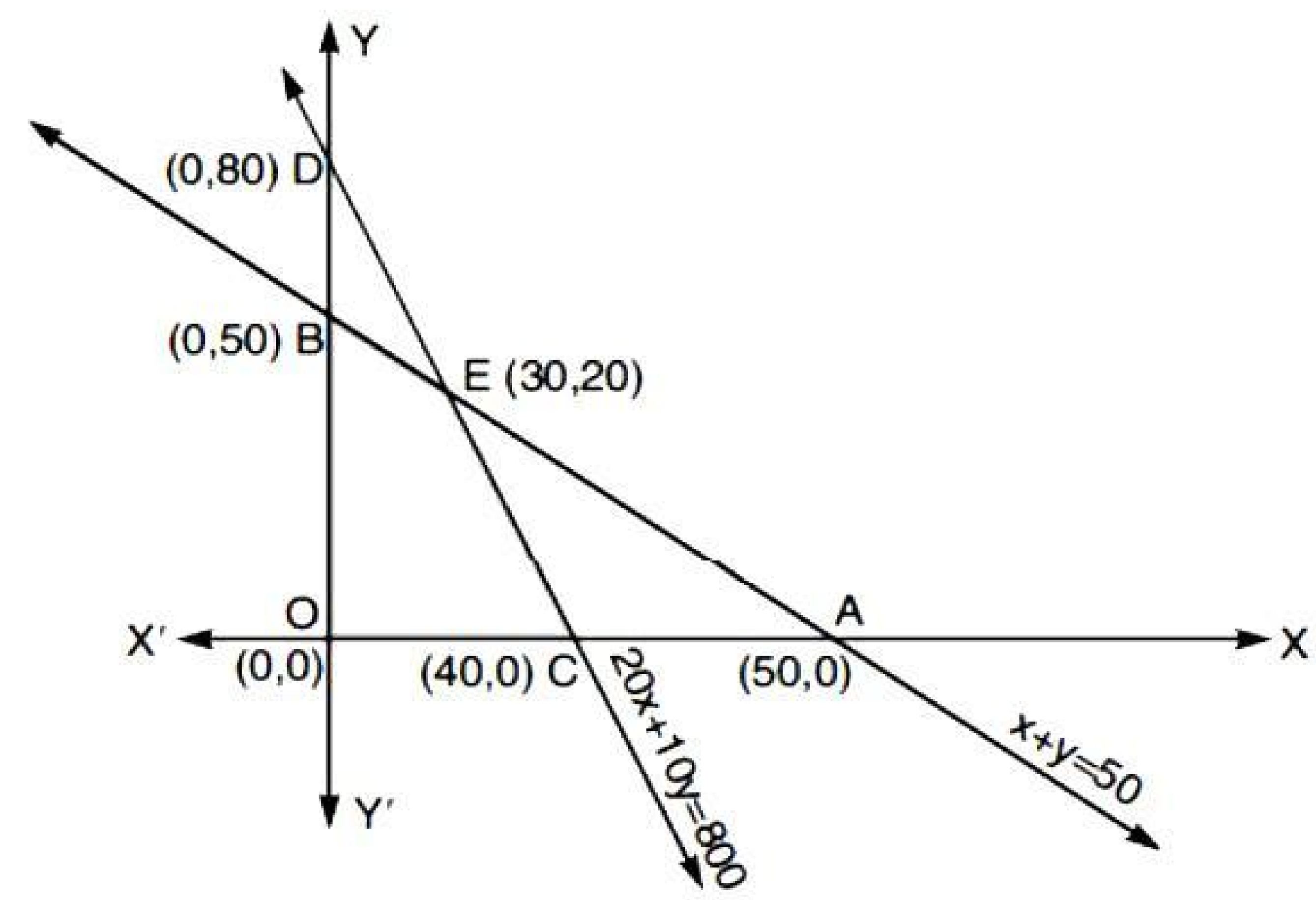


Fig.

For solution set, we draw the lines :

$x = 0, y = 0, x + y = 50$ and $20x + 10y = 800$.

The lines $x + y = 50$ and $20x + 10y = 800$ intersect at E (30, 20)

The shaded portion represents the feasible region, which is bounded.

Applying ***Corner Point Method,*** we have :

Corner Point	Z = 10,500 x + 9,000 y
0 : (0, 0)	0
C : (40, 0)	4,20,000
E : (30, 20)	**4,95,000 (Maximum)**
B : (0, 50)	4,50,000

Hence, to earn maximum profit 30 and 20 hectares of land should be allocated to crops A and B respectively.

Example 11. (Diet Problem) A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods A and B are available at a cost of ₹ 5 and ₹ 4 per unit respectively. One unit of the food A contains 200 units of vitamins, 1 unit of minerals and 40 units of calories, while one unit of the food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the foods A and B should be used to have least cost, but it must satisfy the requirements of the sick persons. Form the question as LPP and solve it graphically.

Solution. Let 'x' units of food A and 'y' units of food B be used.

Then LPP problem is :

Minimize : $Z = 5x + 4y$

subject to : $200x + 100y \geq 4000$ *i.e.* $2x + y \geq 40$,

$x + 2y \geq 50$,

$40x + 40y \geq 1400$ *i.e.* $x + y \geq 35$

and $x \geq 0, y \geq 0$.

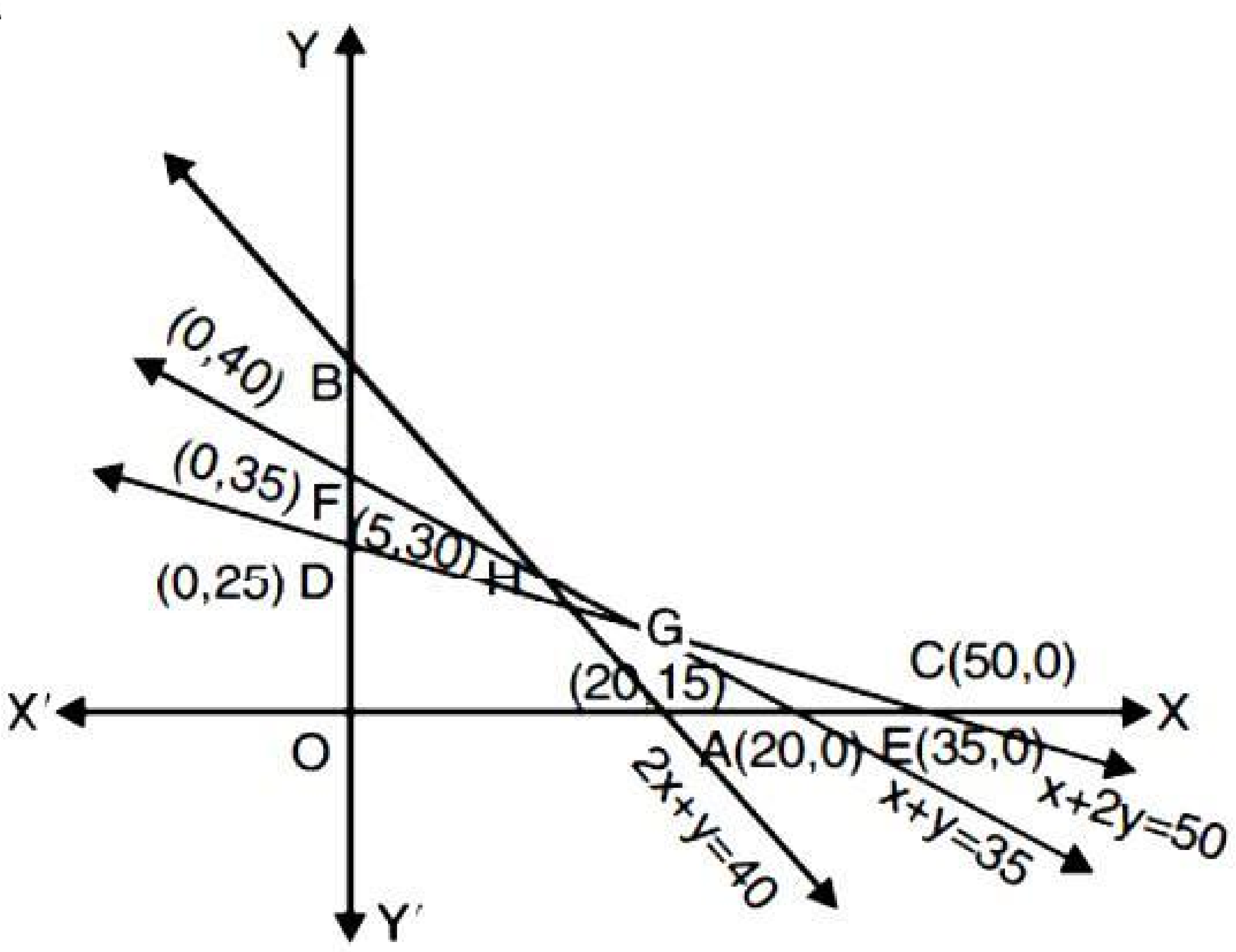


Fig.

The feasible region (shaded) is unbounded.

Let us evaluate Z at the corner points :

C (50, 0), G (20, 15),

[*Point of Intersection of* $x + 2y = 50$, $x + y = 35$]

H (5, 30)

[*Point of Intersection of* $2x + y = 40$, $x + y = 35$]

and B (0, 40).

Applying ***Corner Point Method,*** we have :

Corner Point	Z = 5x + 4y
C : (50, 0)	250
G : (20, 15)	160
H : (5, 30)	**145 (Minimum)**
B : (0, 40)	160

Hence, least cost is ₹ 145 when 5 units of food A and 30 units of food B are used.

Example 12. A factory manufactures two types of screws A and B, each type requiring the use of two machines, an automatic and a hand-operated. It takes 4 miniutes on the automatic and 6 minutes on the hand-operated machines to manufacture a packet of screws 'A' while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture a packet of screws 'B'. Each machine is available for at most 4 hours on any day. The manufacture can sell a packet of screws 'A' at a profit of 70 paise and screws 'B' at a profit of ₹ 1. Assuming that he can sell all the screws he manufactures, how many packets of each type should the factory owner produce in a day in order to maximize his profit ? Formulae the above LPP and solve it graphically and find the maximum profit. *(C.B.S.E. 2018)*

Solution. Let the factory manufactures 'x' of type 'A' and 'y' of type 'B'.

Clearly $x \geq 0$ ...(1) and $y \geq 0$ ...(2)

Since the machines can operate for at the most 4 hours a day,

$\therefore \quad 4x + 6y \leq 240$

i.e., $\quad 2x + 3y \leq 120$

$\therefore \quad 6x + 3y \leq 240$ ...(3)

i.e., $\quad 2x + y \leq 80$ ...(4)

The objective function or the profit, P, is :

$$P = 0.7 + x + y \quad ...(5)$$

We draw the lines :

$$x = 0, y = 0,$$

$$2x + 3y = 120 \text{ and } 2x + y = 80.$$

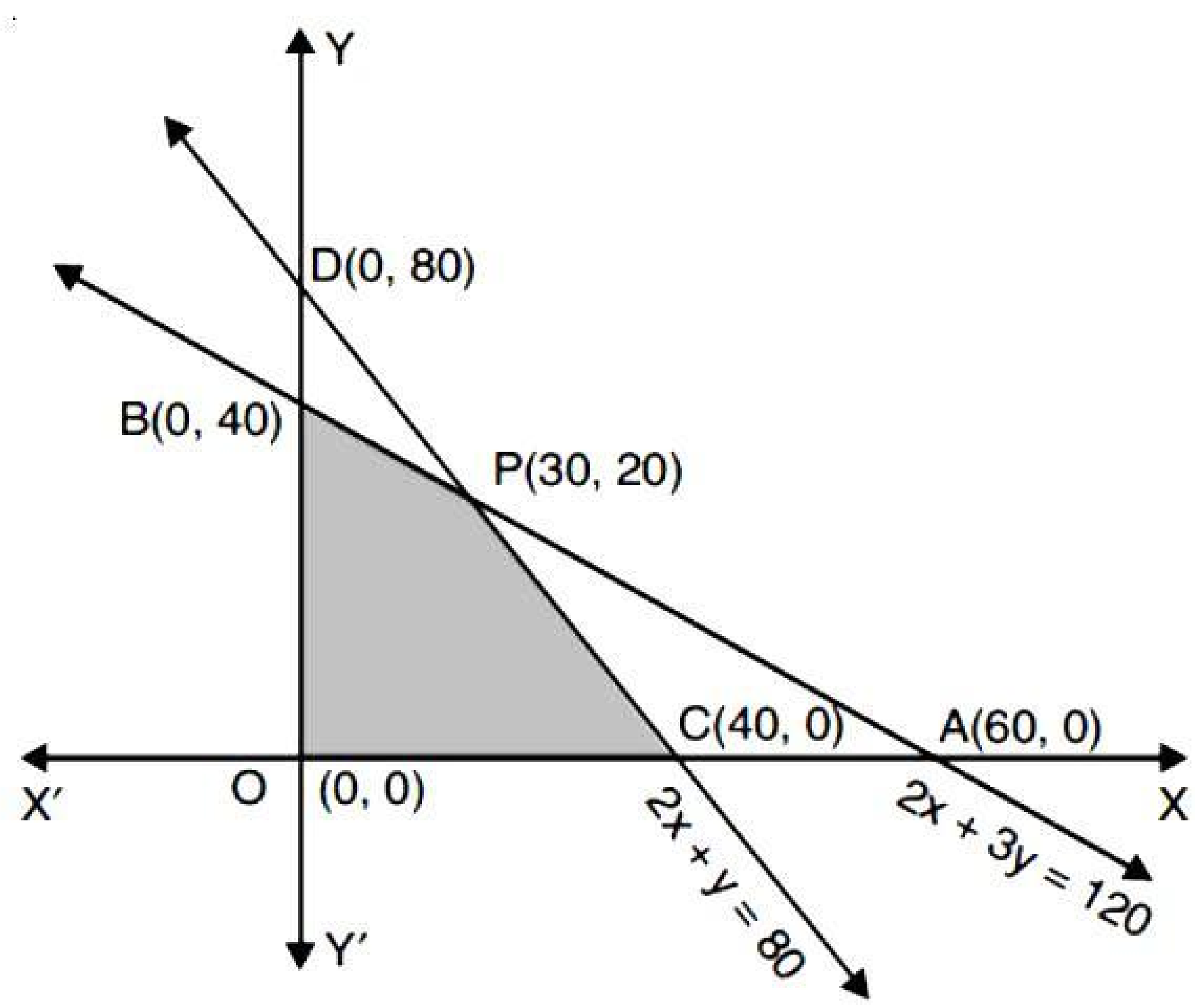


Fig.

The feasible region shown OCPB is bounded, where O is (0, 0), C is (40, 0), B is (0, 40) and P is (30, 20).

[*Solving* $2x + y = 80$ *and* $2x + 3y = 120$]

Applying ***Corner Point Method,*** we have :

Corner point	Z = 0.7x + y
O : (0, 0)	0
A : (40, 0)	40
P : (30, 20)	**41 (Maximum)**
B : (0, 40)	40

Hence, in order to maximise profit 30 packages of screw 'A' and 20 packages of screw 'B' should be manufacturered and maximum profit = ₹ 41.

Example 13. A farmer has a supply of chemical fertilizer of type A, which contains 10% nitrogen and 5% phosphoric acid and type B, which contains 6% nitrogen and 10% phosphoric acid. After testing the soil conditions of the field, it was found that at least 14 kg of nitrogen and 14 kg of phosphoric acid are required for producing a good crop. The fertilizer of type A costs ₹ 5 per kg and type B costs ₹ 3 per kg. How many kg of each type of the fertilizer should be used to meet the requirement at minimum cost ? Using LPP, solve the above problem graphically. *(Mizoram B. 2016)*

Solution. Let 'x' kg of fertilizer A and 'y' kg of fertilizer B be used.

The contents of the fertilizer are as below :

Fertilizer	Nitrogen	Phosphoric Acid	Cost (per kg)
A	10%	5%	₹ 5
B	6%	10%	₹ 3
Minimum Requirement	14 kg	14 kg	

Now LPP problem is :

Minimize : $C = 5x + 3y$

subject to : $\frac{10}{100}x + \frac{6}{100}y \geq 14 \Rightarrow 5x + 3y \geq 700$

$\frac{5}{100}x + \frac{10}{100}y \geq 14 \Rightarrow x + 2y \geq 280$

and $x \geq 0, y \geq 0$.

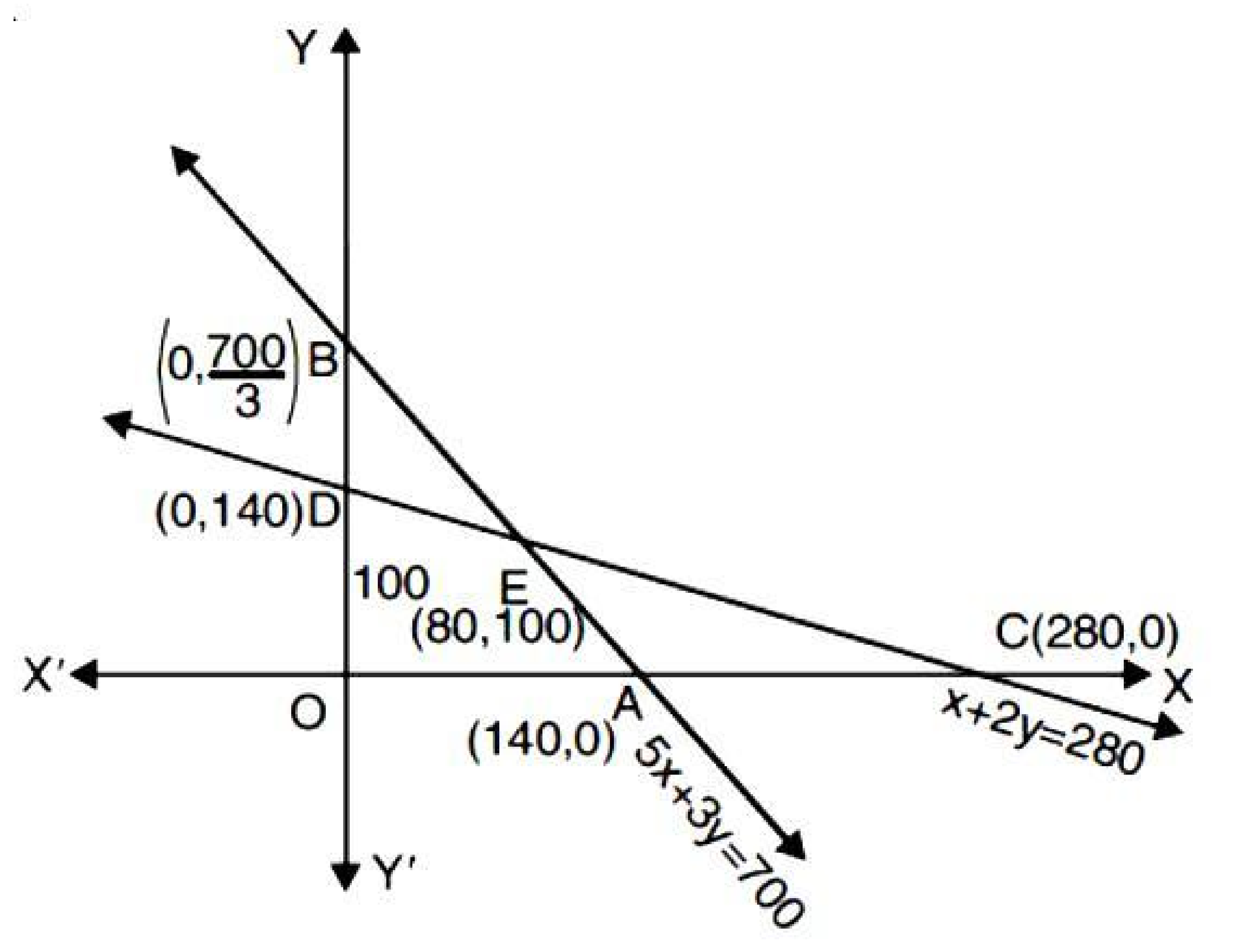


Fig.

The feasible region (shaded) is unbounded.

Let us evaluate C at the corner points :

C (280, 0), B$\left(0, \frac{700}{3}\right)$

and E (80, 100).

[*Solving* $x + 2y = 280$ *and* $5x + 3y = 700$; $x = 80$, $y = 100$]

Applying ***Corner Point Method,*** we have :

Corner Point	C = 5x + 3y	
C : (280, 0)	1400	
E : (80, 100)	**700**	**(Minimum)**
B : $\left(0, \frac{700}{3}\right)$	**700**	

Thus cost is minimum at B$\left(0, \frac{700}{3}\right)$ or at E (80, 100) and minimum cost is ₹ 700.

Since the region is unbounded, therefore, ₹ 700 may or may not be the minimum value of C.

For this, we draw the inequation $5x + 3y < 700$.

L : $5x + 3y = 700$.

x	140	0
y	0	$\frac{700}{3}$

Open half plane has no corner point with the feasible region.

Hence, minimum cost is ₹ 700.

Example 14. An oil company requires 13,000, 20,000 and 15,000 barrels of high grade, medium grade and low grade oil respectively. Refinery A produces 100, 300 and 200 barrels per day of high, medium and low grade oil respectively, whereas refinery B produces 200, 400 and 100 barrels per day respectively. If A costs ₹ 400 per day and B costs ₹ 300 per day to operate, how many days should each be run to minimise the cost of requirement ?

Solution. We have the following data :

Refinery	High-Grade	Medium grade	Low grade	Cost per day
A	100	300	200	₹ 400
B	200	400	100	₹ 300
Minimum requirement	13,000	20,000	15,000	

Let the refineries run for 'x' and 'y' days respectively to minimise the cost of requirement.

Then LPP problem is :

Minimize : $Z = 400x + 300y$

subject to : $100x + 200y \geq 13{,}000$

$300x + 400y \geq 20{,}000$

$200x + 100y \geq 15{,}000$

and $x \geq 0$ and $y \geq 0$.

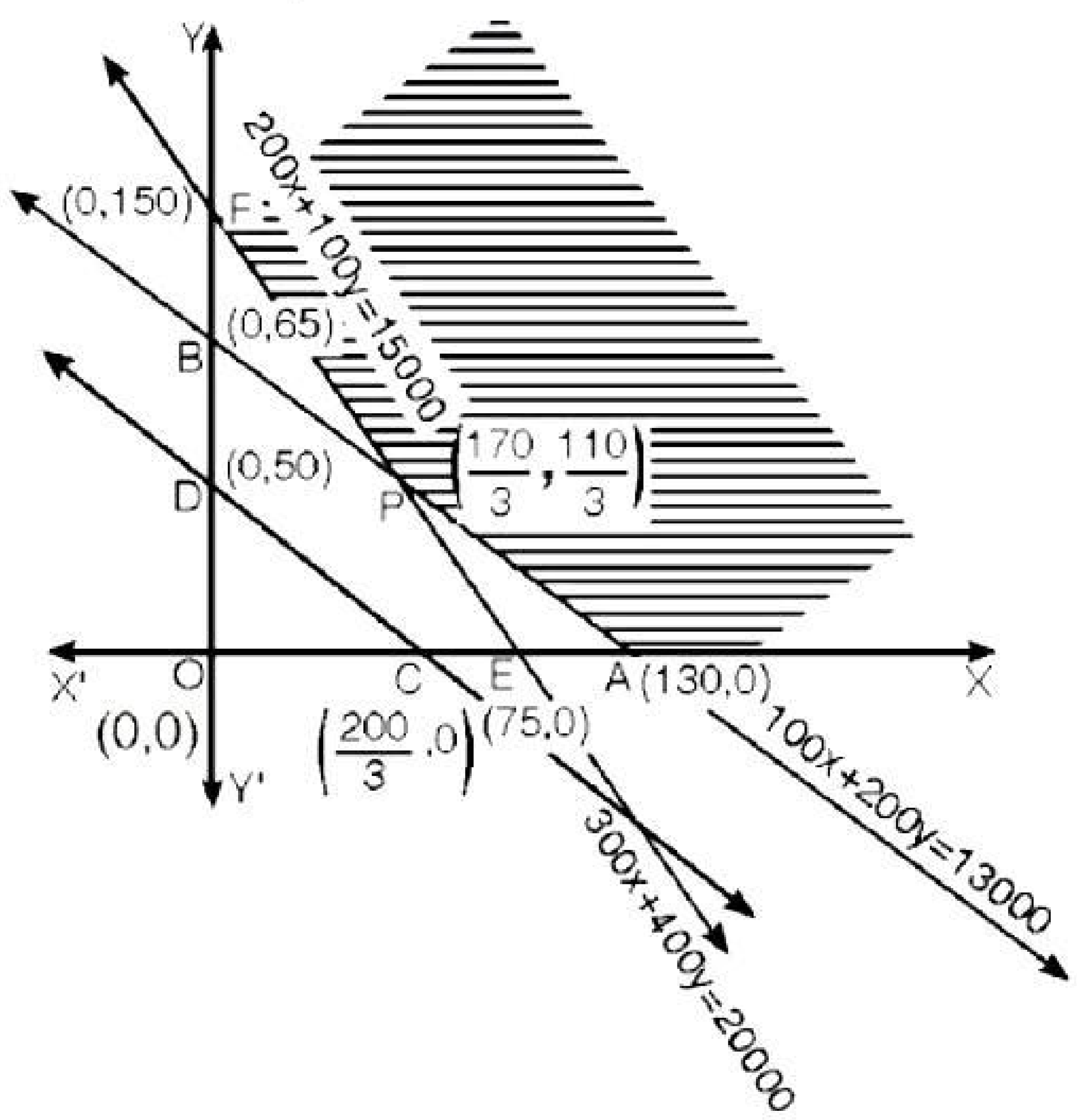


Fig.

The feasible region (shaded) is unbounded.
Let us evaluate Z at the corner points :

A (130, 0), F (0, 150) and P$\left(\dfrac{170}{3}, \dfrac{110}{3}\right)$

[*Solving* $100x + 200y = 13000$ *and* $200x + 100y = 15000$; $x = \dfrac{170}{3}$, $y = \dfrac{110}{3}$]

Applying ***Corner Point Method,*** we have :

Corner point	Z = 400x + 300y
A : (130, 0)	52000
P : $\left(\dfrac{170}{3}, \dfrac{110}{3}\right)$	**$\dfrac{101000}{3}$ (Minimum)**
F : (0, 150)	45000

Hence, the cost is minimum = ₹ $\dfrac{101000}{3}$ when the refineries run for $\dfrac{170}{3}$ and $\dfrac{110}{3}$ days respectively.

Example 15. A dietician wishes to mix two types of foods in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C while food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₹ 5 per kg to purchase food I and ₹ 7 per kg to purchase food II. Determine the minimum cost of such a mixture. Formulate the above as a LPP and solve it graphically. *(A.I.C.B.S.E. 2012)*

Solution. Let 'x' kg of food I and 'y' kg of food II be mixed. We have the table:

Food	Amount	Unit of Vitamin A	Unit of Vitamin C	Cost (in ₹)
I	x kg	$2x$	x	$5x$
II	y kg	y	$2y$	$7y$
Total		$2x + y$	$x + 2y$	$5x+7y$

Thus LPP problem is as below :

Minimize : $Z = 5x + 7y$ ...(1)

Subject to : $2x + y \geq 8$ ...(2)

$x + 2y \geq 10$ ...(3)

and $x, y \geq 0$ ...(4)

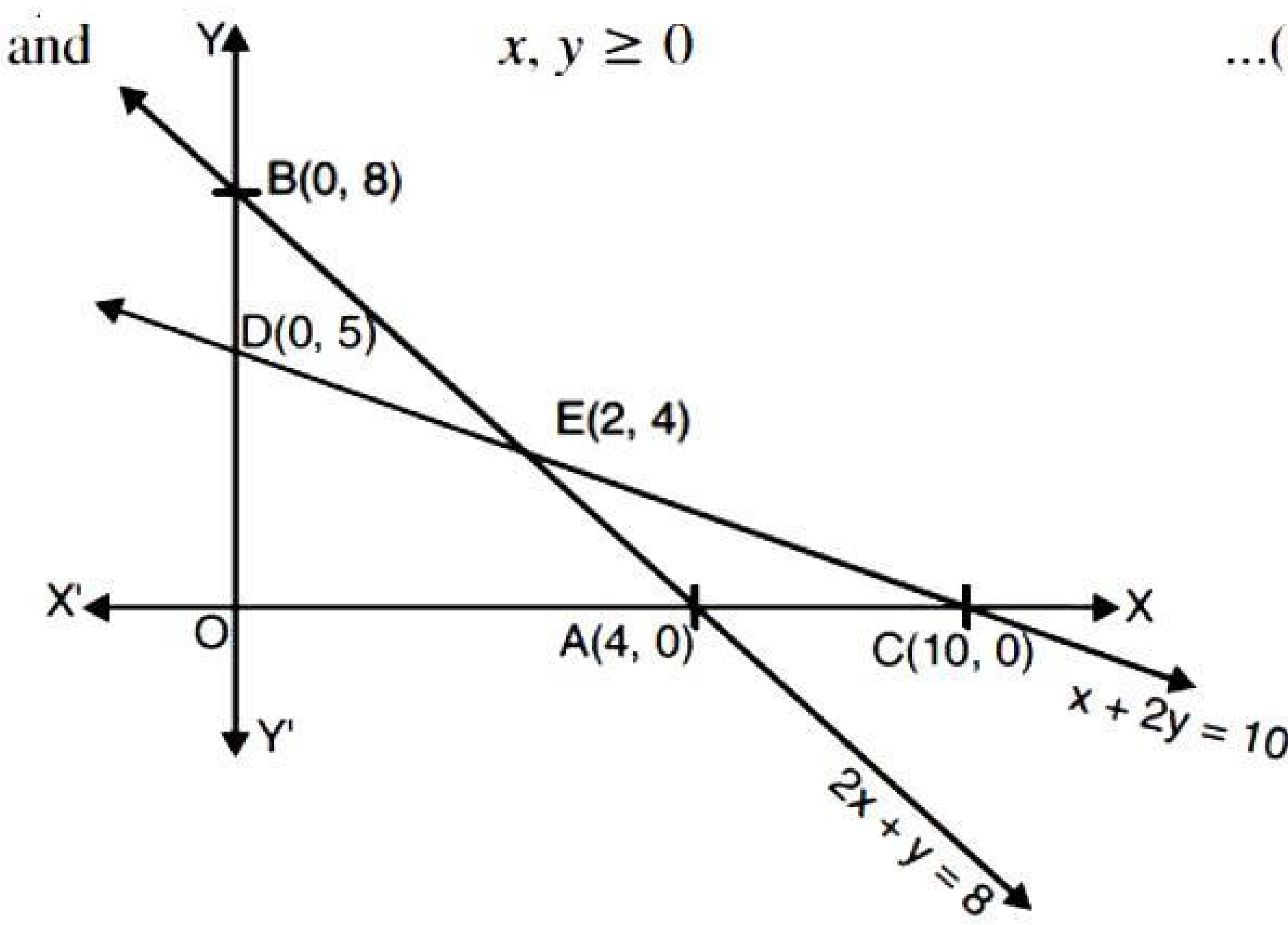


Fig.

First of all, we locate the region represented by (2) – (4). The shaded region, as shown above, is feasible region.

Applying ***Corner Point Method,*** we have :

Corner Point	Z = 5x + 7y
C : (10, 0)	50
E : (2, 4)	**38 (Minimum)**
B : (0, 8)	56

Hence, the minimum cost = ₹ 38 when 2 kg of food I and 4 kg of food II are mixed.

Example 16. (Manufacturing Problem). A manufacturer makes two types of machines. Deluxe sells for ₹ 12,000 and Standard sells for ₹ 8,000. It costs ₹ 9,000 to produce a deluxe and ₹ 6,000 to produce a standard. In one week manufacturer can produce 40 to 60 deluxe machines and 30 to 50 standard machines, but not more than 90 machines. Formulate the problem as LPP to maximize the profit. *(J. & K. B. 2010)*

Solution. Let 'x' and 'y' be the number of Deluxe and Standard machines respectively.

Maximize : $Z = (12000 - 9000)\,x + (8000 - 6000)\,y$

i.e. $Z = 3000x + 2000y$

subject to the constraints :

$x + y \leq 90$ ....(1)

$40 \leq x \leq 60$ ....(2)

and $30 \leq y \leq 50$ ....(3)

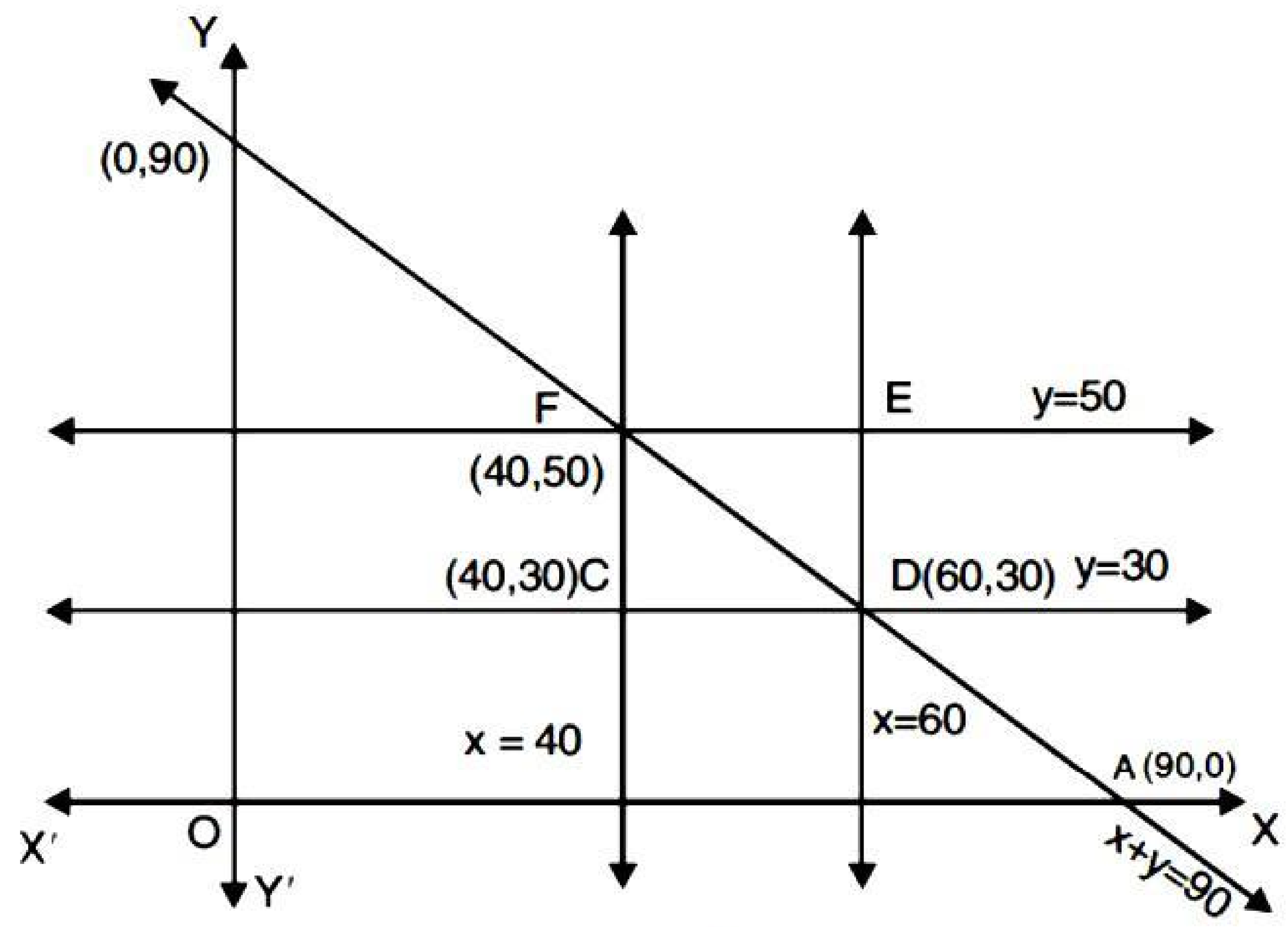


Fig.

The feasible region (shaded) is bounded.

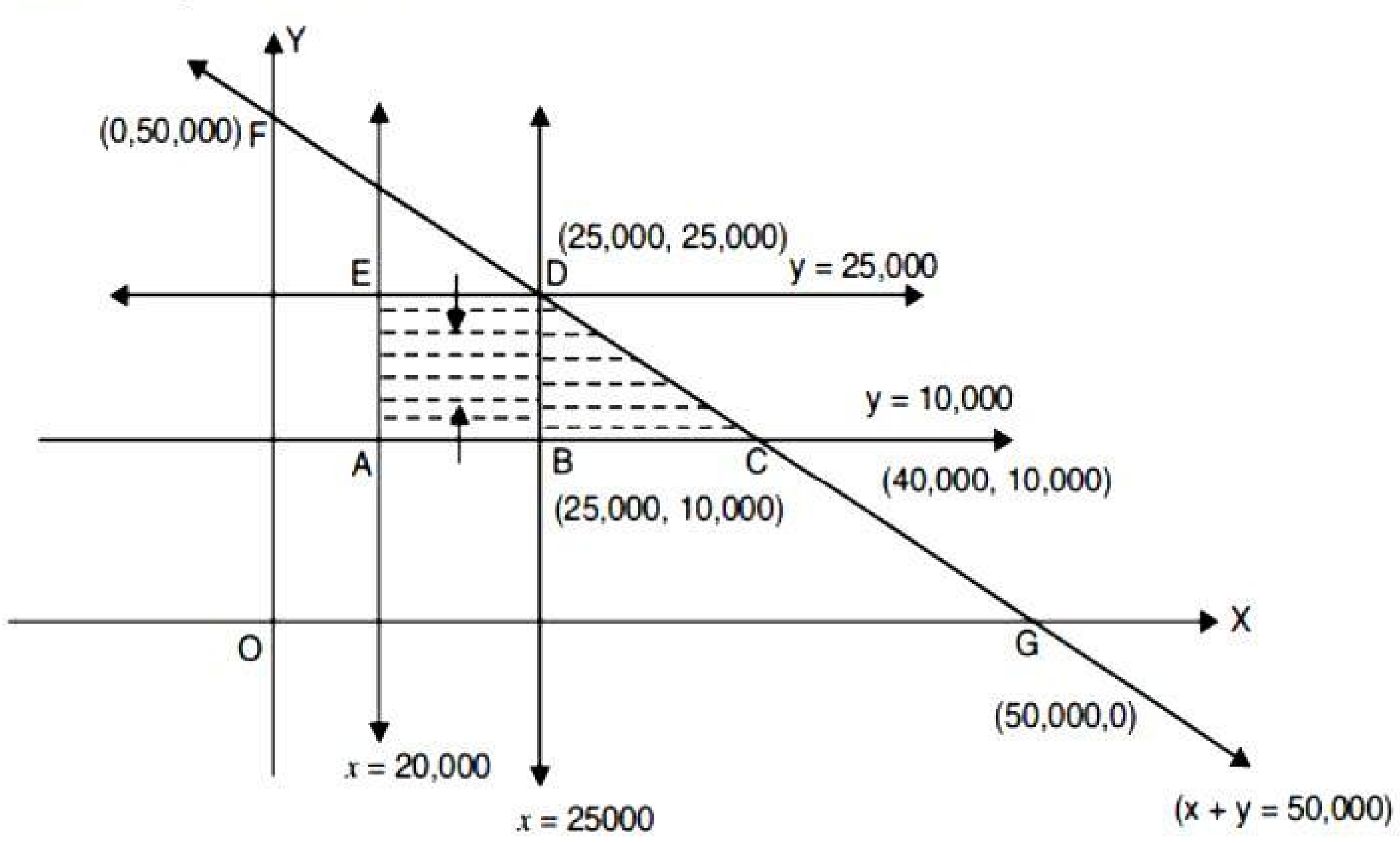
Applying ***Corner Point Method,*** we have :

Corner Point	Z = 23000x + 2000y
C : (40, 30)	180000
D : (60, 30)	**240000 (Maximum)**
F : (40, 50)	220000

Hence, the maximum profit is ₹ 2,40,000 when 60 Deluxe machines and 30 Standard machines are made.

Example 17. A retired person wants to invest an amount of ₹ 50,000. His broker recommends investing in two types of bonds 'A' and 'B' yielding 10% and 9% return respectively on the invested amount. He decides to invest at least ₹ 20,000 in bond 'A' and at least ₹ 10,000 in bond 'B'. He also wants to invest at least as much in bond A as in bond B. Solve this linear programming problem graphically to maximize his returns. *(A.I.C.B.S.E. 2016)*

Solution. Let the retired person invest ₹ x in bond A and ₹ y in bond B.



Then LPP is:

Maximise: $Z = \frac{10x}{100} + \frac{9y}{100}$ subject to:

$$x + y = 50{,}000$$

$20000 \leq x \leq 25000$ and $10000 \leq y \leq 25000$

The co-ordinates of A, B, C, D and E are (20,000, 10,000), (25,000, 10,000), (40,000, 10,000), (25,000, 25,000) and (20,000, 25,000) respectively.

We apply ***Corner Point Method***.

Corner Point	Corresponding value of Z
A : (20,000, 10,000)	2,900
B : (25,000, 10,000)	3,400
C : (40,000, 10,000)	**4,900 (Maximum)**
D : (25,000, 25,000)	4,750
E : (20,000, 25,000)	4,250

Hence, the retired person should invest ₹ 40,000 in bond A and 10,000 in bond B in order to maximize his returns.

Example 18. (Transportation Problem) A catering agency has two kitchens to prepare food at two places A and B. From these places 'Mid-day Meal' is to be supplied to three different schools situated at P, Q, R. The monthly requirements of the schools are respectively 40, 40 and 50 food packets. A packet contains lunch for 1000 students. Preparing capacity of kitchen A and B are 60 and 70 packets per month respectively. The transportation cost per packet for the kitchen to schools is given below :

Transportation cost per packet (in ₹)		
To	**From**	
	A	**B**
P	**5**	**4**
Q	**4**	**2**
R	**3**	**5**

How many packets from each kitchen should be transported to school so that the cost of transportation is minimum ? Also find the minimum cost.

(Mizoram B. 2018)

Solution. Let 'x' food packets and 'y' food packets be transported from kitchen A to schools P and Q respectively.

Then $(60 - x - y)$ food packets will be transported to school R.

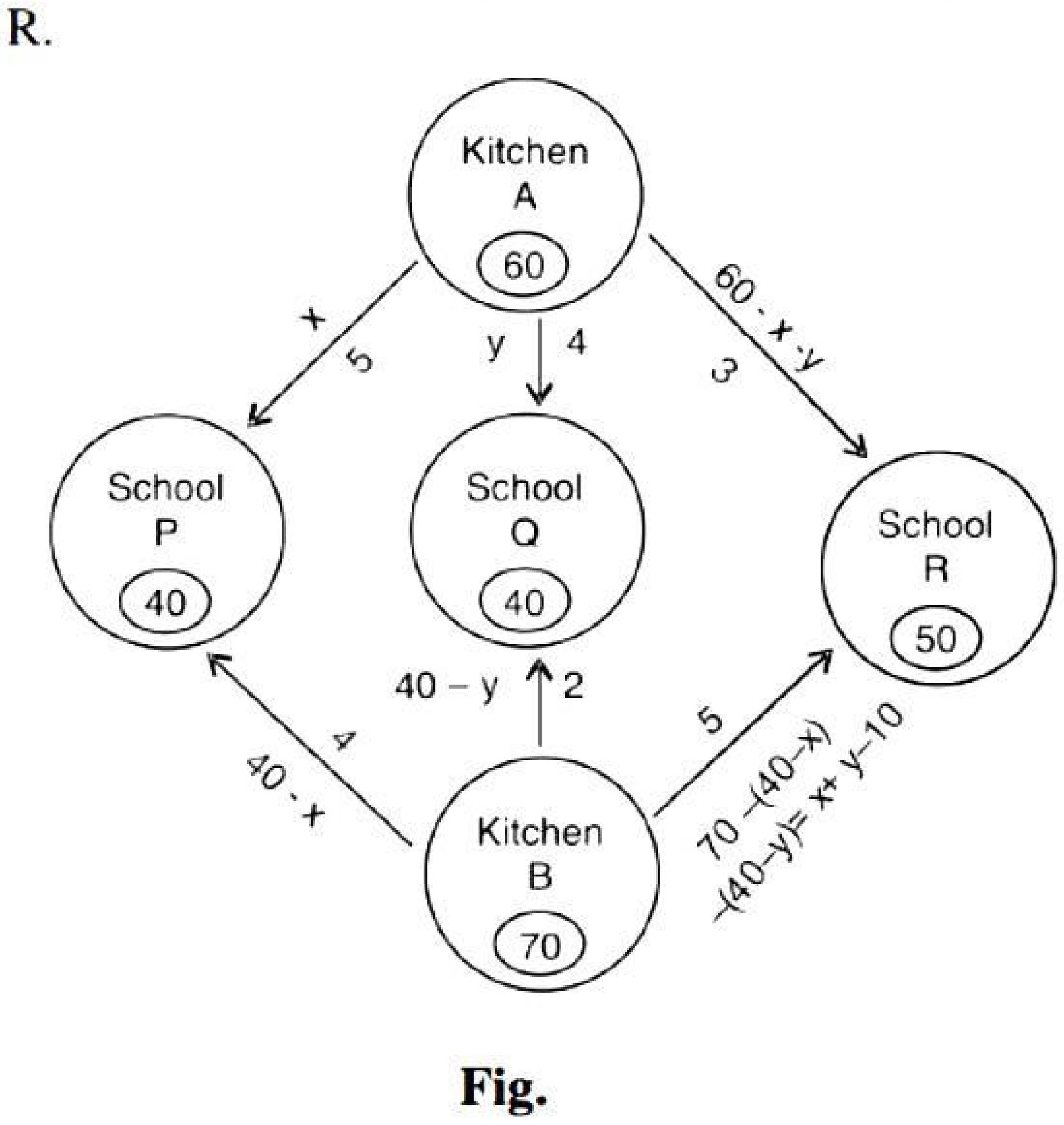


Fig.

Thus we have :

$$x \geq 0,\ y \geq 0 \text{ and } 60 - x - y \geq 0$$

i.e. $x \geq 0,\ y \geq 0$ and $x + y \leq 60$

$$40 - x \geq 0,\ 40 - y \geq 0,\ x + y - 10 \geq 0.$$

Total transportation cost Z is given by :

$$Z = 5x + 4y + 4(40 - x) + 2(40 - y) + 5(x + y - 10) + 3(60 - x - y)$$
$$= 3x + 4y + 270.$$

Thus the problem reduces to :

$$Z = 3x + 4y + 270 \qquad \text{....(1)}$$

Subject to : $x \geq 0,\ y \geq 0$ ....(2)

$x + y \leq 60$ ....(3)

$x \leq 40$ ...(4)

$y \leq 40$ ...(5)

and $x + y \geq 10$ ...(6)

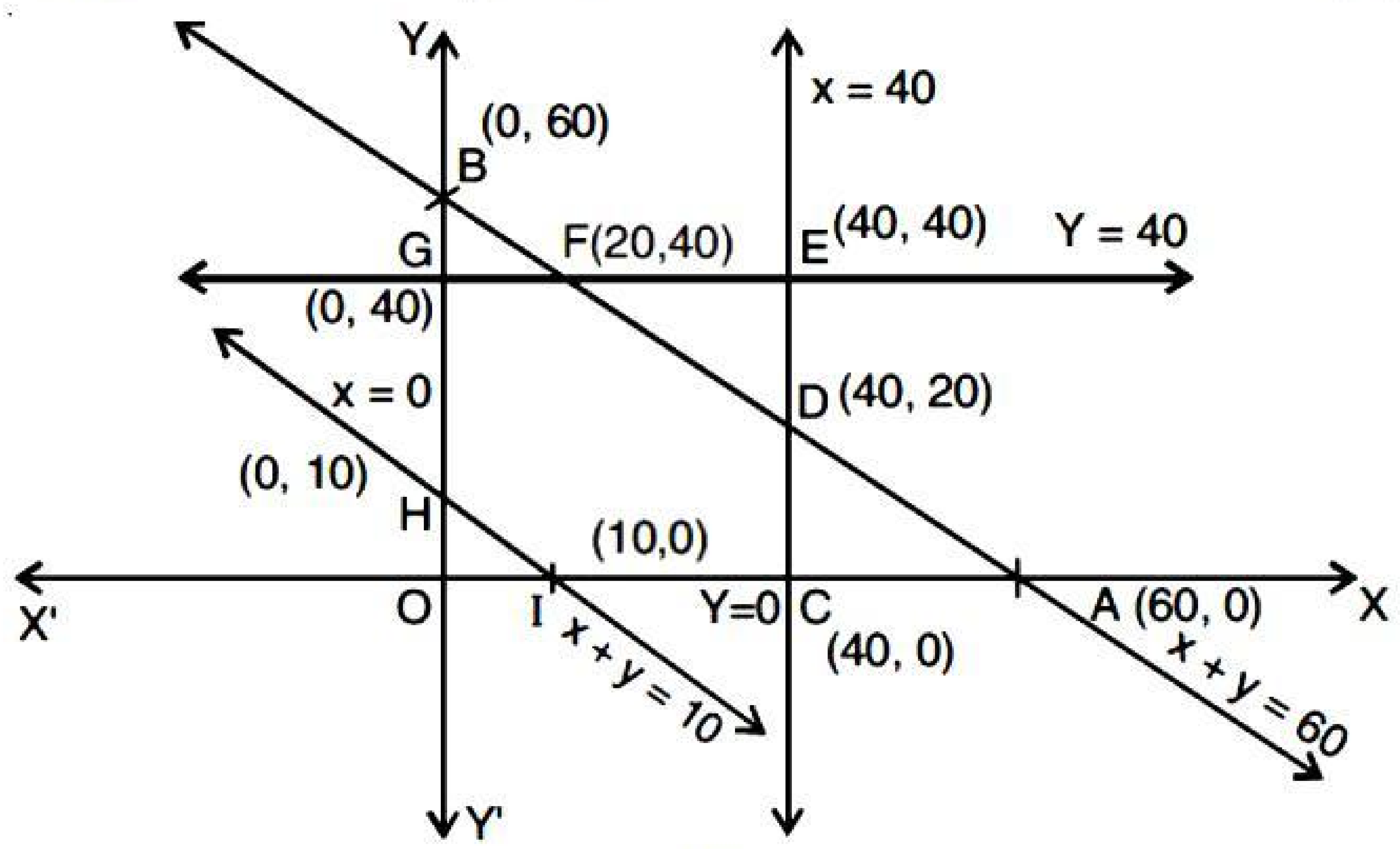


Fig.

The feasible region is as shown shaded in the above figure, which is bounded.

Applying ***Corner Point Method,*** we have :

Corner Point	Z = 3x + 4y + 270
I : (10, 0)	$Z_I = 3(10) + 4(0) + 270 =$ **300** **(Minimum)**
C : (40, 0)	$Z_C = 3(40) + 4(0) + 270 = 390$
D : (40, 20)	$Z_D = 3(40) + 4(20) + 270 = 470$
F : (20, 40)	$Z_F = 3(20) + 4(40) + 270 = 490$
G : (0, 40)	$Z_G = 3(0) + 4(40) + 270 = 430$
H : (0, 10)	$Z_H = 3(0) + 4(10) + 270 = 310$

Minimum cost is ₹ 300 at the point I (10, 0).

∴From Kitchen A : 10 packets, 0 packet and 50 packets to Schools P, Q and R respectively.

From Kitchen B : 30 packets, 40 packets and 0 packet to Schools P, Q and R respectively.

EXERCISE 12 (c)

Long Answer Type Questions

LATQ

1. (*i*) One kind of cake requires 200 g of flour and 25 g of fat and another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes. Form a linear programming problem and solve it graphically. ***(N.C.E.R.T. ; P.B. 2017; Jammu B. 2013)***

(*ii*) One kind of cake requires 300 g of flour and 15 g of fat, another kind of cake requires 150 g of flour and 30 g of fat. Find the maximum number of cakes which can be made from 7·5 kg of flour and 600 g of fat, assuming that there is no shortage of the other ingredients used in making the cakes. Make it as an LPP and solve it graphically. ***(A.I.C.B.S.E. 2010)***

2. Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs ₹ 60/kg and Food Q costs ₹ 80/kg. Food P contains 3 units/kg of Vitamin A and 5 units/kg of Vitamin B while food Q contains 4 units/kg of Vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture. ***(N.C.E.R.T. ; Jammu B. 2013)***

3. A factory makes tennis rackets and cricket bats. A tennis racket takes 1·5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.

(*i*) What number of rackets and bats must be made if the factory is to work at full capacity ?

(*ii*) If the profit on a racket and on a bat is ₹ 20 and ₹ 10 respectively, find the maximum profit of the factory when it works at full capacity. ***(H.B. 2014; C.B.S.E. 2011)***

4. If a young man drives his car at 40 km per hour, he has to spend ₹ 5 per km on petrol; if he drives it at a slower speed of 25 km per hour, the petrol cost decreases to ₹ 2 per km. He has ₹ 100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Express this as a linear programming problem and then solve it. ***(Meghalaya B. 2015)***

5. A dealer deals in two items A and B. He has ₹ 15,000 to invest and a space to store at the most 80 pieces. Item A costs him ₹ 300 and item B costs him ₹ 150. He can sell items A and B at profits of ₹ 40 and ₹ 25 respectively. Assuming that he can sell all that he buys, formulate the above as a linear programming problem for maximum profit and solve it graphically. ***(C.B.S.E. 2010 C)***

6. A man manufactures two types of steel trunks. He has two machines A and B. For completing the first type of the trunk, it requires 3 hours on machine A and 1 hour on machine B, whereas the second type of the trunk requires 3 hours on machine A and 2 hours on machine B. Machine A can work for 18 hours and B for 8 hours only per day. There is a profit of ₹ 30 on the first type of the trunk and ₹ 48 on the second type of the trunk. How many trunks of each type should be manufactured every day to earn maximum profit ? Solve the problem graphically.

(Meghalaya B. 2017)

7. A factory manufactures two types of screws, A and B, Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of ₹ 7 and screws B at a profit of ₹ 10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximise his profit ? Determine the maximum profit.

(N.C.E.R.T)

8. A dealer wishes to purchase a number of fans and sewing machines. He has only ₹ 5,760 to invest and has space for at most 20 items. A fan costs him ₹ 360 and a sewing machine ₹ 240. He expects to gain ₹ 22 on a fan and ₹ 18 on a sewing machine. Assuming that he can sell all the items he can buy, how should he invest the money in order to maximise the profit ? Translate the problem as LPP and solve it graphically.

(Meghalaya B. 2016,13 ; C.B.S.E. 2009 C; A.I.C.B.S.E. 2009 C)

9. A manufacturer produces nuts and bolts for industrial machinery. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts while it takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹ 17·50 per package of nuts and ₹ 7·00 per package of bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates his machine for at the most 12 hours a day ? Form an LPP for the problem and solve it graphically.

(N.C.E.R.T.; Kerala B. 2018; Nagaland B. 2015; C.B.S.E. 2012 ; A.I.C.B.S.E. 2009 C)

10. A shopkeeper manufactures gold rings and chains. The combined number of rings and chains manufactured per day is at most :
(*i*) 16 (*ii*) 24 (*iii*) 24
It takes (*i*) one hour (*ii*) half an hour (*iii*) one hour to make a ring and (*i*) half an hour (*ii*) one hour (*iii*) half an hour for a chain. The maximum number of hours available is (*i*) 12 (*ii*) 16 (*iii*) 16.
If the profit on a ring is :
(*i*) ₹ 300 (*ii*) ₹ 100 (*iii*) ₹ 300
and on a chain is :
(*i*) ₹ 200 (*ii*) ₹ 300 (*iii*) ₹ 190.
How many of each should be manufactured daily, so as to maximize profit ?
Form an LPP and solve it graphically. *(P.B. 2010)*

11. A man has ₹ 1,500 for the purchase of Rice and Wheat. A bag of rice and a bag of wheat costs ₹ 180 and ₹ 120 respectively. He has storage capacity of 10 bags only. He earns a profit of ₹ 11 and ₹ 9 per bag of Rice and Wheat respectively. Formulate the problem as an LPP to find the number of bags of each type he should buy to maximize the profit and solve it graphically.

(J. & K. B. 2011 ; A.I.C.B.S.E. 2009)

12. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 12 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is ₹ 5 and that from a shade is ₹ 3. Assuming that manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit ?

(N.C.E.R.T.)

13. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is ₹ 5 each for type A and ₹ 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximise the profit ? *(N.C.E.R.T.)*

14. A merchant plans to sell two types of personal computers—a desktop model and a portable model that will cost ₹ 25,000 and ₹ 40,000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than ₹ 70 lakhs and his profit on the desktop model is ₹ 4,500 and on portable model is ₹ 5,000. *(N.C.E.R.T.; A.I.C.B.S.E. 2011)*

15. A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of ₹ 8,000 and each piece of Model A and ₹ 12,000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realise a maximum profit ? What is the maximum profit per week ? *(N.C.E.R.T.)*

16. A manufacturing company makes two types of teaching aids A and B of Mathematics for class XII. Each type of A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of ₹ 80 on each piece of type A and ₹ 120 on each piece of type B. How many pieces of type A and type B should be manufactured per week to get a maximum profit ? Make it as an LPP and solve graphically. What is the maximum profit per week ? *(A.I.C.B.S.E. 2014)*

17. Two tailors A and B are paid ₹ 225 and ₹ 300 per day respectively. A can stitch 9 shirts and 6 pants while B can stitch 15 shirts and 6 pants per day. Form a linear programming problem to minimize the labour cost to produce at least 90 shirts and 48 pants. Solve the problem graphically.

18. A Co-operative society of farmers has 50 hectares of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as ₹ 10,500 and ₹ 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres per hectare. Further, no more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects draimage from this land. How much land should be allocated to each crop so as to maximize the total profit of the society ? *(N.C.E.R.T.; H.B. 2016)*

19. A diet for a sick person must contain at least 4,000 units of vitamins, 50 units of minerals and 1,400 of calories. Two foods, X and Y, are available at a cost of ₹ 4 and ₹ 3 per unit respectively. One unit of X contains 200 units of vitamin, 1 unit of mineral and 40 calories, and one unit of food Y contains 100 units of vitamin, 2 units of minerals and 40 calories. Find what combination of foods X and Y should be used to have the least cost, satisfying the requirements. Make its an LPP and solve it graphically ?

20. Food X contains 6 units of vitamin A per gm and 7 units of vitamin B per gm and costs ₹ 2·00 per gm. Food Y contains 8 units of Vitamin A per gm and 12 units of vitamin B per gm and costs ₹ 2·50 per gm. The daily minimum requirement of vitamin A and vitamin B are 100 units and 120 units respectively. Formulate the above as a linear programming problem to minimize the cost.

(W. Bengal B. 2018 ; J. & K. B. 2010)

21. A diet is to contains at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs ₹ 4 per unit food and F_2 costs ₹ 6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

(N.C.E.R.T.; H.B. 2015; C.B.S.E. 2009)

22. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food 'I' contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food 'II' contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₹ 50 per/kg to purchase Food 'I' and ₹ 70 per/kg to purchase Food 'II'. Formulate this problem as a linear programming problem to maximise the cost of such a mixture.

(N.C.E.R.T. ; H.B. 2016, 13)

23. There are two factories located one at place P and the other at place Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 5, 5 and 4 units of the commodity while the production capacity of the factories at P and Q are respectvely 8 and 6 units. The cost of transportation per unit is given below :

From/To	Cost (in ₹)		
	A	B	C
P	160	100	150
Q	100	120	100

How many units should be transported from each factory to each depot in order that the transportable cost is minimum. When will be the minimum transportation cost ? *(N.C.E.R.T.)*

24. A house-wife wishes to mix two kinds of foods X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg of foods is given below :

	Vitamin A	Vitamin B	Vitamin C
Food X	1	2	3
Food Y	2	2	2

If one kg of food X costs ₹ 6 and one kg of food Y costs ₹ 10, find the least cost of the mixture which will produce the diet. Solve the problem graphically, after making it L.P.P. *(C.B.S.E 2009 C)*

Answers

1. (*i*)–(*ii*) Max. number of cakes = 30, 20 cakes of kind one and 10 cakes of another kind.
2. Min. cost = ₹ 160 at all points lying on the line segment joining $\left(\frac{8}{3}, 0\right)$ and $\left(2, \frac{1}{2}\right)$.
3. (*i*) 4 tennis rackets and 12 cricket bats
   (*ii*) Max. profit = ₹ 200.
4. 30 km.
5. A : 20 itesm ; B : 60 itesm;
   Max. Profit = ₹ 2,300.
6. 4 trunks of type I, 2 trunks of type II.
7. 30 packages of screws A and 20 packages of screws B. Max. profit = ₹ 410.
8. 8 fans and 12 sewing machines.
9. 3 packages each of nuts and bolts daily.
10. (*i*) 8 Gold rings and 8 chains ; Max. Profit = ₹ 4,000
    (*ii*) 16 Chains; Max. Profit = ₹ 4,800
    (*iii*) 8 Gold Rings and 16 Chains ;
    Max. Profit = ₹ 5,440.
11. 5 bags of rice and 5 bags of wheat;
    Max. profit = ₹ 100.
12. 4 Pedestal lamps and 4 wooden shades;
    Max. profit = ₹ 32.
13. 8 souvenirs of type A and 20 souvenirs of type B; Max. Profit = ₹ 160.
14. 200 units of desktop model and 50 units of portable model ; Max. Profit = ₹ 11,50,000.
15. 12 Pieces of model A and 6 pieces of Model B; Max. profit = ₹ 1,68,000.
16. 12 pieces of Type A, 6 pieces of Type B;
    Max. profit = ₹ 1,680 per week.
17. A : 5 days; B : 3 days, Min. cost = ₹ 2025.
18. 30 hectares for crop X and 20 hectares of crop Y; Max. Profit = ₹ 4,95,000.
19. A : 5 units, B : 30 units.
20. Minimum cost = ₹ 31·25 when 12·5 gms of food Y is used.
21. Min. cost = ₹ 104 when 24 units of food F_1 and $\frac{4}{3}$ units of food F_2 are mixed.
22. 2 kg of Food 'I' and 4 kg of Food 'II';
    Min. cost = ₹ 380.
23. 0, 5 and 3 from factory P
    5, 0, 1 from factory Q
    Min. cost = ₹ 1550.
24. Min. cost ₹ 52 for 2 kg of food X and 4 kg of food Y.

NCERT-FILE

Questions from NCERT Book

(For each unsolved question, refer : "Solution of Modern's abc of Mathematics")

Exercise 12.1

Solve the following Linear Programming Problems graphically :

1. Maximise Z = $3x + 4y$
subject to the constraints : $x + y \leq 4$, $x \geq 0$, $y \geq 0$.

Solution : The system of constraints is :

$$x + y \leq 4 \quad ...(1)$$

and $x \geq 0$, $y \geq 0$ ...(2)

The shaded region in the following figure is the feasible region determined by the system of constraints (1) – (2) :

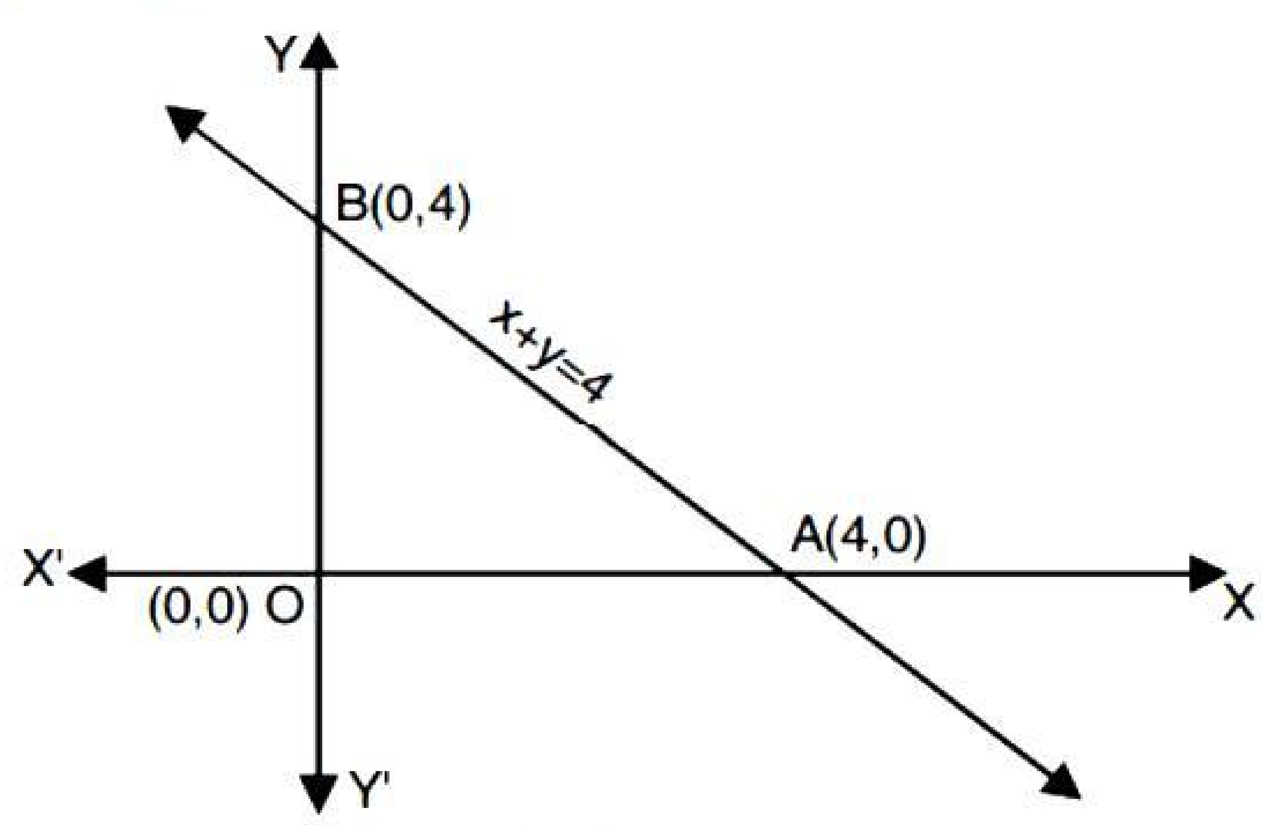


Fig.

It is observed that the feasible region OAB is bounded. Thus we use ***Corner Point Method*** to determine the maximum value of Z, where :

$$Z = 3x + 4y \quad ...(3)$$

The co-ordinates of O, A and B are (0, 0), (4, 0) and (0, 4) respectively.

We evaluate Z at each corner point :

Corner Point	Corresponding value of Z
O : (0, 0)	0
A : (4, 0)	12
B : (0, 4)	**16 (Maximum)**

Hence, Z_{max} = 16 at the point (0, 4).

2. Minimise Z = $-3x + 4y$
subject to $x + 2y \leq 8$, $3x + 2y \leq 12$, $x \geq 0$, $y \geq 0$.
[Solution. Refer Q. 19; Ex. 12(*b*)]

3. Maximise Z = $5x + 3y$
subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.
[Solution. Refer Ex. 3 Page 12/7]

4. Minimise Z = $3x + 5y$
such that $x + 3y \geq 3$, $x + y \geq 2$, $x, y \geq 0$.
[Solution. Refer Q. 20; Ex. 12(*b*)]

5. Maximise Z = $3x + 2y$
subject to $x + 2y \leq 10$, $3x + y \leq 15$, $x, y \geq 0$.
[Solution. Refer Q. 8; Ex. 12(*b*)]

6. Minimise Z = $x + 2y$
subject to $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$.
Show that the minimum of Z occurs at more than two points.

Solution : The system of constraints is :

$$2x + y \geq 3 \quad ...(1)$$
$$x + 2y \geq 6 \quad ...(2)$$

and $x \geq 0$, $y \geq 0$ ...(3)

The shaded region in the following figure is the feasible region determined by the system of constraints (1) – (3) :

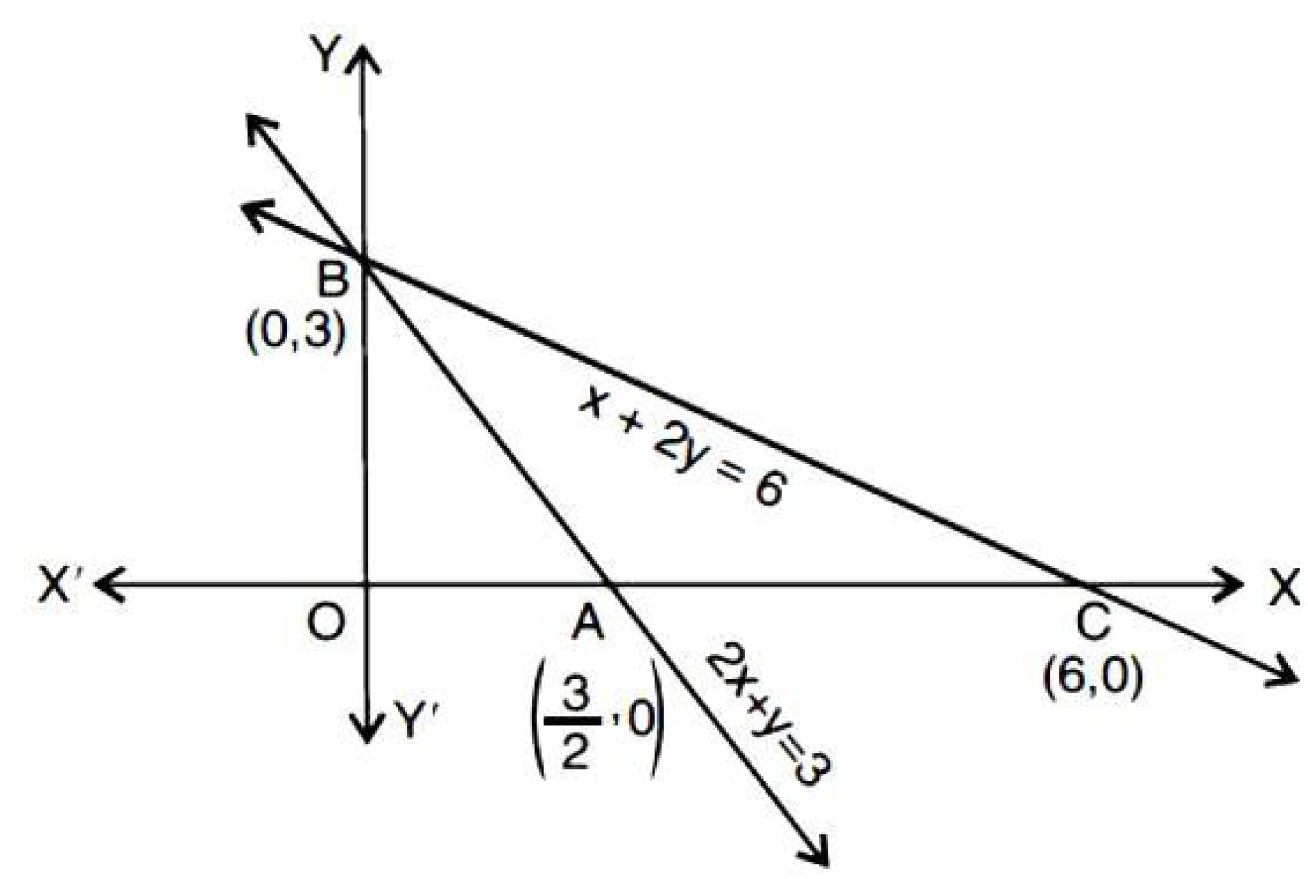


Fig.

It is observed that the feasible region is unbounded.

Applying ***Corner Point Method,*** we evaluate Z = $x + 2y$ at the corner points C (6, 0) and B (0, 3).

Corner Point	Corresponding value of Z	
C : (6, 0)	**6**	**Minimum**
B : (0, 3)	**6**	

Hence, Z_{min} = 6 at all points (more than two points) on the line segment [CB] joining the points C (6, 0) and B (0, 3).

7. Minimise and Maximise Z = $5x + 10y$
subject to $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x, y \geq 0$.
[Solution. Refer Q. 31; Ex. 12(*b*)]

8. Minimise and Maximise Z = $x + 2y$
subject to $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x, y \geq 0$.

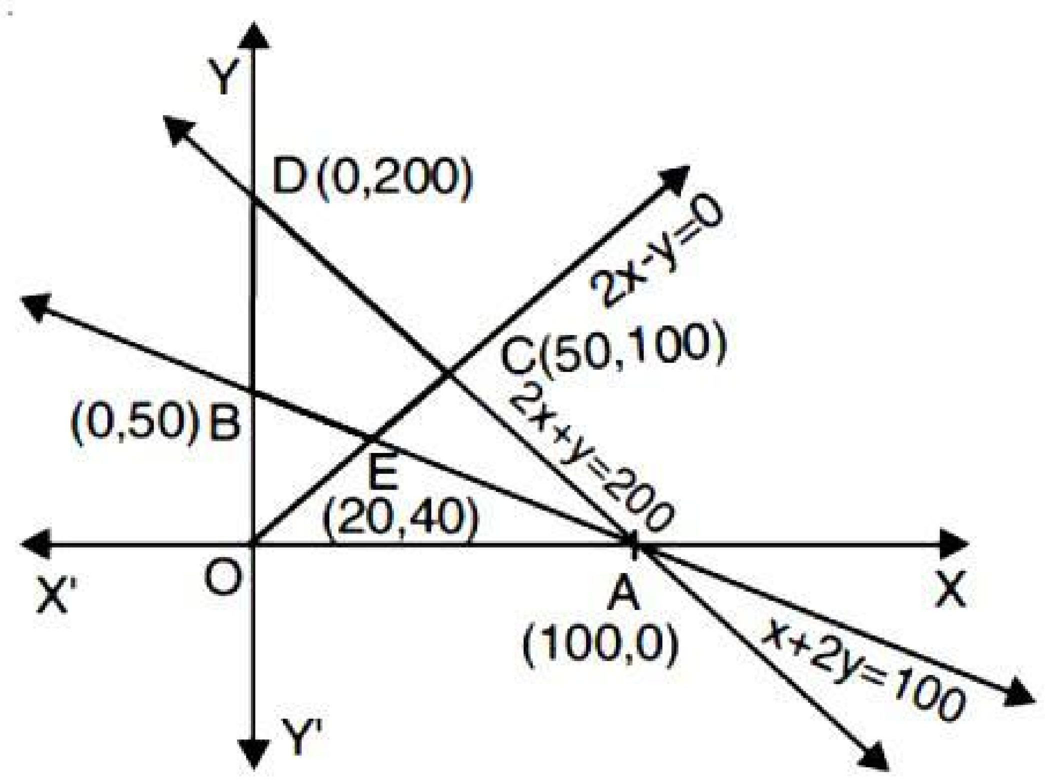
Solution : The system of constraints is :

$x + 2y \geq 100$ ...(1)

$2x - y \leq 0$ ...(2)

$2x + y \leq 200$ ...(3)

and $x, y \geq 0$ ...(4)



The shaded region in the above figure is the feasible region determined by the system of constraints (1)–(4).

It is observed that the feasible region ECDB is bounded. Thus we use ***Corner Point Method*** to determine the maximum of Z, where :

$$Z = x + 2y \quad ...(5)$$

The co-ordinates of E, C, D and B are

(20, 40)(on solving $x + 2y = 100$ and $2x - y = 0$)

(50, 100)(on solving $2x + y = 200$ and $2x - y = 0$)

(0, 200) and (0, 50) respectively.

Corner point	Corresponding Value of Z
E : (20, 40)	**100 (Minimum)**
C : (50, 100)	250
D : (0, 200)	**400 (Maximum)**
B : (0, 50)	**100 (Minimum)**

Hence, $Z_{max} = 400$ at (0, 200)

and $Z_{min} = 100$ at all points on the line segment joining the points B (0, 50) and E (20, 40).

9. Maximise Z = $-x + 2y$, subject to the constraints :
$x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, $y \geq 0$.

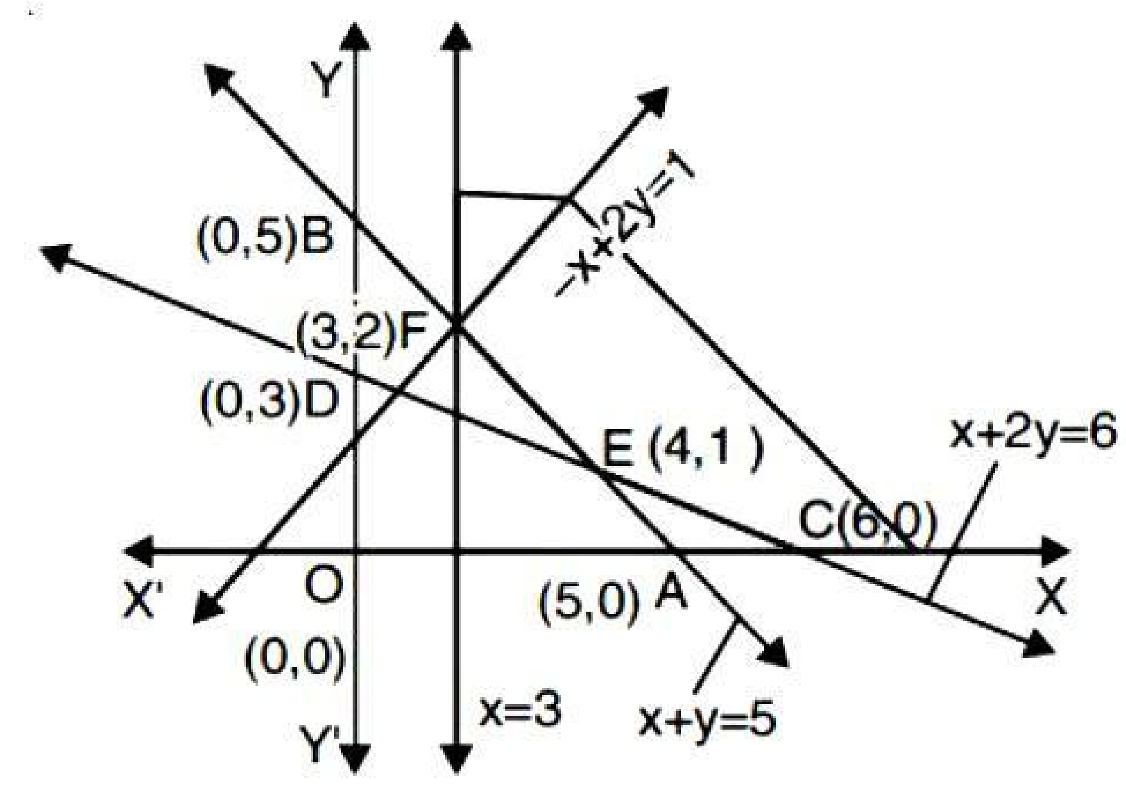
Solution : The system of constraints is :

$x \geq 3$ ...(1)

$x + y \geq 5$ ...(2)

$x + 2y \geq 6$ ...(3)

and $y \geq 0$ ...(4)



The shaded region in the above figure is the feasible region determined by (1)–(4).

The corner points are C (6, 0), E(4, 1) and F (3, 2).

Applying ***Corner Point Method,*** we have :

Corner Point	Corresponding Value of Z
C : (6, 0)	– 6
E : (4, 1)	– 2
F : (3, 2)	1

It appears that $Z_{max} = 1$ at F (3, 2).

But the feasible region is unbounded, therefore, we draw the graph of the inequation $-x + 2y > 1$.

Since the half-plane represented by $-x + 2y > 1$ has points common with the feasible region,

$\therefore \quad Z_{max} \neq 1$.

Hence, Z has no maximum value.

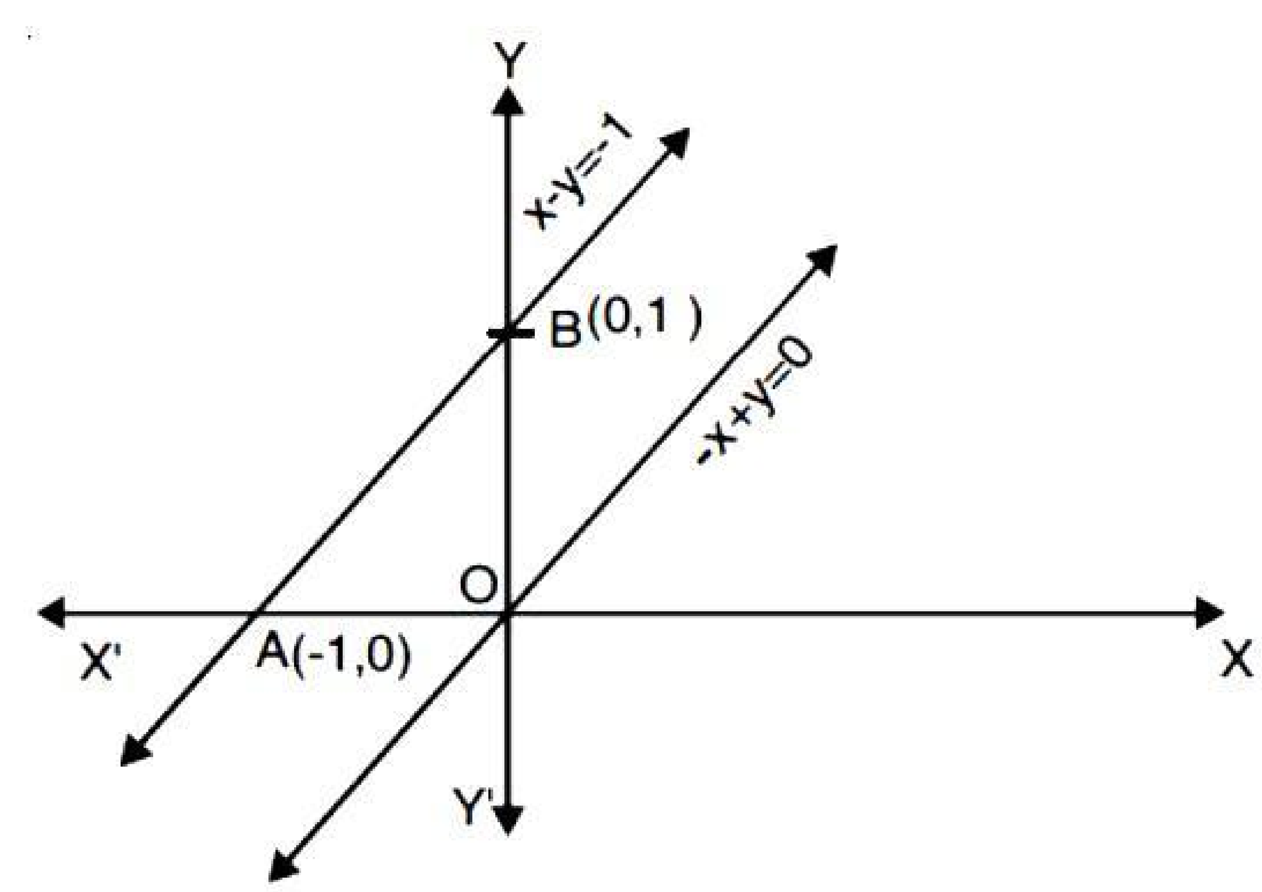
10. Maximise Z = $x + y$,
subject to $x - y \leq -1$, $-x + y \leq 0$, $x, y \geq 0$.

Solution : The system of constraints is :

$x - y \leq -1$ ...(1)

$-x + y \leq 0$ ...(2)

$x, y \geq 0$ ...(3)



Draw the lines $x - y = -1$ and $-x + y = 0$.

Clearly there is no feasible region.

[$\because$ *There is no common region*]

Hence, there is no maximum value of Z.

Exercise 12.2

1. Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs ₹ 60/kg and Food Q costs ₹ 80/kg. Food P contains 3 units/kg of Vitamin A and 5 units/kg of Vitamin B while food Q contains 4 units/kg of Vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture.

Solution : Let Reshma mix 'x' kg of food P and 'y' kg of food Q.

Thus the LPP problem is as follows :

Minimize :

$$Z = 60x + 80y$$

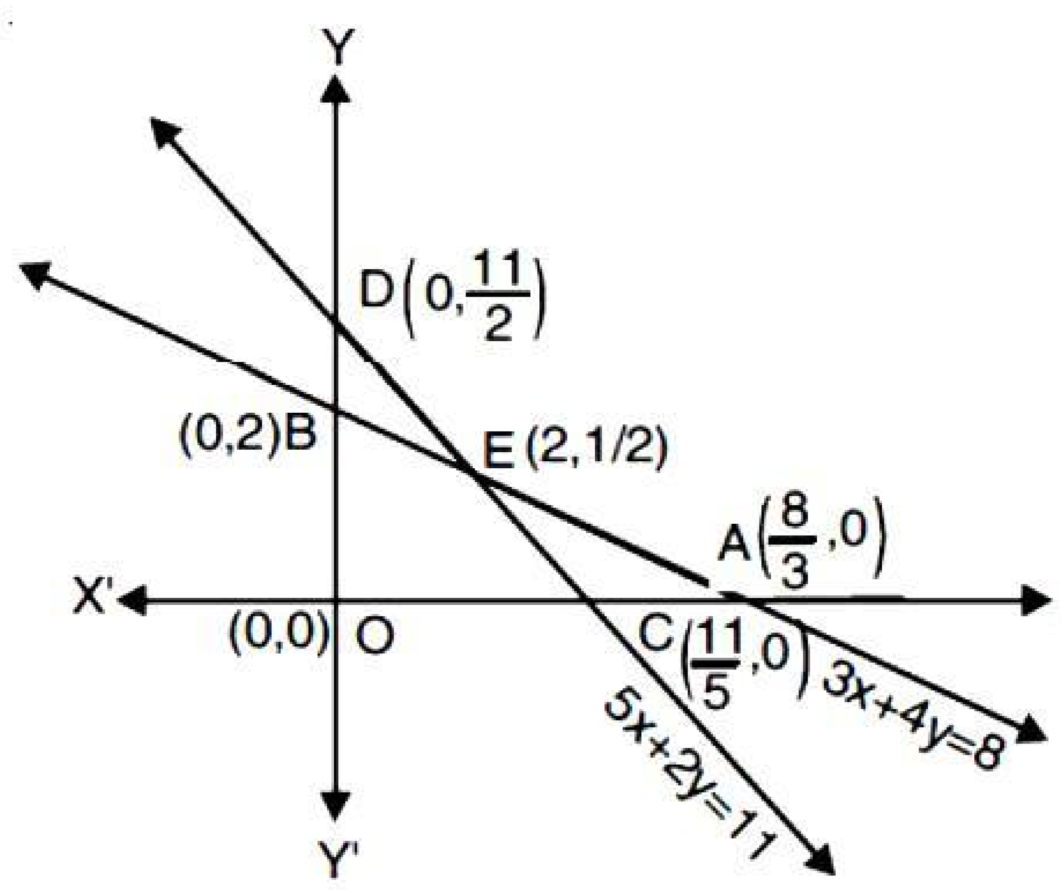
Subject to :

$$3x + 4y \geq 8 \quad ...(1)$$

$$5x + 2y \geq 11 \quad ...(2)$$

and $\quad x, y \geq 0 \quad ...(3)$

The shaded portion represents region, which is unbounded.



Applying ***Corner Point Method,*** we have :

Corner Point	**Z = 60x + 80y**
A : $\left(\frac{8}{3},0\right)$	**160** } **(Minimum)**
E : $\left(2,\frac{1}{2}\right)$	**160**
D : $\left(0,\frac{11}{2}\right)$	440.

Hence, Min. cost = ₹ 160 at all points lying on the line segment joining $\left(\frac{8}{3},0\right)$ and $\left(2,\frac{1}{2}\right)$.

2. One kind of cake reqires 200 g of flour and 25 g of fat, and another kind of cake requires 100 g of flour and 50 g of fat. find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

[Solution. Refer Q. 1; Ex. 12(*c*)]

3. A factory makes tennis rackets and cricket bats. A tennis racket takes 1·5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.

(*i*) What number of rackets and bats must be made if the factory is to work at full capacity ?

(*ii*) If the profit on a racket and on a bat is ₹ 20 and ₹ 10 respectively, find the maximum profit of the factory when it works at full capacity.

[Solution. Refer Q. 3; Ex. 12(*c*)]

4. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹ 17·50 per package on nuts and ₹ 7.00 per package on bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 12 hours a day ?

Solution : Let 'x' and 'y' be the number of packages of nuts and bolts respectively.

We have the following constraints :

$$x \geq 0 \quad ...(1)$$

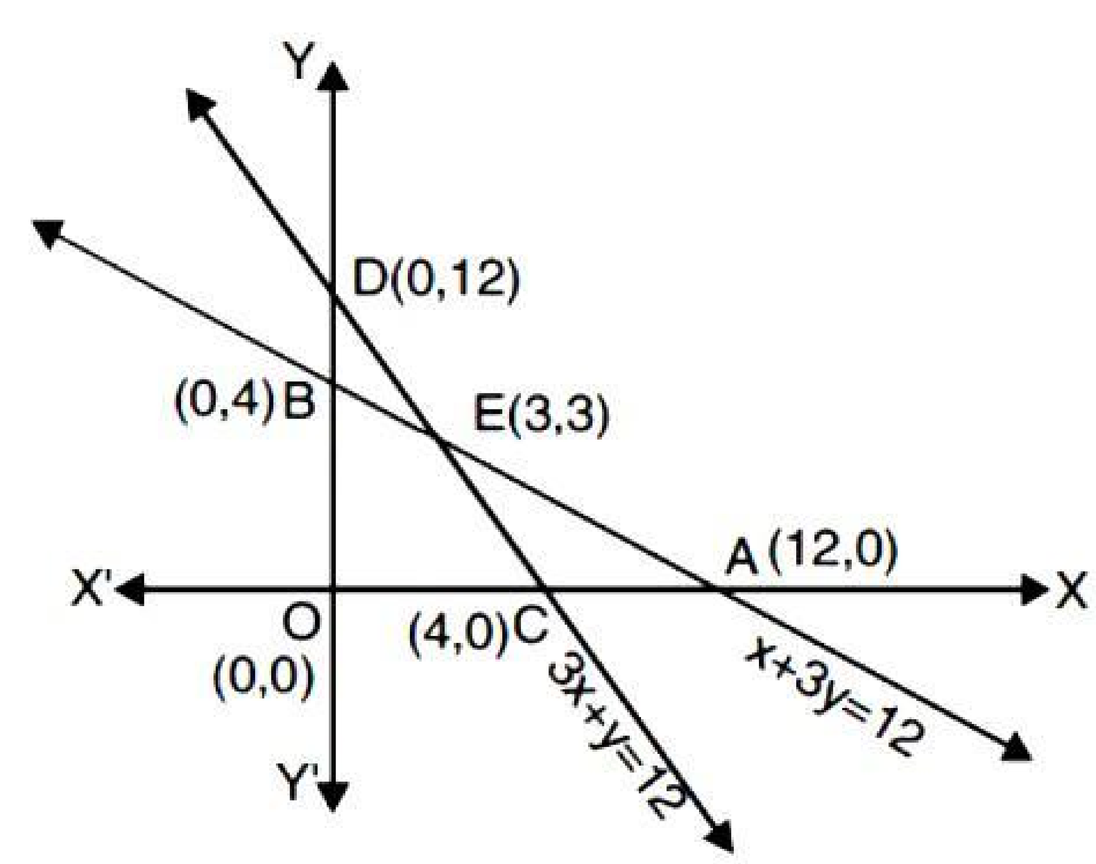
$$y \geq 0 \quad ...(2)$$

$$x + 3y \leq 12 \quad ...(3)$$

$$3x + y \leq 12 \quad ...(4)$$

Now the profit, $\quad P = (17{\cdot}5)x + 7y$

We are to maximize P subject to constraints (1)—(4).



Draw the line AB ($x + 3y = 12$)

Draw the line CD ($3x + y = 12$)

These meet at E(3, 3). **(Solve !)**

The shaded region in the figure represents the feasible region, which is bounded.

Applying ***Corner Point Method,*** we have :

Corner Point	**P = (17·5)x + 7y**
O : (0, 0)	0
C : (4, 0)	70
E : (3, 3)	**73·5 (Maximum)**
B : (0, 4)	28

Hence, max. profit is ₹ 73·50 and it is obtained when 3 packages each of Nuts and Bolts are produced daily.

5. A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of ₹ 7 and screws B at a profit of ₹ 10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximise his profit ? Determine the maximum profit.

[Solution. Refer Q. 7; Ex. 12(c)]

6. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is ₹ 5 and that from a shade is ₹ 3. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit ?

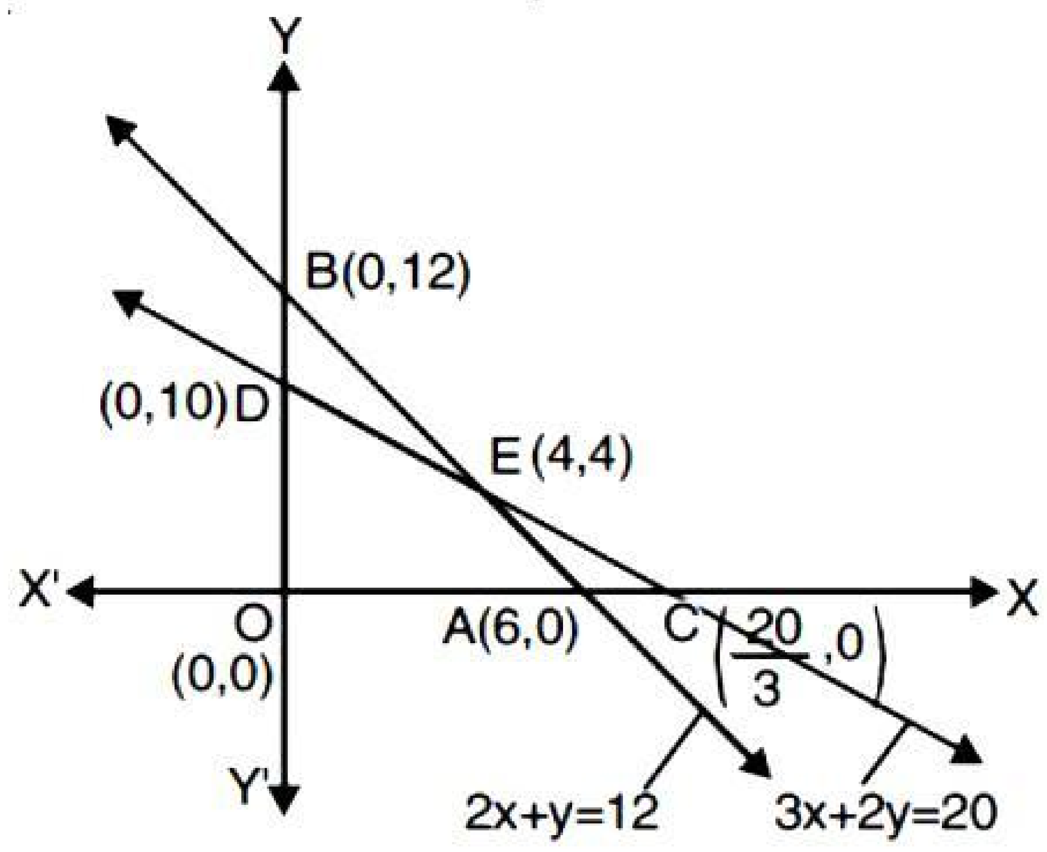
Solution : Let the manufacturer produce 'x' Pedestal lamps and 'y' Wooden shades everyday. Then the LPP problem is as follows :

Maximize : $P = 5x + 3y$ ...(1)

Subject to : $2x + y \le 12$ ...(2)

$3x + 2y \le 20$ ...(3)

and $x \ge 0, y \ge 0$ ...(4)



The shaded portion represents feasible region, which is bounded.

Applying ***Corner Point Method,*** we have :

Corner Point	**P = 5x + 3y**
O : (0, 0)	0
A : (6, 0)	30
E : (4, 4)	**32 (Maximum)**
B : (0, 10)	30

Hence, max. profit is ₹ 32 when 4 Pedestal lamps and 4 Wooden shades are manufactured.

7. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is ₹ 5 each for type A and ₹ 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximise the profit ?

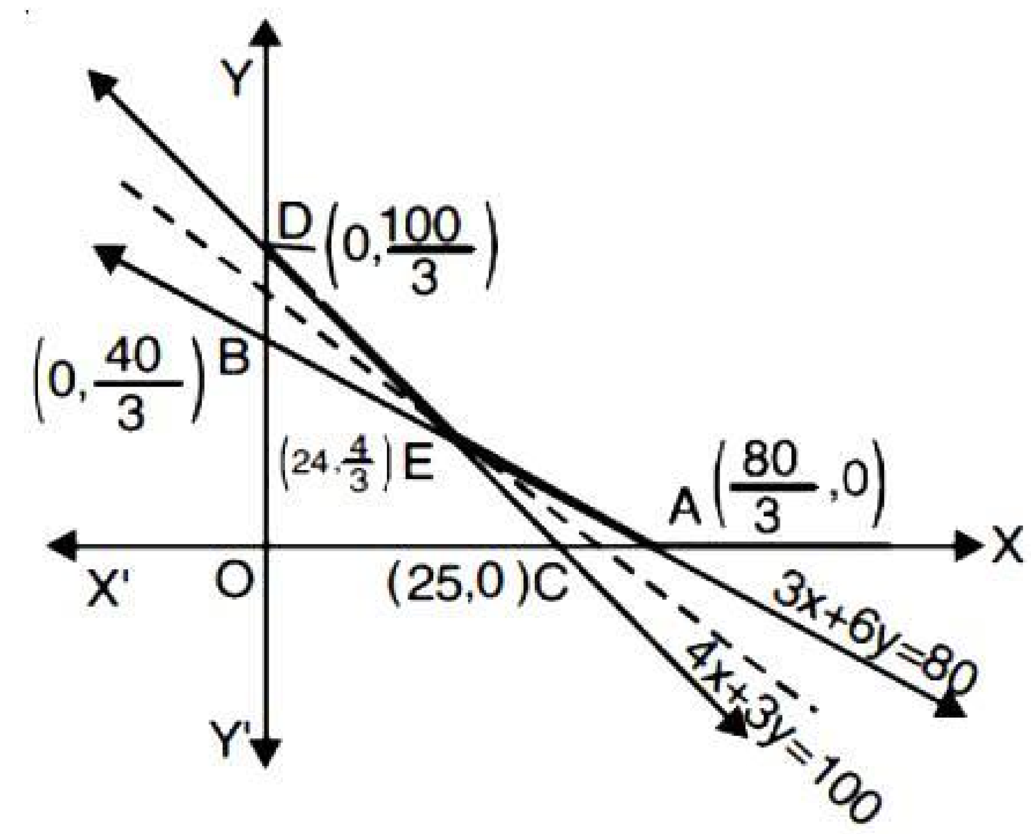
[Solution : Refer Q. 13; Ex. 12(c)]

8. A merchant plans to sell two types of personal computers — a desktop model and a portable model that will cost ₹ 25,000 and ₹ 40,000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than ₹ 70 lakhs and if his profit on the desktop model is ₹ 4,500 and on protable model is ₹ 5,000.

[Solution. Refer Q. 14; Ex. 12(c)]

9. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs ₹ 4 per unit food and F_2 costs ₹ 6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

Solution : Let diet contain 'x' units of food F_1 and 'y' units of food F_2.



Then the LPP problem is as below :

Minimize $C = 4x + 6y$

Subject to : $3x + 6y \geq 80$ ...(1)

$4x + 3y \geq 100$ ...(2)

and $x, y \geq 0$ ...(3)

First of all, let us locate the region represented by (1)—(3).

We apply ***Corner Point Method,*** we have :

Corner point	C = 4x + 6y
A : $\left(\frac{80}{3}, 0\right)$	$\frac{320}{3}$
E : $\left(24, \frac{4}{3}\right)$	**104 (Minimum)**
D : $\left(0, \frac{100}{3}\right)$	200

Hence, minimum cost is ₹ 104 when 24 units of food F_1 and $\frac{4}{3}$ units of food F_2 are mixed.

10. There are two types of fertilisers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs atleast 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 costs ₹ 6/kg and F_2 costs ₹ 5/kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost ?

Solution : Let 'x' kg of fertilizer F_1 and 'y' kg. of fertilizer F_2 be required.

We have :

Fertilizer	Nitrogen	Phosphoric Acid	Quantity	Cost
F_1	10%	6%	x kg	₹ 6
F_2	5%	10%	y kg	₹ 5
Total	14 kg	14 kg		

Thus the LPP problem is as below :

Minimise : $Z = 6x + 5y$ ...(1)

subject to 10% of x + 5% of $y \geq 14$

i.e. $\frac{10}{100}x + \frac{5}{100}y \geq 14$

i.e. $\frac{x}{10} + \frac{y}{20} \geq 14$

i.e. $2x + y \geq 280$ ...(2)

6% of x + 10% of $y \geq 14$

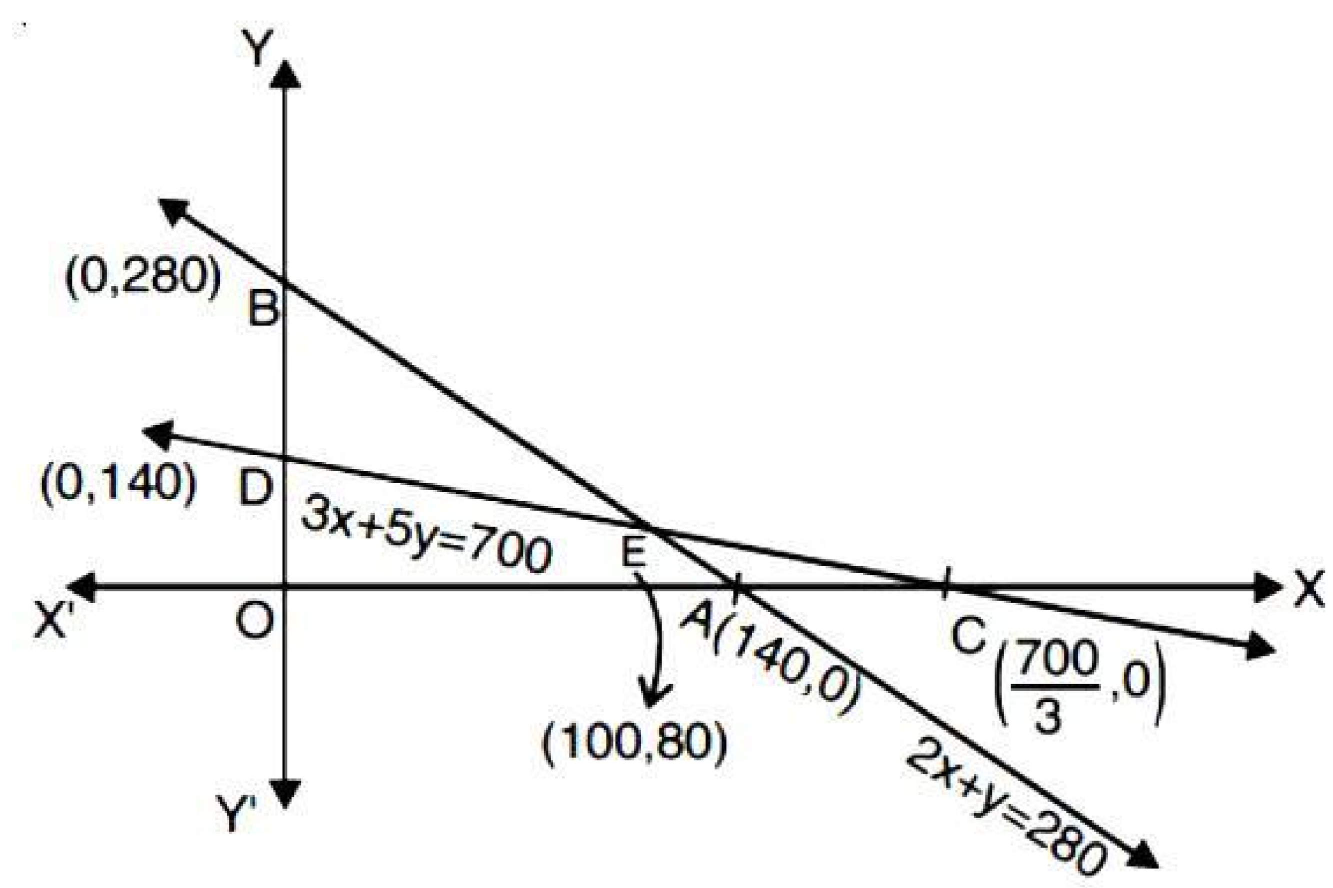
i.e. $\frac{6}{100}x + \frac{10}{100}y \geq 14$

i.e. $6x + 10y \geq 1400$

i.e. $3x + 5y \geq 700$ ...(3)

and $x, y \geq 0$ ...(4)

The feasible region is shown shaded in the following figure :



$2x + y = 280$ and $3x + 5y = 700$ meet at E (100, 80).

Applying ***Corner Point Method,*** we have :

Corner Point	Z = 6x + 5y
C : $\left(\frac{700}{3}, 0\right)$	1400
E : (100, 80)	**1000 (Minimum)**
B : (0, 280)	1400

Thus, $Z_{min} = 1000$ at E (100, 80).

Hence, minimum cost is ₹ 1000 when 100 kg of F_1 and 80 kg of F_2 are used.

11. The corner points of the feasible region determined by the following system of linear inequalities :

$2x + y \leq 10$, $x + 3y \leq 15$, $x, y \geq 0$ are (0, 0), (5, 0), (3, 4) and (0, 5). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5) is

(A) $p = q$ (B) $p = 2q$

(C) $p = 3q$ (D) $q = 3p$ **[Ans.** (D)**]**

Miscellaneous Exercise on Chapter 12

1. Refer to Q. 10 (*a*); Rev. Ex. How many packets of each food should be used to maximise the amount of vitamin A in the diet ? What is the maximum amount of vitamin A in the diet ?

[Solution : Refer Q. 10(*b*); Rev. Ex.]

2. A farmer mixes two brands P and Q of cattle feed. Brand P, costing ₹ 250 per bag, contains 3 units of nutritional element A, 2·5 units of element B and 2 units of element C. Brand Q costing ₹ 200 per bag contains 1·5 units of nutritional element A, 11·25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag ? What is the minimum cost of the mixture per bag ?

Solution. Let 'x' bags of brand P and 'y' bags of brand Q of cattle feed be mixed.

We have :

Brand \ Nutrient	No. of bags	Element A	Element B	Element C
P	x	3 units	2·5 units	2 units
Q	y	1·5 units	11·25 units	3 units
Minimum Requirements		18 units	45 units	24 units.

Then the LPP problem is as follows :

Minimise $Z = 250x + 200y$ ...(1)

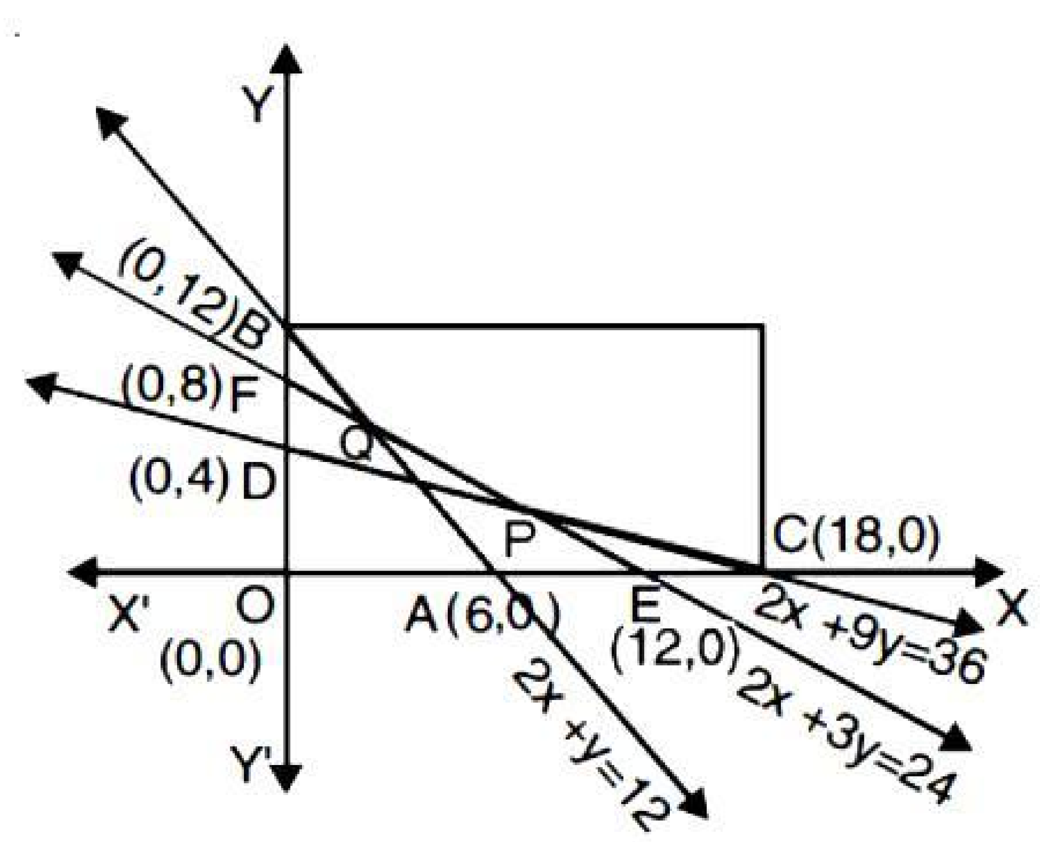
Subject to : $3x + 1{\cdot}5y \geq 18$ *i.e.* $2x + y \geq 12$ ...(2)

$2{\cdot}5x + 11{\cdot}25y \geq 14$ *i.e.* $2x + 9y \geq 36$ ...(3)

$2x + 3y \geq 24$

and $x, y \geq 0$ ...(4)

The shaded portion represents the region, which is unbounded.



$2x + 3y = 24$ and $2x + 9y = 36$ meet at P (9, 2).

$2x + 3y = 29$ and $2x + y = 12$ meet at Q (3, 6)

Applying ***Corner Point Method,*** we have :

Corner Point	Z = 250x + 200y
C : (18, 0)	4500
P : (9, 2)	2650
Q : (3, 6)	**1950 (Minimum)**
B : (0, 12)	2400

Thus $Z_{min} = 1950$ at (3, 6)

Hence, 3 bags of brand P and 6 bags of brand Q of cattle feed are mixed for a minimum cost of ₹ 1950.

3. A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food is given below :

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

One kg of food X costs ₹ 16 and one kg of food Y costs ₹ 20. Find the least cost of the mixture which will produce the required diet ?

[Solution : Refer Q. 9; Rev. Ex.]

4. A manufacturer makes two types of toys A and B. Three machines are needed for this purpose and the time (in minutes) required for each toy on the machines is given below :

Types of Toys	Machines		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is ₹ 7·50 and that on each toy of type B is ₹ 5, show that 15 toys of type A and 30 of type B should be manufactured in a day to get maximum profit.

Solution : Let 'x' toys of type A and 'y' toys of type B be manufactured per day.

Thus LPP is as below :

Maximize : $Z = \frac{15}{2}x + 5y$

Subject to : $12x + 6y \leq 360$

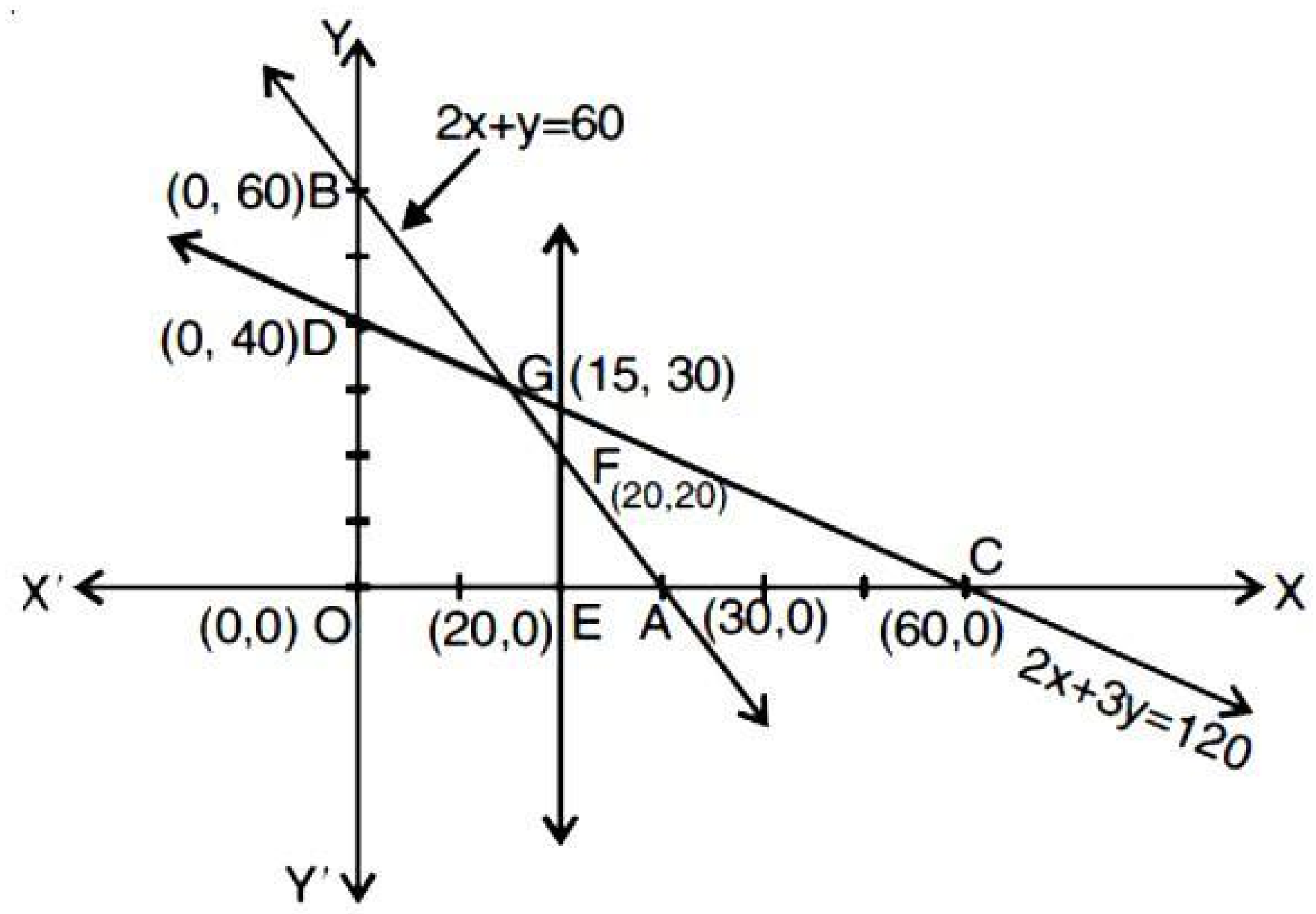
$\Leftrightarrow \quad 2x + y \leq 60 \quad ...(1)$

$18x + 0y \leq 360 \Rightarrow x \leq 20 \quad ...(2)$

$6x + 9y \leq 360 \Rightarrow 2x + 3y \leq 120 \quad ...(3)$

and $x \geq 0, y \geq 0. \quad ...(4)$

The shaded portion is the feasible region, which is bounded.



Applying ***Corner Point Method,*** we have :

Corner Point	$Z = \frac{15}{2}x + 5y$
O : (0, 0)	0
E : (20, 0)	150
F : (20, 20)	250
G : (15, 30)	**262·50 Minimum**
D : (0, 40)	200

Hence, the max. profit is ₹ 262.50 when 15 toys of type A and 30 toys of type B are manufactured.

5. An aeroplane can carry a maximum of 200 passengers. A profit of ₹ 1000 is made on each executive class ticket and a profit of ₹ 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit

[Solution : Refer Q. 2; Rev. Ex.]

6. Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops. D. E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table :

Transportation cost per quintal (in ₹)

From/To	**A**	**B**
D	6	4
E	3	2
F	2·50	3

How should the supplies be transported in order that the transportation cost is minimum ? What is the minimum cost ?

Solution : Let 'x' quintals of grain be transported from Godown A to shop D, 'y' quintals to shop E.

Then $(100 - (x + y))$ quintals will be transported to shop F.

Thus we have the table :

From	**To shop (D)**	**To shop (E)**	**To shop (F)**	**Cost of Transportation (in Rs.)**
Godown (A)	x	y	$100 - (x + y)$	$6x + 3y + \frac{5}{2}(100 - x + y)$
Godown (B)	$60 - x$	$50 - y$	$x + y - 60$	$4(60 - x) + 2(50 - y) + 3(x + y - 60)$
Total	60	50	40	$\frac{5}{2}x + \frac{3}{2}y + 410$

Thus the LPP problem is as below :

Minimize : $C = \frac{5}{2}x + \frac{3}{2}y + 410$

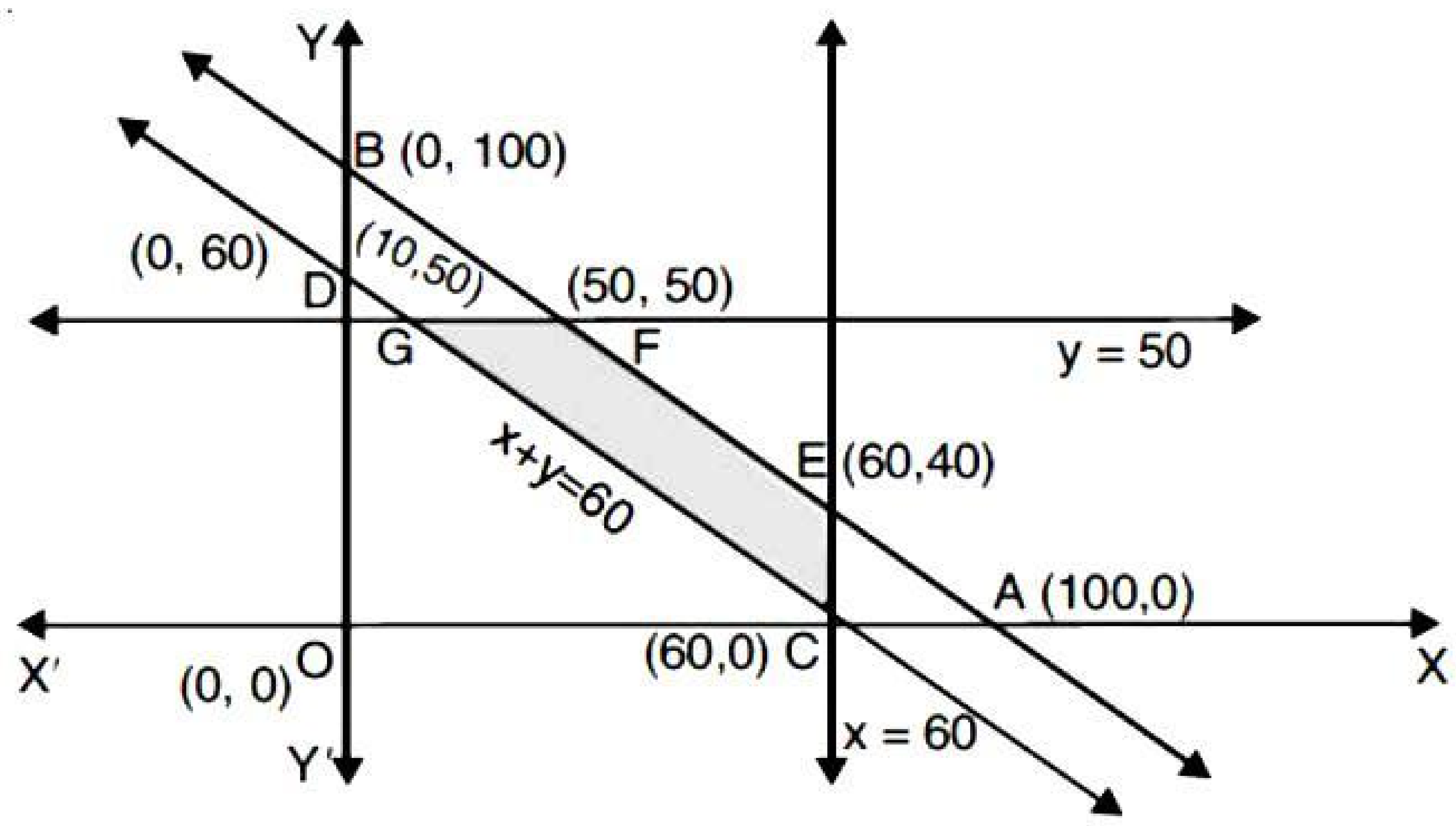
Subject to : $60 - x \geq 0 \Leftrightarrow x \leq 60 \quad ... (1)$

$50 - y \geq 0 \Leftrightarrow y \leq 50 \quad ... (2)$

$100 - (x + y) \geq 0 \Leftrightarrow x + y \leq 100 \quad ... (3)$

$x + y - 60 \geq 0 \Leftrightarrow x + y \geq 60 \quad ... (4)$

and $x, y \geq 0 \quad .... (5)$



The shaded portion represents feasible region, which is bounded

Applying ***Corner Point Method,*** we have :

Corner Point	Corresponding value of C
C : (60, 0)	560
E : (60, 40)	620
F : (50, 50)	610
G : (10, 50)	**510 (Minimum)**

Hence minimum cost = ₹ 510 when from godown A : 10 quintals of grain are sent to shop D, 50 quintals to shop E and 40 quintals to shop F and from godown B : 50 quintals are sent to shop D.

7. An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps, D, E and F whose requirments are 4500 L, 3000 L and 3500 L respectively. The distances (in km) between the depots and the petrol pumps is given in the following table :

Distance in (km)

From/To	A	B
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost of 10 litres of oil is ₹ 1 per km, how should the delivery be scheduled in order that the transportation cost is minimum ? What is the minimum cost ?

Solution : Let, from depot A,

x litres of oil be transported to pump D

y litres of oil to pump E

and $[7000 - (x + y)]$ litres of oil to pump F.

And, from depot B,

$(4500 - x)$ litres of oil be transported to pump D

$(3000 - y)$ litres of oil to pump E

and $[3500 - (7000 - (x + y))] = (x + y - 3500)$ litres to pump F.

Thus we have the table.

Depot	To pump D	To pump E	To pump F	Cost (in ₹) of transportation
A	x	y	$7000 - (x + y)$	$\frac{7x}{10} + \frac{6y}{10} + \frac{3(7000 - x - y)}{10}$
B	$4500 - x$	$3000 - y$	$x + y - 3500$	$\frac{3}{10}(4500 - x) + \frac{4}{10}(3000 - y) + \frac{2}{10}\left(\frac{3x}{10} + \frac{y}{10} + 3950\right)$
Total	4500	3000	3500	$\frac{3x}{10} + \frac{y}{10} + 3950$

Thus LPP problem is as below :

Minimize : $C = \frac{3x}{10} + \frac{y}{10} + 3950$

Subject to : $4500 - x \geq 0 \Leftrightarrow x \leq 4500$ ...(1)

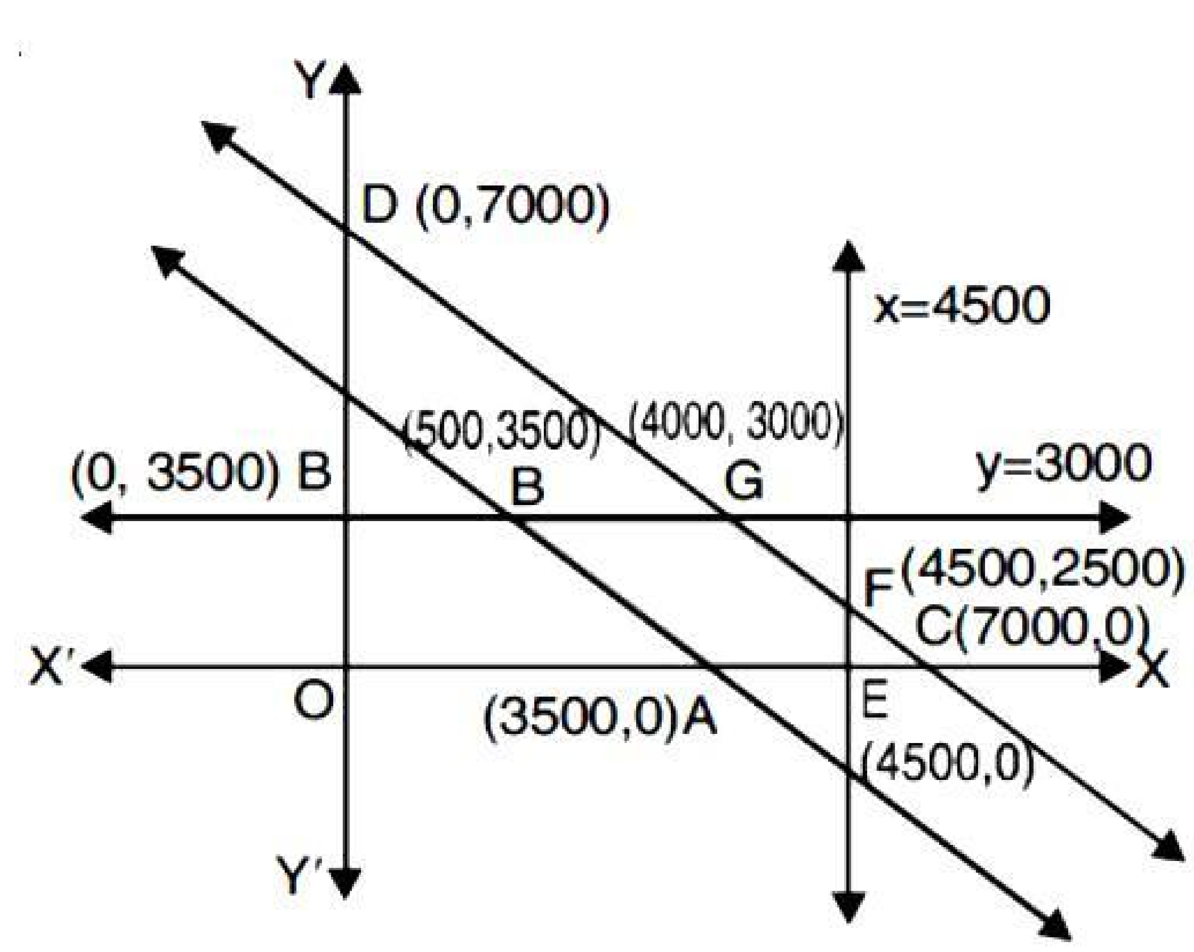
$3000 - y \geq 0 \Leftrightarrow y \leq 3000$ ...(2)

$x + y - 3500 \geq 0 \Leftrightarrow x + y \geq 3500$ ... (3)

$7000 - (x + y) \geq 0 \Leftrightarrow x + y \leq 7000$ ...(4)

and $x, y \geq 0$ ...(5)

The shaded portion represents feasible region, which is bounded.



Applying ***Corner Point Method,*** we have :

Corner Point	Corresponding value of C
A : (3500, 0)	9160
E : (4500, 0)	5300
F : (4500, 2500)	5500
G : (4000, 3000)	5450
H : (500, 3500)	**4400 (Minimum)**

Hence, minimum cost = ₹ 4400 when

From depot A ; 500, 3000 and 3500 litres

and depot B ; 4000, 0, 0 litres

to pumps D, E and F respectively.

8. A fruit grower can use two types of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine.

It the grower wants to minmise the amount of nitrogen added to the garden, how many bags of each brand should be used ? What is the minimum amount of nitrogen added in the garden ?

kg per bag		
	Brand P	**Brand Q**
Nitrogen	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

[Solution : Refer Q. 11(*a*); Rev. Ex.]

9. Refer to Question 8. If the grower wants to maximise the amount of nitrogen added to the garden, how many bags of each brand should be added ? What is the maximum amount of nitrogen added ?

[Solution : Refer Q. 11(*b*); Rev. Ex.]

10. A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of ₹ 12 and ₹ 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximise the profit ?

Solution. Let 'x' dolls of type A and 'y' dolls of type B be manufactured.

Then LPP problem is as below :

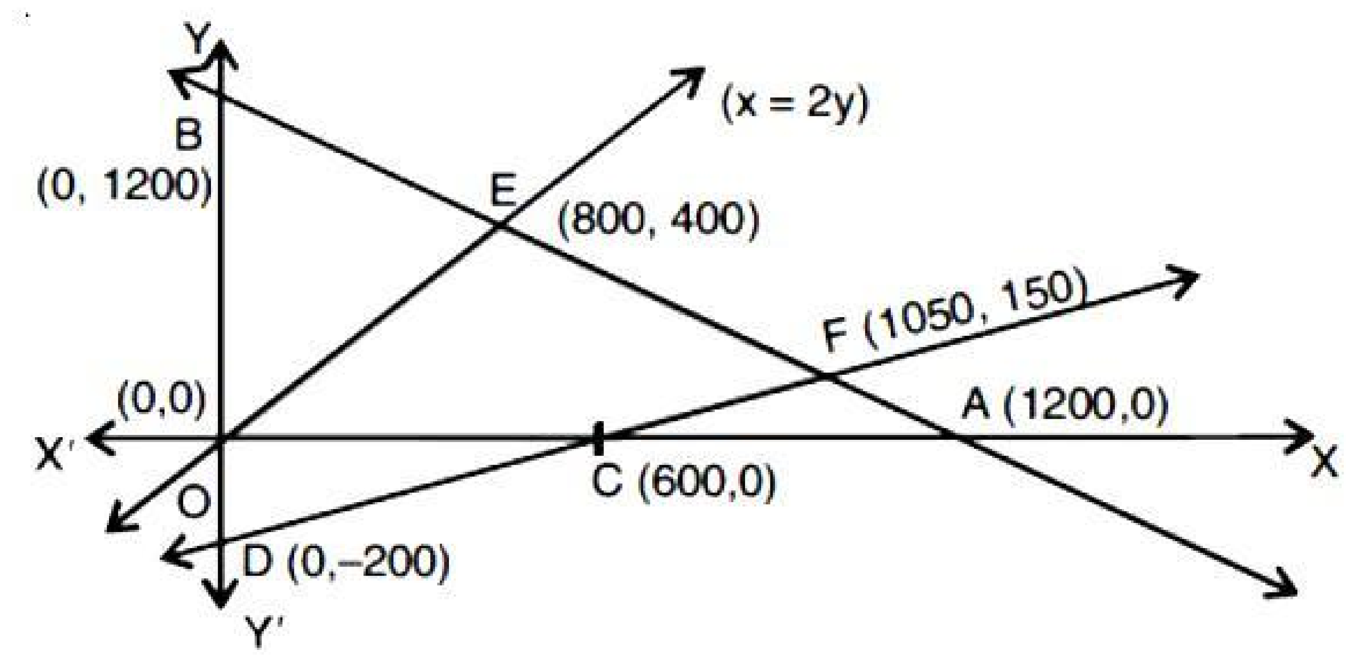
Maximize $P = 12x + 16y$

Subject to : $x + y \leq 1200$ ...(1)

$y \leq \frac{x}{2} \Leftrightarrow x - 2y \geq 0$ ...(2)

$x \leq 3y + 600 \Leftrightarrow x - 3y \leq 600$ ...(3)

and $x, y \geq 0$ ...(4)



The shaded portion is the feasible region, which is bounded.

Applying ***Corner Point Method,*** we have :

Corner Point	P = 12x + 16y
C : (600, 0)	7200
F : (1050, 150)	15000
E : (800, 400)	**16000 (Maximum)**
O : (0, 0)	0

Hence maximum profit is ₹ 16000 when 800 dolls of type A and 400 dolls of type B are manufactured and sold.

Questions From NCERT Exemplar

Example. Determine the minimum value of $Z = 3x + 2y$ (if any), if the feasible region for an LPP is shown in the figure :

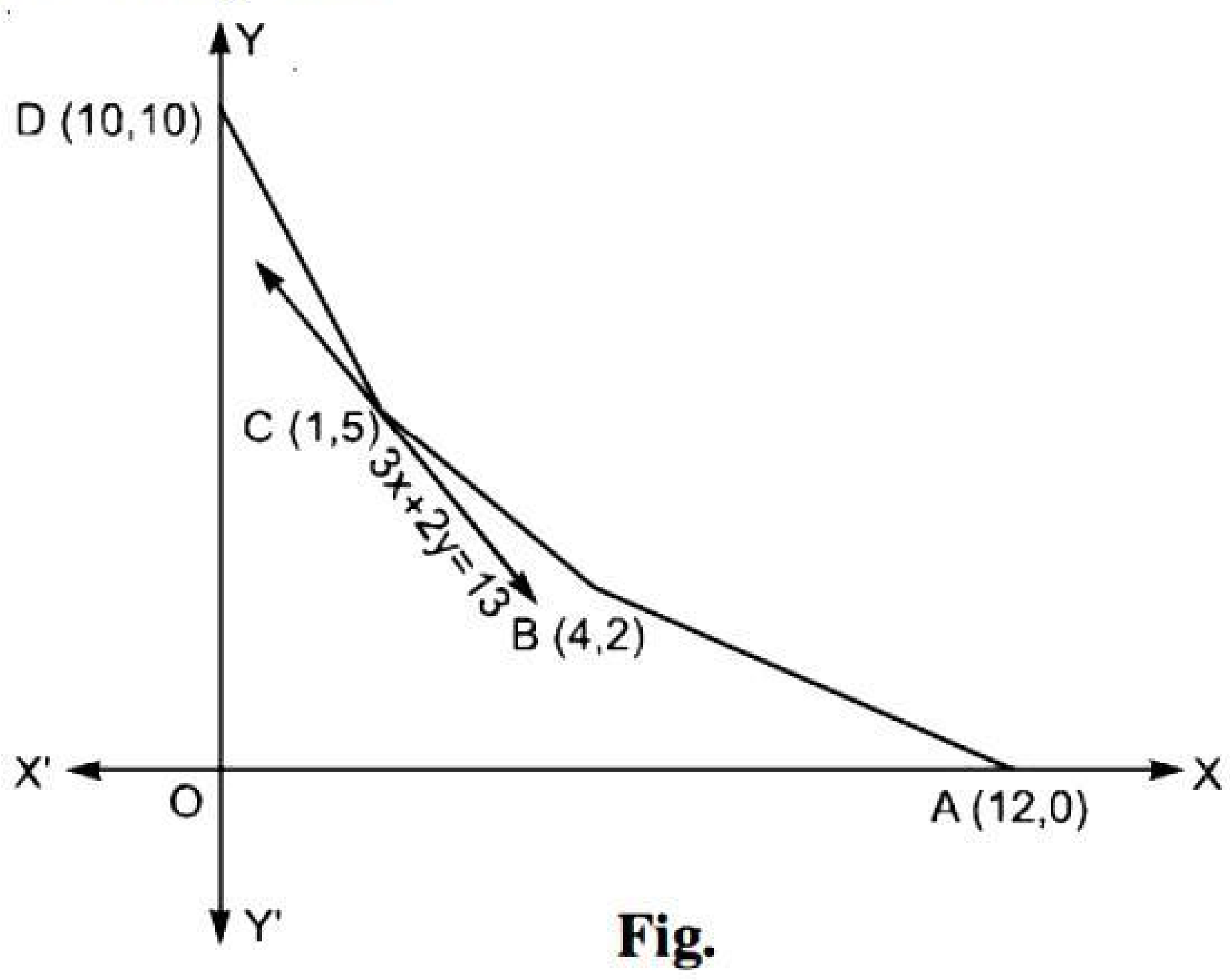


Fig.

Solution. The feasible region is unbounded.

$\therefore$ Minimum value of Z may or may not exist.

Applying ***Corner Point Method*** , we have :

Corner Point	Value of Z
A : (12, 0)	36
B : (4, 2)	16
C : (1, 5)	**13 (Minimum)**
D : (0, 10)	20

We graph $3x + 2y < 13$.

It is observed that the open half plane determined by $3x + 2y < 13$ and R do not have a common point.

Hence, Minimum value of Z = 13.

Exercise

1. Solve the following LPP graphically :

Maximise $Z = 2x + 3y$, subject to :

$x + y \leq 4, x \geq 0, y \geq 0$.

2. Maximum $Z = 3x + 5y$ subject to the constraints :

$x + 2y \geq 10, x + y \geq 6, 3x + y \geq 8, x \geq 0, y \geq 0$.

Answers

1. $Z_{max} = 12$ at (0, 4).

2. $Z_{min} = 26$ at (2, 4).

Revision Exercise

1. Find the maximum and minimum values of f : 2x + y subject to the constraints :

$x + 3y \geq 6$, $x - 3y \leq 3$, $3x + 4y \leq 24$,

$-3x + 2y \leq 6$, $5x + y \geq 5$, $x, y \geq 0$. HOTS

Solution. We have to maximise and minimise for $2x + y$ subject to the constraints :

$x + 3y \geq 6$ ... (1) $\quad x - 3y \leq 3$ ... (2)

$3x + 4y \leq 24$ ... (3) $\quad -3x + 2y \leq 6$ ... (4)

$5x + y \geq 5$ ... (5) $\quad x \geq 0\ y \geq 0$ ... (6)

Consider a set of rectangular axes OXY in the plane.

Clearly any point satisfying $x \geq 0$, $y \geq 0$ lies in first quadrant.

Draw the graph of $x + 3y = 6$.

For $x = 0$, $3y = 6 \Rightarrow y = 2$.

For $y = 0$, $x = 6$.

$\therefore$ the line meets the x-axis at A (6, 0) and y-axis at B (0, 2).

Draw the graph of $x - 3y = 3$.

For $x = 0$, $-3y = 3 \Rightarrow \quad y = -1$.

For $y = 0$, $x = 3$.

$\therefore$ The line meets x-axis at C (3, 0) and y-axis at C (0, – 1).

Draw the graph of $3x + 4y = 24$.

For $x = 0$, $4y = 24 \Rightarrow \quad y = 6$.

For $y = 0$, $3x = 24 \Rightarrow \quad x = 8$.

$\therefore$ The line meets x-axis at E (8, 0) and y-axis at F (0, 6).

Draw the graph of $-3x + 2y = 6$.

For $x = 0$, $2y = 6 \Rightarrow \quad y = 3$.

For $y = 0$, $-3x = 6 \Rightarrow \quad x = -2$.

$\therefore$ The line meets x-axis at G (–2, 0) and y-axis at H (0, 3).

Draw the graph of $5x + y = 5$.

For $x = 0$, $y = 5$.

For $y = 0$, $5x = 5 \Rightarrow \quad x = 1$.

$\therefore$ The line meets x-axis at I (1, 0) and y-axis at J (0, 5).

Since feasible region is the region which satisfies all the constraints, therefore, LMNPQ is the feasible region.

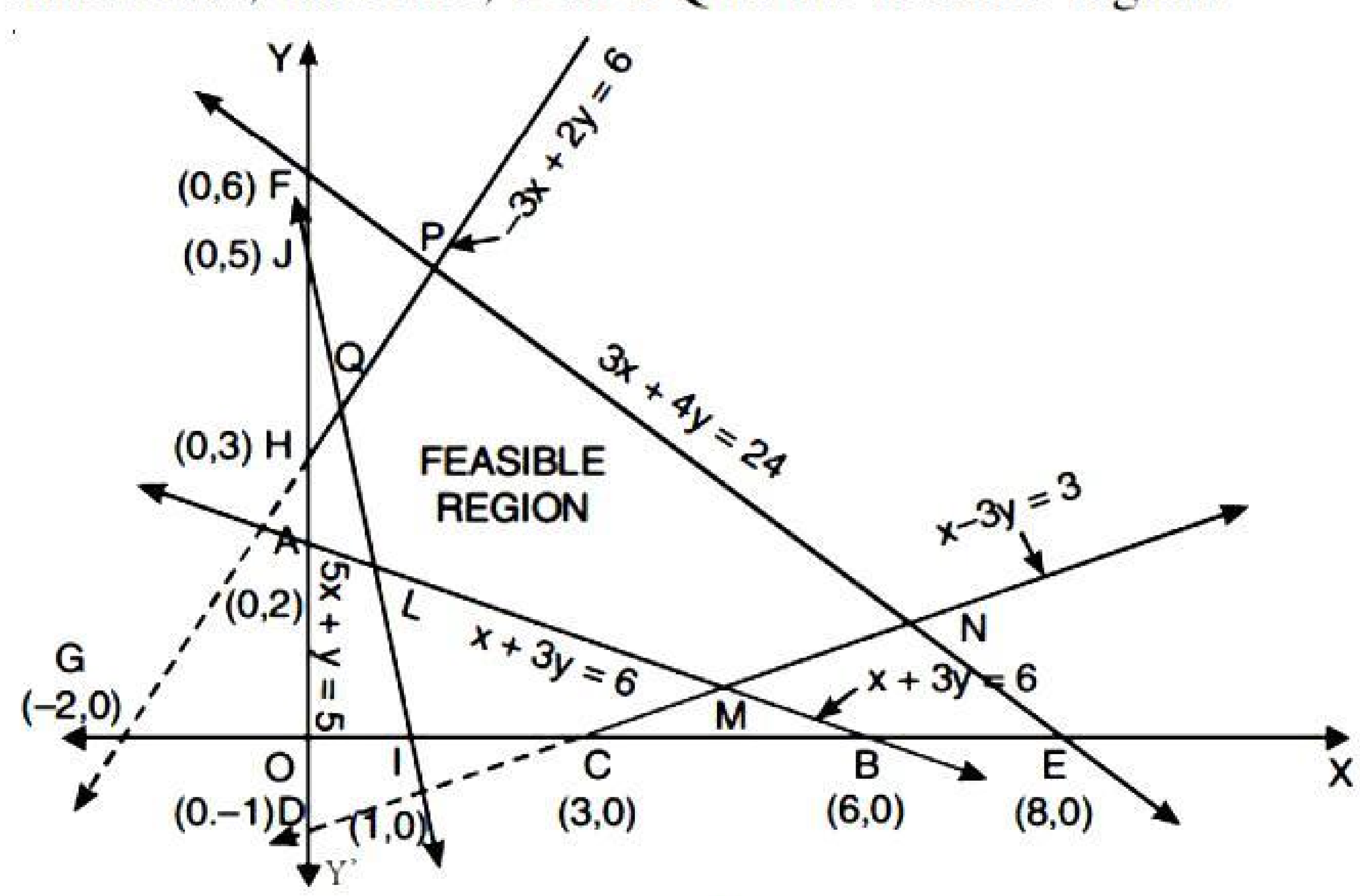


Fig.

The corner points are

$$L\left(\frac{4}{3}, 5\right), M\left(\frac{4}{13}, \frac{45}{13}\right), N\left(\frac{9}{14}, \frac{25}{14}\right),$$

$$P\left(\frac{9}{2}, \frac{1}{2}\right) \text{ and } Q\left(\frac{84}{13}, \frac{15}{13}\right). \qquad [\textit{Solve !}]$$

The shaded region in the figure is the feasible region determined by the system of constraints (1) – (6).

It is observed that feasible region NPQLM is bounded. Thus we use ***Corner Point Method*** to determine the maximum and minimum values $Z = 2x + y$ ...(7)

We evaluate Z at each corner point

Corner Point	Corresponding value of Z
N : $\left(\frac{9}{14}, \frac{25}{14}\right)$	$\mathbf{\frac{43}{14}}$ **(Minimum)**
P : $\left(\frac{9}{2}, \frac{1}{2}\right)$	$\frac{19}{2}$
Q : $\left(\frac{84}{13}, \frac{15}{13}\right)$	$\mathbf{\frac{183}{13}}$ **(Maximum)**
C : $\left(\frac{4}{3}, 5\right)$	$\frac{23}{3}$
M : $\left(\frac{4}{13}, \frac{45}{13}\right)$	$\frac{53}{13}$

Hence, $Z_{max} = \dfrac{183}{13}$ at $\left(\dfrac{84}{13}, \dfrac{15}{13}\right)$ and

$$Z_{min} = \frac{43}{14} \text{ at } \left(\frac{9}{14}, \frac{25}{14}\right).$$

2. An aeroplane can carry a maximum of 200 passengers. A profit of ₹ 1000 is made on each executive class ticket and a profit of ₹ 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit ? Make an LPP and solve it graphically.

(N.C.E.R.T. ; Nagaland B. 2018)

3. A manufacturer of patent medicines is preparing a production plan on medicines, A and B. There are sufficient raw materials available to make 20,000 bottles of A and 40,000 bottles of B, but there are only 45,000 bottles into which either of the medicines can be put. Further, it takes 3 hours to prepare enough material to fill 1000 bottles of A, it takes 1 hour to prepare enough material to fill 1000 bottles of B and there are 66 hours available for this operation. The profit is ₹ 8 per bottle for A and ₹ 7 per bottle for B. How should the manufacturer schedule his production in order to maximize his profit ? *(Mizoram B. 2017)*

4. A medical company has factories at two places, A and B. From these places, supply is made to each of its three agencies situated at P, Q and R. The monthly requirements of the agencies are respectively 40, 40 and 50 packets of the medicines, while the production capacity of the factories A and B, are 60 and 60 packets respectively. The transportation cost per packet from the factories to the agencies are given below :

Transportation cost per packet (in ₹)

To \ From	A	B
P	5	4
Q	4	2
R	3	5

How many packets from each factory be transported to each agency so that the cost of transportation is minimum ? Also find the minimum cost.

5. Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table :

Transportation Cost per Quintal (in ₹)		
From/To	**A**	**B**
D	6	4
E	3	2
F	2·50	3

How should the supplies be transported in order that the transportation cost is minimum ? What is the minimum cost ? *(N.C.E.R.T.)*

6. An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps, D, E and F whose requirements are 4500 L, 3000 L and 3500 L respectively. The distances (in km) between the depots and the petrol pumps are given in the following table :

Distance in (km.)		
From / To	**A**	**B**
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost of 10 litres of oil is ₹ 1 per km, how should the delivery be scheduled in order that the transportation cost is minimum ? What is the minimum cost ? *(N.C.E.R.T.)*

7. A soft drinks firm has two bottling plants, one located at P and the other located at Q. Each plant produces three different soft drinks A, B and C. The capacities of two plants in number of bottles per day, are as follows :

Product	Plant P	Plant Q
A	3000	1000
B	1000	1000
C	2000	6000

A market survey indicates that during the month of April, there will be a demand for 24,000 bottles of A, 16,000 bottles of B and 48,000 bottles of C. The cost of running the two plants P and Q are respectively ₹ 6,000 and ₹ 4,000 per day. Find graphically, the number of days for which either of the two plants P and Q should be run in the month of April so as to minimise production cost while still meeting the market demand.

8. A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of ₹ 12 and ₹ 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximise the profit ? *(N.C.E.R.T.)*

9. A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food is given ahead :

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

One kg of food X costs ₹ 16 and one kg of food Y costs ₹ 20. Find the least cost of the mixture which will produce the required diet ? *(N.C.E.R.T.)*

10. (*a*) **(Diet Problem)** A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A, while each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimize the amount of vitamin A in the diet ? What is the maximum amount of vitamin A ? *(N.C.E.R.T.)*

(*b*) Refer to part (*a*), how many packets of each food should be used to maximise the amount of vitamin A in the diet ? What is the maximum amount of vitamin A in the diet ? *(N.C.E.R.T.)*

11. (*a*) A fruit grower can use two types of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine.

If the grower wants to minimise the amount of nitrogen added to the garden, how many bags of each brand should be used ? What is the minimum amount of nitrogen added in the garden ? *(N.C.E.R.T.)*

	kg per bag	
	Brand P	**Brand Q**
Nitrogen	3	3·5
Phosphoric acid	1	2
Potash	3	1·5
Chlorine	1·5	2

(*b*) Refer to part (*a*). If the grower wants to maximise the amount of nitrogen added to the garden, how many bags of each brand should be added ? What is the maximum amount of nitrogen added ? *(N.C.E.R.T.)*

12. A factory owner purchases two types of machines A and B for his factory. The requirements and limitations for the machines are as follows :

	Area occupied by the machine	Labour force for each machine	Daily output in units
Machine A	1000 sq. m.	12 men	60
Machine B	1200 sq. m.	8 men	40

He has an area of 9,000 sq. m. available and 72 skilled men who can operate the machines. How many machines of each type should he buy to maximize the daily output ?

13. (Manufacturing Problem) A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for at least 5 hours a day. She produces only two items M and N each on the three machines are given in the following table :

Items	Number of hours required on machines		
	I	II	III
M	1	2	1
N	2	1	1·25

She makes a profit of ₹ 600 and ₹ 400 on items M and N respectively. How many of each item should she produce so as to maximise her profit assuming that she can sell all the items that she produced ? What will be the maximum profit ? *(N.C.E.R.T.)*

Solution. Let 'x' and 'y' be the number of items M and N respectively.

Then the profit = ₹ $(600x + 400y)$

Thus the problem is :

Maximize : $Z = 100x + 400y$ subject to the constraints :

$$x + 2y \leq 12$$

$$2x + y \leq 12$$

$$x + \frac{5}{4}y \geq 5$$

and $x \geq 0, y \geq 0.$

Draw the graph of the given constraints.

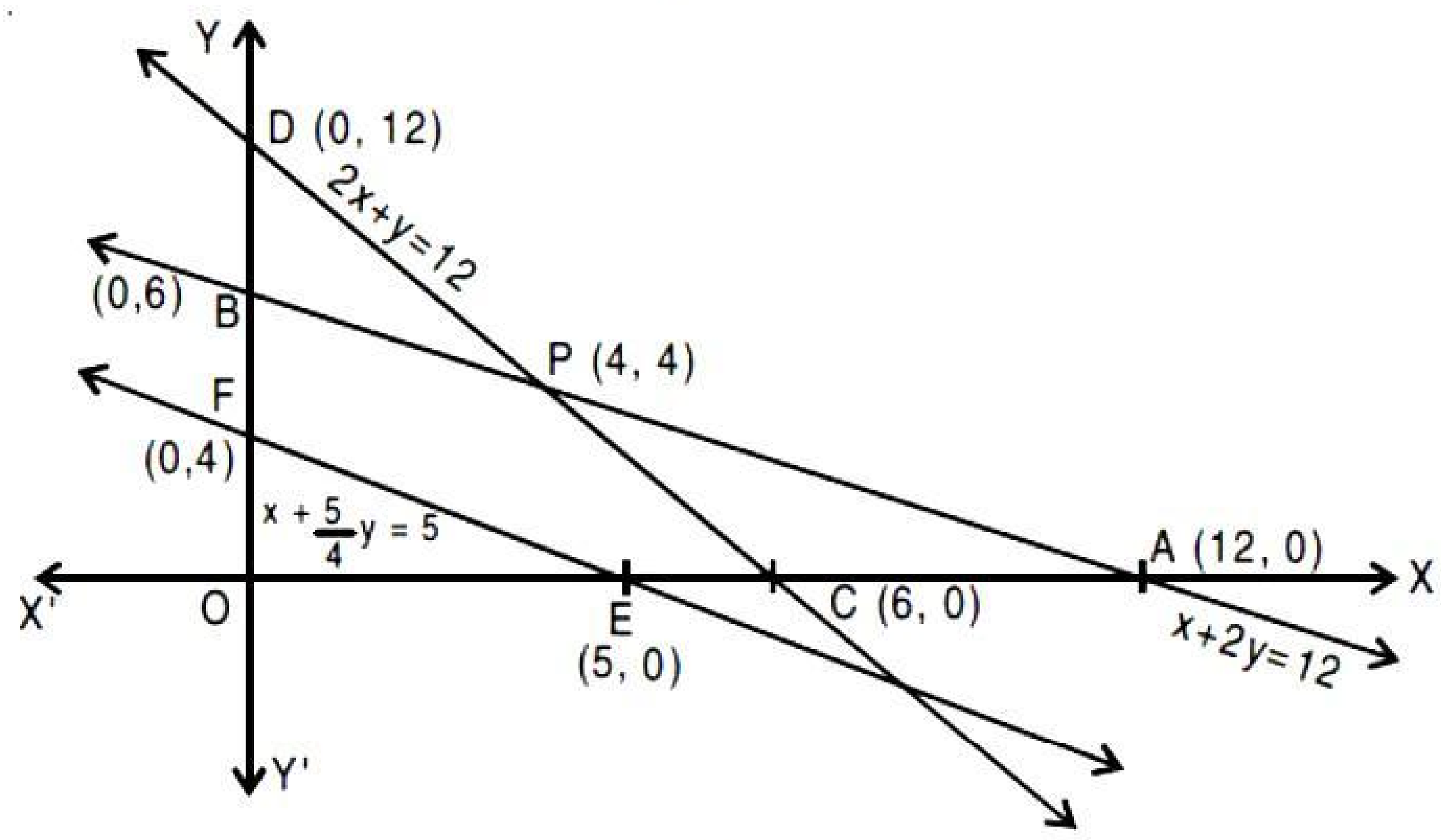


Fig.

It is observed that the feasible region CPBFE (shaded) is bounded whose vertices are :

C (6, 0), B (0, 6), F (0, 4), E (5, 0) and P (4, 4)

[*Solving* $x + 2y = 12$ *and* $2x + y = 12$; $x = 4$, $y = 4$]

Applying ***Corner Point Method,*** we have :

Corner Point	Z = 600x + 400y
C : (6, 0)	3600
D : (4, 4)	**4000** **(Maximum)**
B : (0, 6)	2400
F : (0, 4)	1600
E : (5,0)	3000

Hence, the manufacturer should produce 4 units of each item to get a maximum profit of ₹ 4000.

14. A manufacturer makes two types of toys, A and B, cups. Three machines are needed for the manufacture and the time in minutes required for each cup on the machines is given below :

Machines

Types of Toys	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is ₹ 7.50 and that on each toy of type B is ₹ 5, show that 15 toys of type A and 30 of type B should be manufactured in a day to get the maximum profit. *(N.C.E.R.T.)*

15. A farmer mixes two brands P and Q of cattle feed. Brand P, costing ₹ 250 per bag, contains 3 units of nutritional element A, 2·5 units of element B and 2 units of element C. Brand Q costing ₹ 200 per bag contains 1·5 units of nutritional element A, 11·25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag ? What is the minimum cost of the mixture per bag ? *(N.C.E.R.T.)*

16. A packet of plain biscuits costs ₹ 6 and that of chocolate biscuits costs ₹ 9. A house-wife has ₹ 72 and wants to buy at least three packets of plain biscuits and at least four of chocolate biscuits. How many of each type should she buy so that she can have maximum number of packets ? Make it as an LPP and solve it graphically. *(A.I.C.B.S.E. 2009 C)*

17. A company produces two different products. One of them needs $\frac{1}{4}$ of an hour of assembly work per unit, $\frac{1}{8}$ of an hour is quality control work and ₹1.2 in raw matrials. The other product requires $\frac{1}{3}$ of an hour of assembly work per unit, $\frac{1}{3}$ of an hour in quality control work and ₹0.9 in raw materials. Given the current availability of staff in the company, each day there is at most a total of 90 hours available for assembly and 80 hours for quality control. The first product described has a market value (sale price) of ₹9 per unit. In addition, the maximum amount of daily sales for the first product is estimated to be 200 units, without there being a maximum limit of daily sales for the second product. Formulate and solve graphically the LPP and find the maximum profit. *(C.B.S.E. Sample Paper 2018)*

Answers

2. 120 tickets of executive class and 80 tickets of economy class; Max. Profit = ₹ 1,68,000.

3. A : 20,000 bottles, B : 6,000 bottles; Max. Profit = ₹ 3,25,500.

4. From A : 10 packets, 0 packet and 50 packet to P, Q and R respectively.
From B : 50 packets, 40 packets and 0 packets to P, Q and R respectively.

5. From A : 10,50, 40 units; From B : 50, 0, 0 units D, E and F respectively; Min. cost = ₹ 510.

6. From A : 500, 3000 and 3500 litres
From B : 4000, 0, 0 litres to D, E and F respectively.
Min. cost = ₹ 4,400.

7. 12 days

8. 800 dolls of type A and 400 dolls of type B; Max. profit = ₹ 16,000.

9. Least cost = ₹ 112 (2 kg of food X and 4 kg of food Y).

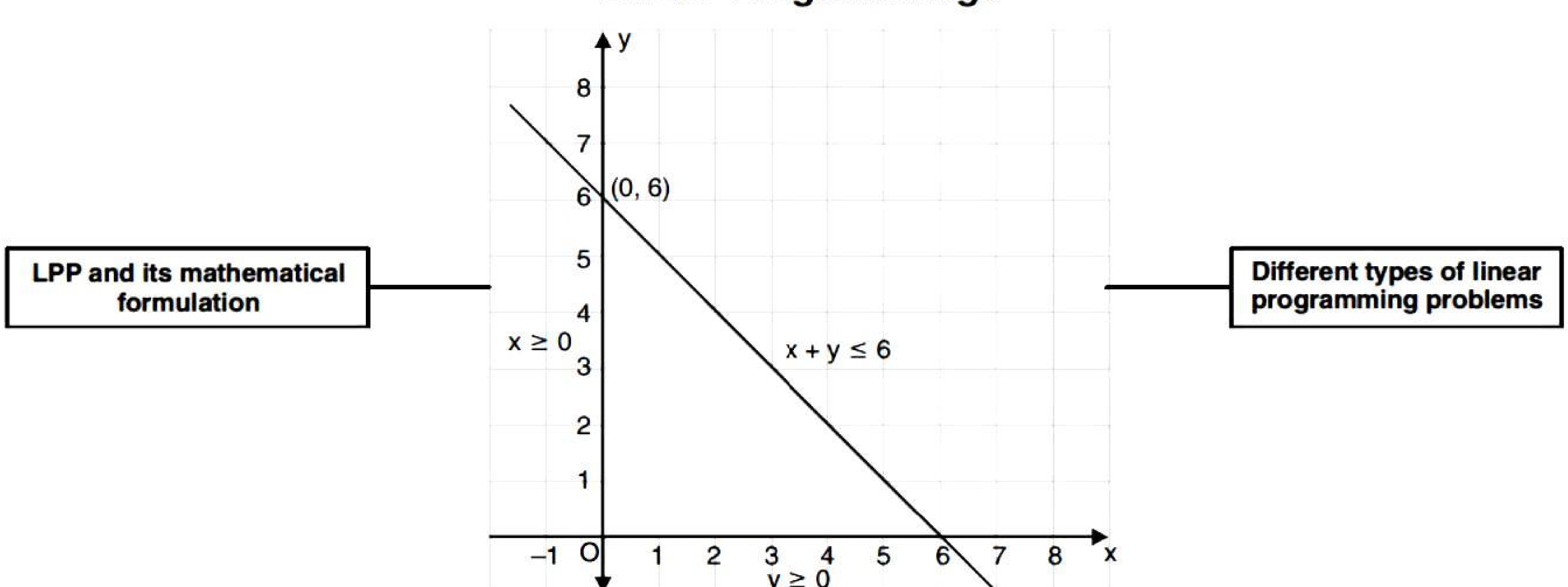
10. (*a*) 150 units
(*b*) 40 packets of food P and 15 packets of food Q; Max. amount of vitamin = 285 units.
11. (*a*) 40 bags of brand P and 100 bags of brand Q. Min. amount of nitrogen = 470 kg.
(*b*) 140 bags of brand P and 50 bags of brand Q. Max. amount of nitrogen. = 595 kg.
12. 6 machines of type A and no machine of type B.
14. 15, 30; ₹ 262·50.
15. 3 bags of brand P and 6 bags of brand Q; Min. cost = ₹1950.
16. 6 Packets : Plain Biscuits
4 Packets : Chocolate Biscuits.
17. ₹3, 240.

CHECK YOUR UNDERSTANDING

1. The graph of the inequation $2x + 3y \geq 6$ does not lie in the first quadrant. **(True/False)**
[**Ans.** False]
2. The graph of the inequation $3x + 2y > 6$ does not lie in the fourth quadrant. **(True/False)**
[**Ans.** False]
3. The objective function is maximum or minimum at a point, which lies on the boundary of the feasible region. **(True/False)**
[**Ans.** True]
4. Maximize $Z = x + 2y$ subject to :
$$x + y \geq 5, x \geq 0, y \geq 0.$$
[**Ans.** 10 at (0, 5)]
5. Maximize $Z = 4x + y$ subject to $x + y \leq 50$, $x, y \geq 0$.
[**Ans.** 200 at (50, 0)]
6. What is optimum solution ?
[**Ans.** Refer Definition]
7. What is objective function ?
[**Ans.** Refer Definition]
8. What is feasible region ?
[**Ans.** Refer Definition]
9. What is feasible solution ?
[**Ans.** Refer Definition]
10. What is optimal solution ?
[**Ans.** Refer Definition]

SUMMARY — LINEAR PROGRAMMING

Linear Programmings



1. Definition : *Linear programming is a tool which is used in decision making in business for obtaining maximum and minimum values of quantities subject to certain constraints.*

2. MATHEMATICAL FORMULATION

Let the linear function Z be defined by :

(*i*) $Z = c_1x_1 + c_2x_2 + \dots\dots\dots + c_nx_n$, where *c*'s are constants.

Let a_{ij} be (mn) constants and b_j be a set of m constants such that :

(*ii*) $a_{11}x_1 + a_{12}x_2 + \dots\dots + a_{1n}x_n\ (\leq, =, \geq)\ b_1$

$a_{21}x_1 + a_{22}x_2 + \dots\dots + a_{2n}x_n\ (\leq, =, \geq)\ b_2$

....

....

$a_{m1}x_1 + a_{m2}x_2 + \dots\dots + a_{mn}x_n\ (\leq, =, \geq)\ b_m$

and let (*iii*) $x_1 \geq 0, x_2 \geq 0, \dots\dots, x_n \geq 0$.

The problem of determining the values of x_1, x_2, ..., x_n, which maximizes (or minimizes) Z and satisfying above (*ii*) is called **General Linear Programming Problem (LPP).**

Note : In (*ii*), there may be any sign ≤, =, or ≥.

3. SOME DEFINITIONS

(*a*) Objective Function. The linear function $Z = ax + by$, where *a, b* are constants, which is to be maximized or minimized is called a **linear objective function** of **LPP.**

The variables *x* and *y* are called **decision variables.**

(*b*) Constraints. The linear inequalities or inequations or restrictions on the variables of a linear programming problem are called **constraints** of **LPP.**

The conditions $x \geq 0$, $y \geq 0$ are called **non-negative restrictions** of **LPP.**

(*c*) Optimization Problem. A problem, in which it is required to maximize or minimize a linear function (say of two variables *x* and *y*) subject to certain constraints, determined by a set of linear inequalities, is called the **optimizations problem.**

(*d*) Solution. The values of *x*, *y* which satisfy the constraints of LPP is called the **solution** of the **LPP.**

(*e*) Feasible Solution. Any solution of LPP, which satisfies the non-negative restrictions of the problem, is called a **feasible solution** to **LPP.**

(*f*) Optimum Solution. Any feasible solution, which optimizes (minimizes or maximizes) the objective function of LPP is called an **optimum solution** of the general **LPP.**

2. (I) Feasible Region. The common region, which is determined by all the constraints including non-negative constraints $x, y > 0$ of a LPP is called feasible region (or **solution region.**)

The region, other the feasible one, is called an **infeasible region.**

(II) Feasible Solution. The points, which lie on the boundary and within the feasible region, represent the **feasible solution** of the constraints.

Any part outside the feasible solution is called **infeasible solution.**

(III) Optimal (Feasible) Solution.

Any point in the feasible region, which gives the optimal value (maximum or minimum) of the objective function, is called **optimal solution.**

These points are **infinitely many.**

Theorem I. Let R be the feasible region (convex polygon) for LPP and $Z = ax + by$, the objective function.

When Z has an optimal value (max. or min.) when *x*, *y* are subject to constraints, this optimal value will occur at a corner point (vertex).

Theorem II. Let R be the feasible region for LPP and

$Z = ax + by$, the objective function.

If R is bounded*, then the objective function Z has both maximum and minimum values of R and each occurs at a corner point** (vertex).

KEY POINT

> When R is unbounded, max. (or min) value of objective function may not exist. In case it exists, it occurs at a corner point of R.

4. CORNER POINT METHOD

In order to solve a Linear Programming Problem we use *Corner Point Method* which is as below :

Step I. Obtain the feasible region of LPP. and determine its corner points (vertices).

Step II. Evaluate the objective function $Z = ax + by$ at each corner point (vertex).

Let M and *m* be the largest and smallest values at these points.

Step III. (*a*) When the feasible region is bounded,

then M and *m* are the maximum and minimum values of Z.

(*b*) When the feasible region is unbounded, then

(*i*) M is the maximum value of Z if the open half-plane determined by $ax + by > M$ has no common point with the feasible region; otherwise Z has no maximum value

(*ii*) *m* is the minimum value of Z if the open half-plane determined by $ax + by < m$ has no common point with the feasible region; otherwise Z has no minimum value.

---

*A feasible region is **bounded** if it can be enclosed within a circle, otherwise it is called unbounded.

**Corner point of a feasible region, is a point in the region, which is the intersection of the boundary lines.

MULTIPLE CHOICE QUESTIONS

For Board Examinations

1. A linear function, which is minimised or maximised is called :
   (A) an objective function (B) an optimal function
   (C) a feasible function (D) None of these. *(Jammu B. 2018)*
2. The maximum value of Z = $3x + 4y$ subject to the constraints : $x + y \le 4, x \ge 0, y \ge 0$ is :
   (A) 0 (B) 12
   (C) 16 (D) 18. *(P.B. 2012)*
3. The maximum value of Z = $2x + 3y$ subject to the constraints :
   $x + y \le 1; 3x + y \le 4; x, y \ge 0$ is :
   (A) 2 (B) 4
   (C) 5 (D) 3. *(H.B. 2007 S)*
4. The point in the half plane $2x + 3y - 12 \ge 0$ is :
   (A) (– 7, 8) (B) (7, – 8)
   (C) (– 7, – 8) (D) (7, 8). *(H.B. 2007 S)*
5. Any feasible solution which maximizes or minimizes the objective function is called :
   (A) A regional feasible solution
   (B) An optimal feasible solution
   (C) An objective feasible solution
   (D) None of these. *(J.& K. B. 2007)*

Answers

1. (A) **2.** (C) **3.** (C) **4.** (D) **5.** (B).

CHAPTER TEST 12

Time Allowed : 1½ Hours **Max. Marks : 34**

Notes : *1. All questions are compulsory.*
2. Marks have been indicated against each question.

1. The graph of the inequation $3x + 2y > 6$ does not lie in the 4th quadrant. (**True/False**) **(1)**
2. Draw the graph of the following LPP :
   $3x + y \le 17, x, y \ge 0.$ **(1)**
3. Maximize Z = $4x + y$ subject to :
   $x + y \le 50, x, y \ge 0.$ **(2)**
4. Maximize Z = $4x + y$ subject to constraints :
   $x + y \le 50, x + y \le 90, x \ge 0, y \ge 0.$ **(2)**
5. Minimize Z = $-3x + 4y$ subject to constraints :
   $x + 2y \le 8, 3x + 2y \le 12, x \ge 0, y \ge 0.$ **(4)**
6. If a man rides his motor cycles at 25 km/hr., he has to spend ₹ 2 per km on petrol, if he rides at a faster speed of 40 km/hr., the petrol cost increases to ₹ 5 per km. He has ₹ 100 to spend on petrol and wishes to find maximum distance he can travel within one hour. Express this as a linear programming problem and then solve it graphically. **(4)**
7. A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of the first machine is 12 hours and that of second machine is 9 hours. Each unit of product A requires 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on the second machine. Each unit of product A is sold at a profit of ₹ 5 and B at a profit of ₹ 6. Find the productive level for maximum profit graphically. **(4)**
8. Two tailors A and B earn ₹ 150 and ₹ 200 per day respectively. A can stitch 6 shirts and 4 pants while B can stitch 10 shirts and 4 pants per day. How many days shall each work if it is desired to produce (at least) 60 shirts and 32 pants at a minimum labour cost ? Make it an LPP and solve the problem graphically. **(4)**
9. A cooperative society of farmers has 50 hectares of land to grow crops A and B. The profits from crops A and B per hectare are estimated as ₹ 10,500 and ₹ 9,000 respectively. To control weeds, a liquid pesticide has to be used for crops A and B at the rate of 20 litres and 10 litres per hectare, respectively. Further not more than 800 litres of pesticide should be used in order to protect fish and wildlife using a pond which collects drainage from this land. Keeping in mind that the protection of fish and other wildlife is more important than earning profit, how much land should be allocated to each crop so as to maximize the total profit ? Form an LPP from the above and solve it graphically. Do you agree with the message that the protection of wildlife is utmost necessary to preserve the balance in environment ? **(6)**
10. A dietician wishes to mix two types of foods in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 10 units of vitamin C. Food I contain 2 units /kg of vitamin A and 1 unit/kg of vitamin C while food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₹ 5 per kg to purchase food I and ₹ 7 per kg to purchase food II. Determine the minimum cost of such a mixture. Formulate the above as a LPP and solve it graphically.
    Why are the people following dieticians these days ?**(6)**

Answers

1. False. **2.** 200 at (50, 0). **3.** 120 at (30,0). **5.** – 12 at (4, 0). **6.** 30 km. **7.** ₹ 36.
8. A : 5 Days; B : 3 Days ; Minimum cost = ₹ 1,350. **9.** Crop A = 30 hectares, Crop B = 20 hectares.
10. Minimum cost ₹ 112 when 2 kg of food X and 4 kg of food Y are mixed.