Shortcuts And Important Results To Remember

1 $|A|$ exists $\Leftrightarrow A$ is square matrix.

2 No element of principal diagonal in a diagonal matrix is zero.

3 If A is a diagonal matrix of order n, then

(a) Number of zeroes in A is $n(n-1)$

(b) If $d_1, d_2, d_3, \dots, d_n$ are diagonal elements, then

$$A = \text{diag}\{d_1, d_2, d_3, \dots, d_n\}$$

and $|A| = d_1 d_2 d_3 \dots d_n$

$$A^{-1} = \text{diag}(d_1^{-1}, d_2^{-1}, d_3^{-1}, \dots, d_n^{-1})$$

(c) Diagonal matrix is both upper and lower triangular.

(d) $\text{diag}\{a_1, a_2, a_3, \dots, a_n\} \times \text{diag}\{b_1, b_2, b_3, \dots, b_n\}$
$= \text{diag}\{a_1b_1, a_2b_2, a_3b_3, \dots, a_nb_n\}$

4 If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, then $A^k = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$ and $B^k = \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$, $\forall\, k \in N$.

5 If A and B are square matrices of order n, then

(a) $|kA| = k^n |A|$, k is scalar

(b) $|AB| = |A||B|$

(c) $|kAB| = k^n |A||B|$, k is scalar

(d) $|AB| = |BA|$

(e) $|A^T| = |A| = |A^\theta|$, where A^θ is conjugate transpose matrix of A

(f) $|A|^m = |A^m|$, $m \in N$

6 Minimum number of zeroes in a triangular matrix is given by $\dfrac{n(n-1)}{2}$, where n is order of matrix.

7 If A is a skew-symmetric matrix of odd order, then $|A| = 0$ and of even order is a non-zero perfect square.

8 If A is involutory matrix, then

(a) $|A| = \pm 1$

(b) $\dfrac{1}{2}(I + A)$ and $\dfrac{1}{2}(I - A)$ are idempotent and $\dfrac{1}{2}(I + A) . \dfrac{1}{2}(I - A) = 0$

9 If A is orthogonal matrix, then $|A| = \pm 1$

10 To obtain an orthogonal matrix B from a skew-symmetric matrix A, then

$B = (I - A)^{-1}(I + A)$ or $B = (I - A)(I + A)^{-1}$

11 The sum of two orthogonal matrices is not orthogonal while the sum of two symmetric (skew-symmetric) matrices is symmetric (skew-symmetric)

12 The product of two orthogonal matrices is orthogonal while the product of two symmetric (skew-symmetric) matrices need not be symmetric (skew-symmetric)

13 The adjoint of a square matrix of order 2 can be easily obtained by interchanging the principal diagonal elements and changing the sign of the other diagonal.

i.e., If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $\text{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

14 If $|A| \neq 0$, then $|A^{-1}| = \dfrac{1}{|A|}$.

15 If A and B are invertible matrices such that $AB = C$, then $|B| = \dfrac{|C|}{|A|}$.

16 Commutative law does not necessarily hold for matrices.

17 If $AB = -BA$, then matrices A and B are called anti-commutative matrices.

18 If $AB = O$, it is not necessary that atleast one of the matrix should be zero matrix.

For example, If $A = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$, then

$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ while neither A nor B is the null matrix.

19 If A, B and C are invertible matrices, then

(a) $(AB)^{-1} = B^{-1}A^{-1}$

(b) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

20 If B is a non-singular matrix and A is any square matrix, then $\det(B^{-1}AB) = \det(A)$

21 If A is a non-singular square matrix of order n, then adj (adj A) $= |A|^{n-2} A$

22 If A is a non-singular square matrix of order n, then

$$|\underbrace{\text{adj}(\text{adj}(\text{adj}\dots(\text{adj}(\text{adj} A))))}_{m \text{ times}}| = |A|^{(n-1)^m}$$

23 If $A = \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix}$; $A^m = O, \forall\, m \geq 2$

$\therefore \quad (A + I)^n = I + nA$

24 If A and B are two symmetric matrices, then $A \pm B$, $AB + BA$ are symmetric matrices and $AB - BA$ is a skew-symmetric matrix.

25 If A and B are two square matrices of order n and λ be a scalar, then

(i) $\text{Tr}(\lambda A) = \lambda \text{Tr}(A)$

(ii) $\text{Tr}(A \pm B) = \text{Tr}(A) \pm \text{Tr}(B)$

(iii) $\text{Tr}(AB) = \text{Tr}(BA)$

(iv) $\text{Tr}(A) = \text{Tr}(A')$

(v) $\text{Tr}(I_n) = n$

(vi) $\text{Tr}(O) = 0$

(vii) $\text{Tr}(AB) \neq \text{Tr}(A) \cdot \text{tr}(B)$

contd...

26 If rank of a matrix A is denoted by $\rho(A)$, then

(i) $\rho(A) = 0$, if A is zero matrix.

(ii) $\rho(A) = 1$, if every element of A is same.

(iii) If A and B are square matrices of order n each and $\rho(A) = \rho(B) = n$, then $\rho(AB) = n$

(iv) If A is a square matrix of order n and $\rho(A) = n - 1$, then $\rho(\text{adj } A) = 1$ and if $\rho(A) < n - 1$, then $\rho(\text{adj } A) = 0$

27 System of planes

$$a_{11}x + a_{12}y + a_{13}z = b_1,$$
$$a_{21}x + a_{22}y + a_{23}z = b_2$$
and $$a_{31}x + a_{32}y + a_{33}z = b_3$$

Augmented matrix $C = [A : B]$ and if Rank of $A = r$ and Rank of $C = s$, then

(i) If $r = s = 1$, then planes are coincident

(ii) If $r = 1$, $s = 2$, then planes are parallel

(iii) If $r = s = 2$, then planes intersect along a single straight line

(iv) If $r = 2$, $s = 3$, then planes form a triangular prism

(v) If $r = s = 3$, then planes meet at a single point

28 If P is an orthogonal matrix, then $\det(P) = \pm 1$

(i) P represents a reflection about a line, then $\det(P) = -1$.

(ii) P represents a rotation about a point, then $\det(P) = 1$.

29 **Cayley-Hamilton Theorem** : Every matrix satisfies its characteristic equation.

For Example, Let A be a square matrix, then $|A - \lambda I| = 0$ is the characteristic equation for A.

If $\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$ is the characteristic equation for A, then $A^3 - 6A^2 + 11A - 6I = O$. Roots of characteristic equation for A are called eigen values of A or characteristic roots of A or latent roots of A. If λ is a characteristic root of A, then λ^{-1} is characteristic root of A^{-1}.