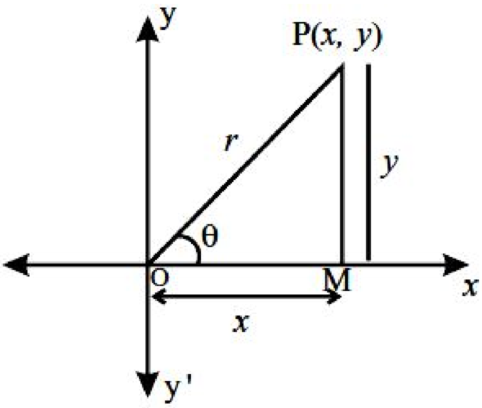
CHAPTER – 8 & 9
TRIGONOMETRY

Trigonometric Ratios (T - Ratios) of an acute angle of a right triangle

In XOY-plane, let a revolving line OP starting from OX, trace out ∠XOP=θ. From P (x, y)draw PM ⊥□ to OX.

In right angled triangle OMP. OM = x (Adjacent side); PM = y (opposite side); OP = r (hypotenuse).



$$\sin\theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{y}{r}, \quad \cos\theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{x}{r}, \quad \tan\theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{y}{x}$$

$$\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}} = \frac{r}{y}, \ \sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} = \frac{r}{x}, \quad \cot\theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}} = \frac{x}{y}$$

Reciprocal Relations

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}, \ \sec\theta = \frac{1}{\cos\theta} \ \textbf{and} \ \cot\theta = \frac{1}{\tan\theta}$$

Quotient Relations

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \ \text{and} \ \cot\theta = \frac{\cos\theta}{\sin\theta}$$

IMPORTANT QUESTIONS

If $\tan A = \frac{4}{3}$, find the value of all T– ratios of θ.

Solution: Given that, In right Δ ABC, $\tan A = \frac{BC}{AB} = \frac{4}{3}$

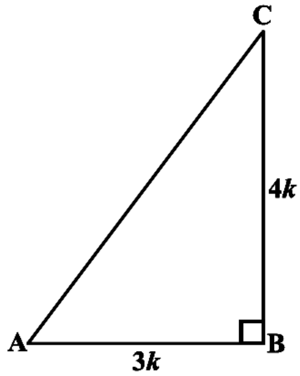
Therefore, if BC = 4k, then AB = 3k, where k is a positive number.

Now, by using the Pythagoras Theorem, we have

$AC^2 = AB^2 + BC^2 = (4k)^2 + (3k)^2 = 25k^2$

So, AC = 5k

Now, we can write all the trigonometric ratios using their definition



$$\sin A = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}, \ \cos A = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

and $\cot A = \frac{1}{\tan A} = \frac{3}{4}$,

$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{5}{4}$,

$\sec A = \frac{1}{\cos A} = \frac{5}{3}$

Questions for Practice

1. If $\sin\theta = \frac{5}{13}$, find the value of all T– ratios of θ.

2. If $\cos\theta = \frac{7}{25}$, find the value of all T– ratios of θ.

3. If $\tan\theta = \frac{15}{8}$, find the value of all T– ratios of θ.

4. If $\cot\theta = 2$, find the value of all T– ratios of θ.

5. If $\text{cosec}\,\theta = \sqrt{10}$, find the value of all T– ratios of θ.

6. In Δ OPQ, right-angled at P, OP = 7 cm and OQ – PQ = 1 cm. Determine the values of sin Q and cos Q.

7. In Δ PQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

Trigonometric ratios for angle of measure.

0^0, 30^0,45^0, 60^0 and 90^0 in tabular form.

∠A	**0^0**	**30^0**	**45^0**	**60^0**	**90^0**
sinA	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosA	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tanA	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosecA	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
secA	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cotA	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

IMPORTANT QUESTIONS

If cos (A – B) = $\frac{\sqrt{3}}{2}$ and sin (A + B) = 1, then find the value of A and B.

Solution: Given that $\cos(A-B) = \frac{\sqrt{3}}{2} = \cos 30^0$

$\Rightarrow A - B = 30^0$ ………………. (1)

and $\sin(A+B) = 1 = \sin 90^0$

$\Rightarrow A + B = 90^0$ …………………… (2)

Solving equations (1) and (2), we get A = 60^0 and B = 30^0.

Questions for Practice

Evaluate each of the following:

1. $\sin 60^0 \cos 30^0 + \cos 60^0 \sin 30^0$

2. $\cos 60^0 \cos 30^0 - \sin 60^0 \sin 30^0$

3. $\cos 45^0 \cos 30^0 + \sin 45^0 \sin 30^0$

4. $\sin 60^0 \sin 45^0 - \cos 60^0 \cos 45^0$

5. $(\sin^2 30^0 + 4\cot^2 45^0 - \sec^2 60^0)(\cos ec^2 45^0 \sec^2 30^0)$

6. If sin (A – B) = $\frac{1}{2}$ and cos(A + B) = $\frac{1}{2}$, then find the value of A and B.

7. If tan (A – B) = $\frac{1}{\sqrt{3}}$ and tan (A + B) = $\sqrt{3}$, then find the value of A and B.

TRIGONOMETRIC IDENTITIES

An equation involving trigonometric ratios of an angle is said to be a trigonometric identity if it is satisfied for all values of θ for which the given trigonometric ratios are defined.

Identity (1) : $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$.

Identity (2) : $\sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \sec^2\theta - \tan^2\theta = 1$ and $\tan^2\theta = \sec^2\theta - 1$.

Identity (3) : $\text{cosec}^2\theta = 1 + \cot^2\theta$

$\Rightarrow \text{cosec}^2\theta - \cot^2\theta = 1$ and $\cot^2\theta = \text{cosec}^2\theta - 1$.

IMPORTANT QUESTIONS

Prove that: $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \cos ecA + \cot A$

Solution: LHS = $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

(Dividing Numerator and Denominator by sinA, we get)

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}} = \frac{\cot A - 1 + \cos ecA}{\cot A + 1 - \cos ecA} \qquad \left[\because \cot A = \frac{\cos A}{\sin A}, \cos ecA = \frac{1}{\sin A}\right]$$

$$= \frac{\cot A + \cos ecA - 1}{\cot A + 1 - \cos ecA} = \frac{\cot A + \cos ecA - (\cos ec^2 A - \cot^2 A)}{\cot A + 1 - \cos ecA} \quad [\because \cos ec^2 A - \cot^2 A = 1]$$

$$= \frac{\cot A + \cos ecA - (\cos ecA + \cot A)(\cos ecA - \cot A)}{\cot A + 1 - \cos ecA}$$

$$= \frac{(\cos ecA + \cot A)(1 - \cos ecA + \cot A)}{\cot A + 1 - \cos ecA} = \cos ecA + \cot A = RHS$$

Questions for Practice

Prove the following identities:

1. sec A (1 – sin A)(sec A + tan A) = 1.

2. $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\cos ecA - 1}{\cos ecA + 1}$

3. $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$

4. $(\cos ec\theta - \cot\theta)^2 = \frac{1 - \cos\theta}{1 + \cos\theta}$

5. $\dfrac{\cos A}{1+\sin A}+\dfrac{1+\sin A}{\cos A}=2\sec A$

6. $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=1+\sec\theta\operatorname{cosec}\theta$

7. $\dfrac{1+\sec A}{\sec A}=\dfrac{\sin^2 A}{1-\cos A}$

8. $\sqrt{\dfrac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

9. $\dfrac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

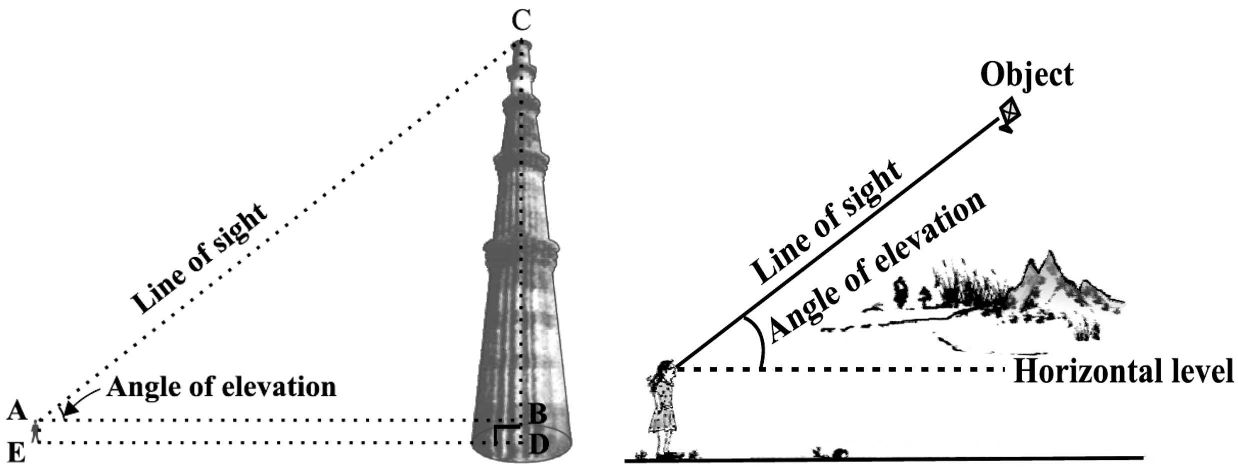
10. $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

11. $(\operatorname{cosec} A-\sin A)(\sec A-\cos A)=\dfrac{1}{\tan A+\cot A}$

12. $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\dfrac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

ANGLE OF ELEVATION

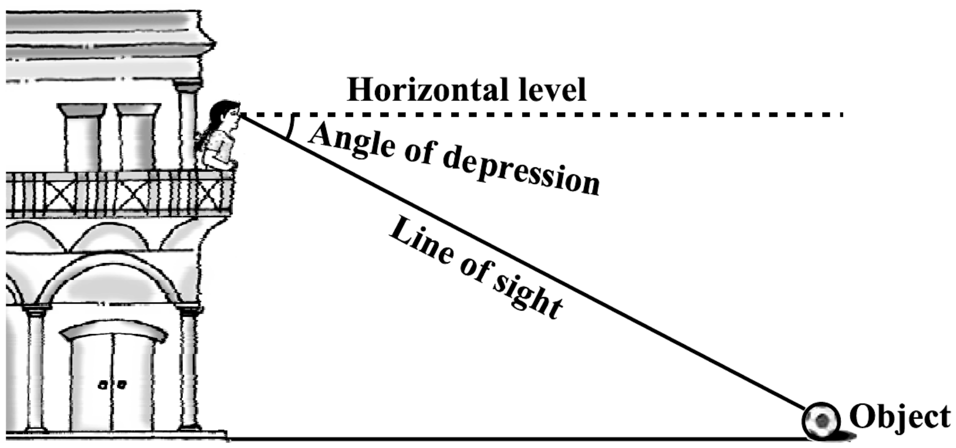
In the below figure, the line AC drawn from the eye of the student to the top of the minar is called the *line of sight*. The student is looking at the top of the minar. The angle BAC, so formed by the line of sight with the horizontal, is called the *angle of elevation* of the top of the minar from the eye of the student. Thus, the **line of sight** is the line drawn from the eye of an observer to the point in the object viewed by the observer.



The **angle of elevation** of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level, i.e., the case when we raise our head to look at the object

ANGLE OF DEPRESSION

In the below figure, the girl sitting on the balcony is *looking down* at a flower pot placed on a stair of the temple. In this case, the line of sight is *below* the horizontal level. The angle so formed by the line of sight with the horizontal is called the *angle of depression*. Thus, the **angle of depression** of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level, i.e., the case when we lower our head to look at the point being viewed

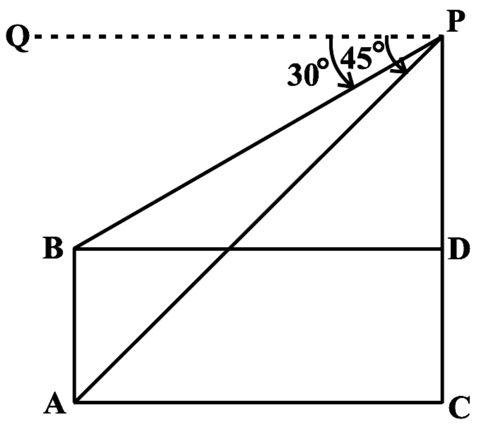


IMPORTANT QUESTIONS

The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45°, respectively. Find the height of the multi-storeyed building and the distance between the two buildings.

Solution : Let PC = h m be the height of multistoryed building and AB denotes the 8 m tall building.
BD = AC = x m, PC = h = PD + DC = PD + AB = PD + 8 m
So, PD = h – 8 m
Now, ∠QPB = ∠PBD = 30°
Similarly, ∠QPA = ∠PAC = 45°.
In right Δ PBD, $\tan 30^0 = \frac{PD}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h-8}{x} \Rightarrow x = (h-8)\sqrt{3}\ m$ ………………. (1)

Also, In right Δ PAC, $\tan 45^0 = \frac{PC}{AC} \Rightarrow 1 = \frac{h}{x}$
$\Rightarrow x = h\ m$ ………………….. (2)



From equations (1) and (2), we get $h = (h-8)\sqrt{3}$
$\Rightarrow h = h\sqrt{3} - 8\sqrt{3} \Rightarrow h\sqrt{3} - h = 8\sqrt{3}$
$\Rightarrow h(\sqrt{3}-1) = 8\sqrt{3} \Rightarrow h = \frac{8\sqrt{3}}{\sqrt{3}-1}$
$\Rightarrow h = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{8\sqrt{3}(\sqrt{3}+1)}{3-1}$
$\Rightarrow h = \frac{8(3+\sqrt{3})}{2} = 4(3+\sqrt{3})m$
Hence, the height of the multi-storeyed building is $4(3+\sqrt{3})m$ and the distance between the two buildings is also $4(3+\sqrt{3})m$.

From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45°, respectively. If the bridge is at a height of 3 m from the banks, find the width of the river.

Solution: Let A and B represent points on the bank on opposite sides of the river, so that AB is the width of the river. P is a point on the bridge at a height of 3 m, i.e., DP = 3 m.
Now, AB = AD + DB
In right Δ APD, $\tan 30^0 = \frac{PD}{AD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{AD}$

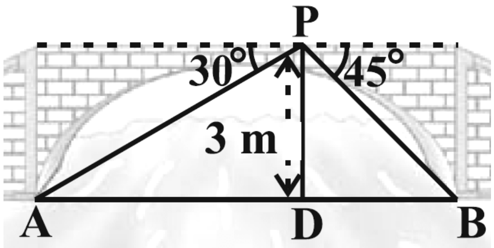
$\Rightarrow AD = 3\sqrt{3}\ \ m$

Also, in right Δ PBD, $\tan 45^0 = \frac{PD}{BD} \Rightarrow 1 = \frac{3}{BD}$

$\Rightarrow BD = 3\ \ m$

Now, AB = BD + AD = $3 + 3\sqrt{3} = 3(1+\sqrt{3})\ \ m$

Therefore, the width of the river is $3(1+\sqrt{3})\ \ m$



Questions for Practice

1. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.
2. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.
3. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.
4. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.
5. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.
6. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.
7. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.
8. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joing this point to the foot of the tower, the angle of elevation of the top of the tower is 30° . Find the height of the tower and the width of the canal.
9. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.
10. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.
11. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30° . Find the distance travelled by the balloon during the interval.
12. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.
13. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

MCQ QUESTIONS (1 mark)

1. The value of (sin30° + cos30°) – (sin60° + cos60°) is
(a) – 1 (b) 0 (c) 1 (d) 2

2. The value of $\frac{\tan 30^0}{\cot 60^0}$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 1

3. The value of (sin 45° + cos 45°) is

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

4. If cos A = $\frac{4}{5}$, then the value of tan A is

(a) $\frac{3}{5}$ (b) $\frac{3}{4}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

5. If sin A = $\frac{1}{2}$, then the value of cot A is

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

6. If cos (α + β) = 0, then sin (α – β) can be reduced to
(a) cos β (b) cos 2β (c) sin α (d) sin 2α

7. If ΔABC is right angled at C, then the value of cos (A+B) is

(a) 0 (b)1 (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

8. If sinA + $\sin^2$A = 1, then the value of the expression ($\cos^2$A + $\cos^4$A) is

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) 3

9. Given that sinα = $\frac{1}{2}$ and cosβ = $\frac{1}{2}$, then the value of (α + β) is

(a) 0° (b) 30° (c) 60° (d) 90°

10. If sinθ – cosθ = 0, then the value of ($\sin^4$θ + $\cos^4$θ) is

(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$