Probability Exercise 1 :
Single Option Correct Type Questions

- This section contains **30 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct

1. There are two vans each having numbered seats, 3 in the front and 4 at the back. There are 3 girls and 9 boys to be seated in the vans. The probability of 3 girls sitting together in a back row on adjacent seats, is

(a) $\frac{1}{13}$ (b) $\frac{1}{39}$ (c) $\frac{1}{65}$ (d) $\frac{1}{91}$

2. The probability that a year chosen at random has 53 Sundays, is

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{28}$ (d) $\frac{5}{28}$

3. The probability that a leap year selected at random contains either 53 Sundays or 53 Mondays, is

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$

4. A positive integer N is selected so as to be $100 < N < 200$. Then, the probability that it is divisible by 4 or 7, is

(a) $\frac{7}{33}$ (b) $\frac{17}{33}$ (c) $\frac{32}{99}$ (d) $\frac{34}{99}$

5. Two numbers a and b are selected at random from $1, 2, 3, \ldots, 100$ and are multiplied. Then, the probability that the product ab is divisible by 3, is

(a) $\frac{67}{150}$ (b) $\frac{83}{150}$ (c) $\frac{67}{75}$ (d) $\frac{8}{75}$

6. Three different numbers are selected at random from the set $A = \{1, 2, 3, \ldots, 10\}$. The probability that the product of two of the numbers is equal to third, is

(a) $\frac{3}{4}$ (b) $\frac{1}{40}$ (c) $\frac{1}{8}$ (d) $\frac{39}{40}$

7. The numbers $1, 2, 3, \ldots, n$ are arranged in a random order. Then, the probability that the digits $1, 2, 3, \ldots, k(k < n)$ appears as neighbours in that order, is

(a) $\frac{1}{n!}$ (b) $\frac{k!}{n!}$ (c) $\frac{(n-k)!}{n!}$ (d) $\frac{(n-k+1)!}{n!}$

8. The numbers $1, 2, 3, \ldots, n$ are arranged in a random order. Then, the probability that the digits $1, 2, 3, \ldots, k(k < n)$ appears as neighbours, is

(a) $\frac{(n-k)!}{n!}$ (b) $\frac{(n-k+1)}{{}^nC_k}$

(c) $\frac{(n-k)}{{}^nC_k}$ (d) $\frac{k!}{n!}$

9. Four identical dice are rolled once. The probability that atleast three different numbers appear on them, is

(a) $\frac{13}{42}$ (b) $\frac{17}{42}$ (c) $\frac{23}{42}$ (d) $\frac{25}{42}$

10. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle formed by these vertices is equilateral, is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{10}$ (d) $\frac{1}{20}$

11. Two small squares on a chess board are chosen at random. Then, the probability that they have a common side, is

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{18}$ (d) $\frac{5}{18}$

12. A letter is known to have come from CHENNAI, JAIPUR, NAINITAL, DUBAI and MUMBAI. On the post mark only two consecutive letters AI are legible. Then the probability that it come from MUMBAI, is

(a) $\frac{42}{319}$ (b) $\frac{84}{403}$ (c) $\frac{39}{331}$ (d) $\frac{42}{331}$

13. Let a die is loaded in such a way that prime number faces are twice as likely to occur as a non-prime number faces. Then, the probability that an odd number will be show up when the die is tossed, is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{9}$ (d) $\frac{5}{9}$

14. One ticket is selected at random from 100 tickets numbered $00, 01, 02, \ldots, 99$.
Suppose, X and Y are the sum and product of the digit found on the ticket $P(X = 7 / Y = 0)$ is given by

(a) $\frac{2}{3}$ (b) $\frac{2}{19}$ (c) $\frac{1}{50}$ (d) None of these

15. All the spades are taken out from a pack of cards. From these cards, cards are drawn one by one without replacement till the ace of spades comes. The probability that the ace comes in the 4th draw, is

(a) $\frac{1}{13}$ (b) $\frac{12}{13}$

(c) $\frac{4}{13}$ (d) None of these

16. A number is selected at random from the first twenty-five natural numbers. If it is a composite number, then it is divided by 5. But if it is not a composite number, then it is divided by 2. The probability that there will be no remainder in the division, is

(a) $\frac{11}{30}$ (b) 0.4

(c) 0.2 (d) None of these

17. If a bag contains 50 tickets, numbered 1, 2, 3, …, 50 of which five are drawn at random and arranged in ascending order of magnitude $(x_1 < x_2 < x_3 < x_4 < x_5)$. The probability that $x_3 = 30$, is

(a) $\frac{{}^{20}C_2 \times {}^{29}C_2}{{}^{50}C_5}$ (b) $\frac{{}^{20}C_2}{{}^{50}C_5}$

(c) $\frac{{}^{29}C_2}{{}^{50}C_5}$ (d) None of these

18. India play two matches each with West Indies and Australia. In any match the probabilities of India getting points 0, 1 and 2 are 0.45, 0.05 and 0.50, respectively. Assuming that the outcomes are independent, then the probability of India getting atleast 7 points, is

(a) 0.8750 (b) 0.0875 (c) 0.0625 (d) 0.0250

19. Three six faced dice are tossed together, then the probability that exactly two of the three numbers are equal, is

(a) $\frac{165}{216}$ (b) $\frac{177}{216}$ (c) $\frac{51}{216}$ (d) $\frac{90}{216}$

. **20.** Three six-faced fair dice are thrown together. The probability that the sum of the numbers appearing on the dice is $k (3 \le k \le 8)$, is

(a) $\frac{(k-1)(k-2)}{432}$ (b) $\frac{k(k-1)}{432}$

(c) $\frac{k^2}{432}$ (d) None of these

21. A book contains 1000 pages. A page is chosen at random. The probability that the sum of the digits of the marked number on the page is equal to 9, is

(a) $\frac{23}{500}$ (b) $\frac{11}{200}$

(c) $\frac{7}{100}$ (d) None of these

22. A bag contains four tickets numbered 00, 01, 10 and 11. Four tickets are chosen at random with replacement, then the probability that sum of the numbers on the tickets is 23, is

(a) $\frac{3}{32}$ (b) $\frac{1}{64}$ (c) $\frac{5}{256}$ (d) $\frac{7}{256}$

23. Fifteen coupons are numbered 1 to 15. Seven coupons are selected at random, one at a time with replacement. Then, the probability that the largest number appearing on a selected coupon be 9, is

(a) $\left(\frac{1}{15}\right)^7$ (b) $\left(\frac{8}{15}\right)^7$

(c) $\left(\frac{3}{5}\right)^7$ (d) None of these

24. A box contains tickets numbered 1 to 20. 3 tickets are drawn from the box with replacement. The probability that the largest number on the tickets is 7, is

(a) $\frac{7}{20}$ (b) $1 - \left(\frac{7}{20}\right)^3$

(c) $\frac{2}{19}$ (d) None of these

25. An unbiased die with faces marked 1, 2, 3, 4, 5 and 6 is rolled four times. Out of four face values obtained, then the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5, is

(a) $\frac{16}{81}$ (b) $\frac{1}{81}$ (c) $\frac{80}{81}$ (d) $\frac{65}{81}$

26. A bag contains four tickets marked with numbers 112, 121, 211 and 222. One ticket is drawn at random from the bag. Let $E_i (i = 1, 2, 3)$ denote the event that ith digit on the ticket is 2. Then, which of the following is not true?

(a) E_1 and E_2 are independent
(b) E_2 and E_3 are independent
(c) E_3 and E_1 are independent
(d) E_1, E_2 and E_3 are not independent

27. Two non-negative integers are chosen at random. The probability that the sum of the square is divisible by 10, is

(a) $\frac{17}{100}$ (b) $\frac{9}{50}$ (c) $\frac{7}{50}$ (d) $\frac{9}{16}$

28. Two positive real numbers x and y satisfying $x \le 1$ and $y \le 1$ are chosen at random. The probability that $x + y \le 1$, given that $x^2 + y^2 \le 1/4$, is

(a) $\frac{8-\pi}{16-\pi}$ (b) $\frac{4-\pi}{16-\pi}$

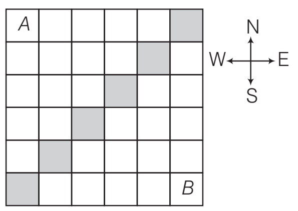
(c) $\frac{4-\pi}{8-\pi}$ (d) None of these

29. If the sides of a triangle are decided by the throw of a single dice thrice, the probability that triangle is of maximum area given that it is an isosceles triangle, is

(a) $\frac{1}{7}$ (b) $\frac{1}{27}$

(c) $\frac{1}{14}$ (d) None of these

30. A and B are persons standing in corner square as shown in the figure. They start to move on same time with equal speed, if A can move only in East or South direction and B can move only in North or West direction. If in each step they reach in next square and their choice of direction are equality. If it is given that A and B meet in shaded region, then the probability that they have met in the top most shaded square, is



(a) $\frac{1}{6}$ (b) $\frac{{}^5C_2}{{}^{10}C_3}$ (c) $\frac{1}{{}^{10}C_5}$ (d) $\frac{1}{2^5 \times 6}$

Probability Exercise 2 :
More than One Correct Option Type Questions

- This section contains **15 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **MORE THAN ONE** may be correct.

31. For two given events A and B, $P(A \cap B)$ is

(a) not less than $P(A) + P(B) - 1$
(b) not greater than $P(A) + P(B)$
(c) equal to $P(A) + P(B) - P(A \cup B)$
(d) equal to $P(A) + P(B) + P(A \cup B)$

32. If E and F are independent events such that $0 < P(E) < 1$ and $0 < P(F) < 1$, then

(a) E and F are mutually exclusive
(b) E and $\overline{F}$ (complement of the event F) are independent
(c) $\overline{E}$ and $\overline{F}$ are independent
(d) $P(E / F) + P(\overline{E} / F) = 1$

33. For any two events A and B in a sample space:

(a) $P\left(\dfrac{A}{B}\right) \geq \dfrac{P(A) + P(B) - 1}{P(B)}$, $P(B) \neq 0$, is always true
(b) $P(A \cap \overline{B}) = P(A) - P(A \cap B)$, does not hold
(c) $P(A \cup B) = 1 - P(\overline{A})\, P(\overline{B})$, if A and B are independent
(d) $P(A \cup B) = 1 - P(\overline{A})\, P(\overline{B})$, if A and B are disjoint

34. Let E and F be two independent events. Then, the probability that both E and F happens is $\dfrac{1}{12}$ and the probability that neither E nor F happens is $\dfrac{1}{2}$. Then,

(a) $P(E) = 1/3$, $P(F) = 1/4$ (b) $P(E) = 1/2$, $P(F) = 1/6$
(c) $P(E) = 1/6$, $P(F) = 1/2$ (d) $P(E) = 1/4$, $P(F) = 1/3$

35. If $\overline{E}$ and $\overline{F}$ are the complementary events of events E and F, respectively and if $0 < P(F) < 1$, then

(a) $P(\overline{E} / F) + P(\overline{E} / F) = 1$ (b) $P(E / F) + P(E / \overline{F}) = 1$
(c) $P(\overline{E} / F) + P(E / \overline{F}) = 1$ (d) $P(E / \overline{F}) + P(\overline{E} / \overline{F}) = 1$

36. Let $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)\, P(B)$. Then,

(a) $P(B - A) = P(B) - P(A)$ (b) $P(A' \cup B') = P(A') + P(B')$
(c) $P((A \cup B)') = P(A')\, P(B')$ (d) $P(A / B) = P(A)$

37. If A and B are two events, then the probability that exactly one of them occurs is given by

(a) $P(A) + P(B) - 2P(A \cap B)$
(b) $P(A \cap B') + P(A' \cap B)$
(c) $P(A \cup B) - P(A \cap B)$
(d) $P(A') + P(B') - 2P(A' \cap B')$

38. If A and B are two independent events such that $P(A) = 1/2$ and $P(B) = 1/5$. Then,

(a) $P(A \cup B) = 3/5$ (b) $P(A / B) = 1/2$
(c) $P(A / A \cup B) = 5/6$ (d) $P(A \cap B) / (A' \cup B') = 0$

39. A student appears for tests I, II and III. The student is successful if he passes either in tests I and II or tests I and III. The probabilities of the student passing in tests I, II and III are p, q and $1/2$, respectively. If the probability that the student is successful is $1/2$, then

(a) $p = 1, q = 0$ (b) $p = 2/3, q = 1/2$
(c) $p = 3/5, q = 2/3$ (d) infinitely values of p and q

40. Let X be a set containing n elements. If two subsets A and B of X are picked at random, then the probability that A and B have same number of elements, is

(a) $\dfrac{{}^{2n}C_n}{2^{2n}}$ (b) $\dfrac{1}{{}^{2n}C_n}$ (c) $\dfrac{1 \cdot 3 \cdot 5 \dots (2n-1)}{2^n \cdot (n!)}$ (d) $\dfrac{3^n}{4^n}$

41. Suppose m boys and m girls take their seats randomly around a circle. The probability of their sitting is $({}^{2m-1}C_m)^{-1}$, when

(a) no two boys sit together (b) no two girls sit together
(c) boys and girls sit alternatively
(d) all the boys sit together

42. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c, respectively. In these subjects, the student has a 75% chance of passing in atleast one, a 50% chance of passing in atleast two and a 40% chance of passing in exactly two. Which of the following relations are true?

(a) $p + m + c = 19/20$ (b) $p + m + c = 27/20$
(c) $pmc = 1/10$ (d) $pmc = 1/4$

43. $(n \geq 5)$ persons are sitting in a row. Three of these are selected at random, the probability that no two of the selected persons are sit together, is

(a) $\dfrac{{}^{n-3}P_2}{{}^{n}P_2}$ (b) $\dfrac{{}^{n-3}C_2}{{}^{n}C_2}$ (c) $\dfrac{(n-3)(n-4)}{n(n-1)}$ (d) $\dfrac{{}^{n-3}C_2}{{}^{n}P_2}$

44. Given that $x \in [0, 1]$ and $y \in [0, 1]$. Let A be the event of (x, y) satisfying $y^2 \leq x$ and B be the event of (x, y) satisfying $x^2 \leq y$, then not true, is

(a) $P(A \cap B) = \dfrac{1}{3}$
(b) A and B are exhaustive
(c) A and B are mutually exclusive
(d) A and B are independent

45. If the probability of chosing an integer 'k' out of $2n$ integers $1, 2, 3, \dots, 2n$ is inversely proportional to $k^4 (1 \leq k \leq n)$. If α is the probability that chosen number is odd and β is the probability that chosen number is even, then

(a) $\alpha > \dfrac{1}{2}$ (b) $\alpha > \dfrac{2}{3}$ (c) $\beta < \dfrac{1}{2}$ (d) $\beta < \dfrac{2}{3}$

Probability Exercise 3 :
Passage Based Questions

- This section contains **9 passages**. Based upon each of the passage **3 multiple choice questions** have to be answered. Each of these questions has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Passage I
(Q. Nos. 46 to 48)

If p and q are chosen randomly from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ *with replacement.*

46. The probability that roots of $x^2 + px + q = 0$ are real and distinct, is
(a) 0.38 (b) 0.03 (c) 0.59 (d) 0.89

47. The probability that roots of $x^2 + px + q = 0$ are equal, is
(a) 0.58 (b) 0.55 (c) 0.38 (d) 0.03

48. The probability that roots of $x^2 + px + q = 0$ are imaginary, is
(a) 0.62 (b) 0.38 (c) 0.59 (d) 0.89

Passage II
(Q. Nos. 49 to 51)

A chess game between two grandmasters X and Y is won by whoever first wins a total of two games. X 's chances of winning, drawing or loosing any particular game are a, b and c, respectively. The games are independent and $a + b + c = 1$.

49. The probability that X wins the match after $(n+1)$ th game $(n \geq 1)$, is
(a) $na^2 b^{n-1}$ (b) $na^2 b^{n-2}(b + (n-1)c)$
(c) $na^2 bc^{n-1}$ (d) $na\, b^{n-1}(b + nc)$

50. The probability that Y wins the match after the 4th game, is
(a) $abc(2a + 3b)$ (b) $bc^2(a + 3b)$
(c) $2ac^2(b + c)$ (d) $3bc^2(2a + b)$

51. The probability that X wins the match, is
(a) $\frac{a^2(a + 2c)}{(a + c)^3}$ (b) $\frac{a^3}{(a + c)^3}$ (c) $\frac{a^2(a + 3c)}{(a + c)^3}$ (d) $\frac{c^3}{(a + c)^3}$

Passage III
(Q. Nos. 52 to 54)

There are n students in a class. Let $P(E_\lambda)$ *be the probability that exactly* λ *out of n pass the examination. If* $P(E_\lambda)$ *is directly proportional to* λ^2 $(0 \leq \lambda \leq n)$.

52. Proportionality constant k is equal to
(a) $\frac{1}{\Sigma n}$ (b) $\frac{1}{\Sigma n^2}$ (c) $\frac{1}{\Sigma n^3}$ (d) $\frac{1}{\Sigma n^4}$

53. If $P(A)$ be the probability that a student selected at random has passed the examination, then $\lim_{x \to \infty} P(A)$, is
(a) 0.25 (b) 0.50
(c) 0.75 (d) 0.35

54. If a selected student has been found to pass the examination, then the probability that he is the only student to have passed the examination, is
(a) $\frac{1}{\Sigma n}$ (b) $\frac{1}{\Sigma n^2}$
(c) $\frac{1}{\Sigma n^3}$ (d) $\frac{1}{\Sigma n^4}$

Passage IV
(Q. Nos. 55 to 57)

A cube having all of its sides painted is cut to be two horizontal, two vertical and other two planes, so as to form 27 cubes all having the same dimensions of these cubes, a cube is selected at random.

55. If P_1 be the probability that the cube selected having atleast one of its sides painted, then the value of $27P_1$, is
(a) 14 (b) 18 (c) 22 (d) 26

56. If P_2 be the probability that the cube selected has two sides painted, then the value of $27P_2$, is
(a) 3 (b) 8 (c) 12 (d) 17

57. If P_3 be the probability that the cube selected has none of its sides painted, then the value of $27P_3$, is
(a) 1 (b) 2
(c) 3 (d) 5

Passage V
(Q. Nos. 58 to 60)

A JEE aspirant estimates that she will be successful with an 80% chance, if she studies 10 h per day with a 60% chance, if she studies 7 h per day and with a 40% chance if she studies 4 h per day. She further believes that she will study 10 h, 7 h and 4 h per day with probabilities 0.1, 0.2, *and* 0.7, *respectively*.

58. The probability that she will be successful, is
(a) 0.28 (b) 0.38 (c) 0.48 (d) 0.58

59. Given that she is successful, the chances that she studied for 4 h, is
(a) $\frac{1}{12}$ (b) $\frac{5}{12}$
(c) $\frac{7}{12}$ (d) $\frac{11}{12}$

60. Given that she does not achieve success, the chance that she studied for 4 h, is

(a) $\frac{15}{26}$ (b) $\frac{17}{26}$ (c) $\frac{19}{26}$ (d) $\frac{21}{26}$

Passage VI

(Q. Nos. 61 to 63)

Suppose E_1, E_2 and E_3 be three mutually exclusive events such that $P(E_i) = p_i$ for $i = 1, 2, 3$.

61. If p_1, p_2 and p_3 are the roots of $27x^3 - 27x^2 + ax - 1 = 0$, the value of a is

(a) 3 (b) 6 (c) 9 (d) 12

62. P (none of E_1, E_2, E_3) equals

(a) 0
(b) $p_1 + p_2 + p_3$
(c) $(1 - p_1)(1 - p_2)(1 - p_3)$
(d) None of the above

63. $P(E_1 \cap \overline{E}_2) + P(E_2 \cap \overline{E}_3) + P(E_3 \cap \overline{E}_1)$ equals

(a) $p_1(1 - p_2) + p_2(1 - p_3) + p_3(1 - p_1)$
(b) $p_1p_2 + p_2p_3 + p_3p_1$
(c) $p_1 + p_2 + p_3$
(d) None of the above

Passage VII

(Q. Nos. 64 to 66)

Let $A = \{1, 2, 3\}$ and $B = \{-2, -1, 0, 1, 2, 3\}$.

64. The probability of increasing functions from A to B, is

(a) $\frac{1}{27}$ (b) $\frac{1}{18}$ (c) $\frac{5}{54}$ (d) $\frac{7}{54}$

65. The probability of non-decreasing functions from A to B, is

(a) $\frac{5}{27}$ (b) $\frac{7}{27}$
(c) $\frac{1}{3}$ (d) $\frac{11}{27}$

66. The probability of onto functions from B to B, such that $f(i) \neq i$, $i = -2, -1, 0, 1, 2, 3$, is

(a) $\frac{53}{144}$ (b) $\frac{35}{144}$ (c) $\frac{29}{72}$ (d) $\frac{25}{72}$

Passage VIII

(Q. Nos. 67 to 69)

A random variable X takes values $0, 1, 2, 3, \ldots$ with probability proportional to $(x+1)\left(\frac{1}{5}\right)^x$.

67. $P(X = 0)$ equals

(a) $\frac{2}{25}$ (b) $\frac{4}{25}$ (c) $\frac{9}{25}$ (d) $\frac{16}{25}$

68. $P(X \geq 2)$ equals

(a) $\frac{11}{25}$ (b) $\frac{13}{25}$ (c) $\frac{11}{125}$ (d) $\frac{13}{125}$

69. The expectation of X i.e., $E(X)$ is equal to

(a) $\frac{1}{4}$ (b) 2 (c) $\frac{1}{2}$ (d) 4

Passage IX

(Q. Nos. 70 to 72)

Let $n = 10\lambda + r$, where $\lambda, r \in N$, $0 \leq r \leq 9$. A number a is chosen at random from the set $\{1, 2, 3, \ldots, n\}$ and let p_n denote the probability that $(a^2 - 1)$ is divisible by 10.

70. If $r = 0$, then np_n equals

(a) 2λ (b) $(\lambda + 1)$
(c) $(2\lambda + 1)$ (d) λ

71. If $r = 9$, then np_n equals

(a) 2λ (b) $2(\lambda + 1)$
(c) $(2\lambda + 1)$ (d) λ

72. If $1 \leq r \leq 8$, then np_n equals

(a) $(2\lambda - 1)$ (b) 2λ
(c) $(2\lambda + 1)$ (d) λ

Probability Exercise 4 :
Single Integer Answer Type Questions

- This section contains **10 questions**. The answer to each question is a **single digit integer**, ranging from **0 to 9** (both inclusive).

73. A bag contains $(n + 1)$ coins. It is known that one of these coins shows heads on both sides, whereas the other coins are fair. One coin is selected at random and tossed. If the probability that the toss result in heads is $\frac{7}{12}$, then the value of n is

74. A determinant of the second order is made with the elements 0 and 1. If $\frac{m}{n}$ be the probability that the determinant made is non-negative, where m and n are relative primes, then the value of $n - m$ is

75. Three students appear in an examination of Mathematics. The probabilities of their success are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$, respectively. If the probability of success of atleast two is $\frac{\lambda}{12}$, then the value of λ is

76. A die is rolled three times, if p be the probability of getting a large number than the previous number, then the value of $54p$ is

77. In a multiple choice question, there are five alternative answers of which one or more than one are correct. A candidate will get marks on the question, if he ticks all the correct answers. If he decides to tick answers at random, then the least number of choices should he be allowed, so that the probability of his getting marks on the question exceeds $\frac{1}{8}$ is

78. There are n different objects $1, 2, 3, \dots, n$ distributed at random in n places marked $1, 2, 3, \dots, n$. If p be the probability that atleast three of the objects occupy places corresponding to their number, then the value of $6p$ is

79. A sum of money is rounded off to the nearest rupee, if $\left(\frac{m}{n}\right)^2$ be the probability that the round off error is atleast ten paise, where m and n are positive relative primes, then the value of $(n - m)$ is

80. A special die is so constructed that the probabilities of throwing 1, 2, 3, 4, 5 and 6 are $(1-k)/6$, $(1+2k)/6$, $(1-k)/6$, $(1+k)/6$, $(1-2k)/6$ and $(1+k)/6$, respectively. If two such dice are thrown and the probability of getting a sum equal to lies between $\frac{1}{9}$ and $\frac{2}{9}$, then the integral value of k is

81. Seven digits from the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 are written in random order. If the probability that this seven-digit number divisible by 9 is p, then the value of $18p$ is

82. 8 players $P_1, P_2, P_3, \dots, P_8$ play a knock out tournament. It is known that all the players are of equal strength. The tournament is held in three rounds where the players are paired at random in each round. If it is given that P_1 wins in the third round. If p be the probability that P_2 loses in the second round, then the value of $7p$ is

Probability Exercise 5 :
Matching Type Questions

- This section contains **6 questions**. Questions 83 to 88 have four statements (A, B C and D) given in **Column I** and four statements (p, q, r and s) in **Column II**. Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II**.

83.

	Column I		Column II
(A)	If $P(\overline{A}) = 0.3$, $P(B) = 0.4$ and $P(A\overline{B}) = 0.5$ and $P[B/(A \cup \overline{B})] = \lambda_1$, then $\frac{1}{\lambda_1}$ is	(p)	A prime number
(B)	The coefficient of a quadratic equation $ax^2 + bx + c = 10\,(a \neq b \neq c)$ are chosen from first three prime numbers, then the probability that roots of the equation are real is λ_2, then $\frac{1}{\lambda_2}$ is	(q)	A composite number
(C)	A fair coin is tossed repeatedly. If tail appears on first four tosses, then the probability of head appearing on the fifth toss is λ_3, then $\frac{1}{\lambda_3}$ is	(r)	A natural number
(D)	Three persons A, B and C are to speak at a function along with 6 other persons. If the persons speak in random order, then the probability that A speaks before B and B speaks before C is λ_4, then $\frac{1}{\lambda_4}$ is	(s)	A perfect number

84. A and B are two events, such that $P(A)=\frac{3}{5}$ and $P(B)=\frac{2}{3}$

	Column I		Column II
(A)	$P(A\cap B)\in$	(p)	$\left[\frac{2}{3},1\right]$
(B)	$P(A\cup B)\in$	(q)	$\left[\frac{4}{9},1\right]$
(C)	$P(A/B)\in$	(r)	$\left[\frac{2}{5},\frac{9}{10}\right]$
(D)	$P(B/A)\in$	(s)	$\left[\frac{4}{15},\frac{3}{5}\right]$

85. Three players A, B and C alternatively throw a die in that order, the first player to throw a 6 being deemed the winner. A's die is fair whereas B and C throw dice with probabilities p_1 and p_2 respectively, of throwing a 6.

	Column I		Column II
(A)	If $p_1=\frac{1}{5}$, $p_2=\frac{1}{4}$ and probability that A wins the game is $\frac{1}{\lambda_1}$, then λ_1 is divisor of	(p)	6
(B)	If $p_1=\frac{1}{5}$, $p_2=\frac{1}{4}$ and probability that C wins the game is $\frac{1}{\lambda_2}$, then λ_2 is divisor of	(q)	8
(C)	If $P(A \text{ wins})=P(B \text{ wins})$ and $\frac{1}{p_1}=\lambda_3$, then λ_3 is divisor of	(r)	12
(D)	If game is equiprobable to all the three players and $\frac{1}{p_1}=\lambda_4$, then λ_4 is divisor of	(s)	15

86. Two numbers a and b are chosen at random from the set $\{1, 2, 3, 4, \ldots, 9\}$ with replacement. The probability that the equation $x^2+\sqrt{2}(a-b)x+b=0$ has

	Column I		Column II
(A)	Real and distinct roots is p_1, then the value of $[9p_1]$, where $[\cdot]$ denotes the greatest integer function, is	(p)	2
(B)	Imaginary roots is p_2, then the value of $[9p_2]$, where $[\cdot]$ denotes the greatest integer function, is	(q)	3
(C)	Equal roots is p_3, then the value of $[81p_3]$, where $[\cdot]$ denotes the greatest integer function, is	(r)	4
(D)	Real roots is p_4, then the value of $[9p_4]$, where $[\cdot]$ denotes the greatest integer function, is	(s)	5

87. Three numbers are chosen at random without replacement from the set $\{\mid x \mid 1\le x\le 10,\ x\in N\}$

	Column I		Column II
(A)	Let p_1 be the probability that the minimum of the chosen numbers is 3 and maximum is 7, then the value of $\frac{2}{5p_1}$, is	(p)	10
(B)	Let p_2 be the probability that the minimum of the chosen numbers is 4 or their maximum is 8, then the value of $80p_2$, is	(q)	14
(C)	Let p_3 be the probability that their minimum is 3, given that their maximum is 7, then the value of $\frac{2}{p_3}$, is	(r)	16
(D)	Let p_4 be the probability that their minimum is 4, given that their maximum is 8, then the value of $\frac{2}{p_4}$, is	(s)	22

88.

	Column I		Column II
(A)	If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form 7^m+7^n is divisible by 5, is	(p)	$\frac{1}{7}$
(B)	A second order determinant is written down at random using the numbers 1, −1 as elements. The probability that the value of the determinant is non-zero, is	(q)	$\frac{1}{5}$
(C)	The probability of a number n showing in a throw of a die marked 1 to 6 is proportional to n. Then, the probability of the number 3 showing in a throw, is	(r)	$\frac{2}{5}$
(D)	A pair of dice is rolled together till a sum of either 5 or 7 is obtained. Then, the probability that 5 comes before 7, is	(s)	$\frac{1}{2}$

Probability Exercise 6 :
Statement I and II Type Questions

- **Directions** (Q. Nos. 89 to 100) are Assertion-Reason type questions. Each of these questions contains two statements:
Statement-1 (Assertion) and
Statement-2 (Reason) Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice as given below.
(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(c) Statement1 is true, Statement-2 is false
(d) Statement-1 is false, Statement-2 is true

89. **Statement-1** If 10 coins are thrown simultaneously, then the probability of appearing exactly four heads is equal to probability of appearing exactly six heads.

Statement-2 ${}^nC_r = {}^nC_s \Rightarrow$ either $r = s$ or $r + s = n$ and $P(H) = P(T)$ in a single trial.

90. **Statement-1** If A is any event and $P(B) = 1$, then A and B are independent.

Statement-2 $P(A \cap B) = P(A) \cdot P(B)$, if A and B are independent.

91. **Statement-1** If A and B be the events in a sample space, such that $P(A) = 0.3$ and $P(B) = 0.2$, then $P(A \cap \overline{B})$ cannot be found.
Statement-2 $P(A \cap \overline{B}) = P(A) - P(A \cap B)$

92. **Statement-1** Let A and B be two events, such that $P(A \cup B) = P(A \cap B)$, then

$$P(A \cap B') = P(A' \cap B) = 0$$

Statement-2 Let A and B be two events, such that $P(A \cup B) = P(A \cap B)$, then $P(A) + P(B) = 1$

93. A fair die is rolled once.
Statement-1 The probability of getting a composite number is $\frac{1}{3}$, is

Statement-2 There are three possibilities for the obtained number.
(i) The number is prime number.
(ii) The number is a composite number and
(iii) The number is 1.

Hence, probabilities of getting a prime number is $\frac{1}{3}$.

94. From a well shuffled pack of 52 playing cards, a card is drawn at random. Two events A and B are defined as
A : Red card is drawn
B : Card drawn is either a Diamond or Heart
Statement-1 $P(A + B) = P(AB)$
Statement-2 $A \subseteq B$ and $B \subseteq A$

95. **Statement-1** The probability that A and B can solve a problem is $\frac{1}{2}$ and $\frac{1}{3}$ respectively, then the probability that problem will be solved is $\frac{5}{6}$.

Statement-2 Above mentioned events are independent events.

96. **Statement-1** Out of 21 tickets with numbers 1 to 21, 3 tickets are drawn at random, the chance that the numbers on them are in AP is $\frac{10}{133}$.

Statement-2 Out of $(2n + 1)$ tickets consecutively numbered three are drawn at random, the chance that the numbers on them are in AP is $(4n - 10)/(4n^2 - 1)$.

97. **Statement-1** If A and B are two events, such that $0 < P(A)$, $P(B) < 1$, then $P\left(\frac{A}{\overline{B}}\right) + P\left(\frac{\overline{A}}{\overline{B}}\right) = \frac{3}{2}$

Statement-2 If A and B are two events, such that $0 < P(A)$, $P(B) < 1$, then
$P(A / B) = \frac{P(A \cap B)}{P(B)}$ and $P(\overline{B}) = P(A \cap \overline{B}) + P(\overline{A} \cap \overline{B})$

98. In a T-20 tournament, there are five teams. Each team plays one match against every other team.
Each team has 50% chance of winning any game it plays. No match ends in a tie.
Statement-1 The probability that there is an undefeated team in the tournament is $\frac{5}{16}$.

Statement-2 The probability that there is a winless team in the tournament is $\frac{3}{16}$.

99. **Statement-1** If p is chosen at random in the closed interval $[0, 5]$, then the probability that the equation $x^2 + px + \frac{1}{4}(p + 2) = 0$ has real is $\frac{3}{5}$.

Statement-2 If discriminant ≥ 0, then roots of the quadratic equation are always real.

100. Let a sample space S contains n elements. Two events A and B are defined on S and $B \neq \phi$.

Statement-1 The conditional probability of the event A given B, is the ratio of the number of elements in AB divided by the number of elements in B.

Statement-2 The conditional probability model given B, is equally likely model on B.

Probability Exercise 7 :
Subjective Type Questions

- In this section, there are **24 subjective** questions.

101. A five digit number is formed by the digits 1, 2, 3, 4 and 5 without repetition. Find the probability that the number formed is divisible by 4.

102. A dice is rolled three times, then find the probability of getting a large number than the previous number.

103. A car is parked among N cars standing in a row but not at either end. On his return, the owner finds that exactly r of the N places are still occupied. What is the probability that both the places neighbouring his car are empty?

104. Two teams A and B play a tournament. The first one to win $(n+1)$ games win the series. The probability that A wins a game is p and that B wins a game is q (no ties). Find the probability that A wins the series. Hence or otherwise prove that $\sum_{r=0}^{n} {}^{n+1}C_r \cdot \frac{1}{2^{n+r}} = 1$.

105. An artillery target may be either at point I with probability $\frac{8}{9}$ or at point II with probability $\frac{1}{9}$. We have 21 shells each of which can be fired either at point I or II. Each shell may hit the target independently of the other shell with probability $\frac{1}{2}$. How many shells must be fired at point I to hit the target with maximum probability?

106. There are 6 red and 8 green balls in a bag. 5 balls are drawn at random and placed in a red box. The remaining balls are placed in a green box. What is the probability that the number of red balls in the green box plus the number of green balls in the red box is not a prime number?

107. An urn contains 'a' green and 'b' pink balls $k\ (< a, b)$ balls are drawn and laid a side, their colour being ignored. Then, one more ball is drawn. Find the probability that it is green.

108. A fair coin is tossed 12 times. Find the probability that two heads do not occur consecutively.

109. Given that $x + y = 2a$, where a is constant and that all values of x between 0 and $2a$ are equally likely, then show that the chance that $xy > \frac{3}{4}a^2$, is $\frac{1}{2}$.

110. A chess game between Kamsky and Anand is won by whoever first wins a out of 2 games. Kamsky's chance of winning, drawing or loosing a particular game are 2. The games are independent and $p + q + r = 1$. Prove that the probability that Kamsky wins the match is $\frac{p^2(p+3r)}{(p+r)^3}$.

111. Of three independent events, the chance that only the first occurs is a, the other that only the second occurs is b and the chance of only third occurs is c. Show that the cases of three events are respectively $a/(a+x)$, $b/(b+x)$, $c/(c+x)$, where x is a root of the equation $(a+x)(b+x)(c+x) = x^2$.

112. A is a set containing n elements. A subset P of A is chosen at random and the set A is reconstructed by replacing the elements of P. Another subset Q of A is now chosen at random. Find the probability that $P \cup Q$ contains exactly r elements, with $1 \leq r \leq n$.

113. An electric component manufactured by 'RASU electronics' is tested for its defectiveness by a sophisticated testing device. Let A denote the even "the device is defective" and B the event "the testing device reveals the component to be defective." Suppose, $P(A) = \alpha$ and $P(B/A) = P(B'/A') = 1 - \alpha$, where $0 < \alpha < 1$. Show that the probability that the component is not defective, given that the testing device reveals it to be defective is independent of α.

114. A bag contains n white and n red balls. Pairs of balls are drawn without replacement until the bag is empty. Show that the probability that each pair consists of one white and one red ball is $2^n / ({}^{2n}C_n)$.

115. If m things are distributed among 'a' men and 'b' women, then show that the probability that the number of things received by men is odd, is $\frac{1}{2} \frac{\{(b+a)^m - (b-a)^m\}}{(b+a)^m}$.

Probability Exercise 8 :
Questions Asked in Previous 13 Year's Exam

- This section contains questions asked in **IIT-JEE, AIEEE, JEE Main & JEE Advanced** from year **2005** to year **2017**.

116. A person goes to office either by car, scooter, bus or train. The probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$, respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$, respectively. Given that he reached office in time, then what is the probability that he travelled by a car.
[IIT-JEE 2005, 2M]

117. A six faced fair die is thrown until 1 comes. Then, the probability that 1 comes in even number of trials, is
[IIT-JEE 2005, 3M]

(a) $\frac{5}{11}$ (b) $\frac{5}{6}$ (c) $\frac{6}{11}$ (d) $\frac{1}{6}$

118. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for complement of event A. Then, events A and B are
[IIT-JEE 2005, 3M]

(a) independent but not equally likely
(b) mutually exclusive and independent
(c) equally likely and mutually exclusive
(d) equally likely but not independent

119. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house, is **[AIEEE 2005, 3M]**

(a) $\frac{8}{9}$ (b) $\frac{7}{9}$ (c) $\frac{2}{9}$ (d) $\frac{1}{9}$

120. A random variable X has Poisson's distribution with mean 2. Then, $P(X > 1.5)$ is equal to **[AIEEE 2005, 3M]**

(a) $1 - \frac{3}{e^2}$ (b) $\frac{3}{e^2}$ (c) $\frac{2}{e^2}$ (d) 0

121. There are n urns each containing $(n + 1)$ balls such that the ith urn contains i white balls and $(n + 1 - i)$ red balls. Let u_i be the event of selecting ith urn, $i = 1, 2, 3, \ldots, n$ and w denotes the event of getting a white ball.
[IIT-JEE 2006, 5+5+5M]

(i) If $P(u_1) \propto i$, where $i = 1, 2, 3, \ldots, n$, then $\lim_{n \to \infty} P(w)$, is

(a) 1 (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$

(ii) If $P(u_i) = c$, where c is a constant, then $P\left(\frac{u_n}{w}\right)$, is

(a) $\frac{2}{n+1}$ (b) $\frac{1}{n+1}$ (c) $\frac{n}{n+1}$ (d) $\frac{1}{2}$

(iii) If n is even and E denotes the event of choosing even numbered urn $\left(P(u_i) = \frac{1}{n}\right)$, then the value of $P\left(\frac{w}{E}\right)$, is

(a) $\frac{n+2}{2n+1}$ (b) $\frac{n+2}{2(n+1)}$ (c) $\frac{n}{n+1}$ (d) $\frac{1}{n+1}$

122. At a telephone enquiry system, the number of phone calls regarding relevant enquiry follow Poisson's distribution with an average of 5 phone calls during 10 min time interval. The probability that there is atmost one phone call during a 10 min time period, is **[AIEEE 2006, 4, 5M]**

(a) $\frac{6}{5^e}$ (b) $\frac{5}{6}$ (c) $\frac{6}{55}$ (d) $\frac{6}{e^5}$

123. One Indian and four American men and their wives are to be seated randomly around a circular table. Then, the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife, is **[IIT-JEE 2007, 3M]**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{1}{5}$

124. Let $H_1, H_2, \ldots, H_n$ be mutually exclusive events with $P(H_i) > 0$, $i = 1, 2, \ldots, n$. Let E be any other event with $0 < P(E) < 1$.

Statement-1 $P(H_i / E) > P(E / H_i)\, P(H_i)$, for $i = 1, 2, \ldots, n$.

Statement-2 $\sum_{i=1}^{n} P(H_i) = 1$
[IIT-JEE 2007, 3M]

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(c) Statement-1 is true, Statement-2 is false
(d) Statement-1 is false, Statement-2 is true

125. Let E^c denote the complement of an event E. Let E, F and G be pairwise independent events with $P(G) > 0$ and $P(E \cap F \cap G) = 0$, then $P(E^c \cap F^c / G)$, is **[IIT-JEE 2007, 3M]**

(a) $P(E^c) + P(F^c)$ (b) $P(E^c) - P(F^c)$
(c) $P(E^c) - P(F)$ (d) $P(E) - P(F^c)$

126. A pair of fair dice is thrown independently three times. Then, the probability of getting a score of exactly 9 twice, is **[AIEEE 2007, 3M]**

(a) $\frac{1}{729}$ (b) $\frac{8}{9}$ (c) $\frac{8}{729}$ (d) $\frac{8}{243}$

127. Two aeroplanes I and II bomb a target in successions. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane, is **[AIEEE 2007, 3M]**

(a) 0.06 (b) 0.14 (c) 0.2 (d) 0.7

128. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, then the number of outcomes that B must have, so that A and B are independent, is **[IIT-JEE 2008, 3M]**

(a) 2, 4 or 8 (b) 3, 6 or 9 (c) 4 or 8 (d) 5 or 10

129. Consider the system of equations $ax + by = 0$ and $cx + dy = 0$, where $a, b, c, d \in \{0, 1\}$.

Statement-1 The probability that the system of equations has a unique solution is 3/8 and **[IIT-JEE 2008, 3M]**

Statement-2 The probability that the system of equations has a solution is 1.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

130. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is **[AIEEE 2008, 3M]**

(a) 0 (b) 1 (c) $\frac{2}{5}$ (d) $\frac{3}{5}$

131. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P\left(\frac{A}{B}\right) = \frac{1}{2}$ and $P\left(\frac{B}{A}\right) = \frac{2}{3}$. Then $P(B)$ is **[AIEEE 2008, 3M]**

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{6}$

■ **Passage for Question Nos. 132 to 134**

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required.

132. The probability that $X = 3$ is

(a) $\frac{25}{216}$ (b) $\frac{25}{36}$ (c) $\frac{5}{36}$ (d) $\frac{125}{216}$

133. The probability that $X \geq 3$ is

(a) $\frac{125}{216}$ (b) $\frac{25}{36}$ (c) $\frac{5}{36}$ (d) $\frac{25}{216}$

134. The conditional probability that $X \geq 6$ given $X > 3$, is

(a) $\frac{125}{216}$ (b) $\frac{25}{216}$ (c) $\frac{5}{36}$ (d) $\frac{25}{36}$

[IIT-JEE 2009, 4+4+4M]

135. In a binomial distribution $B\left(n, p = \frac{1}{4}\right)$, if the probability of atleast one success is greater than or equal to $\frac{9}{10}$, then n is greater than **[AIEEE 2009, 4M]**

(a) $\frac{4}{\log_{10} 4 - \log_{10} 3}$ (b) $\frac{1}{\log_{10} 4 - \log_{10} 3}$

(c) $\frac{1}{\log_{10} 4 + \log_{10} 3}$ (d) $\frac{9}{\log_{10} 4 - \log_{10} 3}$

136. One ticket is selected at random from 50 tickets numbered 00, 01, 02, ..., 49. Then, the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, is **[AIEEE 2009, 4M]**

(a) $\frac{1}{50}$ (b) $\frac{1}{14}$

(c) $\frac{1}{7}$ (d) $\frac{5}{14}$

137. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$, is **[IIT-JEE 2010, 3M]**

(a) $\frac{1}{18}$ (b) $\frac{1}{9}$ (c) $\frac{2}{9}$ (d) $\frac{1}{36}$

138. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green, is **[IIT-JEE 2010, 5M]**

(a) $\frac{3}{5}$ (b) $\frac{6}{7}$ (c) $\frac{20}{23}$ (d) $\frac{9}{20}$

139. Four numbers are chosen at random (without replacement) from the set $\{1, 2, 3, \ldots, 20\}$.

Statement-1 The probability that the chosen numbers, when arranged in some order will form an AP is $\frac{1}{85}$.

Statement-2 If the four chosen number form an AP, then the set of all possible values of common difference is $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$. **[AIEEE 2010, 8M]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is true, Statement-2 is false
(c) Statement-1 is false, Statement-2 is true
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

140. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colour, is **[AIEEE 2010, 4M]**

(a) $\frac{2}{7}$ (b) $\frac{1}{21}$ (c) $\frac{2}{23}$ (d) $\frac{1}{3}$

■ **Passage for Question Nos. 141 and 142**

Let U_1 and U_2 be two urns such that U_1 contains 3 white balls and 2 red balls and U_2 contains only 1 white ball. A fair coin is tossed. If head appears, then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears, then 2 balls are drawn at random from U_1 and put into U_2. Now, 1 ball is drawn at random from U_2.

141. The probability of the drawn ball from U_2 being white, is

(a) $\frac{13}{30}$ (b) $\frac{23}{30}$ (c) $\frac{19}{30}$ (d) $\frac{11}{30}$

142. Given that the drawn ball from U_2 is white, then the probability that head appeared on the coin, is **[IIT-JEE 2011, 3+3M]**

(a) $\frac{17}{23}$ (b) $\frac{11}{23}$ (c) $\frac{15}{23}$ (d) $\frac{12}{23}$

143. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If $P(T)$ denotes the probability of occurrence of the event T, then **[IIT-JEE 2011, 4M]**

(a) $P(E)=\frac{4}{5}, P(F)=\frac{3}{5}$ (b) $P(E)=\frac{1}{5}, P(F)=\frac{2}{5}$

(c) $P(E)=\frac{2}{5}, P(F)=\frac{1}{5}$ (d) $P(E)=\frac{3}{5}, P(F)=\frac{4}{5}$

144. Consider 5 independent Bernoulli's trials each with probability of success P. If the probability of atleast one failure is greater than or equal to $\frac{31}{32}$, then P lies in the interval **[AIEEE 2011, 4M]**

(a) $\left(\frac{3}{4}, \frac{11}{12}\right]$ (b) $\left[0, \frac{1}{2}\right]$ (c) $\left(\frac{11}{12}, 1\right]$ (d) $\left(\frac{1}{2}, \frac{3}{4}\right]$

145. If C and D are two events, such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following, is **[AIEEE 2011, 4M]**

(a) $P\left(\frac{C}{D}\right) \geq P(C)$ (b) $P\left(\frac{C}{D}\right) < P(C)$

(c) $P\left(\frac{C}{D}\right) = \frac{P(D)}{P(C)}$ (d) $P\left(\frac{C}{D}\right) = P(C)$

146. Let A, B and C are pairwise independent events with $P(C) > 0$ and $P(A \cap B \cap C) = 0$. Then, $P\left(\frac{(A^c \cap B^c)}{C}\right)$ is

(a) $P(A^c) - P(B)$ (b) $P(A) - P(B^c)$

(c) $P(A^c) + P(B^c)$ (d) $P(A^c) - P(B^c)$

147. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$, respectively. For the ship to be operational atleast two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true? **[IIT-JEE 2012, 4M]**

(a) $P[X_1^c / X] = \frac{3}{16}$

(b) P [exactly two engines of the ship are functioning $/X] = \frac{7}{8}$

(c) $P[X / X_2] = \frac{5}{16}$

(d) $P[X / X_1] = \frac{7}{16}$

148. Four fair dice D_1, D_2, D_3 and D_4 each having six faces numbered 1, 2, 3, 4, 5 and 6 are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3, is **[IIT-JEE 2012, 3M]**

(a) $\frac{91}{216}$ (b) $\frac{108}{216}$ (c) $\frac{25}{216}$ (d) $\frac{127}{216}$

149. Let X and Y be two events, such that $P(X/Y) = \frac{1}{2}$, $P(Y/X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$. Which of the following is (are) correct? **[IIT-JEE 2012, 4M]**

(a) $P(X \cup Y) = \frac{2}{3}$ (b) X and Y are independent

(c) X and Y are not independent

(d) $P(X^c \cap Y) = \frac{1}{3}$

150. Three numbers are chosen at random without replacement from $\{1, 2, 3, \ldots, 8\}$. The probability that their minimum is 3, given that their maximum is 6, is **[AIEEE 2012, 4M]**

(a) $\frac{1}{4}$ (b) $\frac{2}{5}$ (c) $\frac{3}{8}$ (d) $\frac{1}{5}$

151. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing, is **[JEE Main 2013, 4M]**

(a) $\frac{13}{3^5}$ (b) $\frac{11}{3^5}$ (c) $\frac{10}{3^5}$ (d) $\frac{17}{3^5}$

152. Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}$ and $\frac{1}{8}$. Then, the probability that the problem is solved correctly by atleast one of them, is **[JEE Advanced 2013, 2M]**

(a) $\frac{235}{256}$ (b) $\frac{21}{256}$ (c) $\frac{3}{256}$ (d) $\frac{253}{256}$

153. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2 or E_3 occurs satisfy the equation $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval $(0, 1)$.

Then, $\dfrac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ is

[JEE Advanced 2013, 4M]

■ **Passage for Question Nos. 154 and 155**

A box B_1 contains 1 white ball, 3 red balls and 2 black balls. Another box B_2 contains 2 white balls, 3 red balls and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls and 5 black balls.

154. If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, then the probability that these 2 balls are drawn from box B_2, is

(a) $\frac{116}{181}$ (b) $\frac{126}{181}$ (c) $\frac{65}{181}$ (d) $\frac{55}{181}$

155. If 1 ball is drawn from each of the boxes B_1, B_2 and B_3, then the probability that all 3 drawn balls are of the same colour, is **[JEE Advanced 2013, 3+3M]**

(a) $\frac{82}{648}$ (b) $\frac{90}{648}$ (c) $\frac{558}{648}$ (d) $\frac{566}{648}$

156. Let A and B be two events, such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then, the events A and B are

(a) independent but not equally likely **[JEE Main 2014, 4M]**
(b) independent and equally likely
(c) mutually exclusive and independent
(d) equally likely but not independent

157. Three boys and two girls stand in a queue. The probability that the number of boys ahead of every girl is atleast one more than the number of girls ahead of her, is **[JEE Advanced 2014, 3M]**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

■ **Passage for Question Nos. 158 and 159**

Box 1 contains three cards bearing numbers 1, 2, 3, box 2 contains five cards bearing numbers 1, 2, 3, 4, 5 and box 3 contains seven cards bearing numbers 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the ith box, $i = 1, 2, 3$.

158. The probability that $x_1 + x_2 + x_3$ is odd, is

(a) $\frac{29}{105}$ (b) $\frac{53}{105}$ (c) $\frac{57}{105}$ (d) $\frac{1}{2}$

159. The probability that x_1, x_2 and x_3 are in arithmetic progression, is **[JEE Advanced 2014, 3+3M]**

(a) $\frac{9}{105}$ (b) $\frac{10}{105}$ (c) $\frac{11}{105}$ (d) $\frac{7}{105}$

160. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls, is **[JEE Main 2015, 4M]**

(a) $220\left(\frac{1}{3}\right)^{12}$ (b) $22\left(\frac{1}{3}\right)^{11}$ (c) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (d) $55\left(\frac{2}{3}\right)^{10}$

161. The minimum number of times a fair coin needs to be tossed, so that the probability of getting atleast two heads is atleast 0.96, is **[JEE Advanced 2015, 4M]**

■ **Passage for Question Nos. 162 and 163**

Let n_1 and n_2 be the number of red and black balls respectively, in box I. Let n_3 and n_4 be the number of red and black balls respectively, in box II.

162. One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $1/3$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is (are)

(a) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$ (b) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$
(c) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$ (d) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

163. A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer is $\frac{1}{3}$, then correct option(s) with possible values of n_1 and n_2 is (are) **[JEE Advanced 2015, 4+4M]**

(a) $n_1 = 4$ and $n_2 = 6$ (b) $n_1 = 2$ and $n_2 = 3$
(c) $n_1 = 10$ and $n_2 = 20$ (d) $n_1 = 3$ and $n_2 = 6$

164. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true? **[JEE Main 2016, 4M]**

(a) E_2 and E_3 are independent (b) E_1 and E_3 are independent
(c) E_1, E_2 and E_3 are independent (d) E_1 and E_2 are independent

165. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced 7% of computers produced in the factory turn out to be defective. It is known that P (computer terms out to be defective given that it is produced in plant T_1) = $10P$ (computer terms out to be defective given that it is produced in plant T_2), when $P(E)$ denotes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective. Then, the probability that it is produced in plant T_2 is

[JEE Advanced 2016, 3M]

(a) $\frac{36}{73}$ (b) $\frac{47}{79}$ (c) $\frac{78}{93}$ (d) $\frac{75}{83}$

■ **Passage for Question Nos. 166 and 167**

Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$ respectively. Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2 respectively, after two games. **[JEE Advanced 2016, 3+3M]**

(a) $\frac{1}{4}$ (b) $\frac{5}{12}$ (c) $\frac{1}{2}$ (d) $\frac{7}{12}$

167. $P(X = Y)$ is

(a) $\frac{11}{36}$ (b) $\frac{1}{3}$ (c) $\frac{13}{36}$ (d) $\frac{1}{2}$

168. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one with replacement, then the variance of the number of green balls drawn is **[JEE Main 2017, 4M]**

(a) $\frac{6}{25}$ (b) $\frac{12}{5}$ (c) 6 (d) 4

169. If two different numbers are taken from the set $\{0, 1, 2, 3, \ldots, 10\}$, then the probability that their sum as well as absolute difference are both multiple of 4, is **[JEE Main 2017, 4M]**

(a) $\frac{7}{55}$ (b) $\frac{6}{55}$ (c) $\frac{12}{55}$ (d) $\frac{14}{45}$

170. For three events A, B and C.

P(Exactly one of A or B or C occurs)

$= P$(Exactly one of B or C occurs)

$= P$(Exactly one of C or A occurs) $= \frac{1}{4}$

and P (All the three events occur simultaneously) $= \frac{1}{16}$,

Then the probability that atleast one of the events occurs, is **[JEE Main 2017, 4M]**

(a) $\frac{3}{16}$ (b) $\frac{7}{32}$

(c) $\frac{7}{16}$ (d) $\frac{7}{64}$

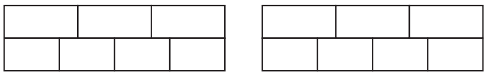
Answers

Chapter Exercise

1. (d) 2. (d) 3. (c) 4. (d) 5. (b) 6. (b)
7. (d) 8. (b) 9. (d) 10. (c) 11. (c) 12. (b)
13. (d) 14. (b) 15. (a) 16. (c) 17. (a) 18. (b)
19. (d) 20. (a) 21. (b) 22. (a) 23. (c) 24. (d)
25. (a) 26. (d) 27. (b) 28. (a) 29. (b) 30. (c)
31. (a,b,c) 32. (b,c,d) 33. (a,c) 34. (a,d) 35. (a,b) 36. (c, d)
37. (a,b,c,d) 38. (a,b,c,d) 39. (a,b,c) 40. (a,c)
41. (a,b,c) 42. (b,c) 43. (a,b,c,d) 44. (b,c,d)
45. (a,c) 46. (c) 47. (d) 48. (b) 49. (b) 50. (d)
51. (c) 52. (b) 53. (c) 54. (c) 55. (d) 56. (c)
57. (a) 58. (c) 59. (c) 60. (d) 61. (c) 62. (a)
63. (c) 64. (c) 65. (b) 66. (a) 67. (d) 68. (d)
69. (c) 70. (a) 71. (b) 72. (c) 73. (5) 74. (3)
75. (2) 76. (5) 77. (4)
78. (1) 79. (1) 80. (0) 81. (2) 82. (2)
83. (A) $\to$ (q,r); (B) $\to$ (p,r); (C) $\to$ (p,r); (D) $\to$ (q,r,s)
84. (A) $\to$ (s); (B) $\to$ (p); (C) $\to$ (r); (D) $\to$ (q)
85. (A) $\to$ (p,r,s); (B) $\to$ (p,r,s); (C) $\to$ (s); (D) $\to$ (q,r)
86. (A) $\to$ (q); (B) $\to$ (r); (C) $\to$ (p); (D) $\to$ (s)
87. (A) $\to$ (r); (B) $\to$ (s); (C) $\to$ (p); (D) $\to$ (q)
88. (A) $\to$ (q); (B) $\to$ (s); (C) $\to$ (p); (D) $\to$ (r)
89. (a) 90. (a) 91. (a) 92. (c) 93. (c)
94. (a) 95. (d) 96. (c) 97. (d) 98. (c)
99. (a) 100. (a)
101. $\left(\frac{1}{5}\right)$ 102. $\left(\frac{5}{54}\right)$ 103. $\left(\frac{(N-r)(N-r-1)}{(N-1)(N-2)}\right)$
105. (12) 106. $\left(\frac{213}{1001}\right)$ 107. $\left(\frac{a}{a+b}\right)$
108. $\left(\frac{377}{4096}\right)$ 109. $\left(\frac{1}{2}\right)$ 112. $\left(\frac{{}^nC_r\, 3^r}{4^n}\right)$
116. $\frac{1}{7}$ 117. (a) 118. (a) 119. (d) 120. (a)
121. (i) (b), (ii) (a), (iii) (b) 122. (d) 123. (c) 124. (d)
125. (c) 126. (d) 127. (b) 128. (d) 129. (b) 130. (b)
131. (a) 132. (a) 133. (b) 134. (d) 135. (b) 136. (b)
137. (c) 138. (c) 139. (b) 140. (a) 141. (b) 142. (d)
143. (a, d) 144. (b) 145. (a) 146. (a) 147. (b,d) 148. (a)
149. (a,b) 150. (d) 151. (b) 152. (a) 153. (b) 154. (d)
155. (a) 156. (a) 157. (a) 158. (b) 159. (b) 160. (c)
161. (8) 162. (a,b) 163. (c,d) 164. (c) 165. (c) 166. (b)
167. (c) 168. (b) 169. (c) 170. (c)

Solutions

1.



$n(S) =$ Number of total ways $= {}^{14}P_{12} = \frac{14!}{2!} = 7 \times 13!$

The girls can be seated together in the back seats leaving a corner seat in $4 \times 3! = 24$ ways and the boys can be seated in the remaining 11 seats in

${}^{11}P_9 = \frac{11!}{2!} = \frac{1}{2} \times 11!$ ways

$\therefore$ $n(E) =$ Number of favourable ways $= 24 \times \frac{1}{2} \times 11! = 12!$

The required probability $= \frac{n(E)}{n(S)} = \frac{12!}{7 \times 13!} = \frac{1}{91}$

2. **For non-leap year**

The probability of 53 Sundays $= \frac{1}{7}$

For leap year

The probability of 53 Sundays $= \frac{2}{7}$

$\therefore$ Required probability $= \frac{3}{4} \times \frac{1}{7} + \frac{1}{4} \times \frac{2}{7} = \frac{5}{28}$

3. Let us consider two events:

A : The leap year contains 53 Sundays.

B : The leap year contains 53 Mondays. We have,

$P(A) = \frac{2}{7}$, $P(B) = \frac{2}{7}$ and $P(A \cap B) = \frac{1}{7}$

$\therefore$ Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{2}{7} + \frac{2}{7} - \frac{1}{7} = \frac{3}{7}$

4. Let us consider two events:

A : Numbers divisible by 4.

B : Numbers divisible by 7.

We have, $A = \{104, 108, \ldots, 196\}$

$\Rightarrow$ $n(A) = 24$

$B = \{105, 112, \ldots, 196\}$

$n(B) = 14$ and $A \cap B = \{112, 140, 168, 196\}$

$\Rightarrow$ $n(A \cap B) = 4$

$n(E) =$ Number of favourable ways $n(A \cup B)$

$= n(A) + n(B) - n(A \cap B) = 34$

$n(S) =$ Total number of ways $= 99$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{34}{99}$

5. $n(S) =$ Total number of ways $= {}^{100}C_2 = 50 \times 99$

The product is divisible by 3, if atleast one of the two numbers is divisible by 3.

Let $n(E) =$ Number of ways, if atleast one of the two numbers is divisible by 3.

and $n(\overline{E}) =$ Number of ways, if none of the two numbers chosen is divisible by 3.

$$= {}^{67}C_2 = \frac{67 \times 66}{1 \times 2} = 67 \times 33$$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{n(S) - n(\overline{E})}{n(S)}$

$= 1 - \frac{67 \times 33}{50 \times 99} = \frac{83}{150}$

6. $n(S) =$ Total number of ways $= {}^{10}C_3 = \frac{10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3} = 120$

The product of two numbers is equal to third number, the favourable cases are 2, 3, 6; 2, 4, 8; 2, 5, 10

$\therefore$ $n(E) =$ The number of favourable cases $= 3$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{3}{120} = \frac{1}{40}$

7. $n(S) =$ Total number of ways $= {}^{n}P_n = n!$

Considering digits 1, 2, 3, 4, …, k as one digit, we have $(n - k + 1)$ digits which can be arranged $= (n - k + 1)!$

$\therefore$ $n(E) =$ Number of favourable ways $= (n - k + 1)!$

Hence, required probability $= \frac{n(E)}{n(S)} = \frac{(n - k + 1)!}{n!}$

8. $n(S) =$ Total number of ways $= {}^{n}P_n = n!$

The number of ways in which the digits 1, 2, 3, 4, …, k $(k < n)$ occur together $= k!(n - k + 1)!$

Hence, required probability $= \frac{n(E)}{n(S)} = \frac{k!(n - k + 1)!}{n!} = \frac{(n - k + 1)}{{}^{n}C_k}$

9. Let a, b, c and d are four different numbers out of $\{1, 2, 3, 4, 5, 6\}$.

$\Rightarrow$ (a, a, a, a) can appear in ${}^{6}C_1 = 6$ ways

$\Rightarrow$ (a, a, a, b) can appear in $2 \times {}^{6}C_2 = 30$ ways

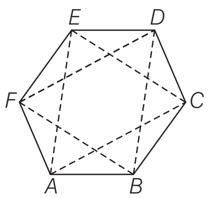
$\Rightarrow$ (a, a, b, b) can appear in ${}^{6}C_2 = 15$ ways

$\Rightarrow$ (a, a, b, c) can appear in $3 \times {}^{6}C_3 = 60$ ways

$\Rightarrow$ (a, b, c, d) can appear in ${}^{6}C_4 = 15$ ways

$\therefore$ Required probability $= \frac{60 + 15}{6 + 30 + 15 + 60 + 15} = \frac{75}{126} = \frac{25}{42}$

10. Let $ABCDEF$ be the regular hexagon.



Number of total triangles $= {}^{6}C_3 = 20$ ways

For the favourable event, the vertices should be either A, C, E or B, D, F

$\therefore$ The required probability $= \dfrac{\text{Favourable ways}}{\text{Total ways}} = \dfrac{2}{20} = \dfrac{1}{10}$

11. Total number of ways to choose two squares

$$= {}^{64}C_2 = \frac{64 \cdot 63}{2} = 32 \cdot 63$$

For favourable ways we must chosen two consecutive small squares for any row or any columns.

$\therefore$ Number of favourable ways $= 7 \cdot 8 + 8 \cdot 7 = 2 \cdot 8 \cdot 7$

$\therefore$ Required probability $= \dfrac{2 \cdot 8 \cdot 7}{32 \cdot 63} = \dfrac{1}{18}$

12. In the word MUMBAI, there are 5 adjacent pairs of letters of which only one gives AI.

$\therefore$ Required probability $= \dfrac{\frac{1}{5}}{\frac{1}{6} + \frac{1}{5} + \frac{1}{7} + \frac{1}{4} + \frac{1}{5}} = \dfrac{84}{403}$

13. Numbers on die are 1, 2, 3, 4, 5, 6.

Prime numbers are 2, 3, 5 and non-prime numbers are 1, 4, 6.

Now, let weight assigned to non-prime numbers is λ, then weight assigned to prime number is 2λ.

$\therefore \lambda + 2\lambda + 2\lambda + \lambda + 2\lambda + \lambda = 1$

$\Rightarrow \qquad \lambda = \dfrac{1}{9}$

$\therefore$ Probability that an odd number will be show up when the die is tossed 1 or 3 or 5.

$$\lambda + 2\lambda + 2\lambda = 5\lambda = \frac{5}{9}$$

14. We have,

$(X = 7) = \{07, 16, 25, 34, 43, 52, 61, 70\}$

and $\quad (Y = 0) = \{00, 01, 02, \ldots, 10, 20, 30, \ldots, 90\}$

Thus, $\quad (X = 7) \cap (Y = 0) = \{07, 70\}$

$\therefore \qquad P\left(\dfrac{X = 7}{Y = 0}\right) = \dfrac{P\{(X = 7) \cap (Y = 0)\}}{P(Y = 0)} = \dfrac{2}{19}$

15. The probability of not drawing the ace in the first draw, in the second draw and in the third draw are (here all spades i.e., 13 cards) $\dfrac{12}{13}, \dfrac{11}{12}, \dfrac{10}{11}$, respectively.

Probability of drawing ace of spades in the 4th draw

$= \dfrac{1}{10}$ (only one ace and remaining cards = 10)

$\therefore$ Required probability $= \dfrac{12}{13} \times \dfrac{11}{12} \times \dfrac{10}{11} \times \dfrac{1}{10} = \dfrac{1}{13}$

16. $n(S) =$ Total number of ways $= {}^{25}C_1 = 25$

Set of composite numbers $= \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25\}$ and set of non-composite numbers $= \{1, 2, 3, 5, 7, 11, 13, 17, 19, 23\}$

Now, set of composite numbers of the form $5k\,(k \in N)$ $= \{10, 15, 20, 25\}$

and set of non-composite numbers of the form $2k\,(k \in N) = \{2\}$

$\therefore$ Required prabability $= \dfrac{{}^4C_1 + {}^1C_1}{{}^{25}C_1} = \dfrac{5}{25} = 0.2$

17. $n(S) =$ Total number of ways $= {}^{50}C_5$

Now, x_3 is fixed to be 30 and x_1, x_2 (two numbers) are to be chosen from first 29 numbers and x_4, x_5 (two numbers) from last 20 numbers are to be chosen.

$\therefore \quad n(E) =$ Number of favourable ways $= {}^{29}C_2 \times {}^{20}C_2$

Hence, required probability $= \dfrac{n(E)}{n(S)} = \dfrac{{}^{29}C_2 \times {}^{20}C_2}{{}^{50}C_5}$

18. The points are 2, 2, 2, 2 or 2, 2, 2, 1

$\therefore$ Required probability

$= (0.5)^4 + {}^4C_1 \times (0.5)^3 \times (0.05)^1 = 0.0875$

19. $n(S) =$ Total number of ways $= 6^3 = 216$

$n(E) =$ Number of favourable ways $= 2 \times {}^6C_2 \times \dfrac{3!}{2!} = 90$

$\therefore$ Required probability $= \dfrac{n(E)}{n(S)} = \dfrac{90}{216}$

20. $n(S) =$ Total number of ways $= 6 \times 6 \times 6 = 216$

$n(E) =$ Number of favourable cases

$= $ Coefficient of x^k in $(x + x^2 + x^3 + x^4 + x^5 + x^6)^3$

$= $ Coefficient of x^{k-3} in $(1 + x + x^2 + x^3 + x^4 + x^5)^3$

$= $ Coefficient of x^{k-3} in $(1 - x^6)^3 (1 - x)^{-3}$

$= $ Coefficient of x^{k-3} in $(1 - x)^{-3}$ $\qquad [\because 0 \le k - 3 \le 5]$

$= $ Coefficient of x^{k-3} in $(1 + {}^3C_1 x + \ldots)$

$= {}^{k-1}C_{k-3} = {}^{k-1}C_2 = \dfrac{(k-1)(k-2)}{2}$

$\therefore$ Required probability $= \dfrac{n(E)}{n(S)} = \dfrac{(k-1)(k-2)}{432}$

21. $n(S) =$ Total number of ways $= 1000$

The favourable cases that the sum of the digits of the marked number on the page is equal to 9 are one digit number or two digits numbers or three digits numbers, if three digit number is abc. Then, $a + b + c = 9$; $0 \le a, b, c \le 9$

$\therefore \qquad n(E) =$ Number of favourable ways

$=$ Number of solutions of the equation

$= {}^{9+3-1}C_{3-1} = {}^{11}C_2 = 55$

$\therefore$ Required probability $= \dfrac{n(E)}{n(S)} = \dfrac{55}{1000} = \dfrac{11}{200}$

22. $n(S) =$ The total number of ways of choosing the tickets

$= 4 \times 4 \times 4 \times 4 = 256$

$n(E) =$ The number of ways in which the sum can be 23

$= $ Coefficient of x^{23} in $(1 + x + x^{10} + x^{11})^4$

$= $ Coefficient of x^{23} in $(1 + x^4) + (1 + x^{10})^4$

$= $ Coefficient of x^{23} in $(1 + 4x + 6x^2 + 4x^3 + x^4)$

$\times (1 + 4x^{10} + 6x^{20})$

$= 4 \times 6 = 24$

The probability of required event $= \dfrac{n(E)}{n(S)} = \dfrac{24}{256} = \dfrac{3}{32}$

23. Total coupons $= 15$

$1 \le$ selected coupon number ≤ 9 i.e., 1, 2, 3, 4, 5, 6, 7, 8, 9

$\therefore$ Probability of one selected coupon $= \frac{9}{15} = \frac{3}{5}$

Hence, the required probability

$$= \left(\frac{3}{5}\right) \times \left(\frac{3}{5}\right) \times \ldots \times 7 \text{ times} = \left(\frac{3}{5}\right)^7$$

24. Let X denote the largest number on the 3 tickets drawn.

We have, $P(X \le 7) = \left(\frac{7}{20}\right)^3$ and $P(X \le 6) = \left(\frac{6}{20}\right)^3$

Thus, $P(X = 7) = P(X \le 7) - P(X \le 6) = \left(\frac{7}{20}\right)^3 - \left(\frac{6}{20}\right)^3$

25. Total number = 6 (i.e., 1, 2, 3, 4, 5, 6)

Favourable number = 2, 3, 4, 5 = 4

Probability of favourable number in one draw $= \frac{4}{6} = \frac{2}{3}$

$\therefore$ Required probability $= \left(\frac{2}{3}\right)^4 = \frac{16}{81}$

26. We have, $P(E_i) = \frac{2}{4} = \frac{1}{2}$ for $i = 1, 2, 3$

Also, for $i \ne j$, $P(E_i \cap E_j) = \frac{1}{4} = P(E_i)\, P(E_j)$

Therefore, E_i and E_j are independent for $i \ne j$.

Also, $P(E_1 \cap E_2 \cap E_3) = \frac{1}{4} \ne P(E_1)P(E_2)P(E_3)$

$\therefore$ E_1, E_2 and E_3 are not independent.

27. Let x and y are two non-negative integers are chosen such that $x^2 + y^2$ is divisible by 10.

By the division algorithm, there exist integers x_1, y_1, a_1 and b_1 such that $x = 10x_1 + a_1$ and $y = 10y_1 + b_1$ with $0 \le a, b_1 \le 9$.

Thus, we can write

$x^2 + y^2 = 100(x_1^2 + y_1^2) + 20(a_1x_1 + b_1y_1) + (a_1^2 + b_1^2)$

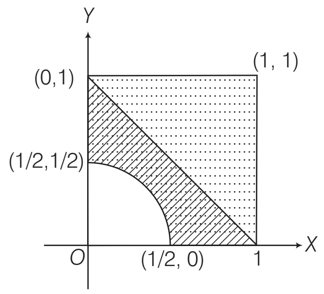
We see that $x^2 + y^2$ will be divisible by 10 if and only if $a_1^2 + b_1^2$ is divisible by 10. Now, there are 10 choices each for a_1 and b_1, so that there are $10 \times 10 = 100$ ways of choosing them. The pairs (a_1, b_1) for which $a_1^2 + b_1^2$ is divisible by 10 are follows:

(0, 0), (1, 3), (1, 7), (2, 4), (2, 6), (3, 1), (3, 9), (4, 2), (4, 8), (5, 5), (6, 2), (6, 8), (7, 1), (7, 9), (8, 4), (8, 6), (9, 3), (9, 7)

Therefore, 18 distinct ways.

$\therefore$ Required probability $= \frac{18}{100} = \frac{9}{50}$

28. Required probability $= \frac{\text{Area of strips region}}{\text{Area of dotted region}}$



$$= \frac{\frac{1}{2} \times 1 \times 1 - \frac{1}{4} \times \pi \left(\frac{1}{2}\right)^2}{1 \times 1 - \frac{1}{4} \times \pi \left(\frac{1}{2}\right)^2} = \frac{8 - \pi}{16 - \pi}$$

29. When the two equal sides are 1 each, then third side could be only 1.

When the two equal sides are 2 each, then third side can take values 1, 2, 3.

When two equal sides are 3 each, then third side can take values 1, 2, 3, 4, 5. When the two equal sides are 4 each, then third side can take values 1, 2, 3, 4, 5, 6 same in the case when two equal sides are 5 and 6.

$\therefore$ Total number of triangles = 1 + 3 + 5 + 6 + 6 + 6 = 27

Required probability $= \frac{1}{27}$

30. Required probability

$$= \frac{\left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^5}{\left(\sum_{r=0}^{5} {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r}\right)^2} = \frac{1}{\left(\sum_{r=0}^{5} {}^5C_r\right)^2} = \frac{1}{{}^{10}C_5}$$

31. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$\therefore 0 \le P(A \cup B) \le 1 - 1 \le -P(A \cup B) \le 0$

$P(A) + P(B) - 1 \le P(A) + P(B) - P(A \cup B) \le P(A) + P(B)$

32. E and F are independent events. Then,

$$P(E \cap F) = P(E) \cdot P(F) \qquad \text{...(i)}$$

Option (a) is obviously not true. So, check for options (b), (c) and (d)

$$\begin{aligned} P(E \cap \bar{F}) &= P(E) - P(E \cap F) \\ &= P(E) - P(E) \cdot P(F) \qquad \text{[from Eq. (i)]} \\ &= P(E)[1 - P(F)] \\ &= P(E) \cdot P(\bar{F}) \end{aligned}$$

$\therefore$ E and $\bar{F}$ are independent events.

Now,
$$\begin{aligned} P(\bar{E} \cap \bar{F}) &= P(\overline{E \cup F}) \\ &= 1 - P(E \cup F) \\ &= 1 - [P(E) + P(F) - P(E \cap F)] \\ &= [1 - P(E)] - P(F) + P(E) \cdot P(F) \\ &= P(\bar{E}) - P(F)[1 - P(E)] \\ &= P(\bar{E})\,[1 - P(F)] \\ &= P(\bar{E}) \cdot P(\bar{F}) \end{aligned}$$

$\therefore$ $\bar{E}$ and $\bar{F}$ are independent events.

Again, $$P\left(\frac{E}{F}\right) + P\left(\frac{\bar{E}}{F}\right) = \frac{P(E \cap F)}{P(F)} + \frac{P(\bar{E} \cap F)}{P(F)}$$
$$= \frac{P(E) \cdot P(F)}{P(F)} + \frac{P(\bar{E}) \cdot P(F)}{P(F)}$$
$$= P(E) + P(\bar{E}) = 1$$

33. We know that, $P(A \cap B) \ge P(A) + P(B) - 1$ ...(i)

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \qquad [P(B) \ne 0]$$

$$\Rightarrow \quad P\left(\frac{A}{B}\right) \geq \frac{P(A)+P(B)-1}{P(B)} \quad \text{[from Eq. (i)]}$$

Option (a) is true.

$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

Option (b) is not true.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are independent events, then $P(A \cap B) = P(A) \cdot P(B)$.

Then, $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

$$= P(A) + P(B)[1 - P(A)] + 1 - 1$$

$$= 1 + P(B)\,P(\bar{A}) - P(\bar{A}) \quad [\because P(\bar{A}) = 1 - P(A)]$$

$$= 1 + P(\bar{A})[P(B) - 1] = 1 - P(\bar{A}) \cdot P(\bar{B})$$

Option (c) is true.

If A and B are disjoint, then $P(A \cap B) = 0$.

Then, $P(A \cup B) = 1 - P(\bar{A})P(\bar{B})$ does not hold.

34. E and F are two independent events

$$P(E \cap F) = P(E) \cdot P(F) \quad \text{...(i)}$$

$$P(E \cap F) = \frac{1}{12} \quad \text{...(ii)}$$

$$P(\bar{E} \cap \bar{F}) = \frac{1}{2}$$

$$\Rightarrow \quad 1 - P(E \cup F) = \frac{1}{2}$$

$$\Rightarrow \quad P(E) + P(F) - P(E \cap F) = \frac{1}{2}$$

$$\Rightarrow \quad P(E) + P(F) - \frac{1}{12} = \frac{1}{2}$$

$$\Rightarrow \quad P(E) + P(F) = \frac{1}{2} + \frac{1}{12}$$

$$\Rightarrow \quad P(E) + P(F) = \frac{7}{12} \quad \text{...(iii)}$$

From Eqs. (i) and (ii), we get,

$$P(E) \cdot P(F) = \frac{1}{12}$$

$$P(E) = \frac{1}{12P(F)}$$

Put this value in Eq. (iii) we get

$$P(F) + \frac{1}{12P(F)} = \frac{7}{12}$$

Let $\quad P(F) = x$

Then, $\quad x + \frac{1}{12x} = \frac{7}{12}$

$$\Rightarrow \quad \frac{12x^2+1}{12x} = \frac{7}{12} \Rightarrow 12x^2 - 7x + 1 = 0$$

$$12x^2 - 4x - 3x + 1 = 0$$

$$\Rightarrow \quad 4x(3x-1) - 1(3x-1) = 0$$

$$(3x-1)(4x-1) = 0 \Rightarrow x = \frac{1}{3} \text{ or } \frac{1}{4}$$

$$\therefore \quad P(E) = \frac{1}{4}, P(F) = \frac{1}{3}$$

or $\quad P(E) = \frac{1}{3}, P(F) = \frac{1}{4}$

35. $P\left(\frac{E}{F}\right) + P\left(\frac{\bar{E}}{F}\right) = \frac{P(E \cap F)}{P(F)} + \frac{P(\bar{E} \cap F)}{P(F)}$

$$= \frac{P(E \cap F) + P(\bar{E} \cap F)}{P(F)} = \frac{P(E \cap F) + P(F) - P(E \cap F)}{P(F)}$$

$$= \frac{P(F)}{P(F)} = 1$$

and $P\left(\frac{E}{\bar{F}}\right) + P\left(\frac{\bar{E}}{\bar{F}}\right) = \frac{P(E \cap \bar{F})}{P(\bar{F})} + \frac{P(\bar{E} \cap \bar{F})}{P(\bar{F})}$

$$= \frac{[P(E) - P(E \cap F)] + [1 - P(E \cup F)]}{P(\bar{F})}$$

$$= \frac{[P(E \cup F) - P(F)] + [1 - P(E \cup F)]}{P(\bar{F})}$$

$$= \frac{1 - P(F)}{P(\bar{F})} = \frac{P(\bar{F})}{P(\bar{F})} = 1$$

36. $P(B - A) = P(B \cap \bar{A}) = P(B) - P(A \cap B)$

Option (a) is not correct.

$P(A' \cup B') = 1 - P(A \cap B) = 1 - P(A) \cdot P(B)$

[by given condition]

$P(A') + P(B') = 1 - P(A) + 1 - P(B) = 2 - [P(A) + P(B)]$

Option (b) is not correct $P[(A \cup B)'] = 1 - P(A \cup B)$

$$= 1 - [P(A) + P(B) - P(A) \cdot P(B)]$$

[by given condition]

$$= 1 - P(A) - P(B)[1 - P(A)]$$

$$= P(\bar{A}) - P(B) \cdot P(\bar{A}) = P(\bar{A}) \cdot P(\bar{B})$$

Option (c) is correct.

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot P(B)}{P(B)} = P(A)$$

37. Required probability $= P(A - B) + P(B - A)$

$= P(A \cap \bar{B}) + P(B \cap \bar{A})$

$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$

$= P(A) + P(B) - 2P(A \cap B)$

So, options (a) and (b) are true.

$P(A - B) + P(B - A) = P(A \cup B) - P(A \cap B)$

[by venn diagram]

So, option (c) is also true.

$P(A') + P(B') - 2P(A' \cap B') = 1 - P(A) + 1 - P(B) - 2 + 2P(A \cup B)$

$= 2P(A \cup B) - P(A) - P(B)$

$= 2P(A) + 2P(B) - 2P(A \cap B) - P(A) - P(B)$

$= P(A) + P(B) - 2P(A \cap B)$

$\therefore$ Option (d) is also true.

38. A and B are independent events, then $P(A \cap B) = P(A) \cdot P(B)$

$$P(A) = \frac{1}{2} \text{ and } P(B) = \frac{1}{5}$$

$$P(A \cap B) = \frac{1}{10}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{1}{5} - \frac{1}{10} = \frac{5+2-1}{10} = \frac{6}{10} = \frac{3}{5}$$

$$P\left(\frac{A}{B}\right) = P(A) = \frac{1}{2}$$

$$P\left(\frac{A}{A \cup B}\right) = \frac{P[A \cap (A \cup B)]}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)} = \frac{\frac{1}{2}}{\frac{3}{5}} = \frac{5}{6}$$

$$P\left(\frac{A \cap B}{A' \cup B'}\right) = P\left(\frac{A \cap B}{(A \cap B)'}\right)$$

$$= \frac{P[(A \cap B) \cap (A \cap B)']}{P(A \cap B)'} = 0$$

39. Let A, B and C be the event that the student is successful in tests I, II and III, respectively.

P (the student is successful)

$= P(A \cap B \cap C') + P(A \cap B' \cap C) + P(A \cap B \cap C)$

$= P(A) \cdot P(B) \cdot P(C') + P(A) \cdot P(B') \cdot P(C) + P(A) \cdot P(B) \cdot P(C)$

$\therefore$ A, B and C are independent events.

$$= pq\left(1 - \frac{1}{2}\right) + p(1-q)\left(\frac{1}{2}\right) + pq\,\frac{1}{2}$$

$$= \frac{1}{2}[pq + p(1-q) + pq] = \frac{1}{2}p(1+q)$$

$$\therefore \quad \frac{1}{2} = \frac{1}{2}p(1+q) \Rightarrow p(1+q) = 1$$

which is satisfied for all pairs of values in (a), (b) and (c). Also, it is satisfied for infinitely many values as p and q. For instance, when $p = \frac{n}{n+1}$ and $q = \frac{1}{n}$, where n is any positive integer.

40. Total number of subset of set contain n elements $= 2^n$

Number of ways choosing A and $B = 2^n \cdot 2^n = 2^{2n}$.

The number of subset of x which contains exactly r elements

$$= {}^nC_r$$

$\therefore$ The number of ways of choosing A and B, so that they have the same number of elements

$$= ({}^nC_0)^2 + ({}^nC_1)^2 + \ldots + ({}^nC_n)^2 = {}^{2n}C_n$$

$$= \frac{1 \cdot 2 \cdot 3 \ldots (2n-1)(2n)}{n!\,n!} = \frac{2^n(1 \cdot 3 \cdot 5 \ldots (2n-1))}{n!}$$

41. The number of ways in which m boys and m girls can take their seats around a circle is $(2m-1)!$

(a) We make the girls sit first around the circle. This can be done in $(m-1)!$ ways, after this boys can take their seats in $(m!)$ ways.

$\therefore$ Favourable number of ways $= m!(m-1)!$

Required probability $= \frac{m!(m-1)!}{(2m-1)!} = \frac{1}{{}^{2m-1}C_m}$

(b) Similarly as (a)

(c) Similarly as (a)

(d) Required probability $= \frac{m!m!}{(2m-1)!} \neq \frac{1}{{}^{2m-1}C_m}$

42. According to the question,

$$(m + p + c) - mp - mc - pc + mpc = \frac{3}{4} \quad \ldots\text{(i)}$$

$$mp(1-c) + mc(1-p) + pc(1-m) = \frac{2}{5}$$

or $$mp + mc + pc - 3mpc = \frac{2}{5} \quad \ldots\text{(ii)}$$

Also, $$mp + pc + mc - 2mpc = \frac{1}{2} \quad \ldots\text{(iii)}$$

From Eqs. (ii) and (iii), we get

$$mpc = \frac{1}{2} - \frac{2}{5} = \frac{1}{10}$$

$$\therefore \quad mp + mc + pc = \frac{2}{5} + \frac{3}{10} = \frac{7}{10}$$

$$m + p + c = \frac{3}{4} + \frac{7}{10} - \frac{1}{10} = \frac{27}{20}$$

43. Favourable number of cases $= {}^{n-3}P_2$

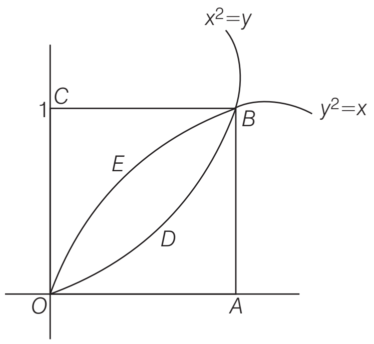
Total number of cases $= {}^nP_2$

$$\therefore \text{ Required probability} = \frac{{}^{n-3}P_2}{{}^nP_2} = \frac{{}^{n-3}C_2 \times 2!}{{}^nC_2 \times 2!} = \frac{{}^{n-3}C_2}{{}^nC_2}$$

$$= \frac{\frac{(n-3)(n-4)}{2 \times 1}}{\frac{n(n-1)}{2 \times 1}} = \frac{(n-3)(n-4)}{n(n-1)}$$

44. $A =$ The event of (x, y) belonging to the area $OEBAO$

$B =$ The event of (x, y) belonging to the area $ODBCO$



$$\therefore \quad P(A) = \frac{\text{area of } OEBAO}{\text{area of } OABCO} = \frac{\int_0^1 \sqrt{x}\,dx}{1 \times 1} = \frac{2}{3}$$

and $$P(B) = \frac{\text{area of } ODBCO}{\text{area of } OABCO} = \frac{\int_0^1 \sqrt{y}\,dy}{1 \times 1} = \frac{2}{3}$$

and $$P(A \cap B) = \frac{\text{area of } ODBEO}{\text{area of } OABCO} = \frac{\int_0^1 \sqrt{x}\,dx - \int_0^1 x^2 dx}{1 \times 1} = \frac{1}{3}$$

$\therefore$ $P(A) + P(B) = \frac{2}{3} + \frac{2}{3} \neq 1.$

So, A and B are not exhaustive,

$$P(A) \cdot P(B) = \frac{2}{3} \cdot \frac{2}{3} \neq \frac{1}{3}.$$

So, A and B are not independent

and $P(A \cup B) = 1$, $P(A) + P(B) \neq P(A \cup B)$.

So, A and B are not mutually exclusive.

45. Let $P(k) \propto \frac{1}{k^4} \Rightarrow P(k) = \frac{\lambda}{k^4}$

where λ is proportionality constant

Then $, \sum_{k=1}^{2n} \frac{\lambda}{k^4} = 1 \Rightarrow \lambda \sum_{k=1}^{2n} \frac{1}{k^4} = 1$

$\therefore \quad \alpha = \sum_{k=1}^{n} P(2k-1) = \lambda \sum_{k=1}^{n} \frac{1}{(2k-1)^4}$

and $\quad \beta = \sum_{k=1}^{n} P(2k) = \lambda \sum_{k=1}^{n} \frac{1}{(2k)^4} < \lambda \sum_{k=1}^{n} \frac{1}{(2k-1)^4}$

$\Rightarrow \quad \beta < \alpha$ and $\alpha + \beta = 1$

Then, $1 - \alpha < \alpha$ and $\beta < 1 - \beta$

$\therefore \quad \alpha > \frac{1}{2}$ and $\beta < \frac{1}{2}$

46. Roots of $x^2 + px + q = 0$ are real and distinct, if $p^2 > 4q$.

Value of p	Possible values of q
1	No value
2	No value
3	1, 2
4	1, 2, 3
5	1, 2, 3, 4, 5, 6
6	1, 2, 3,..., 8
7	1, 2, 3,..., 10
8	1, 2, 3,..., 10
9	1, 2, 3,..., 10
10	1, 2, 3,..., 10

$\therefore$ Number of favourable ways
$= 2 + 3 + 6 + 8 + 10 + 10 + 10 + 10 = 59$
and total ways $= 10 \times 10 = 100$

Hence, the required probability $= \frac{59}{100} = 0.59$

47. Roots of $x^2 + px + q = 0$ are equal, if $p^2 = 4q$

i.e., p^2 must be even.

Value of p	Possible values of q
2	1
4	4
6	9
8	No value
10	No value

$\therefore$ Number of favourable ways $= 1 + 1 + 1 = 3$
and total ways $= 10 \times 10 = 100$

Hence, the required probability $= \frac{3}{100} = 0.03$

48. Roots of $x^2 + px + q = 0$ are imaginary, if $p^2 < 4q$

Hence, the required probability $= 1 -$(Probability that roots of $x^2 + px + q = 0$ are real) $= 1 - (0.59 - 0.03) = 1 - 0.62$
$= 0.38$

49. X can win after the $(n+1)$ th game in the following two mutually exclusive ways.

(i) X wins exactly one of the first n games draws $(n-1)$ games and wins the $(n+1)$th game.

$\therefore$ Probability, $P_1 = ({}^nP_1\, a\, b^{n-1})a = na^2b^{n-1}$

(ii) X losses exactly one of the first n games, wins exactly one of the first n games and draws $(n-2)$ games and wins the $(n+1)$ th game.

$\therefore$ Probability, $P_2 = ({}^nP_2(ac)b^{n-2})a = n(n-1)a^2b^{n-2}c$

Hence, the probability that X wins two match after $(n+1)$th game.

$$P_n = P_1 + P_2 = na^2b^{n-2}[b + (n-1)c]$$

50. Put $n = 3$ in solution of question 4 and interchange a and c, then required probability $= 3c^2 \cdot b^1(b + 2a) = 3bc^2(2a + b)$

51. The probability that X wins the match

$$= \sum_{n=1}^{\infty} Pn = \sum_{n=1}^{\infty} na^2b^{n-1} + \sum_{n=1}^{\infty} n(n-1)a^2b^{n-2}c$$

$$= \frac{a^2}{b} \sum_{n=1}^{\infty} nb^n + \frac{a^2c}{b^2} \sum_{n=1}^{\infty} n(n-1)b^n$$

$$= \frac{a^2}{b} \cdot \frac{b}{(1-b)^2} + \frac{a^2c}{b^2} \cdot \frac{2b^2}{(1-b)^3}$$

[sum of infinite AGS]

$$= \frac{a^2(1-b+2c)}{(1-b)^3} = \frac{a^2(a+3c)}{(a+c)^3} \quad \left[\begin{array}{l} \because a+b+c=1 \\ \because 1-b=a+c \end{array}\right]$$

52. $\because P(E_\lambda) \propto \lambda^2$

$\Rightarrow P(E_\lambda) = k\lambda^2$, where k is proportionality constant.

$\because E_0, E_1, E_2, \ldots, E_n$ are mutually exclusive and exhaustive events.

We have, $P(E_0) + P(E_1) + P(E_2) + \ldots + P(E_n) = 1$

$$0 + k(1)^1 + k(2)^2 + \ldots + k(n)^2 = 1$$

$\Rightarrow \quad k(\sum n^2) = 1$

$\therefore \quad k = \frac{1}{\sum n^2} \quad$ ...(i)

53. $P(A) = \sum_{\lambda=0}^{n} P(E_\lambda) \cdot P\left(\frac{A}{E_\lambda}\right)$

$$= \sum_{\lambda=0}^{n} \left(k\lambda^2 \times \frac{\lambda}{n}\right) = \frac{k}{n} \sum_{\lambda=1}^{n} \lambda^3 = \frac{k}{n} \sum_{\lambda=0}^{n} \lambda^3$$

$$= \frac{k}{n} \sum n^3 = \frac{1}{n} \times \frac{1}{\sum n^2} \times \sum n^3 \quad \text{[from Eq. (i)]}$$

$$= \frac{\left(\frac{n(n+1)}{2}\right)^2}{n \cdot \frac{n(n+1)(2n+1)}{6}} = \frac{3}{2} \cdot \left(\frac{n+1}{2n+1}\right) = \frac{3}{4} \cdot \frac{\left(1 + \frac{1}{n}\right)}{\left(1 + \frac{1}{2n}\right)}$$

$$\therefore \lim_{n\to\infty} P(A) = \frac{3}{4}\lim_{n\to\infty}\left(\frac{1+\frac{1}{n}}{1+\frac{1}{2n}}\right) = \frac{3}{4}\cdot\left(\frac{1+0}{1+0}\right) = \frac{3}{4} = 0.75$$

54. $$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{\sum_{\lambda=0}^{n} P(E_\lambda)\cdot P\left(\frac{A}{E_\lambda}\right)} = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(A)}$$

$$= \frac{\frac{6}{n(n+1)(2n+1)}\times\frac{1}{n}}{\frac{3n(n+1)}{2(2n+1)}} = \frac{4}{[n(n+1)]^2} = \frac{1}{\sum n^3}$$

55. The number of cubes having atleast one side painted is

$9+9+3+3+1+1=26$ and total cubes $=27$

$\therefore$ Required probability, $P_1 = \frac{26}{27} \Rightarrow 27P_1 = 26$

56. The number of cubes having two sides painted is
$4+4+1+1+1+1=12$ and total cubes $=27$

$\therefore$ Required probability, $P_2 = \frac{12}{27} \Rightarrow 27P_2 = 12$

57. Required probability, $P_3 = 1 - p_1 = 1 - \frac{26}{27} = \frac{1}{27} \Rightarrow 27P_3 = 1$

58. A : She gets a success

E_1 : She studies 10 h

$\therefore \quad P(E_1) = 0.1$

E_2 : She studies 7 h

$\therefore \quad P(E_2) = 0.2$

and E_3: She studies 4h

$\therefore \quad P(E_3) = 0.7$

and $P\left(\frac{A}{E_1}\right) = 0.80$, $P\left(\frac{A}{E_2}\right) = 0.60$ and $P\left(\frac{A}{E_3}\right) = 0.40$

$$\therefore \quad P(A) = P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right) + P(E_3)\cdot P\left(\frac{A}{E_3}\right)$$

$$= 0.1\times 0.80 + 0.2\times 0.60 + 0.7\times 0.40 = 0.48$$

59. $$P\left(\frac{E_3}{A}\right) = \frac{P(E_3\cap A)}{P(A)} = \frac{P(E_3)\cdot P\left(\frac{A}{E_3}\right)}{P(A)} = \frac{0.7\times 0.40}{0.48} = \frac{7}{12}$$

60. $$P\left(\frac{E_3}{A}\right) = \frac{P(E_3\cap\overline{A})}{P(\overline{A})} = \frac{P(E_3) - P(E_3\cap A)}{1-P(A)}$$

$$= \frac{0.7-0.28}{1-0.48} = \frac{0.42}{0.52} = \frac{21}{26}$$

61. $\because p_1, p_2$ and p_3 are mutually exclusive events.

$\therefore \quad p_1 + p_2 + p_3 = 1$

Also, p_1, p_2 and p_3 are the roots of

$27x^3 - 27x^2 + ax - 1 = 0$

$\therefore \quad p_1 + p_2 + p_3 = 1 \quad$ ...(i)

$p_1p_2 + p_2p_3 + p_3p_1 = \frac{a}{27} \quad$ ...(ii)

and $\quad p_1p_2p_3 = \frac{1}{27}$

Now, AM of $p_1, p_2, p_3 = \frac{p_1+p_2+p_3}{3} = \frac{1}{3}$

and GM of $p_1, p_2, p_3 = (p_1p_2p_3)^{1/3} = \left(\frac{1}{27}\right)^{1/3} = \frac{1}{3}$

Here, AM = GM

$\therefore \quad p_1 = p_2 = p_3 = \frac{1}{3}$

From Eq. (ii), we get

$$\frac{1}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{1}{3} = \frac{a}{27} \Rightarrow a = 9$$

62. P (none of E_1, E_2, E_3) $= 1 - P(E_1\cup E_2\cup E_3)$

$= 1 - [P(E_1) + P(E_2) + P(E_3)]$

$= 1 - (p_1 + p_2 + p_3) = 0$

[$\because E_1, E_2$ and E_3 are mutually exclusive]

63. $P(E_1\cup\overline{E}_2) + P(E_2\cap\overline{E}_3) + P(E_3\cap\overline{E}_1)$

$= P(E_1) - P(E_1\cap E_2) + P(E_2) - P(E_2\cap E_3) + P(E_3) - P(E_3\cap E_1)$

$= P(E_1) - 0 + P(E_2) - 0 + P(E_3) - 0$

$= p_1 + p_2 + p_3 \quad$ [$\because E_1, E_2$ and E_3 are mutually exclusive]

64. The number of increasing functions $= {}^6C_3 = 20$

and the number of total functions $= 6^3 = 216$

$\therefore$ Required probability $= \frac{20}{216} = \frac{5}{54}$

65. The number of non-decreasing functions $= {}^{6+3-1}C_3 = {}^8C_3 = 56$

the number of total functions $= 6^3 = 216$

$\therefore$ Required probability $= \frac{56}{216} = \frac{7}{27}$

66. The number of onto functions such that $f(i) \neq i$ is

$$6!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!}\right) = 265$$

and number of total functions $= 6! = 720$

$\therefore$ Required probability $= \frac{265}{720} = \frac{53}{144}$

67. $\because P(X = x) \propto (x+1)\left(\frac{1}{5}\right)^x$

$$P(X = x) = k(x+1)\left(\frac{1}{5}\right)^x$$

We have, $k\left[1 + 2\left(\frac{1}{5}\right) + 3\left(\frac{1}{5}\right)^2 + \ldots\right] = 1$

$$\Rightarrow \quad k\left[\frac{1}{\left(1-\frac{1}{5}\right)^2}\right] = 1 \Rightarrow k = \frac{16}{25}$$

Now, $\quad P(X = 0) = k(1)\left(\frac{1}{5}\right)^0 = k = \frac{16}{25}$

68. $P(X \geq 2) = 1 - [P(X=0) + P(X=1)]$

$$= 1 - k\left[1 + \frac{2}{5}\right] = 1 - \frac{7k}{5} = 1 - \frac{7}{5} \times \frac{16}{25} = \frac{13}{125}$$

69. $E(X) = \sum_{x=0}^{\infty} x\, P(X=x) = k \sum_{x=0}^{\infty} x\,(x+1)\left(\frac{1}{5}\right)^x$

$$= k\left[(1)(2)\left(\frac{1}{5}\right)^1 + (2)(3)\left(\frac{1}{5}\right)^2 + (3)(4)\left(\frac{1}{5}\right)^3 + \ldots + \infty\right]$$

$$E(X) = k\left[\frac{2}{5} + \frac{6}{25} + \frac{12}{125} + \ldots + \infty\right] \quad \ldots\text{(i)}$$

and $\frac{1}{5}E(X) = k\left[\frac{2}{25} + \frac{6}{125} + \frac{12}{625} + \ldots + \infty\right] \quad \ldots\text{(ii)}$

On subtracting Eq. (ii) from Eq. (i), we get

$$\frac{4}{5}E(X) = k\left[\frac{2}{5} + \frac{4}{25} + \frac{6}{125} + \ldots + \infty\right] \quad \ldots\text{(iii)}$$

On multiplying both sides by $\frac{1}{5}$ in Eq. (i), we get

$$\frac{4}{25}E(X) = k\left[\frac{2}{25} + \frac{4}{125} + \frac{6}{625} + \ldots + \infty\right] \quad \ldots\text{(iv)}$$

On subtracting Eq. (iv) from Eq. (iii), we get

$$\frac{16}{25}E(X) = k\left[\frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \ldots + \infty\right]$$

$$= k\left[\frac{\frac{2}{5}}{1 - \frac{1}{5}}\right] = \frac{k}{2} = \frac{16}{25} \times \frac{1}{2} \quad \left[\because k = \frac{16}{25}\right]$$

$\therefore \quad E(x) = \frac{1}{2}$

70. $\because (a^2 - 1)$ is divisible by 10, if and only if last digit of a is 1 or 9.

If $r = 0$, then there are 2λ ways to choose a.

$\therefore \quad p_n = \frac{2\lambda}{n} \Rightarrow n p_n = 2\lambda$

71. If $r = 9$, then there are $2(\lambda + 1)$ ways to choose a.

$\therefore \quad p_n = \frac{2(\lambda+1)}{n}$

$\Rightarrow \quad n p_n = 2(\lambda + 1)$

72. If $1 \leq r \leq 8$, then there are $2(\lambda + 1)$ ways to choose a.

$\therefore \quad p_n = \frac{2(\lambda+1)}{n}$

$\Rightarrow \quad x p_n = 2(\lambda + 1)$

73. $\frac{7}{12} = \frac{1}{(n+1)} \cdot 1 + \frac{n}{(n+1)} \cdot \frac{1}{2}$

$\Rightarrow \frac{7}{6} = \frac{(n+2)}{(n+1)} \Rightarrow n = 5$

74. Let S be the sample space, then

$n(S)$ = Total number of determinants that can be made with 0 and 1 $= 2 \times 2 \times 2 \times 2 = 16$

$\because \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, each element can be replaced by two types

i.e., 0 and 1

and let E be the event that the determinant made is non-negative.

Also, E' be the event that the determinant is negative.

$$\therefore \quad E' = \left\{\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}\right\}$$

$\therefore \quad P(E') = 3$

then $P(E') = \frac{n(E')}{n(S)} = \frac{3}{16}$

Hence, the required probability,

$$P(E) = 1 - P(E')$$

$$= 1 - \frac{3}{16} = \frac{13}{16} = \frac{m}{n} \quad \text{[given]}$$

$\Rightarrow \quad m = 13$ and $n = 16$, then $n - m = 3$

75. Let E_1, E_2 and E_3 be the events that first, second and third student get success. Then,

$$P(E_1) = \frac{1}{3}, P(E_2) = \frac{1}{4} \text{ and } P(E_3) = \frac{1}{5}$$

Given, probability of success of atleast two $= \frac{\lambda}{12}$

$$\Rightarrow P(E_1 \cap E_2 \cap \bar{E}_3) + P(\bar{E}_1 \cap E_2 \cap E_3) + P(E_1 \cap \bar{E}_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3) = \frac{\lambda}{12}$$

$$\Rightarrow P(E_1) \cdot P(E_2) \cdot P(\bar{E}_3) + P(\bar{E}_1) \cdot P(E_2) \cdot P(E_3) + P(E_1) \cdot P(\bar{E}_2) \cdot P(E_3) + P(E_1) \cdot P(E_2) \cdot P(E_3) = \frac{\lambda}{12}$$

$$\Rightarrow \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{4}{5} + \frac{2}{3} \cdot \frac{1}{4} \cdot \frac{1}{5} + \frac{1}{3} \cdot \frac{3}{4} \cdot \frac{1}{5} + \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{5} = \frac{\lambda}{12}$$

$\Rightarrow \quad \frac{10}{60} = \frac{\lambda}{12}$

$\therefore \quad \lambda = 2$

76. Let S be the sample space and E be the event of getting a large number than the previous number.

$\therefore \quad n(S) = 6 \times 6 \times 6 = 216$

Now, we count the number of favourable ways. Obviously, the second number has to be greater than 1. If the second number is $i (i > 1)$, then the number of favourable ways $= (i - 1) \times (6 - i)$

$\therefore \quad n(E)$ = Total number of favourable ways

$$= \sum_{i=1}^{6} (i-1) \times (6-i)$$

$$= 0 + 1 \times 4 + 2 \times 3 + 3 \times 2 + 4 \times 1 + 5 \times 0$$

$$= 4 + 6 + 6 + 4 = 20$$

Therefore, the required probability, $P(E) = \frac{n(E)}{n(S)} = \frac{20}{216}$

$$= \frac{5}{54} = p \quad \text{[given]}$$

$\therefore \quad 54p = 5$

77. The number of ways to answer a question $= 2^5 - 1 = 31$.

i.e., In 31 ways only one correct.

Let number of choices $= n$

Now, according to the question $\frac{n}{31} > \frac{1}{8}$

$\Rightarrow \quad n > \frac{31}{8} \Rightarrow n > 3.8$

$\therefore$ Least value of $n = 4$

78. Let E_i denote the event that the ith object goes to the ith place, we have

$$P(E_i \cap E_j \cap E_k) = \frac{(n-3)!}{n!} \text{ for } i < j < k$$

Since, we can choose 3 places out of n is $^nC_3$ ways, the probability of the required event is

$$p = {}^nC_3 \cdot \frac{(n-3)!}{n!} = \frac{n!}{3!(n-3)!} \cdot \frac{(n-3)!}{n!} = \frac{1}{6}$$

$\therefore \quad 6p = 1$

79. The sample space is

$S = \{-0.50, -0.49, -0.48, \ldots, -0.01, 0.00, 0.01, \ldots, 0.49\}$

Let E be the event that the round off error is atleast 10 paise, then E' is the event that the round off error is atmost a paise.

$\therefore \quad E' = \{-0.09, -0.08, \ldots, -0.01, 0.00, 0.01, \ldots, 0.09\}$

$\therefore \quad n(E') = 19$ and $n(S) = 100$

$\therefore \; P(E') = \frac{n(E')}{n(S)} = \frac{19}{100}$

$\therefore$ Required probability, $P(E) = 1 - P(E') = 1 - \frac{19}{100}$

$$= \frac{81}{100} = \left(\frac{m}{n}\right)^2$$

$\therefore \quad m = 9$ and $n = 10$

$\Rightarrow \quad n - m = 1$

80. Let E_1, E_2, E_3, E_4, E_5 and E_6 be the events of occurrence of 1, 2, 3, 4, 5 and 6 on the dice respectively and let E be the event of getting a sum of numbers equal to 9.

$\therefore \quad P(E_1) = \frac{1-k}{6}; P(E_2) = \frac{1+2k}{6}; P(E_3) = \frac{1-k}{6};$

$P(E_4) = \frac{1+k}{6}; P(E_5) = \frac{1-2k}{6}; P(E_6) = \frac{1+k}{6}$

and $\quad \frac{1}{9} \le P(E) \le \frac{2}{9}$

Then, $\quad E \equiv \{(3, 6), (6, 3), (4, 5), (5, 4)\}$

Hence, $P(E) = P(E_3E_6) + P(E_6E_3) + P(E_4E_5) + P(E_5E_4)$

$= P(E_3)P(E_6) + P(E_6)P(E_3) + P(E_4)P(E_5) + P(E_5)P(E_4)$

$= 2P(E_3)P(E_6) + 2P(E_4)P(E_5)$

[since E_1, E_2, E_3, E_4, E_5 and E_6 are independent]

$= 2\left(\frac{1-k}{6}\right)\left(\frac{1+k}{6}\right) + 2\left(\frac{1+k}{6}\right)\left(\frac{1-2k}{6}\right)$

$= \frac{1}{18}[2 - k - 3k^2]$

Since, $\quad \frac{1}{9} \le P(E) \le \frac{2}{9}$

$\Rightarrow \quad \frac{1}{9} \le \frac{1}{18}[2 - k - 3k^2] \le \frac{2}{9}$

$\Rightarrow \quad 2 \le 2 - k - 3k^2 \le 4$

$\Rightarrow \quad 2 \le 2 - k - 3k^2$ and $2 - k - 3k^2 \le 4$

$\Rightarrow \quad 3k\left(k + \frac{1}{3}\right) \le 0$ and $3k^2 + k + 2 \ge 0$

$\Rightarrow \quad -\frac{1}{3} \le k \le 0$ and $k \in R$

$\therefore \quad -\frac{1}{3} \le k \le 0$

Hence, integral value of k is 0.

81. Let $a_1, a_2, a_3, a_4, a_5, a_6$ and a_7 be the seven digits and the remaining two be a_8 and a_9.

Let $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 = 9k, k \in I$ ...(i)

Also, $\quad a_1 + a_2 + a_3 + a_4 + \ldots + a_9 = 1 + 2 + 3 + 4 + \ldots + 9$

$= \frac{9 \times 10}{2} = 45$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$a_8 + a_9 = 45 - 9k$ ...(iii)

Since, $a_1 + a_2 + a_3 + a_4 + \ldots + a_9$ and $a_1 + a_2 + \ldots + a_7$ are divisible by 9, if and only if $a_8 + a_9$ is divisible by 9. Let S be the sample space and E be the event that the sum of the digits a_8 and a_9 is divisible by 9.

$\because \quad a_8 + a_9 = 45 - 9k$

Maximum value of $a_8 + a_9 = 17$ and minimum value of $a_8 + a_9 = 3$

$\therefore \quad 3 \le 45 - 9k \le 17$

$\Rightarrow \quad -42 \le -9k \le -28 \Rightarrow \frac{42}{9} \le k \ge \frac{28}{9}$

or $\quad \frac{28}{9} \le k \ge \frac{42}{9}$

Hence, $\quad k = 4$ [$\because k$ is positive integer]

$\therefore$ From Eq. (iii), we get

$a_8 + a_9 = 45 - 9 \times 4$

$\therefore \quad a_8 + a_9 = 9$

Now, possible pair of (a_8, a_9) can be $\{(1, 8), (2, 7), (3, 6), (4, 5)\}$

$\therefore \quad E = \{(1, 8), (2, 7), (3, 6), (4, 5)\}$

$n(E) = 4$ and $n(S) = {}^9C_2 = 36$

$\therefore$ Required probability, $P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9} = p$ [given]

$\therefore \quad 18p = 2$

82. Let A be the event of P_1 winning in third round and B be the event of P_2 winning in first round but loosing in second round.

We have, $P(A) = \frac{1}{^8C_1} = \frac{1}{8}$

$P(B \cap A)$ = Probability of both P_1 and P_2 winning in first round $\times$ Probability of P_1 winning and P_2 loosing in second round $\times$ Probability of P_1 winning in third round

$$= \frac{^{8-2}C_{4-2}}{^8C_4} \times \frac{^{4-2}C_{2-1}}{^4C_2} \times \frac{^{2-1}C_{1-1}}{^2C_1}$$

$$= \frac{^6C_2}{^8C_4} \times \frac{^2C_1}{^4C_2} \times \frac{^1C_0}{^2C_1} = \frac{1}{28}$$

Hence, $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{1}{28}}{\frac{1}{8}} = \frac{2}{7} = p$ [given]

$\therefore \quad 7p = 2$

83. (A) $\rightarrow$(q,r); (B) $\rightarrow$ (p,r); (C) $\rightarrow$ (p,r); (D) $\rightarrow$ (q,r,s)

(A) $P(\bar{A}) = 0.3 \Rightarrow P(A) = 1 - P(\bar{A}) = 0.7, P(B) = 0.4,$

$\Rightarrow \quad 0.5 = 0.7 - P(AB)$

$\therefore \quad P(AB) = 0.2$

$$\Rightarrow P\left[\frac{B}{(A \cup \bar{B})}\right] = \frac{P[B \cap (A \cup \bar{B})]}{P(A \cup \bar{B})} = \frac{P(A \cap B)}{P(A) + P(\bar{B}) - P(A\bar{B})}$$

$$= \frac{0.2}{0.7 + 0.6 - 0.5} = \frac{1}{4} = \lambda_1 \quad \text{[given]}$$

$\therefore \quad \frac{1}{\lambda_1} = 4$ [composite number and natural number]

(B) First three prime numbers are 2, 3 and 5.

For roots to be real $D \geq 0$

Thus, real roots are obtained by $b = 5, a = 2, c = 3$

and $b = 5, a = 3, c = 2$

i.e., two ways.

Total ways of choosing $a, b, c = 3 \times 2 \times 1 = 6$

$\therefore$ Required probability $= \frac{2}{6} = \frac{1}{3} = \lambda_2$ [given]

$\therefore \quad \frac{1}{\lambda_2} = 3$

[prime number and natural number]

(C) Here, tossing of the coin is an independent event. Thus, the result of 5th trial is independent of outcome of previous trials.

$\therefore \quad \lambda_3 = \frac{1}{2} \Rightarrow \frac{1}{\lambda_3} = 2$

[prime number and natural number]

(D) Clearly, $n(S) = {}^9P_9 = 9!$

Now, 3 positions out of 9 positions can be chosen in ${}^9C_3$ ways and at these positions A, B and C can speak in required order, further remaining 6 persons can speak in 6! ways.

$\therefore$ Required probability $= \frac{{}^9C_3 \times 6!}{9!}$

$$= \frac{9! \times 6!}{3! \times 6! \times 9!} = \frac{1}{6} = \lambda_4 \quad \text{[given]}$$

$\therefore \quad \frac{1}{\lambda_4} = 6$

[a composite number, a natural number and a perfect number]

84. (A) $\rightarrow$ (s); (B) $\rightarrow$ (p); (C) $\rightarrow$ (r); (D) $\rightarrow$ (q)

(A) $\frac{3}{5} \geq P(A \cap B) \geq P(A) + P(B) - 1 = \frac{3}{5} + \frac{2}{3} - 1 = \frac{4}{15}$

$\therefore \quad \frac{3}{5} \geq P(A \cap B) \geq \frac{4}{15}$

$\Rightarrow \quad P(A \cap B) \in \left[\frac{4}{15}, \frac{3}{5}\right]$

(B) $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{3}{5} + \frac{2}{3} - P(A \cap B) = \frac{19}{15} - P(A \cap B)$$

$\Rightarrow \quad \frac{2}{3} \leq P(A \cup B) \leq 1 \quad \left[\because P(A \cap B) \in \left[\frac{4}{15}, \frac{3}{5}\right]\right]$

$\therefore \quad P(A \cup B) \in \left[\frac{2}{3}, 1\right]$

(C) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{3}{2} P(A \cap B)$

$\Rightarrow \quad \frac{2}{5} \leq P\left(\frac{A}{B}\right) \leq \frac{9}{10} \left[\because P(A \cap B) \in \left[\frac{4}{15}, \frac{3}{5}\right]\right]$

$\therefore \quad P\left(\frac{A}{B}\right) \in \left[\frac{2}{5}, \frac{9}{10}\right]$

(D) $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{5}{3} P(A \cap B)$

$\Rightarrow \quad \frac{4}{9} \leq P\left(\frac{B}{A}\right) \leq 1 \quad \left[\because P(A \cap B) \in \left[\frac{4}{15}, \frac{3}{5}\right]\right]$

$\therefore \quad P\left(\frac{B}{A}\right) \in \left[\frac{4}{9}, 1\right]$

85. (A) $\rightarrow$ (p,r,s) ; (B) $\rightarrow$ (p,r,s); (C) $\rightarrow$ (s); (D) $\rightarrow$ (q,r)

Let $q_1 = 1 - p_1$ and $q_2 = 1 - p_2$

(A) $\because$ A can win the game at the 1st, 4th, 7th,... trials.

$$\therefore P(A \text{ wins}) = \frac{1}{6} + \frac{5}{6}(q_1)(q_2)\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^2 (q_1)^2(q_2)^2\left(\frac{1}{6}\right) + \ldots$$

$$= \frac{\frac{1}{6}}{1 - \frac{5}{6}q_1q_2} = \frac{1}{6 - 5q_1q_2} = \frac{1}{6 - 5\left(\frac{4}{5}\right)\left(\frac{3}{4}\right)} = \frac{1}{\lambda_1} \quad \text{[given]}$$

$\therefore \quad \lambda_1 = 3$

(B) $P(C \text{ wins}) = \frac{5}{6}.q_1.p_2 + \left(\frac{5}{6}\right)^2 . q_1^2.q_2 p_2 + \ldots$

$$= \frac{\frac{5}{6}q_1.p_2}{1 - \frac{5}{6}q_1q_2} = \frac{5q_1p_2}{6 - 5q_1q_2} = \frac{5 \times \frac{4}{5} \times \frac{1}{4}}{6 - 5 \times \frac{4}{5} \times \frac{3}{4}} = \frac{1}{3}$$

$$= \frac{1}{\lambda_2} \quad \text{[given]}$$

$\therefore \quad \lambda_2 = 3$

(C) $\because$ P (A wins) $= P$ (B wins)

$\Rightarrow \quad \frac{1}{6 - 5q_1q_2} = \frac{5p_1}{6 - 5q_1q_2}$

$\therefore \quad p_1 = \frac{1}{5} = \frac{1}{\lambda_3}$ [given]

$\therefore \quad \lambda_3 = 5$

(D) $P(A \text{ wins}) = P(B \text{ wins}) = P(C \text{ wins})$

$$\Rightarrow \quad \frac{1}{6-5q_1q_2} = \frac{5p_1}{6-5q_1q_2} = \frac{5q_1p_2}{6-5q_1q_2}$$

$$\Rightarrow \quad 1 = 5p_1 = 5q_1p_2$$

$$\Rightarrow \quad p_1 = \frac{1}{5}, \frac{1}{p_2} = 5q_1 = 5\left(1-\frac{1}{5}\right) = 4 = \lambda_4$$

[given]

$$\therefore \quad \lambda_4 = 4$$

86. (A) → (q); (B) → (r); (C) → (p); (D) → (s)

(A) $\because a, b \in \{1, 2, 3, ..., 9\}$

For real and distinct roots $D > 0$

i.e., $2(a-b)^2 > 4b \Rightarrow (a-b)^2 > 2b$

The possible pairs are

b	**a**	**Total pairs of a and b**
1	3, 4, 5, ..., 9	7
2	5, 6, ..., 9	5
3	6, 7, 8, 9	4
4	1, 7, 8, 9	4
5	1, 9	2
6	1, 2	2
7	1, 2, 3	3
8	1, 2, 3	3
9	1, 2, 3, 4	4
		34

$n(S) = 9 \times 9 = 81$ and $n(E) = 34$

$$\therefore \quad p_1 = \frac{34}{81} \Rightarrow 9p_1 = \frac{34}{9}$$

$$\therefore \quad [9p_1] = \left[\frac{34}{9}\right] = 3$$

(B) For imaginary roots,

$$p_2 = 1 - p_4 = 1 - \frac{5}{9} = \frac{4}{9}$$

$$\therefore \quad [9p_2] = 4$$

(C) For equal roots, there are only 2 possible pairs are

$b = 2, a = 4$ and $b = 8, a = 4$

$$\therefore \quad n(S) = 81, n(E) = 2$$

$$\therefore \quad p_3 = \frac{2}{81}$$

$$\Rightarrow \quad [81p_3] = 2$$

(D) For real roots,

$$p_4 = 1 - (p_1 + p_3) = 1 - \left(\frac{34}{81} + \frac{2}{81}\right)$$

$$= 1 - \frac{36}{81} = 1 - \frac{4}{9} = \frac{5}{9}$$

$$\therefore \quad [9p_4] = 5$$

87. (A) → (r); B → (s); (C) → (p); D → (q)

(A) $n(S) = {}^{10}C_3 = 120$ and $n(E) = {}^3C_1 = 3$, because on selection 3 and 7, we have to select one from 4, 5 and 6.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{120} = \frac{1}{40} = p_1$$ [given]

$$\Rightarrow \quad \frac{2}{5p_1} = 16$$

(B) The probability of 4 being the minimum number

$$= \frac{{}^6C_2}{{}^{10}C_3} = \frac{1}{8}$$ [because after selecting 4 any two can be selected from 5, 6, 7, 8, 9, 10]

and the probability of being the maximum number

$$= \frac{{}^7C_2}{{}^{10}C_3} = \frac{7}{40}$$ [because after selecting 8 any two can be selected from 1, 2, 3, 4, 5, 6, 7]

and the probability of 4 being the minimum number and 8 being the maximum number

$$= \frac{{}^3C_1}{{}^{10}C_3} = \frac{1}{40}$$ [because on selecting 4, 8 and we have to select one from 5, 6, 7]

$\therefore$ Required probability

$$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{8} + \frac{7}{40} - \frac{1}{40} = \frac{11}{40} = p_2$$ [given]

$$\therefore \quad 80p_2 = 22$$

(C) Let A = {maximum of three numbers is 7}

$\therefore \quad A = \{1, 2, 3, 4, 5, 6, 7\}$

and B = {minimum of three numbers is 3}

$\therefore \quad B = \{3, 4, 5, 6, 7, 8, 9, 10\}$

and $A \cap B = \{3, 4, 5, 6, 7\}$

$$\therefore \quad P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{{}^3C_1}{{}^6C_2} = \frac{1}{5} = p_3$$ [given]

$$\therefore \quad \frac{2}{p_3} = 10$$

(D) Let A = { maximum of three numbers is 8 }

$\therefore \quad A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

and B = { minimum of three numbers is 4 }

$\therefore \quad B = \{4, 5, 6, 7, 8, 9, 10\}$ and $A \cap B = \{4, 5, 6, 7, 8\}$

$$\therefore \quad P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{{}^3C_1}{{}^7C_2} = \frac{1}{7} = p_4$$ [given]

$$\therefore \quad \frac{2}{p_4} = 14$$

88. (A) → (q); (B) → (s); C → (p); (D) → (r)

(A) We know that $7^\lambda, \lambda \in N$ has 1, 3, 7, 9 at the unit's place for $\lambda = 4k, 4k-1, 4k-2, 4k-3$ respectively,

when $k = 1, 2, 3, \ldots$

Clearly, $7^m + 7^n$ will be divisible by 5, if 7^m has 3 or 7 in the unit's place and 7^n has 7 or 3 in the unit's place or 7^m has 1 or 9 in the unit's place and 7^n has 9 or 1 in the unit's place.

$\therefore$ For any choice of m, n the digit in the unit's place of $7^m + 7^n$ is 2, 4, 6, 8, 0

It is divisible by 5 only when this digit is 0.

$\therefore$ Required probability $= \frac{1}{5}$

(B) $n(S) = 2 \times 2 \times 2 \times 2 = 16$

[because each of the four places in determinant can be filled in 2 ways]

The zero determinants are

$$\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} -1 & -1 \\ -1 & -1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix}, \begin{vmatrix} -1 & -1 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix},$$

$$\begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix}, \begin{vmatrix} -1 & 1 \\ -1 & 1 \end{vmatrix}, \begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix}$$

Number of zero determinants = 8, number of non-zero determinants

$= 16 - 8 = 8 = n(E)$ [say]

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{8}{16} = \frac{1}{2}$

(C) $\because P(E_n) \propto n$

$\Rightarrow P(E_n) = kn$, where k is proportionality constant.

Clearly,

$P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) + P(E_6) = 1$

$\Rightarrow \quad k(1 + 2 + 3 + 4 + 5 + 6) = 1 \Rightarrow k = \frac{1}{21}$

$\therefore$ Required probability $= P(E_3) = 3k = 3 \times \frac{1}{21} = \frac{1}{7}$

(D) 5 can be thrown in 4 ways and 7 can be thrown in 6 ways in a single throw of two dice.

Number of ways of throwing neither 5 nor 7

$= 36 - (4 + 6) = 26$

Probability of throwing a sum of 5 in a throw $= \frac{4}{36} = \frac{1}{9}$

and probability of throwing neither 5 nor 7 $= \frac{26}{36} = \frac{13}{18}$

$\therefore$ Required probability

$$= \frac{1}{9} + \frac{13}{18}\left(\frac{1}{9}\right) + \left(\frac{13}{18}\right)^2\left(\frac{1}{9}\right) + \dots = \frac{\frac{1}{9}}{1 - \frac{13}{18}} = \frac{2}{5}$$

89. In $\left(\frac{1}{2} + \frac{1}{2}\right)^{10}$

Probability of appearing exactly four heads

$$= {}^{10}C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^6 = {}^{10}C_{10-4}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^4$$

$$= {}^{10}C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^4$$

= Probability of appearing exactly six heads. Both statements are true,

Statement-2 is a correct explanation for Statement-1.

90. If A and B are independent, then

$P(A \cap B) = P(A) \cdot P(B) = P(A)$ $\quad [\because P(B) = 1]$ ...(i)

and $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)$ $\quad$ [from Eq. (i)]

$= P(B) = 1$ which is true.

Hence, both statements are true and Statement-2 is a correct explanation for Statement-1.

91. $\because \quad P(A \cap \overline{B}) = P(A) - P(A \cap B)$

$\Rightarrow \quad P(A \cap \overline{B}) = 0.3 - P(A \cap B)$

$\therefore P(A \cap \overline{B})$ cannot be found. Hence, both statements are true and Statement-2 is a correct explanation for Statement-1.

92. $\because P(A \cup B) = P(A \cap B)$

$\Rightarrow \quad P(A) + P(B) - P(A \cap B) = P(A \cap B)$

$\Rightarrow (P(A) - P(A \cap B)) + (P(B) - P(A \cap B)) = 0$

$\Rightarrow \quad P(A \cap B') + P(A' \cap B) = 0$ ...(i)

$\because \quad A \cap B \subseteq A$ and $A \cap B \subseteq B$

$\Rightarrow \quad P(A \cap B) \le P(A)$ and $P(A \cap B) \le P(B)$

$\Rightarrow P(A) - P(A \cap B) \ge 0$ and $P(B) - P(A \cap B) \ge 0$

$\Rightarrow \quad P(A \cap B') \ge 0$ ...(ii)

and $\quad P(A' \cap B) \ge 0$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$P(A \cap B') = 0$ and $P(A' \cap B) = 0$

or $\quad P(A \cap B') = P(A' \cap B) = 0$

$\Rightarrow$ Statement-1 is true and Statement-2 is false.

93. **Statement-1** There are six equally likely possibilities of which only 2 are favourable (4 and 6)

$\therefore$ Probability that the obtained number is composite $= \frac{2}{6} = \frac{1}{3}$

$\therefore$ Statement-1 is true.

Statement-2 As the given 3 possibilities are not equally likely.

$\therefore$ Statement-2 is false.

94. Total cards = 52 = 26 Red + 26 Black

13 Diamond $\quad$ 13 Heart

Given A : Red card is drawn

B : Card drawn is either a diamond or heart

It is clear that $A \subseteq B$ and $B \subseteq A$

$\therefore$ Statement-2 is true.

and $P(A + B) = P(A \cup B) = P(A \cup A) = P(A)$

$[\because A \subseteq B$ and $B \subseteq A]$

and $\quad P(AB) = P(A \cap B) = P(A \cap A) = P(A)$

$[\because A \subseteq B$ and $B \subseteq A]$

$\therefore \quad P(A + B) = P(AB)$

Statement-1 is true.

Hence, both statements are true and Statement-2 is a correct explanation for Statement-1.

95. Required probability = 1 − Problem will not be solved

$= 1 - P(\overline{A} \cap \overline{B}) = 1 - P(\overline{A})P(\overline{B}) = 1 - (1 - P(A))(1 - P(B))$

$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right) = 1 - \frac{1}{2} \times \frac{2}{3} = 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore$ Statement-1 is false and Statement-2 is true.

96. Total ways $= {}^{2n+1}C_3 = \frac{(2n+1).2n.(2n-1)}{1\cdot2\cdot3} = \frac{n(4n^2-1)}{3}$

Let the three numbers a, b, c are drawn, where $a < b < c$ and given a, b and c are in AP.

$\therefore \quad 2b = a + c$ ...(i)

It is clear from Eq. (i) that a and c both are odd or both are even.

$\therefore$ Favourable ways $= {}^{n+1}C_2 + {}^nC_2$

$$= \frac{(n+1)n}{1\cdot2} + \frac{n(n-1)}{1\cdot2} = n^2$$

$\therefore$ Required probability $= \frac{n^2}{\frac{n(4n^2-1)}{3}} = \frac{3n}{(4n^2-1)}$

$\Rightarrow$ Statement-2 is false.

In Statement-1, $2n + 1 = 21$

$\Rightarrow \quad n = 10$

$\because$ Required probability $= \frac{3\times10}{4(10)^2-1} = \frac{30}{399} = \frac{10}{133}$

$\therefore$ Statement-1 is true.

97. In Statement-2 $P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)}$ [by definition]

$$P(\overline{B}) = P((A\cup\overline{A})\cap\overline{B}) = P((A\cap\overline{B})\cup(\overline{A}\cap\overline{B}))$$
$$= P(A\cap\overline{B}) + P(\overline{A}\cap\overline{B}) \quad \text{...(i)}$$

$\therefore$ Statement-2 is true.

In Statement-1

$$P\left(\frac{A}{\overline{B}}\right) + P\left(\frac{\overline{A}}{\overline{B}}\right) = \frac{P(A\cap\overline{B})}{P(\overline{B})} + \frac{P(\overline{A}\cap\overline{B})}{P(\overline{B})}$$
$$= \frac{P(A\cap\overline{B}) + P(\overline{A}\cap\overline{B})}{P(\overline{B})} = \frac{P(\overline{B})}{P(\overline{B})} = 1 \quad \text{[From Eq. (i)]}$$

$\therefore$ Statement-1 is false.

98. The total number of matches played in the tournament

$$= {}^5C_2 = 10$$

The probability that a particular team (say A) wins all its 4 matches $= \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

$\therefore$ Probability that team is undefeated in the tournament

$$= {}^5C_1\left(\frac{1}{2}\right)^4 = \frac{5}{16}$$

$\Rightarrow$ Statement-1 is true.

Similarly, the probability that there is an winless team $= \frac{5}{16}$

$\Rightarrow$ Statement-2 is false.

99. For real roots, $D \ge 0$

$\Rightarrow \quad p^2 - 4\cdot1\cdot\frac{1}{4}(p+2) \ge 0$

$\Rightarrow \quad p^2 - p - 2 \ge 0$

$\Rightarrow \quad (p-2)(p+1) \ge 0$

$\Rightarrow \quad p \le -1$ or $p \ge 2$

But $\quad p \in [0, 5]$.

So, $\quad E = [2, 5]$

$n(E) =$ length of the interval $[2, 5] = 3$

and $\quad n(S) =$ length of the interval $[0, 5] = 5$

$\therefore \quad$ Required probability $= \frac{n(E)}{n(S)} = \frac{3}{5}$

Hence, both statements are true and Statement-2 is a correct explanation for Statement-1

100. $P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{n(A\cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{n(A\cap B)}{n(B)} = \frac{n(AB)}{n(B)}$

Hence, both statements are true and Statement-2 is a correct explanation for Statement-1.

101. Let S be the sample space, then

$n(S) =$ Total number of numbers of five digits formed with the digits 1, 2, 3, 4 and 5 without repetition $= {}^5P_5 = 5! = 120$

We know that, a number is divisible by 4 if the last two digits of the number is divisible by 4.

Then, for divisible by 4, last two digits 12 or 24 or 32 or 52.

Let E be the event that the number formed is divisible by 4.

$\therefore \quad n(E) = 3! \times 4 = 24$

$\therefore$ Required probability, $P(E) = \frac{n(E)}{n(S)} = \frac{24}{120} = \frac{1}{5}$

102. Let S be the sample space and E be the event of getting a large number than the previous number.

$\therefore \quad n(S) = 6\times6\times6 = 216$

Now, we count the number of favourable ways. Obviously, the second number has to be greater than 1. If the second number is $i(i > 1)$, then the number of favourable ways $= (i-1)\times(6-i)$

$\therefore$ $n(E) =$ Total number of favourable ways

$$= \sum_{i=1}^{6}(i-1)\times(6-i)$$
$$= 0 + 1\times4 + 2\times3 + 3\times2 + 4\times1 + 5\times0$$
$$= 4 + 6 + 6 + 4 = 20$$

Therefore, the required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{20}{216} = \frac{5}{54}$$

103. Finding r cars in N places, there are $(r-1)$ cars other than his own in $(N-1)$ places.

$\therefore \quad$ Total number of ways $= {}^{N-1}C_{r-1} = \frac{(n-1)!}{(r-1)!\,(N-r)!}$

Now, the $(r-1)$ cars must be parked in $N-3$ places (because neighbouring slots are empty).

$\therefore$ Number of favourable ways $= {}^{N-3}C_{r-1}$

$$= \frac{(N-3)!}{(r-1)!\,(N-r-2)!}$$

$\therefore$ Required probability $= \frac{\text{Favourable ways}}{\text{Total ways}}$

$$= \frac{(N-3)!}{(r-1)!\,(N-r-2)!} \times \frac{(r-1)!\,(N-r)!}{(N-1)!}$$
$$= \frac{(N-r)\,(N-r-1)}{(N-1)\,(N-2)}$$

104. A wins the series in $(n + r + 1)$ games (say). He wins the $(n + r + 1)$ th game and

n out of the first $(n + r)$ games.

$$\therefore \quad P(A) = \sum_{r=0}^{n} ({}^{n+r}C_n)\, q^r p^{n+1} \quad [\text{where } p + q = 1]$$

Similarly, $P(B) = \sum_{r=0}^{n} ({}^{n+r}C_n)\, q^{n+1} p^r$

Now, $P(A) + P(B) = 1$

$$\therefore \quad \sum_{r=0}^{n} [q^r p^{n+1} + q^{n+1} p^r]\, {}^{n+r}C_n = 1$$

Now, put $p = q = \frac{1}{2}$

$$\therefore \quad \sum_{r=0}^{n} {}^{n+r}C_n \left[\frac{1}{2^{n+r+1}} + \frac{1}{2^{n+r+1}}\right] = 1$$

$$\Rightarrow \quad \sum_{r=0}^{n} ({}^{n+r}C_n)\, \frac{1}{2^{n+r}} = 1$$

105. Let A denotes the event that the target is hit when x shells are fired at point I.

Let $E_1(E_2)$ denote the event.

We have, $P(E_1) = \frac{8}{9}$, $P(E_2) = \frac{1}{9}$

$$\Rightarrow \quad P\left(\frac{A}{E_1}\right) = 1 - \left(\frac{1}{2}\right)^x \text{ and } P\left(\frac{A}{E_2}\right) = 1 - \left(\frac{1}{2}\right)^{21-x}$$

Now, $P(A) = \frac{8}{9}\left[1 - \left(\frac{1}{2}\right)^x\right] + \frac{1}{9}\left[1 - \left(\frac{1}{2}\right)^{21-x}\right]$

$$\Rightarrow \quad \frac{dP(A)}{dx} = \frac{8}{9}\left[\left(\frac{1}{2}\right)^x \log 2\right] + \frac{1}{9}\left[-\left(\frac{1}{2}\right)^{21-x} \log 2\right]$$

Now, we must have $\frac{dP(A)}{dx} = 0 \Rightarrow x = 12$, also $\frac{d^2P(A)}{dx^2} < 0$

Hence, $P(A)$ is maximum where $x = 12$.

106. The composition of the balls in the red box and in the green box; and the sum suggested in the problem may be one of the following:

Red box		Green box		Sum of Green in Red and Red in Green
Red	Green	Green	Red	
0	5	3	6	11
1	4	4	5	9
2	3	5	4	7
3	2	6	3	5
4	1	7	2	3
5	0	8	1	1

Of these the 2nd and the last correspond to the sum being NOT a prime number. Hence, the required probability

$$= \frac{{}^6C_1 \times {}^8C_4 \times {}^6C_5 \times {}^8C_0}{{}^{14}C_5} = \frac{420 + 6}{2002} = \frac{213}{1001}$$

107. Let E_i denote the event that out of the first k balls drawn, i balls are green. Let A denote the event that $(k + 1)$th ball drawn is also green. We have, now $P(E_i) = \frac{{}^aC_1 \times {}^bC_{k-i}}{{}^{a+b}C_k}$

Here, $0 \le i \le k$ and $P\left(\frac{A}{E_i}\right) = \frac{a - i}{a + b - k}$

Now, $P(A) = \sum_{j=0}^{k} \frac{{}^aC_j \times {}^bC_{k-j}}{{}^{a+b}C_k} \times \frac{a - j}{a + b - k}$

Also, $(1 + x)^{a-1}(1 + x)^b$

$$= [{}^{a-1}C_0 + {}^{a-1}C_1 x + \dots + {}^{a-1}C_{a-1} x^{a-1}] \times [{}^bC_0 + {}^bC_1 x + \dots + {}^bC_b x^b]$$

$$\Rightarrow \quad \sum_{j=0}^{n} ({}^{a-1}C_j)({}^bC_{k-j}) = \text{coefficient of } x^k$$

Hence, $P(A) = \frac{a}{a + b}$

108. Total number of outcomes $= 2 \times 2 \times 2 \dots 12$ times $= 4096$

Let a_n denote the number of outcomes in which two consecutive heads do not occur when the fair coin is tossed n times.

$\Rightarrow \quad a_1 = 2, a_2 = 3$

For $n \ge 3$, if the last outcome is T, then we cannot have two consecutive heads in the first $(n - 1)$ tosses. This can happen in a_{n-1} ways. If the last outcome is H, we must have T the $(n - 1)$th toss and we cannot have two consecutive heads in the first $(n - 2)$ tosses. This can happen in a_{n-2} ways.

$\Rightarrow \quad a_n = a_{n-1} + a_{n-2}$ for $n \ge 3$

$\Rightarrow \quad a_{10} = 144, a_{11} = 233$

$\Rightarrow \quad a_{12} = 377$

Hence, the required probability is $\frac{377}{4096}$.

109. Let PQ be a diameter of a circle with centre O and radius a. Take a point A at random in PQ.

Let, $AP = x$, $AQ = y$, then $x + y = 2a$ and all values of x between 0 and $2a$ are equally likely.

Draw the ordinate AB, then $AB^2 = AP$, $AQ = xy$

If P', Q' are the mid-points of OP, OQ, the ordinates at these points are equal to $a\sqrt{\frac{3}{4}}$.

Hence, $AB > a\sqrt{\frac{3}{4}}$ if and only if, A lies in $P'Q'$.

Hence, the chance that $xy > \frac{3}{4}a^2$ is $\frac{A'B'}{AB}$ i.e., $\frac{1}{2}$.

110. (i) Kamsky wins one of the first n games and draws the remaining [$(n - 1)$ or

(ii) Kamsky wins exactly one of the first n games and draws the remaining] $n - 2$. We have,

$$P(i) = {}^nP_1 pq^{n-1}$$

and $$P(ii) = {}^nP_2\, pq^{n-2} r$$

$\Rightarrow$ The probability that Kamsky wins this match is

$$\sum_{n=1}^{\infty} p^2[nq^{n-1} + n(n-1)\, rq^{n-2}]$$

$$= p^2 \sum_{n=1}^{\infty} nq^{n-1} + p^2 r \sum_{n=1}^{\infty} n(n-1)\, q^{n-2}$$

Differentiating both sides w.r.t. q, we get

$$\sum_{n=1}^{\infty} nq^{n-1} = \frac{1}{(1-q)^2} \text{ and } \sum_{n=1}^{\infty} n(n-1)\, q^{n-2} = \frac{2}{(1-q)^3}$$

Thus, the probability that Kamsky wins the match is

$$\frac{p^2}{(1-q)^2} + \frac{2p^2 r}{(1-q)^3} = \frac{p^2(p+3r)}{(p+r)^3}$$

because $p + q + r = 1$.

111. Let A, B and C be three independent events having probabilities p, q and r, respectively.

Then, according to the question, we have

P (only the first occurs) $= P(A \cap \overline{B} \cap \overline{C})$

[$\because$ A, B, C are independent]

$$= P(A) \cdot P(\overline{B}) \cdot P(\overline{C})$$

$$= p(1-q)(1-r) = a \quad \text{...(i)}$$

P (only the second occurs) $= P(\overline{A} \cap B \cap \overline{C})$

$$= P(\overline{A}) \cdot P(B) \cdot P(\overline{C})$$

$$= (1-p)\, q(1-r) = b \quad \text{...(ii)}$$

and P (only the third occurs) $= P(\overline{A} \cap \overline{B} \cap C)$

$$= P(\overline{A}) \cdot P(\overline{B}) \cdot P(C)$$

$$= (1-p)(1-q)\, r = c \quad \text{...(iii)}$$

Multiplying Eqs. (i), (ii) and (iii), then

$$pqr\, \{(1-p)(1-q)(1-r)\}^2 = abc$$

or $$\frac{abc}{pqr} = [(1-p)(1-q)(1-r)]^2 = x^2 \quad \text{[say] ...(iv)}$$

$$(1-p)(1-q)(1-r) = x \quad \text{...(v)}$$

Dividing Eq. (i) by Eq. (v), then

$$\frac{p}{1-p} = \frac{a}{x} \text{ or } px = a - ap$$

$\therefore$ $$p = \frac{a}{(a+x)}$$

Similarly, $q = \dfrac{b}{b+x}$ and $r = \dfrac{c}{c+x}$

Replacing the values of p, q and r in Eq. (iv), then

$$\left\{\left(1 - \frac{a}{a+x}\right)\left(1 - \frac{b}{b+x}\right)\left(1 - \frac{c}{c+x}\right)\right\}^2 = x^2$$

$$\Rightarrow \quad \frac{(x^3)^2}{(a+x)^2 (b+x)^2 (c+x)^2} = x^2$$

$$\Rightarrow \quad \frac{x^3}{(a+x)(b+x)(c+x)} = x$$

or $$(a+x)(b+x)(c+x) = x^2$$

Hence, x is a root of the equation $(a+x)(b+x)(c+x) = x^2$

112. Let $A = \{a_1, a_2, \ldots, a_n\}$

For each $a_i \in A\ (1 \le i \le n)$, we have the following choices.

(i) $a_i \in P$ and $a_i \in Q$ (ii) $a_i \in P$ and $a_i \notin Q$

(iii) $a_i \notin P$ and $a_i \in Q$ (iv) $a_i \notin P$ and $a_i \notin Q$

Therefore, for one element a_i of A, total number of cases is 4.

Let S be the sample space

$\therefore$ $$n(S) = 4^n$$

and number of cases in which $a_i \in P \cup Q$ is 3, since case $4 \notin P \cup Q$ and let E be the event of favourable cases, then

$n(E)$ = number of ways in which exactly r elements of A will belong to $P \cup Q$

$$= {}^nC_r (3)^r\, 1^{n-r} = {}^nC_r 3^r$$

$\therefore$ Required probability, $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{{}^nC_r 3^r}{4^n}$

113. Given that $P(A) = \alpha$, $P(B / A) = P(B'/A') = 1 - \alpha$

Thus, $P(A') = 1 - P(A) = 1 - \alpha$

and $P(B / A') = 1 - P(B'/A') = 1 - (1-\alpha) = \alpha$ ...(i)

$$\therefore \quad P(A'/B) = \frac{P(A' \cap B)}{P(B)}$$

$$= \frac{P(B) - P(A \cap B)}{P(B)} = \frac{P(B) - P(A) \cdot P(B / A)}{P(B)}$$

$$\left[\because P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}\right]$$

$$= \frac{P(B) - \alpha(1-\alpha)}{P(B)} \quad \text{...(ii)}$$

But $P(B) = P(A) \cdot P(B / A) + P(A') \cdot P(B / A')$

$= \alpha(1-\alpha) + (1-\alpha) \cdot \alpha$ [from Eq. (i)]

$= 2\alpha(1-\alpha)$ ...(3)

Putting the value of $P(B)$ from Eq. (iii) in Eq. (ii), then

$$P\left(\frac{A'}{B}\right) = \frac{2\alpha(1-\alpha) - \alpha(1-\alpha)}{2\alpha(1-\alpha)} = \frac{\alpha(1-\alpha)}{2\alpha(1-\alpha)} = \frac{1}{2}$$

which is independent of α.

114. Let S be the sample space and E be the event that each of the n pairs of balls drawn consists of one white and one red ball.

$\therefore$ $n(S) = ({}^{2n}C_2)({}^{2n-2}C_2)({}^{2n-4}C_2) \ldots ({}^4C_2)({}^2C_2)$

$$= \left\{\frac{(2n)(2n-1)}{1 \cdot 2}\right\}\left\{\frac{(2n-2)(2n-3)}{1 \cdot 2}\right\}\left\{\frac{(2n-4)(2n-5)}{1 \cdot 2}\right\} \ldots \left\{\frac{4 \cdot 3}{1 \cdot 2}\right\}\left\{\frac{2 \cdot 1}{1 \cdot 2}\right\}$$

$$= \frac{1 \cdot 2 \cdot 3 \cdot 4 \ldots (2n-1)\, 2n}{2^n} = \frac{2n!}{2^n}$$

and $n(E) = ({}^nC_1 \cdot {}^nC_1)({}^{n-1}C_1 \cdot {}^{n-1}C_1)({}^{n-2}C_1 \cdot {}^{n-2}C_1) \ldots ({}^2C_1 \cdot {}^2C_1)({}^1C_1 \cdot {}^1C_1)$

$$= n^2 \cdot (n-1)^2 \cdot (n-2)^2 \ldots 2^2 \cdot 1^2 = [1 \cdot 2 \cdot 3 \ldots (n-1)\, n]^2 = (n!)^2$$

$\therefore$ Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{(n!)^2}{(2n)!/2^n} = \frac{2^n}{\dfrac{2n!}{(n!)^2}} = \frac{2^n}{{}^{2n}C_n}$$

115. Let p be the probability that any one thing is received by a men and q be the probability that any one thing is received by a women.

$\therefore$ $$p = \frac{a}{a+b} \text{ and } q = \frac{b}{a+b}$$

Clearly, $p + q = 1$ i.e., $q = 1 - p$

Out of m things, if r are received by a men, then the rest $(m - r)$ will be received by women.

The probability for this to happen is given by

$$P(r) = {}^mC_r p^r q^{m-r} \quad [r = 0, 1, \ldots, m]$$

The probability P that odd number of things are received by men is given by

$$P = P(1) + P(3) + P(5) + \ldots$$

$$= {}^mC_1 pq^{m-1} + {}^mC_3 p^3 q^{m-3} + {}^mC_5 p^5 q^{m-5} + \ldots \quad \text{...(i)}$$

We know that,

$$(q + p)^m = q^m + {}^mC_1 q^{m-1} p + {}^mC_2 q^{m-2} p^2 + \ldots + p^m \quad \text{...(ii)}$$

and $(q - p)^m = q^m - {}^mC_1 q^{m-1} p + {}^mC_2 q^{m-2} p^2 - \ldots + (-1)^m p^m$ ...(iii)

Subtracting Eq.(iii) from Eq. (ii), then

$$(q + p)^m - (q - p)^m = 2\{{}^mC_1 q^{m-1} p + {}^mC_3 q^{m-3} p^3 + \ldots\}$$

$$= 2P \quad \text{[from Eq. (i)]}$$

$$\therefore \quad P = \frac{1}{2}\{(q + p)^m - (q - p)^m\}$$

$$= \frac{1}{2}\left\{1 - \left(\frac{b - a}{b + a}\right)^m\right\} = \frac{1}{2}\left\{\frac{(b + a)^m - (b - a)^m}{(b + a)^m}\right\}$$

116. $\because P(C) = \frac{1}{7}, P(S) = \frac{3}{7}, P(B) = \frac{2}{7}, P(T) = \frac{1}{7}$

Let E be the event that person reaches late

$$\therefore P\left(\frac{E}{C}\right) = \frac{2}{9}, P\left(\frac{E}{S}\right) = \frac{1}{9}, P\left(\frac{E}{B}\right) = \frac{4}{9}, P\left(\frac{E}{T}\right) = \frac{1}{9}$$

To find $P\left(\frac{C}{\bar{E}}\right)$ [$\because$ reaches in time = not late]

Using Baye's Theorem

$$P\left(\frac{C}{\bar{E}}\right) = \frac{P(C).P\left(\frac{\bar{E}}{C}\right)}{P(C).P\left(\frac{\bar{E}}{C}\right) + P(S).P\left(\frac{\bar{E}}{S}\right) + P(B).P\left(\frac{\bar{E}}{B}\right) + P(T).P\left(\frac{\bar{E}}{T}\right)}$$

$$= \frac{\frac{1}{7} \times \left(1 - \frac{2}{9}\right)}{\frac{1}{7} \times \left(1 - \frac{2}{9}\right) + \frac{3}{7} \times \left(1 - \frac{1}{9}\right) + \frac{2}{7} \times \left(1 - \frac{4}{9}\right) + \frac{1}{7} \times \left(1 - \frac{1}{9}\right)}$$

$$= \frac{7}{7 + 3 \times 8 + 2 \times 5 + 8} = \frac{7}{49} = \frac{1}{7}$$

117. Probability of getting 1 is $\frac{1}{6}$ and probability of not getting 1 is $\frac{5}{6}$.

Then, getting 1 in even number of chances = getting 1 in 2nd chance or in 4th chance or in 6th chance and so on.

$\therefore$ Required probability

$$= \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^3 \times \frac{1}{6} + \left(\frac{5}{6}\right)^6 \times \frac{1}{6} + \ldots \infty$$

$$= \frac{\frac{5}{36}}{1 - \frac{25}{36}} = \frac{5}{11}$$

118. $P(\overline{A \cup B}) = \frac{1}{6}; P(A \cap B) = \frac{1}{4},$

$$P(\bar{A}) = \frac{1}{4} \Rightarrow P(A) = \frac{3}{4},$$

$$P(\overline{A \cup B}) = 1 - P(A \cup B)$$

$$= 1 - P(A) - P(B) + P(A \cap B)$$

$$\Rightarrow \quad \frac{1}{6} = \frac{1}{4} - P(B) + \frac{1}{4}$$

$$\Rightarrow \quad P(B) = \frac{1}{3}$$

Since, $P(A \cap B) = P(A) \cdot P(B)$ and $P(A) \neq P(B)$

$\therefore$ A and B are independent but not equally likely.

119. For a particular house being selected, probability $= \frac{1}{3}$

Probability (all the persons apply for the same house)

$$= \left(\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}\right) 3 = \frac{1}{9}$$

120. $P(X > 1.5) = 1 - P(X = 0) - P(X = 1)$

$$P(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}$$

$$\therefore \quad P(x > 1.5) = 1 - \frac{1}{e^2} - \frac{2}{e^2} = 1 - \frac{3}{e^2} \quad [\because \lambda = np = 2]$$

121. (i) $P(u_i) = ki, \sum P(u_i) = 1 \Rightarrow k = \frac{2}{n(n + 1)}$

$$\lim_{n \to \infty} P(w) = \lim_{n \to \infty} \sum_{i = 1}^{\infty} \frac{2i^2}{n(n + 1)^2}$$

$$= \lim_{n \to \infty} \frac{2n(n + 1)(2n + 1)}{n(n + 1)^2 6} = \frac{2}{3}$$

(ii) $$P\left(\frac{u_n}{w}\right) = \frac{P(u_n \cap w)}{P(w)} = \frac{P(u_n) \cdot P\left(\frac{w}{u_n}\right)}{\sum_{i = 1}^{n} P(u_n) \cdot P\left(\frac{w}{u_i}\right)}$$

$$\frac{c\frac{n}{n + 1}}{c\sum_{i = 1}^{n} \frac{i}{(n + 1)}} = \frac{n.2}{n(n + 1)} = \frac{2}{n + 1}$$

(iii) $$E = u_2 \cup u_4 \cup u_6 \cup \ldots \cup u_n$$

$$P(E) = P(u_2) + P(u_4) + \ldots + P(u_n)$$

$$= \frac{1}{n} + \frac{1}{n} + \ldots + \frac{1}{n} = \frac{1}{n}\frac{n}{2} = \frac{1}{2}$$

$$P\left(\frac{w}{E}\right) = \frac{P(w \cap E)}{P(E)}$$

$$= \frac{P(w \cap u_2) + P(w \cap u_4) + \ldots + P(w \cap u_n)}{\frac{1}{2}}$$

$$= 2\left[\frac{1}{n}.\frac{2}{n + 1} + \frac{1}{n} \cdot \frac{4}{n + 1} + \ldots + \frac{1}{n}.\frac{n}{n + 1}\right]$$

$$= \frac{2}{n}.\frac{\frac{n}{4}(2 + n)}{n + 1} = \frac{n + 2}{2(n + 1)}$$

122. $P(X = r) = \dfrac{e^{-m}m^r}{r!}$

$= P(X \le 1) = P(X = 0) + P(X = 1)$

$= e^{-5} + 5 \times e^{-5} = \dfrac{6}{e^5}$ [$\because m = \text{mean} = 5$]

123. Let E be the event when each American man is seated adjacent to his wife and A be the event when Indian man is seated adjacent to his wife.

Now, $n(A \cap E) = (4!) \times (2!)^5$ and $n(E) = (5!) \times (2!)^4$

$\Rightarrow P\left(\dfrac{A}{E}\right) = \dfrac{P(A \cap E)}{P(E)} = \dfrac{n(A \cap E)}{n(E)} = \dfrac{(4!) \times (2!)^5}{(5!) \times (2!)^4} = \dfrac{2}{5}$

124. **Statement-1** If $P(H_i \cap E) = 0$ for some i, then

$$P\left(\frac{H_i}{E}\right) = P\left(\frac{E}{H_i}\right) = 0$$

If $P(H_i \cap E) \ne 0$, $\forall\ i = 1, 2, 3, \ldots, n$, then

$P\left(\dfrac{H_i}{E}\right) = \dfrac{P(H_i \cap E)}{P(E)} = \dfrac{P(H_i \cap E)}{P(H_i)} \times \dfrac{P(H_i)}{P(E)}$

$= P\left(\dfrac{E}{H_i}\right) \times \dfrac{P(H_i)}{P(E)} > P\left(\dfrac{E}{H_i}\right).P(H_i)$ [as $0 < P(E) < 1$]

Hence, Statement-1 may not always be true.

Statement -2 Clearly, $H_1 \cup H_2 \cup \ldots \cup H_n = S$ (Sample space)

$\Rightarrow P(H_1) + P(H_2) + \ldots + P(H_n) = 1$

125. $P\left(\dfrac{E^c \cap F^c}{G}\right) = \dfrac{P(E^c \cap F^c \cap G)}{P(G)}$

$= \dfrac{P(G) - P(E \cap G) - P(F \cap G)}{P(G)}$

$= \dfrac{P(G) - P(E)\cdot P(G) - P(F)\cdot P(G)}{P(G)}$

$= 1 - P(E) - P(F) = P(E^c) - P(F)$

126. Probability of getting sum of nine in a single thrown $= \dfrac{1}{9}$

$\therefore$ Probability of getting sum nine exactly two times out of three draws $= {}^3C_2\left(\dfrac{1}{9}\right)^2\left(\dfrac{8}{9}\right) = \dfrac{8}{243}$

127. $P(I) = 0.3$, $P(II) = 0.2$

$\therefore$ Required probability

$= P(\bar{I})P(II) = (1 - P(I))P(II) = (1 - 0.3) \times 0.2 = 0.7 \times 0.2 = 0.14$

128. Since, $P(A) = \dfrac{2}{5}$

For independent events,

$P(A \cap B) = P(A)P(B) \Rightarrow P(A \cap B) \le \dfrac{2}{5}$

$\Rightarrow P(A \cap B) = \dfrac{1}{10}, \dfrac{2}{10}, \dfrac{3}{10}, \dfrac{4}{10}$

[maximum 4 outcomes may be in $A \cap B$]

(i) Now, $P(A \cap B) = \dfrac{1}{10}$

$\Rightarrow P(A)\cdot P(B) = \dfrac{1}{10}$

$\Rightarrow P(B) = \dfrac{1}{10} \times \dfrac{5}{2} = \dfrac{1}{4}$, not possible

(ii) Now, $P(A \cap B) = \dfrac{2}{10} \Rightarrow \dfrac{2}{5} \times P(B) = \dfrac{2}{10}$

$\Rightarrow P(B) = \dfrac{5}{10}$, outcomes of $B = 5$

(iii) Now, $P(A \cap B) = \dfrac{3}{10}$

$\Rightarrow P(A)P(B) = \dfrac{3}{10} \Rightarrow \dfrac{2}{5} \times P(B) = \dfrac{3}{10}$

$P(B) = \dfrac{3}{4}$, not possible

(iv) Now, $P(A \cap B) = \dfrac{4}{10} \Rightarrow P(A)\cdot P(B) = \dfrac{4}{10}$

$\Rightarrow P(B) = 1$, outcomes of $B = 10$.

129. For unique solution $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \ne 0$,

where $a, b, c, d \in \{0, 1\}$. Total cases = 16

Favourable cases = 6 (Either $ad = 1, bc = 0$ or $ad = 0, bc = 1$)

Probability that system of equations has unique solution is $\dfrac{6}{16} = \dfrac{3}{8}$ and system of equations has either unique solution or infinite solutions, so that probability for system to have a solution is 1.

130. $A = \{4, 5, 6\}$, $B = \{1, 2, 3, 4\}$

$\therefore A \cup B = \{1, 2, 3, 4, 5, 6\}$

$\Rightarrow n(A \cup B) = 6$ and total ways = 6

$\therefore P(A \cup B) = \dfrac{6}{6} = 1$

131. $\because P\left(\dfrac{A}{B}\right) = \dfrac{1}{2} \Rightarrow \dfrac{P(A \cap B)}{P(B)} = \dfrac{1}{2}$ ...(i)

and $P\left(\dfrac{B}{A}\right) = \dfrac{2}{3} \Rightarrow \dfrac{P(A \cap B)}{P(A)} = \dfrac{2}{3}$ ...(ii)

On dividing Eq. (i) by Eq. (ii), we get

$\dfrac{P(A)}{P(B)} = \dfrac{3}{4} \Rightarrow P(B) = \dfrac{4}{3}P(A)$

$= \dfrac{4}{3} \times \dfrac{1}{4} = \dfrac{1}{3}$ $\left[\because P(A) = \dfrac{1}{4}\right]$

132. $P(X = 3) = \left(\dfrac{5}{6}\right)\left(\dfrac{5}{6}\right)\dfrac{1}{6} = \dfrac{25}{216}$

133. $P(X \ge 3) = 1 - P(X \le 2) = 1 - \left\{\dfrac{1}{6} + \dfrac{5}{6} \times \dfrac{1}{6}\right\} = 1 - \dfrac{11}{36} = \dfrac{25}{36}$

134. $P(X \ge 6) = \dfrac{5^5}{6^6} + \dfrac{5^6}{6^7} + \ldots \infty = \dfrac{\dfrac{5^5}{6^6}}{1 - \dfrac{5}{6}} = \left(\dfrac{5}{6}\right)^5$

and $P(X > 3) = \dfrac{5^3}{6^4} + \dfrac{5^4}{6^5} + \dfrac{5^5}{6^6} + \ldots \infty = \dfrac{\dfrac{5^3}{6^4}}{1 - \dfrac{5}{6}} = \left(\dfrac{5}{6}\right)^3$

Hence, the conditional probability $= \dfrac{\left(\dfrac{5}{6}\right)^5}{\left(\dfrac{5}{6}\right)^3} = \dfrac{25}{36}$

135. $1 - q^n \geq \frac{9}{10} \Rightarrow q^n \leq \frac{1}{10} \Rightarrow \left(\frac{3}{4}\right)^n \leq \frac{1}{10}$

$\Rightarrow n(\log_{10} 3 - \log_{10} 4) \leq 0 - 1 \Rightarrow n \geq \frac{1}{(\log_{10} 4 - \log_{10} 3)}$

136. $S = \{00, 01, 02, ..., 49\}$

Let A be the event that sum of the digits on the selected ticket is 8, then $A = \{08, 17, 26, 26, 35, 44\}$

and let B be the event that the product of the digits is zero, then

$B = \{00, 01, 02, ..., 09, 10, 20, 30, 40\}$

$\therefore \quad A \cap B = \{08\}$

$\therefore$ Required probability $= P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{50}}{\frac{14}{50}} = \frac{1}{14}$

137. Required probability $= \frac{2 \times 2 \times 2(3!)}{6^3} = \frac{2}{9}$

138. Let E_1 denote original signal is green, E_2 denote original signal is red and E denote signal received at station B is green.

$$\therefore \quad P\left(\frac{E_1}{E}\right) = \frac{P(E_1) \cdot P\left(\frac{E}{E_1}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)}$$

$$= \frac{\frac{4}{5}\left[\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right]}{\frac{4}{5}\left[\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right] + \frac{1}{5}\left[\frac{3}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{3}{4}\right]} = \frac{40}{46} = \frac{20}{23}$$

139. Total ways $= {}^{20}C_4 = \frac{20 \cdot 19 \cdot 18 \cdot 17}{1 \cdot 2 \cdot 3 \cdot 4} = 4845$

Statement-1

Common difference (d)	Number of cases
1	17
2	14
3	11
4	8
5	5
6	2

$\therefore$ Number of favourable cases $= 17 + 14 + 11 + 8 + 5 + 2$
$= 57$

$\therefore$ Required probability $= \frac{57}{4845} = \frac{1}{85}$

Statement-1 is true and Statement-2 is false.

140. Total ways, ${}^9C_3 = \frac{9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3} = 84$

Favourable ways $= {}^3C_1 \times {}^4C_1 \times {}^2C_1 = 24$

$\therefore$ Required probability $= \frac{24}{84} = \frac{2}{7}$

141. $H \to 1$ ball from U_1 to U_2

$T \to 2$ balls from U_1 to U_2

E : 1 ball drawn from U_2

$$\therefore P(W \text{ from } U_2) = \frac{1}{2} \times \left(\frac{3}{5} \times 1\right) + \frac{1}{2} \times \left(\frac{2}{5} \times \frac{1}{2}\right)$$
$$+ \frac{1}{2} \times \left(\frac{{}^3C_2}{{}^5C_2} \times \frac{1}{3}\right) + \frac{1}{2} \times \left(\frac{{}^3C_1 \cdot {}^2C_1}{{}^5C_2} \times \frac{2}{3}\right) = \frac{23}{30}$$

142. $$P\left(\frac{H}{W}\right) = \frac{P(H) \times P\left(\frac{W}{H}\right)}{P(H) \times P\left(\frac{W}{H}\right) + P(T) \times P\left(\frac{W}{T}\right)} = \frac{\frac{1}{2}\left(\frac{3}{2} \times 1 + \frac{2}{5} \times \frac{1}{2}\right)}{\frac{23}{30}}$$
$$= \frac{12}{23}$$

143. Let $P(E) = e$ and $P(F) = f$

$\Rightarrow \quad P(E \cup F) - P(E \cap F) = \frac{11}{25}$

$\Rightarrow \quad P(E) + P(F) - 2P(E \cap F) = \frac{11}{25}$

$\Rightarrow \quad e + f - 2ef = \frac{11}{25}$ ...(i)

$P(\overline{E} \cap \overline{F}) = \frac{2}{25} \Rightarrow P(\overline{E}).\ P(\overline{F}) = \frac{2}{25} \Rightarrow (1-e)(1-f) = \frac{2}{25}$

$\Rightarrow \quad e + f - ef = \frac{23}{25}$ ...(ii)

From Eqs. (i) and (ii), we get $ef = \frac{12}{25}$ and $e + f = \frac{7}{5}$

On solving, we get $e = \frac{4}{5}, f = \frac{3}{5}$ or $e = \frac{3}{5}, f = \frac{4}{5}$

144. Given probability of atleast one failure $\geq \frac{31}{32}$

$\Rightarrow \quad 1 - P(X = 0) \geq \frac{31}{32}$

$\Rightarrow \quad 1 - {}^5C_0 \cdot Q^0 \cdot P^5 \geq \frac{31}{32}$ $\quad [\because (P + Q)^5]$

$\Rightarrow \quad P^5 \leq \frac{1}{32}$

$\therefore \quad P \leq \frac{1}{2}$ and $P \geq 0 \Rightarrow P \in \left[0, \frac{1}{2}\right]$

145. We have, $C \subset D \Rightarrow C \cap D = C$

$\Rightarrow P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)} \geq P(C)$ $\quad [\because 0 < P(D) \leq 1]$

146. $$P\left(\frac{A^c \cap B^c}{C}\right) = \frac{P(A^c \cap B^c \cap C)}{P(C)}$$
$$= \frac{P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}{P(C)}$$
$$= \frac{P(C) - P(A) \cdot P(C) - P(B) \cdot P(C) + 0}{P(C)}$$

$[\because A, B, C$ are pairwise independent$]$

$= 1 - P(A) - P(B) = P(A^c) - P(B)$

147. $P(X) = P(X_1 \cap X_2 \cap X_3) + P(\overline{X}_1 \cap X_2 \cap X_3)$
$+ P(X_1 \cap \overline{X}_2 \cap X_3) + P(X_1 \cap X_2 \cap \overline{X}_3)$

$$= \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4} = \frac{1}{4}$$

(a) $P\left(\frac{\overline{X}_1}{X}\right) = \frac{\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}}{\frac{1}{4}} = \frac{1}{8}$

(b) P (exactly two engines of the ship are functioning / X)

$$= \frac{\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{4} + \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4}}{1/4} = \frac{7}{8}$$

(c) $P\left(\frac{X}{X_2}\right)$ = Probability that X occurs given that engine E_2 has started

$$\frac{P(X_1 \cap X_2 \cap X_3) + P(\overline{X}_1 \cap X_2 \cap X_3) + P(X_1 \cap X_2 \cap \overline{X}_3)}{P(X_1 \cap X_2 \cap X_3) + P(\overline{X}_1 \cap X_2 \cap X_3) + P(X_1 \cap X_2 \cap \overline{X}_3) + P(\overline{X}_1 \cap X_2 \cap \overline{X}_3)} = \frac{5}{8}$$

(d) $P\left(\frac{X}{X_1}\right)$ = Probability that X occurs given that engine E_1 has started

$$\frac{P(X_1 \cap X_2 \cap X_3)}{P(X_1 \cap X_2 \cap X_3) + P(X_1 \cap \overline{X}_2 \cap X_3) + P(X_1 \cap \overline{X}_2 \cap X_3) + P(X_1 \cap X_2 \cap \overline{X}_3) + P(X_1 \cap X_2 \cap \overline{X}_3) + P(X_1 \cap \overline{X}_2 \cap \overline{X}_3)} = \frac{7}{16}$$

148. ***Case* I** When D_1, D_2, D_3 all show different number and one of the number is shown by D_4, $P(E_1) = \frac{{}^6C_3 \times 3!}{216} \times \frac{3}{6} = \frac{60}{216}$

***Case* II** When D_1, D_2, D_3 all show same number and that number is shown by D_4, $P(E_2) = 6 \times \left(\frac{1}{6}\right)^4 = \frac{1}{216}$

***Case* III** When two numbers shown by D_1, D_2, D_3 are same and one is different and one of the number is shown by D_4,

$$P(E_3) = \frac{{}^6C_1 \times {}^5C_1}{216} \times \frac{3!}{2!} \times \frac{2}{6} = \frac{30}{216}$$

$\therefore$ Required probability $= P(E_1) + P(E_2) + P(E_3) = \frac{91}{216}$

149. $P\left(\frac{X}{Y}\right) = \frac{P(X \cap Y)}{P(Y)} = \frac{1}{2} \Rightarrow P(Y) = \frac{1}{3}$

$P\left(\frac{Y}{X}\right) = \frac{P(X \cap Y)}{P(X)} = \frac{1}{3} \Rightarrow P(X) = \frac{1}{2}$

(a) $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = \frac{2}{3}$

(b) $\because P(X \cap Y) = P(X) \cdot P(Y)$, they are independent. Also, X^c and Y will be independent

Now, $P(X^c \cap Y) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6} \neq \frac{1}{3}$.

150. $\because S = \{1, 2, 3, \ldots, 8\}$

Let A : Maximum of three numbers is 6

$\therefore A = \{1, 2, 3, 4, 5, 6\}$

and B : Maximum of three numbers is 3

$\therefore B = \{3, 4, 5, 6, 7, 8\}$ and $A \cap B = \{3, 4, 5, 6\}$

$$\Rightarrow P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{n(A \cap B)}{n(B)} = \frac{{}^2C_1}{{}^5C_2} = \frac{1}{5}$$

151. Here, $p = \frac{1}{3}$ and $q = 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore$ Required probability $= P(X = 4) + P(X = 5)$ $\left[\because \left(\frac{2}{3} + 1\right)^5\right]$

$$= {}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right) + {}^5C_5 \left(\frac{1}{3}\right)^5 = \frac{11}{3^5}$$

152. Probability of solving the problem correctly by atleast one of them $= 1 - \left(\left(1 - \frac{1}{2}\right)\left(1 - \frac{3}{4}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{8}\right)\right) = \frac{235}{256}$

153. Let x, y and z be the probability of occurrence of E_1, E_2 and E_3, respectively.

Then, $\alpha = x(1-y)(1-z) = \frac{px}{(1-x)}$ $[\because p = (1-x)(1-y)(1-z)]$

Similarly, $\beta = \frac{py}{(1-y)}$ and $\gamma = \frac{pz}{(1-z)}$

Now, $(\alpha - 2\beta)\,p = \alpha\beta$

$\Rightarrow \frac{p}{\beta} - \frac{2p}{\alpha} = 1$

$\Rightarrow \frac{(1-y)}{y} - \frac{2(1-x)}{x} = 1$

$\Rightarrow x = 2y$ ...(i)

and $(\beta - 3\gamma)\,p = 2\beta$

$\Rightarrow \frac{p}{\gamma} - \frac{3p}{\beta} = 2$

$\Rightarrow \frac{(1-z)}{z} - \frac{3(1-y)}{y} = 2$

$\Rightarrow y = 3z$ ...(ii)

From Eqs. (i) and (ii), we get $x = 6z$ $\therefore \frac{x}{z} = 6$

154. Let E = Event that one ball is white and the other ball is red.

$$\text{Then, } P\left(\frac{B_2}{E}\right) = \frac{\frac{{}^2C_1 \times {}^3C_1}{{}^9C_2}}{\frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} + \frac{{}^2C_1 \times {}^3C_1}{{}^9C_2} + \frac{{}^3C_1 \times {}^4C_1}{{}^{12}C_2}} = \frac{55}{181}$$

155. Required probability $= \prod_{i=1}^{3} P(W_i) + \prod_{i=1}^{3} P(R_i) + \prod_{i=1}^{3} P(B_i)$

$$= \frac{1}{6} \times \frac{2}{9} \times \frac{3}{12} + \frac{3}{6} \times \frac{3}{9} \times \frac{4}{12} + \frac{2}{6} \times \frac{4}{9} \times \frac{5}{12}$$

$$= \frac{82}{648}$$

156. $\because P(A \cup B) = 1 - P(\overline{A \cup B}) = 1 - \frac{1}{6} = \frac{5}{6}$

$$P(A) = 1 - P(\overline{A}) = 1 - \frac{1}{4} = \frac{3}{4}$$

and $P(B) = P(A \cup B) - P(A) + P(A \cap B)$

$$= \frac{5}{6} - \frac{3}{4} + \frac{1}{4} = \frac{1}{3}$$

$\Rightarrow A$ and B are not equally likely.

Also, $\quad P(A \cap B) = \frac{1}{4} = \frac{3}{4} \times \frac{1}{3} = P(A) \,.\, P(B)$

$\therefore$ A and B are independent.

Hence, A and B are independent but not equally likely.

157. $n(S) = 5! = 120$ and possible favourable cases are

$(B, G, G, B, B), (G, G, B, B, B), (G, B, G, B, B)$,
$(G, B, B, G, B), (B, G, B, G, B)$

$\therefore$ Number of favourable cases $= n(E) = 5 \times 12 = 60$

$\therefore$ Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{60}{120} = \frac{1}{2}$$

158. $n(S) = 3 \times 5 \times 7 = 105$; $x_1 + x_2 + x_3 =$ odd

***Case* I** All three odd $= 2 \times 3 \times 4 = 24$
***Case* II** Two even and one odd
$= 1 \times 2 \times 4 + 1 \times 3 \times 3 + 2 \times 2 \times 3 = 29 \therefore n(E) = 24 + 29 = 53$

Required probability, $\quad P(E) = \frac{n(E)}{n(S)} = \frac{53}{105}$

159. x_1, x_2, x_3 are in AP.

AP with common difference = 1, (1, 2, 3) (2, 3, 4) (3, 4, 5)
AP with common difference = 2, (1, 3, 5), (2, 4, 6), (3, 5, 7)
AP with common difference = 3, (1, 4, 7)
AP with common difference = 0, (1, 1, 1), (2, 2, 2) (3, 3, 3)

$\therefore \qquad n(E) = 10$

$\therefore$ Required probability, $P(E) = \frac{n(E)}{n(S)} = \frac{10}{105}$

160. We have mentioned that boxes are different and one particular box has 3 balls.

Then number of ways $= \frac{{}^{12}C_3 \times 2^9}{3^{12}} = \frac{55}{3}\left(\frac{2}{3}\right)^{11}$

161. Using Binomial distribution

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1)$$

$$= 1 - \left(\frac{1}{2}\right)^n - \left[{}^nC_1 \cdot \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right)^{n-1}\right]$$

$$= 1 - \frac{1}{2^n} - {}^nC_1 \cdot \frac{1}{2^n} = 1 - \left(\frac{1+n}{2^n}\right)$$

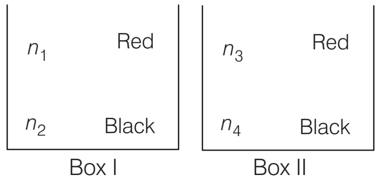
Given, $P(X \geq 2) \geq 0.96$

$$1 - \frac{(n+1)}{2^n} \geq \frac{24}{25}$$

$$\Rightarrow \qquad \frac{(n+1)}{2^n} \leq \frac{1}{25}$$

$$\Rightarrow \qquad n = 8$$

162.



Let $\qquad A$ = Drawing red ball

$\therefore \qquad P(A) = P(B_1) \cdot P(A/B_1) + P(B_2) \cdot P(A/B_2)$

$$= \frac{1}{2}\left(\frac{n_1}{n_1+n_2}\right) + \frac{1}{2}\left(\frac{n_3}{n_3+n_4}\right)$$

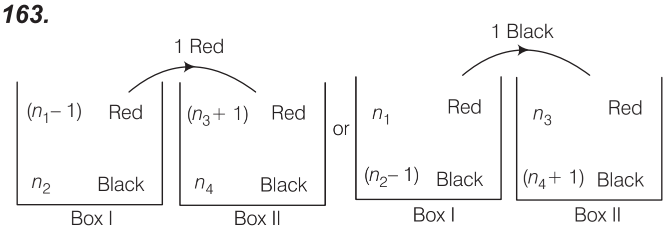
Given, $P(B_2/A) = \frac{1}{3} \Rightarrow \frac{P(B_2) \cdot P(B_2 \cap A)}{P(A)} = \frac{1}{3}$

$$\Rightarrow \qquad \frac{\frac{1}{2}\left(\frac{n_3}{n_3+n_4}\right)}{\frac{1}{2}\left(\frac{n_1}{n_1+n_2}\right) + \frac{1}{2}\left(\frac{n_3}{n_3+n_4}\right)} = \frac{1}{3}$$

$$\Rightarrow \qquad \frac{n_3(n_1+n_2)}{n_1(n_3+n_4) + n_3(n_1+n_2)} = \frac{1}{3}$$

Now, check options, then clearly options (a) and (b) satisfy.

163.



$\therefore$ P (drawing red ball from B_1) $= \frac{1}{3}$

$$\Rightarrow \left(\frac{n_1 - 1}{n_1 + n_2 - 1}\right)\left(\frac{n_1}{n_1 + n_2}\right) + \left(\frac{n_2}{n_1 + n_2}\right)\left(\frac{n_1}{n_1 + n_2 - 1}\right) = \frac{1}{3}$$

$$\Rightarrow \qquad \frac{n_1^2 + n_1 n_2 - n_1}{(n_1 + n_2)(n_1 + n_2 - 1)} = \frac{1}{3}$$

Clearly, options (c) and (d) satisfy.

164. $\because \qquad P(E_1) = \frac{1}{6}, P(E_2) = \frac{1}{6}$,

$$P(E_3) = \frac{2+4+6+4+2}{36} = \frac{1}{2}$$

Also, $\qquad P(E_1 \cap E_2) = \frac{1}{36}$,

$$P(E_1 \cap E_3) = \frac{1}{12}, P(E_3 \cap E_1) = \frac{1}{12}$$

and $\quad P(E_1 \cap E_2 \cap E_3) = 0 \neq P(E_1).P(E_2).P(E_3)$

Hence, E_1, E_2, E_3 are not independent

165. $\qquad P(T_1) = \frac{200}{100} = \frac{1}{5}, P(T_2) = \frac{80}{100} = \frac{4}{5}$,

$P(D) = \frac{7}{100}$. Let $P\left(\frac{D}{T_2}\right) = x$, then

$$P\left(\frac{D}{T_1}\right) = 10 \cdot P\left(\frac{D}{T_2}\right) = 10x$$

$\because$ $$P(D) = P(T_1) \times P\left(\frac{D}{T_1}\right) + P(T_2) \times P\left(\frac{D}{T_2}\right)$$

$\Rightarrow$ $$\frac{7}{100} = \frac{1}{5} \times 10x + \frac{4}{5} \times x$$

$\therefore$ $$x = \frac{1}{40}$$

$\therefore$ $$P\left(\frac{T_2}{\overline{D}}\right) = \frac{P(T_2) \times P\left(\frac{\overline{D}}{T_2}\right)}{P(T_1) \times P\left(\frac{\overline{D}}{T_1}\right) + P(T_2) \times P\left(\frac{\overline{D}}{T_2}\right)}$$

$$= \frac{P(T_2) \times P\left(\frac{\overline{D}}{T_2}\right)}{P(\overline{D})}$$

$$= \frac{\frac{4}{5} \times \frac{39}{40}}{\frac{93}{100}}$$

$$= \frac{78}{93}$$

166. $P(x > y) = P(T_1$ wins 2 games or T_1 wins either of the matches and other is draw)

$$= \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{2}\right)$$

$$= \frac{1}{4} + \frac{1}{6} = \frac{5}{12}$$

167. $P(x = y) = P(T_1$ and T_2 win alternately)

$+P$ (Both matches are draws)

$$= \left(\frac{1}{2} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{2}\right) + \left(\frac{1}{6} \times \frac{1}{6}\right)$$

$$= \frac{1}{3} + \frac{1}{36} = \frac{13}{36}$$

168. $P = \frac{15}{25} = \frac{3}{5}, Q = 1 - P = 1 - \frac{3}{5} = \frac{2}{5}$

and $n = 10$

$\therefore$ Variance $= nPQ = 10 \times \frac{3}{5} \times \frac{2}{5} = \frac{12}{5}$

169. **Cases**: (0, 4), (0, 8), (2, 6), (2, 10), (4, 8), (6, 10)

$\therefore$ Required probability $= \frac{6}{{}^{1}C_2} = \frac{6}{55}$

170. $\because$ $$P(A) + P(B) - 2P(A \cap B) = \frac{1}{4} \quad \text{...(i)}$$

$$P(B) + P(C) - 2P(B \cap C) = \frac{1}{4} \quad \text{...(ii)}$$

$$P(C) + P(A) - 2P(C \cap A) = \frac{1}{4} \quad \text{...(iii)}$$

and $$P(A \cap B \cap C) = \frac{1}{16} \quad \text{...(iv)}$$

Now, adding Eqs. (i), (ii) and (iii), then

$$P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) = \frac{3}{8} \quad \text{...(v)}$$

On adding Eqs. (iv) and (v), we get

$$P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C) = \frac{3}{8} + \frac{1}{16}$$

or $$P(A \cup B \cup C) = \frac{7}{16}$$