VERY SIMILAR PRACTICE TEST 9

Hints and Explanations

1. (c) : Work done by the gas = Area of ΔABC

$$= \frac{1}{2} \times (AB)(BC)$$

$$= \frac{1}{2} \times \frac{(450-200)}{10^6} \times (200-120) \times 1000$$

$$= \frac{(250)(80) \times 1000}{2 \times 10^6} = 10 \text{ J}$$

Work done by the cycle is taken to be negative if the cycle is anticlockwise. $\therefore$ $W = -10$ J

2. (d) : The electric field at a distance r from the line charge of linear density λ is given by

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Hence, the field at the negative charge,

$$E_1 = \frac{(4.0 \times 10^{-4})(2 \times 9 \times 10^9)}{0.02} = 3.6 \times 10^8 \text{ N C}^{-1}$$

The force on the negative charge,

$F_1 = (3.6 \times 10^8)(2.0 \times 10^{-8})$

= 7.2 N towards the line charge

Similarly, the field at the positive charge, *i.e.*, at $r = 0.022$ m is

$E_2 = 3.3 \times 10^8$ N C^{-1}.

The force on the positive charge,

$F_2 = (3.3 \times 10^8) \times (2.0 \times 10^{-8})$

= 6.6 N away from the line charge.

Hence, the net force on the dipole = 7.2 N – 6.6 N

= 0.6 N towards the line charge

3. (c) : Displacement = area of bigger triangle – area of smaller triangle + area of rectangle

$$= \left[\frac{1}{2}(3 \times 2) - \frac{1}{2}(1 \times 2) + (1 \times 2)\right] = 4 \text{ m}$$

4. (b) : Suppose the current is I in the indicated direction. Applying Kirchoff's loop law,

$\varepsilon_1 - Ir_1 + \varepsilon_2 - Ir_2 + \varepsilon_3 - Ir_3 + \ldots + \varepsilon_n - Ir_n = 0$

or, $$I = \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \ldots + \varepsilon_n}{r_1 + r_2 + r_3 + \ldots r_n}$$

$$= \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \ldots + \varepsilon_n}{k(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \ldots \varepsilon_n)} = \frac{1}{k}$$

The potential difference between the terminals of the i^{th} battery is

$$\varepsilon_i - Ir_i = \varepsilon_i - \left(\frac{1}{k}\right)(k\varepsilon_i) = 0$$

5. (b) : Given : $v = ax^{3/2}$ where, $a = 5$ m$^{-1/2}$ s^{-1}

$$\text{Acceleration} = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v\frac{dv}{dx} \quad \left(\because v = \frac{dx}{dt}\right)$$

As $v^2 = a^2x^3$

Differentiating both sides with respect to x, we get

$$2v\frac{dv}{dx} = 3a^2x^2 \text{ or, } \text{Acceleration} = \frac{3}{2}a^2x^2$$

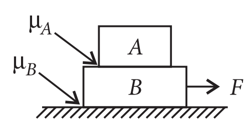
Force, F = Mass × Acceleration = $\frac{3}{2}ma^2x^2$

$$\text{Work done, } W = \int F dx = \int_0^2 \frac{3}{2}ma^2x^2 dx$$

$$W = \frac{3}{2}ma^2\left[\frac{x^3}{3}\right]_0^2 = \frac{3}{2} \times 0.5 \times 5^2 \times \frac{8}{3} = 50 \text{ J}$$

6. (d) : Here, $m_A = \frac{m}{2}$, $m_B = m$

$\mu_A = 0.2$, $\mu_B = 0.1$



Let both the blocks are moving with common acceleration a. Then,

$$a = \frac{\mu_A m_A g}{m_A} = \mu_A g = 0.2g$$

and $F - \mu_B(m_B + m_A)g = (m_B + m_A)a$

$F = (m_B + m_A)a + \mu_B(m_B + m_A)g$

$$= \left(m + \frac{m}{2}\right)(0.2g) + (0.1)\left(m + \frac{m}{2}\right)g$$

$$= \left(\frac{3}{2}m\right)(0.2g) + \left(\frac{3}{2}m\right)(0.1g) = \frac{0.9}{2}mg = 0.45mg$$

7. (b) : Given: $r = 0.1$ m, $N = 10$, $B_H = 0.314 \times 10^{-4}$ T

Magnetic field at the centre of current carrying circular coil

$$B = \frac{\mu_0 NI}{2r}$$

Since at neutral point, the magnetic field due to circular coil is completely cancelled by the horizontal component of earth's magnetic field. Therefore,

$$B = B_H \quad \text{or} \quad \frac{\mu_0 NI}{2r} = B_H$$

$$\therefore \quad I = \frac{2rB_H}{\mu_0 N} = \frac{2\times 0.1\times 0.314\times 10^{-4}}{(4\pi\times 10^{-7})\times 10} = 0.5 \text{ A}$$

8. **(c) :** Here, $l = 50$ cm $= 50 \times 10^{-2}$ m

$m = 50$ g $= 50 \times 10^{-3}$ kg; $I = 5.0$ A

Tension in the wires is zero if the force on the rod due to magnetic field is equal and opposite to the weight of the rod.

i.e., $mg = BIl \Rightarrow B = \frac{mg}{Il}$

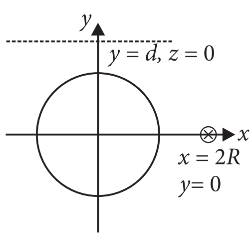
Substituting the given values, we get

$$B = \frac{50\times 10^{-3}\times 10}{5\times 50\times 10^{-2}} = 0.2 \text{ T}$$

9. **(b) :** An axis passing through $x = 2R$, $y = 0$ is in ⊗ direction as shown in figure. Moment of inertia about this axis will be

$$I_1 = \frac{1}{2}mR^2 + m(2R)^2 = \frac{9}{2}mR^2 \quad \text{..(i)}$$

Axis passing through $y = d$, $z = 0$ is shown by dotted line in figure. Moment of inertia about this axis will be



$$I_2 = \frac{1}{4}mR^2 + md^2 \quad \text{...(ii)}$$

By equations (i) and (ii), we get

$$\frac{1}{4}mR^2 + md^2 = \frac{9}{2}mR^2 \text{ or } d = \frac{\sqrt{17}}{2}R$$

10. **(a) :** Here, total length $l = 40$ cm $= 40 \times 10^{-2}$ m,

Resistivity $= 1.7 \times 10^{-8}\ \Omega$ m

The area A of the loop

$$= \left(\frac{40 \text{ cm}}{4}\right)\left(\frac{40 \text{ cm}}{4}\right) = 0.01 \text{ m}^2$$

If the magnetic field at an instant is B, the flux through the frame at that instant will be $\phi = BA$. As the area remains constant, the magnitude of the emf induced will be

$$\varepsilon = \frac{d\phi}{dt} = A\frac{dB}{dt}$$

$= (0.01 \text{ m}^2)\ (0.02 \text{ T s}^{-1}) = 2 \times 10^{-4}$ V

Resistance of the loop,

$$R = \frac{(1.7\times 10^{-8}\,\Omega\text{ m})(40\times 10^{-2}\text{ m})}{3.14\times 1\times 10^{-6}\text{ m}^2} = 2.16\times 10^{-3}\,\Omega$$

Hence, the current induced in the loop will be

$$I = \frac{2\times 10^{-4}\text{ V}}{2.16\times 10^{-3}\,\Omega} = 9.3\times 10^{-2}\text{ A} \approx 0.1 \text{ A}$$

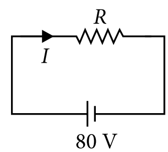
11. **(a) :** The intensity of a plane electromagnetic wave is

$$I = u_{av}\, c = \frac{1}{2}\varepsilon_0 E_0^2 c$$

$$\text{or} \quad E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}} = \sqrt{\frac{2\times 2.0}{8.85\times 10^{-12}\times 3\times 10^8}}$$

$= 38.8$ N C^{-1}

12. **(d) :** For a dc source

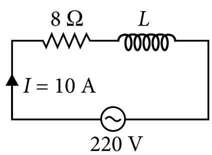


$I = 10$ A, $V = 80$ V

Resistance of the arc lamp,

$$R = \frac{V}{I} = \frac{80}{10} = 8\ \Omega$$

For an ac source,



$\varepsilon_{rms} = 220$ V

$\upsilon = 50$ Hz

$\omega = 2\pi \times 50 = 100\ \pi$ rad s^{-1}

Arc lamp will glow if $I = 10$ A,

$$\therefore \quad I = \frac{\varepsilon_{rms}}{\sqrt{R^2 + \omega^2 L^2}} \text{ or } R^2 + \omega^2 L^2 = \left(\frac{\varepsilon_{rms}}{I}\right)^2$$

$$\text{or } 8^2 + (100\ \pi)^2 L^2 = \left(\frac{220}{10}\right)^2 \text{ or } L^2 = \frac{22^2 - 8^2}{(100\ \pi)^2}$$

$$\therefore \quad L = \frac{\sqrt{30\times 14}}{100\ \pi} = 0.065 \text{ H}$$

13. **(b) :** Time period of revolution of satellite

$$T = 2\pi\sqrt{\frac{(R_e + h)^3}{GM_e}}$$

$$\text{or} \quad \frac{T^2}{4\pi^2} = \frac{(R_e + h)^3}{GM_e} \quad \text{...(i)}$$

Centripetal acceleration, $a = \frac{GM_e}{(R_e + h)^2}$

or $\frac{(R_e + h)^2}{GM_e} = \frac{1}{a}$ ...(ii)

Divide (i) by (ii), we get

$$(R_e + h) = \frac{T^2}{4\pi^2} \times a = \left(\frac{5.26 \times 10^3}{2\pi}\right)^2 \times 9.32$$

$R_e + h = 6.53 \times 10^6$ m

$h = 6.53 \times 10^6$ m – 6.37×10^6 m = 0.16×10^6 m

= 160×10^3 m = 160 km

14. (c) : Distance between the first minimum on the left and the first minimum on the right is also the width of central maximum.

Width of central maximum, $W = \frac{2\lambda D}{a}$

where, λ = Wavelength of light

a = Width of the slit

D = Distance of the screen from the slit

$\therefore \quad a = \frac{2\lambda D}{W}$

Here, λ = 6000 Å = 6000×10^{-10} m,

D = 80 cm = 80×10^{-2} m

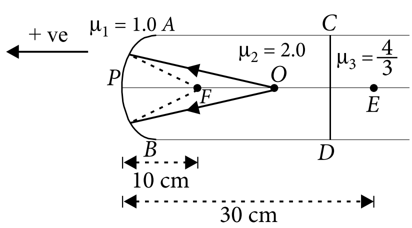
W = 5 mm = 5×10^{-3} m

$$\therefore \quad a = \frac{2 \times 6000 \times 10^{-10}\,\text{m} \times 80 \times 10^{-2}\,\text{m}}{5 \times 10^{-3}\,\text{m}}$$

$= 19.2 \times 10^{-5}\,\text{m} = 0.192 \times 10^{-3}\,\text{m} = 0.192\,\text{mm}$

15. (c) : When light travels from medium of refractive index μ_2 to medium of refractive index μ_1 at a single spherical surface, the formula used is

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$



Direction of light is in positive direction.

$$\therefore \quad \frac{1.0}{v} - \frac{2.0}{-15} = \frac{1.0 - 2.0}{-10}$$

F is centre of curvature of APB.

or $\frac{1}{v} = \frac{1}{10} - \frac{2}{15} = \frac{3-4}{30} = \frac{-1}{30}$ or v = – 30 cm

$\therefore$ The distance of the final image of O from P, as viewed from the left, is 30 cm to right of P. The image formed will be virtual at E.

16. (d) : Here, R_C = 500 Ω, $I_C R_C$ = 0.5 V,

V_{CC} = 5 V, α = 0.96

As $I_C R_C$ = 0.5 V

$$\therefore \quad I_C = \frac{0.5\,\text{V}}{R_C} = \frac{0.5\,\text{V}}{500\,\Omega} = 1 \times 10^{-3}\,\text{A} = 1\,\text{mA}$$

The current gains α and β are related as

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.96}{1-0.96} = 24$$

The base current is

$$I_B = \frac{I_C}{\beta} = \frac{1\,\text{mA}}{24} = \frac{1}{24}\,\text{mA}$$

17. (a) : According to Newton's law of cooling

$$\frac{T_1 - T_2}{t} = K\left(\frac{T_1 + T_2}{2} - T_s\right)$$

where T_s is the surrounding temperature.

For the first case,

T_1 = 91°C, T_2 = 79°C, T_s = 25°C, t = 2 min

$$\therefore \quad \frac{91°\text{C} - 79°\text{C}}{2\ \text{min}} = K\left(\frac{91°\text{C} + 79°\text{C}}{2} - 25°\text{C}\right)$$

or $\frac{12°\text{C}}{2\,\text{min}} = K(60°\text{C})$ ...(i)

For the second case,

T_1 = 91°C, T_2 = 79°C, T_s = 5°C, t = ?

$$\therefore \quad \frac{91°\text{C} - 79°\text{C}}{t} = K\left(\frac{91°\text{C} + 79°\text{C}}{2} - 5°\text{C}\right)$$

or $\frac{12°\text{C}}{t} = K(80°\text{C})$ ...(ii)

Dividing eqn. (i) by equation (ii), we get

$$\frac{t}{2\,\text{min}} = \frac{60°\text{C}}{80°\text{C}} \text{ or } t = \frac{3}{4}(2\,\text{min}) = \frac{3}{2}\,\text{min}$$

18. (c) : Least count of screw gauge

$$= \frac{\text{pitch}}{\text{no. of divisions on circular scale}}$$

$$= \frac{0.5\,\text{mm}}{50} = 0.01\,\text{mm.}$$

Diameter of ball, D = MSR + CSR × LC

= 2.5 mm + 20 × 0.01 mm = 2.7 mm

As density, $\rho = \frac{\text{mass}}{\text{volume}} = \frac{M}{\frac{4\pi}{3}\left(\frac{D}{2}\right)^3}$

Relative error in the density,

$$\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + \frac{3\Delta D}{D}$$

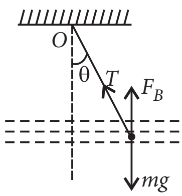
Relative percentage error in the density is

$$\frac{\Delta\rho}{\rho}\times 100 = \left(\frac{\Delta M}{M} + \frac{3\Delta D}{D}\right)\times 100$$

$$= \frac{\Delta M}{M}\times 100 + \frac{3\Delta D}{D}\times 100 \quad = 2 + 3\times\frac{0.01}{2.7}\times 100$$

= 2% + 1.11% = 3.1%

19. (b) : Let us depict the forces acting on the oscillating ball at an arbitrary angular position θ, relative to equilibrium position where F_B is the force of buoyancy.



The equation of motion for ball

$$-mgl\sin\theta + F_B l\sin\theta = ml^2\ddot{\theta} \qquad \text{...(i)}$$

Using $m = \frac{4}{3}\pi r^3\sigma$, $F_B = \frac{4}{3}\pi r^3\rho g$ and $\sin\theta \simeq \theta$ for small θ, in equation (i), we get

$$\ddot{\theta} = -\frac{g}{l}\left(1 - \frac{\rho}{\sigma}\right)\theta$$

Thus the time period of the ball

$$T = 2\pi\frac{1}{\sqrt{\frac{g}{l}\left(1-\frac{\rho}{\sigma}\right)}} = 2\pi\sqrt{\frac{l/g}{1-\frac{1}{3}}}$$

$$= 6.28\sqrt{\frac{0.2/9.8}{2/3}} = 1.1\text{ s}$$

20. (b) : Modulation index, $\mu = \frac{A_m}{A_c}$

Side band frequencies are

USB = $\upsilon_c + \upsilon_m$, LSB = $\upsilon_c - \upsilon_m$

Here, A_m = 10 V, A_c = 20 V

υ_c = 1 MHz = 1000 kHz, υ_m = 10 kHz

$\therefore \quad \mu = \frac{10}{20} = 0.5$

USB = 1000 + 10 = 1010 kHz,

LSB = 1000 – 10 = 990 kHz

21. (5.0)

22. (5.0): Case I : The bus acts as the source u_s = 5 m s^{-1}, u = 335 m s^{-1}, υ = 165 Hz

$$\upsilon' = \frac{u\times\upsilon}{u-u_s} = \frac{335\times 165}{335-5} = \frac{335}{2}\text{ Hz}$$

Case II : For reflected sound from the wall, bus acts as listener u_L = 5 m s^{-1}

$$\upsilon'' = \frac{(u+u_L)\upsilon'}{u} = \frac{(335+5)\times 335}{335\times 2} = 170\text{ Hz}$$

$\therefore$ Number of beats heard per second = $\upsilon'' - \upsilon$ = 170 – 165 = 5

23. (4000) : Maximum kinetic energy

$= \frac{1}{2}mv_{\max}^2 = 9$ eV

Energy of photon emitted during transition of electron in an atom is

$$h\upsilon = E_i - E_f = -\frac{13.6}{n_i^2} - \left(-\frac{13.6}{n_f^2}\right)$$

$$= \frac{-13.6}{3^2} + \frac{13.6}{1^2} = -1.51 + 13.6$$

or $h\upsilon$ = 12.09 eV

By Einstein's photoelectric equation

$$\frac{1}{2}mv_{\max}^2 = h\upsilon - W_0$$

or $W_0 = h\upsilon - \frac{1}{2}mv_{\max}^2 = 12.09 - 9$

or $\frac{hc}{\lambda_0} = 3.09\text{ eV} = 3.09\times 1.6\times 10^{-19}$ J

or $\lambda_0 = \frac{hc}{3.09\times 1.6\times 10^{-19}}$

or $\lambda_0 = 4\times 10^{-7}$ m = 4000 Å

24. (198) : As $E = \frac{1}{2}mv^2$, $v = \sqrt{\frac{2E}{m}} \approx 2500$ m s^{-1}

Time taken to cover a distance of 5 m *i.e.*,

$$dt = \frac{5\text{m}}{2500\text{ m s}^{-1}} = 2\times 10^{-3}\text{ s},$$

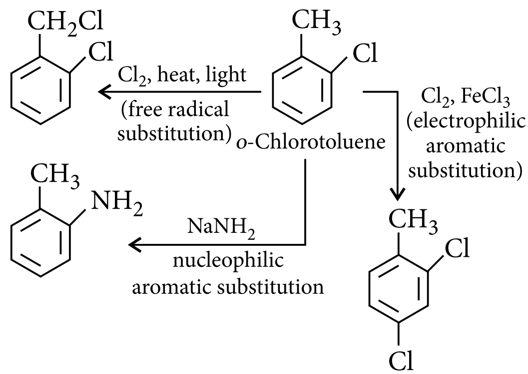
As $-\frac{dN}{dt} = \lambda N$,

$$-\frac{dN}{N} = \lambda dt = \frac{0.693}{T_{1/2}}(2\times 10^{-3}\text{ s})$$

$$= \frac{0.693(2\times 10^{-3}\text{ s})}{700\text{ s}} = 198\times 10^{-8}$$

25. (3.0)

26. (c) :



27. (b) : (ii) is not possible as value of l varies from 0 to $(n-1)$.
(iv) and (v) are not possible as value of m varies from $-l$ to $+l$.

28. (b) : Fe(II) gives blue colour with $K_3[Fe(CN)_6]$ but Fe(III) does not.

$$3Fe^{2+} + 2K_3[Fe(CN)_6] \rightarrow \underset{\text{Blue}}{Fe_3[Fe(CN)_6]} + 6K^+$$

29. (b) : Na and Mg is not an exception. The first ionization enthalpy of elements of group 2 are greater than those of group 1 due to their smaller size and higher nuclear charge.

30. (b) : Solution turns blue due to oxidation of Cu to Cu^{2+} ions in the solution.

31. (c) : Conductivity depends upon solvation of ions present in solution. Greater the solvation of ions, lesser is the conductivity.

32. (a) : Mass of the organic compound $(w) = 0.765$ g
Mass of the carbon dioxide formed $(x) = 0.535$ g
Mass of the water formed $(y) = 0.138$ g

$$\text{Percentage of hydrogen} = \frac{2}{18}\times\frac{y}{w}\times 100 = \frac{2}{18}\times\frac{0.138}{0.765}\times 100 = 2\%$$

$$\text{Percentage of carbon} = \frac{12}{44}\times\frac{x}{w}\times 100 = \frac{12}{44}\times\frac{0.535}{0.765}\times 100 = 19\%$$

So, the ratio of C : H = 19 : 2

33. (b) : $$\underset{(A)}{NH_4NO_3} + NaOH \xrightarrow{\text{boiling}} NaNO_3 + \underset{(B)}{NH_3} + H_2O$$

NH_3 gives brown ppt. with Nessler's reagent (K_2HgI_4).

$$2K_2[HgI_4] + NH_3 + 3KOH \longrightarrow \underset{\text{Brown ppt.}}{H_2N\cdot HgO\cdot HgI} + 7KI + 2H_2O$$

(A) on heating gives N_2O gas (C) which rekindles a glowing splinter but is not converted into NO_2 by air oxidation.

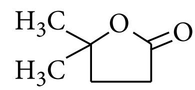
$$\underset{(A)}{NH_4NO_3} \xrightarrow{\Delta} \underset{(C)}{N_2O} + 2H_2O$$

34. (c) :

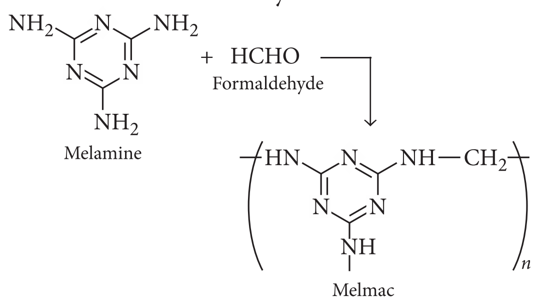
$$CH_3-\overset{O}{\overset{\|}{C}}-CH_2-CH_2-\overset{O}{\overset{\|}{C}}-O-CH_2CH_3 \xrightarrow[\text{1 mole}]{CH_3MgBr}$$

$$CH_3-\underset{CH_3}{\overset{OMgBr}{C}}-CH_2-CH_2-\overset{O}{\overset{\|}{C}}-O-CH_2-CH_3 \xrightarrow{H_3O^+}$$

$$CH_3-\underset{CH_3}{\overset{OH}{C}}-CH_2CH_2\overset{O}{\overset{\|}{C}}OCH_2CH_3 \xrightarrow[-CH_3CH_2OH]{(\Delta)\text{ cyclisation}}$$



35. (b) : Condensation polymerisation of melamine and formaldehyde.



36. (d) : Ideal gas does not show cooling or heating.

37. (c) : Semi-molal $\Rightarrow \frac{1}{2}$ m or mol kg^{-1}

$d = 1.02$ g/mL

1 kg of water contains $\frac{1}{2}$ mole of NaOH *i.e.*, 20 g

Mass of solution = 1000 + 20 = 1020 g

$$\text{Volume of solution} = \frac{1020}{1.02} = 1000\text{ mL} = 1\text{ L}$$

$$M = \frac{\frac{1}{2}\text{ mol}}{1\text{L}} = \frac{1}{2}\ ;\ x_{NaOH} = \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1000}{18}} = \frac{9}{1009}$$

$$\%w/w = \frac{20}{1020} \times 100 = 2\%$$

$$\%w/v = \frac{20}{1000} \times 100 = 2\%$$

38. (c) : $P_{4(g)} + 6Cl_{2(g)} \rightleftharpoons 4PCl_{3(g)}$

One mole of P_4 reacts with 6 moles of Cl_2 *i.e.*, at equilibrium Cl_2 is consumed more than P_4. If we start the reaction with equal number of moles of P_4 and Cl_2 then obviously at equilibrium $[P_4] > [Cl_2]$.

39. (c)

40. (b) : Non-ionic detergents are esters of high molecular mass obtained by the reaction of alcohols with stearic acid.

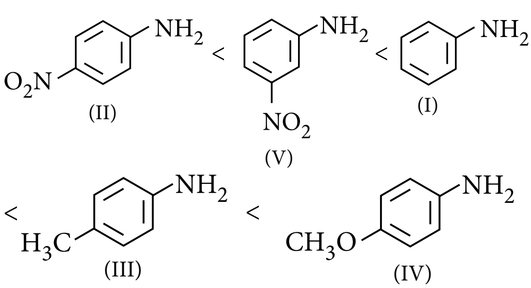
Sodium laurylsulphate – Anionic detergent

Pentaerythrityl stearate – Non-ionic detergent

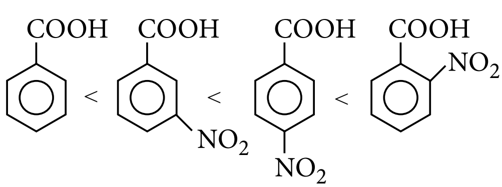
Cetyltrimethylammonium chloride – Cationic detergent

Sodium *n*-dodecylbenzenesulphonate – Anionic detergent

41. (a) : —OCH_3 is strongest electron releasing group (+*M* effect) which opposes most the dispersion of lone pair of electrons of nitrogen into the ring. Thus, —OCH_3 being at *para* position imparts highest basicity. —NO_2 being at *meta* position stabilises the electron pair of nitrogen only by –*I* effect. While —NO_2 being present at *para* position due to –*M* effect and –*I* effect stabilises the lone pair of electrons of nitrogen most and imparts least basicity.



42. (d) : Order of acidity is :



E.W.G. increases the acidity of benzoic acid, *o*-isomer will have higher acidity than corresponding *m*- and *p*-isomers due to *ortho*-effect. In *p*-nitrobenzoic acid, both –*R* effect and –*I* effect of the nitro group increase the acidity while in *m*-nitrobenzoic acid, only the weaker –*I* effect increases the acidity. Therefore the correct order of acidity is ii > iii > iv > i.

43. (c) : A brine solution is electrolysed using a mercury cathode and a carbon anode.

At cathode : $Na^+ + e^- \xrightarrow{Hg}$ Na-amalgam

At anode : $Cl^- \longrightarrow \frac{1}{2}Cl_2 + e^-$

44. (a)

45. (c) : C—Br bond is weaker than C—Cl bond, therefore, alkyl bromide (II) reacts faster than alkyl chloride (III) and (IV). Since CH_2=CH— is electron withdrawing therefore, CH_2 has more +ve charge on III than on IV.

$$\underset{\text{III}}{CH_2{=}CH \leftarrow \overset{\delta\delta+}{C}H_2{-}Cl}, \quad \underset{\text{IV}}{CH_3{-}CH_2 \rightarrow \overset{\delta+}{C}H_2{-}Cl}$$

In other words, nucleophilic attack occurs faster on III than on IV. Further, since Williamson's synthesis occurs by S_N2 mechanism, therefore, due to steric hindrance alkyl bromide (I) is the least reactive. Thus, the decreasing order of reactivity is II > III > IV > I.

46. (2)

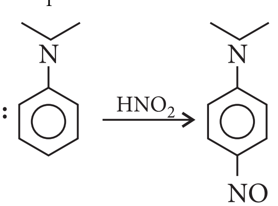
47. (5) : $V_1 = 0.5$ L $\quad V_2 = 0.1$ L

$n_1 = 25$ moles $\quad n_2 = ?$

$$\frac{V_1}{n_1} = \frac{V_2}{n_2}$$

$$n_2 = \frac{V_2 n_1}{V_1} = \frac{0.1 \times 25}{0.5} = 5 \text{ moles}$$

48. (1) :



49. (2) : $Al \longrightarrow Al^{3+} + 3e^-$

$$\frac{4}{3} \text{ mol of Al} = \frac{4}{3} \times 3 \text{ mol of electrons}$$

$$= 4 \text{ mol of electrons}$$

Then, $n = 4$

$\Delta G = -nFE$

$-772 \times 1000 \text{ J mol}^{-1} = -4 \times 96500 \times E$

$\therefore \quad E = \dfrac{772 \times 1000}{4 \times 96500} = 2.0 \text{ V}$

50. (141.4) : $Z = \dfrac{d N_A a^3}{M}$

$= \dfrac{3.115 \times 6.02 \times 10^{23} \times (400)^3 \times 10^{-30}}{30}$

$= 4 \Rightarrow$ *fcc* packing

In *fcc*, $4r = \sqrt{2} \cdot a$

$r = \dfrac{1.414}{4} \times 400 = 141.4 \text{ pm}$

51. (a) : The function $\dfrac{1}{\sqrt{|f(x)| - f(x)}}$ will be defined, when $|f(x)| > f(x)$

$\Rightarrow f(x) < 0 \Rightarrow \log_e x + \log_x e < 0$

$\Rightarrow \dfrac{(\log_e x)^2 + 1}{\log_e x} < 0$

$\Rightarrow \log_e x < 0 \Rightarrow x < e^0 \Rightarrow x < 1 \Rightarrow 0 < x < 1$

52. (c) : We know, $A \cdot \text{adj } A = |A| I$

Here, $|A| = xyz - 8x - 3(z - 8) + 2(2 - 2y)$

$\Rightarrow |A| = xyz - (8x + 3z + 4y) + 28 = 60 - 20 + 28 = 68$

53. (d) : Let $I = \displaystyle\int_1^{e^{17/2}} \frac{\pi \cos(\pi \log x)}{x} dx$

Put $\pi \log x = z \Rightarrow \dfrac{\pi}{x} dx = dz$

Since, $1 \le x \le e^{17/2} \Rightarrow 0 \le z \le \dfrac{17\pi}{2}$

$\therefore \quad I = \displaystyle\int_0^{17\pi/2} \cos z \; dz = [\sin z]_0^{17\pi/2}$

$= \sin\dfrac{17\pi}{2} - \sin 0 = \sin\left(8\pi + \dfrac{\pi}{2}\right) = \sin\dfrac{\pi}{2} = 1$

54. (b) : Here $n = 100$, mean $= 50$ and median $= 52$

$\therefore \quad \bar{x} = \dfrac{1}{n}\displaystyle\sum_{i=1}^{100} x_i = 50 \Rightarrow \sum_{i=1}^{100} x_i = 5000$

Now corrected $\displaystyle\sum_{i=1}^{100} x_i = 5000 - 100 + 110 = 5010$

$\therefore$ Corrected mean $= \dfrac{1}{100}\displaystyle\sum_{i=1}^{100} x_i = \frac{5010}{100} = 50.10$

As median is positional average therefore it will remain same.

55. (a) : $\displaystyle\lim_{x \to \infty}\left(\frac{x^2 + 5x + 3}{x^2 + x + 3}\right)^x \left(1^\infty \text{ form}\right)$

$= \exp\left\{\displaystyle\lim_{x \to \infty}\left(\frac{x^2 + 5x + 3}{x^2 + x + 3} - 1\right)x\right\}$

$= \exp\left\{\displaystyle\lim_{x \to \infty}\frac{4x^2}{x^2 + x + 3}\right\} = e^4.$

56. (a) : According to question, p and q are roots of $3x^2 - 5x - 2 = 0$

$\therefore \quad p + q = \dfrac{5}{3}$ and $pq = \dfrac{-2}{3}$.

We have to find the equation whose roots are $3p - 2q$ and $3q - 2p$.

Clearly, sum of roots $= (3p - 2q) + (3q - 2p) = (p + q) = \dfrac{5}{3}$

and product of roots $= (3p - 2q)(3q - 2p)$

$= 9pq - 6q^2 - 6p^2 + 4pq = 13pq - 2(3p^2 + 3q^2)$

$= 13\left(\dfrac{-2}{3}\right) - 2(5p + 2 + 5q + 2)$

$= 13\left(\dfrac{-2}{3}\right) - 2\left[5\left(\dfrac{5}{3}\right) + 4\right]$

$= \dfrac{-26}{3} - 2\left[\dfrac{25}{3} + 4\right] = \dfrac{-100}{3}$

Hence, required equation is $3x^2 - 5x - 100 = 0$.

57. (b) : If the pair of lines is $ax^2 + 2hxy + by^2 = 0$ and it has slopes m_1 and m_2, then

$m_1 + m_2 = -\dfrac{2h}{b}$ and $m_1 m_2 = \dfrac{a}{b}$

$\therefore \quad (m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1 m_2 = \dfrac{4(h^2 - ab)}{b^2}$

Here $a = \tan^2\theta + \cos^2\theta$, $h = -\tan\theta$ and $b = \sin^2\theta$

$\therefore (m_1 - m_2)^2 = \dfrac{4}{\sin^4\theta}\left[\tan^2\theta - (\tan^2\theta + \cos^2\theta)\sin^2\theta\right]$

$= \dfrac{4}{\sin^2\theta}\left[\dfrac{1}{\cos^2\theta} - \tan^2\theta - \cos^2\theta\right]$

$$= \frac{4}{\sin^2\theta}(1-\cos^2\theta) = 4$$

$\therefore \quad |m_1 - m_2| = 2.$

58. (c) : We have, $\frac{dy}{dx} = (x+3)(y+2)$

Take, $X = x + 3$ and $Y = y + 2 \Rightarrow \frac{dY}{Y} = XdX$

$Y = Ae^{X^2/2} \Rightarrow y = -2 + Ae^{\frac{(x+3)^2}{2}}$

Now, $y(-1) = -2 + Ae^2$, $y(-3) = -2 + A$

$\therefore \quad y(-1) - e^2 y(-3) = -2 + 2e^2 = 2(e^2 - 1).$

59. (c) : $\frac{C_1}{C_0} + 2.\frac{C_2}{C_1} + 3.\frac{C_3}{C_2} + . \quad + n\frac{C_n}{C_{n-1}}$

$$= \frac{n}{1} + 2\frac{n(n-1)/1\cdot 2}{n} + 3\frac{n(n-1)(n-2)/3\cdot 2\cdot 1}{n(n-1)/1\cdot 2} + ... + n\cdot\frac{1}{n}$$

$$= n + (n-1) + (n-2).... + 1 = \sum n = \frac{n(n+1)}{2}$$

60. (a) : The given line passes through $(4, 2, k)$. Now, the line will lie in the plane $2x - 4y + z = 7$, if the point $(4, 2, k)$ lies on the plane.

$\therefore \quad 2\times 4 - 4\times 2 + k = 7 \Rightarrow k = 7$

61. (a) : Clearly, $\frac{m}{n} = \frac{\tan(120°+\theta)}{\tan(\theta-30°)}$

$$\Rightarrow \quad \frac{m+n}{m-n} = \frac{\tan(\theta+120°)+\tan(\theta-30°)}{\tan(\theta+120°)-\tan(\theta-30°)}$$

(By componendo and dividendo)

$$= \frac{\sin(\theta+120°)\cos(\theta-30°)+\cos(\theta+120°)\sin(\theta-30°)}{\sin(\theta+120°)\cos(\theta-30°)-\cos(\theta+120°)\sin(\theta-30°)}$$

$$= \frac{\sin(2\theta+90°)}{\sin(150°)} = \frac{\cos 2\theta}{1/2} = 2\cos 2\theta$$

62. (b) : We have, $f(x) = (x+1)^{1/3} - (x-1)^{1/3}$

$$\therefore \quad f'(x) = \frac{1}{3}\left[\frac{1}{(x+1)^{2/3}} - \frac{1}{(x-1)^{2/3}}\right]$$

$$= \frac{(x-1)^{2/3} - (x+1)^{2/3}}{3(x^2-1)^{2/3}}$$

Clearly, $f'(x)$ does not exist at $x = \pm 1$

Also, $f'(x) = 0$

$\Rightarrow \quad (x-1)^{2/3} = (x+1)^{2/3} \Rightarrow x = 0$

Clearly, $f'(x) \neq 0$ for any other value of $x \in [0, 1]$.

The value of $f(x)$ at $x = 0$ is 2.

Hence, the greatest value of $f(x)$ is 2.

63. (b) : We have, $|A| = 2$, $|B| = 3$ and $|C| = 5$

$$\therefore \det(A^2BC^{-1}) = |A^2BC^{-1}| = \frac{|A|^2|B|}{|C|} = \frac{4\cdot 3}{5} = \frac{12}{5}$$

64. (b) : Let $S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + ...\infty$

$$\Rightarrow \quad S - 1 = \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + ... \infty \quad ...(i)$$

$$\Rightarrow \quad (S-1)\times\frac{1}{3} = \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \frac{14}{3^5} + ...\infty \quad ...(ii)$$

Subtracting (ii) from (i), we get

$$\frac{2}{3}(S-1) = \frac{2}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + ...\infty$$

$$\Rightarrow \quad \frac{2}{3}(S-1) = \frac{2}{3} + \frac{\frac{4}{3^2}}{1-\frac{1}{3}}$$

$$\Rightarrow \quad \frac{2}{3}(S-1) = \frac{2}{3} + \frac{2}{3} \quad \Rightarrow \quad S = 3.$$

65. (d) : Let the equation of the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$.

$\because$ It cuts the two given circles orthogonally

$\therefore \quad -8g - 2f = c + 16$ and $-4g - 4f = c + 1$

$\Rightarrow \quad 8g + 2f + c = -16 \quad ...(i)$

and $4g + 4f + c = -1 \quad ...(ii)$

$\because$ The circle passes through $(1, 1)$

$\therefore \quad 2g + 2f + c = -2 \quad ...(iii)$

Solving (i), (ii) and (iii) we get

$g = -\frac{7}{3}, f = \frac{17}{6}, c = -3$

$\therefore$ The centre is $\left(\frac{7}{3}, -\frac{17}{6}\right) = (a, b)$

$\Rightarrow a = \frac{7}{3}$ and $b = -\frac{17}{6} \Rightarrow a + b = -\frac{1}{2}$

66. (d) : $p \rightarrow (\sim p \vee q) \equiv (\sim p) \vee ((\sim p) \vee q)$

$\equiv ((\sim p) \vee (\sim p)) \vee q \equiv (\sim p) \vee q$

Now, the negation of the given expression

$\equiv \sim((\sim p) \vee q) \equiv \sim(\sim p) \wedge (\sim q) \equiv p \wedge (\sim q)$

67. (c) : Five numbers can be drawn from 40 numbers in ${}^{40}C_5$ ways, therefore total number of cases = ${}^{40}C_5$.

We want that $x_3 = 24$.

$\therefore$ The number of favourable cases are ${}^{23}C_2 \times {}^{16}C_2$

Hence, required probability $= \dfrac{{}^{23}C_2 \times {}^{16}C_2}{{}^{40}C_5}$

68. (c) : $|1-x^2| = \begin{cases} (x^2-1), & \text{if} \quad x < -1 \\ -(x^2-1), & \text{if} \quad -1 \le x < 1 \\ (x^2-1), & \text{if} \quad x \ge 1 \end{cases}$

$\therefore \int_{-2}^{3} |1-x^2|\, dx$

$= \int_{-2}^{-1} (x^2-1)dx + \int_{-1}^{1} -(x^2-1)dx + \int_{1}^{3} (x^2-1)dx$

$= \left[\left(-\frac{1}{3}+1\right) - \left(-\frac{8}{3}+2\right)\right] - \left[\left(\frac{1}{3}-1\right) - \left(-\frac{1}{3}+1\right)\right] + \left[(9-3) - \left(\frac{1}{3}-1\right)\right]$

$= \frac{4}{3} + \frac{4}{3} + 6 + \frac{2}{3} = \frac{28}{3}$

69. (b) : $f'(0) = \lim_{h \to 0} \dfrac{g(0+h)\cos(1/h) - 0}{h}$

$= \lim_{h\to 0} \dfrac{g(h)\cos(1/h)}{h} = \lim_{h \to 0} g'(0)\cos(1/h) = 0$

$[\because g'(-x) = -g'(x) \Rightarrow g'(0) = 0]$

70. (c) : tan9° – tan27° – tan63° + tan81°

= tan9° – tan27° – cot27° + cot9°

= (tan9° + cot9°) – (tan27° + cot27°)

$= \dfrac{2}{\sin 18°} - \dfrac{2}{\sin 54°}$

$= 2\left\{\dfrac{\sin 54° - \sin 18°}{\sin 18° \sin 54°}\right\} = 2.\ \dfrac{2 \cdot \cos 36° \cdot \sin 18°}{\sin 18° \sin 54°} = 4$

71. (20): Since coins are identical and we have to find the number of ways of getting 3 heads and 3 tails.

$\therefore$ Total number of ways $= \dfrac{6!}{3!3!} = 20$

72. (55) : Using Lagrange's identity

$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$

$\Rightarrow |\vec{a} \times \vec{b}|^2 = (5)^2 \times (6)^2 - (-25)^2$

= 25 × 36 – 625 = 900 – 625 = 275 = 11 × 25

$\Rightarrow |\vec{a} \times \vec{b}| = 5\sqrt{11}$

73. (64) : Given that, $a + 2b + 3c = 12$ ...(i)

and a, b, c are positive real numbers.

Now, A.M. ≥ G.M.

$\Rightarrow \dfrac{a+b+b+c+c+c}{6} \ge \sqrt[6]{ab^2c^3}$

$\Rightarrow \dfrac{a+2b+3c}{6} \ge \sqrt[6]{ab^2c^3} \Rightarrow ab^2c^3 \le 2^6$ [From (i)]

74. (3) : We know that, the lines

$\vec{r} = \vec{a} + s\vec{b}, \vec{r} = \vec{c} + t\vec{d}$ are coplanar if $[\vec{c} - \vec{a}\ \vec{b}\ \vec{d}] = 0$

Here, $\vec{a} = (2,9,13), \vec{c} = (a, 1, -2), \vec{b} = (1,2,3), \vec{d} = (1,-2,3)$

$\therefore [\vec{c} - \vec{a}\ \vec{b}\ \vec{d}] = 0 \Rightarrow \begin{vmatrix} a-2 & -8 & -15 \\ 1 & 2 & 3 \\ 1 & -2 & 3 \end{vmatrix} = 0$

$\Rightarrow (a-2)\,12 - 15(-4) = 0$

$\Rightarrow a - 2 + 5 = 0 \Rightarrow |a| = 3.$

75. (5) : $\left|\dfrac{z-25}{z-1}\right| = 5 \Rightarrow |z-25|^2 = 25|z-1|^2$

$\Rightarrow |z|^2 - 25z - 25\bar{z} + 625 = 25\{|z|^2 - z - \bar{z} + 1\}$

$\Rightarrow |z| = 5$