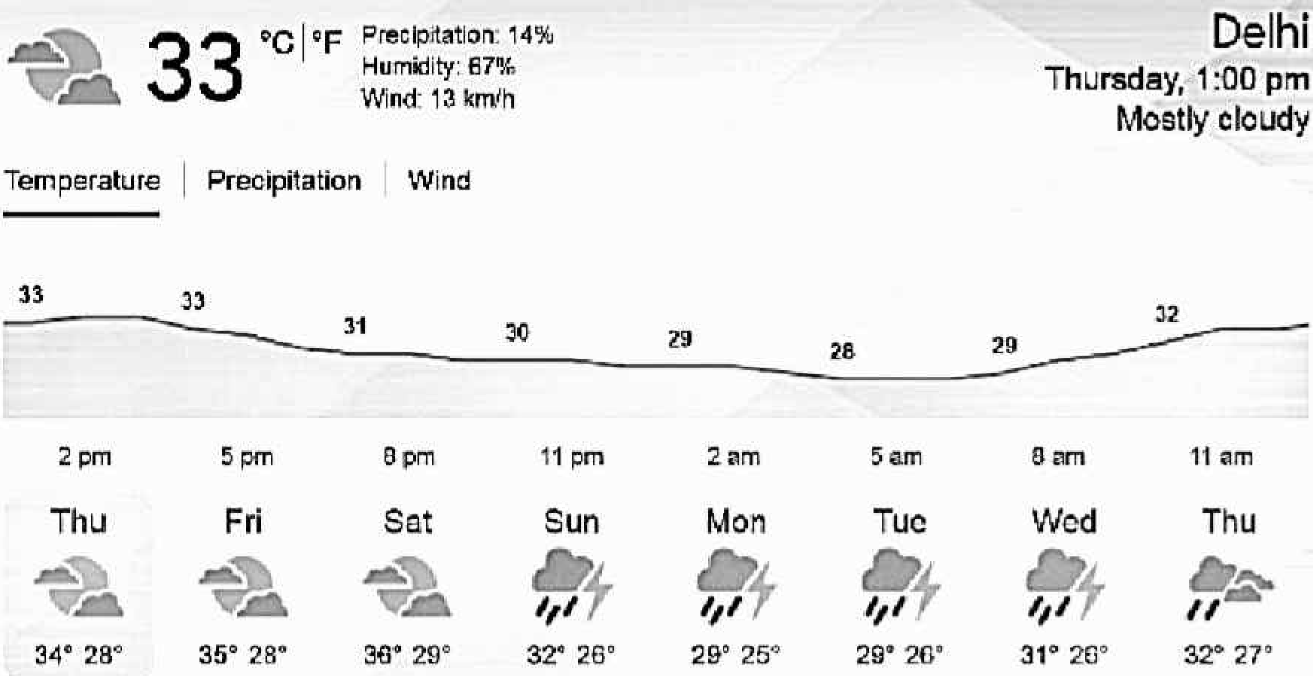
# 1 Relations and Functions




*The concept of Relations and functions is useful in finding how one quantity is related to another. Different countries use different units to measure temperature. While Countries such as the US use the Fahrenheit scale to measure temperature but we in India, use the Celsius scale. Using the function $f(x) = 9/5x + 32$, any temperature in Celsius scale can be converted to Fahrenheit scale.*

## Topic Notes

- *Relations and its Types*
- *Functions and its Types*

# RELATIONS AND ITS TYPES 1

## TOPIC 1

## CARTESIAN PRODUCT OF TWO SETS

Relation between two sets is defined as the collection of ordered pairs formed by containing one element from each set. In an ordered pair, first number should be from domain set and second number should be from range set. Each element of domain set should be related to some element in range set.

Given two non-empty sets A and B, the Cartesian product A × B is the set of all ordered pairs of elements from A and B, *i.e.*, A × B = {(*a*, *b*) : *a* ∈ A and *b* ∈ B}

If either A or B is an empty set, A × B will also be an empty set, *i.e.*, A × B = ϕ.

**Illustration:** Let A = $\{a_1, a_2, a_3\}$ and B = $\{b_1, b_2\}$ be any two non-empty sets.

Then, A × B = $\{(a_1, b_1), (a_1, b_2), (a_2, b_1), (a_2, b_2), (a_3, b_1), (a_3, b_2)\}$

Here, *n*(A) = 3 and *n*(B) = 2. So, *n*(A × B) = 6

## TOPIC 2

## RELATIONS

Let A and B be two sets. Then a relation R from set A to set B is a subset of A × B.

Thus, R is a relation from A to B ⇔ R ⊆ A × B.

Or, in other words, a relation is a collection of ordered pairs from A × B.

Consider the two sets A = {*a*, *b*, *c*} and B = {Ashok, Babita, Chmoli, Dinesh}.

The Cartesian product A × B has 12 ordered pairs.

Let us obtain a subset of A × B by introducing a relation R between the first element *x* and the second element *y* of each ordered pair (*x*, *y*) as

R = {(*x*, *y*) : *x* is the first letter of the name *y*, *x* ∈ A and *y* ∈ B}

Then, R = {(*a*, Ashok), (*b*, Babita), (*c*, Chmoli)}

Thus, a relation R from a non-empty set A to a non-empty set B is a subset of A × B.

The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A × B.

A relation R from A to A is usually used as 'relation R in A'.

### Important

- *The total number of relations that can be defined from a set A to a set B is the number of possible subsets of A × B. If n(A) = p and n(B) = q, then n(A × B) = pq and so the total number of relations from A to B is* $2^{pq}$.

**Example 1.1: Let A = {1, 2} and B = {3, 4}. Find the number of relations from A to B.**

**Ans.** We have

A × B = {(1, 3), (1, 4), (2, 3), (2, 4)}

Since *n*(A × B) = 4, the number of subsets of A × B will be $2^4$.

Therefore, the number of possible relations from A to B will be $2^4$.

**Example 1.2: Let A = {1, 2, 3}. Find the number of relations in A.**

**Ans.** We have

A × A = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

Since, *n*(A × A) = 9, the number of subsets of A × A will be $2^9$.

Therefore, the number of possible relations in A will be $2^9$.

### Methods of Writing a Relation

A relation may be represented algebraically either in the Roster form or in the Set-builder form.

Let A = {1, 2, 3, 4, 5, 6}.

Here, define a relation R on A as : R = {(*x*, *y*) : *y* = *x* + 1, *x*, *y* ∈ A}

Here, the relation is represented in Set-builder form.

In roster form, it is represented as R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}.

### Caution

- *(6, 7) ∉ R as 7 ∉ A.*

### Domain and Range of a Relation

Consider a relation R from set A to set B, *i.e.*, R ⊆ A × B.

The set of all first elements of the ordered pairs in R is called the domain of the relation R and the set of all second elements of the ordered pairs in R is called the range of the relation R. The set B is called the co-domain of R.

**Important**

➥ *The domain of R is a subset of A; and range of R is a subset of B. Further, range of R is a subset of the co-domain of R, i.e., range R $\subseteq$ co-domain R.*

**Illustration:** Let A = {1, 2, 3,..., 14} and R be a relation on A defined as

R = {(x, y) : 3x – y = 0, where x, y ∈ A} *i.e.*, R = {(1, 3), (2, 6), (3, 9), (4, 12)}

Then, domain of R = {1, 2, 3, 4} and range of R = {3, 6, 9, 12}.

# TOPIC 3

# TYPES OF RELATIONS

## Empty Relation

A relation R in a set A is called an empty relation or void relation if no element of set A is related to any element of A *i.e.*, R = ϕ or R = { }.

**Illustration:** Let A = {1, 3, 5}, B = {5, 7} and R = {(*a*, *b*): *a* + *b* is odd}, where *a* ∈ A and *b* ∈ B.

Here, none of the numbers 1 + 5, 3 + 5, 5 + 5, 1 + 7, 3 + 7, 5 + 7, is odd. Thus, R contains no elements. Therefore, R is an empty relation on A × B.

## Universal Relation

A relation R in a set A defined as R = A × A is called an universal relation if each element of A is related to every element of A.

A universal relation on a set A is defined as

$$R = A \times A$$

**Illustration:** Let A = {4, 6}, B = {1, 3, 5}. Consider a relation R from A to B as

R = {(*a*, *b*): *a* + *b* is odd}

Here, each number 4 + 1, 4 + 3, 4 + 5, 6 + 1, 6 + 3, 6 + 5, is odd.

Thus, R = {(4, 1), (4, 3), (4, 5), (6, 1), (6, 3), (6, 5)}. Therefore, R is an universal relation as R = A × B.

**Important**

➥ *Since a relation in a set A is a subset of A × A, the empty set ϕ and A × A are two extreme relations.*

**Illustration:** Consider a relation R in the set A = {1, 2, 3, 4} given by R = {(*a*, *b*) : *a* – *b* = 5}. Here, R = ϕ, as no ordered pair of R satisfy the condition *a* – *b* = 5.

Similarly, a relation S = {(*a*, *b*) : |*a* – *b*| ≥ 0} is the whole set A × A, as all the elements of A satisfy the condition |*a* – *b*| ≥ 0.

These two extreme relations are called empty relation and universal relation respectively. These are also considered as trivial relations.

## Identity Relation

A relation R in a set A defined as R = {(*a*, *a*) : *a* ∈ A} is called an identity relation.

**Illustration:** Let A = {1, 2, 3, 4} and R be a relation on A defined as:

$$R = \{(x, y) : x - y = 0, \text{ where } x, y \in A\}$$

In roster form, R = {(1, 1), (2, 2), (3, 3), (4, 4)}

Thus, R is the identity relation in A.

## Reflexive Relation

A relation R in a set A is called reflexive relation if every element of A is related to itself, *i.e.*, (*a*, *a*) ∈ R, for every *a* ∈ A.

**Illustration:** Let A = {1, 2, 3}. Consider two relations $R_1$ and $R_2$ in A defined as:

$$R_1 = \{(1, 1), (2, 1), (2, 2), (3, 1), (3, 2), (3, 3)\}$$

Since (*a*, *a*) ∈ $R_1$, for every *a* ∈ A, so $R_1$ is a reflexive relation in A.

$$R_2 = \{(1, 2), (1, 3), (2, 3)\}$$

Here $R_2$ is not a reflexive relation on A, because (1, 1), (2, 2), (3, 3) ∉ $R_2$.

**Important**

➥ *Identity and Universal relations are reflexive, but Empty relation is not reflexive.*

## Symmetric Relation

A relation R in a set A is called symmetric relation if (*a*, *b*) ∈ R

⇒ (*b*, *a*) ∈ R, for every *a*, *b* ∈ A.

Or, R is said to be symmetric if *a* R *b* (read as '*a* related to *b*')

⇒ *b* R *a* for every *a*, *b* ∈ A.

**Illustration:** Let A = {1, 2, 3}. Consider two relations $R_1$ and $R_2$ on A defined as

$$R_1 = \{(1, 1), (2, 1), (2, 2), (1, 2)\}$$

$R_1$ is a symmetric relation on A as for every (*a*, *b*) ∈ R ⇒ (*b*, *a*) ∈ R

$$R_2 = \{(1, 1), (2, 2), (1, 2), (2, 1),, (3, 1)\}$$

$R_2$ is not a symmetric relation on A, because (3, 1) ∈ $R_2$, but and (1, 3) ∉ $R_2$.

## Transitive Relation

A relation R in a set A is called transitive relation if *a* R *b* and *b* R *c*

⇒ *a* R *c* for every *a*, *b*, *c* ∈ A, *i.e.*, if (*a*, *b*) ∈ R and (*b*, *c*) ∈ R

⇒ (*a*, *c*) ∈ R, for every *a*, *b*, *c* ∈ A.

**Illustration:** Let A = { 1, 2, 3} and, $R_1$ and $R_2$ in A be defined as:

$$R_1 = \{(1, 1), (2, 1), (2, 2), (1, 2)\}$$
$$R_2 = \{(1, 1), (1, 2), (2, 3)\}$$

Since $(a, b) \in R_1$ and $(b, c) \in R_1$

$\Rightarrow (a, c) \in R_1$, for every $a, b, c \in A$,

So, $R_1$ is a transitive relation in A.

$R_2$ is not a transitive relation in A, because $(1, 2) \in R_2$ and $(2, 3) \in R_2$ but $(1, 3) \notin R_2$.

### Important

➥ *Transitivity is possessed by a relation unless the stated condition is violated i.e., if there is no situation in which $(a, b) \in R$ and $(b, c) \in R$, then the relation is transitive.*

*For example, $R = \{(1, 2), (1, 3)\}$ defined on the set $A = \{1, 2, 3\}$ is transitive.*

**Example 1.3: Check whether the relation R defined in the set A = {1, 2, 3, 4, 5, 6} as $R = \{(a, b) : b = a + 1, a, b \in A\}$ is reflexive, symmetric or transitive. [NCERT]**

**Ans.** In roster form, R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}

Here, for $1 \in A$, $(1, 1) \notin R$. So, R is not a reflexive relation.

For $(2, 3) \in R$, $(3, 2) \notin R$. So, R is not a symmetric relation.

For $(1, 2) \in R$ and $(2, 3) \in R$, $(1, 3) \notin R$. So, R is not a transitive relation.

Thus, the relation R is neither reflexive nor symmetric nor transitive.

**Example 1.4: Show that the relation R in the set R of real numbers, defined as $R = \{(a, b) : a \le b\}$ is reflexive and transitive but not symmetric. [NCERT]**

**Ans.** Is R reflexive?

Let $a \in R$ be any arbitrary real number.

We know that, every real number is equal to itself, *i.e.*, $a = a$

So, we can write $a \le a$

$\Rightarrow (a, a) \in R$

Thus, R is reflexive.

**Is R symmetric?**

Let $(a, b) \in R$. Then $a < b$ or $a = b$

When $a = b$, then $b = a$

But when $a < b$, then $b \nless a \nRightarrow (b, a) \in R$

e.g., $2 < 3$ but $3 \nless 2$

So, R is not symmetric.

**Is R transitive?**

Let $(a, b) \in R$ and $(b, c) \in R$. Then, $a \le b$ and $b \le c$

$\Rightarrow a \le c$

$\Rightarrow (a, c) \in R$

Thus, R is transitive.

# TOPIC 4

# EQUIVALENCE RELATION

A relation R in a set A is said to be an equivalence relation if

(1) R is reflexive, *i.e.*, $a R a$ or $(a, a) \in R$, for every $a \in A$.

(2) R is symmetric, *i.e.*, $a R b$

$\Rightarrow b R a$ or $(a, b) \in R$

$\Rightarrow (b, a) \in R$, where $a, b \in A$.

(3) R is transitive, *i.e.*, $a R b$ and $b R c$

$\Rightarrow a R c$ or $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow (a, c) \in R$, where $a, b, c \in A$.

**Illustration:** Let a relation R in the set of all integers Z be defined as $R = \{(a, b) : 2 \text{ divides } a - b\}$ Then, R is an equivalence relation.

R is reflexive, as 2 divides $(a - a)$, for all $a \in Z$

Further, if $(a, b) \in R$, then 2 divides $(a - b)$

Now, if 2 divides $(a - b)$

$\Rightarrow$ 2 divides $(b - a)$

$\Rightarrow (b, a) \in R$

Thus, R is symmetric.

Similarly, if $(a, b) \in R$ and $(b, c) \in R$ then 2 divides $(a - b)$ and $(b - c)$.

2 divides $(a - b)$ and $(b - c)$

$\Rightarrow$ 2 divides $[(a - b) + (b - c)]$, or $(a - c)$.

So, R is transitive.

Hence, R is an equivalence relation on Z.

**Example 1.5: Let L be the set of all lines in XY plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1 \text{ is parallel to } L_2\}$. Show that R is an equivalence relation.**

**Also, find the set of all lines related to the line $y = 2x + 4$. [NCERT]**

**Ans.** Is R reflexive?

Since every line in XY plane is parallel to itself, $(L, L) \in R$. So, R is reflexive.

**Is R symmetric?**

Let $(L_1, L_2) \in R \Rightarrow L_1$ is parallel to $L_2 \Rightarrow L_2$ is parallel to $L_1 \Rightarrow (L_2, L_1) \in R$

So, R is symmetric.

**Is R transitive?**

Let $(L_1, L_2) \in R$ and $(L_2, L_3) \in R \Rightarrow L_1$ is parallel to $L_2$ and $L_2$ is parallel to $L_3$

$\Rightarrow L_1$ is parallel to $L_3 \Rightarrow (L_1, L_3) \in R$.

So, R is transitive.

Hence, R is an equivalence relation.

The set of all lines related to the line $y = 2x + 4$ is the set of all lines that are parallel to the line $y = 2x + 4$. Slope of the line $y = 2x + 4$ is 2. Since, parallel lines have the same slope. All lines parallel to the line $y = 2x + 4$ are of the form $y = 2x + c$, where $c$ is any real number.

⚠ Caution

➥ *In such types of questions when we have to prove that any of the reflexive or symmetric or transitive relations exists, then we have to prove it in regard of all elements of the given relation. When we have to show that the relation does not exist, then it is sufficient to cite an example which shows that the relation does not exist in that case.*

## TOPIC 5

# EQUIVALENCE CLASSES

Consider an equivalence relation R in a set A and $a \in A$. Then, the set of all those elements of A which are related to $a$, is called the equivalence class of '$a$' and is denoted by $[a]$. Thus, $[a] = \{b \in A : (b, a) \in R\}$.

**Illustration:** For the relation R on the set of integers Z defined as $R = \{(a, b) : 2 \text{ divides } a - b\}$. We have already proved that this relation is an equivalence relation.

Here, $[0] = \{0, \pm 2, \pm 4, \pm 6, \ldots\}$ ; and $[1] = \{\pm 1, \pm 3, \pm 5, \pm 7, \ldots\}$

**Example 1.6: Given that relation R in the set $A = \{x \in Z: 0 \le x \le 12\}$, given by $R = \{(a, b) : |a - b|$ is a multiple of 4, $a, b \in A\}$ is an equivalence relation. Find the set of all elements related to 1 or equivalence class of 1.**

**Ans.** It is given that relation $R = \{(a, b) : |a - b|$ is a multiple of 4$\}$ is an equivalence relation.

Ordered pairs containing 1 are $\{(1, 1), (1, 5), (5, 1), (1, 9), (9, 1)\}$

So, the set of elements related to 1 is $\{1, 5, 9\}$ or equivalence class of $[1] = \{1, 5, 9\}$.

# OBJECTIVE Type Questions

[ 1 mark ]

## Multiple Choice Questions

**1. Let R be a relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$, then:**

**(a) $(2, 4) \in R$** **(b) $(3, 8) \in R$**

**(c) $(6, 8) \in R$** **(d) $(8, 7) \in R$**

**[CBSE Term-1 SQP 2021]**

**2. The maximum number of equivalence relations on the set $A = \{1, 2, 3\}$ are:**

**(a) 1** **(b) 2**

**(c) 3** **(d) 5 [NCERT Exemplar]**

**Ans.** *(d) 5*

**Explanation:** Given, $A = \{1, 2, 3\}$

Possible equivalence relations are:

$R_1 = \{(1, 1), (2, 2), (3, 3)\}$

$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$

$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$

$R_4 = \{(1, 1), (2, 2), (3, 3), (2, 3), (3, 2)\}$

$R_5 = A \times A$

**3. For the set of all straight lines in a plane, the relation of " perpendicularity" is:**

**(a) reflexive but neither symmetric nor transitive**

**(b) symmetric but neither reflexive nor transitive**

**(c) transitive but neither reflexive nor symmetric**

**(d) neither reflexive nor symmetric nor transitive**

**Ans.** *(b) symmetric but neither reflexive nor transitive*

**Explanation:** Let $l$ and $m$ be any two lines in a plane.

Relation is not reflexive, since no line is perpendicular to itself.

Relation is symmetric, since $l \perp m$ implies $m \perp l$.

Relation is not transitive, since $l \perp m$ and $l \perp$ implies $m \parallel n$.

**4. If a relation R on the set {1, 2, 3} be defined by $R = \{(1, 2)\}$, then R is:**

**(a) reflexive** **(b) transitive**

**(c) symmetric** **(d) None of these**

**[NCERT Exemplar]**

**5. A relation R is defined on the set R of all real numbers as follows:**

**$R = \{(a, b) : a, b \in R$ and $a - b + \sqrt{3} \in S\}$, where S is the set of all irrational numbers. Then, this relation R is:**

**(a) reflexive only**

**(b) symmetric only**

**(c) transitive only**

**(d) an equivalence relation**

**Ans.** *(a) reflexive only*

**Explanation:** Let $l$ and $m$ be any two lines in a plane.

R is reflexive, since $(a - a) + \sqrt{3} = \sqrt{3} \in R, \forall a \in R$

R is not symmetric, since $a - b + \sqrt{3} \in S$ does not imply $b - a + \sqrt{3} \in S$, for $a = 1$ and $b = -\sqrt{3}$ {No need to examine for transitivity}

### ⚠ Caution

➥ *In such types of questions, start with examining reflexive, then symmetric and the transitive. If a relation is reflexive but not symmetric, then it is not equivalence. Here, no need to examine transitivity. As all three conditions need to satisfied for being an equivalence relation.*

**6. The relation R in the set {$a$, $b$, $c$} given by R = {($a$, $a$), ($b$, $b$), ($a$, $b$), ($b$, $a$)} is:**

**(a) symmetric and transitive, but not reflexive.**

**(b) reflexive and symmetric, but not transitive.**

**(c) symmetric, but neither reflexive nor transitive.**

**(d) an equivalence relation.**

**[Delhi Gov. 2022]**

**Ans.** *(a) symmetric and transitive, but not reflexive.*

**Explanation:** We have,

$$R = \{(a, a), (b, b), (a, b), (b, a)\}$$

$\because \quad (c, c) \notin R$

$\therefore$ R is not reflexive.

For $(a, b) \in R$, we have $(b, a) \in R$

$\therefore$ R is symmetric.

For $(a, b) \in R$ and $(b, a) \in R$, we have $(a, a) \in R$.

$\therefore$ R is transitive.

**7. A = {1, 2, 3, 4}. A relation R in the set A is given by**

**R = {(1, 1), (2, 3), (3, 2), (4, 3), (3, 4)}, then relation R is:**

**(a) Reflexive (b) Symmetric**

**(c) Transitive (d) Equivalence**

**[Delhi Gov. 2022]**

**8. Let set X = {1, 2, 3} and a relation R is defined in X as : R = {(1, 3), (2, 2), (3, 2)}, then minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are:**

**(a) {(1, 1), (2, 3), (1, 4)}**

**(b) {(5, 3), (3, 1), (1, 2)}**

**(c) {(1, 1), (3, 3), (3, 1), (2, 3)}**

**(d) {(1, 1), (3, 3), (3, 1), (1, 2)}**

**[CBSE Term-1 2021]**

**Ans.** *(c) {(1, 1), (3, 3), (3, 1), (2, 3)}*

**Explanation:** The ordered pairs to be added to R are:

(1, 1), (3, 3) {needed to make R reflexive}

(3, 1), (2, 3) {needed to make R symmetric}

So, {(1, 1) (3, 3), (3, 1), (2, 3)} are required.

**9. Let us define a relation R in R as $a$ R $b$ if $a \geq b$. Then R is:**

**(a) an equivalence relation**

**(b) reflexive, transitive but not symmetric**

**(c) symmetric, transitive but not reflexive**

**(d) neither transitive nor reflexive but symmetric.** **[NCERT Exemplar]**

**10. Let A = {1, 2, 3} and consider the relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}, then R is:**

**(a) reflexive but not symmetric**

**(b) reflexive but not transitive**

**(c) symmetric and transitive**

**(d) neither symmetric nor transitive.**

**[NCERT Exemplar]**

**Ans.** *(a) reflexive but not symmetric*

**Explanation:** Given, R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}

Here, (1, 1), (2, 2) and (3, 3) $\in$ R.

So, R is reflexive.

$(1, 2) \in R$ but $(2, 1) \notin R$, $(2, 3) \in R$ but $(3, 2) \notin R$, $(1, 3) \in R$ but $(3, 1) \notin R$.

So, R is not symmetric.

For $(1, 2) \in R$, $(2, 3) \in R$

$\Rightarrow (1, 3) \in R$.

So, R is transitive.

**11. Let L be the set of all lines in a plane. A relation R in L is given by R = {( $L_1$, $L_2$): $L_1$ and $L_2$ intersect at exactly one point, $L_1$, $L_2$, $\in$ L}. Then, the relation R is:**

**(a) Reflexive (b) Symmetric**

**(c) Transitive (d) Equivalence**

**[Delhi Gov. 2022]**

**12. A relation R in the set of real numbers R is given by R = {($a$, $b$) : $a > b$, $a$, $b \in$ R}. The relation R is:**

**(a) Reflexive (b) Symmetric**

**(c) Transitive (d) Equivalence**

**Ans.** *(c) Transitive*

**Explanation:** We have,

$$R = \{(a, b) : a > b, a, b \in R\}$$

$\because$ For any $a \in R$, $a > a$ is not true.

$\therefore$ R is not reflexive.

Let $(a, b) \in R$, $a, b \in R$.

$\Rightarrow \quad a > b$

or, $\quad b < a$

$\Rightarrow \quad b \not> a$

$\Rightarrow \quad (b, a) \notin R$

$\therefore$ R is not symmetric.

Let $(a, b) \in R$ and $(b, c) \in R$, $a, b, c \in R$

$\therefore \quad a > b$ and $b > c$

$\Rightarrow \quad a > c$

$\Rightarrow \quad (a, c) \in R$

$\therefore$ R is transitive

$\because$ R is neither reflexive nor symmetric, so it is not an equivalence relation.

**13.** **If $R = \{(x, y) : 2x + y = 8\}$ is a relation on N, then the range of R is:**

(a) {2, 4, 6} (b) {1, 2, 3}

(c) {2, 5, 6} (d) {3, 4, 5}

**14.** **If $R = \{(x, y) : x, y \in Z, x^2 + y^2 \leq 4\}$ is a relation in set Z. Then domain of R is:**

(a) {0, 1, 2} (b) {– 2, –1, 0, 1, 2}

(c) [0, –1, –2] (d) {–1, 0, 1}

**[CBSE Term-1 2021]**

**Ans.** *(b) {– 2, –1, 0, 1, 2}*

**Explanation:** $x^2 + y^2 \leq 4$

$\Rightarrow \quad y^2 \leq 4 - x^2$

$\Rightarrow \quad y \leq \sqrt{4 - x^2}$

For Domain: $4 - x^2 \geq 0$

$\Rightarrow \quad x^2 \leq 4$

$\therefore \quad -2 \leq x \leq 2$

So, Domain = {–2, –1, 0, 1, 2}.

**15.** **Let T be the set of all triangles in the Euclidean plane and let a relation R on T be defined as *a* R *b*, if *a* is congruent to *b*, $\forall$ *a*, *b* $\in$ T. Then R is:**

(a) Reflexive but not transitive

(b) Transitive but not symmetric

(c) Equivalence

(d) None of these **[NCERT Exemplar]**

**Ans.** *(c) Equivalence*

**Explanation:** Any triangle is congruent to itself. So, for $a \in T$, $a$ R $a$.

So, R is reflexive.

Now, let $a$ R $b$.

*i.e.*, Triangle *a* is congruent to triangle *b*, then triangle *b* is also congruent to triangle *a* $\forall$ *a*, *b* $\in$ T.

So, R is symmetric.

Let *a* R *b* and *b* R *c*.

$\Rightarrow$Triangle *a* is congruent to triangle *b*, triangle *b* is congruent to triangle *c*.

Then triangle *a* is congruent to triangle *c* $\forall$ *a*, *b*, *c* $\in$ T.

So, R is transitive.

Since, R is reflexive, symmetic and transitive,

Hence, R is an equivalence relation.

**16.** **A relation is a set of all:**

(a) ordered pairs (b) functions

(c) *y*-values (d) none of these

**[DIKSHA]**

**17.** **There are two housing societies "Shivani Apartment" and "Kunj Vihar" in a particular neighbourhood of Dwarka, New Delhi. There are boys and girls of all ages in both the societies. Three boys and three girls were selected from Shivani apartment and Kunj Vihar housing society respectively. The boys were studying in classes 3, 5 and 7 whereas the girls were studying in classes 2, 4 and 9. This can be denoted as A = {3, 5, 7} and B = {2, 4, 9}.**

**Let A = {3, 5, 7} and B = {2, 4, 9} and R be a relation from A to B defined as "is greater than". Then, R in roster form is**

(a) R = {(3, 2), (5, 2), (5, 4), (7, 2), (7, 4)}

(b) R = {(2, 3), (5, 2), (4, 5), (7, 2), (4, 7)}

(c) R = {(3, 2), (2, 5), (4, 5), (2, 7), (4, 7)}

(d) R = {(1, 2), (3, 2), (6, 3), (2, 2), (5, 4)}

**Ans.** *(a) R = {(3, 2), (5, 2) ,(5, 4), (7, 2), (7, 4)}*

**Explanation:** Given, R be a relation from A to B defined as "is greater than". Therefore, in ordered pair, element from set A should be greater than element from set B.

So, R = {(3, 2), (5, 2), (5, 4), (7, 2), (7, 4)}

**Caution**

*In such types of questions, write the elements in proper order i.e., if a relation is X to Y then element of X is written first then element of Y in ordered pair.*

## VERY SHORT ANSWER Type Questions (VSA)

[ 1 mark ]

**18.** **Let the relation R be defined on the set N by *a* R *b* if 2*a* + *b* = 20. Write R in roster form.**

**Ans.** R = {(1, 18), (2, 16), (3, 14), (4, 12), (5, 10), (6, 8), (7, 6), (8, 4), (9, 2)}.

**19.** **If R = {(x, y): x + 2y = 8} is a relation on N, write the range of R.** **[CBSE 2014]**

**Ans.** R = {(x, y): 2x + y = 8} is a relation on N.

Therefore, R = {(3, 2), (2, 4), (1, 6)}

So, Range = {2, 4, 6}.

**20.** **Show that the relation R defined as "is a subset of" on the set of all possible subsets of a universal set U is reflexive and transitive but not symmetric.**

**21.** **Let R = {$(a, a^3)$: $a$ is a prime number less than 5} be a relation. Find the range of R.** **[CBSE 2014]**

**22.** **State the reason for the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} not to be transitive.** **[CBSE 2011]**

**Ans.** As, (1, 2) ∈ R and (2, 1) ∈ R, then for the relation R to be transitive, (1, 1) ∈ R but (1, 1) ∉ R. Hence, R is not transitive.

**23.** **Let the relation R in the set A = {$x$ ∈ Z : $0 \le x \le 12$} given by R = {$(a, b)$ : $|a - b|$ is a multiple of 4}. Then write the equivalence class of [1].** **[CBSE Term-1 SQP 2021]**

## SHORT ANSWER Type-I Questions (SA-I)

[ 2 marks ]

**24.** **Let A = {$a$, $b$, $c$} and the relation R be defined on A as follows:**

**R = {$(a, a)$, $(b, c)$, $(a, b)$}**

**Then, write minimum number of ordered pairs to be added in R to make R reflexive and transitive.** **[NCERT Exemplar]**

**25.** **Let R be a relation defined on Z as follows:**

**R = {$(x, y)$: $|x - y| \le 1$}**

**Show that R is reflexive and symmetric.**

**Ans.** **For reflexive:**

Let $x$ ∈ Z, then $|x - x| = 0 \le 1$.

So, $(x, x)$ ∈ R. Thus, R is a reflexive relation.

**For symmetric:**

Let $x, y$ ∈ Z such that $|x - y| \le 1$, then $|y - x| \le 1$.

So, $(x, y) \in R \Rightarrow (y, x) \in R$.

Thus, R is a symmetric relation.

**26.** **Let $n$ be a fixed positive integer. Define a relation R in Z as follows: ∀ $a, b$ ∈ Z, $a$ R $b$ if and only if $a - b$ is divisible by $n$. Show that R is an equivalence relation.** **[NCERT Exemplar]**

**27.** **Find the maximum number of equivalence relations on the set A = {1, 2, 3}.**

**Ans.** Given, A = {1, 2, 3}

Possible equivalence relations are:

$R_1$ = {(1, 1), (2, 2), (3, 3)}

$R_2$ = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)}

$R_3$ = {(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)}

$R_4$ = {(1, 1), (2, 2), (3, 3), (2, 3), (3, 2)}

$R_5$ = A × A

Hence, the possible number of equivalence relations is 5.

## SHORT ANSWER Type-II Questions (SA-II)

[ 3 marks ]

**28.** **Let R be a relation defined on the set of natural numbers N as R = $(x, y)$ : $x$ ∈ N, $y$ ∈ N and $2x + y = 24$}. Then, find the domain and range of the relation R. Also, find whether R is an equivalence relation or not.** **[CBSE 2014]**

**Ans.** Given,

$2x + y = 24$

$\Rightarrow y = 24 - 2x$

For $x = 1$, $y = 24 - 2 \times 1 = 24 - 2 = 22$

For $x = 2$, $y = 24 - 2 \times 2 = 24 - 4 = 20$

For $x = 3$, $y = 24 - 2 \times 3 = 24 - 6 = 18$

For $x = 4$, $y = 24 - 2 \times 4 = 24 - 8 = 16$

For $x = 5$, $y = 24 - 2 \times 5 = 24 - 10 = 14$

For $x = 6$, $y = 24 - 2 \times 6 = 24 - 12 = 12$

For $x = 7$, $y = 24 - 2 \times 7 = 24 - 14 = 10$

For $x = 8$, $y = 24 - 2 \times 8 = 24 - 16 = 8$

For $x = 9$, $y = 24 - 2 \times 9 = 24 - 18 = 6$

For $x = 10$, $y = 24 - 2 \times 10 = 24 - 20 = 4$

For $x = 11$, $y = 24 - 2 \times 11 = 24 - 22 = 2$

In ordered pair, R = {(1, 22), (2, 20), (3, 18), (4, 16), (5, 14), (6, 12), (7, 10), (8, 8), (9, 6), (10, 4), (11, 2)}

∴ Domain of R = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}

Range of R = {22, 20, 18, 16, 14, 12, 10, 8, 6, 4, 2}

Here, 1 ∈ N but (1, 1) ∉ R, hence R is not reflexive.

Hence, R is not an equivalence relation.

**29.** Show that relation R in the set of real numbers, defined as $R = \{(a, b): a \le b^2\}$ is neither reflexive, nor symmetric, nor transitive.

**30.** Let R be a relation defined on the set of natural numbers N as follows:

$$R = \{(x, y) : x \in N, y \in N, 2x + y = 41\}$$

Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive. [NCERT Exemplar]

**Ans.** Given, $x \in N$, $y \in N$ and $2x + y = 41$

$\therefore$ Domain of R *i.e.*, possible values of $x = \{1, 2, 3, ..., 20\}$

And, Range *i.e.*, possible values of $y = \{39, 37, 35, ..., 5, 3, 1\}$

(1) $(5, 5) \notin R$ as $2 \times 5 + 5 = 15 \neq 41$

So, R is not reflexive.

(2) $(3, 35) \in R$ but $(35, 3) \notin R$. Hence R is not symmetric.

(3) $(15, 11) \in R$ and $(11, 19) \in R$ but $(15, 19) \notin R$. Hence, R is not transitive.

Hence, R is neither reflexive, nor symmetric and nor transitive.

**31.** Let $f : X \to Y$ be a function. Define a relation R on X given by $R = \{(a, b): f(a) = f(b)\}$. Show that R is an equivalence relation on X. [CBSE 2010]

**Ans.** The given relation is $R = \{(a, b): f(a) = f(b)\}$

**Reflexive:** For every $x \in X$, we have

$f(x) = f(x)$

$\Rightarrow (x, x) \in R$

Hence, R is reflexive.

**Symmetric:** Let $(x, y) \in R \ \forall \ x, y \in R$.

Then $f(x) = f(y)$

$\Rightarrow f(y) = f(x)$

$\therefore (y, x) \in R \ \forall \ x, y \in R$

Hence, R is symmetric.

**Transitive:** Let $x, y, z \in X$

Also, $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow f(x) = f(y) \ \forall \ x, y \in X$ ...(i)

and $f(y) = f(z) \ \forall \ y, z \in X$ ...(ii)

From equations (i) and (ii), we get

$f(x) = f(z) \ \forall \ x, z \in X$

$\Rightarrow (x, z) \in R \ \forall \ x, z \in X$

Hence, R is transitive.

Since, R is reflexive, symmetric and transitive, hence, it is an equivalence relation.

**32.** If $A = \{1, 2, 3, 4\}$, define relations on A which have properties of being

(A) Reflexive, transitive but not symmetric.

(B) Symmetric but neither reflexive nor transitive.

(C) Reflexive, symmetric and transitive.

[NCERT Exemplar]

**33.** Let $A = \{x \in Z : 0 \le x \le 12\}$. Show that $R = \{(a, b): a, b \in A, |a - b|$ is divisible by $4\}$ is an equivalence relation. Find the set of all elements related to 1. Also, write the equivalence class [2]. [CBSE 2018]

**Ans. For reflexive:**

Let $a \in Z$, then $|a - a| = 0$, which is divisible by 4.

So, $(a, a) \in R$. Thus, R is a reflexive relation.

**For symmetric:**

Let $a, b \in Z$ such that $|a - b|$ is divisible by 4. Then $|b - a|$ is also divisible by 4 $[\because |b - a| = |a - b|]$

So, $(a, b) \in R$

$\Rightarrow (b, a) \in R$. Thus, R is a symmetric relation.

**For transitive:**

Let $a, b, c \in Z$ and $(a, b) \in R$ and $(b, c) \in R$.

Since $(a, b) \in R$ and $(b, c) \in R$

Therefore, $|a - b| = 4k$

$\Rightarrow (a - b) = \pm 4k$ and $|b - c| = 4l \Rightarrow (b - c) = \pm 4l$

Now, $(a - c) = (a - b) + (b - c) = \pm 4k \pm 4l = 4(\pm k \pm l)$, which is divisible by 4.

So, $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow (a, c) \in R$. Thus, R is transitive.

Since, R is reflexive, symmetric and transitive, hence it is an equivalence relation.

Set of elements related to 1 is $\{(1, 1), (1, 5), (5, 1), (1, 9), (9, 1)\}$

Set of elements related to 2 is $\{(2, 2), (2, 6), (2, 10)\}$

So, equivalence class of [2] is $\{2, 6, 10\}$.

**34.** Let $A = \{1, 2, 3, ..., 9\}$ and R be the relation in $A \times A$ defined by $(a, b)$ R $(c, d)$ if $a + d = b + c$ for $(a, b), (c, d)$ in $A \times A$. Prove that R is an equivalence relation and also obtain equivalence class $[(2, 5)]$. [NCERT Exemplar]

## LONG ANSWER Type Questions (LA)

[ 4 & 5 marks ]

**35. Let N denote the set of all natural numbers and R be a relation on N × N defined by $(a, b)$ R $(c, d)$ if $ad(b + c) = bc(a + d)$. Prove that R is an equivalence relation. [CBSE 2015]**

**Ans. Reflexive:** Let $(a, b) \in N \times N$.

$\because \quad ab(b + a) = ba(a + b)$

$\Rightarrow (a, b)$ R $(a, b)$

$\Rightarrow$ R is reflexive.

**Symmetric:** For $(a, b), (c, d) \in N \times N$ such that $(a, b)$ R $(c, d)$.

$\therefore \quad ad(b + c) = bc(a + d)$

or, $\quad bc(a + d) = ad(b + c)$

or, $\quad cb(d + a) = da(c + b)$

$\Rightarrow (c, d)$ R $(a, b)$

$\Rightarrow$ R is symmetric.

**Transitive:** Let $(a, b), (c, d), (e, f) \in N \times N$ such that $(a, b)$ R $(c, d)$ and $(c, d)$ R $(e, f)$.

$\therefore \quad ad(b + c) = bc(a + d)$

and $\quad cf(d + e) = de(c + f)$

$\Rightarrow \quad adb + adc = bca + bcd \quad$ ...(i)

and $\quad cfd + cfe = dec + def \quad$ ...(ii)

Multiplying (i) by $ef$ and (ii) by $ab$ and then adding them, we get

$adbef + adcef + cfdab + cfeab = bcaef + bcdef + decab + defab$

$\Rightarrow adcef + adcfb = bcdea + bcdef$

$\Rightarrow adcf(e + b) = bcde(a + f)$

$\Rightarrow af(b + e) = be(a + f)$

$\Rightarrow (a, b)$ R $(e, f)$

$\Rightarrow$ R is transitive.

Hence, R is an equivalence relation.

# FUNCTIONS AND ITS TYPES 2

## TOPIC 1

## FUNCTIONS

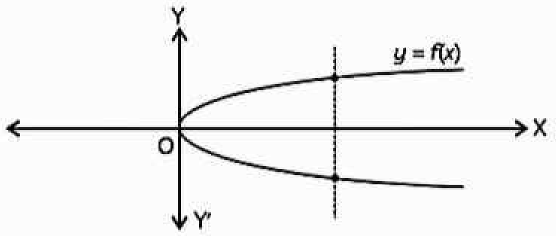
A function $f$ is a relation from a non-empty set A to a non-empty set B such that the domain of $f$ is A and no two distinct ordered pairs in $f$ have the same first element *i.e.*, if $(a, b) \in f$ and $(a, c) \in f$, then $b = c$.

In other words, a relation $f$ from a non-empty set A to a non-empty set B is said to be function if every element of set A has a unique image in set B.

If $f$ is a function from A to B and $(a, b) \in f$, then $b$ is called the image of $a$ under $f$; and $a$ is called the pre-image of $b$ under $f$.

The function $f$ from A to B is denoted by $f: A \to B$.

**Graphical Test Function:** If any straight line, parallel to $y$-axis, cut the curve at more than one point, then that curve does not represents a function.




$y = f(x)$ is not a function

**Illustration:** (1) Consider a relation $R = \{(x, y): y = x + 1\}$ defined in the set $A = \{1, 2, 3, 4, 5, 6\}$.

This relation is not a function, because the element $6 \in A$ has no image.

(2) Consider a relation R from a set $P = \{4, 9, 25\}$ to a set $Q = \{-5, -3, -2, -1, 1, 2, 3, 5\}$ defined as $R = \{(x, y) : x$ is the square of $y$, where $x \in P$ and $y \in Q\}$.

In roster form, $R = \{(4, 2), (4, -2), (9, 3), (9, -3), (25, 5), (25, -3)\}$

This relation is not a function, because $4 \in P$ has two images 2 and $-2$.

(3) consider a relation R in N (set of all natural numbers) defined as $R = \{(x, y) : y = 2x$ where $x, y \in N\}$

This relation is a function.

### Domain, Co-domain and Range of a Function

Let $f: A \in B$ be a function.

The set A is called the domain of $f$ and set B is called the co-domain of $f$.

The set of all $f$-images, corresponding to each element of set A is called the range of $f$.

**Illustration:** Consider the function $f: N \to N$ defined as $f = \{(x, y) : y = 2x, x, y \in N\}$

The domain of $f$ is N. The co-domain is also N. But the range is the set of all even natural numbers.

**Important**

➥ *Range of $f \subseteq$ Co-domain of $f$.*

## TOPIC 2

## REAL VALUED FUNCTION AND REAL FUNCTION

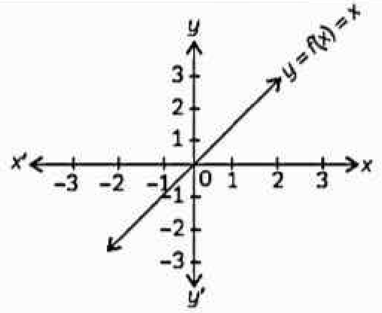
A function whose range is a subset of real numbers is called real valued function.

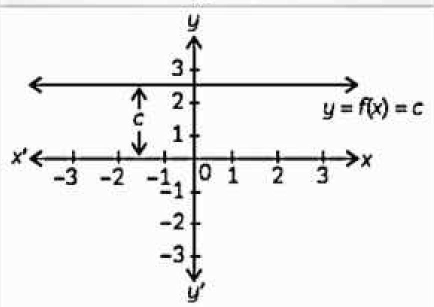
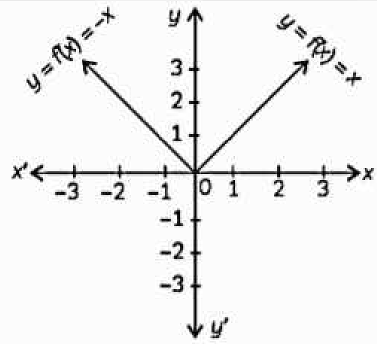
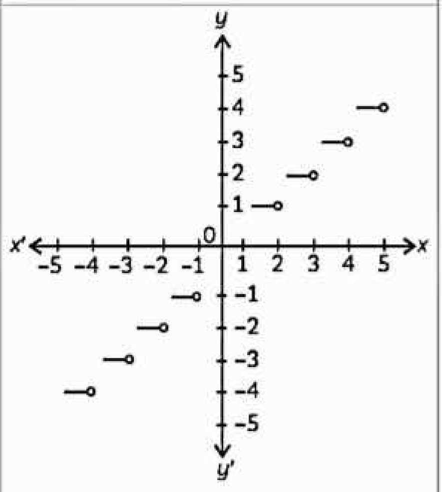
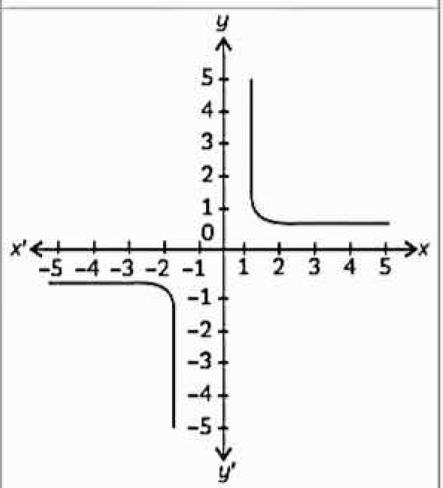
Further, if domain of the function is either real numbers or a subset of real numbers, then it is called a real function.

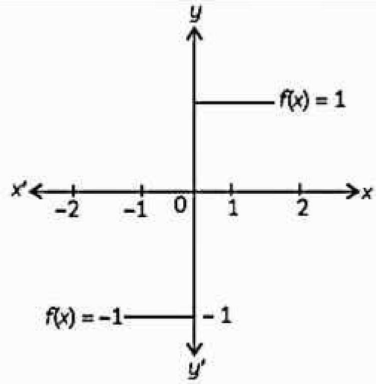
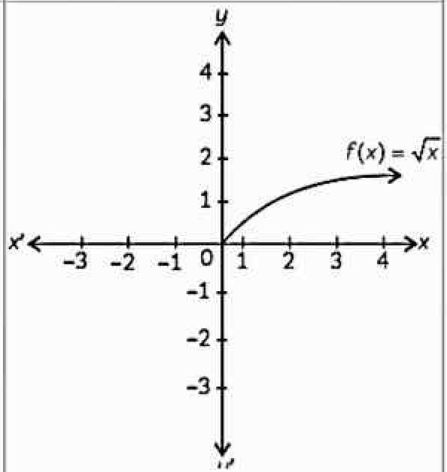
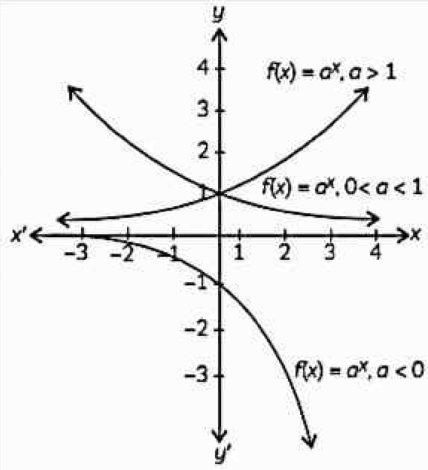
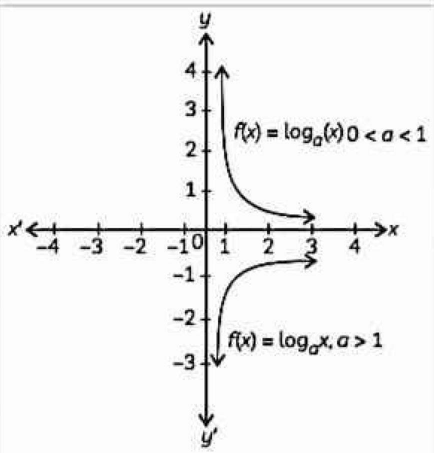
**Important**

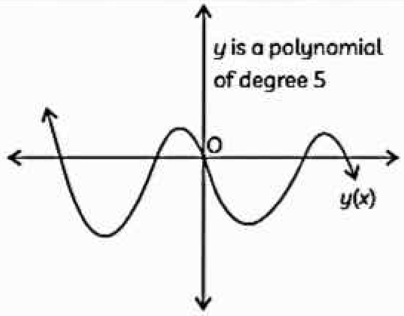
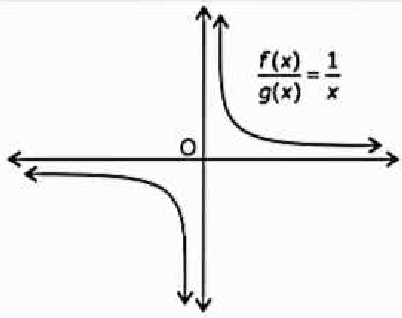
➥ *Every real function is a real valued function but converse is not true, e.g. $f : C \to R$ is a real valued function but not a real function, where C is the set of complex numbers.*

### Some Special Real functions

| Function | Domain | Range | Example | Figure |
|---|---|---|---|---|
| Identity function | R | R | $f(x) = x, \forall x \in R$ |  |

| Function | Domain | Range | Example | Figure |
|---|---|---|---|---|
| Constant function | R | $c$ | $f(x) = c, \forall x \in R$ where, $c$ is any real number |  |
| Modulus function | R | $[0, \infty)$ | $f(x) = \|x\|, \forall x \in R$ OR $f(x) = \begin{cases} x, x \geq 0 \\ -x, x < 0 \end{cases}$ |  |
| Greatest Integer function | R | I | $f(x) = [x], \forall x \in R$ Where, $[x]$ = an integer less or equal to $x$ |  |
| Reciprocal function | R – {0} | R – {0} | $f(x) = \frac{1}{x} \forall x \in R - \{0\}$ |  |

| Function | Domain | Range | Example | Figure |
|---|---|---|---|---|
| Signum function | R | {– 1, 0, 1} | $f(x) = \begin{cases} \dfrac{\lvert x \rvert}{x}, x \neq 0 \\ 0,\ x = 0 \end{cases}$ <br> OR <br> $f(x) = \begin{cases} 1, x > 0 \\ -1, x < 0 \\ 0, x = 0 \end{cases}$ |  |
| Square Root function | $R^+$ | [0, ∞) | $f(x) = \sqrt{x}, \forall\, x \in R^+$ |  |
| Exponential function | R | [0, ∞) | $f(x) = a^x, x \in R,$ |  |
| Logarithmic function | $R^+$ | R | $f(x) = \log_a x, \forall\, x \in R$ |  |

| Function | Domain | Range | Example | Figure |
|---|---|---|---|---|
| Polynomial function | R | R | $y = f(x) = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_nx^n$, where $n$ is a non-negative integer and $a_0, a_1, a_2, \ldots\ldots, a_n \in$ R. |   |
| Rational function | In its domain and $g(x) \neq 0$ | In its image and $g(x) \neq 0$ | $\frac{f(x)}{g(x)}$ |   |

## TOPIC 3

## ALGEBRA OF REAL FUNCTIONS

Let $f: X \to R$ and $g: X \to R$ be any two real functions, where $X \subseteq R$. Then, we define

(1) $(f \pm g): X \to R$ by $(f \pm g)(x) = f(x) \pm g(x)$, for all $x \in X$.

(2) $(f.g): X \to R$ by $(f.g)(x) = f(x).g(x)$, for all $x \in X$.

(3) $\left(\frac{f}{g}\right): X \to R$ by $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$, $x \in X$.

Let $f: X \to R$ and $\alpha$ be a real number. Then, we define $(\alpha f): X \to R$ by $(\alpha f)(x) = \alpha f(x)$, for all $x \in X$.

**Illustration:** Let $f(x) = x^2$ and $g(x) = 2x + 1$ be two real functions. Then,

(1) $(f + g)(x) = x^2 + 2x + 1$ ;

(2) $(f - g)(x) = x^2 - 2x - 1$

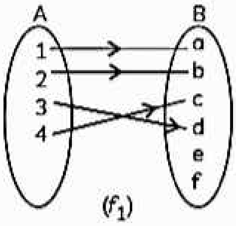
(3) $(f.g)(x) = x^2 (2x + 1) = 2x^3 + x^2$

(4) $\left(\frac{f}{g}\right)(x) = \frac{x^2}{2x+1}, x \neq -\frac{1}{2}$

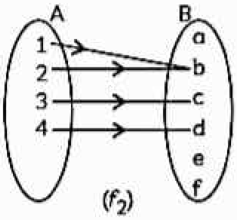
## TOPIC 4

## TYPES OF FUNCTIONS

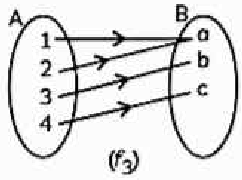
If $f$ function $f$ : A $\to$ B, then $f$ associates all the elements of set A to the elements in set B such that an element of set A is associated to a unique element of set B. But there are some more possibilities which may occur in a function, as demonstrated below in the four diagrams:



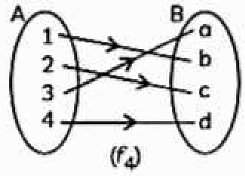

$(f_1)$




$(f_2)$




$(f_3)$




$(f_4)$

We observe that the images of distinct elements of A under the function $f_1$ are distinct, but the image of two distinct elements 1 and 2 of A under the function $f_2$ is same, namely $b$.

Further, there are some elements like $e$ and $f$ in B which are not images of any element of A under the function $f_1$, while all elements of B are images of some elements of A under $f_3$.

## Onto (Surjective) Function

A function $f: X \to Y$ is called an onto (or surjective) function, if every element of Y is the image of some element of X under $f$, *i.e.*, for every $y \in Y$, there exists an element $x \in X$ such that $f(x) = y$.

In other words, $f: X \to Y$ is onto if and only if range of $f = Y$ (Co-domain)

**Illustration:** Let X = {1, 3, 5, 7} and Y = {2, 4, 6, 8} are related under function $f(x) = x + 1$.

Here, every element of Y has pre-image in X. Hence, $f(x)$ is an onto function.

**Important**

➥ *A function $f: X \to Y$ is onto if and only if Range of $f = Y$.*

**Example 2.4: Show that the signum function $f: R \to R$ given by**

$$f(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \\ 0, & x = 0 \end{cases}$$

**is not an onto function. [NCERT]**

**Ans.** Here, the Range of $f$ is $\{-1, 0, 1\} \neq R$ (co-domain). So, $f$ is not onto.

**Example 2.5: Show that the function $f: R \to R$ defined by $f(x) = 3x$ is onto. [NCERT]**

**Ans.** Let $y \in R$ (co-domain) be an arbitrary element.

If possible, there exist a $x \in R$ such that $f(x) = y$, for some $x \in R$ (domain).

Then, $y = 3x$

$\Rightarrow \quad x = \frac{y}{3} \in R$ (Domain)

Thus, for every $y \in R$ (co-domain), we have $x \in R$ (domain).

So, $f$ is onto.

### Method to Check Whether a Function is Onto or Into

To check whether a given function $f: X \to Y$ is onto or into, we use the following steps:

(1) Take $y$ as an arbitrary element of Y (Co – domain).

(2) Put $f(x) = y$ and simplify the equation to get $x$ in terms of $y$.

(3) Now, if for any $y \in Y$, the corresponding value of $x$ does not belong to X, Then $f$ is not onto, *i.e.*, $f$ is into.

(4) But if for all $y \in Y$, the corresponding value of $x$ belongs to X, then $f$ is onto.

**Caution**

➥ *A function $f: N \to N$ (or, $f: Z \to Z$) defined by $f(x) = 3x$ is not onto.*

**Important**

➥ *Let $f: A \to B$ be a function, where $n(A) = m$ and $n(B) = n$. Then,*

*(1) If $n(B) > n(A)$, there is no onto function from A to B.*

*(2) If $n(B) \leq n(A)$, there is no easy formula to determine the number of onto functions. However, if $n(B) = 2$, the number of onto functions is $2^m - 2$.*

*(3) The number of onto functions from A to A is $m!$*

## Graphical Test for Onto (or Surjectivity)

If any horizontal line intersects the graph of a function $f(x)$ at least once, then the function is onto, otherwise not.

## One-One and Onto (Bijective) Function

A function $f: X \to Y$ is called one-one and onto (or bijective) function, if $f$ is both one-one and onto.

**Important**

➥ *For a function to be bijective, it should be both one-one and onto.*

**Example 2.6: Show that the function $f: R \to R$ given by $f(x) = 3 - 4x$ is one–one and onto (or bijective).**

**Ans.** (1) Let $x_1$ and $x_2$ be any two arbitrary elements of R (domain).

Then, $f(x_1) = f(x_2)$ implies $3 - 4x_1 = 3 - 4x_2$

$\Rightarrow \quad x_1 = x_2$

So, $f$ is one – one function.

(2) Let $y \in R$ (co-domain) be an arbitrary element such that $f(x) = y$, for some $x \in R$ (domain).

Then, $y = 3 - 4x$

$\Rightarrow \quad x = \frac{3-y}{4}$

For every $y \in R$ (co-domain), $\frac{3-y}{4}$ is a real number.

Thus, $x \in R$ (domain).

So, $f$ is onto function.

**Example 2.7: Show that the function $f: R \to R$ given by $f(x) = 1 + x^2$ is not bijective.**

**Ans.** Let $x_1$ and $x_2$ be any two arbitrary elements of R (domain).

Then, $f(x_1) = f(x_2)$ implies $1 + {x_1}^2 = 1 + {x_2}^2$

$\Rightarrow \quad {x_1}^2 = {x_2}^2$

$\Rightarrow \quad x_1 = \pm x_2$

Since, $f(1) = f(-1)$, but $1 \neq -1$

So, $f$ is not one – one function and hence not a bijective function.

**Important**

➥ *This function is also not onto.*

*Let $y \in R$ (co-domain) be an arbitrary element such that $f(x) = y$, for some $x \in R$ (domain).*

Thus, in a function, following possibilities may occur:

(1) More than one element of domain (say, A) may have same image in co-domain (say, B);

(2) There may be some elements in B, which are not the images of any element of A.

⚠ **Caution**

*➥ Under a function $f: A \rightarrow B$, no element of A can have two or more images in B.*

The above observations lead to the following definitions:

## One-One (Injective) Function

A function $f: X \rightarrow Y$ is called one-one (or injective) function, if distinct elements of X under f have distinct images in Y, *i.e.*, for every $x_1, x_2 \in X$, $f(x_1) = f(x_2)$ implies $x_1 = x_2$.

or $x_1 \neq x_2$ implies $f(x_1) \neq f(x_2)$, for every $x_1, x_2 \in X$.

**Illustration:** The function $f(x) = 2x + 1$ always gives unique value of $f(x)$ for different values of $x$.

*i.e.*, $f(1) = 2 \times 1 + 1 = 3$, $f(2) = 2 \times 2 + 1 = 5$, $f(-1) = 2 \times (-1) + 1 = -1$ etc.

**Example 2.1: Let A = R - {3} and B = R - {1}. Consider the function $f: A \rightarrow B$ defined by $f(x) = \frac{x-2}{x-3}$. Show that $f$ is one-one function. [NCERT]**

**Ans.** Let $x_1$ and $x_2$ be any two arbitrary elements of A. Also, let

$$f(x_1) = f(x_2)$$

$$\Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$

$$\Rightarrow x_1 x_2 - 3x_1 - 2x_2 + 6 = x_1 x_2 - 2x_1 - 3x_2 + 6$$

$$\Rightarrow x_1 = x_2$$

So, $f$ is one-one function.

**Important**

*➥ Let $f: A \rightarrow B$ be a function, where $n(A) = m$ and $n(B) = n$. Then,*

*(1) The number of functions from A to B is the same as the number of ways of assigning images to every element of A and hence it is $n^m$.*

*(2) If $n(B) < n(A)$, then there is no one - one function possible from A to B.*

*(3) If $n(B) \geq n(A)$, then number of one - one functions from A to B is ${}^nP_m$.*

*(4) The number of one- one functions from A to A is m !*

## Many-One Function

A function $f: X \rightarrow Y$ is called many-one function, if more than one element of X have a common/same image in Y.

**Illustration:** The function $f(x) = x^2$ is a many-one function.

Here, $f(1) = (1)^2 = 1$, $f(-1) = (-1)^2 = 1$. Similarly, $f(2) = 2^2 = 4$, $f(-2) = (-2)^2 = 4$ etc, we can see that 1 and (–1) give same value of $f(x)$, also 2 and (–2) give same value of $f(x)$.

**Example 2.2: Show that the modulus function $f: R \rightarrow R$ defined by $f(x) = |x|$ is a many-one function. [NCERT]**

**Ans.** Here, for two different elements, say 1 and –1 of the domain R of the function $f$, we have $|1| = |-1| = 1$. So, $f$ is a many-one function.

**Example 2.3: Show that the signum function $f: R \rightarrow R$ given by**

$$f(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \\ 0, & x = 0 \end{cases}$$

**is a many-one function. [NCERT]**

**Ans.** Here, all positive real numbers of the domain R of the function $f$ have the same image 1; and all negative real numbers of the domain R of the function $f$ have the same image –1. So, $f$ is a many-one function.

### Method to Check Whether a Function is One-One or Many-One

To check whether a given function is one-one or not, we use the following steps:

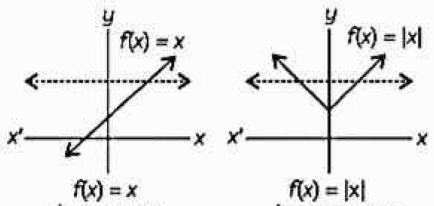
(1) Take any two arbitrary elements, say $x_1, x_2$ from the domain of the function.

(2) Put $f(x_1) = f(x_2)$ and simplify the equation.

(3) If we get $x_1 = x_2$, then the given function is one-one. In case, we get $x_1 \neq x_2$, the given function is many-one.

### Graphical Test for One-One (or injectivity)

If any horizontal line intersects the graph of a function $f(x)$ at only one point, then the function is one-one, otherwise not.




f(x) = x is one-one        f(x) = |x| is many-one

## Into Function

A function $f: X \rightarrow Y$ is called an into function, if there is at least one element of Y which is not the image of any element of X under $f$.

**Illustration:** Let X = {2, 3, 4} and Y = {4, 6, 8, 10} are related under function $f(x) = 2x$.

Here we can see that for element 10 in set Y there is no pre-image in X. Hence, the function $f(x)$ is into.

*Then,* $y = 1 + x^2 \Rightarrow x = \pm\sqrt{y-1}$

*For every* $y = 0 \in R$ *(co-domain), we have* $x \notin R$,
*So, f is not onto function.*

### ⚠ Caution

➥ *Let* $f: A \rightarrow B$ *be a function, where* $n(A) = m$ *and* $n(B) = n$. *Then,*

*(1) the number of bijective functions (or bijections) from A to B* $= \begin{cases} m!, & \text{if } m = n \\ 0, & \text{if } m \neq n \end{cases}$

*(2) the number of bijective functions from A to A* $= m!$

# OBJECTIVE Type Questions

[ 1 mark ]

## Multiple Choice Questions

**1.** **The range of the function** $f(x) = \dfrac{|x+1|}{x+1}$ **is:**

**(a) {-1, 1}** **(b) {0, 1}**
**(c) {1}** **(d) {-1}**

**2. Let A = {1, 2, 3, ..., *n*} and B = {*a*, *b*}. Then the number of surjections from A to B is:**

**(a)** $^nP_2$ **(b)** $2^n - 2$
**(c)** $2^n - 1$ **(d) None of these**

**[NCERT Exemplar]**

**Ans.** *(a)* $^nP_2$

**Explanation:** Given, A = {1, 2, 3, ..., *n*} and B = {*a*, *b*}

Let *p* and *q* be the number of elements of set A and set B respectively.

Number of surjections from A to B is $^pC_q \times q!$ as $p \geq q$.

Here, $q = 2$ (given)

Then, number of surjections from A to B $= {}^nC_2 \times 2!$

$$= \frac{n!}{2!(n-2)!} \times 2!$$

$$= \frac{n(n-1)(n-2)!}{2!(n-2)!} \times 2$$

$$= n(n-1) = n^2 - n = {}^nP_2$$

**3. Let X =** $\{x^2 : x \in N\}$ **and the function** $f: N \rightarrow X$ **is defined by** $f(x) = x^2, x \in N$**. Then this function is:**

**(a) injective only** **(b) not bijective**
**(c) subjective** **(d) bijective**

**[CBSE Term-1 2021]**

**Ans.** *(d) bijective*

**Explanation:**

$$y = x^2$$

**For injective:**

Let $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = x_2$ (– ve rejected)

So, it is injective.

**For subjective:**

Range = X

Co-domain = X

So Range = Co-domain

So, it is subjective.

Hence, it is bijective.

### ⚠ Caution

➥ *In such types of questions, one-one function and onto function can be examined, but many-one and into function cannot be examined easily, so if function is not one-one then it is many-one, and if function is not onto then it is into function.*

**4. Let** $f: R \rightarrow R$ **be defined by** $f(x) = \dfrac{1}{x}\ \forall x \in R$**. Then *f* is:**

**(a) one-one** **(b) onto**
**(c) bijective** **(d) *f* is not-defined**

**[NCERT Exemplar]**

**Ans.** *(d) f is not-defined*

**Explanation:** For $x = 0, f(x) = \dfrac{1}{0}$ (Not defined)

**5. Let** $f: R \rightarrow R$ **be defined as** $f(x) = 7x - 5$**. Then**

**(a) *f* is one-one onto.**
**(b) *f* is many-one onto.**
**(c) *f* is one-one but not onto.**
**(d) *f* is neither one-one nor onto.**

**[Delhi Gov. 2022]**

**Ans.** *(a) f is one-one onto.*

**Explanation:** We have $f: R \to R$ and $f(x) = 7x - 5$

Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow \quad 7x_1 - 5 = 7x_2 - 5$

$\Rightarrow \quad x_1 = x_2$

$\therefore f(x)$ is one-one.

Let $y \in R$ such that $f(x) = y$.

$\therefore \quad y = 7x - 5$

$\Rightarrow \quad x = \frac{y+5}{7}$

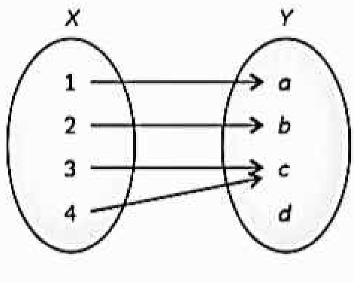
Now, for any $y \in R$, we have $x \in R$.

*i.e.,* Range = Co-domain

$\therefore f(x)$ is onto.

$\therefore f(x)$ is one-one as well as onto.

**6. If $f: X \to Y$ is defined, then $f$ is:**




**(a) Bijective function**
**(b) Many-one and onto**
**(c) Many-one and Into function**
**(d) One-one but not onto [Delhi Gov. 2021]**

**Ans.** *(c) Many-one and Into function*

**Explanation:** From the figure, it is clear that elements 3 and 4 in domain X have the same image *c* in the range Y.

$\therefore f(x)$ is not one-one.

$\Rightarrow f(x)$ is many-one.

Also, the element *d* in the co-domain Y does not has any pre-image in domain X, so it is not onto.

$\Rightarrow f(x)$ is into

Hence, $f(x)$ is many-one and into function.

**7. Set A has 3 elements and the set B has 4 elements, then number of injective functions that can be defined from set A to set B is:**

**(a) 120 (b) 24**
**(c) 144 (d) 64 [DIKSHA]**

**Ans.** *(b) 24*

**Explanation:** Set A has 3 elements and set B has 4 elements.

Now, first element of set A can be mapped to any of the 4 elements of set B.

The second element of set A can be mapped with any of the remaining 3 elements of set B. Similarly, the third element can be mapped with remaining 2 elements of set B

Hence, the total number of injective functions is $4 \times 3 \times 2 = 24$.

Alternatively, number of injective function from a set A containing *m* elements to the set containing *n* elements is ${}^nP_m$, if $n > m$ and is equal to zero, if $n < m$.

**8. A function $f: R \to R$ defined by $f(x) = 2 + x^2$ is**
**(a) not one-one**
**(b) one-one**
**(c) not onto**
**(d) neither one-one nor onto**

**[CBSE Term-1 2021]**

**Ans.** *(d) neither one-one nor onto*

**Explanation:**

For one-one: $f(x_1) = f(x_2)$

$\Rightarrow \quad 2 + x_1^2 = 2 + x_2^2$

$\Rightarrow \quad x_1^1 = x_2^2$

$\Rightarrow \quad x_1 = \pm x_2 \quad \{\because x_1, x_2 \in R\}$

So, it is not one-one.

For onto:

Range = Positive real numbers

Co-domain = R

$\therefore$ Range $\neq$ co-domain

So, it is not onto.

Hence, $f(x)$ is neither one-one nor onto.

**9. Which of the following functions from Z to Z are bijections?**

**(a) $f(x) = x^3$ (b) $f(x) = x + 2$**
**(c) $f(x) = 2x + 1$ (d) $f(x) = x^2 + 1$**

**[NCERT Exemplar]**

**Ans.** *(b) $f(x) = x + 2$*

**Explanation:**

(a) $x^3$ is not onto, as for example, $y = f(x) = 7$, $x \notin Z$.

(b) $x + 2$ is both one-one and onto.

(c) $2x + 1$ is not onto, as for even values of $y, x \notin Z$.

(d) $x^2 + 1$ is not a one-one function.

**10. Let N be the set of natural numbers and function $f: f(n) = 3n + 1, n \in N$. Then $f$ is**

**(a) bijective (b) injective**
**(c) surjective (d) none of these**

**Ans.** *(b) injective*

**Explanation:** Given, $f(n) = 3n + 1, n \in N$.

Let $n_1, n_2 \in N$ such that $f(n_1) = f(n_2)$.

$\Rightarrow 3n_1 + 1 = 3n_2 + 1$

$\Rightarrow 3n_1 = 3n_2$

$\Rightarrow n_1 = n_2$, hence $f$ is one-one.

Here, image of $f$ is all odd numbers, hence $f$ is not onto.

Thus, $f$ is injective.

**11.** Which of the following functions from Z to Z are bijections ?

(a) $f(x)$ $4x + 2$

(b) $f(x) = x + 2$

(c) $f(x) = 3x + 1$

(d) $f(x) = x^2$

**12.** Let $f: [2, \infty) \to R$ be the function defined by $f(x) = x^2 - 4x + 5$, then the range of $f$ is:

(a) R (b) $[1, \infty)$

(c) $[4, \infty)$ (d) $[5, \infty)$

[NCERT Exemplar]

**13.** Let $f: R \to R$ be defined by

$$f(x) = \begin{cases} 2x : & x > 3 \\ x^2 : & 1 < x \le 3 \\ 3x : & x \le 1 \end{cases}$$

Then $f(-1) + f(2) + f(4)$ is:

(a) 9 (b) 14

(c) 5 (d) None of these

[NCERT Exemplar]

**Ans.** *(a) 9*

**Explanation:** Given, $f(x) = \begin{cases} 2x : & x > 3 \\ x^2 : & 1 < x \le 3 \\ 3x : & x \le 1 \end{cases}$

Now, $f(-1) + f(2) + f(4) = 3(-1) + (2)^2 + 2 \times 4$
$= -3 + 4 + 8 = 9$

# CASE BASED Questions (CBQs)

[ **4** & **5** marks ]

**Read the following passages and answer the questions that follow:**

**14.** *For assessing the learning of students on the concept of "Relations", A class room teacher defines some relations on a set A = {1, 2, 3, 4, 5}. The possible number of relations that can be defined on A is $2^5$. Four of them are the following:*

$R_1 = \{(a, b) : a - b = 12\}$;

$R_2 = \{(a, b) : a + b > 0\}$;

$R_3 = \{(a, b) : a - b = 0\}$;

$R_4 = \{(a, b) : b = a + 1, a < 4\}$

**(A) Which of these four relations is an empty relation?**

(a) $R_1$ (b) $R_2$

(c) $R_3$ (d) $R_4$

**(B) Which of these four relations is an universal relation?**

(a) $R_1$ (b) $R_2$

(c) $R_3$ (d) $R_4$

**(C) Which of these four relations is an identity relation?**

(a) $R_1$ (b) $R_2$

(c) $R_3$ (d) $R_4$

**(D) Which of these four relations is not a symmetric relation?**

(a) $R_1$ (b) $R_2$

(c) $R_3$ (d) $R_4$

**(E) Which of these four relations is not a transitive relation?**

(a) $R_1$ (b) $R_2$

(c) $R_3$ (d) $R_4$

**Ans.** (A) *(a) $R_1$*

**Explanation:** Since difference of two numbers of the set A cannot be 12, $R_1$ is an empty relation.

(D) *(d) $R_4$*

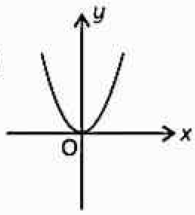
**Explanation:** Let $a = 3$, $b = 4$, then $(a, b) \in R_4$ as $b = a + 1$. But $3 \neq 4 + 1$. So, $(b, a) \notin R_4$

So, $R_4$ is not a symmetric relation.

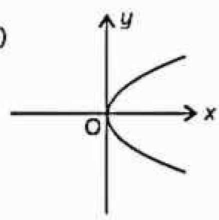
**15.** *In view of Covid-19 pandemic situation, schools are closed for many months. Schools are conducting online classes for teaching and learning activities. Performance of students is also assessed online.*

*A Class XII teacher, after teaching the topic of "Functions and its Types", tries to assess the performance of her students over this topic. She places a chart with seven different graphs given here before them:*

(1)




(2)

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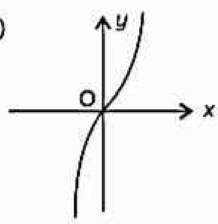


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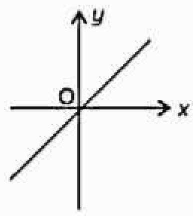

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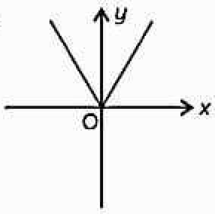
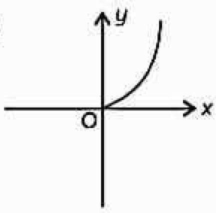
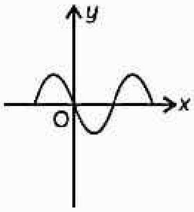


(6)



(7)



**(A) Of the seven graphs, which one does not represent a function?**

(a) Graph (1) (b) Graph (2)

(c) Graph (4) (d) Graph (7)

**(B) Of the seven graphs, which one does not represent an onto function?**

(a) Graph (1) (b) Graph (2)

(c) Graph (3) (d) Graph (4)

**(C) Of the seven graphs, which one does not represent a one-one function?**

(a) Graph (3) (b) Graph (4)

(c) Graph (5) (d) Graph (6)

**(D) Of the seven graphs, which one is a one-one function?**

(a) Graph (2) (b) Graph (4)

(c) Graph (6) (d) Graph (7)

**(E) Graph 3:**

(a) does not represents a function.

(b) represents a one-one function but not onto.

(c) represents an onto function but not a one-one function.

(d) represents a one-one and onto function.

**Ans.** (A) *(b) Graph (2)*

**Explanation:** Any line drawn parallel to $y$-axis will intersect graph (2) more than once. So, Graph (2) does not represent a function.

(C) *(c) Graph (5)*

**Explanation:** It is possible to draw a horizontal line that crosses Graph (5) twice. So, Graph (5) does not represent a one-one function.

**16.** *Mrs. Roopali decided to try yet another engaging style of learning and selected a group of students who would replace her to conduct the class on a particular topic. For this purpose, she writes the following relations, each defined on the set $A = \{1, 2, 3\}$*

$R_1 = \{(2, 3), (3, 2)\}$;

$R_2 = \{(1, 2), (1, 3), (3, 2)\}$;

$R_3 = \{(1, 2), (2, 1), (1, 1)\}$;

$R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$;

$R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$

*The group of students are asked to use their group members to explain the various properties of above relations.*

*All who answer at least four of the following five questions correctly is selected for a homework free week.*

**(A) Which relation is reflexive and transitive but not symmetric?**

(a) $R_3$ (b) $R_4$

(c) $R_2$ (d) $R_5$

**(B) Which relation is reflexive and symmetric but not transitive?**

(a) $R_3$ (b) $R_4$

(c) $R_2$ (d) $R_5$

**(C) Which relation is symmetric and transitive but not reflexive?**

(a) $R_3$ (b) $R_4$

(c) $R_2$ (d) $R_5$

**(D) Which relation is symmetric but neither reflexive nor transitive?**

(a) $R_1$ (b) $R_4$

(c) $R_2$ (d) $R_3$

**(E) Which relation is transitive but neither reflexive nor symmetric?**

(a) $R_3$ (b) $R_4$

(c) $R_2$ (d) $R_5$

**Ans.** (A) *(b) $R_4$*

**Explanation:** $R_4$ is a reflexive and transitive relation but not a symmetric relation, as

$(1, 2) \in R_4$, but $(2, 1) \notin R_4$

(B) *(d) $R_5$*

**Explanation:** $R_5$ is a reflexive and symmetric relation but not a transitive relation, as

$(3, 2)$ and $(2, 1) \in R_5$, but $(3, 1) \notin R_5$

**17.** *For the teacher's day, the class room teacher wishes to select one student who would replace her on the teacher's day and conduct the class. For this purpose, she writes the following functions :*

*$f : R \rightarrow R$ defined by $f(x) = |x|$;*

*$g : R \rightarrow R$ defined by $g(x) = [x]$;*

*$h : Z \rightarrow Z$ defined by $h(x) = 2x + 3$;*

*$p : N \rightarrow N$ defined by $p(x) = x^2$*

**A student who answers at least four of the following five questions correctly, is to be selected for the teachers' day.**

**(A) The function $f$ is:**
- (a) one-one
- (b) onto
- (c) one-one and onto
- (d) neither one-one nor onto

**(B) The function $g$ is:**
- (a) one-one
- (b) onto
- (c) one-one and onto
- (d) neither one-one nor onto

**(C) The function $h$ is:**
- (a) one-one
- (b) onto
- (c) one-one and onto
- (d) neither one-one nor onto

**(D) The function $p$ is:**
- (a) one-one
- (b) onto
- (c) both one-one and onto
- (d) neither one-one nor onto

**(E) How many functions are both one-one and onto?**

(a) none (b) one
(c) two (d) three

**Ans.** (A) *(d) neither one-one nor onto*

**Explanation:** $f: R \to R$ defined by $f(x) = |x|$ is not one–one, because $f(-1) = f(1) = 1$, despite $-1 \neq 1$.

The function is not onto, because range of $f$ is $R^+$ (and not R)

(C) *(a) one-one*

**Explanation:** $h: Z \to Z$ defined by $h(x) = 2x + 3$ is one–one, as

$h(x_1) = h(x_2) \Rightarrow 2x_1 + 3 = 2x_2 + 3 \Rightarrow x_1 = x_2$

The function is not onto, because for $y \in Z$ (co-domain), $x = \dfrac{y-3}{2} \notin Z$ (domain).

**18. Let $f: A \to R$ be defined by $f(x) = \tan x$. Then,**

**(A) Calculate the domain A of $f$.**

**(B) A is replaced by which limit, so that function $f$ becomes one-one and onto?**

**Ans.** (A) By definition, $R - \left\{x : x = (2n+1)\frac{\pi}{2}, n \in Z\right\}$ is the domain of $f$.

(B) Though no trigonometric function is one-one, but $f(x) = \tan x$ can be made one-one by restricting domain A to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

**19.** *A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote and voter turnout was about 67%, the highest ever.*

ONE – NATION
ONE – SELECTION
FESTIVAL OF
DEMOCRACY
GENERAL ELECTION – 2019


*Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2019. A relation 'R' is defined on I as follows:*

*$R = \{(V_1, V_2) : V_1, V_2 \in I\}$ and both use their voting right in general election–2019 exercise.*

**(A) Two neighbors X and Y $\in$ I. X exercise his voting right while Y did not cast her vote in general election–2019. Which of the following is true?**

(a) $(X, Y) \in R$ (b) $(Y, X) \in R$
(c) $(X, X) \notin R$ (d) $(X, Y) \notin R$

**(B) Mr. 'X' and his wife 'W' both exercised their voting right in general election-2019. Which of the following is true?**
- (a) both $(X, W)$ and $(W, X) \in R$
- (b) $(X, W) \in R$ but $(W, X) \notin R$
- (c) both $(X, W)$ and $(W, X) \notin R$
- (d) $(W, X) \in R$ but $(X, W) \notin R$

**(C) Three friends $F_1$, $F_2$ and $F_3$ exercised their voting right in general election–2019, then which of the following is true?**
- (a) $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \in R$
- (b) $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \notin R$
- (c) $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_3, F_3) \notin R$
- (d) $(F_1, F_2) \notin R$, $(F_2, F_3) \notin R$ and $(F_1, F_3) \notin R$

**(D) The above defined relation R is ..............**
- (a) symmetry and transitive but not reflexive
- (b) universal relation
- (c) equivalence relation
- (d) reflexive but not symmetric and transitive

**(E) Mr. Shyam exercised his voting right in General Election–2019, then Mr. Shyam is related to which of the following?**
- (a) All those eligible voters who cast their votes

(b) **Family members of Mr. Shyam**

(c) **All citizens of India**

(d) **Eligible voters of India**

**[CBSE Question Bank 2021]**

**Ans.** (A) *(d)* $(X, Y) \notin R$

**Explanation:** Since, R is a set of persons who use their voting right. But Y being eligible for voting, did not cast her vote.

So, $(X, Y) \notin R$

(C) *(a)* $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \in R$

**Explanation:** It is a transitive relation, if $(X, Y) \in R$, $(Y, Z) \in R$ then $(X, Z) \in R$.

*i.e.*, $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ then $(F_1, F_3) \in R$.

**20.** *Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set {1, 2, 3, 4, 5, 6}. Let A be the set of players while B be the set of the possible outcomes.*

$\therefore A = \{S, D\}, B = \{1, 2, 3, 4, 5, 6\}$

**(A) Let R : B → B be defined by R = {(x, y) : y is divisible by x}. Check whether R is reflexive, symmetric or transitive.**

**(B) Raji wants to know the number of functions from A to B. How many number of functions are possible?**

**[CBSE Question Bank 2021]**

**Ans.** (A) We have,

$R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)\}$

As from the above set, it is clear that $(a, a) \in R$, so the relation is reflexive.

And if $a$ divides $b$ and $b$ divides $c$. Then, $a$ divides $c$.

*i.e.*, If 1 divides 2 and 2 divides 4. Then, 2 divides 4. So, the relation is transitive.

But if $(1, 2) \in R$, then $(2, 1) \notin R$.

Hence, the relation is reflexive and transitive but not symmetric.

**21.** *An organization conducted bike race under two different categories-boys and girls. Totally there were 28 participants. Among all of them, finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.*

*Let* $B = \{b_1, b_2, b_3\}$, $G = \{g_1, g_2\}$ *where B represents the set of boys selected and G the set of girls who were selected for the final race.*

*Ravi decides to explore these sets of various types of relations and functions.*

**(A) Let R : B → B be defined by R = {(x, y) : x and y are students of same gender}. Then show that this relation R is an equivalence relation.**

**(B) Let R : B → G be defined by R = {$(b_1, g_1)$, $(b_2, g_2)$, $(b_3, g_1)$}. Check whether R is injective, surjective or bijective.**

**[CBSE Question Bank 2021]**

**Ans.** (A) For any $b \in B$, $(b, b) \in R$.

Also, if $(b_1, b_2) \in R$

Then, $(b_2, b_1) \in R$

and if $(b_1, b_2) \in R$ and $(b_2, b_3) \in R$

Then, $(b_1, b_3) \in R$

So, the relation is reflexive, symmetric as well as transitive. So, it is equivalence relation.

**22.** *Students of Grade 12, planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the saplings along the line* $y = x - 4$. *Let L be the set of all lines which are parallel on the ground and R be a relation on L.*

Answer the following using the above information.

(A) Let relation R be defined by R = $\{(L_1, L_2) : L_1 \parallel L_2$ where $L_1, L_2 \in L\}$ then show that R is an equivalence relation.

(B) Prove that the function $f: R \to R$ defined by $f(x) = x - 4$ is bijective function.

**[CBSE Question Bank 2021]**

**Ans.** (B) We have,

$$f(x) = x - 4$$

One-one:

$$f(x_1) = f(x_2)$$

$\therefore \quad x_1 - 4 = x_2 - 4 \Rightarrow x_1 = x_2$

So, it is one-one.

Onto: Since,

$$f(x) = x - 4$$

$\therefore$ Range = R

$\Rightarrow$ Range = Co-domain.

So, it is onto. Since, it is both one-one and onto, so it is a bijective function.

## VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

**23.** Let $f : R \to R$ be the function defined by $f(x) = \tan x \ \forall \ x \in R$, then show that $f$ is not one-one.

**Ans.** Given, $f(x) = \tan x \ \forall \ x \in R$

We know that $\tan x = \tan y$

$\Rightarrow x = n\pi \pm y$, where $n = 1, 2, 3, ....$

Hence, $f(x)$ is not one-one.

**24.** What is the range of the function $f(x) = \dfrac{|x-1|}{x-1}, x \neq 1$? **[CBSE 2010]**

**Ans.** Given function, $f(x) = \dfrac{|x-1|}{x-1}, x \neq 1$

The above function can be written as

$$f(x) = \begin{cases} \dfrac{x-1}{x-1}, & \text{if } x > 1 \\ \dfrac{-(x-1)}{x-1}, & \text{if } x < 1 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 1, & \text{if } x > 1 \\ -1, & \text{if } x < 1 \end{cases}$$

Therefore, range of $f(x)$ is the set $\{-1, 1\}$.

**25.** Show that the function $f : R \to R$ defined by $f(x) = \dfrac{1}{x}$ is one-one but not onto.

**26.** Let D be the domain of the real valued function $f$ defined by $f(x) = \sqrt{25 - x^2}$. Then write D. **[NCERT Exemplar]**

**27.** Are the following set of ordered pairs functions? If so, examine whether the mapping is injective or surjective.

(A) $\{(x, y) : x$ is a person, $y$ is the mother of $x\}$

(B) $\{(a, b) : a$ is a person, $b$ is an ancestor of $a\}$ **[NCERT Exemplar]**

**Ans.** (A) It represents a function.

The images of distinct elements of $x$ under $f$ are not distinct.

Therefore, it is not injective but it is surjective.

(B) It does not represent a function as every domain under mapping does not have a unique image.

**28.** Population Census is the total process of collecting, compiling, analyzing or otherwise disseminating demographic, economic and social data pertaining, at a specific time, of all persons in a country or a well-defined part of a country. The citizens are given a unique enrolment number after entering all particulars. A family had three members denoted by the set A = {1, 2, 3} and let the set of enrolment numbers be given by B = {4, 5, 6, 7}.

Let A = {1, 2, 3}, B = {4, 5, 6, 7} and $f$ = {(1, 4), (2, 5), (3, 6)} be a function from A to B. State whether '$f$' is one-one or not. **[CBSE 2014, 11]**

**Ans.** Given that A = {1, 2, 3}, B = {4, 5, 6, 7}

Now, $f : A \to B$ is defined as $f$ = {(1, 4), (2, 5), (3, 6)}.

$f(1) = 4$, $f(2) = 5$, $f(3) = 6$, so $f$ is one-one for $x_1, x_2 \in A$.

## SHORT ANSWER Type-I Questions (SA-I)

[ 2 marks ]

**29. Is $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ a function? If $g$ is described by $g(x) = ax + b$, then what value should be assigned to $a$ and $b$? [NCERT Exemplar]**

**Ans.** Given function, $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$

Here, for every unique value of domain *i.e.*, $\{1, 2, 3, 4\}$, we have a unique image *i.e.* $\{1, 3, 5, 7\}$ respectively. Hence, $g$ is a function.

Now, $g(x) = ax + b$

For $(1, 1) : g(1) = a \times 1 + b$

$\Rightarrow \quad 1 = a + b \quad ...(i)$

For $(2, 3) : g(2) = a \times 2 + b$

$\Rightarrow \quad 3 = 2a + b \quad ...(ii)$

Subtracting equation (i) from equation (ii), we get

$a = 2$

Putting value of $a$ in equation (i), we get

$1 = 2 + b$

$\Rightarrow \quad b = 1 - 2 = -1$

Thus, $a = 2, b = -1$

**⚠ Caution**

➥ *In such types of questions, to state whether a given relationship is a function or not, check whether the given relationship is one-one or not. If given relationship is one-one, then this is a function as domain and range already given and we assume that all elements of domain set and range set are given in function.*

**30. State whether the function $f : N \to N$ given by $f(x) = 5x$ is injective, surjective or both.**

**Ans.** Given function is $f(x) = 5x$

As $f(x_1) = f(x_2)$

$\Rightarrow \quad 5x_1 = 5x_2$

$\Rightarrow \quad x_1 = x_2$

So, $f(x)$ is injective function.

Also, range of $f(x) = 5x, x \in N$

But co-domain = N

$\because$ Range = Co-domain.

$\therefore f(x)$ is not surjective.

Hence, the given function is injective.

**31. Let C be the set of complex numbers. Prove that the mapping $f : C \to R$ given by $f(z) = |z|$, $\forall z \in C$, is neither one-one nor onto.**

**[NCERT Exemplar]**

**Ans. For one-one:**

Given, $f(z) = |z| \ \forall z \in C$

$f(1) = |1| = 1$ and $f(-1) = |-1| = 1$

We can see that $f(1) = f(-1)$ but $1 \neq -1$

Thus, it is not one-one.

**For onto:**

Let $f(z) = |z| = y$.

There is no pre-image of negative numbers as modulus of positive and negative numbers are always positive. Hence, it is not onto.

**32. Given A = {2, 3, 4}, B = {2, 5, 6, 7}. Construct an example of each of the following:**

**(A) an injective mapping from A to B.**

**(B) a mapping from A to B which is not injective**

**(C) a mapping from B to A. [NCERT Exemplar]**

**Ans.** Given, A = {2, 3, 4} and B = {2, 5, 6, 7}

(A) Injective mapping from A to B, $f = \{(2, 2), (3, 5), (4, 6)\}$

(B) A mapping from A to B which is not injective, $g = \{(2, 6), (3, 6), (4, 5)\}$

(C) Mapping from B to A, $h = \{(2, 3), (5, 4), (6, 3), (7, 3)\}$

**33. Show that the function $f$: R → R defined by $f(x) = 3x - 4$ is bijective. [DIKSHA]**

**Ans.** Given, $f$: R → R and $f(x) = 3x - 4$

Let $x_1, x_2 \in R$ such that

$f(x_1) = f(x_2)$

$\Rightarrow \quad 3x_1 - 4 = 3x_2 - 4$

$\Rightarrow \quad 3x_1 = 3x_2$

$\Rightarrow \quad x_1 = x_2$

So, $\quad f(x_1) = f(x_2)$

$\Rightarrow \quad x_1 = x_2$

Therefore, $f(x)$ is one to one function.

$f(x)$ is a linear equation, hence there exist a value of $x$ for every value of $f(x) \in R$.

Therefore, $f$ is onto.

Hence, $f(x)$ is one-one and onto or $f(x)$ is bijective.

**34. Let the function $f$: R → R be defined by $f(x) = \cos x, \forall x \in R$. Show that $f$ is neither one-one nor onto. [NCERT Exemplar]**

## SHORT ANSWER Type-II Questions (SA-II)

[ 3 marks ]

**35. Let $f: N \to N$ be defined as**

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases} \quad \text{for all } n \in N.$$

**Find whether the function $f$ is bijective or not.**

**Ans.** The given function is $f: N \to N$ such that

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases} \quad \text{for all } n \in N$$

We shall verify whether $f(x)$ is one-one and onto.

**One-One:** From the definition of $f(n)$

$$f(1) = \frac{1+1}{2} = 1 \text{ and } f(2) = \frac{2}{2} = 1$$

$f(n)$ is not an one-one function because at two distinct values from domain (N), $f(n)$ has same image.

**Onto:** For onto function, we check whether.

Range of $f(n)$ = Co-domain of $f(n)$

Now, if $n$ is an odd natural number, then $(2n - 1)$ is also $n$ odd natural number.

Now, $$f(2n-1) = \frac{2n-1+1}{2} = n \quad ...(i)$$

Again, if $n$ is an even natural number, then $2n$ is also an even natural number. Then,

$$f(2n) = \frac{2n}{2} = n \quad ...(ii)$$

From equations, (i) and (ii), we observe that for each $n$ (whether even or odd), there exists its pre-image, N.

*i.e.*, Range of $f(n)$ = Co-domain of $f(n)$.

Hence, $f$ is onto.

Since, $f(n)$ is onto but not one-one, it is not a bijective function.

**36. Let $f: R \to R$ be the function defined by $f(x) = \frac{1}{2-\cos x}$ $\forall x \in R$. Then, find the range of $f$.** **[NCERT Exemplar]**

**Ans.** Given, $f(x) = \frac{1}{2-\cos x}$, $\forall x \in R$

We know that range of $\cos x$ is $[-1, 1]$.

$$\Rightarrow \quad -1 \le \cos x \le 1$$

Multiplying by (–1), we get

$$1 \ge -\cos x \ge -1$$

Adding 2, we get

$$3 \ge 2 - \cos x \ge 1$$

Taking reciprocal, we get

$$\frac{1}{3} \le \frac{1}{2-\cos x} \le 1$$

Hence, range of $f$ is $\left[\frac{1}{3}, 1\right]$.

**37. If $f: R \to R$ be the function defined by $f(x) = 4x^3 + 7$. Then show that $f$ is a bijection.** **[CBSE 2011]**

**Ans.** The given function is $f: R \to R$ such that $f(x) = 4x^3 + 7$

**One-one:** Given that,

$$f(x) = 4x^3 + 7$$

Let $f(x_1) = f(x_2)$ $\forall x_1, x_2 \in R$

$\Rightarrow$ $4x_1^3 + 7 = 4x_2^3 + 7$

$\Rightarrow$ $4x_1^3 = 4x_2^3$

$\Rightarrow$ $x_1^3 = x_2^3$

$\Rightarrow$ $x_1^3 - x_2^3 = 0$

$\Rightarrow (x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2) = 0$

Either $(x_1 - x_2) = 0$ or $(x_1^2 + x_1x_2 + x_2^2) = 0$

$\Rightarrow$ $x_1 = x_2$ $[\because (x_1^2 + x_1x_2 + x_2^2) \ne 0]$

Hence, $f(x)$ is one-one function.

**Onto:** To show that $f(x)$ is an onto function, we show that Range of $f(x)$ = Co-domain of $f(x)$

Given that co-domain of $f(x)$ = R.

Now, let $y = 4x^3 + 7$

$$\Rightarrow \quad x^3 = \frac{y-7}{4}$$

$$\Rightarrow \quad x = \left(\frac{y-7}{4}\right)^{1/3} \quad ...(i)$$

It is clear from equation (i) that for every $y \in R$, we get $x \in R$

$\therefore$ Range of $f(x)$ = R = Co-domain of $f(x)$

$\Rightarrow f(x)$ is an onto function.

Since, $f(x)$ is both one-one and onto, it is bijective function.

**38. Give an example of a map:**

**(A) which is one-one but not onto.**

**(B) which is not one-one but onto.**

**(C) which is neither one-one nor onto.**

**[NCERT Exemplar]**

**Ans.** (A) Let $f: N \rightarrow N$ and $f(x) = x^2$

Let $x_1, x_2 \in N$

such that, $f(x_1) = f(x_2)$

$\Rightarrow \quad x_1^2 = x_2^2$

$\Rightarrow \quad x_1^2 - x_2^2 = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2) = 0$

$\Rightarrow \quad (x_1 - x_2) = 0 \quad [\because (x_1 + x_2) \neq 0]$

$\Rightarrow \quad x_1 = x_2$

Therefore, $f(x)$ is one to one function.

Now, $2 \in N$ has no pre-image as if $f(x) = 2$, then

$x^2 = 2$

$\Rightarrow \quad x = \pm\sqrt{2} \notin N$

Therefore, $f$ is not onto.

Hence, $f(x) = x^2$ is one-one but not onto.

(B) Let $f: R \rightarrow [0, \infty)$, such that

$$f(x) = \begin{cases} |x|, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Now, $f(-1) = 1$ and $f(1) = 1$

Since, two different domains has same range, therefore, $f$ is not one-one.

Now, for any value of $x$, $f(x) \geq 0$ *i.e.*

Range $= [0, \infty)$

$\because$ Range = Co-domain

So, $f$ is onto.

Therefore, the function $f$ is not one-one but onto.

(C) Let $f: R \rightarrow R$ and let $f(x) = x^2$.

Let $x_1 = 1$ and $x_2 = -1$.

So, $f(x_1) = 1^2 = 1$ and $f(x_2) = (-1)^2 = 1$

$f(x_1) = f(x_1)$ but $x_1 \neq x_2$

So, $f$ is not one-one.

$f$ is not onto as $-1$ has no pre-image. Because if $f(x) = -1 \Rightarrow x^2 = -1$

$\Rightarrow x = \pm\sqrt{-1} \notin R$

So function is not onto.

Hence, the function $f$ is neither one-one nor onto.

### ⚠ Caution

➥ *In such types of questions where we have to prove whether the given function is onto or not, it is sufficient to find an element which has no pre-image.*

**39.** Ⓐ **Let $f: N \rightarrow N$ given by**

$$f(x) = \begin{cases} x + 1, & \text{if } x \text{ is odd} \\ x - 1, & \text{if } x \text{ is even} \end{cases}$$ **is bijective (both one-one and onto).**

**40.** Ⓐ**Let $A = R - \{3\}$, $B = R - \{1\}$. Let $f: A \rightarrow B$ be defined by $f(x) = \frac{x-2}{x-3}$, $\forall x \in A$. Then, show that $f$ is bijective. [NCERT Exemplar]**

**41. Show that the function $f$ in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. [CBSE 2013]**

**Ans.** Given that, $f(x) = \frac{4x+3}{6x-4}$

Let $x_1, x_2 \in A = R - \left\{\frac{2}{3}\right\}$; $x_1 \neq x_2$

Consider, $f(x_1) = f(x_2)$

$\Rightarrow \quad \frac{4x_1+3}{6x_1-4} = \frac{4x_2+3}{6x_2-4}$

$\Rightarrow (4x_1 + 3)(6x_2 - 4) = (4x_2 + 3)(6x_1 - 4)$

$\Rightarrow 24x_1x_2 - 16x_1 + 18x_2 - 12 = 24x_1x_2 - 16x_2 + 18x_1 - 12$

$\Rightarrow \quad -34x_1 = -34x_2$

$\Rightarrow \quad x_1 = x_2$

$\therefore f(x)$ is a one-one function.

Let $\quad y = \frac{4x+3}{6x-4}$

$\Rightarrow \quad 6xy - 4y = 4x + 3$

$\Rightarrow \quad (6y - 4)x = 3 + 4y$

$\Rightarrow x = \frac{4y+3}{6y-4}$ and $y \neq \frac{4}{6}$, *i.e.*, $y \neq \frac{2}{3}$

$\therefore y = R - \left\{\frac{2}{3}\right\}$

Thus, $f(x)$ is an onto function.

## LONG ANSWER Type Questions (LA)

[ **4** & **5** marks ]

**42. Let A = [-1, 1], then discuss whether the following functions defined on A are one-one, onto or bijective.**

**(A)** $f(x) = \frac{x}{2}$ **(B)** $g(x) = |x|$

**(C)** $h(x) = x|x|$ **(D)** $k(x) = x^2$

**[NCERT Exemplar]**

**Ans.** (A) Let $x_1, x_2 \in$ A such that

$$f(x_1) = f(x_2)$$

$$\Rightarrow \quad \frac{x_1}{2} = \frac{x_2}{2}$$

$$\Rightarrow \quad x_1 = x_2$$

So $f(x)$ is one-one.

Now, let $\quad f(x) = y$

$$\Rightarrow \quad \frac{x}{2} = y$$

$$\Rightarrow \quad x = 2y$$

Here, for $y = 1, x = 2 \times 1 = 2 \notin [-1, 1]$

So, function is not onto.

Hence, $f(x)$ is not a bijective function.

(B) Given, $g(x) = |x|$

Let $x_1, x_2 \in$ A, such that

$$g(x_1) = g(x_2)$$

$$\Rightarrow \quad |x_1| = |x_2|$$

$$\Rightarrow \quad x_1 = \pm x_2$$

i.e., for a unique value of $x_2$, we have two images of $x_1$.

So, $g(x)$ is not one-one function.

$g(x)$ is not an onto function because $-1 \in$ A has no pre-image under $g$.

Hence, $g(x)$ is not a bijective function.

(C) Given, $\quad h(x) = x|x|$

Let $x_1, x_2 \in$A such that

$$h(x_1) = h(x_2)$$

$$\Rightarrow \quad x_1|x_1| = x_2|x_2|$$

If $x_1, x_2 > 0$, then

$$x_1^2 = x_2^2$$

$$\Rightarrow \quad x_1^2 - x_2^2 = 0$$

$$\Rightarrow (x_1 - x_2)(x_1 + x_2) = 0$$

$$\Rightarrow \quad x_1 - x_2 = 0 \qquad [\because x_1 + x_2 \neq 0]$$

$$\Rightarrow \quad x_1 = x_2$$

Similarly, if $x_1, x_2 < 0$, then $x_1 = x_2$

And, if $x_1 > 0$, $x_2 < 0$ and vice-versa, then $x_1 \neq x_2$ and $h(x_1) \neq h(x_2)$.

So, $h(x)$ is one-one.

Now, for $x \in [-1, 0]$, $h(x) \in [-1, 0]$

and, for $x \in [0, 1]$, $h(x) \in [0, 1]$

$\Rightarrow$ Range of $h(x) = [-1, 0] \cup [0, 1] = [-1, 1]$ = Co-domain

So, $h(x)$ is onto.

Since, $h(x)$ is both one-one and onto,

Hence, $h(x)$ is a bijective function.

(D) Given, $k(x) = x^2$

Let $x_1, x_2 \in$A such that

$$k(x_1) = k(x_2)$$

$$\Rightarrow \quad x_1^2 = x_2^2$$

$$\Rightarrow \quad x_1 = \pm x_2$$

i.e., for a unique value of $x_2$, we have two images of $x_1$.

So, $k(x)$ is not one-one function.

Now, let $\quad k(x) = y$

$$\Rightarrow \quad x^2 = y$$

$$\Rightarrow \quad x = \pm\sqrt{y}$$

For $y = -1$, $x = \pm\sqrt{-1} \notin$ A.

So $k(x)$ is not onto.

Hence, $k(x)$ is not a bijective function.

**43. Consider $f : R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that $f$ is both one-one and onto.**

**Ans.** Given function is $f: R_+ \to [-5, \infty)$ given by

$$f(x) = 9x^2 + 6x - 5$$

**For one-one:** We know that, a function of $f: A \to B$ is said to be one-one, if

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \ \forall \ x_1, x_2 \in A$

So, for $x_1, x_2 \in R_+$

Let $f(x_1) = f(x_2)$

$$\Rightarrow \quad 9x_1^3 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$$

$$\Rightarrow 9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$$

$$\Rightarrow (x_1 - x_2)[9x_1 + 9x_2 + 6] = 0$$

Now, either $(x_1 - x_2) = 0$ or $[9x_1 + 9x_2 + 6] = 0$ (Which is not possible)

$$\Rightarrow \quad x_1 - x_2 = 0$$

$$\Rightarrow \quad x_1 = x_2$$

Hence, $f(x)$ is one-one function.

**For onto:** To show that $f(x)$ is onto, we show that range of $f(x)$ = co-domain of $f(x)$

Let $\quad y = 9x^2 + 6x - 5$

$$\Rightarrow \quad 9x^2 + 6x = y + 5$$

$$\Rightarrow 9\left(x^2 + \frac{2x}{3} + \frac{1}{9} - \frac{1}{9}\right) = y + 5$$

$$\Rightarrow \quad 9\left(x + \frac{1}{3}\right)^2 - 1 = y + 5$$

$$\Rightarrow \quad \left(x + \frac{1}{3}\right)^2 = \frac{y+6}{9}$$

$$\Rightarrow \quad x + \frac{1}{3} = \sqrt{\frac{y+6}{9}}$$

[Taking positive sign as $x \in$ R]

$$\Rightarrow \quad x = \frac{\sqrt{y+6} - 1}{3}$$

From above equation, we get for every $y \in [-5, \infty)$, we have $x \in R_+$

$\therefore$ Range of $f(x) = [-5, \infty)$ = Co-domain

Hence, $f(x)$ is onto.

# TOPPER'S CORNER

## VERY SHORT ANSWER Type Questions (VSA)

[ **1** mark ]

**1. If R = {(x, y) : x + 2y = 8} is a relation on N, write the range of R.**

**Ans.**

$x + 2y = 8$

Range = {1, 3, 2}

**[CBSE Topper 2014]**

## SHORT ANSWER Type-II Questions (SA-II)

[ **3** marks ]

**2. Show that the relation R on defined as R = {(a, b) : a ≤ b}, is reflexive, and transitive but not symmetric.**

**Ans.**

$R = \{(a, b) : a \le b\}$

REFLEXIVE :

Every element a ∈ R is equal to itself

⇒ a = a

⇒ a ≤ a is true

∴ (a, a) ∈ R for all a ∈ R — where ℝ = set of Real nos

The relation is REFLEXIVE — R → Relation.

TRANSITIVE :

For all (a, b) ∈ R and (b, c) ∈ R

a ≤ b and b ≤ c — where a, b, c ∈ ℝ

⇒ a ≤ c

⇒ (a, c) ∈ R

∴ The ~~set is tr~~ relation is TRANSITIVE

∵ for (a, b), (b, c) ∈ R, ~~(a, c) ∈ R~~ (a, c) ∈ R

SYMMETRIC : For relation to be symmetric, for all (a, b) ∈ R, (b, a) should also exist in R

~~a ≤ b ⇒~~ a ≤ b

b ≰ a → This relation is true only a = b = 1.

For eg: $\frac{1}{2} \le 1 \Rightarrow (\frac{1}{2}, 1) \in R$

but $1 \not\le \frac{1}{2}$ ∴ $(1, \frac{1}{2}) \notin R$

∴ Relation is NOT SYMMETRIC

**[CBSE Topper 2019]**