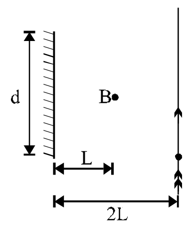
GUIDED REVISION

SECTION-I

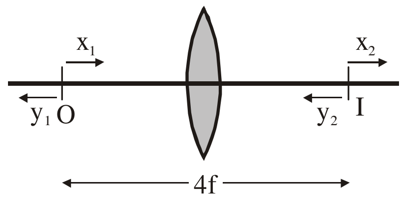
Single Correct Answer Type **11 Q. [3 M (–1)]**

1. A point source of light B is placed at a distance L in front of the centre of a mirror of width d hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance $2L$ from it as shown. The greatest distance over which he can see the image of the light source in the mirror is :- **[IIT-JEE '2000 (Scr)]**



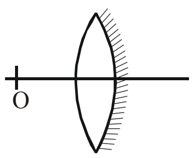
(A) d/2 (B) d (C) 2d (D) 3d

2. In a converging lens of focal length f and the distance between real object and its real image is $4f$. If the object moves x_1 distance towards lens its image moves x_2 distance away from the lens and when object moves y_1 distance away from the lens its image moves y_2 distance towards the lens, then choose the correct option:-



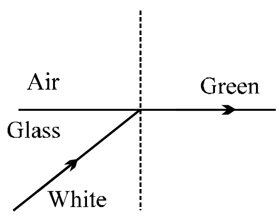
(A) $x_1 > x_2$ and $y_1 > y_2$
(B) $x_1 < x_2$ and $y_1 < y_2$
(C) $x_1 < x_2$ and $y_1 > y_2$
(D) $x_1 > x_2$ and $y_2 > y_1$

3. An equiconvex lens of refractive index μ and radius of curvature R has its one surface silvered. A point source O is placed before the silvered lens so that its image is coincident with it, the distance of the object from the lens is :-



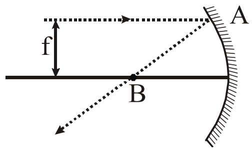
(A) $\frac{R}{(\mu-1)}$ (B) $\frac{2R}{(\mu-1)}$ (C) $\frac{R}{(2\mu-1)}$ (D) $\frac{2R}{(2\mu-1)}$

4. White light is incident on the interface of glass and air as shown in the figure. If green light is just totally internally reflected then the emerging ray in air contains **[IIT-JEE 2004 (Scr)]**



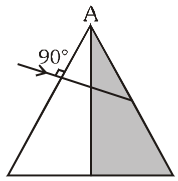
(A) yellow, orange, red
(B) violet, indigo, blue
(C) all colours
(D) all colours except green

5. A ray of light is incident on a concave mirror. It is parallel to the principal axis and its height from principal axis is equal to the focal length of the mirror. The ratio of the distance of point B to the distance of the focus from the centre of curvature is (AB is the reflected ray) :-



(A) $\frac{2}{\sqrt{3}}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{2}{3}$ (D) $\frac{1}{2}$

6. A ray of light is incident normally on the first refracting face of the prism of refracting angle A. The ray of light comes out at grazing emergence. If one half of the prism (shaded position) is knocked off, the same ray will :–



(A) Emerge at an angle of emergence $\sin^{-1}\left(\frac{1}{2}\sec A/2\right)$

(B) Not emerge out of the prism

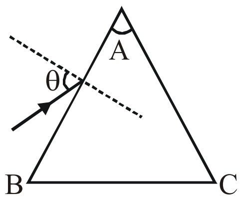
(C) Emerge at an angle of emergence $\sin^{-1}\left(\frac{1}{2}\sec A/4\right)$

(D) None of these

7. A person has D cm wide face and his two eyes are separated by d cm. The minimum width of a mirror required for the person to view his complete face is

(A) $\frac{D+d}{2}$ (B) $\frac{D-d}{4}$ (C) $\frac{D+d}{4}$ (D) $\frac{D-d}{2}$

8. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided : **[JEE-Main- 2015]**



(A) $\theta > \cos^{-1}\left[\mu\sin\left(A+\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$ (B) $\theta < \cos^{-1}\left[\mu\sin\left(A+\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

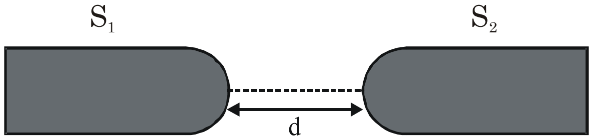
(C) $\theta > \sin^{-1}\left[\mu\sin\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$ (D) $\theta < \sin^{-1}\left[\mu\sin\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

9. In an experiment for determination of refractive index of glass of a prism by $i-\delta$, plot, it was found that a ray incident at angle 35°, suffers a deviation of 40° and that it emerges at angle 79°. In that case which of the following is closest to the maximum possible value of the refractive index?

[JEE-Main- 2016]

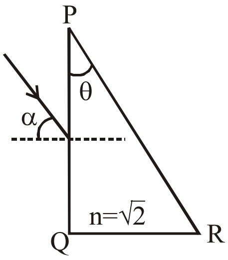
(A) 1.8 (B) 1.5 (C) 1.6 (D) 1.7

10. Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes (shown by the dashded line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is : **[JEE-Advance-2015]**



(A) 60 cm (B) 70 cm (C) 80 cm (D) 90 cm

11. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. The angle θ of the prism is : **[JEE-Advance-2016]**



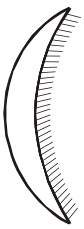
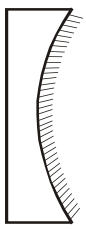
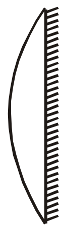
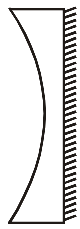
(A) 15° (B) 22.5° (C) 30° (D) 45°

Multiple Correct Answer Type 6 Q. [4 M (–1)]

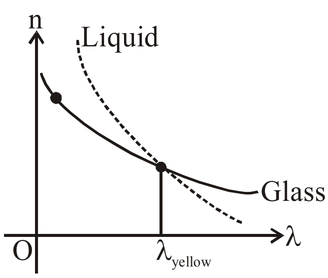
12. Optical axis of a thin equi-convex lens is the X-axis. The co-ordinate of a point object and its image are (–20 cm, 1 cm) and (25 cm, –2 cm) respectively

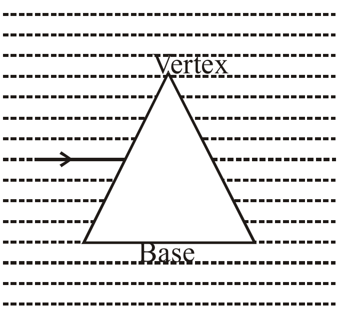
(A) the lens is located at x = 5 cm
(B) the lens is located at x = –5 cm
(C) the focal length of the lens is 10 cm
(D) the focal length of the lens is 15 cm

13. Which of the following silvered lenses kept in air may form real image of a real object ?

(A)  (B)  (C)  (D) 

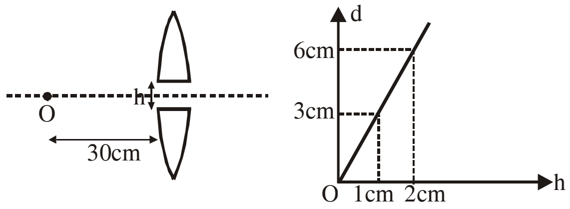
14. A glass prism is immersed in a hypothetical liquid. The curves showing the refractive index n as a function of wavelength λ for glass and liquid are as shown in the figure. When a ray of white light is incident on the prism parallel to the base :





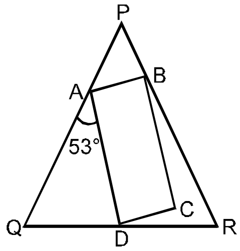
(A) yellow ray travels without deviation
(B) blue ray is deviated towards the vertex
(C) red ray is deviated towards the base
(D) there is no dispersion

15. Figure shows a convex lens cut symmetrically into two equal halves and separated laterally by a distance h. A point object O placed symmetrically at a distance 30 cm, from the lens halves, forms two images separated by a distance d. A plot of d versus h is shown in figure. The focal length of the lens is :-



(A) 22.5 cm (B) 40 cm (C) 45 cm (D) 20 cm

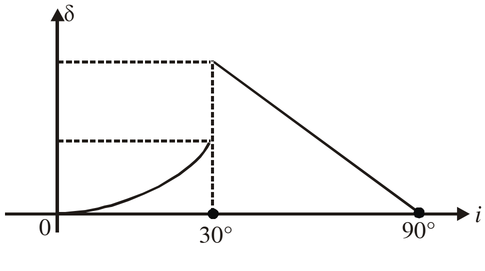
16. A rectangular cavity ABCD is carved inside an equilateral prism PQR of refractive index $\sqrt{2}$ shown in the figure. For a ray entering at face PQ and emerging at face PR, without passing through AB and CD:



(A) there will not be any effect of cavity on δ (B) there may be effect of cavity on δ

(C) for cavity filled with water $\delta_{min} = 30°$ (D) for cavity filled with air $\delta_{min} = 30°$

17. Figure shows graph of angle of deviation v/s angle of incidence for a light ray. Incident ray goes from medium 1 (μ_1) to medium 2 (μ_2). Mark the **correct** option(s).



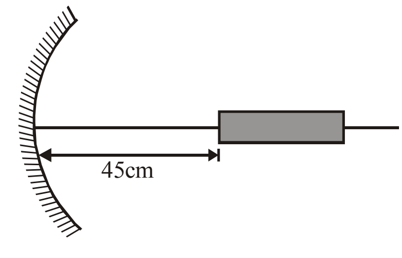
(A) $\frac{\mu_1}{\mu_2} = \frac{1}{2}$ (B) Critical angle is 30°

(C) $\mu_1 > \mu_2$ (D) Maximum deviation is 120°

SECTION-III

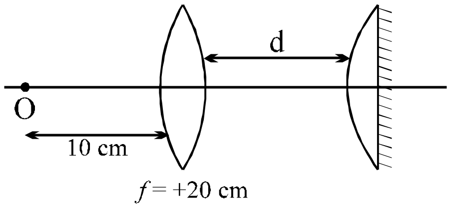
Numerical Grid Type (Ranging from 0 to 9) 6 Q. [4 M (0)]

1. An object of length 30 cm is placed on principal axis of a concave mirror of focal length 30 cm. Its one end at a distance of 45 cm as shown. If length of image is 10x (in cm) find the value of x.

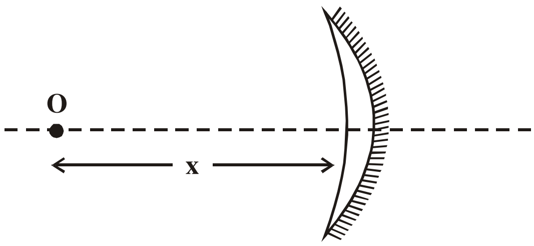


2. A lens is held directly above a coin lying on a table and forms an image of it. After the lens has been moved vertically a distance equal to its focal length, it forms another image of the coin equal in size to previous image. If the diameter of the coin is 4.0 cm, what is the diameter (in cm) of the image?

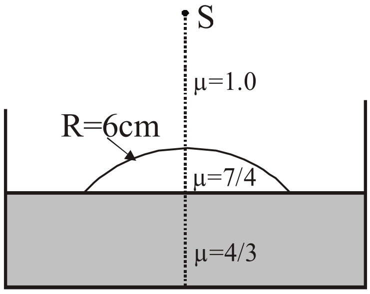
3. A convex lens of focal length 20 cm and another plano convex lens of focal length 40 cm are placed co-axially (see fig.). The plano convex lens is silvered on plane surface. What should be the 5d (in m) so that final image of the object 'O' is formed on O itself ?



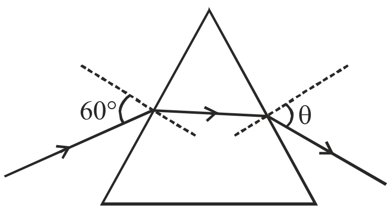
4. Radii of curvature of a concavo-convex lens (refractive index = 1.5) are 40 cm and 20 cm as shown. The convex side is silvered. The distance x (in cm) on the principal axis where an object is placed so that its image is created on the object itself, is given as 4β. Find the value of β.



5. Water (with refractive index = $\frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature 'R = 6 cm' as shown. Consider oil to act as a thin lens. An object 'S' is placed 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. Then 'x' is **[IIT-JEE 2011]**

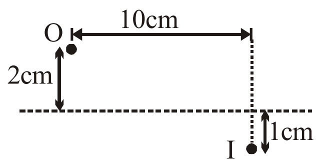


6. A monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle θ(n) with the normal (see the figure). For n = $\sqrt{3}$ the value of θ is 60° and $\frac{d\theta}{dn} = m$. The value of m is **[JEE-Advance-2015]**

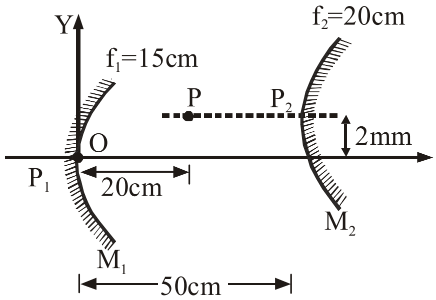


Subjective Type   5 Q. [4 M (0)]

1. The principal axis of a spherical mirror is shown by dotted line. O is the point object whose real image is I. Find the distance of the pole and centre of curvature of the mirror from object measured along principal axis by drawing ray diagram.

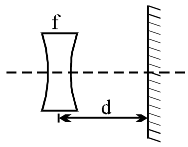


2. Find the co-ordinates of image of point object P formed after two successive reflection in situation as shown in figure considering first reflection at concave mirror and then at convex.

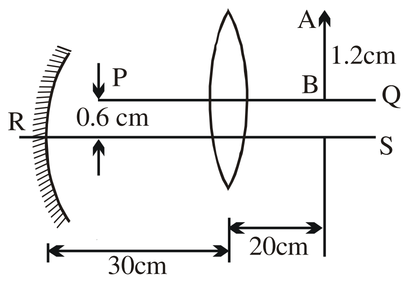


3. The x-y plane is the boundary between two transparent media. Medium-1 with z > 0 has refractive index $\sqrt{2}$ and medium – 2 with z < 0 has a refractive index $\sqrt{3}$. A ray of light in medium –1 given by the vector $A = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. Find the unit vector in the direction of refracted ray in medium -2. **[1999, 10M]**

4. A diverging lens of focal length 10 cm is placed 10 cm in front of a plane mirror as shown in the figure. Light from a very far away source falls on the lens. Find the image of source due to plane mirror (before hitting lens again) at a distance from mirror?



5. A convex lens of focal length 15 cm and a concave mirror of focal length 30 cm are kept with their optic axes *PQ* and *RS* parallel but separated in vertical direction by 0.6 cm as shown. The distance between the lens and mirror is 30 cm. An upright object *AB* of height 1.2 cm is placed on the optic axis *PQ* of the lens at a distance of 20 cm from the lens. If $A'B'$ is the image after refraction from the lens and reflection from the mirror, find the distance of $A'B'$ from the pole of the mirror and obtain its magnification. Also locate positions of A' and B' with respect to the optic axis *RS*. **[IIT-JEE 2000]**



SECTION-I

Single Correct Answer Type 11 Q. [3 M (–1)]

1. Ans. (D) **2. Ans. (C)** **3. Ans. (C)** **4. Ans. (A)**
5. Ans. (A) **6. Ans. (A)** **7. Ans. (D)** **8. Ans. (C)**
9. Ans. (B) **10. Ans. (B)** **11. Ans. (A)**

Multiple Correct Answer Type 6 Q. [4 M (–1)]

12. Ans. (B,C) **13. Ans. (A, C)** **14. Ans. (A,B,C)** **15. Ans. (B,D)**
16. Ans. (A,C,D) **17. Ans. (B,C,D)**

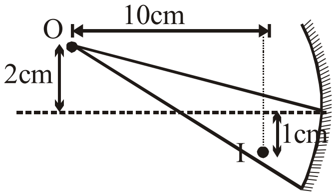
SECTION-III

Numerical Grid Type (Ranging from 0 to 9) 6 Q. [4 M (0)]

1. Ans. 4 **2. Ans. 8** **3. Ans. 1** **4. Ans. 4**
5. Ans. 2 **6. Ans. 2**

Subjective Type 5 Q. [4 M (0)]

1. Ans. 20 cm from object, 20/3 cm from object



2. Ans. (30 cm, – 14 mm)

3. Ans. $\vec{r} = \frac{3}{5\sqrt{2}}\hat{i} + \frac{2\sqrt{2}}{5}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}$ (angle of incidence = 60^0; $r = 45^0$)

4. Ans. 20 cm behind the mirror

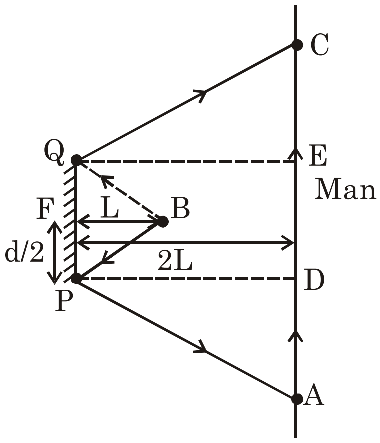
5. Ans. 15 cm right of the mirror, magnification = 1.5; (–15 cm, –1.5 cm; – 15 cm, 0.3 cm)

SOLUTION
SECTION-I

Single Correct Answer Type **11 Q. [3 M (–1)]**

1. Ans. (D)

Sol.



According to the ray diagram in the figure the man will be able to see light source from A to C, by using the law of bidirectionality of light.

Triangles BFP and PDA are similar

So, $\frac{L}{2L} = \frac{d/2}{AD} \Bigg| \; AD = d$

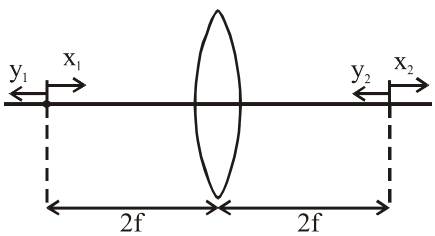
So, AD = d, DE = d, EC = d

So, AC = 3d

2. Ans. (C)

Sol. By the formula of lateral magnification we know, $\frac{\Delta\ell_{image}}{\Delta\ell_{object}} = m^2 = \left(\frac{V}{U}\right)^2$

So we have the following



$$\frac{\text{displacement of image}}{\text{displacement of object}} = \left(\frac{\text{position of image}}{\text{position of object}}\right)^2$$

3. Ans. (C)

Sol. The object must be placed on the centre of curvature of effective mirror which will be developed by silvering the lens. So, object must be placed at U = 2f and $f = \frac{1}{P_{Net}}$

So we have the solution below.

$P = P_L + P_m + P_L$

$$P_{net} = \left(\frac{4\mu - 2}{R}\right)$$

$$f = \frac{1}{2}\left(\frac{R}{2\mu - 1}\right)$$

4. Ans. (A)

Sol. By Cauchy's theorem we know

$\Rightarrow \mu(\lambda) = A + \frac{B}{\lambda^2}$ where $\lambda\uparrow$, $\mu\downarrow$

$\Rightarrow$ we know that $\lambda_{Red} > \lambda_{violet}$

$\Rightarrow$ V I B G Y O R

$\Rightarrow \longrightarrow$ (increasing wavelength)

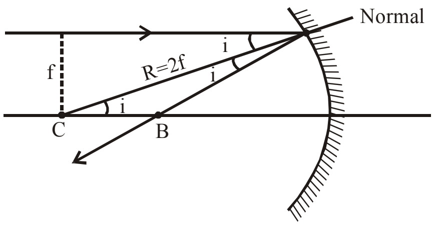
$\Rightarrow \longrightarrow$ (decreasing refractive index)

$\Rightarrow$ So, light rays with greater refractive index than green will be back in glass i.e. violet, indigo and blue.

$\Rightarrow$ and light rays with smaller refractive index than green will emerge out of glass in air i.e. yellow, orange and red.

5. Ans. (A)

Sol. $\sin i = \frac{f}{R} = \frac{1}{2}$; i = 30°



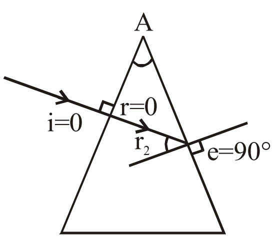
Not a paraxial ray

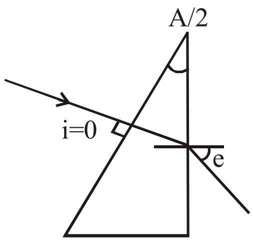
$$CB = \frac{R}{2\cos i} = \frac{f}{\left(\sqrt{3}/2\right)} = f\left(\frac{2}{\sqrt{3}}\right)$$

$$\frac{CB}{f} = \frac{2}{\sqrt{3}}$$

6. Ans. (A)

Sol.





$r_2 = A$

$\mu \sin A = 1 \sin 90°$

$$r_2 = \frac{A}{2}$$

$$\mu \sin\frac{A}{2} = 1 \sin e$$

dividing the two equations.

$$2\cos\mu_2 = \frac{1}{\sin e}$$

$$e = \sin^{-1}\left(\frac{1}{2}\sec\frac{A}{2}\right)$$

7. Ans. (D)

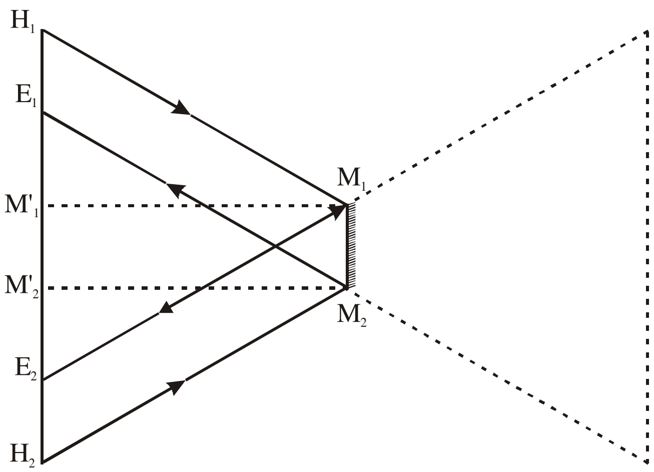
Sol. Let's say $H_1H_2 = D$, $E_1E_2 = d$

H_1H_2 = width of persons face.

E_1E_2 = separation between the eyes.

Let's say M_1M_2 = Width of mirror required.

Then following will be the ray diagram and calculations.



$$H_1M'_1 = \frac{H_1E_2}{2} \ \& \ H_2M'_2 = \frac{H_2E_1}{2}$$

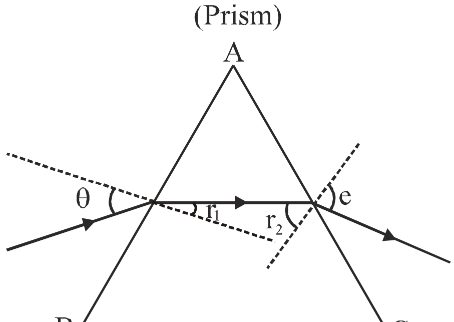
$$H_1E_2 = D - \frac{1}{2}(D-d) = \frac{D+d}{2} = H_2E_1$$

$$M'_1M'_2 = D - H_1M'_1 - H_2M'_2$$

$$= D - \left(\frac{D+d}{2}\right) = \frac{D-d}{2}$$

8. Ans. (C)

Sol. We wish to find out the condition on θ for which all the incident rays are transmitted through the other face.



⇒ we know that when
⇒ $\theta\uparrow, r_1\uparrow, r_2\downarrow, e\downarrow$
So $\theta\uparrow, e\downarrow$
When θ increases, e decreases. $[r_1 + r_2 = A]$
⇒ Now we want all the rays from AC to emerge out, for this we will increase e till 90°, and at this condition θ (→ min.) will tend to minimum θ_{min}.
⇒ If $\theta < \theta_{min}$ then all rays at AC will reflect back and TIR will happen at AC.
⇒ If $\theta \geq \theta_{min}$ then all the rays will refract out from face AC.
⇒ So, we need to find out θ_{min} and our answer will be then, $\theta \geq \theta_{min}$ .... (i)
⇒ Now when $\theta \to \theta_{min}$, $e \to \pi/2$
So, $1 \times \sin(\theta_{min}) = \mu \sin r_1$ ... (ii)
⇒ $\mu \sin(r_2) = 1 \times \sin \pi/2$ ... (iii)
⇒ $r_1 + r_2 = A$ ... (iv)
By solving these equations we get the answer.

9. Ans. (B)

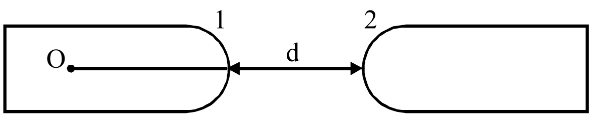
Sol. $i = 35°, \delta = 40°, e = 79°$
$\delta = i + e - A$
$40° = 35° + 79° - A$
$A = 74°$
and $r_1 + r_2 = A = 74°$
solving these, we get $\mu = 1.5$
Since $\delta_{min} < 40°$

$$\mu < \frac{\sin\left(\frac{74+40}{2}\right)}{\sin 37}$$

$\mu_{max} = 1.44$

10. Ans. (B)

Sol.



For first surface

$$\frac{1}{V} - \frac{1.5}{-50} = \frac{1-1.5}{-10}$$

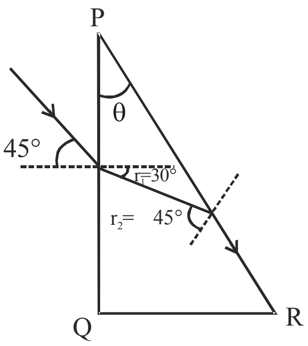
$V = 50$ cm
for second surface

$$\frac{1.5}{\infty} - \frac{1}{-(d-50)} = \frac{1.5-1}{10}$$

$d = 70$ cm
∴ (B)

11. Ans. (A)

Sol. This case of prism is unlike to the cases we are familiar with. In this case the light ray is coming from above, so our conventional formulas would not be applicable. In the ray diagram below we can say that:
⇒ $(r_1 + 90°) + (90 - r_2) + \theta = 180°$
⇒ $r_2 - r_1 = \theta$. Rest of the solution is as below.



$1 \sin 45° = \sqrt{2} \sin r_1$

$r_2 - r_1 = \theta$

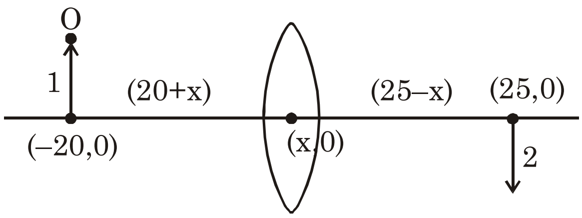
$\theta = 45° - 30°$

$\Rightarrow \theta = 15°$

Multiple Correct Answer Type 6 Q. [4 M (–1)]

12. Ans. (B,C)

Sol.



Let's say the lens is located at (x, 0), then following is the representations of object and image. Assuming x > 0.

$$\Rightarrow \frac{1}{(25-x)} + \frac{1}{(20+x)} = \frac{1}{f} \qquad \text{... (i)}$$

$$\Rightarrow \frac{25-x}{20+x} = \frac{2}{1} = 2 \qquad \text{... (ii)}$$

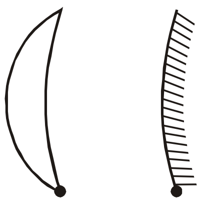
By solving above two equations we get the answer.

13. Ans. (A, C)

Sol. For a system to form real image of a real object, the system should be converging in nature. A diverging system cannot do that.

Now let's check with options. The power of system should be +ve for it to converge.

(A)



(i) (ii)

(i) Can have +ve or –ve power depending on radius of curvature.

(ii) Will have –ve power

Resultant power $\Rightarrow P_{Net} = P_1 = P_2 + P_1$

This may be +ve as well so there is a possibility that this system may form real image of real object.

(B) Both the lens and mirror are diverging so resultant will be diverging.

(C) Lens is converging and mirror's power is zero. So, final system will be converging.

(D) Lens $\Rightarrow$ Diverging, Mirror $\Rightarrow$ zero power $\Rightarrow$ Final system = Diverging

14. Ans. (A,B,C)

Sol. We know that, $\lambda_R > \lambda_y > \lambda_B$

By Cauchy's theorem we known that :

$$\mu(\lambda) = A + \frac{B}{\lambda^2}$$

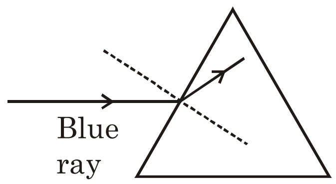
So, when $\lambda\uparrow$, $\mu\downarrow$. Now let's check with options.

(A) For λ_{yellow} $(= \lambda_Y)$, $\mu_{liquid} = \mu_{Glass}$

So ray will pass undeviated.

(B) For blue ray, $\mu_{liquid} > \mu_{Glass}$

as can be seen from the graph, so the light ray will deviate away from the normal



(C) For red ray, $\mu_{Glass} > \mu_{liquid}$, so light ray will bend towards the base.

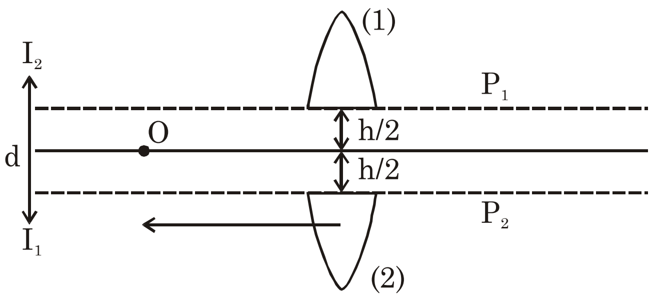
15. Ans. (B,D)

Sol. In this question, two cases are possible.

(1) Object is placed between pole and focus

(2) Object is placed beyon focus, away from lesn.

Case-I : When object is placed between pole and focus.



$\Rightarrow$ P_1 = Principal axis of lens (1)

$\Rightarrow$ P_2 = Principal axis of lens (2)

In this case the image will be virtual i.e. on the left side.

$\Rightarrow$ The object will be considered as an extended object for lens (1) placed below the principal axis by h/2.

$\Rightarrow$ Images by lenses (1) and (2) will be I_1 and I_2 as shown in the diagram.

$\Rightarrow$ Let's solve for lens (1)

$$\Rightarrow \frac{1}{(-V)} - \frac{1}{(-30)} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{30} - \frac{1}{V} = \frac{1}{f} \qquad \text{... (i)}$$

$\Rightarrow$ from the graph given we know, $\frac{d}{h} = \frac{3}{1}$ .... (ii)

magnifications of image will be :

$\Rightarrow m = \frac{d/2 + h/2}{h/2} = \frac{V}{30}$ ... (iii)

$\Rightarrow$ from (i) $\frac{1}{V} = \frac{1}{30} - \frac{1}{f}$

$V = \frac{30f}{f-30}$

$\Rightarrow$ from (iii) $\Rightarrow$ $\frac{d+h}{h} = \frac{30f}{f-30} \times \frac{1}{30}$ $\Rightarrow$ using (ii) here.

$\Rightarrow \frac{3+1}{1} = \frac{f}{f-30}$

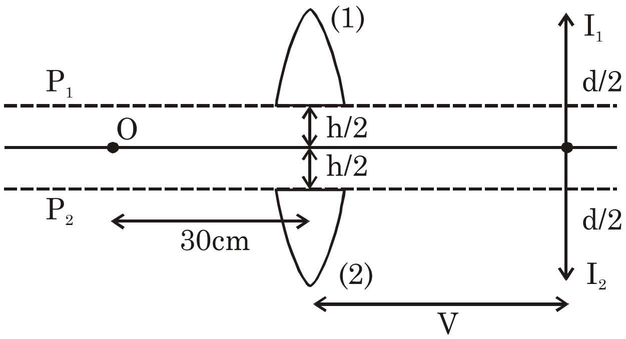
$\Rightarrow f = 4f - 120$

$\Rightarrow 3f = 120$

$f = 40$ cm

Case-2 :

Ray diagram for case 2 is as shown.



$\Rightarrow \frac{1}{V} + \frac{1}{30} = \frac{1}{f}$ ... (i)

$\Rightarrow \frac{d}{h} = 3$ ... (ii) [From graph]

$\Rightarrow \frac{d/2 - h/2}{h/2} = m = \frac{V}{30}$ .... (iii)

$\Rightarrow$ From (iii) $\Rightarrow \frac{3-1}{1} = \frac{V}{30} \Rightarrow V = 60$ ... (iv)

$\Rightarrow$ From (i) $\Rightarrow \frac{1}{60} + \frac{1}{30} = \frac{1}{f} \Rightarrow f = \frac{30 \times 60}{90} = 20$ cm

16. Ans. (A,C,D)

Sol. The cavity will behave like a thick parallel slab and we know that it does not bring any deviations to light, it just laterally displaces the light and that does not matter for our case.

So independent of what is filled in the cavity the minimum angle of deviation will not change.

$\Rightarrow$ We know that, $u = \dfrac{\sin\left(\dfrac{A+\delta_{min}}{2}\right)}{\sin(A/2)}$

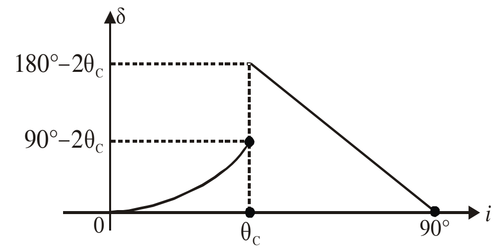
$$\Rightarrow \sqrt{2} = \frac{\sin\left(\frac{60+\delta_{min}}{2}\right)}{\sin\left(\frac{60}{2}\right)}$$

$\Rightarrow$ $\delta_{min} = 30°$ So, (A,C,D)

17. Ans. (B,C,D)

Sol. Here, $\theta_C = 30°$

and $\sin\theta_C = \dfrac{\mu_r}{\mu_d}$



or $\sin 30° = \dfrac{\mu_1}{\mu_2}$

so, $\dfrac{\mu_1}{\mu_2} = \dfrac{1}{2}$

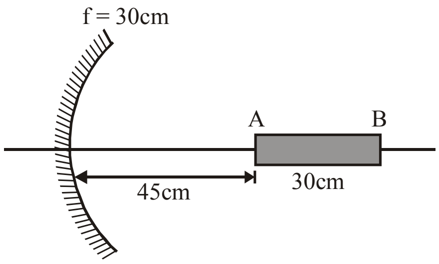
and δ_{max} $(180° - 2\theta_C) = 120°$ during TIR

SECTION-III

Numerical Grid Type (Ranging from 0 to 9) 6 Q. [4 M (0)]

1. Ans. 4

Sol.



Let's find the image of end A.

$$\Rightarrow \frac{1}{V_1} + \frac{1}{45} = \frac{1}{30}$$

$\Rightarrow V_1 = 90$ cm ... (i)

$\Rightarrow$ Let's find the image of end B.

$$\Rightarrow \frac{1}{V_2} + \frac{1}{(45+30)} = \frac{1}{30}$$

$V_2 = 50$ cm ... (ii)

$\Rightarrow$ So length of image $= |V_1 - V_2| = 40$ cm

2. Ans. 8

Sol. Object distance = – u, Focal length = f & Image distance = v

By lens formula $\longrightarrow$ $\dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u} \Rightarrow \dfrac{1}{f} = \dfrac{1}{v} + \dfrac{1}{u} \Rightarrow \dfrac{1}{v} = \dfrac{u-f}{uf} \Rightarrow v = \dfrac{uf}{u-f}$

So, $m_1 = \frac{v}{u} = \frac{f}{u-f}$ ....(1)

if lens has been moved by f vertically down, then Object distance (u') = – (u – f),
Focal length = f & image distance = v'

$\frac{1}{f} = \frac{1}{v'} + \frac{1}{(u-f)} \Rightarrow \frac{1}{v'} + \frac{u-f-f}{f(u-f)} \Rightarrow v' = \frac{f(u-f)}{(u-2f)}$

So, $m_2 = \frac{v'}{(u-f)} = \frac{f}{(u-2f)}$ ......(2)

$m_1 = m_2$ ....(3)

$\frac{f}{(u-f)} = \frac{f}{(u-2f)}$

$u - 2f = -u + f$

$2u = 3f$

$u = \frac{3}{2}f$ ....(4)

Put u in (1) from (4)

$v = 3f$ .....(5)

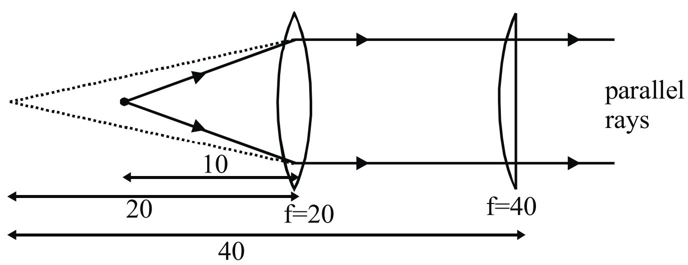
$m = \frac{v}{u} = 2$

$I = 2 \times 4 = 8$ cm

3. Ans. 1

Sol. The image will form at O itself if the image formed by first lens comes at the center of curvature of second system which will effectively be a concave mirror.

$\Rightarrow$ Following ray diagram shows the condition.



d= 20 cm; 5d = 100 cm = 1 meter

4. Ans. 4

Sol. $\frac{1}{F_{lens}} = (1.5-1)\left[\frac{1}{-40} - \frac{1}{-20}\right] = \frac{1}{80}$; $F_\ell = 80$ cm

$F_m = -\frac{20}{2} = -10$ cm; $\frac{1}{F_{eq}} = \frac{1}{f_m} - \frac{2}{f_\ell} = \frac{1}{-10} - \frac{2}{80}$; $f_{eq} = -8$ cm

Hence object should be placed at x = 16 cm, i.e. at the centre of curvature.

5. Ans. 2

Sol. Refraction from a spherical surface is given by,

$\frac{\mu_2}{v} = \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

For refraction from the first surface,

$$\frac{7}{4v_1}-\frac{1}{-24}=\frac{\frac{3}{4}}{6}$$

For refraction from second surface,

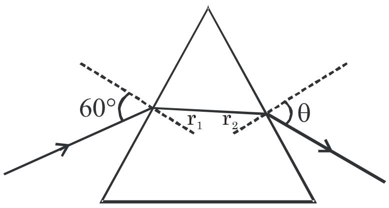
$$\frac{4}{3v_2}-\frac{7}{4v_1}=\frac{\frac{3}{4}-\frac{7}{4}}{\infty}$$

Adding the two equations gives,

$v_2 = 16$ cm

Thus distance from the bottom = depth – v_2 = 18 cm – 16 cm = 2 cm

6. Ans. 2

Sol. 

From the geometry we have $r_1 + r_2 = 60°$

Using Snells law at the left side we have $\frac{\sin 60°}{\sin r_1} = n$

or

$$\sin r_1 = \frac{\sqrt{3}}{2n}$$

Using Snells law at the right end

$$\frac{\sin r_2}{\sin\theta}=\frac{1}{n}$$

or

$\sin\theta = n \sin r_2$

$= n\sin(60° - r_1)$

$$= n\left(\frac{\sqrt{3}}{2}\cos r_1 - \frac{1}{2}\sin r_1\right)$$

or

$$= n\left(\frac{\sqrt{3}}{2}\sqrt{1-\sin^2 r_1} - \frac{1}{2}\sin r_1\right)$$

or

$$= n\left(\frac{\sqrt{3}}{2}\sqrt{1-\frac{3}{4n^2}} - \frac{1}{2}\frac{\sqrt{3}}{2n}\right)$$

Solving we get

$$\sin\theta = \frac{\sqrt{3}}{4}\sqrt{4n^2-3}-\frac{\sqrt{3}}{4}$$

Differentiating

$$\cos\theta d\theta = \frac{\sqrt{3}}{4} \times \frac{1}{2\sqrt{4n^2-3}} \times 8ndn$$

or

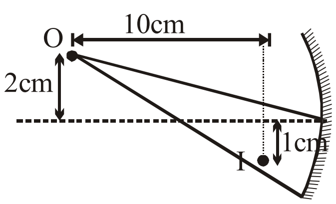
$$\frac{d\theta}{dn} = \frac{\sqrt{3}n}{\sqrt{4n^2-3}} \times \frac{1}{\cos\theta}$$

$$= \frac{\sqrt{3}\times\sqrt{3}}{\sqrt{4\times3-3}} \times \frac{1}{1/2}$$

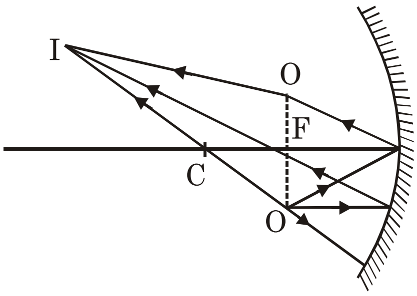
= 2

Subjective Type 5 Q. [4 M (0)]

1. **Ans.** 20 cm from object, 20/3 cm from object



Sol.



2. **Ans. (30 cm, – 14 mm)**

Sol. Consider concave mirror (M_1) from figure.
Object distance, $u_1 = -20$ cm
Focal length of the mirror, $f_1 = -15$ cm
Applying mirror formula,

$$\frac{1}{f_1} = \frac{1}{v_1} + \frac{1}{u_1}$$

$$\Rightarrow \frac{1}{v_1} = \frac{1}{f_1} - \frac{1}{u_1}$$

$$\Rightarrow \frac{1}{v_1} = \frac{1}{-15} - \frac{1}{-20}$$

$$\Rightarrow \frac{1}{v_1} = \frac{-4+3}{60}$$

$$\Rightarrow v_1 = -60 \text{ cm}$$

Magnification, $m_1 = -\frac{v_1}{u_1} = -\frac{(-60)}{(-20)} = -3$ (image is inverted)

The height at which the image point will be $m_1 \times AP = 3 \times 2 = 6$ mm
The image formed by concave mirror acts as object for convex mirror.
Now, at convex mirror (M_2) from figure
Object distance, $u_2 = +10$ cm
Focal length of the mirror, $f_2 = 20$ cm
Applying mirror formula, we get

$$\frac{1}{f_2} = \frac{1}{v_2} + \frac{1}{u_2}$$

$$\Rightarrow \frac{1}{v_2} = \frac{1}{f_2} - \frac{1}{u_2}$$

$$\Rightarrow \frac{1}{v_2} = \frac{1}{20} - \frac{1}{10}$$

$$\Rightarrow \frac{1}{v_2} = \frac{1-2}{20}$$

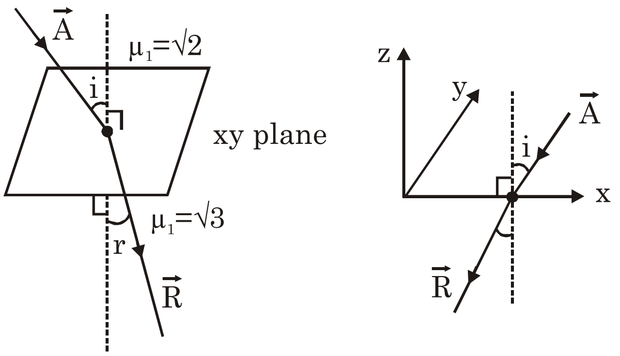
$\Rightarrow v_2 = -20$ cm
Hence the final position of image after two successive reflection at concave mirror and then at convex 20 cm in front of convex mirror, M_2

Magnification $m_2 = -\frac{v_2}{u_2} = -\frac{(-20)}{10} = +2$

So the image of the point will be formed above, $m_2 \times C'P' = 2 \times 8 = 16$ mm
So the required coordinates of the image is, P(30, –14)

3. **Ans.** $\vec{r} = \frac{3}{5\sqrt{2}}\hat{i} + \frac{2\sqrt{2}}{5}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}(\text{angle of incidence} = 60^0; r = 45^0)$

Sol.



From the figure, it can be seen that normal to the boundary separating the two media will be in z-direction, as incident ray, refracted ray and normal are in the same plane
The angle made by the incident ray with the normal is given as

$$\cos i = \frac{A_z}{A} = \frac{10}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + 100^2}} = \frac{10}{\sqrt{108 + 192 + 100}} = \frac{10}{20}$$

$$\Rightarrow \cos i = \frac{-1}{2} \Rightarrow i = \cos^{1}\left(\frac{1}{2}\right)$$

$$\Rightarrow i = 60°$$

Applying Snell's law of refraction at the interface of two media

$n_1 \sin i = n_2 \sin r$

$$\sqrt{2}\sin 60^\circ = \sqrt{3}\sin r$$

$$\sin r = \frac{\sqrt{2}\times\sqrt{3}}{\sqrt{3}\times 2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow r = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) \Rightarrow r = 45^\circ$$

As the X, Y components remain unchanged, so unit vector along direction of refracted ray is

$$\hat{R} = \frac{6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j}}{\sqrt{\left(6\sqrt{3}\right)^2 + \left(8\sqrt{3}\right)^2}}\sin 45 + \cos 45\left(-\hat{k}\right)$$

$$\hat{R} = \frac{3}{5\sqrt{2}}\hat{i} + \frac{4}{5\sqrt{2}}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}$$

$$\hat{R} = \frac{1}{5\sqrt{2}}\left(3\hat{i} + 4\hat{j} - 5\hat{k}\right)$$

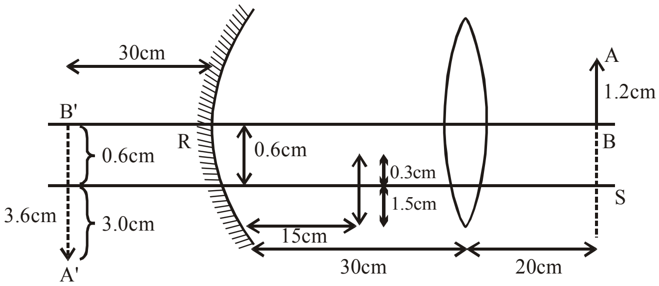
Hence, the answer is $\frac{1}{5\sqrt{2}}\left(3\hat{i} + 4\hat{j} - 5\hat{k}\right)$

4. **Ans.** 20 cm behind the mirror

Sol. Parallel ray on equi-concave lens will make the image on the left side of the lens at it's focal length. This will be considered an object for the plane mirror. This object will be at the distance of $|d + f|$ from the plane mirror on the left so image will be $|d + f|$ on the right side of mirror.

5. **Ans.** 15 cm right of the mirror, magnification = 1.5; (–15 cm, –1.5 cm; – 15 cm, 0.3 cm)

Sol.



For lens,

$u = -20$ cm, $f = +15$ cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow v_1 = \frac{uf}{u+f}$$

= 60 cm left of lens

This $A'B'$ will act as object for mirror.

So, $u_2 = +30$ cm

$f_2 = -30$ cm

$$\frac{1}{v_2} + \frac{1}{u_2} = \frac{1}{f_2} \Rightarrow v_2 = \frac{u_2 f_2}{u_2 - f_2} = -15 \text{ cm right of mirror}$$

Position of $A'' = (15 \text{ cm}, -1.5 \text{ cm})$

$B'' = (15 \text{ cm}, +0.3 \text{ cm})$