JEE(Main + Advanced) : LEADER & ENTHUSIAST COURSE SCORE(ADVANCED)

ANSWER KEY — PAPER-1

PART-1 : PHYSICS

Section									
SECTION-I (i)	Q.	1	2	3	4	5	6		
	A.	A,C	A,B,C	A,C	B	A,C	C,D		
SECTION-I (ii)	Q.	7	8	9	10				
	A.	C	C	C	B				
SECTION-II	Q.	1	2	3	4	5	6	7	8
	A.	2.00	200.00	3.00	25.00	1.00	3.00	4.00	6.00

PART-2 : CHEMISTRY

Section									
SECTION-I (i)	Q.	1	2	3	4	5	6		
	A.	D	B	A,B,C	B,C,D	A,B	A,C,D		
SECTION-I (ii)	Q.	7	8	9	10				
	A.	A	A	C	B				
SECTION-II	Q.	1	2	3	4	5	6	7	8
	A.	2.00	5.00	2.00	3.00	840.00	3.00	7.00	1.11

PART-3 : MATHEMATICS

Section									
SECTION-I (i)	Q.	1	2	3	4	5	6		
	A.	B,C	A,C	A,C	A,B	A,B,C	A,B		
SECTION-I (ii)	Q.	7	8	9	10				
	A.	C	B	C	C				
SECTION-II	Q.	1	2	3	4	5	6	7	8
	A.	7.00	1.00	9.00	4.00	6.00	7.00	19.00	20.00

HINT – SHEET

PART-1 : PHYSICS

SECTION-I (i)

1. **Ans (A,C)**

$$\frac{\mu}{v}-\frac{1}{\infty}=\frac{(\mu-1)}{10}$$

$$v=\frac{10\mu}{\mu-1}=\frac{10}{1-\frac{1}{\mu}}$$

$v_v < v_r$

$$v_v=\frac{10}{0.615}\times 1.615$$

$$v_r=10\times\frac{1.6}{0.6}$$

$$v_v-v_r=\frac{16.15}{0.615}-\frac{16}{0.6}=\frac{16.15\times 0.6-16\times 0.615}{0.6\times 0.615}$$

$= 0.40\text{cm}$

2. **Ans (A,B,C)**

$$I_{BD}=\frac{1}{2}\left[\frac{4}{3}m\ell^2\right]+\left[\frac{1}{2}\left(\frac{4}{3}m\ell^2\right)+4m\ell^2\right]+2\left[\frac{m\ell^2}{3}+m\left(\frac{\ell^2}{12}\right)\right]$$

(B) $I_{HI}=8\left(\frac{mL^2}{12}+\frac{mL^2}{4}\right)+4m\left(\frac{\ell}{\sqrt{2}}\right)^2$

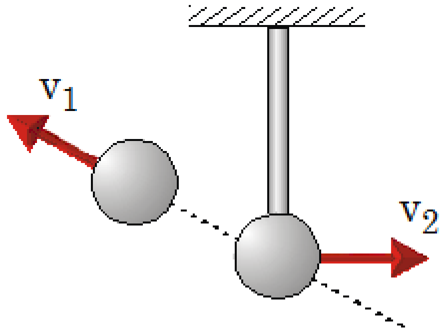
(C) $4\left(\frac{mL^2}{3}\right)+2\left(m\ell^2\right)+$

$m(\sqrt{2}\ell)^2+4\left(\frac{m\ell^2}{12}+m\left(\frac{5\ell^2}{4}\right)\right)$

(D) $\frac{2m\ell^2}{12}+2m\left(\frac{\ell}{2}\right)^2+2\left[\frac{5m\ell^2}{4}\right]+$

$2\left(\frac{m\ell^2}{12}+m\ell^2\right)+4\left(\frac{m\ell^2}{12}+\frac{m\ell^2}{4}\right)$

3. **Ans (A,C)**



$v_2 \cos 60 + v_1 = 5e$

$m(5) \sin 30 = -\frac{mv_1}{2} + mv_2$

4. **Ans (B)**

Conceptual

5. **Ans (A,C)**

Use $\int \left(\frac{1}{2}\varepsilon_0 E^2\right) dv$

6. **Ans (C,D)**

v = 10 m/s, f = 5 Hz, $\lambda = 2m$

Point P can be antinode.

SECTION-I (ii)

7. **Ans (C)**

$4(7.1) - 2(2 \times 1.15) = Q$

8. **Ans (C)**

$117\ (2 \times 8.5) - 236\ (7.6) = Q$

9. **Ans (C)**

Since coherence time is finite, as we move away from O, interfering waves will not be able to maintain a definite phase relationship for a appreciable time.

10. **Ans (B)**

$d\sin\theta = (10^{-10})\ (3 \times 10^8) = 3 \times 10^{-2} m = 3$ cm

(A) for d = 0.3 cm, we will get fringes on entire screen.

(B) for d = 0.03 cm, we will get fringes on entire screen.

(D) for d = 3 cm, $\theta = 90°$

SECTION-II

1. **Ans (2.00)**

$E = B(\omega r) = 2(2)\ (1/2) = 2$ N/C

2. **Ans (200.00)**

$\sigma_m = \alpha\ \theta m\ Y \Rightarrow \theta_m = 200°C$

3. **Ans (3.00)**

$\sigma = -\eta \frac{dv}{dy}$

$\Rightarrow \sigma = (1.5)\ [4 - 2y]$

$\sigma = \frac{3}{2}[4 - 2] = 3$

4. **Ans (25.00)**

$v = \sqrt{\frac{2Mg}{A\rho}} = 10\sqrt{2}\,m/s$

$t = \frac{V}{av} = 25$ sec

5. **Ans (1.00)**

$\varepsilon = \frac{\frac{2}{2} + \frac{4}{6}}{\frac{1}{2} + \frac{1}{6}} = \frac{\frac{5}{3}}{\frac{2}{3}} = 2.5$ volt

$\left(\frac{12.5}{4 + 16}\right) \frac{16}{4} y = 2.5 \Rightarrow y = 1m$

6. **Ans (3.00)**

$ma = -Kx_1$

$2Kx_1 = 2Kx_2$

$2x_2 + x_1 = x$

$x_1 = x/3$

$T = 2\pi\sqrt{\frac{3m}{K}}$

7. **Ans (4.00)**

$u_b = 4$ m/s; $\beta = 30°$

$\frac{240}{u\cos\beta + 2} + \frac{240}{u\cos\beta - 2} = 120\sqrt{3}\,s$

$u \sin\beta = 2$

8. **Ans (6.00)**

Conservation of momentum

$1 \times 8 + 3 \times 4 = (1 + 3)v$ ....(i)

$v = 5$

Applying work energy theorem

$w_f = \frac{1}{2} \times 4 \times 5^2 - \frac{1}{2} \times 1 \times 8^2 - \frac{1}{2} \times 3 \times 4$

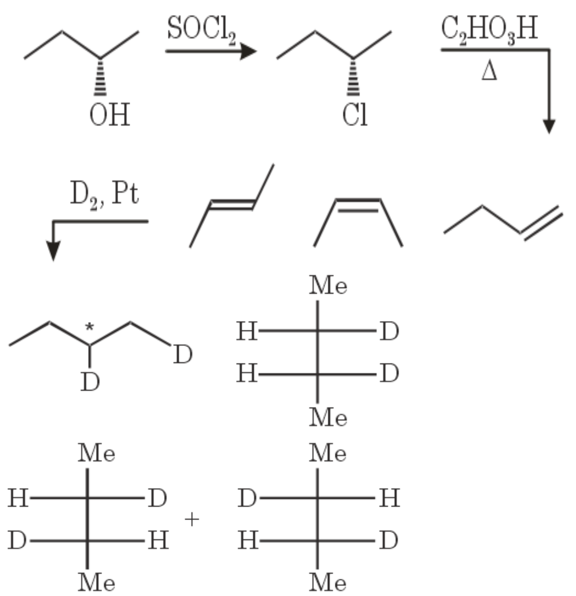
$w_f = -6$

$w_{against} = 6$

PART-2 : CHEMISTRY

SECTION-I (i)

1. **Ans (D)**



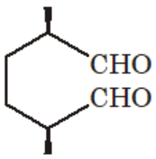
2. **Ans (B)**

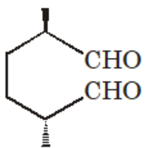
$\underbrace{Sc > Ti > V > Cr > Mn}_{\text{size}\downarrow} > \underbrace{Fe \approx Co \approx Ni}_{\text{size constant}} < \underbrace{Cu < Zn}_{\text{size}\uparrow}$

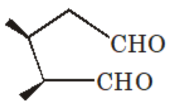
3. **Ans (A,B,C)**

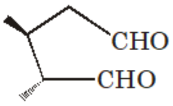
Orthorhombic crystal system have 4 bravias lattice.

4. **Ans (B,C,D)**

(I)  (Optically Inactive)

(II)  (Optically active)

(II)  (Optically active)

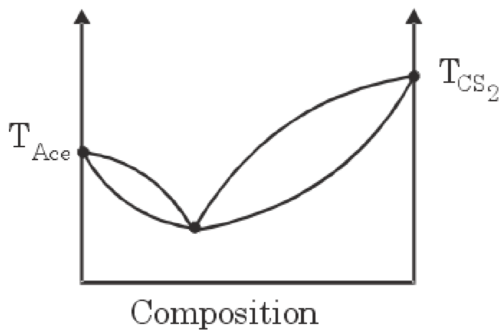
(IV)  (Optically active)

5. **Ans (A,B)**

In parke's process Ag – Zn are separated by distillation

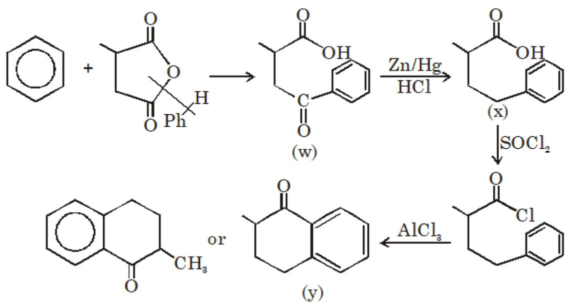
In amalgamation Hg – Au/Hg – Ag are separated by distillation

6. **Ans (A,C,D)**



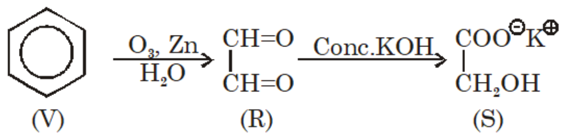
SECTION-I (ii)

7. **Ans (A)**



As given in option.

8. **Ans (A)**



(Cannizaro reaction)

9. **Ans (C)**

IF (polar and planar)

IF_3 (polar and planar)

IF_5 (polar and non-planar)

IF_7 (non-polar and non-planar)

10. Ans (B)

Molecule electronic geometry   molecular geometry

(A) NCl_3 Tetrahedral   Trigonal pyramidal

(B) PCl_5 Trigonal bi-pyramidal Trigonal bi-pyramidal

(C) SF_4 Trigonal bi-pyramidal See-saw

(D) XeF_4 Square bi-pyramidal Square planar

SECTION-II

1. Ans (2.00)

$$0.441 = E^{o}_{OP} - \frac{0.06}{2} \cdot \log \frac{[H^+]^2_a}{1} +$$

$$+E^{o}_{RP} - \frac{0.06}{2} \times \log \frac{2}{[H^+]^2_c}$$

$$= -\frac{0.06}{2}\log[H^+]^2_a - \frac{0.06}{2}\log 2 + \frac{0.06}{2}\log[H^+]^2_c$$

$$= [H^+]_a = \frac{10^{-14}}{\sqrt{\frac{10^{-14}}{10^{-5}} \times 1}} = 10^{-9.5},$$

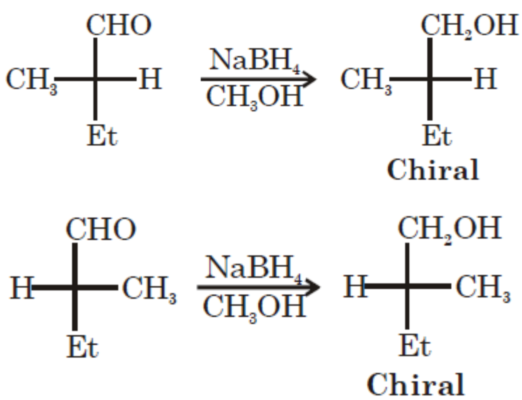
$[H^+]c = C$, hence $C = 10^{-2}$, $x = 2$

2. Ans (5.00)

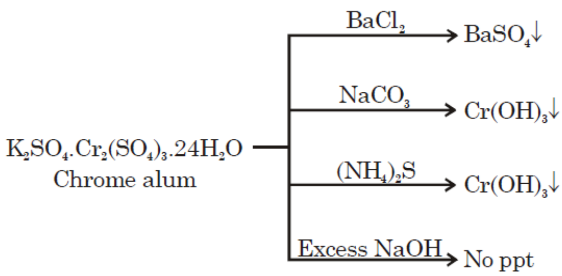
Theory based.

i), (ii), (iv), (v) & (vi) are correct.

3. Ans (2.00)



4. Ans (3.00)



5. Ans (840.00)

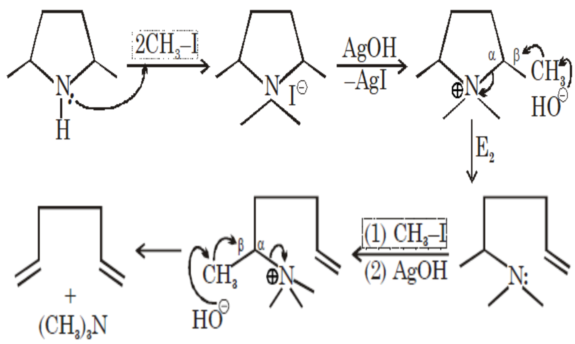
$$\Delta S_{sys} = nR \ln \frac{V_2}{V_1} = 2R \ln 2 = 2.8 \text{ cal K}^{-1}$$

$\Delta S_{surr} = 0$

$\Delta S_{univ} = 2.8$ cal /K

$|\Delta G_{sys}| = T\Delta S_{univ.} = 300 \times 2.8 = 840$

6. Ans (3.00)



7. Ans (7.00)

'10' NH_3 (g) + '3'$Cl_2O \rightarrow 6\,NH_4Cl + 2N_2 + 3H_2O$

$10 - 3 = 7$

8. Ans (1.11)

$P(V_m - b) = R.T$

$$V_m = 10\,b \Rightarrow P\left(V_m - \frac{V_m}{10}\right) = RT$$

$$\frac{9}{10} P.V_m = RT \Rightarrow \left(\frac{PV_m}{RT}\right) = Z = \frac{10}{9}$$

PART-3 : MATHEMATICS

SECTION-I (i)

1. **Ans (B,C)**

Observe that the lines L_1, L_2 & L_3 are parallel to the vector $\hat{i} - \hat{j} - \hat{k}$.

Also, $\Delta = 0 = \Delta_1$ & $b_1c_2 \neq b_1c_1$ Hence the three planes intersect in a line.

$P_1 = 2x + y + z + 4 = 0$

$P_2 = 0x + y - z + 4 = 0$

$P_3 = 3x + 2y + z + 8 = 0$

P_2 and P_3 gives line L_1

vector parallel to line $L_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -1 \\ 3 & 2 & 1 \end{vmatrix}$

$= 3\hat{i} - 3\hat{j} - 3\hat{k}$

$= 3\left[\hat{i} - \hat{j} - \hat{k}\right]$

Similarly

vector parallel to L_2, P_3 and $P_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{vmatrix}$

$= -\hat{i} + \hat{j} + \hat{k} = -\left[\hat{i} - \hat{j} - \hat{k}\right]$

Similarly

vector parallel to L_3, P_1 and $P_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix}$

$= (-2)\hat{i} - (-2)\hat{j} + 2\hat{k}$

$= -2\hat{i} + 2\hat{j} + 2\hat{k}$

$= -2\left(\hat{i} - \hat{j} - \hat{k}\right)$

We can see all the lines are parallel to vector $\left(\hat{i} - \hat{j} - \hat{k}\right)$

Also $2x + y + z = -4$

$0x + y - z = -4$

$3x + 2y + z = -8$

$$\Delta = \begin{vmatrix} 2 & 1 & 1 \\ 0 & 1 & -1 \\ 3 & 2 & 1 \end{vmatrix}$$

$\Rightarrow 2(1 + 2) - 1(0 + 3) + 1(0 - 3)$

$\Rightarrow 2(3) - 3 - 3 = 0 \quad \Delta_1 = \begin{vmatrix} -4 & 1 & 1 \\ -4 & 1 & -1 \\ -8 & 2 & 1 \end{vmatrix} = 0$

$$\Delta_2 = \begin{vmatrix} 2 & -4 & 1 \\ 0 & -4 & -1 \\ 3 & -8 & 1 \end{vmatrix} = 0$$

$$= -4\begin{vmatrix} 2 & 1 & 1 \\ 0 & 1 & -1 \\ 3 & 2 & 1 \end{vmatrix} = 0$$

$\Delta_3 = 0$

So all planes intersection in line L.

2. **Ans (A,C)**

$$\int f'(x)\,(1 + f(x)) = \int 1 + x$$

$$\frac{(1 + f(-x))^2}{2} = \frac{(1 + x)^2}{2} + c$$

Here $f(0) = 1$

$$\frac{(1 + 1)^2}{2} = \frac{1}{2} + c \quad \therefore \frac{3}{2} = c$$

$$\frac{(1 + f(x))^2}{2} = \frac{(1 + x)^2}{2} + \frac{3}{2}$$

$(1 + f(x))^2 = (1 + x)^2 + 3$

$$1 + f(x) = \pm\sqrt{(1 + x)^2 + 3}$$

$$f(x) = -1 \pm \sqrt{(1 + x)^2 + 3}$$

$f(0) = 1 \quad \therefore f(x) = -1 + \sqrt{(1 + x)^2 + 3}$

$f(x) = -1 + \sqrt{(1 + x)^2 + 3} \quad \Rightarrow f(x) > -1 \ \forall x > 0$

$f'(x) = \dfrac{2(1 + x)}{\sqrt{(1 + x)^2 + 3}} \ ; \ f'(x) > 0 \ \forall x > 0$

$\therefore$ option (A) and (C) are correct

3. **Ans (A,C)**

For real roots $4a^2 - 4b(a-1) \geq 0$

$\Rightarrow \quad a^2 - ba + b \geq 0.$

Now for above inequality to hold for all values of a, $b^2 - 4b \leq 0$.

or $\quad 0 \leq b \leq 4$.

4. **Ans (A,B)**

Let r be common ratio of G.P. a, b, c we have

$$z = \frac{\frac{a}{b}+i}{\frac{c}{b}-i} = \frac{\frac{1}{r}+i}{r-i} = \frac{i}{r}$$

Hence $z = \dfrac{ib}{c}$ or $\dfrac{ia}{b}$

5. **Ans (A,B,C)**

$$\angle B = \sec^{-1}\left(\frac{5}{4}\right) + \operatorname{cosec}^{-1}\sqrt{5}$$

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{2}\right)$$

$$= \tan^{-1}\frac{\frac{3}{4}+\frac{1}{2}}{1-\frac{3}{4}\cdot\frac{1}{2}} = \tan^{-1}2$$

$$\angle C = \operatorname{cosec}^{-1}\left(\frac{25}{7}\right) + \cot^{-1}\left(\frac{9}{13}\right)$$

$$= \tan^{-1}\left(\frac{7}{24}\right) + \tan^{-1}\left(\frac{13}{9}\right)$$

$$= \tan^{-1}\left(\frac{\frac{7}{24}+\frac{13}{9}}{1-\frac{7}{24}\cdot\frac{13}{9}}\right) = \tan^{-1}3$$

$\therefore \quad \angle A = \pi - \angle B - \angle C = \pi - \tan^{-1}2 - \tan^{-1}3$

$= \tan^{-1}1$

$$\therefore \sin A = \frac{1}{\sqrt{2}} \; \sin B = \frac{2}{\sqrt{5}} \; \text{ and } \sin C = \frac{3}{\sqrt{10}}$$

$$\therefore a = \sin A \cdot \frac{c}{\sin C} = \frac{1}{\sqrt{2}} \cdot \frac{3}{\left(\frac{3}{\sqrt{10}}\right)} = \sqrt{5}$$

and

$$b = \sin B \cdot \frac{c}{\sin C} = \frac{2}{\sqrt{5}} \cdot \frac{3}{\left(\frac{3}{\sqrt{10}}\right)} = \sqrt{2}$$

(1) tan A = 1, tan B = 2, tan C = 3 are in A.P.

Ans. (A)

(3) Area of ΔABC, $\Delta = \dfrac{1}{2}ab\sin C = 3$

All other parameters are irrational.

6. **Ans (A,B)**

$$a_2 + b_2 = \int_0^{\pi/4} \ln(\sin x \cos x)dx$$

$$\Rightarrow a_2 + b_2 = \int_0^{\pi/4} \ln(\sin 2x)dx - \int_0^{\pi/4} \ln 2\, dx$$

$$2x = t \Rightarrow a_2 + b_2 = \frac{1}{2}\int_0^{\pi/2} \ln(\sin t)dt - \frac{\pi}{4}\ln 2$$

$$\text{Now } a_1 = \int_0^{\pi/2} \ln(\sin t)dt = \int_0^{\pi/2} \ln(\cos t)dt$$

$$\Rightarrow 2I = \int_0^{\pi/2} \ln(\sin t \cos t)dt$$

$$\Rightarrow 2a_1 = \int_0^{\pi/2} \ln(\sin 2x)dx - \int_0^{\pi/2} \ln 2dx$$

$$= \frac{1}{2}\int_0^{\pi} \ln(\sin x)dx - \frac{\pi}{2}\ln 2$$

$$= \int_0^{\pi} \ln(\sin x)dx - \frac{\pi}{2}\ln 2$$

$$\Rightarrow a_1 = -\frac{\pi}{2}\ln 2 \quad \Rightarrow a_1 + b_2 = -\frac{\pi}{2}\ln 2$$

SECTION-I (ii)

7. **Ans (C)**

Let f(x) has degree n so

$n^2 = 1 + n + 1 \Rightarrow n = 2$

$\Rightarrow$ f(x) is quadratic with f(0) = 0 let $f(x) = ax^2 + bx$

so $a(ax^2 + bx)^2 + b(ax^2 + bx) = x\left[\dfrac{ax^3}{3} + \dfrac{bx^2}{2}\right] \; \forall$

$x \in R$

$$\Rightarrow \quad a^3 - \frac{a}{3} = 0; 2a^3b - \frac{b}{2} = 0 \; ; \; ab^2 + ab = 0$$

and $b^2 = 0$

$\Rightarrow b = 0$ and $a = \pm\dfrac{1}{\sqrt{3}}$

$\because$ given leading coefficient is positive

$$\Rightarrow \quad f(x) = \frac{x^2}{\sqrt{3}}.$$

8. Ans (B)

Let f(x) has degree n so

$n^2 = 1 + n + 1 \Rightarrow n = 2$

$\Rightarrow$ f(x) is quadratic with f(0) = 0 let $f(x) = ax^2 + bx$

so $a(ax^2 + bx)^2 + b(ax^2 + bx) = x\left[\frac{ax^3}{3} + \frac{bx^2}{2}\right]$ $\forall$ $x \in R$

$\Rightarrow a^3 - \frac{a}{3} = 0 \; ; 2a^3b - \frac{b}{2} = 0 \; ; ab^2 + ab = 0$ and $b^2 = 0$

$\Rightarrow b = 0$ and $a = \pm\frac{1}{\sqrt{3}}$

$\because$ given leading coefficient is positive

$\Rightarrow f(x) = \frac{x^2}{\sqrt{3}}$.

9. Ans (C)

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} ; B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

$$A + \alpha B = \begin{bmatrix} a + \alpha p & b + \alpha q \\ c + \alpha r & d + \alpha s \end{bmatrix}$$

$$|A + \alpha B| = \begin{vmatrix} a + \alpha p & b + \alpha q \\ c + \alpha r & d + \alpha s \end{vmatrix}$$

$\Rightarrow |A| + \alpha^2|B| + \alpha(as + pd - br - qc)$

$$|\alpha A + B| = \begin{vmatrix} \alpha a + p & \alpha b + q \\ \alpha c + r & \alpha d + s \end{vmatrix}$$

$= \alpha^2|A| + |B| + \alpha[as + pd - br - qc]$

$|A + \alpha B| - |\alpha A + B|$

$= (1 - \alpha^2)|A| + (\alpha^2 - 1)|B|$

$= (1 - \alpha^2)(|A| - |B|)$

$= \alpha^2 - 1$

(C) option correct

10. Ans (C)

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} ; B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

$$A - \alpha B = \begin{bmatrix} a - \alpha p & b - \alpha q \\ c - \alpha r & d - \alpha s \end{bmatrix}$$

$$|A - \alpha B| = \begin{vmatrix} a - \alpha p & b - \alpha q \\ c - \alpha r & d - \alpha s \end{vmatrix}$$

$\Rightarrow |A| + \alpha^2|B| - \alpha(as + pd - br - qc)$

$$|-\alpha A + B| = \begin{vmatrix} -\alpha a + p & -\alpha b + q \\ -\alpha c + r & -\alpha d + s \end{vmatrix}$$

$= \alpha^2|A| + |B| - \alpha[as + pd - br - qc]$

$|A - \alpha B| - |-\alpha A + B|$

$= (1 - \alpha^2)|A| + (\alpha^2 - 1)|B|$

$= (1 - \alpha^2)(|A| - |B|)$

$= \alpha^2 - 1$

(C) option correct

SECTION-II

1. Ans (7.00)

The distance of the point 'a' from the plane $\vec{r} \cdot \vec{n} = q$ measured in the direction of the unit vector $\hat{b}$ is

$$= \frac{q - \vec{a} \cdot \vec{n}}{\vec{b} \cdot \vec{n}}.$$

Here $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{n} = \hat{i} + \hat{j} + \hat{k}$ and q = 5

Also

$$\vec{b} = \frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{\sqrt{(2)^2 + (3)^2 + (-6)^2}} = \frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{7}$$

$\therefore$ The required distance =

$$\frac{5 - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) \cdot \left(\hat{i} + \hat{j} + \hat{k}\right)}{\frac{1}{7}\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right) \cdot \left(\hat{i} + \hat{j} + \hat{k}\right)} = 7.$$

2. Ans (1.00)

For more than 2 roots the given equation must be an identity. Hence p = 1 as for p = 1 each term becomes zero independent of value of x.

3. **Ans (9.00)**

Area = ab = 36 gives 9 hyperbolas.

4. **Ans (4.00)**

Let P be (h, k), then chord of contact will be hx = –4(y + k) or hx + 4y + 4k = 0.

For being tangential to $x^2 + y^2 = 4$,

$$\left|\frac{4k}{\sqrt{h^2+16}}\right| = 2 \text{ or } 4k^2 - h^2 = 16.$$

5. **Ans (6.00)**

$$52 \xrightarrow{\text{face card removed}} 40 \xrightarrow{\text{20 drawn randomly}}$$

Let E_0 : 20 cards randomly removed has 4 aces.

E_1 : 20 cards randomly removed has exactly 3 aces.

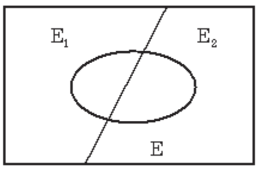
E_2 : 20 cards randomly removed has exactly 2 aces.

E : event that 2 drawn from the remaining 20 cards has both the aces.

$$P(E) = P(E \cap E_0) + P(E \cap E_1) + P(E \cap E_2)$$

$$= P(E_0) \cdot P\left(\frac{E}{E_0}\right) + P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)$$

$$= 40 \left[\begin{array}{l} 4 \text{ aces} \\ 36 \text{ other} \end{array}\right.$$



$$= \frac{{}^4C_0 \cdot {}^{36}C_{20}}{{}^{40}C_{20}} \cdot \frac{{}^4C_2}{{}^{20}C_2} + \frac{{}^4C_1 \cdot {}^{36}C_{19}}{{}^{40}C_{20}} \cdot \frac{{}^3C_2}{{}^{20}C_2} + \frac{{}^4C_2 \cdot {}^{36}C_{18}}{{}^{40}C_{20}} \cdot \frac{{}^2C_2}{{}^{20}C_2}$$

$$= \frac{({}^{36}C_{20} \cdot {}^4C_2) + ({}^4C_1 \cdot {}^{36}C_{19} \cdot {}^3C_2) + ({}^4C_2 \cdot {}^{36}C_{18} \cdot {}^2C_2)}{{}^{40}C_{20} \cdot {}^{20}C_2}$$

$$= \frac{6 \cdot {}^{36}C_{20} + 12 \cdot {}^{36}C_{19} + 6 \cdot {}^{36}C_{18}}{{}^{40}C_{20} \cdot {}^{20}C_2}$$

$$= \frac{6\left[{}^{36}C_{20} + 2\,{}^{36}C_{19} + {}^{36}C_{18}\right]}{{}^{40}C_{20} \cdot {}^{20}C_2}$$

$$= \frac{6\left({}^{37}C_{20} + {}^{37}C_{19}\right)}{{}^{40}C_{20} \cdot {}^{20}C_2} = \frac{6\left({}^{38}C_{20}\right)}{{}^{40}C_{20} \cdot {}^{20}C_2} \Rightarrow p = 6.$$

6. **Ans (7.00)**

As 3x + 4y = 7 is normal to y = f(x) at (1, 1) hence f(1) = 1 and $f'(1) = \frac{4}{3}$.

Also 4x – 3y = 1 will be the tangent and hence $f(x) = \frac{4x-1}{3}$ at x = 1. Now

$f(x) \geqslant \frac{4x-1}{3}$ for $x \geq 1$ & $f(x) \leqslant \frac{4x-1}{3}$ for $x \leq 1$ implies on right hand of x = 1 curve of y = f(x) lies above its tangent and on left hand of x = 1 curve of y = f(x) lies below its tangent. Therefore x = 1 is a point of inflection, hence f″(1) = 0.

Now $\lim\limits_{x \to 1} \frac{3f'(x) - 2f(x) - 2x}{f(x) - x^2}$ is in $\frac{0}{0}$ indeterminate form. Applying L'hospital Rule gives $= \lim\limits_{x \to 1} \frac{3f'(x) - 2f(x) - 2x}{f(x) - x^2} = \lim\limits_{x \to 1} \frac{3f''(x) - 2f'(x) - 2}{f'(x) - 2x}$

Hence the required limit is 7.

7. **Ans (19.00)**

a + c = 2b ⇒ 3b = 30 ⇒ b = 10 ⇒ a + c = 20.

Number of positive integer solution is ${}^{19}C_1 = 19$.

8. **Ans (20.00)**

$(1 - x + 2x^2)^{12} = ((1 - x) + 2x^2)^{12}$

$= (1 - x)^{12} + {}^{12}C_1(1 - x)^{11}\, 2x^2 + {}^{12}C_2\,(1 - x)^{10}\, 4x^4 + ...$

co-efficient of x^4 = co-efficient of x^4 in $(1 - x)^{12}$ + 24 co-efficient of x^2 in $(1 - x)^{11} + 4 \times {}^{12}C_2$

$= {}^{12}C_4 + 24 \times {}^{11}C_2 + 4 \times {}^{12}C_2 = {}^{12}C_4 + \frac{2 \cdot 12 \cdot 11 \cdot 10 \cdot 3}{1 \cdot 2 \cdot 3} + 4 \cdot {}^{12}C_2$

$= {}^{12}C_4 + 6 \times {}^{12}C_3 + 4 \times {}^{12}C_2$

$= {}^{12}C_4 + 2 \times {}^{12}C_3 + 4({}^{12}C_3 + {}^{12}C_2)$

$= {}^{12}C_4 + 2 \times {}^{12}C_3 + 4 \times {}^{13}C_3$

$= {}^{12}C_4 + {}^{12}C_3 + {}^{12}C_3 + 4 \times {}^{13}C_3$

$= {}^{13}C_4 + {}^{13}C_3 + {}^{12}C_3 + 3 \times {}^{13}C_3$

$= {}^{14}C_4 + 3 \times {}^{13}C_3 + {}^{12}C_3 \Rightarrow n = 12, r = 3$

Hence the required coeff. is ${}^6C_3$ i.e. 20.

JEE(Main + Advanced) : LEADER & ENTHUSIAST COURSE SCORE(ADVANCED)

ANSWER KEY — PAPER-2

PART-1 : PHYSICS

SECTION-I (i)	Q.	1	2	3	4	5	6		
	A.	A,D	A,B,D	A,B,C,D	B,C	A,C	A,B		
SECTION-I (ii)	Q.	7	8	9	10				
	A.	B	C	D	A				
SECTION-II	Q.	1	2	3	4	5	6	7	8
	A.	1.94	0.40	216.85 to 217.00	15.00	33.60	2.00	2.00	4.00

PART-2 : CHEMISTRY

SECTION-I (i)	Q.	1	2	3	4	5	6		
	A.	A,C	B,C	A,B	C	B,C	B		
SECTION-I (ii)	Q.	7	8	9	10				
	A.	B	A	B	B				
SECTION-II	Q.	1	2	3	4	5	6	7	8
	A.	3.00	8.00	6.00	4.00	6.00	6.00	5.00	1.00

PART-3 : MATHEMATICS

SECTION-I (i)	Q.	1	2	3	4	5	6		
	A.	A,C	A,B,C,D	A,B,C	A,B,C	A,B	B,C		
SECTION-I (ii)	Q.	7	8	9	10				
	A.	D	C	D	C				
SECTION-II	Q.	1	2	3	4	5	6	7	8
	A.	91.00	0.00	4.00	51.00	4.00	75.00	2.00	5.00

HINT – SHEET

PART-1 : PHYSICS

SECTION-I (i)

1. **Ans (A,D)**

$$\frac{\rho g\left(h_1-h_2\right)\pi r^4}{8\eta L}=\frac{Adh_2}{dt}=\frac{-Adh_1}{dt}=k$$

$$dh_1=\frac{-kdt}{A} \qquad d\left(h_1-h_2\right)=-\frac{2kdt}{A}$$

$$dh_2=\frac{+kdt}{A}$$

$$\frac{-\rho g\left(h_1-h_2\right)\pi r^4}{8\eta L}=\frac{Ad\left(h_1-h_2\right)}{2dt}$$

$$\frac{d\left(\Delta h\right)}{dt}=\frac{-\rho g\pi r^4 2\Delta h}{8\eta LA}$$

$$\Delta h=he^{-\lambda t} \qquad \left(\lambda=\frac{\rho g\pi r^4 2}{8\eta LA}\right) \qquad \frac{h}{e}=he^{-\lambda t}$$

$$t=\frac{1}{\lambda}$$

$$T=\frac{4\eta\ell R^2}{\rho g r^4}=1.6\times10^4\ s$$

Since viscous forces are dissipative, so mechanical energy of the fluid is not conserved water level decreases exponentially, hence it will take infinite time for water level in the two containers to become equal

2. **Ans (A,B,D)**

$$\frac{1}{2}mv^2 - Q\left(\frac{p\cos\theta}{4\pi\epsilon_0 r^2}\right) = 0 \text{ or } v = \sqrt{\frac{2Q\,p\cos\theta}{4\pi\epsilon_0 m r^2}}$$

circular motion of bead requires a centripeal force

$$E_r = -\frac{\partial V}{\partial r} = \frac{2p\cos\theta}{4\pi\epsilon_0 r^3}\text{ ; note that } QE_r = \frac{mv^2}{r}$$

thus wire frame does not exert any force on the bead to sustain circular motion. Bead will reach the point opposite its starting position and then repeatedly retrace its path executing a periodic motion.

3. **Ans (A,B,C,D)**

For a point charge q moving with a velocity $\vec{v}$,

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

$$\vec{r} = \left[-3\vec{i} - 4\vec{j}\right](m)$$

$$r = \sqrt{(-3m)^2 + (-4m)^2} = 5m$$

Thus : $\hat{r} = \frac{\vec{r}}{r} = \frac{\left[-3\vec{i} - 4\vec{j}\right](m)}{5m} = -0.6\vec{i} - 0.8\vec{j}$

Substituting the above results into the equation for $\vec{B}$ we obtain:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{q\left(v\vec{i}\right)\times\left[-0.6\vec{i} - 0.8\vec{j}\right]}{r^2}$$

$$= -\frac{\mu_0}{4\pi}\frac{qv\left(0.8\vec{k}\right)}{r^2}$$

$$= -\left(10^{-7}T.m/A\right)\frac{\left(6\times10^{-6}C\right)\left(5\times10^4 m/s\right)(0.8)}{(5m)^2}\vec{k}$$

$$= -9.6\times10^{-10}\vec{k}\,(T)$$

Both the electric & magnetic fields at origin will decrease as the charge moves away

4. **Ans (B,C)**

$\overrightarrow{\boxed{X}}$ $\qquad$ $\overleftarrow{\boxed{Y}}$

$$f_Y = f_0\left[\frac{V + V_Y}{V - V_X}\right]$$

for lowest frequency 650 Hz

$$f_B = 650\left[\frac{340+11}{340-15}\right] = 702\text{ Hz}$$

for highest frequency

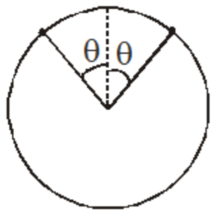
$$f_B = 800\left[\frac{340+11}{340-15}\right] = 864\text{ Hz}$$

$\therefore$ B and width = 864 – 702 = 162 Hz

$\underset{340\text{ ms}}{\xrightarrow{\text{Sound}}}$ $\qquad$ $\underset{11\text{ m/s}}{\xleftarrow{Y}}$

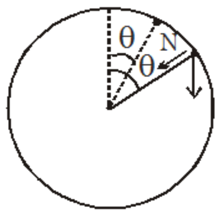
$\therefore$ Urel = 340 + 11 = 351 m/s

5. **Ans (A,C)**



At $\cos\theta = \frac{2}{3}$

N between bead and hoop will be zero.



$$N + mg\cos\theta = \frac{mv^2}{R}$$

$$\frac{1}{2}mv^2 = mgR(1-\cos\theta)$$

$$mv^2 = 2mgR(1-\cos\theta)$$

$$N = 2mg - 3mg\cos\theta$$

Net upward force on hoop

$$2N\cos\theta = 2\cos\theta\, mg\,[2 - 3\cos\theta]$$

$$F_U = 2mg\cos\theta\,(2-\cos\theta)$$

for $F_{u_{max}}$, $-2\sin\theta - 3(2)\cos\theta(-\sin\theta) = 0$

$$3\cos\theta = 1$$

$$\cos\theta = 1/3$$

$$F_u = 2mg\frac{1}{3}(2-1)$$

$$F_u = \frac{2}{3}mg$$

for hoop to rise off

$$F_u > Mg$$

$$\frac{2}{3}mg > Mg$$

$$\left[\frac{m}{M} > \frac{3}{2}\right]$$

6. **Ans (A,B)**

$X_L = wL = (100)\dfrac{\sqrt{3}}{10} = 10\sqrt{3}$

$X_C = \dfrac{1}{100\frac{\sqrt{3}}{2}\times 10^{-3}} \Rightarrow \dfrac{20}{\sqrt{3}}$

$\tan\phi_1 = \dfrac{10\sqrt{3}}{10} = \sqrt{3}$

$\tan\phi_2 = \dfrac{20/\sqrt{3}}{20} = \dfrac{1}{\sqrt{3}}$

$\phi_1 = \dfrac{\pi}{3} \qquad \phi_2 = \dfrac{\pi}{6}$

(A) $\Delta\phi = \dfrac{\pi}{3}\left(1-\dfrac{1}{2}\right) \Rightarrow \dfrac{\pi}{2}$

(B) $I_1 = \dfrac{200\sqrt{2}}{z_1}\sin\left(100t-\dfrac{\pi}{3}\right)$

$z_1 = \sqrt{100+300} = 20$

$I_1 = 10\sqrt{2}\ \sin\left(100t-\dfrac{\pi}{3}\right)$

$I_2 = \dfrac{200\sqrt{2}}{z_2}\ \sin\left(100t-\dfrac{\pi}{6}\right)$

$z_2 = \sqrt{400+\dfrac{400}{3}} = \dfrac{20\times 2}{\sqrt{3}} = \dfrac{40}{\sqrt{3}}$

$I_2 = \dfrac{200\sqrt{2}\sqrt{3}}{40}\sin\left(100t+\dfrac{\pi}{6}\right)$

$I_2 = 5\sqrt{6}\sin\left(100t+\dfrac{\pi}{6}\right)$

$\dfrac{1}{\sqrt{2}} = \sin\left(100t-\dfrac{\pi}{3}\right)$

$\dfrac{\pi}{4} = 100t-\dfrac{\pi}{3}$

$100t = \pi\dfrac{7}{12}$

$I_2 = 5\sqrt{6}\sin\left(\dfrac{7\pi}{12}+\dfrac{\pi}{6}\right)$

$= 5\sqrt{6}\ \sin\ \dfrac{\pi}{6}\left[\dfrac{\pi 3}{2}\right]$

$= 5\sqrt{6}\ \ -\dfrac{1}{\sqrt{2}} = 5\sqrt{3}$

(D) $P_1 = 200\,(10)\ \cos\dfrac{\pi}{3} = \dfrac{2000}{2} = 1000W$

$P_2 = \dfrac{(200)\ 5\sqrt{6}}{\sqrt{2}}\ .\ \cos\dfrac{\pi}{6} = 200\ 5\sqrt{2}.\ \dfrac{3}{2}$

$= \dfrac{1500\sqrt{2}}{\sqrt{2}} = 1500$

$P_{net} = 2500W$

SECTION-I (ii)

7. **Ans (B)**

(A) $m_1g - T_1 = m_1a$ .....(1)

$T_1 - m_2g - T_3 = m_2a$ .....(2)

$T_3 + m_3g - T_2 = m_3a$ .....(3)

$T_2 - m_4g = m_4a$ .....(4)

(B) $mg - T_1 = 2ma$ .....(1)

$T_1 - 2mg - T_3 = 2ma$ .....(2)

$T_3 + 4mg - T_2 = 4ma$ .....(3)

$T_2 - 3mg = 3ma$ .....(4)

In this way equation for all options can be obtained]

Alternative :

(A) m_4 will dominate and move down and m_1 will move up.

(B) In pair m_1 and m_2 will come down and m_3 and m_4 , m_3 will come down. Hence $T_3 = 0$ and like at wood machine

$T_1 = \dfrac{2m_1m_2}{m_1+m_2}$ g and $T_2 = \dfrac{m_3m_4}{m_3+m_4}$ g

(C) In pair m_1 and m_2 will come down with acceleration g/3 and m_3 and m_4, m_3 will go up with acceleration g/7.

Hence $T_3 = 0$ and like at wood machine

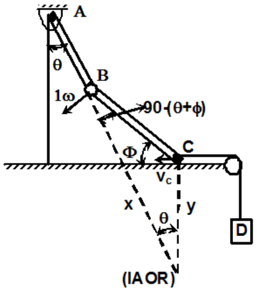
$T_1 = \dfrac{2m_1m_2}{m_1+m_2}$ g and $T_2 = \dfrac{m_3m_4}{m_3+m_4}$ g

(D) In pair m_1 and m_2, m_2 will go up with acceleration g/7 and m_3 and m_4, m_3 will come down with acceleration g/3.

Hence $T_3 = 0$ and like at wood machine

$T_1 = \dfrac{2m_1m_2}{m_1+m_2}$ g and $T_2 = \dfrac{m_3m_4}{m_3+m_4}$ g.

8. **Ans (C)**



Angular velocity of rod BC $\omega' = -\dfrac{d\phi}{dt}$

$$\omega' = \frac{v_c}{y} = \frac{l\omega}{x}$$

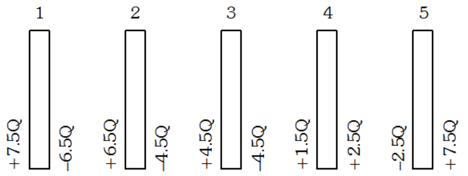
$$\frac{x}{\cos\phi} = \frac{y}{\cos(\theta+\phi)} = \frac{l}{\sin\theta}$$

$$\omega' = \omega\frac{\sin\theta}{\cos\phi}$$

Angular acceleration of rod

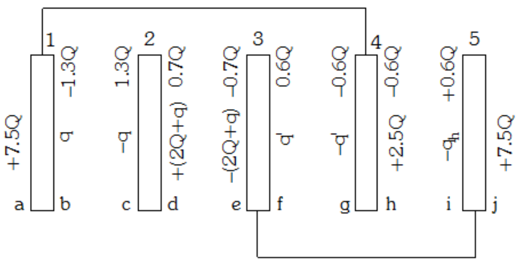
$$BC = \alpha = \frac{d\omega'}{dt} = -\omega^2\frac{(\cos^2\phi\cos\theta + \sin\phi\sin^2\theta)}{\cos^3\phi}$$

9. **Ans (D)**



$$E_{23} = \frac{1}{2A\varepsilon_0}(4.5Q)\times 2Q = \frac{9Qd}{2A\varepsilon_0}$$

$$V_3 - V_2 = \left(\frac{9Qd}{2A\varepsilon_0}\right)$$



$7.5Q + q - q' + q_h = 5Q$ ....(1)

$q' = -q_h$ ....(2)

$$\frac{1}{2A\varepsilon_0}[2q + 2(2Q+q) + 2q'] = 0 \text{ ....(3)}$$

Solving $q = -1.3\ Q$

$q' = 0.6Q$

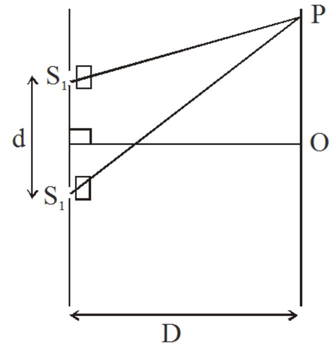
$q_h = -0.6Q$

10. **Ans (A)**

From phase diagram, in case 'C' & 'D' particle are able to reach at highest point, while in 'a' & ' they don't.

SECTION-II

1. **Ans (1.94)**



$$Y = \frac{D}{d}[(\mu_1 - 1)t_1 - (\mu_2 - 1)t_2]$$

Case-I

$$Y = \frac{D}{d}\left[(\mu_1 - 1 - \mu_2 + 1)\frac{t_1 + t_2}{2}\right]$$

$$5mm = \frac{D}{d}\,\frac{0.2}{2}(t_1 + t_2)$$

Case-II

$$8mm = \frac{D}{d}\left[\left(\frac{\mu_1 + \mu_2 - 1}{2}\right)(t_1 + t_2)\right]$$

$$8mm = \frac{D}{d}[0.5\,(t_1 - t_2)]$$

$$\frac{t_1}{t_2} = \frac{33}{17} = 1.94$$

2. **Ans (0.40)**

Applying Bernouli's equation outside and just inside the orifice.

$$P_0 + \rho_{oil}g\,(15) + \frac{1}{2}\rho_w V^2$$

$$= P_0 + \rho_w g\,(10) + \rho_{oil} g\,(5)$$

$$\therefore V = \sqrt{40} = 6.3 m/s$$

(b) Let h be the height when flow through orifice stops.

$$\therefore P_0 + \rho_{oil}g\,(5) + \rho_w gh = P_0 + \rho_{oil} g\,(15)$$

$$\Rightarrow h = \frac{\rho_{oil}}{\rho_w}10 = 8m$$

(c) $P_0 + \rho_{oil}g\,(5) + \rho_w gh$

$$= P_0 + \rho_{oil}g\,(15) + \frac{1}{2}\rho_w V^2$$

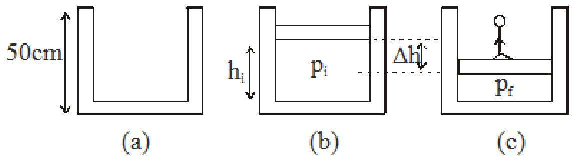
$$\Rightarrow V = \sqrt{20\,(h-8)}$$

$$\Rightarrow -\frac{dh}{dt} = \sqrt{20\,(h-8)}$$

$$\Rightarrow \int_{10}^{8}\frac{dh}{\sqrt{20\,(h-8)}} = \int_0^t dt$$

$$\therefore t = \sqrt{\frac{2}{5}}\,s$$

3. **Ans (216.85 to 217.00)**



$p_i.h_i = p_0H_0$

$$p_f(h_i - \Delta h) = p_ih_i = p_0H_0$$

$$\frac{p_iAh_i}{T_i} = \frac{p'_fAh_i}{T_f}$$

$$\frac{p_ih_i}{T_i} = \frac{p'_fh_i}{T_f}$$

$$\frac{p'_fh_i}{p_i} = \frac{T_f}{T_i}$$

$$\frac{p_0 + (mg + Mg)/A}{p_0 + mg} = \frac{T_f}{T_i}$$

$$\frac{10^5 + \frac{100g}{A}}{10^5 + \frac{20g}{A}} = \frac{273 + x}{273 + 21}$$

$$\frac{10^5 + \frac{10^3}{100\times10^{-4}}}{10^5 + \frac{200}{100\times10^{-4}}}$$

$$= \frac{10^5 + 10^5}{10^5 + 0.2\times10^5} = \frac{273 + x}{294}$$

$$\frac{2\times10^5}{1.2\times10^5} = \frac{273 + x}{294}$$

$$273 + x = \frac{294\times2}{1.2}$$

$$x = \frac{294\times2}{1.2} - 273$$

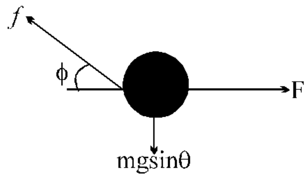
$x = 490 - 273 = 217$

4. **Ans (15.00)**

There are four forces acting on the object; a normal force N perpendicularly away from the incline, gravity mg vertically downward, the applied force F in the direction of motion and a static frictional force f before the object begins to move. Since the object is about to slide along the plane, the frictional force must be at its maximum value, $f = \mu_sN$. The sum of the force components perpendicular to the incline is zero, so,

$N = mg\cos\theta \Rightarrow f = \mu_s mg\cos\theta$ ...(1)

On the other hand, the force components parallel to the surface of the incline are sketched in the following free-body diagram.



Note that the frictional force must make some angle ϕ as drawn because it initially balances the other two forces on this diagram,

$F = f\cos\phi$ ...(2)

for the minimum applied force to get the object to start sliding, and

$f\sin\phi = mg\sin\theta \Rightarrow f = \dfrac{mg\sin\theta}{\sin\phi}$ ...(3)

Substitute eq. (3) into (2) along with

$\cos\phi = \sqrt{1 - \sin^2\phi}$

to obtain

$F = mg\sin\theta\sqrt{\dfrac{1}{\sin^2\phi} - 1}$ ...(4)

Also substitute eq. (3) into (1) and rearrange to find

$\dfrac{1}{\sin\phi} = \dfrac{\mu_s\cos\theta}{\sin\theta}$ ...(5)

Now put eq. (5) into (4) to get

$F = mg\sqrt{\mu_s^2\cos^2\theta - \sin^2\theta}$ ...(6)

5. **Ans (33.60)**

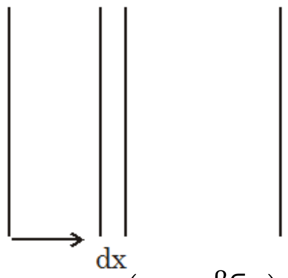
$P_{solar} = \eta P_{sun}$

$= \eta I_{sun} A = 0.1 \times 1000 \times \frac{1}{16} = \frac{25}{4}$

$\rightarrow E_{solar} = E_{battery}$

$\frac{25}{4} \times t = 3.5 \times 1 \times 60 \rightarrow t = 33.6 \text{ min}$

6. **Ans (2.00)**



$$dC_1 = \frac{A\left(\epsilon_0 + \frac{\beta\epsilon_0 x}{d}\right)}{dx}$$

$$\frac{1}{C_1} = \int \frac{1}{dC_1} = \int_0^{d/2} \frac{dx}{A\epsilon_0\left(1 + \frac{\beta x}{d}\right)}$$

$$\frac{1}{C_2} = \int \frac{1}{dC_2} = \int_{d/2}^{d} \frac{dx}{A\epsilon_0\left(1 + \frac{\beta(d-x)}{d}\right)}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{d}{\beta A\epsilon_0} = \frac{1}{C_1} + \frac{1}{C_2}$$

7. **Ans (2.00)**

$nf = x + n\sqrt{(f-x)^2 + y^2}$

$(nf-x)^2 = n^2\left|(f-x)^2 + y^2\right|$

$\left(n^2 - 1\right)x^2 + n^2y^2 - 2n^2fx + 2nfx = 0$

$2n^2fx - 2nfx = Bn(n-1)fx$

$2nfx(n-1) = Bn(n-1)fx$

$B = 2$

8. **Ans (4.00)**

$$\frac{\Delta y}{y} \times 100 = \frac{2\Delta d}{d} \times 100 + \frac{\Delta \ell}{\ell} \times 100$$

$$= \frac{2 \times 5 \times 10^{-3} \times 100}{5 \times 10^{-1}} + \frac{x5 \times 10^{-3} \times 100}{250 \times 10^{-3}}$$

$= 2 + 2 = 4$

PART-2 : CHEMISTRY

SECTION-I (i)

2. **Ans (B,C)**

$W \rightarrow IE_3$ & IE_4 gap is largest. W^{3+} stable.

$X \rightarrow IE_2$ & IE_3 gap is largest. X^{2+} stable.

$Y \rightarrow Y^{-2}$

$Z \rightarrow Z^{-1}$

$W^{3+}Y^{2-} \rightarrow W_2Y_3$

$X^{2+}Y^{2-} \rightarrow XY$

$W^{3+}Z^{-1} \rightarrow WZ_3$

$X^{2+}Z^{-1} \rightarrow XZ_2$

4. **Ans (C)**

Polymer names are :

(A) Neoprene

(B) Buna-N

(C) P.H.B.V

(D) Nylon-6,6

P.H.B.V : Poly hydroxy-b-valerate is a biodegradable polymer

5. **Ans (B,C)**

(A) Around T = 300 K , gypsum is not dehydrated to Plaster of paris

(B) $Ca^{2+} + Na_2CO_3 \rightarrow CaCO3 \downarrow + 2Na^+$

(C) $Na_2O_2 + H_2O \rightarrow 2NaOH + H_2O_2$

(D) $Li^+ < Na^+ < K^+ < Kb^+ < Cs^+$ mobility in aqueous solution.

6. **Ans (B)**

$\sqrt{3}a = r_c + 2r_A + 2r_B + 2r_A + r_c$

$= 2r_C + 4r_A + 2r_B = 2r_C + 4 \times 0.225\ r_C + 2 \times 0.414\ r_C$

$= 3.728\ r_C$

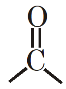
$\sqrt{2}a = 4rC$

$$\text{Fraction occupied} = \frac{3.728\ r_C}{\sqrt{3} \times \frac{4r_C}{\sqrt{2}}} = 0.7609$$

Fraction of body diagonal not covered = 0.24

SECTION-I (ii)

7. Ans (B)

a-hydrogen containing compound show aldol condensation. without α-hydrogen  compound show cannizzaro reaction (Disproportionation) Aldehyde show Tollen test but ketone does not show Acetaldehyde & acetone show haloform reaction.

9. Ans (B)

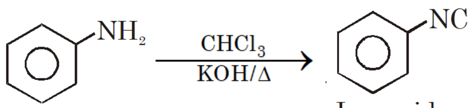
(A) $\underset{J}{CH_3COOH}$ and $\underset{M}{CH_3CH_2OH}$ (NaOI)
× ✓

(B) $\underset{J}{CH_3COOH}$ and $\underset{K}{CH_3-\overset{O}{\overset{\|}{C}}-CH_3}$ (2, 4-DNP)
× ✓

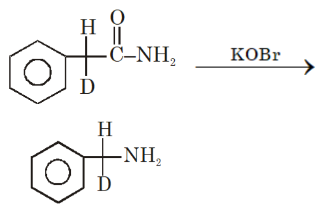
(C) $\underset{K}{CH_3-\overset{O}{\overset{\|}{C}}-CH_3}$ and $\underset{M}{CH_3CH_2OH}$ $(NaHSO_3)$
✓ ×

(D) $\underset{L}{CH_4}$ and $\underset{M}{CH_3CH_2OH}$ (Na-metal)
× ✓

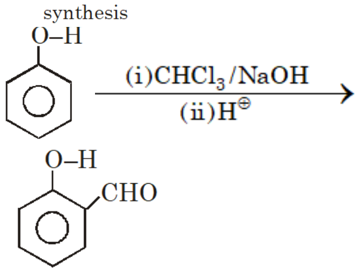
10. Ans (B)

(P) 

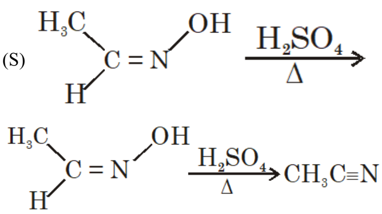
Isocynide

(Q) 

By Hoffman bromamide synthesis

(R) 

By Reimer tiemenn reaction

(S) 

Aldoxime gives cynide

SECTION-II

1. Ans (3.00)

In paramagnetic species incoming e^- enters into H.O.M.O and they are B_2, O_2, S_2.

2. Ans (8.00)

Given millimoles of salt/compound

$= 40 \times 0.05 = 2$ mms

(i) Using Hph (phenolphthalein)

2 mms of HCl consumed to convert only Na_2CO_3 portion to $NaHCO_3$.

(ii) Using MeOH (Methyl orange)

6 mms of HCl consumed to convert entire salt to H_2CO_3.

$$\text{So,} \quad X = \frac{2}{0.05} = 40 \text{ mL}$$

$$Y = \frac{6}{0.05} = 120 \text{ mL}$$

$$\text{Hence,} \quad \frac{|Y - X|}{10} = \frac{120-40}{10} = 8$$

3. Ans (6.00)

$\Delta_rG = \Delta_rG° + RTlnQ$

$$Q = \frac{10^{--6}}{10^{--12}} = 10^6$$

$\Delta_rG° = (\Delta_fG°)_P - (\Delta_fG°)_R$

$= -150$ kJ

$$\Delta_rG = -150 + 10^{-3} \times \frac{8.314 \times 600}{2.303 \times 0.8314} \ln 10^6$$

$= -114$

$$z = \frac{114}{19} = 6$$

4. Ans (4.00)

Ag_2S : No roasting [Mac arthur forest cyanide process]

5. Ans (6.00)

$\Delta T_f = i \times K_f m$

$K_{sp} = 4s^3 = 4 \times 10^{-6}$

$s = 10^{-2}M$

$\Delta T_f = 3 \times 2 \times 10^{-2}$

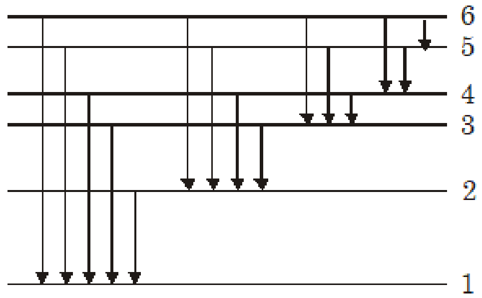
$$= \frac{6}{100}$$

6. Ans (6.00)

(i), (ii), (iii), (iv), (v) and (vii).

The compounds which are more acidic than water are soluble is aqueous NaOH

8. **Ans (1.00)**



$$\frac{n(n-1)}{2} = 15, \; n = 6$$

From given condition, 9 transition having lesser energy and 5 transition having larger energy

$\therefore$ Initial excited state = 1

(Transition $2 \rightarrow 6$)

PART-3 : MATHEMATICS

SECTION-I (i)

1. **Ans (A,C)**

$f(x) = 2x^3 - 15x^2 + 36x - 23 \Rightarrow f'(x) = 6(x^2 - 5x + 6)$

. f(x) has a local maxima at x = 2 and local minima at x = 3. Also f(2) = 5

Now

$$g(x) = \begin{cases} 2x^3 - 15x^2 + 36x - 23, & \text{if } 1 \leqslant x < 2 \\ 5, & \text{if } 2 \leqslant x < \frac{7}{2} \\ 12 - 2x, & \text{if } \frac{7}{2} \leqslant x \leqslant 6 \end{cases}$$

The greatest value of g(x) = 5

2. **Ans (A,B,C,D)**

$z_n - z_1 = z_n - z_{n-1} + ... + z_3 - z_2 + z_2 - z_1$

$\Rightarrow |z_n - z_1| \leq |z_n - z_{n-1}| + |z_{n-1} - z_{n-2}| + ... + |z_3 - z_2| + |z_2 - z_1|$

$$\Rightarrow \frac{|z_2 - z_1|^2 + |z_3 - z_2|^2 + ... + |z_n - z_{n-1}|^2}{(n-1)} \geqslant \left(\frac{|z_n - z_{n-1}| + |z_{n-1} - z_{n-2}| + ... + |z_2 - z_1|}{(n-1)}\right)^2$$

$$\geqslant \frac{|z_n - z_1|^2}{(n-1)^2}$$

$$\Rightarrow \frac{|z_2 - z_1|^2 + |z_3 - z_2|^2 + ... + |z_n - z_{n-1}|^2}{|z_n - z_1|^2} \geqslant \frac{1}{n-1}$$

3. **Ans (A,B,C)**

Tangents to the given parabola in standard form are

$$y = m(x - 3) + \frac{2}{m} \;\&\; y - 3 = mx - 2m^2$$

For common tangents

$$\frac{2}{m} - 3m = 3 - 2m^2 \Rightarrow 2m^3 - 3m^2 - 3m + 2 = 0$$

This gives $m = -1, \; 2$ and $\frac{1}{2}$

4. **Ans (A,B,C)**

Intersection of line with both the planes are the same

$$\Rightarrow \frac{3}{3\beta^2 + 6(1 - 2\alpha) + 3} = \frac{-6}{6\alpha^2 + 6(1 - 2\beta) + 6}$$

$\Rightarrow 2(\beta - 1)^2 + 3(\alpha - 2)^2 = 0 \Rightarrow \alpha = 2, \beta = 1.$

5. **Ans (A,B)**

$$I_n = \int_0^\infty x^n e^{-x^2} dx = \int_0^\infty x^{n-1} x e^{-x^2} dx$$

$$\Rightarrow I_n = \left[x^{n-1} \int xe^{-x^2} dx\right]_0^\infty - \int_0^\infty \left\{\frac{d}{dx}(x^{n-1}) \int xe^{-x^2} dx\right\} dx$$

$$-x^2 = t \Rightarrow I_n = -\left[\frac{x^{n-1}}{2e^{x^2}}\right]_0^\infty + \frac{n-1}{2} \int_0^\infty x^{n-2} e^{-x^2} dx$$

$$\Rightarrow I_n = \frac{n-1}{2} I_{n-2}$$

Case I : $n = 2m \Rightarrow I_n = \frac{(2m-1)(2m-3)(2m-5)....1}{2^m} I_0 = \frac{(2m-1)!}{2^{2m-1}(m-1)!}a$

6. **Ans (B,C)**

We have $2\cos\frac{A+B}{2}\cos\frac{A-B}{2} = 4\sin^2\frac{C}{2}$

or $\cos\frac{A-B}{2} = 2\sin\frac{C}{2} \quad \left(\sin\frac{C}{2} = \cos\frac{A+B}{2}\right)$

or $\cos\frac{A-B}{2} = 2\cos\frac{A+B}{2}$

or

$\cos\frac{A-B}{2} - \cos\frac{A+B}{2} = \cos\frac{A+B}{2}$

$2\sin\frac{A}{2}\sin\frac{B}{2} = \cos\frac{A}{2}\cos\frac{B}{2} - \sin\frac{A}{2}\sin\frac{B}{2}$

$3\sin\frac{A}{2}\sin\frac{B}{2} = \cos\frac{A}{2}\cos\frac{B}{2}$

or $\tan\frac{A}{2}\tan\frac{B}{2} = \frac{1}{3}$

Now

$$\frac{\Delta}{s(s-a)} \cdot \frac{\Delta}{s(s-b)} = \frac{1}{3}; \quad \frac{s-c}{s} = \frac{1}{3}$$

$\therefore 2s = 3c \quad a + b + c = 3c$

$\therefore a + b = 2c \Rightarrow$ a, c, b are in A.P.

SECTION-I (ii)

7. **Ans (D)**

(P) $a_{ij} = i^2 + j^2 \Rightarrow a_{ij} = a_{ji}$

$$\text{(Q) } |A| = \begin{vmatrix} 1 & 1/3 & 1/9 \\ 3 & 1 & 1/3 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

(R) $a_{ij} = i^2 - j^2 \Rightarrow a_{ij} = -a_{ji}$

(S) $|A| = 2 \times (3\times(-3) - (-2)\times 4) - (-2)((-1)\times(-3) - 1\times 4) - 4\times((-1)(-2) - 1\times 3) = 0$

8. Ans (C)

(P) continuity must be checked at x = 1, e, 3, $\sqrt{10}$, $\sqrt{11}$, $\sqrt{12}$, 3.5

Further, f(1) = 0 and $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} \operatorname{sgn}(x-2) \times [\log_e x] = 0$

Hence, f(x) is continuous at x = 1

$f(e) = 1,\ f(e^-) = 0$

Hence f(x) is discontinuous at x = e

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} \operatorname{sgn}(x-2) \times [\log_e x] = 1$$

$$\Rightarrow \lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} \{x^2\} = 0$$

Hence, f(x) is discontinuous at x = 3

Also $\{x^2\}$ is discontinuous $x = \sqrt{10}$, $\sqrt{11}$, $\sqrt{12}$ therefore,

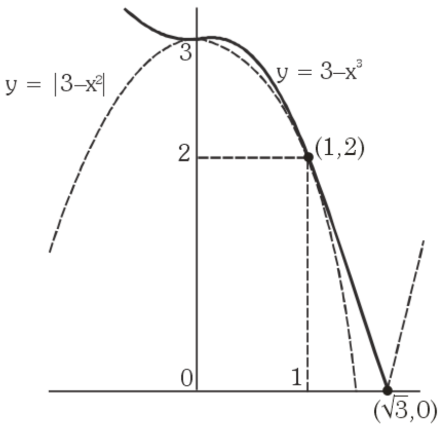
$$\lim_{x \to \frac{7}{2}^-} f(x) = \lim_{x \to 3^-} \{x^2\} = 0.25 = f(3.5)$$

Hence, f(x) is discontinuous at $x = 3, \sqrt{10}, \sqrt{11}, \sqrt{12}$, e

(Q) $f'(x) = \begin{cases} -\frac{1}{x^2}\left(\sin\frac{1}{x} + \cos\frac{1}{x} + \sqrt{2}\right) e^{\frac{1}{x}} & \text{if } x < 0 \\ \frac{1}{x^2}\left(\sin\frac{1}{x} - \cos\frac{1}{x} + \sqrt{2}\right) e^{-\frac{1}{x}} & \text{if } x > 0 \end{cases}$

clearly there is one point of extremum.

(R) not differentiable at x = 1 and x = $\sqrt{3}$



(S) $\lim\limits_{x \to 0} x^2 \left(1 + 2 + 3 + \ldots\ldots + \left[\frac{1}{|x|}\right]\right)$

$$= \lim_{x \to 0} x^2 \frac{\left[\frac{1}{|x|}\right]\left(\left[\frac{1}{|x|}\right] + 1\right)}{2}$$

$$= \lim_{x \to \infty} \frac{[y]([y]+1)}{2y^2}$$

$$= \lim_{y \to \infty} \frac{(y - \{y\})(y - \{y\} + 1)}{2y^2} = \frac{1}{2}$$

9. Ans (D)

(P) $\frac{\pi}{2} - \cos^{-1}|\cos x| - \frac{\pi}{2} + \sin^{-1}|\sin x| = a$

$\sin^{-1}|\sin x| - \cos^{-1}|\cos x| = a$

$\{\sin^{-1}|\sin x|\} = \{\cos^{-1}|\cos x|\} \Rightarrow a = 0$

(Q) $\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x^2-1} + \tan^{-1}\tan y = a$

$0 \leqslant x \leqslant 1,\ 0 \leqslant x^2 - 1 \leqslant 1,\ 1 \leqslant x^2 \leqslant 2$

$\Rightarrow x = 1, \Rightarrow \frac{\pi}{2} + \frac{\pi}{2} + \tan^{-1}\tan y = a$

$-\frac{\pi}{2} < \tan^{-1}\tan y < \frac{\pi}{2} \Rightarrow \frac{\pi}{2} < a < \frac{3\pi}{2}$

(R) $|\sin^{-1}|\sin x|| = |\tan^{-1}|\tan x||$ $\forall x \in R - (2n+1)\frac{\pi}{2}, n \notin I$

$(\because a \geq 0)$

$\sqrt{\left|\sin^{-1}|\sin x|\right|} = \sqrt{a}$

$\left|\sin^{-1}|\sin x|\right| = a \Rightarrow a \in \left[0, \frac{\pi}{2}\right)$

(S) $\sin^{-1}(x^2+y^2) + \tan^{-1}\sqrt{4y^2-1} + \sec^{-1}x = a$

$\sin^{-1}(x^2 + y^2) \Rightarrow 0 \leq (x^2 + y^2) \leq 1$ ... (1)

for $\tan^{-1}\sqrt{4y^2-1} \Rightarrow 4y^2 - 1 \geq 0$

$\Rightarrow y \in \left(-\infty, \frac{-1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$ ... (2)

for $\sec^{-1}x \Rightarrow x \geq 1$ or $x \leq 1$, ... (3)

from equation (i), (ii) and (iii) we get x, y ∈ ϕ ⇒ a ∈ R.

10. Ans (C)

(P) Slopes of sides are 1, 8, –6 hence $\tan A = \frac{7}{8}$, $\tan B = \frac{14}{47}$, $\tan C = -\frac{7}{5}$

centroid is $\left(2, \frac{2}{3}\right)$

(Q) Vertices are (2, 1), (1, 2) and (–4, –1)

Slopes of the sides are $-1, \frac{3}{5}. \frac{1}{3}$ hence tan A = –4, tan B = $\frac{1}{3}$, tan C = 2

Also for each of the three vertices the point (1, 1) gives the same sign when put in the equations of opposite sides.

(R) for λ = –7 lines become concurrent.

SECTION-II

1. **Ans (91.00)**

$S = {}^{n+1}C_2 + 2\left({}^3C_3 + {}^3C_2 + \ldots\ldots + {}^nC_2\right)$

$= {}^{n+1}C_2 + 2\left({}^{n+1}C_3\right)$, now use

${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$, to get

$S = {}^{n+2}C_3 + {}^{n+1}C_3 = \dfrac{n(n+1)(2n+1)}{6}$,

hence for n = 6, S= 91

2. **Ans (0.00)**

$4s^2 = 2025$

$\because 2025 - 45 = 1980$

$\therefore T_{2003} = 2025 + 23 = 2048$

Hence remainder is 0

3. **Ans (4.00)**

$$2(f(x))^2 - f''(x)f(x) + (f'(x))^2 = 0$$

$$\Rightarrow \frac{f''(x)f(x) - (f'(x))^2}{(f(x))^2} = 2$$

$$\Rightarrow \frac{d}{dx}\left(\frac{f'(x)}{f(x)}\right) = 2 \Rightarrow \frac{f'(x)}{f(x)} = 2x + a$$

$$\Rightarrow \frac{df(x)}{f(x)} = (2x + a)\,dx \Rightarrow \ell n|f(x)| = x^2 + ax + b$$

$$\Rightarrow |f(x)| = e^{x^2 + ax + b}$$

Required area = $\int_0^1 \left|(2x-1)e^{x^2-x}\right| dx = -\int_0^{1/2} (2x-1)e^{x^2-x}dx + \int_{1/2}^1 (2x-1)e^{x^2-x}dx$

$$A = -e^{x^2-x}\Big|_0^{1/2} + e^{x^2-x}\Big|_{1/2}^1 = 2\left(\frac{e^{1/4} - 1}{e^{1/4}}\right)$$

4. **Ans (51.00)**

The n strings have a total of 2n ends. One boy picks up one end, this leaves (2n –1) ends for the second boy to choose, of which only one is correct.

$$\therefore p = \frac{1}{2n-1} \Rightarrow \frac{1}{2n-1} = \frac{1}{101}$$

$\Rightarrow 2n - 1 = 101 \Rightarrow n = 51$

5. **Ans (4.00)**

For equilateral Δ $r = \dfrac{R}{2}$. Hence R = 8

Now inradius r of ABC is the circum radius for second Δ. Hence $x_1 = \dfrac{r}{2} = \dfrac{R}{4}$

Similarly

$$x_2 = \frac{x_1}{2} = \frac{R}{8},\ x_3 = \frac{x_2}{2} = \frac{R}{16},\ \ldots\ldots,\ x_n = \frac{x_{n-1}}{2} = \frac{R}{2^{n+1}}$$

$x_1 + x_2 + x_3 + \ldots x_n + \ldots$ upto ∞ term

$= \dfrac{R}{4} + \dfrac{R}{8} + \dfrac{R}{16} + \ldots + \dfrac{R}{2^{n+1}} + \ldots$ upto ∞ terms

Hence sum of the radii of all the is

$$\frac{\frac{R}{4}}{1 - \frac{1}{2}} = 4$$

6. **Ans (75.00)**

Vertices of the trapezium will be $\left(t_1^2, \pm 2t_1\right)$ and $\left(t_2^2, \pm 2t_2\right)$

Given that $\sqrt{(t_1^2 - t_2^2)^2 + (2t_1 + 2t_2)^2} = \dfrac{25}{4}$

Also $t_1t_2 = 1 \Rightarrow t_1 = 2$ and $t_2 = \dfrac{1}{2}$

7. **Ans (2.00)**

For one root to be less than 1 and other to be greater than 2, P(1) < 0 and P(2) < 0

Hence $p < \dfrac{7}{3}$ and $p < \dfrac{11}{4}$, which implies $p < \dfrac{7}{3}$

Greatest integral values of p is 2

8. **Ans (5.00)**

$$|\vec{A}| = 3,\ |\vec{B}| = 4,\ |\vec{C}| = 5$$

$$\vec{A}.(\vec{B} + \vec{C}) = \vec{B}.(\vec{C} + \vec{A}) = \vec{C}.(\vec{A} + \vec{B}) = 0$$

Now, $\left|\vec{A} + \vec{B} + \vec{C}\right|^2$

$$= A^2 + B^2 + C^2 + \vec{A}(\vec{B} + \vec{C}) + \vec{B}(\vec{C} + \vec{A}) + \vec{C}(\vec{A} + \vec{B})$$

$= 50 + 0 + 0 + 0 = 50 \Rightarrow \left|\vec{A} + \vec{B} + \vec{C}\right| = 5\sqrt{2}$