VERY SIMILAR PRACTICE TEST 5

Hints and Explanations

1. **(a) :** $M_1 = 10$ g $\quad M_2 = 1$ kg
$L_1 = 10$ cm $\quad L_2 = 1$ m
$T_1 = 0.1$ s $\quad T_2 = 1$ s
$n_1 = 1 \quad n_2 = ?$

The dimensional formula of force is [MLT^{-2}].

$\therefore \quad a = 1, b = 1, c = -2$

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$$

$$= 1\left(\frac{10\text{ g}}{1\text{ kg}}\right)^1\left(\frac{10\text{ cm}}{1\text{ m}}\right)^1\left(\frac{0.1\text{ s}}{1\text{ s}}\right)^{-2}$$

$$= 1\left(\frac{10^{-2}\text{ kg}}{1\text{ kg}}\right)^1\left(\frac{10^{-1}\text{ m}}{1\text{ m}}\right)^1\left(\frac{0.1\text{ s}}{1\text{ s}}\right)^{-2}$$

$$= \frac{1\times10^{-2}\times10^{-1}}{10^{-2}} = 10^{-1} = 0.1$$

Hence, the unit of force in a given system will equivalent to 0.1 N.

2. **(d) :** Given : $\upsilon = 24 \times 10^6$ Hz, $R = 0.60$ m

We know that, $R = \dfrac{mv}{qB}$

$$\therefore \quad B = \frac{mv}{qR} \quad \text{...(i)}$$

where $v = \omega R = 2\pi\upsilon R$

$\Rightarrow \quad v = 2\pi \times 24 \times 10^6 \times 0.60 = 9.04 \times 10^7$ m s^{-1}

From (i), we get

$$B = \frac{(3.34\times10^{-27})(9.04\times10^7)}{1.6\times10^{-19}\times0.60} = 3.2\text{ T}$$

3. **(c) :** Let u be the initial speed and θ is the angle of the projection.

As per question,

Speed at the maximum height

$$v_H = u\cos\theta = \frac{\sqrt{3}}{2}u$$

$$\therefore \quad \cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30°$$

Range, $R = \dfrac{u^2\sin2\theta}{g}$

Maximum height, $H = \dfrac{u^2\sin^2\theta}{2g}$

As $R = PH$ (Given)

$$\therefore \quad \frac{u^2\sin2\theta}{g} = P\frac{u^2\sin^2\theta}{2g}$$

$$\Rightarrow \quad 2\sin\theta\cos\theta = \frac{P}{2}\sin^2\theta \Rightarrow \tan\theta = \frac{4}{P}$$

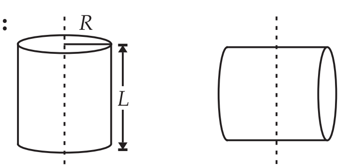
$$\text{or} \quad P = \frac{4}{\tan30°} = 4\sqrt{3}$$

4. **(a) :** Equator, $\theta = 0°$, $B_H = B_e\cos\theta = 5\cos0° = 5$

At poles, $\theta = 90°$, $V = B_e\sin\theta = 5\sin90° = 5$.

Thus, total intensity of earth's magnetic field at poles is also 5 units, but in the vertical direction. At equator, value is the same, but the direction is horizontal.

5. **(b) :**



(I) $\qquad$ (II)

(I) Moment of inertia of a cylinder about an axis passing through its centre and normal to its circular face $= \dfrac{MR^2}{2}$

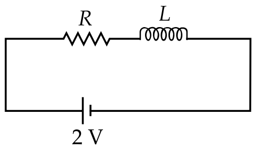
(II) Moment of inertia of the same cylinder about an axis passing through its centre and perpendicular to its length $= M\left(\dfrac{L^2}{12} + \dfrac{R^2}{4}\right)$

According to the question,

$$\frac{ML^2}{12} + \frac{MR^2}{4} = \frac{MR^2}{2} \quad \text{or} \quad L = \sqrt{3}R$$

6. **(b) :** The current at any instant is given by

$\Rightarrow \quad I = I_0(1 - e^{-Rt/L})$



$$\Rightarrow \quad \frac{I_0}{2} = I_0(1 - e^{-Rt/L})$$

$$\text{or} \quad \frac{1}{2} = (1 - e^{-Rt/L})$$

$$\text{or} \quad e^{-Rt/L} = \frac{1}{2} \quad \text{or} \quad \frac{Rt}{L} = \ln 2$$

$$\therefore \quad t = \frac{L}{R}\ln 2 = \frac{300 \times 10^{-3}}{2} \times 0.693$$

$$= 150 \times 0.693 \times 10^{-3} = 0.10395 \text{ s} = 0.1 \text{ s}$$

7. **(b) :** Let v be the velocity of projection of the body from the surface of the earth.
According to conservation of energy, we get

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{(R+h)}$$

$$\Rightarrow \quad v^2 - \frac{2GM}{R} = -\frac{2GM}{(R+h)}$$

As per question,

$$v = \frac{3}{4}v_e = \frac{3}{4}\sqrt{\frac{2GM}{R}} \qquad \left(\because v_e = \sqrt{\frac{2GM}{R}}\right)$$

$$\therefore \quad \left(\frac{3}{4}\sqrt{\frac{2GM}{R}}\right)^2 - \frac{2GM}{R} = -\frac{2GM}{(R+h)}$$

$$\Rightarrow \quad \frac{9}{16}\left(\frac{2GM}{R}\right) - \frac{2GM}{R} = -\frac{2GM}{(R+h)}$$

$$\Rightarrow \quad \frac{9}{16R} - \frac{1}{R} = -\frac{1}{(R+h)}$$

$$\Rightarrow \quad \frac{7}{16R} = -\frac{1}{R+h} \quad \Rightarrow 16R = 7R + 7h$$

$$\therefore \quad h = \frac{9}{7}R$$

8. **(b)**

9. **(d) :** For 16 g of helium, $\mu_1 = \dfrac{16}{4} = 4$

For 16 g of oxygen, $\mu_2 = \dfrac{16}{32} = \dfrac{1}{2}$

For mixture of gases,

$$C_V = \frac{\mu_1 C_{V_1} + \mu_2 C_{V_2}}{\mu_1 + \mu_2} \qquad \text{where } C_V = \frac{f}{2}R$$

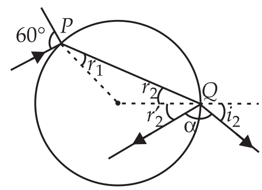
$$C_P = \frac{\mu_1 C_{P_1} + \mu_2 C_{P_2}}{\mu_1 + \mu_2} \qquad \text{where } C_P = \left(\frac{f}{2} + 1\right)R$$

For helium, $f = 3$, $\mu_1 = 4$

For oxygen, $f = 5$, $\mu_2 = \dfrac{1}{2}$

$$\therefore \quad \frac{C_P}{C_V} = \frac{\left(4 \times \frac{5}{2}R\right) + \left(\frac{1}{2} \times \frac{7}{2}R\right)}{\left(4 \times \frac{3}{2}R\right) + \left(\frac{1}{2} \times \frac{5}{2}R\right)} = \frac{47}{29} = 1.62.$$

10. **(c) :**



For refraction at P, $\dfrac{\sin 60°}{\sin r_1} = \sqrt{3}$

$$\Rightarrow \quad \sin r_1 = \frac{1}{2} \quad \Rightarrow \quad r_1 = 30°$$

Since $r_2 = r_1 \quad \therefore \quad r_2 = 30°$

For refraction at Q, $\dfrac{\sin r_2}{\sin i_2} = \dfrac{1}{\sqrt{3}}$

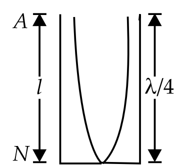
$$\Rightarrow \quad \frac{\sin 30°}{\sin i_2} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \sin i_2 = \frac{\sqrt{3}}{2}$$

$\Rightarrow i_2 = 60°$

For refraction at Q, $r_2' = r_2 = 30°$

$$\therefore \quad \alpha = 180° - (r_2' + i_2) = 180° - (30° + 60°) = 90°$$

11. **(a) :** When air is blown at the open end of a closed pipe a longitudinal wave travels in the air of the pipe from the closed end to the open end. When λ is wavelength and l is the length of pipe and υ is the frequency of note emitted and v is the velocity of sound in air, then



$\upsilon = \dfrac{v}{\lambda}$, (fundamental note)

Given, $\upsilon = 166$ Hz, $v = 332$ m s^{-1}

$$\therefore \quad \lambda = \frac{332}{166} = 2 \text{ m}$$

But, $\lambda = 4l$

$$\therefore \quad l = \frac{\lambda}{4} = \frac{2}{4} = 0.5 \text{ m}$$

12. **(c) :** Intensity $\propto$ (Amplitude)^2

$\Rightarrow \quad I \propto A^2$

When two waves (beams) of amplitude A_1 and A_2 superimpose, at maxima and minima, the amplitude of the resulting wave are $(A_1 + A_2)$ and $(A_1 - A_2)$ respectively. If the maximum and minimum possible intensities are I_{max} and I_{min} respectively, then

$$I_{max} \propto (A_1 + A_2)^2 \text{ and } I_{min} \propto (A_1 - A_2)^2$$

$$\Rightarrow \frac{I_{max}}{I_{min}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \left\{\frac{\frac{A_1}{A_2} + 1}{\frac{A_1}{A_2} - 1}\right\}^2$$

Given, $\frac{A_1}{A_2} = \frac{\sqrt{I}}{\sqrt{4I}} = \frac{1}{2}$

$$\Rightarrow \frac{I_{max}}{I_{min}} = \frac{9}{1}$$

$\therefore$ $I_{max} = 9I$ and $I_{min} = I$

13. (a) : Stretching force, $F = \frac{Y\pi r^2 \Delta l}{l}$

where, Y is Young's modules, Δl is change in length of the wire and l is original length of the wire.
As both wires are of same material, Y will be equal, extension in both the wires is same, so Δl will be equal.

$$\therefore \quad F \propto \frac{r^2}{l}$$

$$\therefore \quad \frac{F'}{F} = \frac{(2r)^2}{(l/2)} \times \frac{l}{r^2} = 8$$

or $F' = 8F$ ...(i)

$\because$ Work done in stretching a wire,

$$W = \frac{1}{2} \times F \times \Delta l$$

For same extension $W \propto F$

$$\therefore \quad \frac{W'}{W} = \frac{F'}{F} = 8 \qquad \text{(Using (i))}$$

$W' = 8W = 8 \times 2 \text{ J} = 16 \text{ J}$

14. (b) : Pressure exerted by absorbed light

$$= \frac{1}{2}\left(\frac{I}{c}\right)$$

Pressure exerted by reflected light $= \frac{1}{2}\left(\frac{2I}{c}\right)$

Total radiation pressure on the surface is,

$$P_{rad} = \frac{\frac{3}{2}I}{c} = \frac{1.5 \times 10^3}{3 \times 10^8} = 5 \times 10^{-6} \text{ Pa}$$

15. (a) : Initial energy, $U_1 = \frac{1}{2}CV^2$

$$= \frac{1}{2} \times 100 \times 10^{-6} \times (50)^2$$

$= 125 \times 10^{-3}$ J

When the distance is doubled, the capacitance decreases to 1/2, *i.e.*, $C' = 50$ μF

$\therefore$ Final energy,

$$U_2 = \frac{1}{2}C'V^2 = \frac{1}{2} \times 50 \times 10^{-6} \times (50)^2$$

$$= \frac{125 \times 10^{-3}}{2}$$

Additional energy, $U = U_1 - U_2 = \frac{125 \times 10^{-3}}{2}$ J

16. (c) : Energy of a photon $E = \frac{hc}{\lambda}$

or $E \propto \frac{1}{\lambda}$ $\therefore$ $\frac{E_2}{E_1} = \frac{\lambda_1}{\lambda_2}$

or $E_2 = E_1 \times \frac{\lambda_1}{\lambda_2} = 1.23 \times \frac{10000}{5000} = 2.46 \text{ eV}$

According to Einstein's photoelectric equation

$$h\upsilon - \phi_0 = \frac{1}{2}mv_{max}^2 = eV_s$$

or $\phi_0 = h\upsilon_2 - eV_s = E_2 - eV_s$
$= 2.46 - 1.36 = 1.10$ eV

17. (d) : Given : $d = 0.28$ mm $= 0.28 \times 10^{-3}$ m
$r = 0.14 \times 10^{-3}$ m
Surface tension, $S = 0.07$ N m^{-1}
Atmospheric pressure $= 10^5$ N m^{-2}

$$S = \frac{rh\rho g}{2} = 0.07$$

$$0.07 = \frac{P \times 0.14 \times 10^{-3}}{2} \qquad (\because P = h\rho g)$$

$P = 1 \times 10^3$ N m^{-2}
Total pressure $= P +$ Atmospheric pressure
$= 10^3$ N m^{-2} $+ 10^5$ N m^{-2}
$= 10^3(1 + 100)$ N m^{-2}
$= 101 \times 10^3$ N m^{-2}

18. (b) : The smallest frequency and largest wavelength in ultraviolet region will be for transition from $n_2 = 2$ to $n_1 = 1$

$$\therefore \quad \frac{1}{\lambda_{max}} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \frac{1}{\lambda_{max}} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = R\left[1 - \frac{1}{4}\right] = \frac{3R}{4} \qquad \text{...(i)}$$

The highest frequency and smallest wavelength for infrared region will be for transition from $n_2 = \infty$ to $n_1 = 3$

$$\therefore \quad \frac{1}{\lambda_{min}} = R\left(\frac{1}{3^2} - \frac{1}{\infty}\right) = \frac{R}{9} \qquad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{\lambda_{min}}{\lambda_{max}} = \frac{3R}{4} \times \frac{9}{R} = \frac{27}{4}$$

$$\lambda_{min} = \frac{27}{4} \times \lambda_{max}$$

$$= \frac{27}{4} \times 122 \text{ nm} = 823.5 \text{ nm}$$

19. **(a) :** Mean free path, $\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$

where, n = number of molecules per unit volume
d = diameter of the molecule
As $PV = k_B NT$

$$n = \frac{N}{V} \quad \frac{P}{k_B T} \quad \therefore \ \lambda = \frac{k_B T}{\sqrt{2}\pi d^2 P}$$

where,
k_B = Boltzmann constant
P = pressure of gas
T = absolute temperature of the gas

20. **(c) :** $I = \frac{MR^2}{2} + MR^2$ (By parallel axis theorem)

$$T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{3}{2}\frac{R}{g}}$$

21. **(1.0):** Let $\lambda_A = \lambda \therefore \lambda_B = 2\lambda$.
If N_0 is total number of atoms in A and B at $t = 0$, then initial rate of disintegration of $A = \lambda N_0$, and initial rate of disintegration $B = 2\lambda N_0$

As $\quad \lambda_B = 2\lambda_A \quad \therefore T_B = \frac{1}{2} T_A$

i.e., Half life of B is half the half life of A.
After one half life of A

$$\left(-\frac{dN}{dt}\right)_A = \frac{\lambda N_0}{2}$$

Equivalently, after two half lives of B

$$\left(-\frac{dN}{dt}\right)_B = \frac{2\lambda N_0}{4} = \frac{\lambda N_0}{2}$$

After $n = 1$ *i.e.*, one half life of $A \left(-\frac{dN}{dt}\right)_A = -\left(\frac{dN}{dt}\right)_B$.

22. **(4.0):** Force on 10 kg mass = 10 × 12 = 120 N
The mass of 10 kg will pull the mass of 20 kg in the backward direction with a force of 120 N.
$\therefore$ Net force on mass 20 kg = 200 – 120 = 80 N

So, acceleration for 20 kg mass $= \frac{80 \text{ N}}{20 \text{ kg}} = 4 \text{ m s}^{-2}$

23. **(5.0):** Loss in elastic potential energy
= Gain in kinetic energy

$$\frac{1}{2}k(l - l_0)^2 = \frac{1}{2}mv^2$$

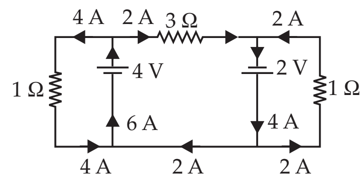
$$v = (l - l\ )\sqrt{\frac{k}{m}} = \left(\frac{l_0}{\cos 37^\circ} - l_0\right)\sqrt{\frac{k}{m}}$$

$$= l_0\left[\frac{1}{\cos 37^\circ} - 1\right]\sqrt{\frac{k}{m}} = (0.1)\left[\frac{5}{4} - 1\right]\sqrt{\frac{80}{2 \times 10^{-3}}}$$

$$= 0.1 \times \frac{1}{4} \times 2 \times 10^2 = 5 \text{ m s}^{-1}$$

24. **(6.0):** Current through 1 Ω resistor parallel to 4 V battery = 4 A
Current through 1 Ω resistor parallel to 2 V battery = 2 A

Current through $3\,\Omega$ resistor $= \frac{4+2}{3} = 2$ A



$\therefore$ Current through 4 V battery = 4 + 2 = 6 A

25. **(1.0):** The velocity of longitudinal wave is,

$$v_L = \sqrt{\frac{Y}{\rho}}$$

The velocity of transverse wave is

$$v_T = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\pi r^2 \rho}}$$

$$\therefore \quad \frac{v_L}{v_T} = \sqrt{\frac{Y}{\rho} \times \frac{\pi r^2 \rho}{T}} = \sqrt{\frac{Y}{T/\pi r^2}} = \sqrt{\frac{Y}{\text{Stress}}}$$

$$\therefore \quad \text{Stress} = \frac{Y}{(v_L/v_T)^2} = \frac{1 \times 10^{11}}{(100)^2} = 1 \times 10^7 \text{ N m}^{-2}$$

Hence, $x = 1$.

26. **(a) :** $\frac{1}{2}A \longrightarrow 2B$

$$-\frac{1}{1/2}\frac{d[A]}{dt} = \frac{1}{2}\frac{d[B]}{dt}$$

or $\quad -\frac{2d[A]}{dt} = \frac{1}{2}\frac{d[B]}{dt} \Rightarrow \frac{-d[A]}{dt} = \frac{1}{4}\frac{d[B]}{dt}$

27. **(c) :** The substances which have lower reduction potentials are strong reducing agents while those which have higher reduction potentials are stronger oxidising agents.

$\because \ E^\circ_{M^{n+}|M}$ for V, Fe and Hg are lower than that of NO_3^-, so NO_3^- will oxidise V, Fe and Hg.

28. (b) : (a) d^6 (High spin); $t_{2g}^4 e_g^2$

CFSE = $(-0.4 \times 4 + 0.6 \times 2)\Delta_o = -0.4\,\Delta_o$

(b) d^5 (Low spin); t_{2g}^5

CFSE = $(-0.4 \times 5 + 0.6 \times 0)\Delta_o = -2.0\,\Delta_o$

(c) d^4 (Low spin); t_{2g}^4

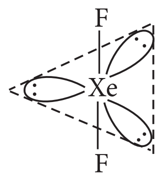
CFSE = $(-0.4 \times 4 + 0.6 \times 0)\Delta_o = -1.6\,\Delta_o$

(d) d^7 (High spin); $t_{2g}^5 e_g^2$

CFSE = $(-0.4 \times 5 + 0.6 \times 2)\Delta_o = -0.8\,\Delta_o$

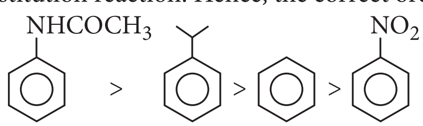
Magnitude of CFSE is maximum for d^5 (low spin) complex.

29. (b) :



Linear, sp^3d

30. (d) : The more nucleophilic compound among the given will undergo more rapid electrophilic substitution reaction. $-NHCOCH_3$ group is more electron releasing group than $-CH(CH_3)_2$ group, exhibit more rapid electrophilic substitution reaction. Hence, the correct order is



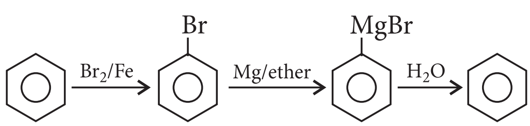
(iii) (iv) (i) (ii)

31. (c) : The value of van't Hoff factor for the given solutions will be the same, *i.e.*, $i_A = i_B = i_C$ due to complete dissociation of NaCl (strong electrolyte) in dilute solutions. On complete dissociation value of i for NaCl is 2.

32. (a) : $2H_2\overset{-2}{S} + \underset{\text{Oxidising agent}}{\overset{+4}{SO_2}} \longrightarrow 2N_2O + 3\overset{0}{S}$

33. (a) : Benzene can be prepared by cyclisation of long chain alkanes on heating at 500 - 550°C under high pressure in presence of catalyst CrO_3 supported on alumina or Pt – Al_2O_3 (*i.e.*, catalytic reforming).

$$\underset{n\text{-hexane}}{C_6H_{14}} \xrightarrow[\text{773 K, at high pressure}]{CrO_3/Al_2O_3} \underset{(A)}{C_6H_6} + 4H_2$$



(A) (B) (C) (D)

34. (d)

35. (b) : $\underset{(X)}{2NaHCO_3} \xrightarrow{\Delta} \underset{(Y)\text{ white residue, soluble in water}}{Na_2CO_3} + CO_2 + H_2O$

$\underset{(Y)}{Na_2CO_3} + \underset{\text{(dil.)}}{2HCl} \longrightarrow 2NaCl + CO_2 + H_2O$

36. (a) : The change given is occurring at the boiling point of the liquid, where, at given pressure and temperature, the liquid-vapour system virtually remains at equilibrium and hence, $\Delta G = 0$. Also due to absorption of heat as latent heat of vaporisation, or due to change from liquid to gaseous state where randomness has also increased, $\Delta S > 0$.

37. (b) : Physical adsorption results into multimolecular layers on adsorbent surface under high pressure. While chemical adsorption results into unimolecular layer.

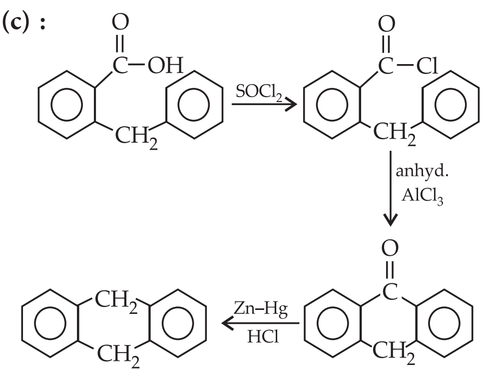
38. (a) : $\underset{\text{Methyl amine}}{CH_3NH_2} + \underset{\text{Phosgene}}{COCl_2} \xrightarrow{-2HCl} \underset{\text{Methyl isocyanate}}{CH_3{-}N{=}C{=}O}$

39. (d) : 2-Chloro-2-phenylethane racemises *i.e.*, it proceeds *via* the formation of carbocation.

40. (d) : (+)-Lactose is a reducing sugar and shows mutarotation.

41. (b)

42. (c) :



43. (b) **44. (c)**

45. (a) : Among the following options, only $K_2Cr_2O_7$ exhibits triclinic crystal system in which $\alpha \neq \beta \neq \gamma \neq 90°$ and $a \neq b \neq c$.

46. (2) : Williamson's synthesis requires that the alkyl halide should be 1° and alkoxide ion may be 1°, 2° or 3°. Thus, two ethers which cannot be prepared by Williamson's synthesis are $(C_6H_5)_2O$, $(CH_3)_3COC(CH_3)_3$.

47. (4) : Given $T_1 = 400$ K, $T_2 = 60$ K
Moleculer weight of X, $M_1 = 40$
Moleculer weight of Y, $M_2 = ?$

$$U_{rms(X)} = \sqrt{\frac{3RT_1}{M_1}},\ U_{mp(Y)} = \sqrt{\frac{2RT_2}{M_2}}$$

Given, $U_{rms(X)} = U_{rms(Y)}$

$$\therefore \quad \sqrt{\frac{3R\times 400}{400}} = \sqrt{\frac{2R\times 60}{M_2}}$$

$$30 = \frac{120}{M_2} \Rightarrow M_2 = 4$$

48. (2.67) :

	$I_{2(g)} \rightleftharpoons$	$2I_{(g)}$
Initial moles	1	0
Moles at equilibrium	$1-x$	$2x$

Total moles at equilibrium $= 1 - x + 2x = 1 + x$

But $\frac{2x}{1+x} = 0.4$ (given) $\therefore\ x = \frac{1}{4}$

So, mole fraction of $I_2 = \frac{1-\frac{1}{4}}{1+\frac{1}{4}} = \frac{3}{5} = 0.6$,

Mole fraction of $I = \frac{2\times\frac{1}{4}}{1+\frac{1}{4}} = \frac{2}{5} = 0.4$

$$K_p = \frac{p_I^2}{p_{I_2}} = \frac{(0.4\times 10^5)^2}{0.6\times 10^5} = 2.67\times 10^4$$

49. (9) : Mole fraction of H_2SO_4 in the solution $= 1 - 0.86 = 0.14$

$$\frac{n_2}{n_1+n_2} = 0.14$$

$$\therefore \quad \frac{n_2}{55.55+n_2} = 0.14 \text{ or } n_2 = 0.14\,n_2 + 7.777$$

or $0.86\,n_2 = 7.777$ or $n_2 = 9.0$

50. (7.45) : *K.E.* of photoelectrons $= e \times$ stopping potential
$= 1.602 \times 10^{-19} \times 300 = 4.806 \times 10^{-17}$ J

K.E. of photoelectrons $= h(\nu - \nu_0) = h\left(\nu - \frac{c}{\lambda_0}\right)$
(ν_0 = Threshold frequency)

$$4.806 \times 10^{-17} = 6.626\times 10^{-34}\left(\nu - \frac{3\times 10^8}{1500\times 10^{-10}}\right)$$

$$= 6.626 \times 10^{-34}\,(\nu - 2 \times 10^{15})$$

$$\nu - 2 \times 10^{15} = \frac{4.806\times 10^{-17}}{6.626\times 10^{-34}} = 7.25 \times 10^{16} \text{ Hz}$$

$\nu = 7.25 \times 10^{16} + 0.2 \times 10^{16} = (7.25 + 0.2)\, 10^{16}$
$\nu = 7.45 \times 10^{16}$

51. (b) : Clearly, $(A-B)\cup(B-C)\cup(C-A)$

$=(A\cup B\cup C)-(A\cap B\cap C)$

$=(A\cap B\cap C)'$ $[\because A\cup B\cup C = U]$

$\therefore\ [(A-B)\cup(B-C)\cup(C-A)]^c$

$=\left[(A\cap B\cap C)'\right]' = A\cap B\cap C.$

52. (b) : $\sum_{n=1}^{13}(i^n + i^{n+1}) = \sum_{n=1}^{13} i^n(1+i) = (1+i)\sum_{n=1}^{13} i^n$

$= (1 + i)(i + i^2 + i^3 + \ldots + i^{13})$

$=(1+i)\left\{\frac{i(1-i^{13})}{1-i}\right\} = (1 + i)i = -1 + i$

53. (d) : $f(x) = e^{ax} + e^{-ax}$
$f'(x) = ae^{ax} - ae^{-ax} = a(e^{ax} - e^{-ax})$
If $f(x)$ is monotonically increasing, then $f'(x) > 0$
$\Rightarrow\ a(e^{ax} - e^{-ax}) > 0 = (e^{ax} - e^{-ax}) > 0 \qquad (\because a > 0)$
$\Rightarrow\ e^{ax} > e^{-ax} = e^{2ax} > 1 = e^0 \Rightarrow 2ax > 0$
$\Rightarrow\ x > 0 \qquad (\because a > 0)$
$\therefore$ Function $f(x)$ is increasing when $x > 0$.

54. (a) : Applying $R_3 \to R_3 - R_2$ and $R_2 \to R_2 - R_1$ and then $C_1 \to C_1 - C_2$ we get

$$\begin{vmatrix} -1 & a+2 & a+p \\ 0 & 1 & q-p \\ 0 & 1 & r-q \end{vmatrix} = 0$$

Now, on expanding along C_1 we get $p + r = 2q$
$\Rightarrow\ p, q, r$ are in AP.

55. (d) : According to the given condition,
$\sin\alpha+\sin\beta = -a$ and $\cos\alpha + \cos\beta = -c$.

$$\Rightarrow\quad 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -a$$

and $2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -c \Rightarrow \tan\frac{\alpha+\beta}{2} = \frac{a}{c}$

$$\text{Now, } \sin(\alpha+\beta) = \frac{2\tan\left(\frac{\alpha+\beta}{2}\right)}{1+\tan^2\left(\frac{\alpha+\beta}{2}\right)} = \frac{2ac}{a^2+c^2}$$

56. (b) : By L' Hospital's Rule,

$$\text{Given limit} = \lim_{x\to 0}\cos(\pi\cos^2 x)(-2\pi\cos x)\frac{\sin x}{2x}$$

$= (\cos\pi)(-\pi) = \pi.$

57. (d) : Let $\bar{x}_1$ and $\bar{x}_2$ are the arithmetic means of two distributions with n_1 and n_2 observations respectively.

Clearly, the combined mean $\bar{x} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

Now, $\bar{x} - \bar{x}_1 = \dfrac{n_2}{n_2 + n_1}(\bar{x}_2 - \bar{x}_1)$

$\Rightarrow \quad \bar{x} - \bar{x}_1 > 0 \text{ as } \bar{x}_2 > \bar{x}_1$ ...(i)

Similarly, $\bar{x} - \bar{x}_2 = \dfrac{n_1}{n_2 + n_1}(\bar{x}_1 - \bar{x}_2)$

$\Rightarrow \quad \bar{x} - \bar{x}_2 < 0 \text{ as } \bar{x}_2 > \bar{x}_1$ ...(ii)

(i) and (ii) $\Rightarrow \bar{x}_1 < \bar{x} < \bar{x}_2$

58. (c) : We have, $\int_0^x f(t)dt = x + \int_x^1 t\, f(t)dt$

$\Rightarrow \quad \dfrac{d}{dx}\int_0^x f(t)dt = \dfrac{d}{dx}(x) + \dfrac{d}{dx}\int_x^1 t\, f(t)dt$

$\Rightarrow \quad f(x)\dfrac{d}{dx}(x) - f(0)\cdot\dfrac{d}{dx}(0)$

$= \dfrac{d}{dx}(x) + 1 \times f(1)\dfrac{d}{dx}(1) - xf(x)\dfrac{d}{dx}(x)$

$\Rightarrow \quad f(x) - 0 = 1 + 0 - x f(x) \Rightarrow f(x) = 1 - x f(x)$

$\Rightarrow \quad f(x) + x f(x) = 1 \Rightarrow f(x)(1 + x) = 1$

$\Rightarrow \quad f(x) = \dfrac{1}{1 + x}$

59. (c) : We have,

$$f(x) = \begin{cases} 3(1+|\tan x|)^{\frac{\alpha}{|\tan x|}} & ; \dfrac{-1}{2} < x < 0 \\ \beta & ; x = 0 \\ 3\left(1+\left|\dfrac{\sin x}{3}\right|\right)^{\frac{6}{|\sin x|}} & ; \mathbf{0} < x < \dfrac{2}{3} \end{cases}$$

Now, R.H.L $= \lim\limits_{x\to 0^+} f(x)$

$= \lim\limits_{x\to 0^+} 3\left(1+\dfrac{|\sin x|}{3}\right)^{\frac{6}{|\sin x|}} = 3e^2$ ...(i)

Similarly, L.H.L $= \lim\limits_{x\to 0^-} f(x) = 3e^{\alpha}$ ...(ii)

$\because$ $f(x)$ is continuous at $x = 0$

$\therefore \quad \lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^-} f(x) = f(0)$

$\Rightarrow \quad 3e^{\alpha} = 3e^2 = \beta.$

So, $\alpha = 2$ and $\beta = 3e^2$.

60. (c) : Given equations of straight lines, are

$4x - 3y + 7 = 0$ and $3x - 4y + 14 = 0$

Here, $a_1 = 4$, $b_1 = -3$ and $a_2 = 3$, $b_2 = -4$

Now, $a_1 a_2 + b_1 b_2 = 4 \times 3 + (-3)(-4) = 24 > 0$

So, the bisector of the acute angle is given by

$$\frac{4x - 3y + 7}{\sqrt{4^2 + (-3)^2}} = -\frac{3x - 4y + 14}{\sqrt{3^2 + (-4)^2}} \Rightarrow x - y + 3 = 0$$

61. (c) : We have, $y\dfrac{dy}{dx} = x\left[\dfrac{y^2}{x^2} + \dfrac{\phi\left(\dfrac{y^2}{x^2}\right)}{\phi'\left(\dfrac{y^2}{x^2}\right)}\right]$

Putting $\dfrac{y}{x} = v \Rightarrow y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$, we get

$$vx\left(v + x\frac{dv}{dx}\right) = x\left[v^2 + \frac{\phi(v^2)}{\phi'(v^2)}\right]$$

$\Rightarrow \quad v^2 + vx\dfrac{dv}{dx} = v^2 + \dfrac{\phi(v^2)}{\phi'(v^2)}$

$\Rightarrow \quad v\dfrac{\phi'(v^2)}{\phi(v^2)}dv = \dfrac{dx}{x}$

On integrating, we get

$$\int\frac{dx}{x} = \int\frac{v\phi'(v^2)}{\phi(v^2)}dv$$

$\Rightarrow \quad \ln|x| + \ln|c_1| = \dfrac{1}{2}\ln|\phi(v^2)|$

$\Rightarrow \quad \ln|c_1 x| = \ln|\phi(v^2)|^{1/2}$

$\Rightarrow \quad |c_1 x| = \phi(v^2)^{1/2} \Rightarrow c_1^2 x^2 = \phi(v^2)$

$\Rightarrow \quad cx^2 = \phi\left(\dfrac{y^2}{x^2}\right)$ [where $c_1{}^2 = c$]

62. (a) : Since, x_1, x_2 are roots of $x^2 + 2x - 3 = 0$

$\therefore \quad x_1 + x_2 = -2 \quad \Rightarrow \quad \dfrac{x_1 + x_2}{2} = -1$

Also, y_1, y_2 are roots of $y^2 + 4y - 12 = 0$

$\therefore \quad y_1 + y_2 = -4 \Rightarrow \dfrac{y_1 + y_2}{2} = -2$

Centre of circle $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right) = (-1, -2)$

63. (b) : $\sim(p \wedge (q \to \sim r)) \equiv \sim(p \wedge (\sim q \vee \sim r))$

$\equiv \sim p \vee (q \wedge r)$

64. (a) : We have, $\frac{S_n}{S'_n}=\frac{2n+3}{6n+5}$, where S_n and S'_n are the sum of n terms of given arithmetic series.

$$\Rightarrow \frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2a'+(n-1)d']}=\frac{2n+3}{6n+5}$$

$$\Rightarrow \frac{a+\left(\frac{n-1}{2}\right)d}{a'+\left(\frac{n-1}{2}\right)d'}=\frac{2n+3}{6n+5}$$

Now, put $\frac{n-1}{2}=12$ or $n=25$ then $\frac{a+12d}{a'+12d'}=\frac{2(25)+3}{6(25)+5}$

$$\Rightarrow \frac{T_{13}}{T'_{13}}=\frac{53}{155}$$

65. (b) : The total number of selections of two numbers x and y from the numbers 0 to 10 $= 11 \times 11 = 121$

Now, if $|x-y|>5$, then the possible values for (x, y) are

(0, 6), (0, 7), (0, 8), (0, 9), (0 10), (1, 7), (1, 8), (1, 9), (1, 10), (2, 8), (2, 9), (2, 10), (3, 9), (3, 10), (4, 10), (6, 0), (7, 0), (8, 0), (9, 0), (10, 0), (7, 1), (8, 1), (9, 1), (8, 2), (10, 1), (9, 2), (10, 2), (9, 3), (10, 3), (10, 4)

So, there are 30 pairs of values of x and y.

Hence, the required probability $=\frac{30}{121}$.

66. (d) : Clearly,

$$\tan(100°+125°)=\frac{\tan100°+\tan125°}{1-\tan100°\tan125°}$$

$$\Rightarrow \tan225°=\frac{\tan 100°+\tan 125°}{1-\tan100°\tan125°}$$

$$\Rightarrow 1=\frac{\tan100°+\tan 125°}{1-\tan 100°\tan 125°}$$

$$\Rightarrow \tan100° + \tan125° + \tan100° \tan125° = 1$$

67. (b) : For infinitely many solutions, the two equations must be identical

$$\Rightarrow \frac{k+1}{k}=\frac{8}{k+3}=\frac{4k}{3k-1}$$

$\Rightarrow$ $(k+1)(k+3) = 8k$ and $8(3k-1) = 4k(k+3)$

$\Rightarrow$ $k^2-4k+3=0$ and $k^2-3k+2=0$

By cross multiplication method,

$$\frac{k^2}{-8+9}=\frac{k}{3-2}=\frac{1}{-3+4}$$

$\Rightarrow$ $k^2 = 1$ and $k = 1$

$\therefore$ $k = 1$.

68. (b) : $\int\frac{5\tan x}{\tan x-2}dx=x+a\ln|\sin x-2\cos x|+k$

On differentiating both sides, we get

$$\frac{5\tan x}{\tan x-2}=1+\frac{a(\cos x+2\sin x)}{\sin x-2\cos x}$$

$$\Rightarrow \frac{5\sin x}{\sin x-2\cos x}=\frac{\sin x(1+2a)+\cos x(a-2)}{\sin x-2\cos x}$$

$\Rightarrow$ $a = 2$

69. (c) : Let T_{r+1} term containing x^{32}.

Clearly, $T_{r+1}={}^{15}C_r(x^4)^{(15-r)}\left(\frac{-1}{x^3}\right)^r$

$={}^{15}C_r\,x^{60-4r}(-1)^r x^{-3r}={}^{15}C_r(-1)^r x^{60-7r}$

$\Rightarrow$ $60-7r=32$ $\Rightarrow$ $7r=28$ $\Rightarrow$ $r=4$

Thus, coefficient of x^{32} is ${}^{15}C_4$.

70. (a) : The lines are $\frac{x-1}{1}=\frac{y+3}{-\lambda}=\frac{z-1}{\lambda}$ and $\frac{x}{1/2}=\frac{y-1}{1}=\frac{z-2}{-1}$

The lines are coplanar if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

$$\begin{vmatrix} 0-1 & 1-(-3) & 2-1 \\ 1 & -\lambda & \lambda \\ \frac{1}{2} & 1 & -1 \end{vmatrix}=\begin{vmatrix} -1 & 4 & 1 \\ 1 & -\lambda & \lambda \\ \frac{1}{2} & 1 & -1 \end{vmatrix}=0$$

$$\Rightarrow -1(\lambda-\lambda)-4\left(-1-\frac{\lambda}{2}\right)+1+\frac{\lambda}{2}=0$$

$$\Rightarrow 5+\frac{5\lambda}{2}=0 \Rightarrow \lambda=-2.$$

71. (6) : No. of 4 digit numbers greater than $3000 = 3\times5\times4\times3 = 180$

No. of 5 digit numbers $= 5\times5\times4\times3\times2 = 600$

No. of 6 digit numbers $= 5\times5\times4\times3\times2\times1 = 600$

$\therefore$ $n = 1380$

$$\Rightarrow \frac{n}{230}=6$$

72. (1.6) : Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = m\hat{i} - \hat{j} + 2\hat{k}$.

Then $\vec{a}, \vec{b}, \vec{c}$, will be coplanar if $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

$$\Rightarrow \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ m & -1 & 2 \end{vmatrix} = 0$$

$\Rightarrow$ 2(4 –1) – (–3) (2 + m) + 4 (–1–2m) = 0

$\Rightarrow$ 6 + 6 + 3m – 4 – 8m = 0 $\Rightarrow$ 5m = 8

$\Rightarrow$ m = 8/5 = 1.6.

73. (9) : Let $a = A - 2d$, $x = A - d$, $y = A$, $z = A + d$ and $b = A + 2d$

Now, $x + y + z = 15 \Rightarrow A = 5 \Rightarrow a = 5 - 2d$ and $b = 5 + 2d$

Also, it is given that $\frac{1}{a}, \frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}, \frac{1}{b}$ are in A.P

$$\Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{3}{2}\left[\frac{1}{a} + \frac{1}{b}\right] = \frac{5}{3}$$

or $\frac{3}{2}\left(\frac{1}{5-2d} + \frac{1}{5+2d}\right) = \frac{5}{3} \Rightarrow d = \pm 2$

Take d = –2 since $a > b$. Hence a = 9

74. (9) : Given equations of planes are $x - 2y + 2z - 8 = 0$ and $x - 2y + 2z + 19 = 0$

Now, the distance between them is given by

$$\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{|-8 - 19|}{\sqrt{1 + 4 + 4}} = \frac{27}{3} = 9.$$

75. (1.5): We have, $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$

$$\Rightarrow 2^{2x} + 2^{2x-1} = 3^{x+\frac{1}{2}} + 3^{x-\frac{1}{2}}$$

$$\Rightarrow 2^{2x}\left(1 + \frac{1}{2}\right) = 3^{x-\frac{1}{2}}(1 + 3)$$

$$\Rightarrow 2^{2x-3} = 3^{x-\frac{3}{2}}$$

Taking log on both sides, we get

$(2x - 3)\log 2 = (x - 3/2)\log 3$

$$\Rightarrow 2x\log 2 - 3\log 2 = x\log 3 - \frac{3}{2}\log 3$$

$$\Rightarrow x\log 4 - x\log 3 = 3\log 2 - \frac{3}{2}\log 3$$

$$\Rightarrow x\log\left(\frac{4}{3}\right) = \log 8 - \log 3\sqrt{3}$$

$$\Rightarrow \left(\frac{4}{3}\right)^x = \frac{8}{3\sqrt{3}} \Rightarrow \left(\frac{4}{3}\right)^x = \left(\frac{4}{3}\right)^{3/2}$$

$$\Rightarrow x = \frac{3}{2} = 1.5.$$