13 PROBABILITY

Objectives

After studying the material of this chapter, you should be able to :

- Understand Conditional Probability and its Applications.
- Understand Independent events and their Applications.
- Understand Bayes' Theorem and its Applications.
- Understand Random variable and its Probability distribution along with Mean (Expectation), Variance and Standard deviation.
- Understand Binomial Distribution.

The theory of probabilities is simply the Science of logic quantitatively treated.

—*C.S. Peirce*

INTRODUCTION

In Class XI, we discussed Probability under axiomatic approach and treated it as a function of outcomes of the experiment. In this chapter, we will learn the following concepts :

- Conditional Probability of the event, given that another event has already occurred.
- Bayes' Theorem.
- Multiplication rule of Probability and independence of events.
- Random variable and its Probability distribution along with Mean (Expectation), Variance and Standard deviation.
- Binomial Distribution.

SUB CHAPTER 13.0 Review

13.1. RANDOM EXPERIMENTS AND SAMPLE SPACES

When a die is thrown, we get any one of the following on the top face :

1, 2, 3, 4, 5 or 6.

Thus 1 or 2 or 3 or 4 or 5 or 6 is called an **outcome** of the above activity.

The set S of all possible outcomes *i.e.* S = {1, 2, 3, 4, 5, 6} is called the **sample space** of the said activity.

Element of a sample space is called a **sample point.**

The activity (of throwing a die) is called a **random experiment** or simply an **experiment.**

Thus we have the definitions :

(*i*) **Random Experiment.** *An experiment whose outcomes can't be predicted in advance is called a random experiment.*

Chapter at Glance

(*ii*) **Sample Space.** The set of all possible outcomes of an experiment is called a sample space.

This is generally denoted by S.

For Examples : (I) When a coin is tossed, S = {(H, T)}, where H ≡ Head and T ≡ Tail.

(II) When a die is thrown, S = {1, 2, 3, 4, 5, 6}.

13.2. EVENTS

(*i*) **Events.** *The possible outcomes of a trial are called events.* These are also known as **cases.**

For Examples : (I) When a coin is tossed, the outcome of a head or tail is an event.

(II) When a die is thrown, the outcome of getting 1 or 2 or 3 or 4 or 5 or 6 is an event.

(*ii*) **Equally Likely Events.** *The events are said to be equally likely if there is no reason to expect any one in preference to any other.*

For Examples : (I) When a die is thrown, then all the six faces are equally likely to come.

(II) When a card is drawn from a well shuffled pack, then all the 52 cases are equally likely to come.

(*iii*) **Exhaustive Events.** *It is the total number of all possible outcomes of any trial.*

For Examples : (I) When a coin is tossed, there are two exhaustive events *i.e.* head and tail.

(II) When a die is thrown, there are six exhaustive events *i.e.* any one of the six faces may appear.

(*iv*) **Mutually Exclusive Events.** *Two or more events are said to be mutually exclusive if they cannot happen simultaneously in a trial.*

These are also called **incompatible events.**

For Examples : (I) When a coin is tossed, either head or tail will appear.

(II) When a die is thrown, any one of the six faces will appear.

(*v*) **Favourable Events.** *The cases which ensure the occurrence of the particular event are called favourable.*

For Example : When two dice are thrown, the number of the cases favourable for getting a sum 6 is 5 *viz.* (1,5) ; (5, 1) ; (4, 2) ; (2, 4) ; (3, 3).

(*vi*) **Independent and Dependent Events.** *Two or more events are said to be independent if the happening or non-happening of any one does not depend on the happening or non-happening of any other, otherwise they are said to be dependent.*

For Example : When a card is drawn from a pack of well shuffled cards and replaced before drawing the second card, the result of second draw is independent of the first one.

However, if the first card is not replaced, the second draw is dependent on the first one.

(*vii*) **Simple (Elementary) Event.** *An event, which contains only a single simple point, is called a* **simple event.** Events other than simple (elementary) are called **compound or mixed events.**

For Example : *When we toss two coins.*

Here S = {HT, TH, HH, TT}. Then A = {TT} of getting both tails is a simple event,

while B = {HT, TH, TT} of getting at least one tail is a compound event.

13.3. AXIOMATIC APPROACH TO PROBABILITY

So far we have considered sample space and events associated with these experiments. In our daily life we use many words about the chances of occurrence of different events. The theory of Probability tries to quantify these chances of occurrence/non-occurrence of events.

Axiomatic approach is a different way of describing the probability of an event. Here we depict some rules and axioms to assign probabilities.

Let S be the sample space of a random experiment. The probability P is a real valued function whose domain is P (S), *i.e.* the power set of S and range is the interval [0, 1] so as to satisfy the following axioms :

Axiom I. For any event E, $P(E) \geq 0$.

Axiom II. $P(S) = 1$.

Axiom III. When E and F are disjoint events, then :

$$P(E \cup F) = P(E) + P(F).$$

Deduction. $P(\phi) = 0$.

13.4. CLASSICAL (OR A PRIORI) PROBABILITY

Let us perform a random experiment.

Let there be n exhaustive, mutually exclusive and equally likely cases. Out of these, let m be favourable to the happening of an event A.

Then the Probability of occurrence of events A, denoted by P (A), is defined as :

$$P(A) = \frac{\text{Number of favourable cases}}{\text{Number of exhaustive cases}} = \frac{n(A)}{n(S)} = \frac{m}{n}.$$

Notes. 1. When m is the number of favourable cases out of n exhaustive, mutually exclusive and equally likely cases, then the number of unfavourable cases = $n - m$.

2. Odds in favour of an event A. *It is defined as the ratio of the number of favourable cases to the number of failures.*

Thus odds in favour of an event A = $\frac{m}{n-m}$ and odds against the event A = $\frac{n-m}{m}$.

3. The probability P ($\overline{A}$) that the event A will not happen is given by :

$$P(\overline{A}) = \frac{\text{Number of unfavourable cases}}{\text{Number of exhaustive cases}}$$

$$= \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A).$$

Hence, $\mathbf{P(A) + P(\overline{A}) = 1}$.

It may also be noted that P(A) and P ($\overline{A}$) are non-negative and cannot exceed unity.

Hence, $\mathbf{0 \le P(A) \le 1}$ and $\mathbf{0 \le P(\overline{A}) \le 1}$.

4. (I) *When P (A) = 1*, then A is called a **certain event** or **sure event.**

In this case, the chance of the happening of an event A is cent-per–cent.

Q. *The probability of a sure event is 1.* **(True/False)** *(Kashmir B. 2016)*

Ans : True.

(II) *When P (A) = 0,* then P ($\overline{A}$) = 1, then A is called an **impossible event.**

In this case, the chance of non-happening of an event A is cent-per–cent.

Example : *In the throw of die, sample space, S = {1, 2, 3, 4, 5, 6}.*

Let A : Event of getting a number less than 1 and B : Event of getting a number less than 7.

Clearly A is an impossible event. [∵ *No outcome is less than 1*]

and B is a sure event. [∵ *Each outcome is less than 7*]

ILLUSTRATIVE EXAMPLES

Example 1. A four– digit number is formed, using the digits 1, 2, 3, 5 with no repetitions. Find the probability that the number is divisible by 5.

Solution.

Here m, the number of favourable cases

= Number of four–digit numbers, which are divisible by 5 = 6

and n, the no. of exhaustive cases = ${}^4C_4$

= 4! = 24.

∴ Reqd. probability $= \frac{m}{n} = \frac{6}{24} = \frac{1}{4}$.

Example 2. If a leap year is selected at random, what is the chance that it will contain 53 Tuesdays ? *(N.C.E.R.T.)*

Solution. A leap year has 366 days and thus leap year has 52 weeks and 2 days over.

The two over (successive) days have the following likely cases :

(*i*) Sunday and Monday
(*ii*) Monday and Tuesday
(*iii*) Tuesday and Wednesday
(*iv*) Wednesday and Thursday
(*v*) Thursday and Friday
(*vi*) Friday and Saturday
(*vii*) Saturday and Sunday.

$\therefore$ n, the number of exhaustive cases= 7.

Out of these, the favourable cases are (*ii*) and (*iii*).

Thus m, the number of favourable cases = 2.

$\therefore$ Required probability $= \frac{m}{n} = \frac{2}{7}$.

13.5. THEOREMS ON PROBABILITY

Theorem I. *In a random experiment, if S be the sample space and A an event, then :*

(*i*) $P(A) \geq 0$ (*ii*) $P(\phi) = 0$ (*iii*) $P(S) = 1$.

KEY POINT

1. **Probability of occurrence of an event is always non-negative.**
2. **Probability of an impossible event is 0.**
3. **Probability of sure event is 1.**

Theorem II. *If A and B are two mutually exclusive events, then* $P(A \cap B) = 0$.
Theorem III. *If A and B are two mutually exclusive events, then* $P(A) + P(B) = 1$.
Theorem IV. Addition Law. *If A and B are mutually exclusive events, then* $P(A \cup B) = P(A) + P(B)$.
Extension. If $A_1, A_2, \ldots\ldots, A_k$ are mutually exclusive events,

then $P(A_1 \cup A_2 \cup \ldots\ldots \cup A_k) = P(A_1) + P(A_2) + \ldots\ldots\ldots + P(A_k) = \sum_{i=1}^{k} P(A_i)$.

Theorem V. *For any two events A and B,* $P(A - B) = P(A) - P(A \cap B)$.
Theorem VI. Addition Law. *For any two events A and B,* $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
Theorem VII. Addition Law for three events. *If A, B, C be three events associated with a random experiment, then :*
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$.
Cor. If A, B, C are mutually exclusive events, then $P(A \cap B) = P(B \cap C) = P(A \cap C) = P(A \cap B \cap C) = 0$.
$\therefore \quad P(A \cup B \cup C) = P(A) + P(B) + P(C)$.

Theorem VIII. *For each event A,* $P(\overline{A}) = 1 - P(A)$, *where* $\overline{A}$ *is the complementary event.*
Theorem IX. *If A and B be two events such that* $A \subseteq B$, *then* $P(A) \leq P(B)$.
Theorem X. *If A is an event associated with a random experiment, then* $0 \leq P(A) \leq 1$.

SUB CHAPTER 13.1 Conditional Probability

13.6. CONDITIONAL PROBABILITY

Conditional Probability. *Let A and B be two events associated with the same sample space.* Then we *can talk of conditional probability* of the occurrence of an event A, given that the event B has already occurred and $P(B) \neq 0$ is called conditional probability and is denoted by P (A/B). ***(Kashmir B. 2013)***

In order to calculate P (A/B), we take elementary events favourable to the occurrence of B as the new sample space and then we find how many of these are favourable to the occurrence of A.

$$\therefore P(A/B) = \frac{\text{No. of elementary events favourable to B which are also favourable to A}}{\text{No. of elementary events favourable to B}}.$$

For Example : *Consider a pack of cards and a card is drawn.*

B : "the card drawn is black"

A : "the card drawn is king".

Here there are 26 elementary events favourable to B, out of which 2 are favourable to A.

$\therefore \quad P(A/B) = \frac{2}{26} = \frac{1}{13}.$

Thus $P(A/B) = \frac{\text{No. of elementary events favourable to A and B}}{\text{No. of elementary events favourable to B}}.$

So we have the following theorem :

Multiplication Theorem. *Let A and B be two events associated with the same sample space. Then :*

$$P(A \cap B) = P(B) \,.\, P(A/B), \text{ where } P(B) \neq 0.$$

Proof. Let S be the sample space and A, B be the events associated with it.

Then $\quad P(A \cap B) = \frac{n(A \cap B)}{n(S)} \quad$ ...(1)

Suppose A occurs only when B has already occurred. Then for the event (A/B), favourable outcomes are the elements of $(A \cap B)$ and all possible outcomes are the elements of B.

$\therefore \quad P(A/B) = \frac{n(A \cap B)}{n(B)} \quad$ ...(2)

$\therefore \quad P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{n(B)}{n(S)} \times \frac{n(A \cap B)}{n(B)}$

$= P(B) \times P(A/B).$ [*Using* (2)]

Hence, $\quad P(A \cap B) = P(B) \times P(A/B)$, where $P(B) \neq 0$.

Similarly, $\quad P(A \cap B) = P(A) \times P(B/A)$, where $P(A) \neq 0$.

Cor. $\mathbf{P(A/B) = \frac{P(A \cap B)}{P(B)}}$ provided $P(B) \neq 0$ and $\mathbf{P(B/A) = \frac{P(A \cap B)}{P(A)}}$ provided $P(A) \neq 0$.

Question. *Write conditional probability P(A/B) =.................* ***(Jammu B. 2014)***

Ans : $\quad P(A/B) = \frac{P(A \cap B)}{P(B)}.$

13.6.1. PROPERTIES OF CONDITIONAL PROBABILITY

Let A and B be events of sample space S of an experiment.

Property I. $0 \leq P(A/B) \leq 1$.

The conditional probability of an event A, given that B has occurred, is always greater than or equal to 0 and less than or equal to 1.

Proof. We know that $A \cap B \subseteq B$

$\Rightarrow \quad P(A \cap B) \leq P(B)$

$\Rightarrow \quad \frac{P(A \cap B)}{P(B)} \leq 1$ when $P(B) \neq 0$

$\Rightarrow \quad P(A/B) \leq 1 \quad$ ...(1)

Also $\quad P(A \cap B) \geq 0$ and $P(B) > 0$

$\Rightarrow \quad \frac{P(A \cap B)}{P(B)} \geq 0$

$\Rightarrow \quad P(A/B) \geq 0 \quad$ ...(2)

Combining (1) and (2), $0 \leq P(A/B) \leq 1$, which is true.

Property II. P(S/B) = P(B/B) = 1.

Proof. We know that $P(S/B) = \frac{P(S \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1.$

Also $P(B/B) = \dfrac{P(B \cap B)}{P(B)} = \dfrac{P(B)}{P(B)} = 1$.

Hence, $P(S/B) = P(B/B) = 1$.

Property III. If A and B are any two events of a sample space S and F is an event of S such that P (F) ≠ 0, then : P(A ∪ B/F) = P (A/F) + P(B/F) – P (A ∩ B/F).

Proof. We have :

$$P(A \cup B/F) = \frac{P[(A \cup B) \cap F]}{P(F)} \quad \text{[By def.]}$$

$$= \frac{P[(A \cap F) \cup (B \cap F)]}{P(F)} \quad \text{[Distributive Law]}$$

$$= \frac{P(A \cap F) + P(B \cap F) - P(A \cap B \cap F)}{P(F)}$$

$$= \frac{P(A \cap F)}{P(F)} + \frac{P(B \cap F)}{P(F)} - \frac{P[(A \cap B) \cap F]}{P(F)}$$

$= P(A/F) + P(B/F) - P((A \cap B)/F)$, which is true.

Particular Case. When A and B are disjoint events, then P[A ∪ B/F] = P(A/F) + P (B/F).

Here $P((A \cap B)/F)) = 0$ [∵ *A and B are disjoint events*]

∴ From above property, we have : $P(A \cup B/F) = P(A/F) + P(B/F)$.

Property IV. P(A′ / B) = 1 – P (A/B).

Proof. From Property II, we have : $P(S/B) = 1$

⇒ $P(A \cup A'/B) = 1$ [∵ $S = A \cup A'$]

⇒ $P(A/B) + P(A'/B) = 1$ [∵ *A and A′ are disjoint events*]

⇒ $P(A'/B) = 1 - P(A/B)$, which is true.

ILLUSTRATIVE EXAMPLES

Example 1. If $P(A) = \dfrac{1}{5}$ and $P(A - B) = \dfrac{1}{6}$, find the value of P(A ∩ B). *(Tripura B. 2016)*

Solution. We know that $P(A - B) = P(A) - P(A \cap B)$.

∴ $P(A \cap B) = P(A) - P(A - B)$

$$= \frac{1}{5} - \frac{1}{6} = \frac{6-5}{30} = \frac{1}{30}.$$

Example 2. If P(A) = 0.6, P(B) = 0.5 and P(A/B) = 0.3, then find P(A ∪ B). *(C.B.S.E. Sample Paper 2019)*

Solution. We have : $P(A) = 0.6$, $P(B) = 0.5$ and $P(A/B) = 0.3$.

Now, $P(A/B) = 0.3$

⇒ $\dfrac{P(A \cap B)}{P(B)} = 0.3$

⇒ $P(A \cap B) = 0.3 \times 0.5 = 0.15$.

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

∴ $P(A \cup B) = 0.6 + 0.5 - 0.15 = 0.95$.

Examples 3. If A and B are two events such that P(A) = 0.4, P(B) = 0.8 and P(B/A) = 0.6, then find P(A/B). *(C.B.S.E. Sample Paper 2018)*

Solution. We have : $P(A) = 0.4$ and $P(B) = 0.8$.

Now, $P(B/A) = \dfrac{P(B \cap A)}{P(A)}$

⇒ $0.6 = \dfrac{P(A \cap B)}{0.4}$

⇒ $P(A \cap B) = 0.6 \times 0.4 = 0.24$.

Hence, $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$= \dfrac{0.24}{0.8} = 0.3$.

Example 4. If P(A/B) = 0.8, P(B/A) = 0.6 and $P(A^C \cup B^C) = 0.7$, then find the value of P(A ∪ B). *(W. Bengal B. 2018)*

Solution. We have : $P(A^C \cup B^C) = 0.7$

⇒ $P(A \cap B)^C = 0.7$

⇒ $P(A \cap B) = 1 - 0.7 = 0.3$.

Now, $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

⇒ $0.8 = \dfrac{0.3}{P(B)}$

⇒ $P(B) = \dfrac{3}{8}$.

And, $$P(B/A) = \frac{P(B \cap A)}{P(A)}$$

$$\Rightarrow \quad 0.6 = \frac{0.3}{P(A)}$$

$$\Rightarrow \quad P(A) = \frac{3}{6}.$$

Now, $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{6} + \frac{3}{8} - \frac{3}{10}$$

$$= \frac{60 + 45 - 36}{120} = \frac{69}{120}$$

$$= \frac{23}{40} = 0.57.$$

Example 5. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4. *(C.B.S.E. 2018)*

Solution. Let the events be as :

E : Sum of numbers is 8

and F : Number of red die less than 4.

Thus, E = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

and F = {(1, 1), (2, 1), ..., (6, 1)},
(1, 2), (2, 2), ..., (6, 2),
(1, 3), (2, 3), ..., (6, 3)}.

Also, $E \cap F$ = {(5, 3), (6, 2)}.

$$\therefore \quad P(E) = \frac{5}{36}, \; P(F) = \frac{18}{36}$$

and $P(E \cap F) = \frac{2}{36}$.

Hence, $$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{2/36}{18/36} = \frac{2}{18} = \frac{1}{9}.$$

Example 6. Ten cards numbered 1 through 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, what is the probability that it is an even number. *(N.C.E.R.T)*

Solution. Let the events be :

A : Number on the card drawn is even

B : Number on the card is more than 3.

We are to find P(A/B).

We have : S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

Then A = {2, 4, 6, 8, 10}

and B = {4, 5, 6, 7, 8, 9, 10}.

$\therefore$ $A \cap B$ = {4, 6, 8, 10}.

Hence, $$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{4/10}{7/10} = \frac{4}{7}.$$

Example 7. Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4', given that 'there is at least one tail'. *(N.C.E.R.T.)*

Solution. We represent the outcomes of the experiment in the following diagram :

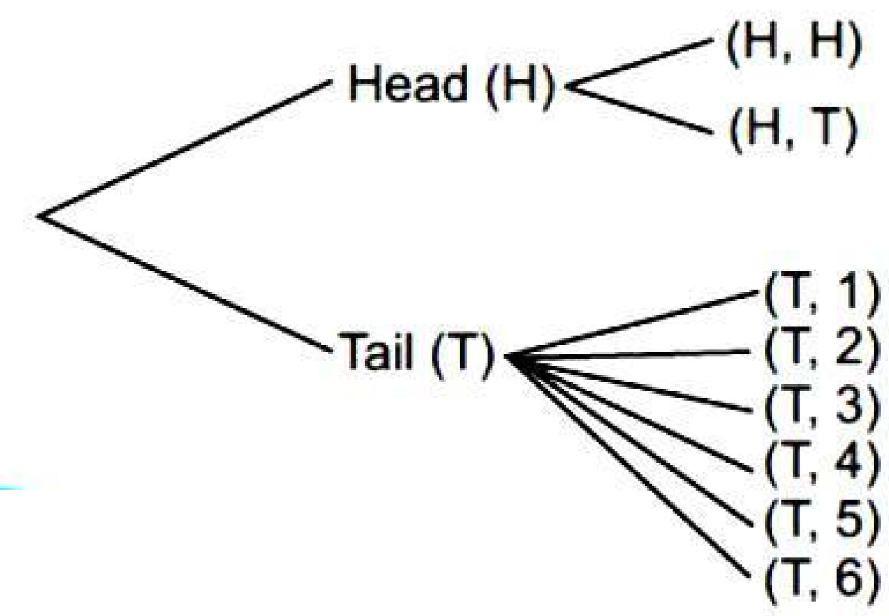

Fig.

Thus the sample space is :

S = {(H, H), (H, T), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)},

where H ≡ Head and Tail ≡ Tail and (H, H) denotes both the tosses result into head and (T, *i*) denotes the first toss results into a tail and the number *i* appears on the die for *i* = 1, 2, 3, 4, 5, 6.

$\therefore$ The probabilities of these 8 elementary events :

(H, H), (H, T), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)

are $\frac{1}{4}, \frac{1}{4}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}$ respectively.

These are exhibited in the following diagram :

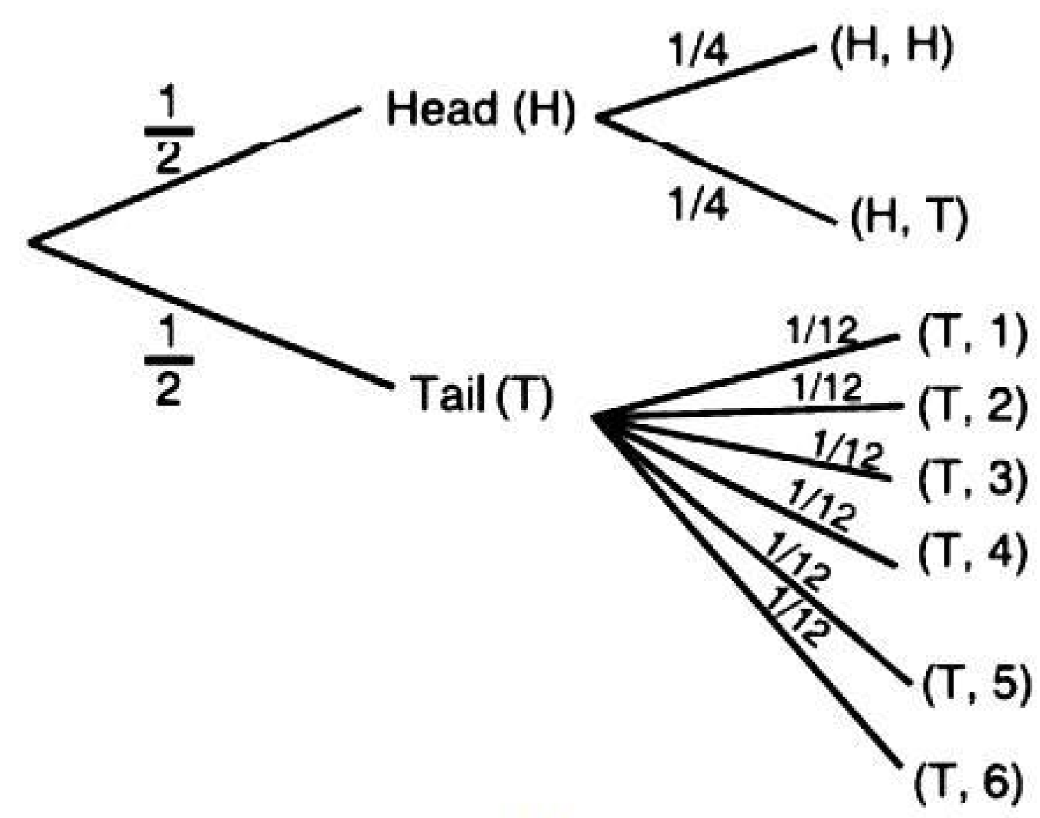

Fig.

Let the events be :

E : 'die shows a number greater than 4'

and F : 'At least one tail'

i.e. E = {(T, 5), (T, 6)}

and F = {(H, T), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)}

Also $E \cap F$ = {(T, 5), (T, 6)}.

Now $$P(F) = P(\{HT\}) + P(\{T, 1\}) + P(\{T, 2\}) + P(\{T, 3\}) + P(\{T, 4\}) + (\{T, 5\}) + P(\{T, 6\})$$

$$= \frac{1}{4} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{3}{4}$$

and $$P(E \cap F) = P(\{T, 5\}) + P(\{T, 6\})$$

$$= \frac{1}{12} + \frac{1}{12} = \frac{1}{6}.$$

Hence, $$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{1/6}{3/4} = \frac{2}{9}.$$

EXERCISE 13 (a)

Fast Track Answer Type Questions

FTATQ

1. (*a*) The probability of 'Ace of spade' is ....

(Fill in the blank) *(Jammu B. 2018)*

(*b*) Compute P (A/B) if P(B) = 0·5 and P(A ∩ B) = 0·32.

(N.C.E.R.T.; Kashmir B. 2017, 16; Jammu B. 2017)

2. (*a*) If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, evaluate P (A/B). *(N.C.E.R.T.)*

(*b*) If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, find P(not A and not B).

(Jammu B. 2015 W)

(*c*) (*i*) If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{13}$, find (*i*) P(A ∩ B) *(Kashmir B. 2017)*

(*ii*) P(B/A). *(Kashmir B. 2017, 13; Jammu B. 2017)*

3. If P(A) = 0·8, P(B) = 0·5 and P(B/A) = 0·4, find :

(*i*) P(A ∩ B) *(Kashmir B. 2017, 16, 12)*

(*ii*) P(A/B) *(Jammu B. 2017; Kerala B. 2014; Kashmir B. 2012)*

(*iii*) P(A ∪ B). *(N.C.E.R.T.; Jammu B. 2017)*

4. If A and B are two events such that P(A) = 0·3, P(B) = 0·6 and P(B/A) = 0·5, find P(A/B).

(Kashmir B. 2011)

5. (*a*) If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$, find

(*i*) P(A ∩ B) (*ii*) P(A/B) (*iii*) P(B/A).

(N.C.E.R.T. ; Jammu B. 2018, 17, 15, 13; Meghalaya B. 2016; Kashmir B. 2013)

(*b*) If P(A) = 0·6, P(B) = 0·7, and P(A ∪ B) = 0·9, then find P (A/B) and P (B/A). *(Kerala B. 2015)*

Very Short Answer Type Questions

VSATQ

Determine P(E/F) in the following (6-7) :

6. (*a*) A coin is tossed three times.

(*i*) E : heads on third toss ; F : heads on first two tosses

(*ii*) E : at least two heads;
F : at most two heads

(*iii*) E : at most two tails ; F : at least one tail.

(N.C.E.R.T.)

(*b*) A die is thrown three times.

E : 4 appears on third toss ; F : 6 and 5 appear respectively on first two tosses. *(N.C.E.R.T.)*

7. Mother, father and son line up at random for a family picture :

E : Son on one end ; F : father in the middle.

(N.C.E.R.T.)

8. A black and a red die are rolled. Find the conditional probability of obtaining :

(*a*) a sum greater than 9, given that the black die resulted in a 5

(*b*) the sum 8, given that the red die resulted in a number less than 4. *(N.C.E.R.T.)*

9. Let E and F be events with :

$P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \text{ and } F) = \frac{1}{5}$.

Are E and F independent ? *(H.P.B. 2010 S)*

Short Answer Type Questions

SATQ

10. A fair die is rolled. Consider the events :

E = {1, 3, 5}, F = {2, 3} and G = {2, 3, 4, 5}.

Find : (*i*) P(E/F) and P(F/E) *(H.P.B. 2012)*

(*ii*) P(E/G) and P (G/E) *(H.P.B. 2012)*

(*iii*) P(E ∪ F/G) and P(E ∩ F/G).

(N.C.E.R.T.; H.P.B. 2012)

11. (*a*) A family has 2 children. Find the probability that both are boys, if it is known that :

(*i*) at least one of the children is a boy

(N.C.E.R.T. ; H.P.B. 2017; Jharkhand B. 2016; A.I.C.B.S.E. 2010)

(*ii*) the elder child is a boy. *(A.I.C.B.S.E. 2010)*

(*b*). A family has two children. If it is known that at least one of the children is a girl, then find the probability that both of the children are girls. *(Uttarakhand B. 2015)*

12. Two coins are tossed. What is the probability of getting 2 heads if it is known that at least one head comes up ?

13. In a school there are 1,000 students, out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII. What is the probability that a student chosen randomly studies in class XII, given that the chosen student is a girl ? *(N.C.E.R.T.; A.I.C.B.S.E. 2013, 09 C)*

14. A die is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once ?

(N.C.E.R.T.; Assam B. 2018; H.B. 2017; H.P.B. 2010)

15. Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'. *(N.C.E.R.T.)*

16. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls, given that :

(*i*) the youngest is a girl

(*ii*) at least one is a girl ?

(N.C.E.R.T.; H.P.B. 2017; C.B.S.E. 2014)

Answers

1. (*a*) $\frac{1}{52}$ (*b*) $\frac{6}{25}$.
2. (*a*) $\frac{4}{9}$ (*b*) $\frac{3}{8}$ (*c*) (*i*) $\frac{6}{13}$ (*ii*) $\frac{11}{13}$.
3. (*i*) 0·32 (*ii*) 0·64 (*iii*) 0·98. **4.** 0·25.
5. (*a*) (*i*) $\frac{4}{11}$ (*ii*) $\frac{4}{5}$ (*iii*) $\frac{2}{3}$ (*b*) $\frac{4}{7}, \frac{2}{3}$.
6. (*a*) (*i*) $\frac{1}{2}$ (*ii*) $\frac{3}{7}$ (*iii*) $\frac{6}{7}$ (*b*) $\frac{1}{6}$.
7. 0. **8.** (*a*) $\frac{1}{3}$ (*b*) $\frac{1}{9}$.
9. No. **10.** (*i*) $\frac{1}{2}, \frac{1}{3}$ (*ii*) $\frac{1}{2}, \frac{2}{3}$ (*iii*) $\frac{3}{4}, \frac{1}{4}$.
11. (*a*) (*i*) $\frac{1}{3}$ (*ii*) $\frac{1}{2}$ (*b*) $\frac{1}{3}$. **12.** $\frac{1}{3}$. **13.** $\frac{1}{10}$.
14. $\frac{2}{5}$. **15.** $\frac{1}{15}$. **16.** (*i*) $\frac{1}{2}$ (*ii*) $\frac{1}{3}$.

Hints to Selected Questions

13. Reqd. probability $= \dfrac{\frac{100}{1000} \times 430}{430} = \frac{1}{10}$.

15. A = {(1, 3), (3, 1), (2, 2)}

B = { (1,2),........ (1, 6) ;........; (6, 1),..........., (6, 6)}

−{(1, 1), (2, 2),.......(6, 6)}

A ∩ B = {(1, 3), (3, 1)}.

∴ $P(A) = \frac{3}{36}$, $P(B) = \frac{30}{36}$ and $P(A \cap B) = \frac{2}{36}$.

∴ Reqd. probability $= \dfrac{2/36}{30/36} = \frac{1}{15}$.

16. (*i*) S = {B*b*, B*g*, G*b*, G*g*}.

A : Both children are girls = {G*g*}

B : Younger child is a girl = {B*g*, G*g*}

and (A ∩ B) = {G*g*}.

∴ Reqd. probability $= \dfrac{P(A \cap B)}{P(B)} = \dfrac{1/4}{1/2} = \frac{1}{2}$.

13.7. COMPOUND EVENTS

Compound Event. *The simultaneous happening of two or more events is called a compound event if they occur in connection with each other.*

For Example : *Consider an urn containing 5 black and 6 white balls.*

The event of drawing 2 black balls is a simple event while the event of drawing 2 black and 3 white balls is a compound event.

13.8. INDEPENDENT EVENTS

Definition

(i) Two events are said to be ***independent*** *if the occurrence of one does not depend upon the occurrence of the other.*

(ii) Two random experiments are said to be ***independent*** *if for each pair of events A and B, associated with the* ***first and second*** *experiment respectively, the probability of the simultaneous occurrence of A and B is the product of P (A) and P (B).*

Consider an experiment of drawing a card from a pack of 52 cards, in which the elementary events can be supposed to be equally likely.

Let the event A denote "the card drawn is a spade" and the event B denote "the card drawn is an ace."

∴ $P(A) = \dfrac{\text{No. of spade cards}}{52} = \frac{13}{52} = \frac{1}{4}$

and $P(B) = \dfrac{\text{No. of aces}}{52} = \frac{4}{52} = \frac{1}{13}$.

Also "A and B" is the event "the card drawn is the ace of spade."

$\therefore \quad P(A \cap B) = \frac{1}{52}$.

$\therefore \quad P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/52}{1/13} = \frac{1}{4} = P(A)$.

Also $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/52}{1/4} = \frac{1}{13} = P(B)$.

Thus the event A that has occurred does not change the probability of the occurrence of the event B and vice-versa. Such events are independent events.

Theorem I. *Two events A and B, defined on the sample space S of a random experiment, are said to be independent if and only if :*

$$P(A \text{ and } B) = P(A \cap B) = P(A) \,.\, P(B).$$

Proof. Since A and B are independent events,

$\therefore$ the occurrence of A does not depend on the occurrence of B

$\Rightarrow \quad P(A/B) = P(A)$ ...(1)

$\therefore P(A \cap B) = P(A/B) \cdot P(B)$ [*Multiplication Theorem*]

$= P(A).\, P(B).$ [*Using (1)*]

Conversely. If A and B be two events such that $P(A \cap B) = P(A) \,.\, P(B)$ ...(2)

By *Multiplication Theorem,* we have :

$$P(A \cap B) = P(A/B) \,.\, P(B) \quad ...(3)$$

From (2) and (3), $P(A) \,.\, P(B) = P(A/B) \,.\, P(B)$

$\Rightarrow \quad P(A) = P(A/B)$

$\Rightarrow$ A and B are independent events.

Theorem II. *If A and B are independent, show that :*

$$P(A \cup B) = 1 - P(\overline{A})\, P(\overline{B}).$$

Proof. Since A and B are independent,

$\therefore \quad P(A \cap B) - P(A)\, P(B)$ ...(1)

Now $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)\, P(B)$ [*Using (1)*]

$= (1 - P(\overline{A})) + (1 - P(\overline{B})) - (1 - P(\overline{A}))(1 - P(\overline{B}))$

$= 1 - P(\overline{A}) + 1 - P(\overline{B}) - 1 + P(\overline{A}) + P(\overline{B}) - P(\overline{A})\, P(\overline{B})$

$= 1 - P(\overline{A})\, P(\overline{B}).$

Hence, the theorem.

Theorem III. *If A and B are two independent events associated with a random experiment, prove that the following pairs of events are also independent :*

(*i*) $\overline{A}$, B **(*C.B.S.E. Sample Paper 2019*)** (*ii*) A, $\overline{B}$ **(*C.B.S.E. 2017; Uttarakhand B. 2015; Kerala B. 2013*)**

(*iii*) $\overline{A}$, $\overline{B}$. **(*N.C.E.R.T.; Kashmir B. 2017; Bihar B. 2012*)**

Proof. (*i*) $P(B)\, P(\overline{A}) = P(B)(1 - P(A)) = P(B) - P(B)\, P(A)$

$= P(B) - P(B \cap A)$ [$\because$ *A and B are independent*]

$= P(B - (B \cap A)) = P(B - A) = P(B \cap \overline{A}).$

Hence, $\overline{A}$ and B are independent.

(*ii*) Interchanging A and B in part (*i*), we get :

$P(A)\, P(\overline{B}) = P(A)(1 - P(B)) = P(A) - P(A)\, P(B)$

$= P(A) - P(A \cap B)$ [$\because$ *A and B are independent*]

$= P(A - (A \cap B)) = P(A - B) = P(A \cap \overline{B}).$

Hence, A and $\overline{B}$ are independent.

(*iii*) Since A and B are independent,

$\therefore$ $\overline{A}$ and B are also independent [*Part (i)*]

$\Rightarrow$ $\overline{A}$ and $\overline{B}$ are also independent. [*Part (ii)*]

Theorem IV. (Multiplication Law). *If the events A and B defined on the sample space S of a random experiment are independent, then :*

$P(A/B) = P(A)$ and $P(B/A) = P(B)$.

Proof. Since A and B are independent events,

$\therefore$ P (A and B) = P (A). P (B) [*Def.*]

$$\therefore \quad P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A).P(B)}{P(B)} = P(A)$$

and $$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A).P(B)}{P(A)} = P(B).$$

Hence, the theorem.

Note. General Case. When A and B may or may not be independent, then P (A and B) = P (A) . P (B/A) = P (B) . P (A/B).

Frequently Asked Questions

FAQs

Example 1. Events A and B are such that :

$P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and P(not A or not B) = $\frac{1}{4}$.

State whether A and B are independent. *(H.P. B. 2018, 15)*

Solution. We have : $P(A) = \frac{1}{2}$ and $P(B) = \frac{7}{12}$.

$$\therefore \quad P(\bar{A}) = 1 - \frac{1}{2} = \frac{1}{2} \text{ and } P(\bar{B}) = 1 - \frac{7}{12} = \frac{5}{12}$$

and, $P(\bar{A} \text{ or } \bar{B}) = P(A \cup B)'$.

Example 2. A fair coin and unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event '3 on the die'. Check whether A and B are independent events or not. *(H.P. B. 2018)*

Solution. Here, A : Head appears on the coin

and B : 3 on the die.

$$\therefore \quad P(A) = \frac{1}{2} \text{ and } P(B) = \frac{1}{6}.$$

And, P (A ∩ B) = P (Head on coin and 3 on die)

$$= \frac{1}{12}.$$

Thus, $P(A \cap B) = P(A)\,P(B)$. $\left[\because \frac{1}{12} = \frac{1}{2} \times \frac{1}{6}\right]$

Hence, A and B are independent.

Example 3. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event 'the number is even' and B be the event 'the number is red'. Are A and B independent ? *(H.P. B. 2018)*

Solution. Here, A : the number is even

and B : the number is red.

$$\therefore \quad P(A) = \frac{3}{6} = \frac{1}{2} \text{ and } P(B) = \frac{3}{6} = \frac{1}{2}.$$

And, P (A ∩ B) = P (number is even and red)

$$= \frac{1}{6}.$$

Thus, $P(A \cap B) \neq P(A)\,P(B)$ $\left[\because \frac{1}{6} \neq \frac{1}{2} \times \frac{1}{2}\right]$

Hence, the events A and B are not independent.

Example 4. If A and B are independent events such that $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$, then find :

(*i*) P(A and B) and (*ii*) P(A or B). *(Meghalaya B. 2016)*

Solution. (*i*) Since A and B are independent events,

$$\therefore \quad P(A \text{ and } B) = P(A \cap B) = P(A)\,P(B) = \left(\frac{3}{10}\right)\left(\frac{2}{5}\right) = \frac{6}{50} = \frac{3}{25}.$$

(*ii*) $$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{10} + \frac{2}{5} - \frac{3}{25} \quad [\textit{Using part (i)}]$$

$$= \frac{15 + 20 - 6}{50} = \frac{29}{50}.$$

Example 5. One card is drawn from a pack of 52 cards so that each card is equally likely to be selected. Prove that the following cases are independent :

(a) A : "The card drawn is a spade"

B : "The card drawn is an ace." *(N.C.E.R.T.)*

(b) A : "The card drawn is black"

B : "The card drawn is a king." *(N.C.E.R.T.)*

Solution. (*a*) $P(A) = \frac{13}{52} = \frac{1}{4}$, $P(B) = \frac{4}{52} = \frac{1}{13}$

$$P(A \cap B) = \frac{1}{52} = \frac{1}{4} \cdot \frac{1}{13} = P(A).P(B).$$

Hence, the events A and B are independent.

(*b*) $P(A) = \frac{26}{52} = \frac{1}{2}, P(B) = \frac{4}{52} = \frac{1}{13}$

$$P(A \cap B) = \frac{2}{52} = \frac{1}{26}$$

$$= \frac{1}{2} \cdot \frac{1}{13} = P(A) \cdot P(B).$$

Hence, the events A and B are independent.

Example 6. An unbiased die is thrown twice. Let the event A be 'odd number on the first throw' and B the event 'odd number on the second throw'. Check the independence of the events A and B.

(N.C.E.R.T.; H.B. 2017; H.P.B. 2011)

Solution. If all the elementary events are considered to be equally likely, then :

$$P(A) = \frac{18}{36} = \frac{1}{2} \quad \text{and} \quad P(B) = \frac{18}{36} = \frac{1}{2}.$$

Also P (A ∩ B) = P (odd number on both throws)

$$= \frac{9}{36} = \frac{1}{4}. \qquad [(1, 1)\ (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)]$$

Now $P(A \cap B) = \frac{1}{4} = \frac{1}{2} \times \frac{1}{2} = P(A)P(B).$

Hence, A and B are independent events.

Example 7. Three coins are tossed simultaneously. Consider the events :

E : 'three heads or three tails'

F : 'at least two heads'

G : 'at most two heads'.

Of the pairs (E, F), (E, G) and (F, G), which are independent, which are dependent ? *(N.C.E.R.T.)*

Solution. Here sample space is given by :

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.

Also E = {HHH, TTT},

F = {HHH, HHT, HTH, THH}

and G = {HHT, HTH, THH, HTT, THT, TTH, TTT}.

∴ E ∩ F = {HHH}, E ∩ G = {TTT} and

F ∩ G = {HHT, HTH, THH}.

$$\therefore \quad P(E) = \frac{2}{8} = \frac{1}{4}, P(F) = \frac{4}{8} = \frac{1}{2}, \quad P(G) = \frac{7}{8},$$

$$P(E \cap F) = \frac{1}{8}, P(E \cap G) = \frac{1}{8}, \quad P(F \cap G) = \frac{3}{8}.$$

$$P(E).P(F) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8},$$

$$P(E).P(G) = \frac{1}{4} \times \frac{7}{8} = \frac{7}{32},$$

and $P(F).P(G) = \frac{1}{2} \times \frac{7}{8} = \frac{7}{16}.$

Clearly $P(E \cap F) = \frac{1}{8} = \frac{1}{4} \times \frac{1}{2} = P(E).P(F)$

$$P(E \cap G) = \frac{1}{8} \neq \frac{7}{32} = P(E).P(G)$$

and $P(F \cap G) = \frac{3}{8} \neq \frac{7}{16} = P(F).P(G).$

Hence, the events (E and F) are independent while the events (E and G) and (F and G) are dependent.

Example 8. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find if A and B are independent events.

(A.I.C.B.S.E. 2017)

Solution. Here S, sample space = {1, 2, 3, 4, 5, 6}.

The events are :

A = Number is even = {2, 4, 6}

and B = Number is red = {1, 2, 3}.

∴ A ∩ B = {2}.

$\therefore P(A) = \frac{3}{6} = \frac{1}{2}$ and $P(B) = \frac{3}{6} = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{6}.$

Now $P(A \cap B) = \frac{1}{6}$ and $P(A)\,P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$

Thus P(A ∩ B) ≠ P(A) P(B).

Hence, the events A and B are not independent.

Example 9. A husband and his wife appear for an interview for two posts. The probability of husband's selection is $\frac{1}{7}$ and that of wife's selection is $\frac{1}{5}$. What is the probability that only one of them is selected ?

Solution. Here $P(A) = \frac{1}{7}$ and $P(B) = \frac{1}{5}$,

where A ≡ Husband and B ≡ Wife.

$$\therefore \quad P(\overline{A}) = 1 - \frac{1}{7} = \frac{6}{7}$$

and $P(\overline{B}) = 1 - \frac{1}{5} = \frac{4}{5}.$

$\therefore$ Reqd. probability $= P(A\overline{B}) + P(\overline{A}B)$

$$= P(A)\,P(\overline{B}) + P(\overline{A})\,P(B)$$

[$\because$ *A & B are independent events ; so are A,* $\overline{B}$ *and* $\overline{A}$, *B*]

$$= \frac{1}{7}\times\frac{4}{5}+\frac{6}{7}\times\frac{1}{5}$$

$$= \frac{4}{35}+\frac{6}{35}=\frac{10}{35}=\frac{2}{7}.$$

Example 10. In a hockey match two teams A and B scored same number of goals upto the end of the game, so to decide the winner, the referee asked both the captains to throw a die alternately and decided that the team, whose captain gets a six first, will be declared the winner. If the captain of team A was asked to start, find their respective probabilities of winning the match.

(A.I.C.B.S.E. 2013)

Solution. P(A), probability of A's getting a six $= \frac{1}{6}$

$P(\overline{A})$, probability of A's not getting a six $= 1-\frac{1}{6}=\frac{5}{6}$.

Thus, $P(A) = \frac{1}{6}, P(\overline{A}) = \frac{5}{6}$.

Similarly, $P(B) = \frac{1}{6}, P(\overline{B}) = \frac{5}{6}$.

$\therefore$ P (A wins)

$$= \frac{1}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+........\text{to}\,\infty$$

$$= \frac{1}{6}\left[1+\left(\frac{5}{6}\right)^2+\left(\frac{5}{6}\right)^4+........\text{to}\,\infty\right] \qquad |\text{G.P.}$$

$$= \frac{1}{6}\left[\frac{1}{1-\left(\frac{5}{6}\right)^2}\right] \qquad \left[\because S_\infty = \frac{a}{1-r}\right]$$

$$= \frac{1}{6}.\frac{1}{1-\frac{25}{36}}=\frac{1}{6}.\frac{36}{11}=\frac{6}{11}.$$

And P (B wins) $= \frac{5}{6}\times\frac{1}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+........\text{to}\,\infty$

$$= \frac{5}{36}\left(1+\left(\frac{5}{6}\right)^2+\left(\frac{5}{6}\right)^4+........\text{to}\,\infty\right) \qquad |\text{G.P.}$$

$$= \frac{5}{36}\left[\frac{1}{1-\left(\frac{5}{6}\right)^2}\right] \qquad \left[\because S_\infty = \frac{a}{1-r}\right]$$

$$= \frac{5}{36}\times\frac{36}{11}=\frac{5}{11}.$$

Example 11. A speaks truth in 60% cases, while B in 90% cases. In what percent of cases are they likely to contradict each other in stating the same fact ?

(C.B.S.E. 2013)

Solution. Let the events be :

E : A speaks truth and

F : B speaks truth.

Then E and F are independent events such that :

$$P(E) = \frac{60}{100}=\frac{6}{10}=\frac{3}{5} \text{ and } P(F) = \frac{90}{100}=\frac{9}{10}.$$

$$\therefore P(\overline{E}) = 1-\frac{3}{5}=\frac{2}{5} \quad\text{and}\quad P(\overline{F}) = 1-\frac{9}{10}=\frac{1}{10}.$$

Now, P (A and B contradict each other)

$$= P(E\cap\overline{F})\cup P(\overline{E}\cap F)$$

$$= P(E\cap\overline{F})+P(\overline{E}\cap F)$$

[$\because$ $(E\cap\overline{F})$ *and* $(\overline{E}\cap F)$ *are mutually exclusive*]

$$= P(E)P(\overline{F})+P(\overline{E})P(F)$$

[$\because$ *E and F are independent, so E and* $\overline{F}$ *and* $\overline{E}$ *and F are also independent*]

$$= \left(\frac{3}{5}\right)\left(\frac{1}{10}\right)+\left(\frac{2}{5}\right)\left(\frac{9}{10}\right)$$

$$= \frac{3}{50}+\frac{18}{50}=\frac{21}{50}=\frac{42}{100}.$$

Hence, A and B contradict each other in 42% cases.

Example 12. The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events, 'A coming in time' and 'B coming in time' are independent, find the probability of only one of them coming to the school in time.

(A.I.C.B.S.E. 2013)

Solution. We have :

P(A) = Probability of student A coming to school in time = $\frac{3}{7}$

P(B) = Probability of student B coming to school in time = $\frac{5}{7}$.

$\therefore\ P(\overline{A}) = 1-\frac{3}{7}=\frac{4}{7}$ and $P(\overline{B}) = 1-\frac{5}{7}=\frac{2}{7}$.

$\therefore$ Probability that only one of the students coming to school in time

$= P(A\cap\overline{B})+P(\overline{A}\cap B)$

$= P(A)P(\overline{B})+P(\overline{A})P(B)$

[$\because$ *A and B are independent $\Rightarrow$ A and $\overline{B}$ and $\overline{A}$ and B are also independent*]

$= \left(\frac{3}{7}\right)\left(\frac{2}{7}\right)+\left(\frac{4}{7}\right)\left(\frac{5}{7}\right)=\frac{26}{49}$.

Example 13. A and B throw a pair of dice alternately. A wins the game if he gets a total of 7 and B wins if he gets a total of 10. If A starts the game, find the probability that B wins. *(C.B.S.E. 2016)*

Solution. Number of outcomes = 6 × 6 = 36.

Favourable outcomes for winning of A are :
{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)}.

$\therefore$ P(A wins) = $P(A) = \frac{6}{36}=\frac{1}{6}$

and P(A loses) = $P(\overline{A}) = 1-\frac{1}{6}=\frac{5}{6}$.

Favourable outcomes for winning of B are :
{(4, 6), (6, 4), (5, 5)}

$\therefore$ P(B wins) = $P(B) = \frac{3}{36}=\frac{1}{12}$

and P(B loses) = $P(\overline{B}) = 1-\frac{1}{12}=\frac{11}{12}$.

P(B wins) = $P(\overline{A})\,P(B) + P(\overline{A})\,P(\overline{B})\,P(\overline{A})\,P(B) + P(\overline{A})\,P(\overline{B})\,P(\overline{A})\,P(\overline{B})\,P(\overline{A})\,P(B) + \ldots$

$= \frac{5}{6}\times\frac{1}{12}+\frac{5}{6}\times\frac{11}{12}\times\frac{5}{6}\times\frac{1}{12}+\frac{5}{6}\times\frac{11}{12}\times\frac{5}{6}\times\frac{11}{12}\times\frac{5}{6}\times\frac{1}{12}$ | **G.P.**

$= \frac{\frac{5}{72}}{1-\frac{5}{6}\times\frac{11}{12}}=\frac{5}{72-55}=\frac{5}{17}$.

Example 14. If A and B are two independent events such that :

$P(\overline{A}\cap B)=\frac{2}{15}$ and $P(A\cap\overline{B})=\frac{1}{6}$, then find P(A) and P(B).

(C.B.S.E. 2015)

Solution. Since A and B are independent events,

$\therefore$ $\overline{A}$ and B, A and $\overline{B}$ are also independent events.

Now $P(\overline{A}\cap B) = \frac{2}{15}$ $\Rightarrow$ $P(\overline{A})P(B)=\frac{2}{15}$

$\Rightarrow$ $(1-P(A))P(B)=\frac{2}{15}$ $\Rightarrow$ $P(B)-P(A)P(B)=\frac{2}{15}$ ...(1)

And $P(A\cap\overline{B}) = \frac{1}{6}$ $\Rightarrow$ $P(A)P(\overline{B})=\frac{1}{6}$

$\Rightarrow$ $P(A)(1-P(B))=\frac{1}{6}$ $\Rightarrow$ $P(A)-P(A)P(B)=\frac{1}{6}$ ...(2)

Putting P(A) = x and P(B) = y in (1) and (2), we get

$y-xy = \frac{2}{15}$ ...(1)′

and $x-xy = \frac{1}{6}$ ...(2)′

Subtracting (2)′ find (1)′, $y-x=\frac{2}{15}-\frac{1}{6}$ $\Rightarrow$ $y-x=-\frac{1}{30}$

$\Rightarrow$ $y = x-\frac{1}{30}$ ...(3)

Putting in (1)′, $x-\frac{1}{30}-x\left(x-\frac{1}{30}\right)=\frac{2}{15}$

$\Rightarrow$ $x-\frac{1}{30}-x^2+\frac{x}{30} = \frac{2}{15}$

$\Rightarrow 30x - 1 - 30x^2 + x = 4 \Rightarrow 30x^2 - 31x + 5 = 0.$

Solving, $x = \frac{31\pm\sqrt{961-600}}{60}=\frac{31\pm19}{60}=\frac{5}{6},\frac{1}{5}$.

When $x = \frac{5}{6}$, then from (3), $y = \frac{5}{6}-\frac{1}{30}=\frac{24}{30}=\frac{4}{5}$.

When $x = \frac{1}{5}$, then from (3), $y = \frac{1}{5}-\frac{1}{30}=\frac{5}{30}=\frac{1}{6}$.

Hence, $P(A) = \frac{5}{6}$, $P(B) = \frac{4}{5}$ or $P(A) = \frac{1}{5}$, $P(B) = \frac{1}{6}$.

EXERCISE 13 (b)

Fast Track Answer Type Questions

FTATQ

1. (*a*) If A and B are independent events, then :
$P(A \cap B)$ = ... **(Fill in the blank)** *(Kashmir B. 2017)*

(*b*) Let E and F be events with :
$P(E) = \frac{3}{5}, P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$.
Are E and F independent ? *(N.C.E.R.T.; H.P.B. 2014, 11, 10 S)*

(*c*) Two events E and F are such that :
$P(E) = 0{\cdot}6$, $P(F) = 0{\cdot}2$ and $P(E \cup F) = 0{\cdot}68$.
Are E and F independent ? *(Kerala B. 2014)*

2. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event 'number is even' and B be the event 'number is red'. Are A and B independent ? *(N.C.E.R.T.)*

3. Let A and B be independent events with :
$P(A) = 0{\cdot}3$ and $P(B) = 0{\cdot}4$.
Find : (*i*) $P(A \cap B)$ *(Meghalaya B. 2017)*
(*ii*) $P(A \cup B)$ *(Meghalaya B. 2017)*
(*iii*) $P(A/B)$ (*iv*) $P(B/A)$. *(N.C.E.R.T.)*

4. Events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and P (not A or not B) = $\frac{1}{4}$. State whether A and B are independent. *(N.C.E.R.T.; H.P.B. 2015, Jammu B. 2013, 12)*

5. If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{1}{2}$, show that A and B are independent events. *(Nagaland B. 2016)*

6. (*a*) Given two independent events A, B such that $P(A) = 0{\cdot}3$, $P(B) = 0{\cdot}6$. Find :
(*i*) P (A and B) *(H.P.B. 2018; Rajasthan B. 2013)*
(*ii*) P (A and not B) *(Kashmir B. 2015; H.B. 2013; Rajasthan B. 2013 ; Jammu B. 2012)*
(*iii*) P (A or B) *(H.B. 2013; Jammu B. 2012)*
(*iv*) P (neither A nor B). *(N.C.E.R.T.; H.B. 2018, 16; H.P.B. 2015)*

(*b*) (*i*) If $P(A) = 0{\cdot}2$, $P(A \cup B) = 0{\cdot}6$, find P (B) *(P.B. 2010)*
(*ii*) If $P(A) = 0{\cdot}35$, $P(A \cup B) = 0{\cdot}60$, find P (B), where A and B are independent events. *(Bihar B. 2014; P.B. 2010)*

(*c*) (*i*) If A and B are two independent events such that :
$P(A \cup B) = 0{\cdot}60$ and $P(A) = 0{\cdot}2$,
then find the value of P (B). *(H.B. 2012)*

(*ii*) If $P(\overline{A}) = 0{\cdot}4$, $P(A \cup B) = 0{\cdot}7$ and A and B are given to be independent events, then find the value of P (B). *(Jammu B. 2013 ; Rajasthan B. 2012)*

(*iii*) Given that the events A and B are such that :
$P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$.
Find the value of '*p*' if the events are independent. *(P.B. 2016; Jammu B. 2013 ; Rajasthan B. 2012)*

(*d*) If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{5}$ and $P(A \cup B) = \frac{11}{30}$, then find P (A/B). *(P.B. 2010 S)*

7. Let A and B be two independent events such that :
$P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$, find :
(*i*) P(A or B)
(*ii*) P (neither A nor B) *(Jammu B. 2013)*
(*iii*) P(not A and not B). *(H.P.B. 2018)*

8. A coin is tossed thrice and all eight outcomes are assumed equally likely. In which of the following cases are the events A and B independent ?
(*a*) (*i*) A : "the first throw results in head"
B : "the last throw results in tail"
(*ii*) A : "the number of heads is two"
B : "the last throw results in head".

(*b*) A die is thrown. If E is the event 'the number appearing is a multiple of 3' and F be the event 'the number appearing is even', then find whether E and F are independent. *(Kashmir B. 2011 ; H.P.B. 2010 S)*

9. One card is drawn at random from a pack of well-shuffled deck of 52 cards. In which of the following cards are the events E and F independent ?
(*i*) E : the card drawn is spade"
F : the card drawn is an ace"
(*ii*) E : "the card drawn is black"
F : "the card drawn is a king" *(H.P.B. 2014)*
(*iii*) E : "the card drawn is a king or queen"
F : "the card drawn is a queen or jack." *(N.C.E.R.T.)*

10. The odds in favour of an event are 3 : 4. Find the probability of :
(*a*) Occurrence (*b*) Non-occurrence of the event. *(J. & K.B. 2010)*

Very Short Answer Type Questions

VSATQ

11. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

(N.C.E.R.T.; H.P.B. 2018, 16, 10 S, 10; Rajasthan B. 2013)

12. The probability of student A passing an examination is $\frac{3}{7}$ and of student B passing is $\frac{5}{7}$. Assuming the two events "A passes, B passes", as independent, find the probability of :

(*i*) only A passing the examination

(*ii*) only one of them passing the examination.

13. Given that two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the die is 4'. *(H.P.B. 2010 S)*

Short Answer Type Questions

SATQ

14. A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale otherwise it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale. *(N.C.E.R.T.)*

15. A die is thrown once. If E is the event "the number appearing is a multiple of 3" and F the event" the number appearing is even", then find whether E and F are independent ? *(N.C.E.R.T.)*

16. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that :

(*i*) both balls are red *(H.P.B. 2013, 12)*

(*ii*) first ball is black and second is red

(H.P.B. 2016, 13)

(*iii*) one of them is black and other is red.

(N.C.E.R.T.; H.P.B. 2016, 13, 10)

17. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are black ? *(N.C.E.R.T.)*

18. A bag contains 10 white and 15 black balls. Two balls are drawn out in succession without replacement. What is the probability that the first is white and the second is black ? *(Mizoram B. 2015)*

19. One bag contains 3 red and 5 black balls. Another bag contains 6 red and 4 black balls. A ball is transferred from first bag to the second bag and then a ball is drawn from the second bag. Find the probability that the ball drawn is red.

(C.B.S.E. Sample Paper 2019)

20. Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. What is the probability that first two cards are kings and the third card drawn is an ace. *(N.C.E.R.T.)*

21. A bag 'A' contains 6 white and 7 black balls while the other bag 'B' contains 4 white and 5 black balls. A ball is transferred from the bag 'A' to the bag 'B'. Then a ball is drawn from the bag 'B'. Find the probability that the ball drawn is white. *(Type : W. Bengal 2018; P.B. 2010)*

22. There are three urns A, B and C. Urn A contains 4 red balls and 3 black balls. Urn B contains 5 red balls and 4 black balls. Urn C contains 4 red balls and 4 black balls. One ball is drawn from each of these urns. What is the probability that the 3 balls drawn consist of 2 red balls and a black ball.

23. (*i*) P speaks truth in 70% of the cases and Q in 80% of the cases. In what percentage of cases are they likely to agree in stating the same fact ? *(C.B.S.E. 2013)*

(*ii*) A speaks truth in 75% of the cases, while B in 90% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact ?

(C.B.S.E. 2013)

24. (*i*) A and B toss a coin alternately till one of them tosses a head and wins the game. If A starts the game, find their respective probability of winning. *(H.B. 2017)*

(*ii*) A and B throw a coin alternately till one of them gets a '*head*' and wins the game. If A starts the game, find the probability of his winning at his third throw.

25. (*i*) A and B throw a die alternately till one of them gets a '6' and wins the game. Find their respective probability of winning if A starts first.

(N.C.E.R.T.; Assam B. 2018; H.B. 2017)

(*ii*) A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning if A starts first.

(H.P.B. 2017; A.I.C.B.S.E. 2016)

26. A, B and C in turn throw a die and one who gets a 6 first ,wins the game. A takes the first chance followed by B and C, and the process is repeated till one of them who gets a 6, wins the game. Find the probabilities of each for winning the game.

27. A, B and C play a game and chances of their winning it in an attempt are $\frac{2}{3}, \frac{1}{2}$ and $\frac{1}{4}$ respectively. A has the first chance, followed by B and then by C. The cycle is repeated till one of them wins the game. Find their respective chances of winning the game.

28. Three ships A, B, C sail from England to India. Odds in favour of their arriving safely are 2 : 5, 3 : 7, 6 : 1 respectively. Find the chance that they will arrive safely.

29. (*i*) A husband and wife appear in an interview for two vacancies in the same post. The probability of husband's selection is $\frac{1}{7}$ and that of wife's selection is $\frac{1}{5}$. Find the probability that both of them will be selected. ***(H.B. 2012)***

(*ii*) Amit and Nisha appear for an interview for two vacancies in a company. The probability of Amit's selection is $\frac{1}{5}$ and that of Nisha's selection is $\frac{1}{6}$. What is the probability that both of them are selected ? ***(Meghalaya B. 2015)***

30. The probabilities of A, B, C solving a question are $\frac{1}{3}, \frac{2}{7}$ and $\frac{3}{8}$ respectively. Find the probability that exactly one of them will solve it.

31. A bag contains 50 tickets numbered 1, 2, 3,........., 50 of which five are drawn at random and arranged in ascending order of the number appearing on the tickets $(x_1 < x_2 < x_3 < x_4 < x_5)$. Find the probability that $x_3 = 30$.

32. A bag contains 100 bolts and 300 nuts, 50% of each have been rusted. One item is chosen at random. Find the probability that chosen item is rusted or a bolt.

(J. & K.B. 2010)

Answers

1. (*a*) $P(A \cap B) = P(A)\,P(B)$ (*b*) No (*c*) Yes.

2. No.

3. (*i*) 0·12 (*ii*) 0·58 (*iii*) 0·3 (*iv*) 0·4.

4. Not independent.

6. (*a*) (*i*) 0·18 (*ii*) 0·12 (*iii*) 0·72 (*iv*) 0·28

(*b*) (*i*) 0·5 (*ii*) $\frac{5}{13}$

(*c*) (*i*) $\frac{1}{2}$ (*ii*) $\frac{1}{4}$ (*iii*) $p = \frac{1}{5}$ (*d*) $\frac{5}{6}$.

7. (*i*) $\frac{5}{8}$ (*ii*) $\frac{3}{8}$ (*iii*) $\frac{3}{8}$.

8. (*a*) (*i*) (*b*) Yes. **9.** (*i*), (*ii*). **10.** (*a*) $\frac{3}{7}$ (*b*) $\frac{4}{7}$.

11. $\frac{25}{102}$. **12.** (*i*) $\frac{6}{49}$ (*ii*) $\frac{26}{49}$.

13. $\frac{1}{18}$. **14.** $\frac{44}{91}$.

15. Yes.

16. (*i*) $\frac{16}{81}$ (*ii*) $\frac{20}{81}$ (*iii*) $\frac{40}{81}$.

17. $\frac{3}{7}$.

18. $\frac{1}{4}$. **19.** $\frac{51}{88}$. **20.** $\frac{2}{5525}$.

21. $\frac{29}{65}$. **22.** $\frac{17}{42}$. **23.** (*i*) 62%. (*ii*) 30%.

24. (*i*) $\frac{2}{3}, \frac{1}{3}$ (*ii*) $\frac{1}{32}$.

25. (*i*) $\frac{6}{11}, \frac{5}{11}$ (*ii*) $\frac{12}{23}, \frac{11}{23}$.

26. $\frac{36}{91}, \frac{30}{91}, \frac{25}{91}$. **27.** $\frac{16}{21}, \frac{4}{21}, \frac{1}{21}$.

28. $\frac{18}{245}$. **29.** (*i*) $\frac{1}{35}$ (*ii*) $\frac{1}{30}$.

30. $\frac{25}{56}$. **31.** $\frac{{}^{29}C_2 \times {}^{20}C_2}{{}^{50}C_5}$. **32.** $\frac{5}{8}$.

Hints to Selected Questions

10. $P(\text{Occurrence}) = \frac{3}{3+4} = \frac{3}{7}$

$P(\text{Non-occurrence}) = \frac{4}{3+4} = \frac{4}{7}$.

11. Reqd. probability $= \frac{26}{52} \times \frac{25}{51}$.

14. Reqd. probability $= \frac{{}^{12}C_3}{{}^{15}C_3}$.

17. Reqd. probability $= \frac{10}{15} \times \frac{9}{14}$.

21. As in Ex. 12.

23. (*i*) $P(P) = \frac{70}{100}$ and $P(Q) = \frac{80}{100}$.

Find $P(P \cap Q) + P(\overline{P} \cap \overline{Q})$.

30. Reqd. prob. $= P(A\overline{B}\overline{C}) + P(\overline{A}B\overline{C}) + P(\overline{A}\overline{B}C)$

$= P(A)\,P(\overline{B})\,P(\overline{C}) + P(\overline{A})\,P(B)\,P(\overline{C})$

$+ P(\overline{A})\,P(\overline{B})\,P(C)$.

31. Reqd. probability $= \frac{{}^{29}C_2 \times {}^{20}C_2}{{}^{50}C_5}$.

32. $P(E) = \frac{100}{400} = \frac{1}{4}$

$P(F) = \frac{50+150}{400} = \frac{1}{2}$

$P(E \cap F) = \frac{50}{400} = \frac{1}{8}$.

Reqd. Probability $= P(E \cup F)$

$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8}$.

13.9. PROBABILITY OF HAPPENING OF AT LEAST ONE OF INDEPENDENT EVENTS

Let $A_1, A_2, \ldots, A_r$ be r independent events.

Let $p_1, p_2, \ldots, p_r$ be their respective probabilities of happening.

Thus we have :

$$P(A_1) = p_1, \quad P(A_2) = p_2, \ldots, \quad P(A_r) = p_r$$

$$\Rightarrow \quad P(\overline{A}_1) = 1 - p_1,\ P(\overline{A}_2) = 1 - p_2, \ldots, P(\overline{A}_r) = 1 - p_r.$$

$\therefore$ Probability when no event happens $= (1 - p_1)(1 - p_2) \ldots (1 - p_r)$.

Hence, the probability when at least one event happens

$$= 1 - (1 - p_1)(1 - p_2) \ldots (1 - p_r) = 1 - P(\overline{A}_1)\,P(\overline{A}_2) \ldots P(\overline{A}_r).$$

Frequently Asked Questions

FAQs

Example 1. A pair of coins is tossed once. Find the probability of showing at least one head.

(J. & K. B. 2010)

Solution. S, Sample space = {HH, HT, TH, TT},

where H ≡ Head and T ≡ Tail.

$\therefore$ P (at least one head) $= \frac{3}{4}$.

Example 2. How many times must a fair coin be tossed so that the probability of getting at least one head is more than 80% ? *(A.I.C.B.S.E. 2015 ; C.B.S.E. 2012)*

Solution. When the coin is fair,

then $P(H) = P(T) = \frac{1}{2}$, where H ≡ Head and T ≡ Tail.

Let the coin be tossed n times.

$\therefore$ Reqd. probability $= 1 - P(\text{all tails}) = 1 - \left(\frac{1}{2}\right)^n$.

By the question, $1 - \frac{1}{2^n} > \frac{80}{100}$

$\Rightarrow \quad 1 - \frac{1}{2^n} > \frac{4}{5} \Rightarrow 1 - \frac{4}{5} > \frac{1}{2^n}$

$$\Rightarrow \quad \frac{1}{2^n} < \frac{1}{5} \Rightarrow n = 3.$$

Hence, the fair coin is to be tossed 3 times in order to get the desired situation.

Example 3. If A and B are two independent events, then the probability of occurrence of at least one of A or B is given by $1 - P(\overline{A})P(\overline{B})$.

(N.C.E.R.T.; Kerala B. 2013; Uttarakhand B. 2013 ; Jammu B. 2012)

Solution. P (at least one of A and B) = P (A ∪ B)

$$= P(A) + P(B) - P(A \cap B)$$
$$= P(A) + P(B) - P(A)P(B)$$
[∵ *A, B are independent*]
$$= P(A) + P(B)(1 - P(A))$$
$$= P(A) + P(B)\,P(\overline{A})$$
$$= (1 - P(\overline{A})) + P(B)P(\overline{A})$$
$$= 1 - P(\overline{A})\,[1 - (P(B))]$$
$$= 1 - P(\overline{A})\,P(\overline{B}).$$

Example 4. A and B appeared for an interview. The probability of their selection is $\frac{1}{2}$ and $\frac{1}{3}$ respectively. Find the probability that :

(i) both selected

(ii) at least one of them selected. *(P.B. 2013)*

Solution. Here $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$.

$$\therefore \quad P(\overline{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$$

and $$P(\overline{B}) = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}.$$

(*i*) P(Both selected) = $P(A \cap B) = P(A)P(B)$

$$= \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}.$$

(*ii*) P (At least one of A and B is selected)

$$= 1 - P(\overline{A})P(\overline{B})$$
$$= 1 - \frac{1}{2} \times \frac{2}{3} = 1 - \frac{1}{3} = \frac{2}{3}.$$

Example 5. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. What is the probability in the following cases ?

(i) that the problem is solved *(H.B. 2018)*

(ii) only one (exactly one) of them solves it correctly *(P.B. 2017)*

(iii) at least one of them may solve it.

Solution. Let A, B, C be three events when a problem in Mathematics is solved by three students.

Given : $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(C) = \frac{1}{4}.$

$$\therefore \quad P(\overline{A}) = 1 - \frac{1}{2} = \frac{1}{2}, \; P(\overline{B}) = 1 - \frac{1}{3} = \frac{2}{3}$$

and $$P(\overline{C}) = 1 - \frac{1}{4} = \frac{3}{4}.$$

(*i*) Probability that the problem is solved

= Probability that the problem is solved by at least one student

$$= 1 - P(\overline{A})\,P(\overline{B})\,P(\overline{C})$$
$$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}.$$

(*ii*) Probability that only one solves it correctly

$$= P(A \cap \overline{B} \cap \overline{C}) + P(\overline{A} \cap B \cap \overline{C}) + P(\overline{A} \cap \overline{B} \cap C)$$
$$= P(A)\,P(\overline{B})\,P(\overline{C}) + P(\overline{A})\,P(B)\,P(\overline{C}) + P(\overline{A})\,P(\overline{B})\,P(C)$$
$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4}$$
$$= \frac{1}{4} + \frac{1}{8} + \frac{1}{12} = \frac{11}{24}.$$

(*iii*) Probability that atleast one of them may solve the problem

$$= 1 - P(\overline{A})\,P(\overline{B})\,P(\overline{C})$$
$$= 1 - \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right) = 1 - \frac{1}{4} = \frac{3}{4}.$$

EXERCISE 13 (c)

Very Short Answer Type Questions

VSATQ

1. If A and B are two events such that :

$$P(A) = \frac{1}{4}, \; P(B) = \frac{1}{2} \text{ and } P(A \cap B) = \frac{1}{8},$$

find P (not A and not B). *(N.C.E.R.T.)*

2. Three coins are tossed once. Find the probability of getting :

(*i*) at least two tails

(*ii*) at most two heads.

3. The probability of A hitting a target is $\frac{4}{5}$ and that of B hitting it is $\frac{2}{3}$. They both fire at the target. Find the probability that :

(*i*) at least one of them will hit the target

(*ii*) only one of them will hit the target.

4. A dice is tossed thrice. Find the probability of getting an odd number at least once. *(H.P.B. 2016)*

5. (*i*) A problem in Mathematics is given to three students whose chances of solving it are :

$\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{5}$.

What is the probability that at least one of them may solve it ?

(*ii*) A problem is given to three students, whose chances of solving it are :

$\frac{1}{3}, \frac{1}{5}$ and $\frac{1}{6}$. *(P.B. 2012, 10)*

What is the probability that exactly one of them may solve it. *(P.B. 2009 S)*

6. A and B try to solve the problem independently. The probability that A solves the problem is $\frac{1}{2}$ and that B solves the problem is $\frac{1}{3}$. Find the probability that :

(*a*) Both of them solve the problem

(*b*) The problem is solved. *(Kerala B. 2013)*

7. A and B appeared for interview. The probability of their selection is :

$\frac{1}{3}$ and $\frac{1}{4}$ respectively.

Find the probability that :

(*i*) both selected

(*ii*) at least one of them selected. *(P.B. 2013)*

8. A husband and his wife appear for an interview for two posts. The probability of the husband's selection is $\frac{1}{7}$ and that of wife's selection is $\frac{1}{5}$. What is the probability that :

(*i*) both of them will be selected

(*ii*) only one of them will be selected

(*iii*) at least one of them will be selected

(*iv*) none of them will be selected ?

9. A can solve 90% of the problems given in a book and B can solve only 70% problems. What is the probability that at least one of them will solve the problem selected at random from the book ?

If 100 enemy planes, one after the other, are fired by the gunner, how many planes are likely to be shot down ?

10. An anti-air craft gun can take maximum four shots at an enemy plane, moving away from it. The probabilities of hitting the plane at first, second, third and fourth shot are 0·4, 0·3, 0·2 and 0·1 respectively. What is the probability that the gun hits the plane ?

11. In a lot of 12 microwave ovens, there are 3 defective units. A person has ordered 4 of these units and since each is identically packed. What is the probability that :

(*i*) all units are good

(*ii*) exactly 3 units are good.

Answers

1. $\frac{3}{8}$. **2.** (*i*) $\frac{1}{2}$ (*ii*) $\frac{7}{8}$.

3. (*i*) $\frac{14}{15}$ (*ii*) 0·4. **4.** $\frac{7}{8}$.

5. (*i*) $\frac{7}{10}$ (*ii*) $\frac{19}{45}$. **6.** (*a*) $\frac{1}{6}$ (*b*) $\frac{2}{3}$.

7. (*i*) $\frac{1}{12}$ (*ii*) $\frac{1}{2}$.

8. (*i*) $\frac{1}{35}$ (*ii*) $\frac{2}{7}$ (*iii*) $\frac{11}{35}$ (*iv*) $\frac{24}{35}$.

9. $\frac{97}{100}$.

Out of 100 enemy planes, 97 of planes will be shot down.

10. 0·6976.

11. (*i*) $\frac{14}{55}$ (*ii*) $\frac{28}{55}$.

Hints to Selected Questions

8. $P(H) = \frac{1}{7}, P(W) = \frac{1}{5}$.

(*i*) Reqd. Probability $= P(H) \times P(W)$

$$= \frac{1}{7} \times \frac{1}{5} = \frac{1}{35}.$$

10. Reqd. prob. $= 1 - P(\overline{I})\, P(\overline{II})\, P(\overline{III})\, P(\overline{IV})$.

11. (*i*) P (all are good)

$$= \frac{9}{12} \times \frac{8}{11} \times \frac{7}{10} \times \frac{6}{9} = \frac{14}{55}.$$

(*ii*) P (exactly 3 are good)

$$= 4\left(\frac{9}{12} \times \frac{8}{11} \times \frac{7}{10} \times \frac{3}{9}\right) = \frac{28}{55}.$$

SUB CHAPTER 13.2 Bayes' Theorem

13.10. INTRODUCTION

Thomas Bayes, an English philosopher, gave the theorem known as Bayes' theorem in 1763 AD. This theorem gives a unique method for calculating conditional probabilities.

13.11. PARTITION OF SAMPLE SPACE

Thomas Bayes

The set of events $E_1, E_2, ..., E_n$ is said to represent a partition of the sample space S if :

(*a*) $E_i \cap E_j = \phi, i \neq j ; i, j = 1, 2, 3, ... , n$

(*b*) $E_1 \cup E_2 \cup ... \cup E_n = S$

and (*c*) $P(A_i) > 0$ for all $i = 1, 2, ..., n$.

Thus the events $E_1, E_2, ..., E_n$ represent a partition of the sample space S if they are disjoint, pair-wise exhaustive and have non-zero probabilities.

13.12. LAW OF TOTAL PROBABILITY AND BAYES' THEOREM

(*i*) Let S be the sample space and let E_1, E_2 be two mutually exclusive events (See Fig.). Let A be another event that occurs with E_1 or E_2.

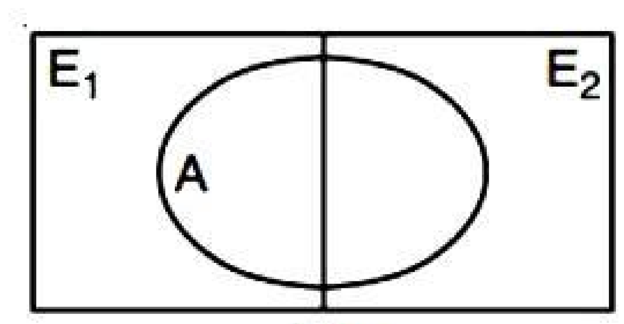

Fig.

Now $$S = E_1 \cup E_2$$

$$S \cap A = (E_1 \cup E_2) \cap A = (E_1 \cap A) \cup (E_2 \cap A).$$

$E_1 \cap A$ and $E_2 \cap A$ are mutually exclusive.

Hence, $$P(A) = P(S \cap A) = P[(E_1 \cap A) \cup (E_2 \cap A)]$$
$$= P(E_1 \cap A) + P(E_2 \cap A)$$
$$= P(E_1). P(A/E_1) + P(E_2). P(A/E_2).$$

This is a particular case of Law of Total Probability.

Law of Total Probability.

If $E_1, E_2, ..., E_n$ are mutually exclusive events and A is any event that occurs with E_1 or E_2 ...... or E_n, then :

$$P(A) = P(E_1). P(A/E_1) + P(E_2) P(A/E_2) + ... + P(E_n) . P(A/E_n)$$

$$= \sum_{j=1}^{n} P(E_j) P(A/E_j).$$

(*ii*) From the Fig., we deduce that :

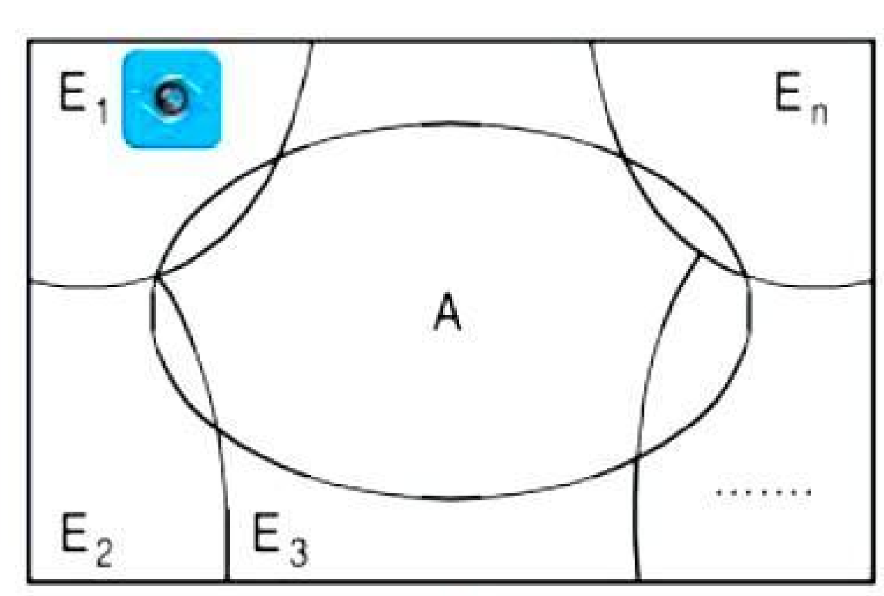

Fig.

$$P(E_1 \cap A) = P(E_1). P(A/E_1).$$

Interchanging E_1 and A, we have : $P(A \cap E_1) = P(A) . P(E_1/A)$.

Since $$E_1 \cap A = A \cap E_1,$$

$\therefore$ $$P(A) . P(E_1/A) = P(E_1) . P(A/E_1)$$

$\therefore$ $$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(A)}$$

$$= \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) P(A/E_1) + P(E_2) P(A/E_2)}.$$

This is a particular case of **Bayes' Theorem*,** which can be extended as :

*This theorem is named after **Reverend Thomas Bayes** (1720 - 61) and was published in 1763 A.D.

Bayes' Theorem

If $E_1, E_2, ..., E_n$ are mutually exclusive and exhaustive events and A is any event that occurs with $E_1, E_2,....., E_n$, then :

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + ... + P(E_n)\,P(A/E_n)}.$$

Hence,
$$P(E_i/A) = \frac{P(E_i)\,P(A/E_i)}{\sum P(E_i)\,P(A/E_i)} \text{ for } i = 1, 2, ... n.$$

Definition

(1) **Bayes' Theorem** enables us to connect the conditional probability of E_i given A, with the conditional probability of A, given E_i and the probability of E_i, themselves.

(2) The events $E_1, E_2, ..., E_n$ are called **hypothesis**.

(3) The probabilities P (E_i), $i = 1, 2, ..., n$ are called **'priori probabilities'.**

(4) The probabilities P (E_i/A), $i = 1, 2, ..., n$ are called **'posteriori probabilities'.**

(5) The probabilities P (A/E_i), $i = 1, 2, ..., n$ are called **'likelihoods'.**

KEY POINT

1. Bayes' Theorem can be applied if :
   (*i*) **the events E_i, i = 1, 2,......, n are mutually exclusive**
   (*ii*) **the sum of their probabilities is equal to 1.**
2. In a problem, if we read **'is found to be',** then it is likely to be a problem on Bayes' Theorem.

Frequently Asked Questions

FAQs

Example 1. In a set of 10 coins, 2 coins with heads on both sides. A coin is selected at random from this set and tossed five times. Of all the five times, the result was head, find the probability that the selected coin had heads on both sides. *(A. I. C.B.S.E. 2015)*

Solution. Let the events be :

E_1 : Selecting a coin with two heads

E_2 : Selecting a normal coin and

A : The coin falls head all the times.

Since E_1 and E_2 are mutually exclusive and by the data given in the problem, we have :

$$P(E_1) = \frac{2}{10} = \frac{1}{5}, \quad P(E_2) = \frac{8}{10} = \frac{4}{5}; \; P(A/E_1) = 1$$

and
$$P(A/E_2) = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{32}.$$

Now
$$P(A) = P(A \cap E_1) + P(A \cap E_2)$$
$$= P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)$$
$$= \frac{1}{5}\times 1 + \frac{4}{5}\times\frac{1}{32}$$
$$= \frac{1}{5} + \frac{1}{40} = \frac{8+1}{40} = \frac{9}{40}.$$

Example 2. A person has undertaken a construction job. The probabilities are 0·65 that there will be strikes 0·80, that the construction job will be completed on time if there is no strike and 0·32 that the construction job will be completed on time if there is a strike. Determine the probability that the construction job will be completed on time. *(N.C.E.R.T.)*

Solution. Let the events be as below :

A : Construction job will be completed on time

and B : There is strike.

We are to find P (A).

We have : P (B) = 0·65.

$$P(\text{no strike}) = P(\overline{B}) = 1 - P(B) = 1 - 0{\cdot}65 = 0{\cdot}35.$$

$$\therefore \quad P(A/B) = 0{\cdot}32, \; P(A/\overline{B}) = 0{\cdot}80.$$

Since the events B and $\overline{B}$ form a sample space S,

$$\therefore \quad P(A) = P(B)\,P(A/B) + P(\overline{B})\,P(A/\overline{B})$$

[*Law of Total Probability*]

$$= (0{\cdot}65)(0{\cdot}32) + (0{\cdot}35)(0{\cdot}80)$$
$$= 0{\cdot}208 + 0{\cdot}280 = 0{\cdot}488.$$

Hence, the probability that the construction job will be completed on time = 0·488.

Example 3. There are two bags I and II. Bag I contains 4 white and 3 red balls while another Bag II contains 3 white and 7 red balls. One ball is drawn at random from one of the bags and it is found to be white. Find the probability that it was drawn from bag I. *(C.B.S.E. 2010)*

Solution. Let the events E_1, E_2 and A be as below :

E_1 : Ball drawn from bag I

E_2 : Ball drawn from bag II and

A : Ball drawn is white.

We have to find $P(E_1/A)$.

Since both the bags are equally likely to be selected,

$$\therefore \quad P(E_1) = P(E_2) = \frac{1}{2}.$$

Also $P(A/E_1) = \frac{4}{7}$ and $P(A/E_2) = \frac{3}{10}$.

By *Bayes' Theorem,*

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{4}{7}\right)}{\left(\frac{1}{2}\right)\left(\frac{4}{7}\right)+\left(\frac{1}{2}\right)\left(\frac{3}{10}\right)} = \frac{\frac{2}{7}}{\frac{2}{7}+\frac{3}{20}}$$

$$= \frac{2}{7}\times\frac{140}{40+21} = \frac{40}{61}.$$

Example 4. A bag X contains 4 white balls and 2 black balls, while another bag Y contains 3 white balls and 3 black balls. Two balls are drawn (without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag Y. *(A.I.C.B.S.E. 2016)*

Solution. Let the events E_1, E_2 and A be as below :

E_1 : Balls drawn from bag X

E_2 : Balls drawn from bag Y and

A : Balls drawn – one white and one black.

Since both the bags are equally likely to be selected,

$$\therefore \quad P(E_1) = P(E_2) = \frac{1}{2}.$$

Also $$P(A/E_1) = \frac{4}{6}\times\frac{2}{5} = \frac{4}{15}$$

and $$P(A/E_2) = \frac{3}{6}\times\frac{3}{5} = \frac{3}{10}.$$

By *Bayes' Theorem*, $P(E_2/A)$

$$= \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{3}{10}\right)}{\left(\frac{1}{2}\right)\left(\frac{4}{15}\right)+\left(\frac{1}{2}\right)\left(\frac{3}{10}\right)}$$

$$= \frac{3}{20}\times\frac{60}{8+9} = \frac{9}{17}.$$

Example 5. Assume that the chances of a patient having a heart attack is 40%. Assume that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces the chance by 25%. At a time a patient can choose any one of two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga. *(N.C.E.R.T ; Assam B. 2015; C.B.S.E. 2013)*

Solution. Let the events be :

E_1 : Patient follows meditation and yoga

E_2 : Patient uses certain drug and

A : Patient suffers a heart attack.

$$\therefore \quad P(E_1) = P(E_2) = \frac{1}{2}.$$

Also $$P(A/E_1) = \frac{40}{100}\left(1-\frac{30}{100}\right) = \frac{28}{100}$$

and $$P(A/E_2) = \frac{40}{100}\left(1-\frac{25}{100}\right) = \frac{30}{100}.$$

By *Bayes' Theorem,*

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{28}{100}\right)}{\left(\frac{1}{2}\right)\left(\frac{28}{100}\right)+\left(\frac{1}{2}\right)\left(\frac{30}{100}\right)}$$

$$= \frac{28}{28+30} = \frac{28}{58} = \frac{14}{29}.$$

Example 6. An insurance company insured 2000 cyclists, 4000 scooter drivers and 6000 motorbike drivers. The probability of an accident involving a cyclist, scooter driver and a motorbike driver is 0·01, 0·03, 0·15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver. *(N.C.E.R.T.)*

Solution. Let the events be as below :

E_1 : Insured person is a cyclist

E_2 : Insured person is a scooter driver

E_3 : Insured person is a motorbike driver and

A : Insured person meets with an accident.

$$\therefore \quad P(E_1) = \frac{2000}{2000+4000+6000} = \frac{2}{2+4+6} = \frac{2}{12} = \frac{1}{6};$$

$$P(E_2) = \frac{4000}{2000+4000+6000} = \frac{4}{2+4+6} = \frac{4}{12} = \frac{1}{3}$$

and
$$P(E_3) = \frac{6000}{2000+4000+6000} = \frac{6}{2+4+6} = \frac{6}{12} = \frac{1}{2}.$$

Also $P(A/E_1) = 0{\cdot}01 = \frac{1}{100}$; $P(A/E_2) = 0{\cdot}03 = \frac{3}{100}$;

$$P(A/E_3) = 0{\cdot}15 = \frac{15}{100}.$$

By *Bayes' Theorem,*

$P(E_2/A)$

$$= \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$$

$$= \frac{\left(\frac{1}{3}\right)\left(\frac{3}{100}\right)}{\left(\frac{1}{6}\right)\left(\frac{1}{100}\right)+\left(\frac{1}{3}\right)\left(\frac{3}{100}\right)+\left(\frac{1}{2}\right)\left(\frac{15}{100}\right)}$$

$$= \frac{1}{\frac{1}{6}+1+\frac{15}{2}} = \frac{6}{1+6+45} = \frac{6}{52} = \frac{3}{26}.$$

Example 7. A card from a pack of 52 playing cards is lost. From the remaining cards of the pack, three cards are drawn at random (without replacement) and are found to be all spades. Find the probability that the lost card being a spade. *(C.B.S.E. 2014)*

Solution. Let the events be as below :

E_1 : Missing card is a diamond

E_2 : Missing card is a spade

E_3 : Missing card is a club

E_4 : Missing card is a heart

and A : Three spade cards are drawn.

Now $P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{13}{52} = \frac{1}{4}.$

Also $P(A/E_2) = \frac{12}{51} \times \frac{11}{50} \times \frac{10}{49}$

and $P(A/E_1) = P(A/E_3) = P(A/E_4) = \frac{13}{51} \times \frac{12}{50} \times \frac{11}{49}.$

By *Bayes' Theorem,*

$$P(E_2/A) = \frac{P(E_2)\,P(A/E_2)}{\sum_{i=1}^{4} P(E_i)\,P(A/E_i)}$$

$$= \frac{\frac{1}{4} \times \frac{12}{51} \times \frac{11}{50} \times \frac{10}{49}}{\frac{1}{4}\left[\frac{13\times12\times11+12\times11\times10+13\times12\times11+13\times12\times11}{51\times50\times49}\right]}$$

$$= \frac{12\times11\times10}{12\times11\times10+3(13\times12\times11)} = \frac{10}{10+3(13)}$$

$$= \frac{10}{10+39} = \frac{10}{49}.$$

Example 8. Three persons A, B and C apply for a job of manager in a Private Company. Chances of their selection (A, B and C) are in the ratio 1 : 2 : 4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0·8, 0·5 and 0·3 respectively. If the change does not take place, find the probability that it is due to the appointment of C.

(C.B.S.E. 2016)

Soluction. Let 'E' be the event when change takes place.

Now, we have :

$$P(A) = \frac{1}{7},\ P(B) = \frac{2}{7} \text{ and } P(C) = \frac{4}{7}$$

and $P(E/A) = 1 - 0{\cdot}8 = 0{\cdot}2$, $P(E/B) = 1 - 0{\cdot}5 = 0{\cdot}5$ and $P(E/C) = 1 - 0{\cdot}3 = 0{\cdot}7$.

By *Bayes' Theorem,*

$$P(C/E) = \frac{P(C)\,P(E/C)}{P(A)\,P(E/A) + P(B)\,P(E/B) + P(C)\,P(E/C)}$$

$$= \frac{\left(\frac{4}{7}\right)(0{\cdot}7)}{\left(\frac{1}{7}\right)(0{\cdot}2) + \left(\frac{2}{7}\right)(0{\cdot}5) + \left(\frac{4}{7}\right)(0{\cdot}7)}$$

$$= \frac{2{\cdot}8}{0{\cdot}2 + 1 + 2{\cdot}8} = \frac{2{\cdot}8}{4} = 0{\cdot}7,$$

which is the required probability.

Example 9. Often it is taken that a truthful person commands more respect in the seciety. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. Do you also agree that the value of truthfulness leads to more respect in the society ? *(C.B.S.E. 2017)*

Solution. Let the events be :

E_1 : 6 occurs

E_2 : 6 does not occur and

A : The man reports that it is '6'.

Now $P(E_1) = \frac{1}{6}$ and $P(E_2) = \frac{5}{6}$.

And $P(A/E_1)$ = Probability that the man speaks truth = $\frac{1}{5}$

and $P(A/E_2) = 1 - \frac{4}{5} = \frac{1}{5}$.

By *Bayes' Theorem,*

$$P(E_1/A) = \frac{P(E_1).\,P(A/E_1)}{P(E_1).\,P(A/E_1) + P(E_2).\,P(A/E_2)}$$

$$= \frac{\frac{1}{6} \times \frac{4}{5}}{\frac{1}{6} \times \frac{4}{5} + \frac{5}{6} \times \frac{1}{5}}$$

$$= \frac{4}{4+5} = \frac{4}{9}.$$

Yes, we agree that the value of truthfulness leads to more respect in the society.

Example 10. In a bolt factory, machines A, B and C manufacture respectively 25%, 35%, 40% of the total. Of their output 5, 4 and 2% are defective. A bolt is drawn at random from the product.

(i) What is the probability that the bolt drawn is defective ?

(ii) If the bolt drawn is found to be defective, find the probability that it is a product of machine :

***(a)* B** *(N.C.E.R.T.; H.B. 2016; C.B.S.E. 2010 C)*

***(b)* C ?** *(Meghalaya B. 2017)*

Solution. Here $P(A) = \frac{25}{100}$; $P(B) = \frac{35}{100}$

and $P(C) = \frac{40}{100}$.

Also $P(D/A) = \frac{5}{100}$, $P(D/B) = \frac{4}{100}$ and $P(D/C) = \frac{2}{100}$.

and $P(D/C) = \frac{2}{100}$,

where D denotes defective bolt.

(i) $P(D) = P(A)\,P(D/A) + P(B)\,P(D/B) + P(C)\,P(D/C)$

$$= \frac{25}{100}\left(\frac{5}{100}\right) + \frac{35}{100}\left(\frac{4}{100}\right) + \frac{40}{100}\left(\frac{2}{100}\right)$$

$$= \frac{1}{10000}(125 + 140 + 80) = \frac{1}{10000}(345)$$

$$= 0{\cdot}0345.$$

(ii) (a) By *Bayes' Theorem,*

$$P(B/D) = \frac{P(B)\,P(D/B)}{P(A)\,P(D/A) + P(B)\,P(D/B) + P(C)\,P(D/C)}$$

$$= \frac{\frac{35}{100}\left(\frac{4}{100}\right)}{\frac{25}{100}\left(\frac{5}{100}\right) + \frac{35}{100}\left(\frac{4}{100}\right) + \frac{40}{100}\left(\frac{2}{100}\right)}$$

$$= \frac{140}{125 + 140 + 80} = \frac{140}{345} = \frac{28}{69}.$$

(b) By *Bayes' Theorem,*

$$P(C/D) = \frac{P(C)\,P(D/C)}{P(A)\,P(D/A) + P(B)\,P(D/B) + P(C)\,P(D/C)}$$

$$= \frac{\frac{40}{100}\left(\frac{2}{100}\right)}{\frac{25}{100}\left(\frac{5}{100}\right) + \frac{35}{100}\left(\frac{4}{100}\right) + \frac{40}{100}\left(\frac{2}{100}\right)}$$

$$= \frac{80}{125 + 140 + 80} = \frac{80}{345} = \frac{16}{69}.$$

Example 11. In answering a question on MCQ test with 4 choices per question, a student knows the answer, guesses or copies the answer. Let $\frac{1}{2}$ be the probability that he knows the answer, $\frac{1}{4}$ be the probability that he guesses and $\frac{1}{4}$ that he copies it. Assuming that a student, who copies the answer, will be correct with the probability $\frac{3}{4}$, what is the probability that the student knows the answer, given that he answered it correctly ?

Solution. Let the events be as below :

E_1 : He knows the answer

E_2 : He guesses

E_3 : He copies

and A : He answered correctly.

Then $P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{4}$ and $P(E_3) = \frac{1}{4}$.

Also $P(A/E_1) = 1$,

$$P(A/E_2) = \frac{1}{4} \text{ and } P(A/E_3) = \frac{3}{4}.$$

By *Bayes' Theorem,*

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$= \frac{\frac{1}{2}\times 1}{1\times\frac{1}{2}+\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{3}{4}}$$

$$= \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{16}+\frac{3}{16}} = \frac{1}{2}\times\frac{16}{8+1+3}$$

$$= \frac{8}{12} = \frac{2}{3}.$$

Example 12. Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 and 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die ? *(C.B.S.E. 2018)*

Solution. Let the events be as :

E_1 : The girl gets 1 or 2 on the roll

E_2 : The girl gets 3, 4, 5 or 6 on the roll

and A : She obtained exactly 1 tail.

$$\therefore \quad P(E_1) = \frac{2}{6} = \frac{1}{3} \text{ and } P(E_2) = \frac{4}{6} = \frac{2}{3}.$$

If the girl tossed a coin 3 times and exactly 1 tail shown, then

{HTH, HHT, THH} = 3.

$$\therefore \quad P(A/E_1) = \frac{3}{8}.$$

Let A be the event that the girl obtained exactly one tail. If the girl tossed a coin only once and exactly 1 tail.

$$\therefore \quad P(A/E_2) = \frac{1}{2}.$$

By *Bayes' Theorem,*

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$

$$= \frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}}$$

$$= \frac{1/3}{\frac{1}{8}+\frac{1}{3}} = \frac{1}{3}\times\frac{24}{11} = \frac{8}{11}.$$

Example 13. A girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin two times and notes the number of heads obtained. If she obtained exactly two heads, what is the probability that she throws 1, 2, 3 or 4 with the die. *(C.B.S.E. 2012)*

Solution. Let the events be :

E_1 : The girl gets 1, 2, 3 and 4 on the die

E_2 : The girl gets 5 or 6 on the die and

A : Exactly two heads show up.

$$\therefore \quad P(E_1) = \frac{4}{6} = \frac{2}{3},\ P(E_2) = \frac{2}{6} = \frac{1}{3}.$$

Also $P(A/E_1)$ = P (Exactly two heads show up when a coin is tossed two times)

= P({HH})

$= \frac{1}{4}$

$P(E_2)$ = P (exactly two heads show up when a coin is tossed 3 times)

$= P(\{HHT, HTH, THH\}) = \frac{3}{8}.$

By *Bayes' Theorem,*

$$P(A/E_1) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$

$$= \frac{\left(\frac{2}{3}\right)\left(\frac{1}{4}\right)}{\left(\frac{2}{3}\right)\left(\frac{1}{4}\right)+\left(\frac{1}{3}\right)\left(\frac{3}{8}\right)}$$

$$= \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{8}}$$

$$= \frac{1}{6}\times\frac{24}{4+3} = \frac{4}{7}.$$

Example 14. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. Two balls are transferred at random from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred balls were both black. ***(C.B.S.E. 2012)***

Solution. Let the events be as below :

E_1 : 2 red balls are transferred from Bag I to Bag II

E_2 : 2 black balls are transferred from Bag I to Bag II

E_3 : 1 red and 1 black balls are transferred from Bag I to Bag II

and A : 1 red ball is drawn from Bag II.

$$\therefore \quad P(E_1) = \frac{{}^3C_2}{{}^7C_2} = \frac{3\times2}{7\times6} = \frac{1}{7}$$

$$P(E_2) = \frac{{}^4C_2}{{}^7C_2} = \frac{4\times3}{7\times6} = \frac{2}{7}$$

$$P(E_3) = \frac{{}^3C_1\times{}^4C_1}{{}^7C_2} = \frac{3\times4}{\frac{7\times6}{1\times2}} = \frac{3\times4}{7\times3} = \frac{4}{7}$$

and $P(A/E_1) = \frac{6}{11}$, $P(A/E_2) = \frac{4}{11}$ and $P(A/E_3) = \frac{5}{11}$.

By *Bayes' Theorem,*

$P(A/E_2)$

$$= \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$= \frac{\left(\frac{2}{7}\right)\left(\frac{4}{11}\right)}{\left(\frac{1}{7}\right)\left(\frac{6}{11}\right)+\left(\frac{2}{7}\right)\left(\frac{4}{11}\right)+\left(\frac{4}{7}\right)\left(\frac{5}{11}\right)}$$

$$= \frac{8}{6+8+20} = \frac{8}{34} = \frac{4}{17}.$$

Example 15. There are three coins. One is a two-headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tails 40% of the times. One of the three coins is chosen at random and tossed, and it shows head. What is the probability that it was the two-headed coin ? ***(A.I.C.B.S.E. 2014)***

Solution. Let the events be :

E_1 : coin is two headed

E_2 : coin is biased (heads 75%)

E_3 : coin is biased (tails 40%)

and A : coin shows up head.

$$\therefore \quad P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}.$$

Also $P(A/E_1) = 1$, $P(A/E_2) = \frac{75}{100} = \frac{3}{4}$, $P(E_3) = \frac{60}{100} = \frac{3}{5}$.

[$\because$ *40% tails means 60% heads*]

By Bayes' Theorem,

$P(E_1/A) =$

$$\frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1)+P(E_2)\,P(A/E_2)+P(E_3)\,P(A/E_3)}$$

$$= \frac{\frac{1}{3}\times1}{\frac{1}{3}\times1+\frac{1}{3}\times\frac{3}{4}+\frac{1}{3}\times\frac{3}{5}}$$

$$= \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{4}+\frac{1}{5}} = \frac{1}{3}\times\frac{60}{20+15+12} = \frac{20}{47}.$$

Example 16. Suppose that reliability of a HIV test is specified as follows :

Of people having HIV, 90% of the test detected but 10% go undetected. Of people free of HIV, 99% of the test are judged HIV –ive but 1% are diagnosed as showing HIV + ve. From a large population of which only 0·1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV +ve. What is the probability that the person actually has HIV ? ***(N.C.E.R.T.)***

Solution. Let the events be as below :

E : Person selected is actually having HIV

and A : Person's HIV test is diagnosed as + ve.

We have to find D (E/A).

Now $\overline{E}$ denotes the event when the person selected is actually not having HIV.

Then $\{E, \overline{E}\}$ is a partition of sample space of all people of the population.

We have : $P(E) = 0{\cdot}1\% = \frac{0{\cdot}1}{100} = 0{\cdot}001$

$$P(\overline{E}) = 1 - P(E) = 1 - 0{\cdot}001 = 0{\cdot}999$$

and $P(A/E) =$ P (Person tested as HIV + ve given that he/she is having HIV actually)

$$= 90\% = \frac{90}{100} = 0{\cdot}9$$

and $P(A/\overline{E}) =$ P (Person tested as HIV + ve given that he/she is not having HIV actually)

$$= 1\% = \frac{1}{100} = 0{\cdot}01.$$

By *Bayes' Theorem,*

$$P(E/A) = \frac{P(E)\,P(A/E)}{P(E)\,P(A/E) + P(\overline{E})\,P(A/\overline{E})}$$

$$= \frac{0{\cdot}001 \times 0{\cdot}9}{0{\cdot}001 \times 0{\cdot}9 + 0{\cdot}999 \times 0{\cdot}01}$$

$$= \frac{90}{1089} = 0{\cdot}083 \text{ app.}$$

Example 17. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance ? Is regularity required only in school ? Justify your answer. *(A.I.C.B.S.E. 2017)*

Solution. Let the events E_1, E_2 and A be as below :

E_1 : Students having 100% attendance

E_2 : Students which are irregular

and A : Students getting A-grade.

We have $P(E_1) = \frac{30}{100} = \frac{3}{10}$ and $P(E_2) = \frac{70}{100} = \frac{7}{10}$.

Also $P(A/E_1) = \frac{70}{100} = \frac{7}{10}$ and $P(A/E_2) = \frac{10}{100} = \frac{1}{10}$.

By *Bayes' Theorem,*

$$P(E_1/A) = \frac{P(E_1).\,P(A/E_1)}{P(E_1).\,P(A/E_1) + P(E_2)\,P(A/E_2)}$$

$$= \frac{\frac{3}{10} \times \frac{7}{10}}{\frac{3}{10} \times \frac{7}{10} + \frac{7}{10} \times \frac{1}{10}}$$

$$= \frac{21}{21+7} = \frac{21}{28} = \frac{3}{4}.$$

Irregularity is required not only in school and should be everywhere in office attendance; etc.

EXERCISE 13 (d)

Short Answer Type Questions

SATQ

1. (*i*) A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond. *(N.C.E.R.T.; Kerala B. 2016)*

(*ii*) A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to be both clubs. Find the probability of the lost card being of clubs. *(C.B.S.E. 2010)*

2. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. *(Uttarakhand B. 2013 ; H.B. 2012)*

3. Two bags contain :

(*i*) 4 red and 4 black, 2 red and 6 black balls

(*ii*) 6 red and 3 black, 5 red and 5 black balls

(*iii*) 6 red and 4 black, 3 red and 3 black balls.

One ball is drawn at random from one of the bags and found to be red. Find the probability that it was drawn from the second bag. *(P.B. 2011)*

4. (*i*) Bag I contains 3 red and 4 black balls while another bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from bag II. *(N.C.E.R.T.; H.P.B. 2013 S; C.B.S.E. 2011)*

(*ii*) There are two bags I and II. Bag I contains 4 white and 3 red balls and bag II contains 6 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag II. *(Assam B. 2017; P.B. 2014 S)*

5. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bag is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag. ***(N.C.E.R.T.; Assam B. 2018; H.P.B. 2016, 13; Jammu B. 2016; P.B. 2014 S; C.B.S.E. 2009 C)***

6. (*a*) (*i*) Bag I contains 3 red and 4 black balls, Bag II contains 5 red and 6 black balls. One bag is chosen at random and a ball is drawn which is found to be red. Find the probability that it was drawn from (I) Bag I, (II) Bag I. ***(P.B. 2015)***

(*ii*) Bag I contains 3 red and 5 white balls and bag II contains 4 red and 6 white balls. One of the bags is selected at random and a ball is drawn from it. The ball is found to be red. Find the probability that ball is drawn from Bag II. ***(P.B. 2016)***

(*b*) Bag I contains 4 black and 6 red balls, bag II contains 7 black and 3 red balls and bag III contains 5 black and 5 red balls. One bag is chosen at random and a ball is drawn from it which is found to be red. Find the probability that it was drawn from bag II. ***(P. B. 2017)***

7. Given three identical boxes I, II and III, each containing two coins. In box I both coins are gold coins, in box II both are silver coins and in the third box III there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold ? ***(N.C.E.R.T.)***

8. In a tape recorder factory three machines A, B and C produced 50%, 30% and 20% of total production. The percentage of the defective output of these machines are 3%, 4% and 5% respectively. A tape recorder is selected randomly and found to be defective, find the probability that it is produced by machine A. ***(P.B. 2013)***

9. A company has two plants to manufacture motor cycles. Plant 1 manufactures 70% of the motor cycles and Plant 2 manufactures 30%. At Plant 1, 80% of the motor cycles are rated of standard quality and at Plant 2, 90% of the motor cycles are rated of standard quality. A motor cycle is chosen at random and is found to be of standard quality. Find the probability that it has come from (*i*) Plant 1 (*ii*) Plant 2. ***(Mizoram B. 2016)***

10. An insurance company insured 2,000 scooters and 3,000 motor cycles. The probability of an accident involving a scooter is 0·01 and that of a motor cycle is 0·02. An insured vehicle met with an accident. Find the probability that the accidented vehicle was a motor cycle. ***(Meghalaya B. 2013)***

11. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$, if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives he is late. What is the probability that he comes by train ? ***(N.C.E.R.T.)***

12. (*i*) A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is 6. Find the probability that it is actually a 6. ***(N.C.E.R.T.; H.B. 2017; W. Bengal B. 2017)***

(*ii*) A man is known to speak the truth 3 out of 5 times. He throws a die and reports that it is number greater than 4. Find the probability that it is actually a number greater than 4. ***(A.I.C.B.S.E. 2009)***

13. A bag contains 4 balls. Two balls are drawn at random and are found to be white. What is the probability that all balls are white ? ***(A.I.C.B.S.E. 2010)***

14. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of the students who reside in hostel attain 'A' grade and 20% of day scholars attain 'A' grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an 'A' grade, what is the probability that the student is a hostlier ? ***(N.C.E.R.T. ; H.P.B. 2013 S ; C.B.S.E. 2012)***

15. A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0·5% of the healthy person tested. *i.e.* if a healthy person is tested, then, with probability 0·005, the test will imply he has the disease. If 0·1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test results is positive ? ***(N.C.E.R.T.)***

16. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she attained exactly one head, what is the probability that she threw 1, 2, 3, or 4 with the die ? ***(N.C.E.R.T. ; A.I.C.B.S.E. 2012)***

17. There are three coins. One is two headed coin (having head on both faces), another is biased coin that comes up fails 25% of the times and third is unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two-headed coin ? ***(C.B.S.E. 2009)***

18. In a certain college, 4% of boys and 1% of girls are taller than 1·75 metres. Furthermore, 60% of the students in the college are girls. A student is selected at random from the college and is found to be taller than 1·75 metres. Find the probability that the selected student is girl. ***(C.B.S.E. (F) 2012)***

19. Bag I contains 1 white, 2 black and 3 red balls; Bag II contains 2 white, 1 black and 1 red balls; Bag III contains 4 white, 3 black and 2 red balls. A bag is chosen at random and two balls are drawn from it with replacement. They happen to be one white and one red. What is the probability that they come from bag III ? ***(C.B.S.E. Sample Paper 2018)***

20. (*i*) Coloured balls are distributed in three bags as shown in the following table :

Bag	Colour of the ball		
	Black	White	Red
I	1	2	3
II	2	4	1
III	4	5	3

A bag is selected at random and then two balls are randomly drawn from the selected bag. They happen to be black and red. Find the probability that they come from bag I ? *(A.I.C.B.S.E. 2009)*

(*ii*) Three bags contain balls as shown in the table below :

Bag	Number of white balls	Number of Black balls	Number of Red balls
I	1	2	3
II	2	1	1
III	4	3	2

A bag is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they come from the III bag ? *(C.B.S.E. 2009)*

21. The members of a consulting firm rent cars from three rental agencies :

50% from agency X, 30% from agency Y and 20% from agency Z. From past experience, it is known that 9% of the cars from agency X need a service and tunning before renting, 12% of the cars from agency Y need a service and tunning before renting and 10% of the cars from agency Z need a service and tunning before renting. If the renting car delivered to the firm needs service and tunning, find the probability that agency Z is not to be blamed. *(C.B.S.E. Sample Paper 2019)*

Answers

1. (*i*) – (*ii*) $\frac{11}{50}$. **2.** $\frac{3}{8}$.

3. (*i*) $\frac{1}{3}$ (*ii*) $\frac{3}{7}$ (*iii*) $\frac{5}{11}$.

4. (*i*) – (*ii*) $\frac{35}{68}$. **5.** $\frac{2}{3}$.

6. (*a*) (*i*) (I) $\frac{33}{68}$ (II) (*ii*) $\frac{16}{31}$ (*b*) $\frac{3}{14}$.

7. $\frac{2}{3}$. **8.** $\frac{15}{37}$.

9. (*i*) $\frac{56}{83}$ (*ii*) $\frac{27}{83}$.

10. $\frac{3}{4}$. **11.** $\frac{1}{2}$.

12. (*i*) $\frac{3}{8}$ (*ii*) $\frac{3}{7}$.

13. $\frac{3}{5}$. **14.** $\frac{9}{13}$.

15. $\frac{22}{23}$. **16.** $\frac{8}{11}$. **17.** $\frac{4}{9}$.

18. $\frac{3}{11}$. **19.** $\frac{64}{199}$.

20. (*i*) $\frac{231}{551}$ (*ii*) $\frac{5}{17}$. **21.** $\frac{81}{101}$.

Hints to Selected Questions

11. Let A be the event that the doctor visits late and E_1, E_2, E_3, E_4 be the different events.

Then $P(E_1) = \frac{3}{10}$, $P(E_2) = \frac{1}{5}$, $P(E_3) = \frac{1}{10}$,

$P(E_4) = \frac{2}{5}$

and $P(A/E_1) = \frac{1}{4}$; etc.

$$\text{Now } P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3) + P(E_4)\,P(A/E_4)}\text{ ;etc.}$$

SUB CHAPTER 13.3 Random Variable and Probability Distribution

13.13. INTRODUCTION

Consider two coins, which are tossed simultaneously.

Here the sample space is given by : S = {HH, HT, TH, TT}.

If X denotes the number of heads, then X (HH) = 2, X (HT) = 1, X (TH) = 1, X (TT) = 0.

Thus X takes the values 0, 1, 2.

We call X as the **random variable** or **stochastic variable.**

Definition

A random variable is a real valued function X defined over the sample space of an experiment.

(I) If the random variable takes only a finite or an infinite but countable values, then it is called **a discrete variable.**

Its distribution is called a **discrete probability distribution.**

(II) If the random variable takes values in an interval, then it is called **continuous random variable.**

Its distribution is called **continuous probability distribution.**

13.14. PROBABILITY DISTRIBUTION OF A RANDOM VARIABLE

Let 'X' be the random variable, which denotes the number of heads in a simultaneous throw of two coins. Then X takes the values 0, 1, 2 as :

X(HH) = 2, X(HT) = 1, X(TH) = 1, X(TT) = 0.

Let P(X) be the probability of the variable X, then :

$$P(0) = P\text{ (no head) } = \frac{1}{4}$$

$$P(1) = P\text{ (one head) } = \frac{2}{4} = \frac{1}{2} \text{ and } P(2) = P\text{(two heads)} = \frac{1}{4}.$$

Note. P(X = 0) + P(X = 1) + P(X = 2) = 1.

Graphically. *Consider the above Example* :

We represent the values of random variable along X–axis and the corresponding probabilities along Y–axis.

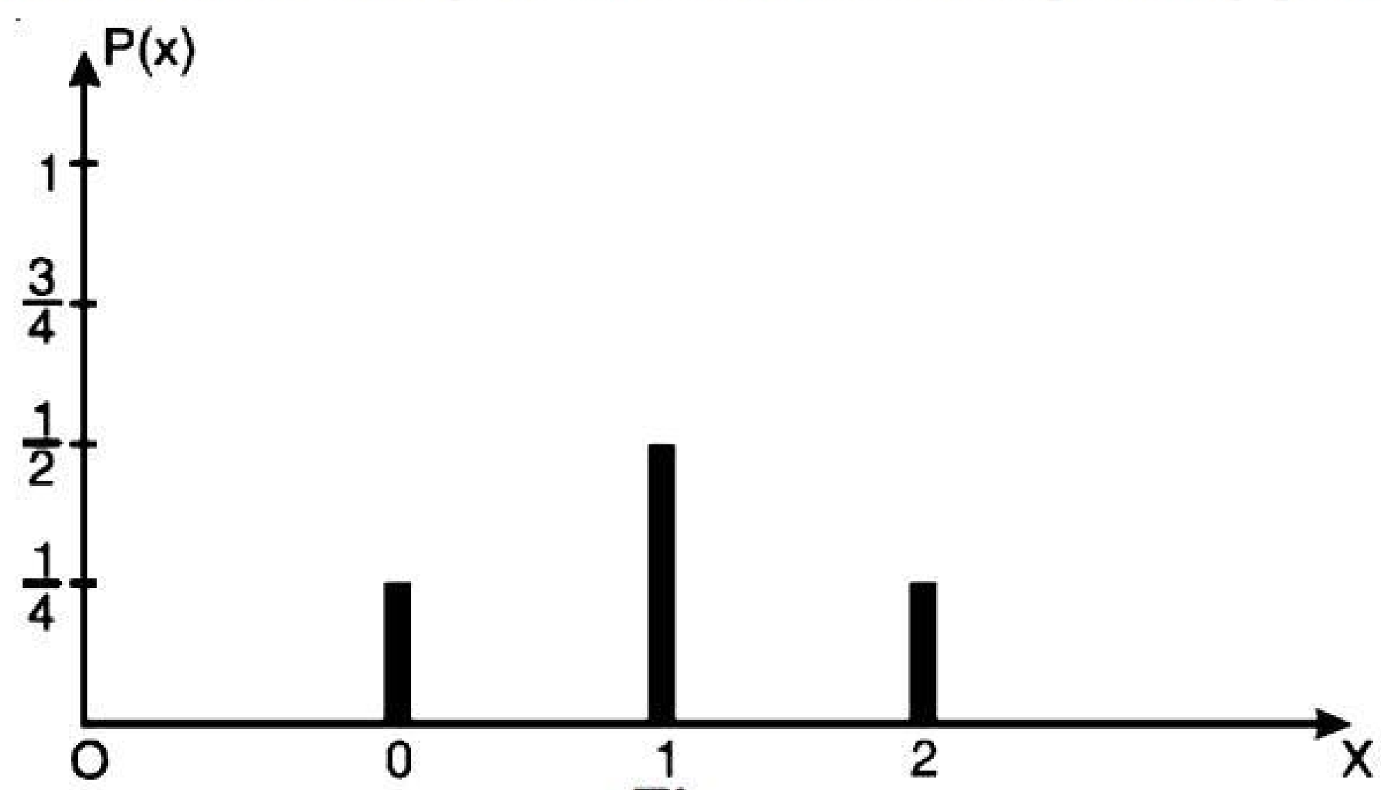

Fig.

At each value of 'X', we draw a line segment, parallel to Y–axis with its length equal to the corresponding probability.

Let a discrete random variable X assume values $x_1, x_2, \ldots\ldots, x_n$ with probabilities $p_1, p_2, \ldots\ldots, p_n$ respectively, then :

$$p_1 + p_2 + \ldots\ldots + p_n = 1.$$

Hence, the following is the table of Probability Distribution of X :

X :	x_1	x_2	....	x_n
P (X) :	p_1	p_2	....	p_n

The values $x_1, x_2, \ldots, x_n$ together with their corresponding probabilities $p_1, p_2, \ldots, p_n$ form a **probability distribution** of the random variable X.

Graphically. The graphical representation of the probability distribution is as below :

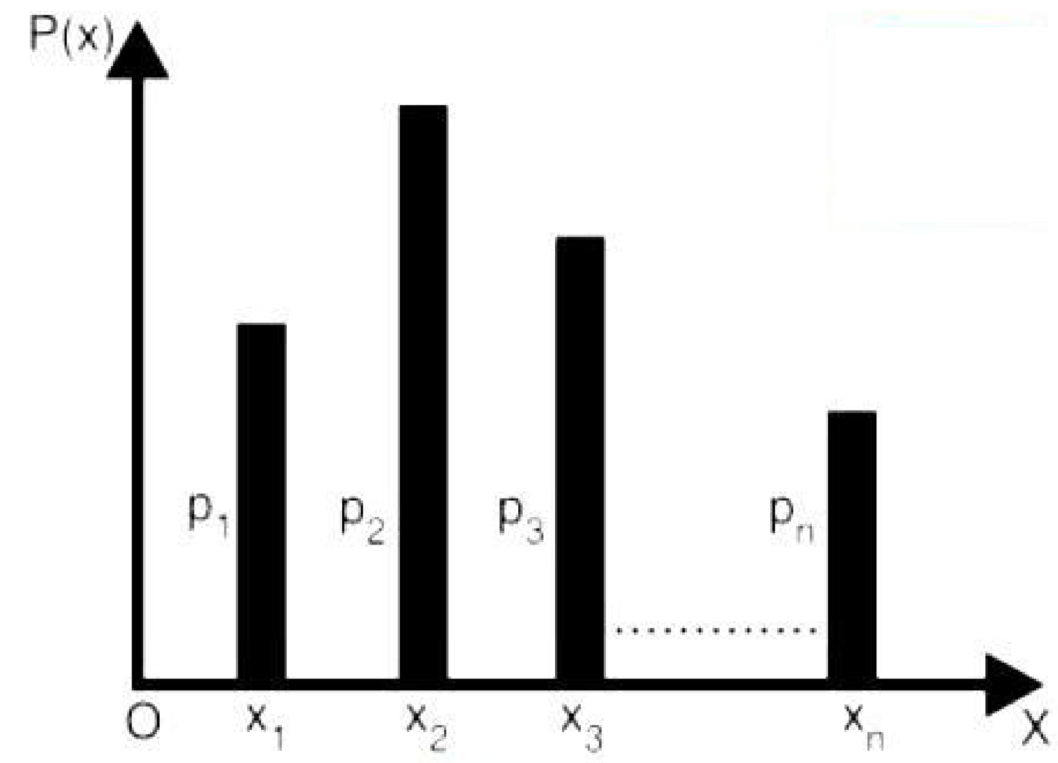

Fig.

Note. (*i*) $P(X \le x_i) = P(X = x_1) + P(X = x_2) + \ldots + P(X = x_i) = p_1 + p_2 + \ldots + p_i$

(*ii*) $P(X \ge x_i) = P(X = x_i) + P(X = x_{i+1}) + \ldots + P(X = x_n) = p_{i+1} + p_{i+2} + \ldots + p_n$.

ILLUSTRATIVE EXAMPLES

Example 1. A random variable 'X' has a probability distribution P(X) of the following form (K is constant) :

X :	0	1	2	3
P(X) :	3K	2K	K	0

Find K. *(Tripura B. 2016)*

Solution. Here 'X', a random variable takes values 0, 1, 2 and 3.

$\therefore \quad P(0) + P(1) + P(2) + P(3) = 1$

$\Rightarrow \quad 3K + 2K + K + 0 = 1$

$\Rightarrow \quad 6K = 1.$

Hence, $K = \frac{1}{6}$.

Example 2. The following is a probability distribution function of a random variable :

X:	–5	–4	–3	–2	–1	0	1	2	3	4	5
P(X):	*k*	2*k*	3*k*	4*k*	5*k*	7*k*	8*k*	9*k*	10*k*	11*k*	12*k*

(*i*) Find *k* (*ii*) Find P(X > 3) (*iii*) Find P(–3 < X < 4) (*iv*) Find P(X < – 3). *(Kerala B. 2018)*

Solution. (*i*) Here X is a random variable.

$\therefore P(-5) + P(-4) + P(-3) + P(-2) + P(-1) + P(0) + P(1) + P(2) + P(3) + P(4) + P(5) = 1$

$\Rightarrow k + 2k + 3k + 4k + 5k + 7k + 8k + 9k + 10k + 11k + 12k = 1$

$\Rightarrow \quad 72k = 1 \quad \Rightarrow \quad k = \frac{1}{72}$.

(*ii*) $P(X > 3) = P(X = 4) + P(X = 5)$

$= 11k + 12k = 23k = \frac{23}{72}$.

(*iii*) $P(-3 < X < 4) = P(-2) + P(-1) + P(0) + P(1) + P(2) + P(3)$

$= 4k + 5k + 7k + 8k + 9k + 10k$

$= 43k = \frac{43}{72}$.

(*iv*) $P(X < -3) = P(X = -5) + P(X = -4)$

$= k + 2k = 3k = \frac{3}{72} = \frac{1}{24}$.

Example 3. A bag contains 2 white and 1 red ball. One ball is drawn at random and then put back in the box after noting its colour. The process is repeated again. If 'X' denotes the number of red balls recorded in the two draws, describe 'X'. *(N.C.E.R.T.)*

Solution. Let the balls be w_1, w_2, r,

where $w \equiv$ white ball and $r \equiv$ red ball.

$\therefore$ Sample space $S = \{w_1w_1, w_1w_2, w_2w_1, w_2w_2, w_1r, rw_1, w_2r, rw_2, rr\}$.

For $\omega \in S$, $X(\omega)$ = Number of red balls.

$\therefore \quad X(\{w_1 w_1\}) = X(\{w_1 w_2\}) = X(\{w_2 w_1\})$

$= X(\{w_2 w_2\}) = 0$

$X(\{w_1 r\}) = X(\{rw_1\}) = X(\{w_2 r\})$

$= X(\{rw_2\}) = 1$ and $X(\{rr\}) = 2$.

Hence, 'X' is a random variable, which takes values 0, 1, 2.

Example 4. A person plays a game of tossing a coin thrice. For each head he is given ₹ 2 by the organiser of the game and for each tail he has to give ₹ 1·50 to the organiser. Let 'X' denote the amount gained or lost by the person. Show that 'X' is a random variable and exhibit it as a function on the sample space of the experiment. *(N.C.E.R.T.)*

Solution. Since 'X' is a number whose values are defined by the outcomes of the random experiment,

∴ 'X' is a random variable.

Now sample space is given by :

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\},$$

where $H \equiv$ Head and $T \equiv$ Tail.

Thus X (HHH) = 2 × 3 = ₹ 6

X (HHT) = X (HTH) = X (THH) = 2 × 2 – 1 × 1·5 = ₹ 2·50

X (HTT) = X (THT) = X (TTH) = 1 × 2 – 2 × 1·5 = – ₹ 1

and X (TTT) = – (3 × 1·5) = – ₹ 4·50.

Thus for each element of S, X takes a unique value.

∴ 'X' is a function on the sample space S having range = {6, 2·50, – 1, – 4·50}.

Example 5. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed three times, find the probability distribution of number of tails.

Solution. By the question, $P(H) = \frac{3}{4}$ and $P(T) = \frac{1}{4}$,

where $H \equiv$ Head and $T \equiv$ Tail.

If 'X' be the random variable, "number of tails", then 'X' takes values 0, 1, 2, 3.

$$\therefore \quad P(X = 0) = P(\text{No tail}) = P(HHH) = \frac{3}{4}\times\frac{3}{4}\times\frac{3}{4} = \frac{27}{64}$$

$$P(X = 1) = P(1\text{ tail}) = P(HHT) + P(HTH) + P(THH)$$

$$= \frac{3}{4}\times\frac{3}{4}\times\frac{1}{4}+\frac{3}{4}\times\frac{1}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{3}{4}\times\frac{3}{4}$$

$$= \frac{9}{64}+\frac{9}{64}+\frac{9}{64} = \frac{27}{64}$$

$$P(X = 2) = P(2\text{ tails}) = P(HTT) + P(THT) + P(TTH)$$

$$= \frac{3}{4}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{4}\times\frac{3}{4}$$

$$= \frac{3}{64}+\frac{3}{64}+\frac{3}{64} = \frac{9}{64}$$

$$\text{and} \quad P(X = 3) = P(3\text{ tails}) = P(TTT) = \frac{1}{4}\times\frac{1}{4}\times\frac{1}{4} = \frac{1}{64}.$$

Hence, the probability distribution is :

X :	0	1	2	3
P (X) :	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

Example 6. Find the probability distribution of 'X', the number of heads in two tosses of a coin (or a simultaneous toss of two coins). Sketch its graph.

(P.B. 2014 S, 12; Karnataka B. 2014; H.P.B. 2012)

Solution. Here 'X' is the random variable, which is the number of heads.

Now 'X' takes values 0, 1, 2.

Now $p = \frac{1}{2}, q = 1 - \frac{1}{2} = \frac{1}{2}$,

where 'p' is the probability of success and 'q' that of failure.

$$\therefore \quad P(X = 0) = P(TT) = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4},$$

$$P(X = 1) = P(HT) + P(TH) = \frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2} = \frac{1}{2}$$

$$\text{and} \quad P(X = 2) = P(HH) = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}.$$

Hence, the probability distribution is :

X :	0	1	2
P (X) :	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Graphically, the probability distribution is as shown below :

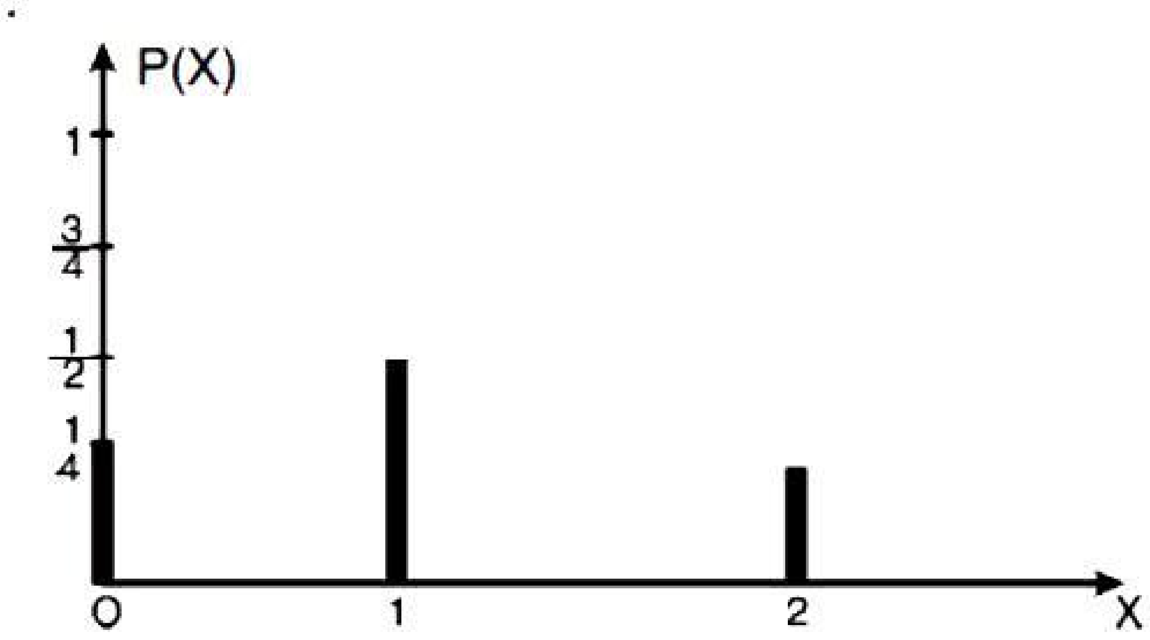

Fig.

Example 7. Four defective oranges are accidently mixed with sixteen good ones and by looking at them it is not possible to differentiate between them. Three oranges are drawn at random from the lot. Find the probability distribution of X, the number of defective oranges.

Solution. Let 'X' be the random variable, which is the number of defective oranges.

Here 'X' takes value 0, 1, 2, 3.

Total number of oranges = 4 + 16 = 20.

Number of defective oranges = 4.

$$\therefore \quad P(X = 0) = P(\text{No defective orange}) = \frac{^{16}C_3}{^{20}C_3} = \frac{16\times15\times14}{20\times19\times18} = \frac{140}{285},$$

$$P(X = 1) = P(\text{One defective orange}) = \frac{^4C_1 \times {^{16}C_2}}{^{20}C_3}$$

$$= \frac{4\times16\times15\times6}{2\times20\times19\times18} = \frac{120}{285},$$

$P(X = 2) = P(\text{Two defective oranges})$

$$= \frac{{}^4C_2 \times {}^{16}C_1}{{}^{20}C_3} = \frac{4 \times 3 \times 16 \times 6}{2 \times 20 \times 19 \times 18} = \frac{24}{285}$$

and P (X = 3) = P (Three defective oranges)

$$= \frac{{}^4C_3}{{}^{20}C_3} = \frac{4 \times 3 \times 2}{20 \times 19 \times 18} = \frac{1}{285}.$$

Hence, the probability distribution is :

X :	0	1	2	3
P (X) :	$\frac{140}{285}$	$\frac{120}{285}$	$\frac{24}{285}$	$\frac{1}{285}$

Example 8. The random variable 'X' can take only the values {0, 1, 2, 3}. Given that :
P(X = 0) = P(X = 1) = p and P(X = 2) = P(X = 3) such that $\Sigma p_i x_i^2 = 2\Sigma p_i x_i$, find the value of p.

(C.B.S.E. 2017)

Solution. We have : P(X = 0) = P(X = 1) = *p*.

Let P(X = 2) = P(X = 3) = *k*.

Since X is a random variable,

∴ X takes values 0, 1, 2 and 3.

∴ P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 1

$\Rightarrow p + p + k + k = 1 \Rightarrow 2p + 2k = 1$

$\Rightarrow p + k = \frac{1}{2}$ ...(1)

Now, $\Sigma p_i x_i^2 = 2\Sigma p_i x_i$

$\Rightarrow p(0) + p(1) + k(4) + k(9) = 2[p(0) + p(1) + k(2) + k(3)]$

$\Rightarrow p + 13k = 2p + 10k$

$\Rightarrow p - 3k = 0$ ...(2)

Subtracting (2) from (1), $4k = \frac{1}{2} \Rightarrow k = \frac{1}{8}$.

Putting in (1), $p + \frac{1}{8} = \frac{1}{2} \Rightarrow p = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$.

Hence, $p = \frac{3}{8}$.

EXERCISE 13 (e)

Very Short Answer Type Questions

VSATQ

1. An urn contains 5 red and 2 black balls. Two balls are randomly selected. Let 'X' represent the number of black balls. What are the possible values of 'X' ? Is X a random variable ? *(N.C.E.R.T.)*

2. Let 'X' represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of 'X' ? *(N.C.E.R.T.)*

3. A random variable 'X' has the following probability distribution :

X :	– 2	– 1	0	1	2	3
P (X) :	0·1	K	0·2	2K	0·3	K

Find : (*i*) the value of K (*ii*) P (X ≤ 1)

(*iii*) P (X ≥ 0).

4. A random variable 'X' has the following probability distribution :

X :	0	1	2	3	4	5	6	7
P (X) :	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2 + K$

Determine (*i*) K (*ii*) P (X < 3) (*iii*) P (X > 6) (*iv*) P (0 < X < 3). *(Jammu B. 2018)*

5. A fair die is tossed once. If the random variable is the number of "getting an even number" (denoted by X), find the probability distribution of 'X'. Sketch the graph.

6. (*i*) A coin is tossed two times. Find the probability distribution of the number of

(*i*) heads (tails).

(H.P.B. 2017; Karnataka B. 2017; P.B. 2011)

(*ii*) Find the probability distribution of the number of tails when two coins are tossed. *(P.B. 2012)*

7. A coin is tossed 5 times. 'X' is the number of heads observed. Find the probability distribution of 'X'.

8. (*a*) (*i*) Find the probability distribution of the number of heads when three coins are tossed simultaneously.

(*ii*) Find the probability distribution of the number of tails in the simultaneous tosses of three coins. *(H.P.B. 2017, 12)*

(*iii*) Find the probability distribution of the number of heads in the simultaneous toss of four coins. *(Assam B. 2013)*

(*b*) (*i*) Find the probability distribution of the number of heads in three tosses of a coin. *(Assam B. 2013)*

(*ii*) Find the probability distribution of the number of sixes in two tosses of a die.

(*c*) Find the probability distribution of the number of heads (tails) in four tosses of a coin.

(H.B. 2018; H.P.B. 2017)

(*d*) Find the probability distribution of the number of doublets in three throws of a pair of dice.

(Kashmir B. 2016, 15)

9. Find the probability distribution of the number of successes in two tosses of a die when a success is defined as :

(*i*) 'number greater than 4'

(H.P.B. Model Paper 2018; H.P.B. 2016, 12 ; Jammu B. 2012)

(*ii*) six appears on at least one die.

(N.C.E.R.T.; Jammu B. 2012)

10. (*i*) Find the probability distribution of the number of doublets in three throws of a pair of dice.

(N.C.E.R.T.; Jammu B. 2014S, 13 ; Kashmir B. 2012 ; C.B.S.E. 2010 C; H.P.B. 2009)

(*ii*) A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of number of successes.

Short Answer Type Questions

SATQ

11. Let 'X' denote the number of hours you study during a randomly selected school day. The probability that X can take the values of x has the following form, where k is some unknown constant.

$$P(X = x) = \begin{cases} 0.1, & \text{if } x = 0 \\ kx, & \text{if } x = 1 \text{ or } 2 \\ k(5 - x), & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{otherwise.} \end{cases}$$

(*a*) Find the value of k.

(*b*) What is the probability that you study at least two hours ? Exactly two hours ? At most two hours ? ***(N.C.E.R.T.)***

12. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of :

(I) aces ***(N.C.E.R.T.; H.P.B. 2010 S)***

(II) kings.

13. Two cards are drawn from a well shuffled pack of 52 cards. Find the probability distribution of queens if cards are drawn at random. ***(J.&K. B. 2010)***

14. Two cards are drawn one by one without replacement from a well shuffled deck of 52 cards. Find the probability distribution of the number of (I) Aces ***(Assam B. 2016)*** (II) kings (III) face cards (IV) spades.

15. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Obtain the probability distribution of 'the number of aces' obtained.

(Jammu B. 2017)

16. Two cards are drawn without replacement from a well shuffled pack of 52 cards. Obtain the probability distribution of the number of face cards (Jack, Queen, King and Ace).

17. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

(N.C.E.R.T.; H.P.B. 2016, 14, 11; Jammu B. 2013 ; Assam B. 2013)

18. Three cards are drawn at random successively with replacement from a well shuffled pack of 52 cards. Getting 'a card of spades' is regarded a success. Obtain the probability distribution of the number of successes. ***(C.B.S.E. 2009 C)***

19. An urn contains 4 white and 3 red balls. Find the probability distribution of the number of red balls if 3 balls are drawn at random from the urn. ***(Jammu B. 2013)***

20. A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

(N.C.E.R.T; H.P.B. 2010)

21. A die is tossed twice. 'Getting a number greater than 4' is considered a success, find the probability distribution of the number of successes.

(Nagaland B. 2015 ; Jammu B. 2013 ; P.B. 2010 S)

22. Find the probability distribution of the number of green balls drawn when 3 balls are drawn one by one without replacement from a bag containing 3 green and 5 white balls.

23. 3 defective bulbs are mixed up with 7 good ones. 3 bulbs are drawn at random. Find the probability distribution of the defective bulbs.

24. A coin is biased so that the head is 3 times as likely to occur as tail. If a coin is tossed twice, find the probability distribution of number of tails. ***(H.P.B. 2012)***

25. We take 8 identical slips of paper, write the number 0 on one of them, the number 1 on three of the slips, the number 2 on three of the slips and the number 3 on one of the slips. These slips are folded, put in a box and thoroughly mixed. One slip is then drawn at random from the box. If Z is the random variable denoting the number written on the drawn slip, find the probability distribution of Z.

26. From a lot of 10 bulbs, which includes 3 defectives, a sample of 2 bulbs is drawn at random. Find the probability distribtuion of the number of defective bulbs. ***(C.B.S.E. 2010)***

27. A coin is tossed 5 times. If 'X' is the number of heads observed, find the probability distribution of 'X'.

(J. & K.B. 2010)

Answers

1. X = 0, 1, 2 ; Yes. **2.** X = 0, 2, 4, 6.

3. (*i*) K = 0·1 (*ii*) P (X ≤ 1) = 0.6

(*iii*) P (X ≥ 0) = 0·8.

4. (*i*) K = 1 (*ii*) P (X < 3) = 0.3

(*iii*) P (X > 6) = 0.17

(*iv*) P (0 < X < 3) = 0.3.

5. $P(0) = \frac{1}{2}, P(1) = \frac{1}{2}$.

6. (*i*) – (*ii*) $P(0) = \frac{1}{4}, P(1) = \frac{2}{4}, P(2) = \frac{1}{4}$.

7. $P(0) = \frac{1}{32}, P(1) = \frac{5}{32}, P(2) = \frac{10}{32}$,

$P(3) = \frac{10}{32}, P(4) = \frac{5}{32}, P(5) = \frac{1}{32}$.

8. (*a*) (*i*) – (*ii*) $P(0) = \frac{1}{8}, P(1) = \frac{3}{8}, P(2) = \frac{3}{8}, P(3) = \frac{1}{8}$

(*iii*) $P(0) = \frac{1}{16}, P(1) = \frac{1}{4}, P(2) = \frac{3}{8}$,

$P(3) = \frac{1}{4}, P(4) = \frac{1}{16}$

(*b*) (*i*) $P(0) = \frac{1}{8}, P(1) = \frac{3}{8}, P(2) = \frac{3}{8}, P(3) = \frac{1}{8}$

(*ii*) $P(0) = \frac{25}{36}, P(1) = \frac{5}{18}, P(2) = \frac{1}{36}$

(*c*) $P(0) = \frac{1}{16}, P(1) = \frac{1}{4}, P(2) = \frac{3}{8}, P(3) = \frac{1}{4}, P(4) = \frac{1}{16}$

(*d*) $P(0) = \frac{125}{216}, P(1) = \frac{25}{72}, P(2) = \frac{5}{72}, P(3) = \frac{1}{216}$.

9. (*i*) $P(0) = \frac{4}{9}, P(1) = \frac{4}{9}, P(2) = \frac{1}{9}$

(*ii*) $P(1) = \frac{10}{36}, P(2) = \frac{1}{36}$.

10. (*i*) $P(0) = \frac{125}{216}, P(1) = \frac{75}{216}, P(2) = \frac{15}{216}, P(3) = \frac{1}{216}$

(*ii*) $P(0) = \frac{625}{1290}, P(1) = \frac{500}{1296}, P(2) = \frac{25}{216}$

$P(3) = \frac{5}{324}, P(4) = \frac{1}{1296}$.

11. (*a*) $k = 0{\cdot}15$ (*b*) (*i*) 0·75 (*ii*) 0·3 (*iii*) 0·55.

12. (I) – (II) $P(0) = \frac{144}{169}, P(1) = \frac{24}{169}, P(2) = \frac{1}{169}$.

13. $P(0) = \frac{188}{221}, P(1) = \frac{32}{221}, P(2) = \frac{1}{221}$.

14. (I)-(II) $P(0) = \frac{188}{221}, P(1) = \frac{32}{221}, P(2) = \frac{1}{221}$

(III) $P(0) = \frac{105}{221}, P(1) = \frac{96}{221}, P(2) = \frac{20}{221}$

(IV) $P(0) = \frac{19}{34}, P(1) = \frac{13}{34}, P(2) = \frac{2}{34}$.

15. $P(0) = \frac{144}{169}, P(1) = \frac{24}{169}, P(2) = \frac{1}{169}$.

16. $P(0) = \frac{105}{221}, P(1) = \frac{96}{221}, P(2) = \frac{20}{221}$.

17. $P(0) = \frac{256}{625}, P(1) = \frac{256}{625}$,

$P(2) = \frac{96}{625}, P(3) = \frac{16}{625}, P(4) = \frac{1}{625}$.

18. $P(0) = \frac{27}{64}, P(1) = \frac{27}{64}, P(2) = \frac{9}{64}, P(3) = \frac{1}{64}$.

19. $P(0) = \frac{64}{343}, P(1) = \frac{144}{343}, P(2) = \frac{108}{343}, P(3) = \frac{27}{343}$.

20. $P(0) = \frac{9}{16}, P(1) = \frac{3}{8}, P(2) = \frac{1}{16}$.

21. $P(0) = \frac{4}{9}, P(1) = \frac{4}{9}, P(2) = \frac{1}{9}$.

22. $P(0) = \frac{5}{28}, P(1) = \frac{15}{28}, P(2) = \frac{15}{56}, P(3) = \frac{1}{56}$.

23. $P(0) = \frac{727}{1000}, P(1) = \frac{243}{1000}, P(2) = \frac{27}{1000}$,

$P(3) = \frac{1}{1000}$.

24. $P(0) = \frac{9}{16}, P(1) = \frac{6}{16}, P(2) = \frac{1}{16}.$

25. $P(0) = \frac{1}{8}, P(1) = \frac{3}{8}, P(2) = \frac{3}{8}, P(3) = \frac{1}{8}.$

26. $P(0) = \frac{49}{100}, P(1) = \frac{42}{100}, P(2) = \frac{9}{100}.$

27. $P(0) = \frac{1}{32}, P(1) = \frac{5}{32}, P(2) = \frac{10}{32},$

$P(3) = \frac{10}{32}, P(4) = \frac{5}{32}, P(5) = \frac{1}{32}.$

Hints to Selected Questions

11. (*a*) $P(X = 0) + P(X = 1) + ... + P(X = 4) = 1$

$\Rightarrow 0{\cdot}1 + k + 2k + 2k + k = 1 \Rightarrow 6k = 0{\cdot}9$

$\Rightarrow k = 0{\cdot}15.$

(*b*) $P(X \geq 2) = P(X = 2) + (X = 3) + P(X = 4).$

14. (IV) $P(X = 0) = \frac{^{39}C_2}{^{52}C_2}$; etc.

17. $P(X = 0) = \frac{24}{30} \times \frac{24}{30} \times \frac{24}{30} \times \frac{24}{30}$; etc.

20. $P(H) = \frac{3}{4}$ and $P(T) = \frac{1}{4}$.

13.15. MEAN OF A RANDOM VARIABLE

In certain problems it is useful to describe some features of the random variable by means of a single number, which can be computed by probability distribution. Such numbers are mean, median and mode. But here we shall discuss mean only. Mean is a measure of location (or central tendency) that it locates roughly a *middle* (or *average*) value of the random variable.

Definition

Let X be a random variable whose possible values $x_1, x_2, ..., x_n$ occur with probabilities $p_1, p_2, ..., p_n$ respectively. Then mean of X (denoted by μ) is $\sum_{i=1}^{n} x_i p_i$.

Thus the mean of 'X' is the weighted average of the possible values of 'X', each value being weighted by its probability with which it occurs.

Thus $$\mu = \frac{p_1 x_1 + p_2 x_2 + ... + p_n x_n}{p_1 + p_2 + ... + p_n} = \frac{\sum x_i p_i}{\sum p_i} = \sum_{i=1}^{n} x_i p_i. \qquad \left[\because \sum p_i = 1\right]$$

The mean of a random variable 'X' is also called **expectation of 'X'** and is denoted by E (X).

Thus

$$E(X) = \mu = \sum_{i=1}^{n} x_i p_i = x_1 p_1 + x_2 p_2 + ... + x_n p_n.$$

In Words : The mean (or expectation) of a random variable 'X' is the sum of the products of all possible values of 'X' by their respective probabilities.

13.16. VARIANCE OF A RANDOM VARIABLE

The mean of a random variable does not indicate about the variability in the values of the random variable. As in Statistics, the variability (or spread) in the values of a random variable may be measured by variance.

Definition

Let 'X' be a random variable whose possible values $x_1, x_2, ..., x_n$ occur with probabilities $p_1(x_1), p_2(x_2), ..., p_n(x_n)$ respectively. Then variance of 'X', denoted by Var (X) or σ_x^2 is defined as:

$$\sigma_x^2 = Var(X) = \sum_{i=1}^{n} (x_i - \mu)^2 \, p(x_i) \text{ or equivalently } \sigma_x^2 = E(x-\mu)^2.$$

The non-negative number $\sigma_x = \sqrt{\text{Var (X)}} = \sqrt{\sum_{i=1}^{n} (x_i - \mu)^2 \, p(x_i)}$

is called the **standard deviation** of the random variable 'X'.

Another Form of Variance :

We know that : $\text{Var (X)} = \sum_{i=1}^{n} (x_i - \mu)^2 \, p(x_i)$

$$= \sum_{i=1}^{n} (x_i^2 + \mu^2 - 2\mu x_i) \, p(x_i)$$

$$= \sum_{i=1}^{n} x_i^2 \, p(x_i) + \sum_{i=1}^{n} \mu^2 \, p(x_i) - \sum_{i=1}^{n} 2\mu x_i \, p(x_i)$$

$$= \sum_{i=1}^{n} x_i^2 \, p(x_i) + \mu^2 \sum_{i=1}^{n} p(x_i) - 2\mu \sum_{i=1}^{n} x_i \, p(x_i)$$

$$= \sum_{i=1}^{n} x_i^2 \, p(x_i) + \mu^2 (1) - 2\mu(\mu) \qquad \left[\because \sum_{i=1}^{n} p(x_i) = 1 \text{ and } \mu = \sum_{i=1}^{n} x_i \, p(x_i)\right]$$

$$= \sum_{i=1}^{n} x_i^2 \, p(x_i) - \mu^2 .$$

Hence,

$$\text{Var (X)} = \sum_{i=1}^{n} x_i^2 \, p(x_i) - \left[\sum_{i=1}^{n} x_i \, p(x_i)\right]^2$$

$$= E(X^2) - [E(X)]^2.$$

Note. The second form is useful when μ is a fraction.

Notations. σ^2 and σ are denoted for **variance** and **standard deviation** respectively.

ILLUSTRATIVE EXAMPLES

Example 1. Let a pair of dice be thrown and the random variable 'X' be the sum of the numbers that appear on the two dice. Find the mean (or expectation) of X. *(N.C.E.R.T. ; H.P.B. 2013)*

Solution. Clearly the sample space consists of 36 elementary events = $\{(x_i, y_i) \; ; x_i, y_i = 1, 2, ..., 6\}$.

X, the random variable = Sum of the numbers on the two dice.

∴ 'X' takes values 2, 3, 4, ..., or 12.

Now $P(X=2) = P(\{1, 1\}) = \frac{1}{36}$

$$P(X=3) = P(\{1, 2\}, \{2, 1\}) = \frac{2}{36}$$

$$P(X=4) = P\{(1, 3), (2, 2), (3, 1)\} = \frac{3}{36}$$

$$P(X=5) = P\{(1, 4), (2, 3), (3, 2), (4, 1)\} = \frac{4}{36}$$

$$P(X=6) = P\{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\} = \frac{5}{36}$$

$$P(X=7) = P\{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\} = \frac{6}{36}$$

$$P(X=8) = P\{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\} = \frac{5}{36}$$

$$P(X=9) = P\{(3, 6), (4, 5), (5, 4), (6, 3)\} = \frac{4}{36}$$

$$P(X=10) = P\{(4, 6), (5, 5), (6, 4)\} = \frac{3}{36}$$

$$P(X=11) = P\{(5, 6), (6, 5)\} = \frac{2}{36}$$

$$P(X=12) = P\{(6, 6)\} = \frac{1}{36}.$$

Thus the probability distribution is :

X :	2	3	4	5	6	7	8	9	10	11	12
P (X) :	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$\therefore \quad \mu = E(X) = \sum_{i=1}^{n} x_i p_i$

$$= 2 \times \frac{1}{36} + 3 \times \frac{2}{36} + 4 \times \frac{3}{36} + 5 \times \frac{4}{36} + 6 \times \frac{5}{36} + 7 \times \frac{6}{36} + 8 \times \frac{5}{36} + 9 \times \frac{4}{36} + 10 \times \frac{3}{36} + 11 \times \frac{2}{36} + 12 \times \frac{1}{36}$$

$$= \frac{1}{36}(2 + 6 + 12 + 20 + 30 + 42 + 40 + 36 + 30 + 22 + 12) = \frac{1}{36}(252) = 7.$$

Hence, the reqd. mean = 7.

Example 2. Find the mean and variance of the numbers obtained on a throw of an unbiased die.

(N.C.E.R.T.; P.B. 2014; H.P.B. 2013)

Solution. Here sample space, S = {1, 2, 3, 4, 5, 6}.

Let 'X' denote the number obtained on the throw.

Thus 'X' takes values 1, 2, 3, 4, 5 or 6.

$\therefore \quad P(1) = P(2) = \ldots\ldots\ldots = P(6) = \frac{1}{6}.$

∴ Probability distribution is :

X :	1	2	3	4	5	6
P (X) :	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$\therefore \quad \mu = E(X) = \sum_{i=1}^{n} x_i p(x_i)$$

$$= 1\times\frac{1}{6}+2\times\frac{1}{6}+3\times\frac{1}{6}+4\times\frac{1}{6}+5\times\frac{1}{6}+6\times\frac{1}{6} = \frac{1}{6}(1+2+3+4+5+6) = \frac{1}{6}(21) = \frac{7}{2}.$$

Also, $E(X^2) = 1^2\times\frac{1}{6}+2^2\times\frac{1}{6}+3^2\times\frac{1}{6}+4^2\times\frac{1}{6}+5^2\times\frac{1}{6}+6^2\times\frac{1}{6}$

$$= \frac{1}{6}(1+4+9+16+25+36) = \frac{1}{6}(91) = \frac{91}{6}.$$

Hence, $Var(X) = E(X^2) - (E(X))^2$

$$= \frac{91}{6} - \left(\frac{7}{2}\right)^2 = \frac{91}{6} - \frac{49}{4} = \frac{182-147}{12} = \frac{35}{12}.$$

Example 3. Find the mean and variance of the number of heads in two tosses of a coin.

Solution. Let 'X' denote the number of heads obtained in two tosses of a coin. Thus 'X' takes values 0, 1, 2.

Now p, the probability of getting a head $= \frac{1}{2}$

and q, the probability of not getting a head $= 1 - \frac{1}{2} = \frac{1}{2}$.

$$\therefore \quad P(X=0) = q\times q = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$$

$$P(X=1) = p\times q + q\times p = 2\left(\frac{1}{2}\times\frac{1}{2}\right) = \frac{1}{2}$$

and $P(X=2) = p\times p = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$.

Thus we have :

x_i	p_i	$p_i x_i$	x_i^2	$p_i x_i^2$
0	$\frac{1}{4}$	0	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$
2	$\frac{1}{4}$	$\frac{2}{4}$	4	1
Total		1		3/2

Hence, the mean, $\mu = \sum_{i=1}^{n} p_i x_i = 0 + \frac{1}{2} + \frac{2}{4} = 1$

and the variance, σ_x^2

$$= \Sigma p_i x_i^2 - \mu^2 = \frac{3}{2} - (1)^2 = \frac{3}{2} - 1 = \frac{1}{2}.$$

Example 4. Two cards are drawn (without replacement) from a well shuffled deck of 52 cards. Find the probability distribution table and mean of number of kings.

(P.B. 2018)

Solution. Let 'X' be the random variable, which is the number of kings.

Here, 'X' takes values 0, 1 and 2.

$$\therefore \quad P(X=0) = \frac{48}{52}\times\frac{47}{51} = \frac{188}{221}$$

$$P(X=1) = \frac{4}{52}\times\frac{48}{51} + \frac{48}{52}\times\frac{4}{51} = \frac{32}{221}$$

and $P(X=2) = \frac{4}{52}\times\frac{3}{51} = \frac{1}{221}$.

Hence, the probability distribution is :

X :	0	1	2
P(X) :	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Now, we have :

x_i	p_i	$p_i x_i$
0	$\frac{188}{221}$	0
1	$\frac{32}{221}$	$\frac{32}{221}$
2	$\frac{1}{221}$	$\frac{2}{221}$
Total		$\frac{34}{221}$

∴ Mean, $\mu = \Sigma p_i x_i = \frac{34}{221}$.

Example 5. Two numbers are selected at random (without replacement) from the first five positive integers. Let X denote the larger of the two numbers obtained. Find the mean and variance of X. *(C.B.S.E. 2018)*

Solution. The first five positive integerse are 1, 2, 3, 4 and 5.

We select two positive numbers in 5 × 4 = 20 ways.

Out of three, two numbers are selected at random.

Let 'X' denote the larger of the two numbers.

X can be 2, 3, 4 or 5.

$\therefore \quad P(X = 2) = P$ (Larger number is 2)

$$= \frac{2}{20}\{(1, 2), (2, 1)\}$$

Similarly $\quad P(X = 3) = \frac{4}{20}$,

$$P(X = 4) = \frac{6}{20}$$

and $\quad P(X = 5) = \frac{8}{20}$.

Hence, the probability distribution is :

X :	2	3	4	5
P(X) :	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$
X.P(X)	$\frac{2}{10}$	$\frac{6}{10}$	$\frac{12}{10}$	$\frac{20}{10}$
X^2 P(X)	$\frac{4}{10}$	$\frac{18}{10}$	$\frac{48}{10}$	$\frac{100}{10}$

$\therefore \quad$ Mean $= \Sigma X P(X)$

$$= 2\times\frac{2}{20}+3\times\frac{4}{20}+4\times\frac{6}{20}+5\times\frac{8}{20}$$

$$= \frac{4+12+24+40}{20} = \frac{80}{20} = 4$$

and $\quad$ Variance $= \Sigma X^2 P(X) - [\Sigma P(X)]^2$

$$= \frac{170}{10} - (1)^2 = 17 - 1 = 16.$$

Example 6. Two cards are drawn simultaneously (without replacement) from a well shuffled pack of 52 cards. Find the mean and variance of the number of red cards. *(A.I.C.B.S.E. 2012)*

Solution. Here 'X' takes values 0, 1, 2.

$$\therefore \quad P(X = 0) = \frac{26}{52}\times\frac{25}{51} = \frac{25}{102}$$

$$P(X = 1) = \frac{26}{52}\times\frac{26}{51}+\frac{26}{52}\times\frac{26}{51} = \frac{13}{51}+\frac{13}{51} = \frac{26}{51}$$

$$P(X = 2) = \frac{26}{52}\times\frac{25}{51} = \frac{25}{102}.$$

Thus we have :

x_i	p_i	p_ix_i	x_i^2	$p_ix_i^2$
0	$\frac{25}{102}$	0	0	0
1	$\frac{26}{51}$	$\frac{26}{51}$	1	$\frac{26}{51}$
2	$\frac{25}{102}$	$\frac{25}{51}$	4	$\frac{50}{51}$
Total		1		$\frac{76}{51}$

(I) Mean, $\mu = \sum x_i p_i = 1$.

(II) Variance, $\quad \sigma^2 = \sum p_i x_i^2 - \mu^2$

$$= \frac{76}{51} - (1)^2 = \frac{76-51}{51} = \frac{25}{51}.$$

Example 7. There are 4 cards numbered 1, 3, 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let 'X' denote the sum of the numbers on the two drawn cards. Find the mean and variance of X. *(A.I.C.B.S.E. 2017)*

Solution. The sample space consists of 12 events.

X, the random variable = Sum of numbers.

$\therefore$ X takes values 4, 6, 8, 10, 12.

Now $P(X = 4) = P\{(1, 3), (3, 1)\} = \frac{2}{12} = \frac{1}{6}$

$P(X = 6) = P\{(1, 5), (5, 1)\} = \frac{2}{12} = \frac{1}{6}$

$P(X = 8) = P\{(1, 7), (7, 1), (3, 5), (5, 3)\} = \frac{4}{12} = \frac{1}{3}$

$P(X = 10) = P\{(3, 7), (7, 3)\} = \frac{2}{12} = \frac{1}{6}$

$P(X = 12) = P\{(5, 7), (7, 5)\} = \frac{2}{12} = \frac{1}{6}$.

Thus the probability distribution is :

X :	4	6	8	10	12
P(X) :	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

$\therefore$ Mean, $\mu = E(X)$

$$= 4\times\frac{1}{6}+6\times\frac{1}{6}+8\times\frac{1}{3}+10\times\frac{1}{6}+12\times\frac{1}{6}$$

$$= \frac{2}{3}+1+\frac{8}{3}+\frac{5}{3}+2 = 3+\frac{15}{3}$$

$$= 3 + 5 = 8.$$

Also $E(X^2) = 4^2 \times \frac{1}{6} + 6^2 \times \frac{1}{6} + 8^2 \times \frac{1}{3} + 10^2 \times \frac{1}{6} + 12^2 \times \frac{1}{6}$

$= \frac{1}{6}(16 + 36 + 64 + 100 + 144)$

Hence, $Var(X) = E(X^2) - (E(X))^2$

$= 60 - (8)^2 = 60 - 64 = -4.$

Example 8. From a lot of 10 items containing 3 defective items a sample of 4 items is drawn at random. Let the random variable 'X' denote the number of defective items in the sample. If the sample is drawn without replacement, find :

(i) The Probability Distribution of X

(ii) Mean of X (iii) Variance of X. *(J. & K.B. 2009)*

Solution. (*i*) Here 'X' is random variable '*Defective item*'.

∴ 'X' takes values 0, 1, 2, 3.

∴ $P(X = 0) = \frac{7}{10} \times \frac{6}{9} \times \frac{5}{8} \times \frac{4}{7} = \frac{1}{6}$

$P(X = 1) = 4\left(\frac{3}{10} \times \frac{7}{9} \times \frac{6}{8} \times \frac{5}{7}\right) = \frac{1}{2}$

$P(X = 2) = 6\left(\frac{3}{10} \times \frac{2}{9} \times \frac{7}{8} \times \frac{6}{7}\right) = \frac{3}{10}$

and $P(X = 3) = 4\left(\frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} \times \frac{7}{7}\right) = \frac{1}{30}.$

∴ The Probability distribution is :

X	0	1	2	3
P (X) :	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

(*ii*) We have :

x_i	p_i	$p_i x_i$	x_i^2	$p_i x_i^2$
0	$\frac{1}{6}$	0	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$
2	$\frac{3}{10}$	$\frac{3}{5}$	4	$\frac{6}{5}$
3	$\frac{1}{30}$	$\frac{1}{10}$	9	$\frac{3}{10}$
		$\frac{6}{5}$		2

Hence, the mean of X, $\mu = \frac{6}{5}.$

(*iii*) Variance of X $= \sigma_x^2 = \Sigma p_i x_i^2 - \mu^2$

$= 2 - \left(\frac{6}{5}\right)^2 = 2 - \frac{36}{25} = \frac{14}{15}.$

Example 9. Three numbers are selected at random (without replacement) from first six positive integers. Let X denote the largest of the three numbers obtained. Find the probability distribution of X. Also, find the mean and variance of the distribution.

(A.I.C.B.S.E. 2016)

Solution. Here 'X' takes values 3, 4, 5, 6.

[∵ *1 and 2 cannot be greater than other selected numbers*]

∴ $P(X = 3) = P(\{1, 2, 3\}) = \frac{1}{{}^6C_3} = \frac{1}{\frac{6\times5\times4}{3\times2\times1}} = \frac{1}{20}$

$P(X = 4) = P(\{1, 2, 4\}, \{2, 3, 4\}, \{1, 3, 4\})$

$= \frac{3}{{}^6C_3} = \frac{3}{20}$

$P(X = 5) = P(\{1, 2, 5\}, (\{1, 3, 5\}, \{1, 4, 5\},$

$\{2, 3, 5\}, \{2, 4, 5\}, \{3, 4, 5\}) = \frac{6}{{}^6C_3} = \frac{6}{20}$

$P(X = 6) = P(\{1, 2, 6\}, \{1, 3, 6\}, \{1, 4, 6\}, \{1, 5, 6\},$
$\{2, 3, 6\}, \{2, 4, 6\}, \{2, 5, 6\}, \{3, 4, 6\}$
$\{3, 5, 6\}, \{4, 5, 6\}) = \frac{10}{20}.$

∴ $E(X) = \Sigma p_i x_i = 3\times\frac{1}{20} + 4\times\frac{3}{20} + 5\times\frac{6}{20} + 6\times\frac{10}{20}.$

Hence, Mean $= \frac{1}{20}[3 + 12 + 30 + 60]$

$= \frac{105}{20} = \frac{21}{4}.$

Also $E(X^2) = 3^2 \times \frac{1}{20} + 4^2 \times \frac{3}{20} + 5^2 \times \frac{6}{20} + 6^2 \times \frac{10}{20}$

$= \frac{1}{20}[9 + 48 + 150 + 360] = \frac{567}{20}.$

Hence, Variance $= E(X^2) - (\Sigma(X))^2$

$= \frac{567}{20} - \frac{441}{16} = \frac{2268 - 2205}{80}$

$= \frac{63}{80} = 0.787.$

EXERCISE 13 (f)

Fast Track Answer Type Questions

FTATQ

1. The mean of the number of heads in the two tosses of a coin is.............. . **(Fill in the blank)** *(Kashmir B. 2013)*

Very Short Answer Type Questions

VSATQ

2. A random variable has the following distribution :

X :	– 1	0	1	2
P(X) :	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$

(*i*) Does it represent a probability function ?

(*ii*) If yes, find its mean and variance.

3. Find mean μ, variance σ^2 for the following probability distribution :

X :	0	1	2	3
P (X) :	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{5}$	$\frac{1}{8}$

4. Find μ, σ_x^2 and σ_x for each of the following probability distributions :

(*a*)

X :	0	1	3	5
P (X) :	0·2	0·5	0·2	0·1

(*b*)

X :	–2	–1	0	1	2
P(X) :	0·2	0·2	0·4	0·2	0·1

(*c*)

X :	–3	–1	0	4
P(X) :	0·2	0·4	0·3	0·1

(*d*)

X :	1	2	3	–2	–1
P(X) :	0·10	0·35	0·20	0·30	0·05

5. Two dice are thrown simultaneously. If 'X' denotes the number of sixes, find the expectation of 'X'.

(N.C.E.R.T.; H.P.B. 2010)

Short Answer Type Questions

SATQ

6. Find the mean and variance of the number obtained on a throw of an unbiased die.

(Assam B. 2018; Kerala B. 2014)

7. Let 'X' denote the sum of the numbers, obtained when two fair dice are rolled. Find the variance and standard deviation of 'X'.

(N.C.E.R.T. ; Kashmir B. 2013; Assam B. 2013)

8. (*a*) Find the mean and variance of the probability distribution of the number of sixes in three tosses of a die.

(*b*) Find the mean of the probability distribution of the number of doublets in three throws of a pair of dice.

(C.B.S.E. 2010 C)

9. Two cards are drawn simultaneously from a well shuffled deck of 52 cards. Find the mean and standard deviation of the number of kings.

10. (*a*) Two cards are drawn simultaneously (or successively, without replacement) from a well shuffled deck of 52 cards. Compute the mean, variance and standard deviation for the number of

(*i*) kings *(N.C.E.R.T; H.P.B. 2013 ; A.I.C.B.S.E. 2009 C)*

(*ii*) queen *(Assam B. 2017)* (*iii*) aces. *(N.C.E.R.T)*

(*b*) Two cards are drawn (without replacement) from a well shuffled deck of 52 cards. Find probablility distribution and mean of number of cards numbered 2. *(P.B. 2017)*

11. A coin is tossed 4 times. Let X denote the number of heads. Find the probability distribution of X, its mean and variance. *(Nagaland B. 2016)*

12. Find the mean, variance and standard deviation of the number of heads in three tosses of a coin (or a simultaneous toss of three coins).

(N.C.E.R.T.; C.B.S.E. (F) 2011; H.P.B. 2016, 11)

13. Two numbers are selected at random (without replacement) from the first six positive integers. Let 'X' denote the larger of the two numbers obtained. Find the probability distribution of the random variable 'X' and hence find the mean of the distribution. *(N.C.E.R.T.; A.I.C.B.S.E. 2014)*

14. Two bad eggs are mixed accidentally with 10 good ones. Find the probability distribution of the number of bad eggs in 3 eggs drawn at random in succession, without replacement from the lot. Find the mean number of bad eggs drawn.

15. A class has 15 students whose ages are :

14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age 'X' of the selected student is recorded. What is the probability distribution of the random variable 'X' ? Find the mean.

(N.C.E.R.T.)

16. There are 40 scholars in a class, out of which 10 are sports-persons. Three scholars are selected at random out of them. Find the probability distribution for the selected persons who are sports-persons. Find the mean of the distribution.

17.

X :	1	2	3	4	5
P(X) :	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	p

The probability distribution of a random variable 'X', taking values 1, 2, 3, 4, 5 is given :

(*a*) Find the value of p.

(*b*) Find the mean of X.

(*a*) Find the variance of X. ***(Kerala B. 2015)***

Long Answer Type Questions

LATQ

18. Four bad oranges are accidently mixed with 16 good ones. Find the probability distribution of the number of bad oranges when two oranges are drawn at random from this lot. Find the mean and variance of the distribution. ***(C.B.S.E. Sample Paper 2018)***

19. From a lot of 15 bulbs, which includes 5 defectives, a sample of 4 bulbs is drawn one by one with replacement. Find the probability distribution of the number of defective bulbs. Hence, find the mean of the distribution. ***(C.B.S.E. 2014)***

20. Find the probability distribution of the number of white balls drawn in a random of 3 balls without replacement from a bag of 4 white and 6 red balls. Also find the mean and variance of the distribution. **HOTS**

21. Two numbers are selected at random (without replacement) from first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of X. Find the mean and variance of this distribution. ***(A.I.C.B.S.E. 2015)***

22. An urn contains 3 white and 6 red balls. Four balls are drawn one by one with replacement from the urn. Find the probability distribution of the number of red balls drawn. Also find mean and variance of the distribution. ***(C.B.S.E. 2016)***

Answers

1. 1. **2.** (*i*) Yes, because $P(X) \geq 0$, for all X and $\Sigma p_i = 1$

(*ii*) $\mu = \frac{1}{2}, \sigma^2 = \frac{19}{12}$. **3.** $\mu = \frac{3}{2}, \sigma^2 = \frac{3}{4}$.

4. (*a*) $\mu = 1{\cdot}6,\ \sigma_x^2 = 2{\cdot}24,\ \sigma_x = 1{\cdot}497$

(*b*) $\mu = -0{\cdot}2,\ \sigma_x^2 = 1{\cdot}2,\ \sigma_x = 1{\cdot}095$

(*c*) $\mu = -0{\cdot}6,\ \sigma_x^2 = 3{\cdot}44,\ \sigma_x = 1{\cdot}85$

(*d*) $\mu = 0{\cdot}75,\ \sigma_x^2 = 3{\cdot}9875,\ \sigma_x = 2$.

5. $\frac{1}{3}$. **6.** $\mu = 3{\cdot}5,\ \sigma_x^2 = 2{\cdot}91$.

7. $\sigma_x^2 = 5{\cdot}83,\ \sigma_x = 2{\cdot}415$.

8. (*a*) $\mu = \frac{19}{36}, \sigma_x^2 = \frac{2}{9}$ (*b*) $\mu = \frac{91}{216}$.

9. $\mu = \frac{2}{13}, \sigma_x = 0{\cdot}373$.

10. (*a*) (*i*) – (*iii*) $\frac{34}{221}, \frac{6800}{(221)^2}, 0{\cdot}37$.

(*b*) $P(0) = \frac{192}{221}, P(1) = \frac{32}{221}, P(2) = \frac{1}{221}; \frac{34}{221}$.

11. $P(0) = \frac{1}{16}, P(1) = \frac{1}{4}, P(2) = \frac{3}{8}$,

$P(3) = \frac{1}{4}, P(4) = \frac{1}{10}$; 2, 1.

12. $\mu = \frac{3}{2}, \sigma_x^2 = \frac{3}{4}, \sigma_x = 0{\cdot}87$.

13. (*i*)

X :	2	3	4	5	6
P(X) :	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

(*ii*) $E(X) = \frac{14}{3}$. **14.** $\frac{1}{2}$.

15. $P(14) = \frac{2}{15}, P(15) = \frac{1}{15}, P(16) = \frac{2}{15}, P(17) = \frac{3}{15}$,

$P(18) = \frac{1}{15}, P(19) = \frac{2}{15}, P(20) = \frac{3}{15}, P(21) = \frac{1}{15}$;

$\mu = 17{\cdot}53$.

16. Mean $= \frac{3}{4}$.

17. (*a*) $\frac{1}{16}$ (*b*) $\frac{31}{16}$ (*c*) $\frac{367}{256}$.

18. $P(0) = \frac{16}{25}, P(1) = \frac{8}{25}, P(2) = \frac{1}{25}$;

Mean $= \frac{2}{5}$; Variance $= \frac{8}{25}$.

19. $P(0) = \frac{16}{81}, P(1) = \frac{32}{81}, P(2) = \frac{24}{81}, P(3) = \frac{8}{81}$,

$P(4) = \frac{1}{81}$; Mean $= \frac{4}{3}$.

20. $P(0) = \frac{1}{6}, P(1) = \frac{1}{2}, P(2) = \frac{3}{10}, P(3) = \frac{1}{30}$;

Mean $= \frac{6}{5}$; Variance $= \frac{14}{25}$.

21. $\frac{14}{3}, \frac{14}{9}$.

22. $P(0) = \frac{1}{81}$, $P(1) = \frac{8}{81}$, $P(2) = \frac{24}{81}$, $P(3) = \frac{32}{81}$, $P(4) = \frac{16}{81}$; Mean $= \frac{8}{3}$; Variance $= \frac{8}{9}$.

Hints to Selected Questions

13. P (X = 2) = P (2 and a number less than 2)

$$= P(1 \text{ and } 2) = \frac{1}{^6C_2} = \frac{1}{15}$$

P (X = 3) = P (3 and a number less than 3)

$$= P(\{(1, 3), (2, 3)\} = \frac{2}{^6C_2} = \frac{2}{15}$$

....

P (X = 6) = P (6 and a number less than 6)

$$= P(\{(1, 6), (2, 6), (3, 6), (4, 6), (5, 6)\}$$

$$= \frac{5}{^6C_2} = \frac{5}{15}.$$

$$\therefore \quad E(X) = 2 \times \frac{1}{15} + 3 \times \frac{2}{15} + ... + 6 \times \frac{5}{15}.$$

SUB CHAPTER 13.4 Binomial Distribution

13.17. INTRODUCTION

We classify frequency distribution into two heads :
(*i*) Observed frequency distribution
and (*ii*) Theoretical (or Expected) frequency distribution.
So far we have done observed frequency distributions, which are based on observation and experimentation.
Amongst theoretical or expected frequency distributions, the following are very important :
(*i*) Binomial distribution (*ii*) Poisson distribution (*iii*) Normal distribution.
The first two are of discrete type while the last one is of continuous type.
Here we shall discuss only the first one.

13.18. BERNOULLI TRIALS

The independent trials having only two outcomes (usually '*success*' or '*failure*') are called **Bernoulli trials.**

Definition

Trials of a random experiment are said to be Bernoulli trials if they satisfy the following :

(i) Number of trials should be finite
(ii) Trials should be independent
(iii) Every trial has exactly two outcomes viz. success or failure
(iv) Probability of success remains the same in each trial.

13.19. BINOMIAL DISTRIBUTION

James Bernoulli

It was discovered by a Swiss Mathematician **James Bernoulli** (1654 – 1705). This is a probability distribution, which expresses the probability of one set of dichotomous alternative *i.e.* success or failure.

Let us perform a random experiment. Let p be the probability of occurrence of an event and q that of its failure, then $p + q = 1$.

Let us repeat the above experiment n times, assuming that all the trials are independent and the probabilities of success and failure remain to be the same.

We are to find the probability of successes 0, 1, 2,........, n.

Let there be r successes in n trials.

Then there will be $n - r$ failures.

Thus we have as follows :

SSS...... r times FFF...... $(n - r)$ times, where S ≡ Success and F ≡ Failure.

Since each trial is independent,

∴ the probability of r successes and $(n - r)$ failures is $p^r q^{n-r}$. *[Theorem of Compound Probability]*

But r successes in n trials can occur in ${}^nC_r$ ways.

∴ The probability of occurrence of the event in one of ${}^nC_r$ ways is ${}^nC_r q^{n-r} p^r$. *[Theorem of Total Probability]*

For Example : *Let the two coins be tossed.*

If H ≡ Head and T ≡ Tail, the following will occur as :

TT ; TH, HT ; HH.

Let the three coins be tossed.

The following will occur as :

TTT ; TTH, THT, HTT ; THH, HTH, HHT ; HHH.

[The first set having 3 tails, second two tails, third one tail, fourth no tail]

The respective probabilities of the occurrence of these events are :

q^3, (q^2p, q^2p, q^2p), (qp^2, qp^2, qp^2), p^3.

The probability when one of these events occurs $= q^3 + 3qp^2 + 3pq^2 + p^3 = {}^3C_0q^3 + {}^3C_1q^2p + {}^3C_2qp^2 + {}^3C_3p^3$.

The probability of 0 success is ${}^3C_0q^3$.

The probability of 1 success is ${}^3C_1q^2p$.

The probability of 2 successes is ${}^3C_2\, qp^2$.

The probability of 3 successes is ${}^3C_3p^3$.

These form the successive terms in the expansion of $(q + p)^3$.

Let n coins be tossed. As above, the probabilities of successes 0, 1, 2,......,n are the successive terms in the expansion of $(q + p)^n$.

This is called **Binomial Probability Distribution** or **Binomial Distribution.**

General Term. P(r) , the probability of r successes, is given by :

$$\mathbf{P\,(r) = {}^nC_r\, q^{n-r}\, p^r.}$$

Notation. A binomial distribution with n-Bernoulli trials and probability of success in each trial as p, is denoted by B (n, p).

Cor. Excepted Frequency.

We are to obtain the probable frequencies of the various outcomes, in N sets of n trials, the following expression shall be used :

$$N\,(q + p)^n,$$

where $$N\,(q + p)^n = N\,(q^n + {}^nC_1\, q^{n-1}\, p + {}^nC_2\, q^{n-2}\, p^2 + ... + {}^nC_r\, q^{n-r}\, p^r + ... + p^n).$$

For application, see Solved **Examples 9–10.**

Frequently Asked Questions

FAQs

Example 1. Six balls are drawn successively from an urn containing 7 red and 9 black balls. Tell whether or not the trials of drawing black balls are Bernoulli trials when after each draw the ball drawn is :

(i) replaced

(ii) not replaced

in the urn. *(N.C.E.R.T.)*

Solution. No. of trials is 6, which is finite.

(*i*) *When the drawing is done with replacement.*

Here the probability of success (say black ball) $= \frac{9}{16}$,

which is same in each of six trials (draws).

Hence, the trials are Bernoulli trials.

(*ii*) *When the drawing is done without replacement.*

Here the probability of success (black ball) in first trial $= \frac{9}{16}$ in second trial $\frac{8}{15}$ if first ball drawn in black or $\frac{9}{15}$ if first ball drawn is red.

Thus the probability of success is not the same for all trials.

Hence, the trials are not Bernoulli trials.

Example 2. An experiment succeeds thrice as often as it fails. Find the probability that in the next five trials, there will be at least 3 successes. *(A.I.C.B.S.E. 2014)*

Solution. Let 'p' be the probability of success and 'q' be the probability of failure.

Then $p + q = 1$ and $p = 3q$.

Solving, $p = \frac{3}{4}$ and $q = \frac{1}{4}$.

Also $n = 5$.

$\therefore$ Reqd. probability $= P(X \geq 3) = P(3) + P(4) + P(5)$

$$= {}^5C_3\, q^2p^3 + {}^5C_4\, q^1p^4 + {}^5C_5\, q^0p^5$$

$$= 10\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3 + 5\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 + (1)(1)\left(\frac{3}{4}\right)^5$$

$$= \left(\frac{3}{4}\right)^3\left[\frac{10}{16} + \frac{15}{16} + \frac{9}{16}\right]$$

$$= \frac{34}{16}\left(\frac{3}{4}\right)^3.$$

Example 3. An unbiased coin is tossed 6 times. Using binomial distribution, find the probability of getting at least 5 heads. *(W. Bengal B. 2018)*

Solution. Here 'p', probability of success (getting a head) $= \frac{1}{2}$ and

'q', probability of failure (not getting a head) $= 1 - \frac{1}{2} = \frac{1}{2}$

and $n = 6$.

$\therefore$P (at least 5 heads) = P (5) + P (6)

$$= {}^6C_5\, q\, p^5 + {}^6C_6\, q^0\, p^6$$

$$= (6)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^5 + (1)(1)\left(\frac{1}{2}\right)^6$$

$$= \frac{6}{64} + \frac{1}{64} = \frac{7}{64}.$$

Example 4. A pair of dice is thrown 4 times. If getting a doublet is considered as a success,

(*i*) find the probability of getting a doublet

(*ii*) hence, find the probability of two succeses.

(Kerala B. 2016)

Solution. Let 'p' be the probability of success and 'q' be the probability of failure.

And $n = 4$.

(*i*) p, probability of success $= \frac{6}{36} = \frac{1}{6}$.

[(1, 1), (2, 2), ..., (6, 6)]

(*ii*) We have : $p = \frac{1}{6}$, $q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$ and $n = 4$.

$\therefore$ $P(X = 2) = {}^4C_2\, q^2p^2$

$$= \frac{4 \times 3}{1 \times 2} \times \left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right)^2$$

$$= 6 \times \frac{25}{36 \times 36} = \frac{25}{216}.$$

Example 5. The probability that a student entering the university will graduate is 0·4. Find the probability that out of 3 students of the University :

(i) none will graduate (ii) only one will graduate

(iii) all will graduate.

Solution. We have : $n = 3$, $p = 0{\cdot}4$

and $q = 1 - 0{\cdot}4 = 0{\cdot}6$.

Now $P(r) = {}^3C_r q^{3-r} p^r$.

(*i*) $P(0) = {}^3C_0\, (0{\cdot}6)^3\, (0{\cdot}4)^0 = (0{\cdot}6)^3$.

(*ii*) $P(1) = {}^3C_1\, (0{\cdot}6)^2\, (0{\cdot}4)^1 = \frac{6}{5}\, (0{\cdot}6)^2$.

(*iii*) $P(3) = {}^3C_3\, (0{\cdot}6)^0\, (0{\cdot}4)^3 = (0{\cdot}4)^3$.

Example 6. Ten eggs are drawn successively with replacement, from a lot containing 10% defective eggs. Find the probability that there is at least one defective egg. *(N.C.E.R.T.)*

Solution. Here $p = \frac{10}{100} = \frac{1}{10}$.

$\therefore$ $q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$ and $n = 10$.

$\therefore$ P (at least one defective egg)

$$= P(X \geq 1) = 1 - P(X = 0)$$

$$= 1 - {}^{10}C_0 \left(\frac{9}{10}\right)^{10} \left(\frac{1}{10}\right)^0$$

$$= 1 - \frac{9^{10}}{10^{10}}.$$

Example 7. The probability of a shooter hitting a target is $\frac{3}{4}$. How many minimum number of times must he/she fire so that the probability of hitting the target at least once is more than 0·99 ? *(N.C.E.R.T.; H.B. 2017)*

Solution. Let the shooter fire n times.

In each trial, 'p', probability of hitting the target $= \frac{3}{4}$

and 'q', probability of not hitting the target $= 1 - \frac{3}{4} = \frac{1}{4}$.

Now $P(X = r) = {}^nC_r\, q^{n-r} p^r = {}^nC_r \left(\frac{1}{4}\right)^{n-r} \left(\frac{3}{4}\right)^r$

$$= {}^nC_r \frac{3^r}{4^n}.$$

By the question, $P(X \geq 1) > 0{\cdot}99$

$\Rightarrow$ $1 - P(0) > 0{\cdot}99$

$\Rightarrow$ $1 - {}^nC_0 \left(\frac{1}{4}\right)^n \left(\frac{3}{4}\right)^0 > 0{\cdot}99$

$\Rightarrow$ ${}^nC_0 \frac{1}{4^n} < 0{\cdot}01$ $\Rightarrow$ $\frac{1}{4^n} < 0{\cdot}01$

$\Rightarrow$ $4^n > \frac{1}{0{\cdot}01}$ $\Rightarrow$ $4^n > 100$

$\Rightarrow$ minimum value of $n = 4$.

Hence, the shooter must fire at least 4 times.

Example 8. Five dice are thrown 729 times. How many times do you expect that at least four dice to show five or six ?

Solution. Here p, probability of getting 5 or 6 with one dice

$= \frac{2}{6} = \frac{1}{3}$, $q = 1 - \frac{1}{3} = \frac{2}{3}$, $n = 5$, N = 729.

The dice are in sets of 5 and there are 729 sets.

The binomial distribution is $N(q + p)^n$

$$= 729 \left(\frac{2}{3} + \frac{1}{3}\right)^5.$$

The expected number of times at least four dice showing five or six

$$= 729\left[{}^5C_4 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^4 + {}^5C_5 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^5\right]$$

$$= 729\left[(5)\left(\frac{2}{3}\right)\left(\frac{1}{81}\right) + (1)(1)\left(\frac{1}{243}\right)\right]$$

$$= 729\left[\frac{10}{243} + \frac{1}{243}\right] = 3(10 + 1) = 3(11) = 33.$$

Example 9. In a backward state, there are 729 families having six children each. If probability of survival of a girl is $\frac{1}{3}$ and that of a boy is $\frac{2}{3}$, find the number of families having 2 girls and 4 boys.

Solution. Let N be the number of families having six children.

Thus N = 729.

Let 'p' be the probability of survival of a girl $= \frac{1}{3}$ and 'q' that of boy $= \frac{2}{3}$.

Let 'X' be number of girls in the family.

$\therefore$ 'X' takes values 0, 1, 2, 3, 4, 5, 6.

Now $P(r) = {}^nC_r q^{n-r} p^r$

$$= {}^6C_r \left(\frac{2}{3}\right)^{6-r} \left(\frac{1}{3}\right)^r.$$

$\therefore$ Probability of 2 girls and 4 boys in the family,

$$P(2) = {}^6C_2 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 = \left(\frac{6 \times 5}{2 \times 1}\right)\left(\frac{16}{81}\right)\left(\frac{1}{9}\right) = \frac{80}{243}.$$

$\therefore$ Number of families having 2 girls and 4 boys

$$= N \times P(2)$$

$$= 729 \times \frac{80}{243} = 3 \times 80 = 240.$$

Example 10. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth trial of the die. *(C.B.S.E. 2009)*

Solution. Here 'p', probability of getting a six $= \frac{1}{6}$

and 'q', probability of not getting a six $= 1 - \frac{1}{6} = \frac{5}{6}$.

$\therefore$ Reqd. probability = (Two sixes in first 5 throws) + (six in the 6th throw)

$$= \frac{{}^5C_2\, q^3 \times p^2}{6^5} \times \frac{1}{6}$$

$$= \frac{(10)\left(\frac{5}{6}\right)^3 \left(\frac{1}{6}\right)^2}{6^6} = \frac{(10)(5^3)}{6^{11}}.$$

Example 11. In an examination, 10 questions of true-false type are asked. A student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers true and if it falls tails, he answers 'false'. Show that the probability that he answers at most 7 questions correctly is $\frac{121}{128}$.

Solution. P (Answer is true) $= \frac{1}{2} (= p)$.

P (Answer is false) $= \frac{1}{2} (= q)$.

And $n = 10$.

Now P (at most 7 correct answers)

$$= 1 - (P(8) + P(9) + P(10))$$

$$= 1 - \left({}^{10}C_8 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^8 + {}^{10}C_9 \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^9 + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \right)$$

$$= 1 - \left({}^{10}C_2 + {}^{10}C_1 + {}^{10}C_0\right)\left(\frac{1}{2}\right)^{10}$$

$$= 1 - \left(\frac{10 \times 9}{1 \times 2} + 10 + 1\right)\left(\frac{1}{1024}\right)$$

$$= 1 - (45 + 10 + 1)\left(\frac{1}{1024}\right)$$

$$= 1 - \frac{56}{1024} = 1 - \frac{7}{128} = \frac{121}{128},$$

which is true.

EXERCISE 13 (g)

Fast Track Answer Type Questions

FTATQ

1. (*a*) Obtain binomial probability distribution, if :

(*i*) $n = 6, p = \frac{1}{3}$ (*ii*) $n = 5, p = \frac{1}{6}$.

(*b*) Suppose X has a binomial distribution $B\left(6, \frac{1}{2}\right)$. Show that X = 3 is the most likely outcome.

2. (*i*) A coin is tossed 7 times. What is the probability that head appears an odd number of times ?

(*ii*) A coin is tossed 7 times. What is the probability that tail appears an odd number of times ?

(*iii*) A coin is tossed 5 times. What is the probability that head appears an odd number of times ?

Very Short Answer Type Questions

VSATQ

3. (*i*) A coin is tossed 5 times. What is the probability of getting :

(*a*) at least 3 heads (*b*) at most 2 heads

(*c*) no head (*d*) 3 heads ?

(*ii*) If a fair coin is tossed 10 times, find the probability of :

(*a*) exactly four heads ***(Rajasthan B. 2012)***

(*b*) exactly six heads ***(H.P.B. 2018, 16, 10 S, 10)***

(*c*) at least six heads ***(H.P.B. 2018, 16, 10 S, 10)***

(*d*) at most six heads. ***(N.C.E.R.T.; H.P.B. 2018, 16, 10)***

4. Find the probability of :

(*i*) getting 5 exactly twice in 7 throws of a die

(*ii*) throwing at most 2 sixes in 6 throws of a single die.

(Jammu B. 2012 ; A.I.C.B.S.E. 2011)

5. (*i*) A die is thrown 5 times. If getting an 'odd number' is success, find the probability of getting at least 4 successes.

(*ii*) A die is thrown 6 times. If getting an 'odd (even) number' is a success, what is the probability of :

(I) 5 successes (II) at least 5 successes (III) at most 5 successes (IV) no success ?

(N.C.E.R.T.; H.P.B. 2018, 15, 10 ; Jammu B. 2018, 17, 16, 15 W, 15, 13, 10; Kashmir B. 2017, 16; Kerala B. 2014; H.B. 2012)

(*iii*) A die is thrown 10 times. If getting an even number is considered a success, find the probability of at least 9 successes.

6. A pair of dice is thrown 7 times. If 'getting a total of 7' is considered 'success', find the probability of getting :

(*i*) no success ***(Nagaland B. 2016)***

(*ii*) at least 6 successes.

(Mizoram B. 2016; Nagaland B. 2016)

7. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least one defective egg. ***(H.P.B. 2011)***

8. Find the probability of :

(*i*) getting 5 exactly twice in 7 throws of a die

(H.P.B. 2011)

(*ii*) throwing at most 2 sixes in 6 throws of a single die.

(N.C.E.R.T.)

9. Probability of a shooter of hitting the target is $\frac{3}{4}$. If he shoots 10 times, find the probability of hitting 8 targets successfully. ***(H.B. 2016)***

10. Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right–handed ? ***(N.C.E.R.T.; H.B. 2016)***

11. Four dice are thrown simultaneously. If the occurrence of :

(*i*) 2, 4 or 6 (*ii*) 2, 3 or 4

in single die is considered a success, find the probability of at least three successes. ***(P.B. 2011)***

12. A bag consists of 10 balls each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0 ? *(N.C.E.R.T.)*

13. An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark noted down and is replaced. If 6 balls are drawn in this way, find the probability that :

(*i*) all will bear 'X' mark

(*ii*) not more than 2 will bear 'Y' mark *(H.P.B. 2013 S)*

(*iii*) at least one ball will bear 'Y' mark *(H.P.B. 2013 S)*

(*iv*) the number of balls with 'X' mark and 'Y' mark will be equal. *(N.C.E.R.T. ; H.P.B. 2013 S)*

14. There are 5 per cent defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item ? *(N.C.E.R.T.)*

15. The items produced by a company contain 10% defective items. Show that the probability of getting 2 defective items in a sample of 8 items is $\frac{28 \times 9^6}{10^8}$.

16. In a box containing 100 bulbs, 10 are defective. What is the probability that out of a sample of 5 bulbs, (*i*) none is defective and (*ii*) exactly 2 are defective ? *(Kashmir B. 2011)*

17. The probability that a bulb produced by a factory will fuse after 160 days of use is 0·06. Find the probability that out of 5 such bulbs at the most one bulb will fuse after 160 days of use.

18. In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles ? *(N.C.E.R.T.)*

19. Assume that on an average one telephone number out of 15 called between 2 P.M. and 3 P.M. on week days is busy. What is the probability that if six randomly selected telephone numbers are called, at least three of them will be busy ?

20. If getting a '5' or a '6' in the throw of an unbiased die is a 'success' and the random variable 'X' denotes the number of succeses in six throws of the die, find $P_X[X \geq 4]$.

21. On a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate should get four or more correct answers just by guessing ? *(N.C.E.R.T.; C.B.S.E. 2010; A.I.C.B.S.E. 2009)*

Long Answer Type Questions

LATQ

22. Calculate P (*r*) for *r* = 1, 2, 3, 4 and 5 by using the recurrence formula of the binomial distribution for the following. Hence, draw the histogram for the distribution :

(*i*) $p = \frac{1}{3}$; $n = 5$ (*ii*) $p = \frac{1}{6}$; $n = 5$.

23. Six dice are thrown 729 times. How many times do you expect at least three dice to show a five or a six ?

Answers

1. (*a*) (*i*) $\left(\frac{2}{3}+\frac{1}{3}\right)^6$ (*ii*) $\left(\frac{5}{6}+\frac{1}{6}\right)^5$.

2. (*i*) – (*iii*) $\frac{1}{2}$.

3. (*i*) (*a*) – (*b*) $\frac{1}{2}$ (*c*) $\frac{1}{32}$ (*d*) $\frac{5}{16}$

(*ii*) (*a*) – (*b*) $\frac{105}{512}$ (*c*) $\frac{193}{512}$ (*d*) $\frac{53}{64}$.

4. (*i*) $\frac{21 \times 5^5}{6^7}$ (*ii*) $\frac{70 \times 5^4}{6^6}$.

5. (*i*) $\frac{3}{16}$ (*ii*) (I) $\frac{3}{32}$ (II) $\frac{7}{64}$ (III) $\frac{63}{64}$ (IV) $\frac{1}{64}$

(*iii*) $\frac{11}{1024}$. **6.** (*i*) $\left(\frac{5}{6}\right)^7$ (*ii*) $\frac{1}{6^5}$.

7. $1-\frac{9^{10}}{10^{10}}$. **8.** (*i*) $\frac{7}{12}\left(\frac{5}{6}\right)^5$ (*ii*) $\frac{35}{18}\left(\frac{5}{6}\right)^4$.

9. $\frac{45\ (3)^8}{4^{10}}$. **10.** $1-\sum_{r=7}^{10} {}^{10}C_r\,(0{\cdot}9)^r\,(0{\cdot}1)^{10-r}$.

11. (*i*) – (*ii*)

12. $\left(\frac{9}{10}\right)^4$. **13.** (*i*) $\left(\frac{2}{5}\right)^6$ (*ii*) $7\left(\frac{2}{5}\right)^4$

(*iii*) $1-\left(\frac{2}{5}\right)^6$ (*iv*) $20\left(\frac{6}{25}\right)^3$.

14. $\left(\frac{19}{20}\right)^9 \times \frac{29}{20}$.

16. (*i*) $\left(\frac{9}{10}\right)^5$ (*ii*) 0·0729.

17. $\frac{31}{25}\left(\frac{47}{50}\right)^4$.

18. $\frac{5}{2}\left(\frac{5}{6}\right)^9$.

19. $1 - 295\frac{(14)^4}{(15)^6}$.

20. $P_4 = \frac{20}{243}, P_5 = \frac{4}{243}, P_6 = \frac{1}{729}$.

21. $\frac{11}{3^5}$.

22. (*i*) P (1) = 0·33, P (2) = 0·33, P (3) = 0·16, P (4) = 0·04, P (5) = 0

(*ii*) $P(1) = \frac{3125}{7776}, P(2) = \frac{1250}{7776}, P(3) = \frac{250}{7776}, P(4) = \frac{25}{7776}, P(5) = \frac{1}{7776}$.

23. 233.

Hints to Selected Questions

1. (*b*) P (X = 3) is maximum among all $P(x_i), x_i = 0, 1, 2, 3, 4, 5, 6$.

2. (*i*) $n = 7, p = \frac{3}{6} = \frac{1}{2}, q = 1 - \frac{1}{2} = \frac{1}{2}$.

Find P (1), P (3), P (5).

14. $p = \frac{5}{100} = \frac{1}{20}, q = 1 - \frac{1}{20} = \frac{19}{20}, n = 10$.

Find P (0) + P (1).

20. $p = \frac{2}{6} = \frac{1}{3}, q = 1 - \frac{1}{3} = \frac{2}{3}, n = 6$.

Find P (4) + P (5) + P (6).

13.20. MEAN AND VARIANCE OF BINOMIAL DISTRIBUTION

Find the mean and variance of a binomial distribution with parameters 'p' and 'n'.

[**Moment.** The *r*th moment μ_r of a variate *x* about any arbitrary point '*a*' is defined as :

$$\mu_r = \sum_{i=1}^{n} f_r (x_i - a)^r, \text{ where } N = \sum_{i=1}^{n} f_i.]$$

(i) Mean of binomial distribution is given by :

$$\mu_r = \sum_{x=0}^{n} xP(x) = \sum_{x=0}^{n} x\,{}^nC_x q^{n-x} p^x = 0 + 1.\,{}^nC_1 q^{n-1}p + 2.\,{}^nC_2 q^{n-2}p^2 + 3.\,{}^nC_3 q^{n-3}p^3 + \ldots\ldots + n.\,p^n$$

$$= 1.\,\frac{n}{1} q^{n-1} p + 2.\,\frac{n(n-1)}{1\cdot 2} q^{n-2} p^2 + 3\,\frac{n(n-1)(n-2)}{1\cdot 2\cdot 3} q^{n-3} p^3 + \ldots\ldots + np^n$$

$$= np\left[q^{n-1} + (n-1)q^{n-2}p + \frac{(n-1)(n-2)}{1\cdot 2} q^{n-3}p^2 + \ldots\ldots + p^{n-1}\right]$$

$$= np\left[q^{n-1} + {}^{n-1}C_1 q^{n-2} p + {}^{n-1}C_2 q^{n-3} p^2 + \ldots\ldots + p^{n-1}\right]$$

$$= np(q+p)^{n-1} = np(1)^{n-1} \qquad [\because q + p = 1]$$

$$= np.$$

Hence, **Mean = np.**

(ii) Variance is given by :

$$\therefore \qquad \sigma_x^2 = \sum_{x=0}^{n} x^2 P(x) - \bar{x}^2.$$

Now $$\sum_{x=0}^{n} x^2 P(x) = \sum_{x=0}^{n} x^2\, {}^nC_x q^{n-x} p^x$$

$$= \sum_{x=0}^{n} [x(x-1) + x]\, {}^nC_x\, q^{n-x} p^x \qquad [\because x^2 = x(x-1) + x]$$

$$= \sum_{x=0}^{n} x(x-1)\, {}^nC_x\, q^{n-x} p^x + \sum_{x=0}^{n} x\, {}^nC_x\, q^{n-x} p^x$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n(n-1) \ldots (n-x+1)}{1.2 \ldots x} q^{n-x} p^x + np \qquad [Using\ part\ (i)]$$

$$= \sum_{x=0}^{n} n(n-1) \frac{(n-2)(n-3) \ldots (n-x+1)}{1.2 \ldots (x-2)} q^{n-x} p^x + np$$

$$= n(n-1) \sum_{x=2}^{n} \frac{(n-2)(n-3) \ldots (n-x+1)}{1.2 \ldots (x-2)} q^{n-x} p^x + np$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)(n-3) \ldots (n-x+1)}{1.2 \ldots (x-2)} q^{n-x} p^{x-2} + np$$

$$= n(n-1)p^2 [q^{n-2} + {}^{n-2}C_1\, q^{n-3} p + {}^{n-2}C_2\, q^{n-4} p^2 + \ldots + p^{n-2}] + np$$

$$= n(n-1)p^2 [q+p]^{n-2} + np = n(n-1)p^2 (1) + np \qquad [\because q + p = 1]$$

$$= n(n-1)p^2 + np$$

Also, $\bar{x} = np$ ...(2)

$\therefore$ From (1), using (2) and (3), $\sigma^2 = n(n-1)p^2 + np - n^2p^2$ ...(3)

$$= n^2p^2 - np^2 + np - n^2p^2 = -np^2 + np = np(1-p)$$

$$= npq. \qquad [\because p + q = 1 \Rightarrow 1 - p = q]$$

Hence, **Variance = npq.**

Also, S. D., standard deviation $= \sigma$.

Hence, **S.D. = $\sqrt{\mathbf{npq}}$.**

Note. Mean is also called the **average, expected value** or the **expectation** of the distribution and is usually denoted by E (x).

Thus $\quad E(x) = \mu = \sum_{i=1}^{n} x_i p_i.$

KEY POINT

In Binomial distribution, variance is always less than mean.

13.21. FITTING OF BINOMIAL DISTRIBUTION

Suppose a random experiment consists of n trials, which satisfy the conditions of binomial distribution and further suppose that this experiment is repeated N times. Then the frequency of r successes is given by the following formula :

$$N \times P(X = r) = N \times {}^nC_r\, q^{n-r} p^r \;;\; r = 0, 1, 2, \ldots, n.$$

Putting $r = 0, 1, 2, \ldots, n$; we get the **expected** or theoretical frequencies of the Binomial distribution, which are given in the following table :

No. of successes (r)	*Expected or Theoretical frequencies* $N \times P(X = r)$
0	Nq^n
1	$N \times {}^nC_1\, q^{n-1}\, p$
2	$N \times {}^nC_2\, q^{n-2}\, p^2$
:	:
:	:
:	:
n	Np^n

If 'p', the probability of success is not known, we first find the mean of the given frequency distribution by the formula $\overline{X} = \dfrac{\Sigma fx}{\Sigma f}$ and equate it to np, which is the mean of the binomial probability distribution.

Hence, p can be calculated by the relation :

$np = \overline{X}$ or $p = \dfrac{\overline{X}}{n}$, then $q = 1 - p$.

ILLUSTRATIVE EXAMPLES

Example 1. Find the mean of the binomial distribution B $\left(4, \frac{1}{3}\right)$. (*N.C.E.R.T.*)

Solution. We have :

$$n = 4,\ p = \frac{1}{3} \text{ and } q = 1 - \frac{1}{3} = \frac{2}{3}.$$

$$\text{Now } P(X = r) = {}^4C_r \left(\frac{2}{3}\right)^{4-r} \left(\frac{1}{3}\right)^r ;\ r = 0, 1, 2, 3, 4.$$

The distribution of X is as below :

X	p	np
0	${}^4C_0 \left(\frac{2}{3}\right)^4$	0
1	${}^4C_1 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)$	${}^4C_1 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)$
2	${}^4C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^2$	$2\left({}^4C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^2\right)$
3	${}^4C_3 \left(\frac{2}{3}\right) \left(\frac{1}{3}\right)^3$	$3\left({}^4C_3 \left(\frac{2}{3}\right) \left(\frac{1}{3}\right)^3\right)$
4	${}^4C_4 \left(\frac{1}{3}\right)^4$	$4\left({}^4C_4 \left(\frac{1}{3}\right)^4\right)$

$\therefore$ Mean (μ) = np

$$= 0 + {}^4C_1 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right) + 2\left({}^4C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^2\right) + 3\left({}^4C_3 \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^3\right) + 4\left({}^4C_4 \left(\frac{1}{3}\right)^4\right)$$

$$= 4 \times \frac{2^3}{3^4} + 2 \times 6 \times \frac{2^2}{3^4} + 3 \times 4 \times \frac{2}{3^4} + 4 \times 1 \times \frac{1}{3^4}$$

$$= \frac{1}{3^4}(32 + 48 + 24 + 4)$$

$$= \frac{108}{81} = \frac{4}{3}.$$

Example 2. The sum of mean and variance of a binomial distribution is 15 and the sum of their squares is 117. Find the binomial distribution. (*P.B. 2012*)

Solution. We have : $np + npq = 15$ ...(1)

and $n^2p^2 + n^2p^2q^2 = 117$ ...(2)

From (1), $np(1 + q) = 15.$

Squaring, $n^2p^2(1 + q)^2 = 225$ ...(3)

From (2), $n^2p^2(1 + q^2) = 117$ ...(4)

Dividing (3) by (4), $\dfrac{(1+q)^2}{1+q^2} = \dfrac{225}{117}$

$\Rightarrow \dfrac{(1+q)^2}{1+q^2} = \dfrac{25}{13}$

$\Rightarrow 13 + 26q + 13q^2 = 25 + 25q^2$

$\Rightarrow 12q^2 - 26q + 12 = 0$

$\Rightarrow \quad 12q^2 - 18q - 8q + 12 = 0$

$\Rightarrow \quad 6q(2q-3) - 4(2q-3) = 0$

$\Rightarrow \quad (6q-4)(2q-3) = 0$

$\Rightarrow \quad q = \frac{4}{6}, \frac{3}{2} = \frac{2}{3}, \frac{3}{2}.$

Thus $\quad q = \frac{2}{3}. \quad [\because q \not> 1]$

$\therefore \quad p = 1 - \frac{2}{3} = \frac{1}{3}.$

Putting in (1), $n\left(\frac{1}{3}\right) + n\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = 15$

$\Rightarrow \quad 3n + 2n = 135$

$\Rightarrow \quad 5n = 135$

$\Rightarrow \quad n = 27.$

Hence, the binomial distribution is $\left(\frac{2}{3} + \frac{1}{3}\right)^{27}$.

Example 3. The mean and variance of the binomial distribution are 4 and $\frac{4}{3}$ respectively. Find P (X ≥ 1).

(Mizoram B. 2015)

Solution. We have : Mean $= np = 4$ ...(1)

Variance $= npq = \frac{4}{3}$ ...(2)

Dividing (2) by (1), $q = \frac{4}{3} \times \frac{1}{4} \Rightarrow q = \frac{1}{3}.$

$\therefore \quad p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}.$

$\therefore$ From (1), $n\left(\frac{2}{3}\right) = 4 \Rightarrow n = 6.$

Hence, $P(X \geq 1) = 1 - P(X = 0) = 1 - q^n = 1 - \left(\frac{1}{3}\right)^6.$

EXERCISE 13 (h)

Short Answer Type Questions

SATQ

1. An unbiased coin is tossed 4 times. Find the mean and variance of the number of heads obtained. *(C.B.S.E. 2015)*

2. If the mean and variance of a Binomial distribution are 9 and 6 respectively, find the number of trials.

(Kashmir B. 2011)

3. If the sum of the mean and variance of a binomial distribution of 18 trials is 10, determine the distribution.

(P.B. 2010 S)

4. If the sum of the mean and variance of a binomial distribution for 5 trials is $\frac{35}{16}$, find the binomial distribution.

(Kashmir B. 2011)

5. The mean and variance of a Binomial variable X are respectively 4 and $\frac{4}{5}$. Find $P(X \geq 3)$. *(Tripura B. 2016)*

6. If the sum of mean and variance of a binomial distribution is 1·8 for five trials, find the distribution.

(W. Bengal B. 2017)

7. Determine the binomial distribution whose mean is 10 and whose standard deviation is $2\sqrt{2}$.

8. Find the binomial distribution whose :

(*i*) mean is 4 and variance is 3

(*ii*) mean is 9 and variance is 6.

9. If 12 dice are rolled at random, obtain the mean and variance of the distribution of successes, if 'getting a number greater than 4' is considered a success.

10. A die is thrown 20 times and getting a number 'greater than 4' is considered a success. Find the mean and the variance of the number of successes.

11. 10 coins are tossed at random. Obtain the mean and variance of the number of heads obtained.

12. The sum and product of the mean and variance of a binomial distribution are 3·5 and 3 respectively. Find the binomial distribution.

13. A die is thrown 6 times. Find the mean and variance of the number of aces.

14. Eight dice are rolled at random. Find the mean and variance of number of successes if :

(*i*) Getting an odd number is success

(*ii*) Getting a number less than 3 is success.

15. Two dice are rolled at random 5 times. Obtain the mean and variance of the distribution of doublets obtained.

16. The mean and variance of a binomial distribution is 4 and $\frac{4}{3}$ respectively. Find $P(X \geq 1)$.

17. If the sum of mean and variance of a binomial distribution is 4·8 for 5 trials. Find the distribution.

18. A discrete random variable 'X' has mean equal to 3 and variance equal to 2. Assuming that the underlying distribution of 'X' is binomial, find the distribution and hence obtain :

(*i*) P (X = 0)

(*ii*) Draw a histogram for the distribution.

19. (*a*) Determine the binomial distribution whose mean is 10 and variance is 8.

(*b*) Write its probability function.

20. The screws produced by a certain machine were checked by examining samples of 7. The following table shows the distribution of 128 samples according to the number of defective items they contained.

No. of defectives in a sample of 7 is :

:	0	1	2	3	4	5	6	7
No of samples :	7	6	19	35	30	23	7	1

N = 128.

Fit a binomial distribution and find the expected frequencies if the chance of screw being defective is $\frac{1}{2}$. Also find the mean and variance of the fitted distribution.

Answers

1. 2, 1. **2.** 27. **3.** $\left(\frac{2}{3}+\frac{1}{3}\right)^{18}$.

4. $\left(\frac{3}{4}+\frac{1}{4}\right)^{5}$. **5.** $\frac{2944}{3125}$.

6. $\left(\frac{4}{5}+\frac{1}{5}\right)^{5}$. **7.** $\left(\frac{4}{5}+\frac{1}{5}\right)^{50}$.

8. (*i*) $\left(\frac{3}{4}+\frac{1}{4}\right)^{16}$ (*ii*) $\left(\frac{2}{3}+\frac{1}{3}\right)^{27}$.

9. 4, 2·5. **10.** 6·67, 4·44. **11.** Mean = 5, Variance = 2·5.

12. $\left(\frac{3}{4}+\frac{1}{4}\right)^{8}$. **13.** 1, $\frac{5}{6}$. **14.** (*i*) 4, 2 (*ii*) $\frac{8}{3}, \frac{16}{9}$.

15. $\frac{5}{6}, \frac{25}{36}$. **16.** $\frac{728}{729}$.

17. $\left(\frac{1}{5}+\frac{4}{5}\right)^{5}$.

18. $\left(\frac{2}{3}+\frac{1}{3}\right)^{9}$ (*i*) $\left(\frac{2}{3}\right)^{9}$.

19. (*a*) $\left(\frac{4}{5}+\frac{1}{5}\right)^{50}$

(*b*) $P(r) = {}^{50}C_r\left(\frac{4}{5}\right)^{50-r}\left(\frac{1}{5}\right)^{r}$; $r = 0, 1, 2, \ldots\ldots\ldots, 50$.

20. 3·5; 1·75; 1, 7, 21, 35, 35, 21, 7, 1.

Hints to Selected Questions

20. $n = 7$, N = 128 ; $p = \frac{1}{2}$, $q = \frac{1}{2}$.

Mean = $np = 7 \times \frac{1}{2} = 3{\cdot}5$.

Variance = $npq = 7 \times \frac{1}{2} \times \frac{1}{2} = 1{\cdot}75$.

Expected frequencies : 1, 7, 21, 35, 35, 21, 7, 1.

NCERT-FILE

Questions from NCERT Book

(For each unsolved question, refer : "Solution of Modern's abc of Mathematics")

Exercise 13.1

1. Given that E and F are events such that P(E) = 0·6, P(F) = 0·3 and P (E ∩ F) = 0·2, find P (E/F) and P (F / E).

Solution : (*i*) $P(E/F) = \frac{P(E \cap F)}{P(F)}$

$= \frac{0.2}{0.3} = \frac{2}{3}$.

(*i*) $P(F/E) = \frac{P(E \cap F)}{P(E)}$

$= \frac{0.2}{0.6} = \frac{2}{6} = \frac{1}{3}$.

2. Computer P (A/B), if P(B) = 0·5 and P (A ∩ B) = 0·32.

[Solution. Refer Q.1 (*b*), Ex. 13(*a*)]

3. If P(A) = 0·8, P(B) = 0·5 and P (B/A) = 0·4, find
(*i*) P (A ∩ B) (*ii*) P (A / B) (*iii*) P (A ∪ B).
[Solution. Refer Q.3, Ex. 13(*a*)].

4. Evaluate P (A ∪ B), if 2P(A) = P(B) = $\frac{5}{13}$ and

$P(A / B) = \frac{2}{5}$.

Solution : We have :

$$2\,P(A) = P(B) = \frac{5}{13}.$$

$$\therefore\ P(A) = \frac{5}{26} \text{ and } P(B) = \frac{5}{13}.$$

$$\text{And } P(A/B) = \frac{2}{5}$$

$$\Rightarrow \quad \frac{P(A\cap B)}{P(B)} = \frac{2}{5} \Rightarrow \frac{P(A\cap B)}{\frac{5}{13}} = \frac{2}{5}$$

$$\Rightarrow \quad P(A\cap B) = \frac{2}{5}\times\frac{5}{13} = \frac{2}{13}.$$

Now P (A ∪ B) = P (A) + P (B) – P (A ∩ B)

$$= \frac{5}{26}+\frac{5}{13}-\frac{2}{13} = \frac{5+10-4}{26} = \frac{11}{26}.$$

5. If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$, find :

(*i*) P (A ∩ B) (*ii*) P (A / B) (*iii*) P (B / A).
[Solution. Refer Q.5 (*a*) ; Ex. 13(*a*)]

6. A coin is tossed three times, where :

(*i*) E : head on third toss ,
F : heads on first two tosses
(*ii*) E : at least two heads,
F : at most two heads
(*iii*) E : at most two tails ,
F : at least one tail

[Solution. Refer Q. 6 (*a*); Ex. 13 (*a*)].

7. Two coins are tossed once, where :

(*i*) E : tail appears on one coin,
F : one coin shows head
(*ii*) E : no tail appears
F : no head appears

Solution :

E : tail appears on one coin = {TH, HT}
F : one coin shows head = {HT, TH}
∴ E ∩ F = {TH, HT}.

No. of exhaustive cases = 2 × 2 = 4.

$$\therefore \quad P(E\cap F) = \frac{2}{4} = \frac{1}{2}$$

$$\text{and } P(F) = \frac{2}{4} = \frac{1}{2}.$$

$$\therefore \quad P(E/F) = \frac{P(E\cap F)}{P(F)} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1.$$

(*ii*) E : no tail appears = {HH}
F : no head appears = {TT}.
No. of exhaustive cases = 2 × 2 = 4.

$$\therefore \quad E\cap F = \phi.$$

$$\therefore \quad P(E/F) = \frac{P(E\cap F)}{P(F)}$$

$$= \frac{0}{P(F)} = 0.$$

8. A die is thrown three times,
E : 4 appears on the third toss,
F : 6 and 5 appears respectively on first two tosses.
[Solution. Refer Q. 6 (*b*); Ex. 13 (*a*)].

9. Mother, father and son line up at random for a family picture
E : son on one end,
F : father in middle.
[Solution. Refer Q.7 ; Ex. 13 (*a*)].

10. A black and a red dice are rolled.

(*a*) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(*b*) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

[Solution. Refer Q.8 ; Ex. 13(*a*)]

11. A fair die is rolled. Consider events E = {1, 3, 5}, F = {2, 3} and G = {2, 3, 4, 5}. Find :

(*i*) P (E / F) and P (F / E) (*ii*) P (E / G) and P (G / E)
(*iii*) P (E ∪ F) / (G) and P (E ∩ F) / (G)

[Solution. Refer Q. 10 ; Ex. 13(*a*)].

12. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (*i*) the youngest is a girl, (*ii*) at least one is a girl ?

[Solution. Refer Q.16 ; Ex. 13(*a*)].

13. An instructor has a question bank consisting of 300 easy True/False questions, 200 difficult True/False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question ?

Solution : We have :

	Easy	Difficult	Total
True/False	300	200	500
Multiple Choice	500	400	900
Total	800	600	1400

Let E, D, T and M denote Easy, Difficult, True/False and Multiple Choice Questions respectively.

Total number of questions = 1400

Number of easy multiple choice questions = 500

$$\therefore \quad P(E \cap M) = \frac{500}{1400}.$$

Total number of Multiple Choice Questions = 900

$$\therefore \quad P(M) = \frac{900}{1400}$$

$$\therefore \quad P(E/M) = \frac{P(E \cap M)}{P(M)}$$

$$= \frac{\frac{500}{1400}}{\frac{900}{1400}} = \frac{5}{9}.$$

14. Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

[Solution. Refer Q. 15 ; Ex. 13(*a*)].

15. Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.

Solution : Let n be the number of throws when a multiple of 3 occurs.

$\therefore$ P (a multiple of 3 *i.e.* 3 or 6) in one throw $= \frac{2}{6} = \frac{1}{3}$

P (a multiple of 3) in n throws $= \left(\frac{1}{3}\right)^n$.

P (getting a 6) in one throw $= \frac{1}{6}$

P (getting a 6) in n throws $= \left(\frac{1}{6}\right)^n$.

$\therefore$ Probability of getting at least a 3 in n throws

$$= \left(\frac{1}{3}\right)^n - \left(\frac{1}{6}\right)^n.$$

Let a multiple of 3 do not occur in $(n + 1)$th throw.

P (getting 1, 2, 4, 5) in $(n + 1)$th throw

$$= \frac{4}{6} = \frac{2}{3}.$$

In the next throw a coin is tossed and tail occurs.

$$\therefore \quad P(\text{getting a tail}) = \frac{1}{2}$$

P (getting at least a 3 and a tail) in $(n + 2)$th throw

$$= \left[\left(\frac{1}{3}\right)^n - \left(\frac{1}{6}\right)^n\right]\frac{2}{3} \times \frac{1}{2}$$

Since $n \to \infty$, the probability of getting at least a 3 till tail is obtained

$$= \sum_{n=1}^{\infty}\left[\left(\frac{1}{3}\right)^n - \left(\frac{1}{6}\right)^n\right] \times \frac{2}{3} \times \frac{1}{2}$$

$$= \left[\left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right) - \left(\frac{\frac{1}{6}}{1-\frac{1}{6}}\right)\right] \times \frac{1}{3}$$

$$= \left(\frac{1}{3} \times \frac{3}{2} - \frac{1}{6} \times \frac{6}{5}\right) \times \frac{1}{3} = \left(\frac{1}{2} - \frac{1}{5}\right) \times \frac{1}{3}$$

$$= \frac{3}{10} \times \frac{1}{3} = \frac{1}{10}.$$

In each of the Exercises 16 and 17 choose the correct answer :

16. If $P(A) = \frac{1}{2}$, $P(B) = 0$, then P (A / B) is

(A) 0 (B) $\frac{1}{2}$

(C) not defined (D) 1 **[Ans.** (C)**]**

17. If A and B are events such that P(A / B) = P(B / A), then

(A) $A \subset B$ but $A \neq B$ (B) A = B

(C) $A \cap B = \phi$ (D) P(A) = P(B)

[Ans. (D)**]**

Exercise 13.2

1. If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, find $P(A \cap B)$ if A and B are independent events.

Solution : Since A and B are independent events,

$$\therefore \quad P(A \cap B) = P(A)\,P(B)$$

$$= \left(\frac{3}{5}\right)\left(\frac{1}{5}\right) = \frac{3}{25}.$$

2. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

[Solution. Refer Q. 11; Ex. 13(*b*)].

3. A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

[Solution. Refer Q. 14 ; Ex. 13(*b*)].

4. A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event '3 on the die'. Check whether A and B are independent events or not.

Solution : $P(A) = P(\text{Head}) = \frac{1}{2}$

$$P(B) = P(3 \text{ on the die}) = \frac{1}{6}.$$

When a die and a coin are tossed,

then sample space = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

$$P(A \cap B) = P(\text{Head and } 3) = \frac{1}{12}.$$

$$\text{Now } P(A \cap B) = \frac{1}{12} = \frac{1}{2} \times \frac{1}{6} = P(A)\,P(B).$$

Hence, A and B are independent events.

5. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'the number is even,' and B be the event, 'the number is red'. Are A and B independent ?

[Solution. Refer Q. 2 ; Ex. 13(*b*)].

6. Let E and F be events with $P(E) = \frac{3}{5}, P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$. Are E and F independent ?

[Solution. Refer Q. 1(*b*) ; Ex. 13(*b*)].

7. Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$. Find p if they are (*i*) mutually exclusive (*ii*) independent.

Solution : Given $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$.

$$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \quad \frac{3}{5} = \frac{1}{2} + p - P(A \cap B)$$

$$\Rightarrow \quad P(A \cap B) = p + \frac{1}{2} - \frac{3}{5} = p - \frac{1}{10} \quad \text{....(1)}$$

(*i*) When A and B are mutually exclusive,

then $P(A \cap B) = 0 \Rightarrow p - \frac{1}{10} = 0$

$\Rightarrow p = \frac{1}{10}$.

(*ii*) When A and B are independent,

then $P(A \cap B) = P(A)\,P(B)$

$$\Rightarrow \quad p - \frac{1}{10} = \frac{1}{2}p \Rightarrow p - \frac{p}{2} = \frac{1}{10}$$

$$\Rightarrow \quad \frac{p}{2} = \frac{1}{10}.$$

Hence, $p = \frac{1}{5}$.

8. Let A and B be independent events with P(A) = 0·3 and P(B) = 0·4. Find

(*i*) $P(A \cap B)$ (*ii*) $P(A \cup B)$

(*iii*) $P(A / B)$ (*iv*) $P(B / A)$

[Solution. Refer Q.3 ; Ex. 13(*b*)].

9. If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, find P (not A and not B).

[Solution. Refer Q.1 ; Ex. 13(*c*)].

10. Events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and P (not A or not B) $= \frac{1}{4}$. State whether A and B are independent ?

[Solution. Refer Q.4 ; Ex. 13(*b*)].

11. Given two independent events A and B such that P(A) = 0·3, P(B) = 0·6. Find :

(*i*) P (A and B) (*ii*) P (A and not B)

(*iii*) P (A or B) (*iv*) P (neither A nor B).

[Solution. Refer Q.6(*a*) ; Ex. 13(*b*)].

12. A die is tossed thrice. Find the probability of getting an odd number at least once.

Solution : P (getting an odd number) $= \frac{3}{6} = \frac{1}{2}$

[$\because$ *1, 3, 5 are odd*]

$\therefore$ P (getting an even number) $= 1 - \frac{1}{2} = \frac{1}{2}$.

$\therefore$ Probability of getting an odd number on throw of a die 3 times

= Probability of getting an even number in a throw of a die 3 times

$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}.$$

Hence, probability of at least one odd number in a throw of a die 3 times

$$= 1-\frac{1}{8}=\frac{7}{8}.$$

13. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the porobability that

(*i*) both balls are red

(*ii*) first ball is black and second is red

(*iii*) one of them is black and other is red.

[Solution. Refer Q.16 ; Ex. 13(*b*)].

14. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that

(*i*) the problem is solved

(*ii*) exactly one of them solves the problem.

Solution : Given : $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$.

$$\therefore \quad P(\overline{A}) = 1-\frac{1}{2}=\frac{1}{2}$$

and $$P(\overline{B}) = 1-\frac{1}{3}=\frac{2}{3}.$$

(*i*) The probability that the problem is solved.

= Probability that the problem is solved by at least one of A and B

$$= 1-P(\overline{A})\,P(\overline{B})$$

$$= 1-\frac{1}{2}\times\frac{2}{3} = 1-\frac{1}{3}=\frac{2}{3}.$$

(*ii*) The probability that the problem is solved exactly by one of A and B

$$= P(A\cap\overline{B})+P(\overline{A}\cap B)$$

$$= P(A)\,P(\overline{B})+P(\overline{A})\,P(B)$$

$$= \frac{1}{2}\times\frac{2}{3}+\frac{1}{2}\times\frac{1}{3}$$

$$= \frac{1}{3}+\frac{1}{6}=\frac{3}{6}=\frac{1}{2}.$$

15. One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent ?

(*i*) E : 'the card drawn is a spade'
F : 'the card drawn is an ace'

(*ii*) E : 'the card drawn is black'
F : 'the card drawn is a king'

(*iii*) E : 'the card drawn is a king or queen'
F : 'the card drawn is a queen or jack'.

[Solution. Refer Q.9, Ex. 13 (*b*)].

16. In a hostel, 60% of the students read Hindi news paper, 40% read English news paper and 20% read both Hindi and English news papers. A student is selected at random.

(*a*) Find the probability that she reads neither Hindi nor English news papers.

(*b*) If she reads Hindi news paper, find the probability that she reads English news paper.

(*c*) If she reads English news paper, find the probability that she reads Hindi news paper.

Solution : We have : $P(H) = \frac{60}{100} = \frac{3}{5}$,

$$P(E) = \frac{40}{100}=\frac{2}{5}$$

and $$P(H\cap E)= \frac{20}{100} = \frac{1}{5}.$$

(*a*) $P(H\cup E) = P(H) + P(E) - P(H\cap E)$

$$= \frac{3}{5}+\frac{2}{5}-\frac{1}{5} = \frac{4}{5}.$$

$\therefore$ P (neither Hindi nor English)

$$= 1 - P(H\cup E)$$

$$= 1-\frac{4}{5} = \frac{1}{5}.$$

(*b*) $$P(E/H) = \frac{P(E\cap H)}{P(H)} = \frac{\frac{1}{5}}{\frac{3}{5}}=\frac{1}{3}.$$

(*c*) $$P(H/E) = \frac{P(E\cap H)}{P(E)} = \frac{\frac{1}{5}}{\frac{2}{5}}=\frac{1}{2}.$$

Choose the correct answer in Exercises 17 and 18.

17. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is :

(A) 0 (B) $\frac{1}{3}$

(C) $\frac{1}{12}$ (D) $\frac{1}{36}$. **[Ans.** (D)**]**

18. Two events A and B will be independent, if :

(A) A and B are mutually exclusive

(B) $P(A'B') = [1 - P(A)]\,[1 - P(B)]$

(C) $P(A) = P(B)$

(D) $P(A) + P(B) = 1$. **[Ans.** (B)**]**

Exercise 13.3

1. An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red ?

Solution : The reqd. probability

= P (second ball is red)

= P (a red ball drawn and returned along with 2 red balls and then a red ball is drawn) + P (A black ball drawn and returned along with 2 black balls and then a red ball is drawn)

$$= \frac{5}{10} \times \frac{7}{12} + \frac{5}{10} \times \frac{5}{12} = \frac{35}{120} + \frac{25}{120}$$

$$= \frac{60}{120} = \frac{1}{2}.$$

2. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

[**Solution.** Refer Q. 5 ; Ex. 13(*d*)]

3. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostlier ?

[**Solution.** Refer Q. 14 ; Ex. 13(*d*)]

4. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly ?

Solution : Let the events be :

E_1 : "the student knows the answer"
E_2 : "the student guesses the answer" and
A : "the student answers correctly".

$$\therefore \quad P(E_1) = \frac{3}{4}, \; P(E_2) = \frac{1}{4}.$$

$$P(A/E_2) = \frac{1}{4} \text{ and}$$

$$P(A/E_1) = 1 - \frac{1}{4} = \frac{3}{4}.$$

By *Bayes' Theorem,*

P (E_1/A)

$$= \frac{P(E_1) \,.\, P(A/E_1)}{P(E_1) \,.\, P(A/E_1) + P(E_2) \,.\, P(A/E_2)}$$

$$= \frac{\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)}{\left(\frac{3}{4}\right)\left(\frac{3}{4}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)} = \frac{9}{9+1} = \frac{9}{10}.$$

5. A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0·5% of the healthy person tested (*i.e.* if a healthy person is tested, then, with probability 0·005, the test will imply he has the disease). If 0·1 per cent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

[**Solution.** Refer Q. 15 ; Ex. 13(*d*)]

6. There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin ?

Solution. Let the events be :

E_1 : coin is two headed,
E_2 : coin is biased
and E_3 : coin is unbiased
And A : coin shows up head.

$$\therefore \quad P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

and $P(A/E_1) = 1,$

$$P(A/E_2) = \frac{75}{100} = \frac{3}{4}, \; P(A/E_3) = \frac{1}{2}.$$

By *Bayes' Theorem,*

$$P(E_1/A) = \frac{P(E_1)\, P(A/E_1)}{P(E_1)\, P(A/E_1) + P(E_2)\, P(A/E_2) + P(E_3)\, P(A/E_3)}$$

$$= \frac{\frac{1}{3} \times 1}{\frac{1}{3} \times 1 + \frac{1}{3} \times \frac{3}{4} + \frac{1}{3} \times \frac{1}{2}}$$

$$= \frac{4}{4+3+2}$$

$$= \frac{4}{9}.$$

7. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of the accidents are 0·01, 0·03 and 0·15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver ?

(Kashmir B. 2017; Jammu B. 2015 W)

Solution : Let E_i ($i = 1, 2, 3$) be the event that the insured person is a scooter driver, car driver and truck driver and A, the event when an insured person meets with an accident.

$$\therefore \quad P(E_1) = \frac{2000}{2000+4000+6000}$$

$$= \frac{2}{2+4+6} = \frac{2}{12} = \frac{1}{6}$$

$$P(E_2) = \frac{4000}{2000+4000+6000}$$

$$= \frac{4}{2+4+6} = \frac{4}{12} = \frac{1}{3}$$

and $$P(E_3) = \frac{6000}{2000+4000+6000}$$

$$= \frac{6}{2+4+6} = \frac{6}{12} = \frac{1}{2}.$$

And $P(A/E_1) = \dfrac{1}{100}$;

$P(A/E_2) = \dfrac{3}{100}$; $P(A/E_3) = \dfrac{3}{20} = \dfrac{15}{100}$.

By *Bayes' Theorem,*

$P(E_1/A)$

$$= \frac{P(E_1) \,.\, P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)}$$

$$= \frac{\frac{1}{6}\left(\frac{1}{100}\right)}{\frac{1}{6}\left(\frac{1}{100}\right) + \frac{1}{3}\left(\frac{3}{100}\right) + \frac{1}{2}\left(\frac{15}{100}\right)}$$

$$= \frac{\frac{1}{6}}{\frac{1}{6} + 1 + \frac{15}{2}} = \frac{\frac{1}{6}}{\frac{52}{6}} = \frac{1}{52}.$$

8. A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B ?

Solution : Let the events be :

E_1 : Item from machine A

E_2 : Item from machine B

and A : Item is defective.

$$\therefore \quad P(E_1) = \frac{60}{100} = \frac{3}{5}$$

and $$P(E_2) = \frac{40}{100} = \frac{2}{5}.$$

Also $P(A/E_1) = \dfrac{2}{100}$ and $P(A/E_2) = \dfrac{1}{100}$.

By *Bayes' Theorem,*

$$P(E_2/A) = \frac{P(E_2)\,P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$

$$= \frac{\left(\frac{2}{5}\right)\left(\frac{1}{100}\right)}{\left(\frac{3}{5}\right)\left(\frac{2}{100}\right) + \left(\frac{2}{5}\right)\left(\frac{1}{100}\right)}$$

$$= \frac{2}{6+2} = \frac{2}{8} = \frac{1}{4}.$$

9. Two groups are competing for the position on the Board of directors of a corporation. The probabilities that the first and the second groups will win are 0·6 and 0·4 respectively. Further, if the first group wins, the probability of introducing a new product is 0·7 and the corresponding probability is 0·3 if the second group wins. Find the probability that the new product introduced was by the second group.

Solution : Let the events E_1, E_2 and A be as below :

E_1 : First group wins

E_2 : Second group wins

A : New product is introduced.

We have : $P(E_1) = 0{\cdot}6$, $P(E_2) = 0{\cdot}4$,

$P(A/E_1) = 0{\cdot}7$, $P(A/E_2) = 0{\cdot}3$.

By *Bayes' Theorem,*

$P(E_2/A)$

$$= \frac{P(E_2)\,P(A/E_2)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$$

$$= \frac{(0{\cdot}4)\,(0{\cdot}3)}{(0{\cdot}6)\,(0{\cdot}7) + (0{\cdot}4)\,(0{\cdot}3)}$$

$$= \frac{12}{42+12} = \frac{12}{54} = \frac{2}{9}.$$

10. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die ?

[Solution. Refer Q. 16 ; Ex. 13(*d*)]

11. A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A ?

Solution : Let the events be :

E_1 : Item is from machine A

E_2 : Item is from machine B

E_3 : Item is from machine C

and A : Items is defective.

$$\therefore \quad P(E_1) = \frac{50}{100},\ P(E_2) = \frac{30}{100} \text{ and}$$

$$P(E_3) = \frac{20}{100}.$$

$$\text{Also } P(A/E_1) = \frac{1}{100},\ P(A/E_2) = \frac{5}{100}$$

$$\text{and } P(A/E_3) = \frac{7}{100}.$$

By *Bayes' Theorem*,

$P(E_1/A)$

$$= \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)}$$

$$= \frac{\frac{50}{100} \times \frac{1}{100}}{\frac{50}{100} \times \frac{1}{100} + \frac{30}{100} \times \frac{5}{100} + \frac{20}{100} \times \frac{7}{100}}$$

$$= \frac{50}{50 + 150 + 140} = \frac{50}{340} = \frac{5}{34}.$$

12. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

[Solution. Refer Q. 1(*i*) ; Ex. 13(*d*)]

13. Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was head is :

(A) $\frac{4}{5}$ (B) $\frac{1}{2}$ (C) $\frac{1}{5}$ (D) $\frac{2}{5}$. **[Ans.** (A)]

14. If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct ?

(A) $P(A/B) = \frac{P(B)}{P(A)}$ (B) $P(A/B) < P(A)$

(C) $P(A/B) \geq P(A)$ (D) None of these

[Ans. (C)]

Exercise 13.4

1. State which of the following are not the probability distributions of a random variable. Give reasons for your answer.

(*i*)

X	0	1	2
P(X)	0.4	0.4	0.2

(*ii*)

X	0	1	2	3	4
P(X)	0.1	0.5	0.2	– 0.1	0.3

(*iii*)

Y	–1	0	1
P(Y)	0.6	0.1	0.2

(*iv*)

Z	3	2	1	0	–1
P(Z)	0.3	0.2	0.4	0.1	0.05

Solution : (*i*) This distribution is probability distribution.

$$\left[\because 0\cdot4 + 0\cdot4 + 0\cdot2 = 1\right]$$

(*ii*) This distribution is not a probability distribution.

$$\left[\begin{array}{l}\because \textit{One of the probabilities i.e.} \\ -0\cdot1 \textit{ is } -ve\end{array}\right]$$

(*iii*) This distribution is not a probability distribution.

$$\left[\because 0\cdot6 + 0\cdot1 + 0\cdot2 = 0\cdot9 \neq 1\right]$$

(*iv*) This distribution is not a probability distribution.

$$\left[\begin{array}{r}\because 0\cdot3 + 0\cdot2 + 0\cdot4 + 0\cdot1 + 0\cdot05 \\ = 1\cdot05 > 1\end{array}\right]$$

2. An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls. What are the possible values of X ? Is X a random variable ?

[Solution. Refer Q. 1 Ex. 13(*e*)].

3. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X ?

[Solution. Refer Q. 2. Ex. 13(*e*)].

4. Find the probability distribution of :

(*i*) number of heads in two tosses of a coin

(*ii*) number of tails in the simultaneous tosses of three coins

(*iii*) number of heads in four tosses of a coin.

Solution : (*i*) Here S = {HH, HT, TH, TT},

where H ≡ Head and T ≡ Tail.

Let 'X' be the random variable, which is the number of heads.

Here X takes values 0, 1 and 2.

$\therefore$ $P(X = 0) = P(TT) = \frac{1}{4}$.

$P(X = 1) = P(\{TH, HT\}) = \frac{2}{4} = \frac{1}{2}$

and $P(X = 2) = P(HH) = \frac{1}{4}$.

Hence, the probability distribution is :

X:	0	1	2
P(X):	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(*ii*) Here S = {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT},

where H ≡ Head and T ≡ Tail.

Let 'X' be the random variable, which is the number of tails.

Here X takes values 0, 1, 2 and 3.

$\therefore$ $P(X = 0) = P(\{HHH\}) = \frac{1}{8}$,

$P(X = 1) = P(\{TTH, THT, HTT\})$

$= \frac{3}{8}$,

$P(X = 2) = P(\{HHT, HTH, THH\})$

$= \frac{3}{8}$

and $P(X = 3) = P(TTT) = \frac{1}{8}$.

Hence, the probability distribution is :

X:	0	1	2	3
P(X):	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(*iii*) Here S = {HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THHT, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT},

where H ≡ Head and T ≡ Tail.

Let 'X' be random variable, which is the number of heads.

Here X takes values 0, 1, 2 and 3.

$\therefore$ $P(X = 0) = P(\{TTTT\}) = \frac{1}{16}$,

$P(X = 1) = P(\{HTTT, THTT, TTHT, TTTH\}) = \frac{4}{16} = \frac{1}{4}$,

$P(X = 2) = (\{HHTT, HTHT, HTTH, THHT, THTH, TTHH\}) = \frac{6}{16} = \frac{3}{8}$,

$P(X = 3) = P(\{HHHT, HHTH, HTHH, THHH\})$

$= \frac{4}{16} = \frac{1}{4}$

and $P(X = 4) = P(\{HHHH\}) = \frac{1}{16}$.

Hence, the probability distribution is :

X:	0	1	2	3	4
P(X):	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

5. Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as

(*i*) number greater than 4

(*ii*) six appears on at least one die

[Solution. Refer Q. 9 ; Ex. 13(*e*)]

6. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

[Solution. Refer Q. 17 ; Ex. 13(*e*)]

7. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

[Solution. Refer Q. 20. ; Ex. 13(*e*)]

8. A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
P(X)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Determine

(*i*) k (*ii*) $P(X < 3)$

(*iii*) $P(X > 6)$ (*iv*) $P(0 < X < 3)$.

Solution : (*i*) Since $\Sigma P(X) = 1$,

$\therefore 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow$ $10k^2 + 9k - 1 = 0$

$\Rightarrow$ $k = \frac{-9 \pm \sqrt{81+40}}{20}$

$= \frac{-9 \pm 11}{20} = \frac{1}{10}, -1$.

Since the probability is ≥ 0, therefore, rejecting K = – 1,

we have : $K = \frac{1}{10}$.

(*ii*) (I) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$$= 0 + k + 2k = 3k = \frac{3}{10}.$$

(II) $P(X > 6) = P(7)$

$$= 7k^2 + k$$

$$= \frac{7}{100} + \frac{1}{10} = \frac{17}{100}.$$

(III) $P(0 < X < 3) = P(X = 1) + P(X = 2)$

$$= k + 2k = 3k = \frac{3}{10}.$$

9. The random variable X has a probability distribution P(X) of the following form, where k is some number :

$$P(X) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

(*a*) Determine the value of k.

(*b*) Find $P(X < 2)$, $P(X \leq 2)$, $P(X \geq 2)$.

Solution : The probability distribution of X is :

X :	0	1	2
P (X) :	k	$2k$	$3k$

(*a*) Since $\sum_{i=1}^{n} p_i = 1$,

$\therefore \quad k + 2k + 3k = 1 \Rightarrow 6k = 1 \Rightarrow k = \frac{1}{6} = 0{\cdot}17.$

(*b*) (*i*) $P(X < 2) = P(X = 0) + P(X = 1)$

$$= k + 2k = 3k$$

$$= 3\left(\frac{1}{6}\right) = \frac{1}{2} = 0{\cdot}5.$$

(*ii*) $P(X \leq 2)$

$$= P(X = 0) + P(X = 1) + P(X = 2)$$

$$= k + 2k + 3k = 6k = 6\left(\frac{1}{6}\right) = 1.$$

(*iii*) $P(X \geq 2) = P(2) = 3k = 3\left(\frac{1}{6}\right) = \frac{1}{2} = 0{\cdot}5.$

10. Find the mean number of heads in three tosses of a fair coin.

[Solution. Refer Q.12 ; Ex. 13(*f*)].

11. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

Solution : Here p, the probability of getting six $= \frac{1}{6}$

and q, the probability of not getting six $= 1 - \frac{1}{6} = \frac{5}{6}.$

$\therefore \quad P(X = 0) = q \times q = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

$P(X = 1) = p \times q + q \times p$

$$= \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} = \frac{10}{36} = \frac{5}{18}$$

and $\quad P(X = 2) = p \times p = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}.$

Thus we have :

x_i	p_i	$p_i x_i$
0	$\frac{25}{36}$	0
1	$\frac{5}{18}$	$\frac{5}{18}$
2	$\frac{1}{36}$	$\frac{1}{18}$
Total		$\frac{6}{18} = \frac{1}{3}$

$$\therefore \quad \mu = \sum_{i=1}^{n} p_i x_i = \frac{1}{3}.$$

Hence, the expectation of X $= \frac{1}{3}$.

12. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E(X).

[Solution. Refer Q. 13 ; Ex. 13(*f*)]

13. Let X denote the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X.

[Solution. Refer Q.7 ; Ex. 13(*f*)]

14. A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X ? Find mean, variance and standard deviation of X.

[Solution. Refer Q.15 ; Ex. 13(*f*)]

15. In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take X = 0 if he opposed, and X = 1 if he is in favour. Find E(X) and Var (X).

Solution : Here $X = 0$, $P(X) = \frac{30}{100} = \frac{3}{10}$

$X = 1$, $P(X) = \frac{70}{100} = \frac{7}{10}.$

Thus we have :

x_i	p_i	$p_i x_i$	x_i^2	$p_i x_i^2$
0	$\frac{3}{10}$	0	0	0
1	$\frac{7}{10}$	$\frac{7}{10}$	1	$\frac{7}{10}$
Total		$\frac{7}{10}$		$\frac{7}{10}$

$$\therefore \quad E(X) = \sum_{i=1}^{n} x_i p_i = x_1 p_1 + x_2 p_2$$

$$= 0 + \frac{7}{10} = \frac{7}{10}.$$

Also $E(X^2) = 0^2 \times \frac{3}{10} + 1^2 \times \frac{7}{10} = \frac{7}{10}$.

Hence, Var. (X) $= E(X^2) - (E(X))^2$

$$= \frac{7}{10} - \left(\frac{7}{10}\right)^2 = \frac{7}{10} - \frac{49}{100}$$

$$= \frac{70-49}{100} = \frac{21}{100}.$$

Choose the correct answer in each of the following :

16. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is :

(A) 1 (B) 2 (C) 5 (D) $\frac{8}{3}$. **[Ans. (B)]**

17. Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of E(X) is :

(A) $\frac{37}{221}$ (B) $\frac{5}{13}$ (C) $\frac{1}{13}$ (D) $\frac{2}{13}$. **[Ans. (D)]**

Exercise 13.5

1. A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of

(*i*) 5 successes ? (*ii*) at least 5 successes ? (*iii*) at most 5 successes ?

[Solution. Refer Q.5(*ii*) ; Ex. 13(*g*)]

2. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes.

Solution : Here p, the probability of throwing doublet with a pair of dice $= \frac{6}{36} = \frac{1}{6}$.

$$\therefore \quad q = 1 - \frac{1}{6} = \frac{5}{6} \text{ and } n = 4.$$

$\therefore$ P (r), the probability of r successes $= {}^nC_r\, q^{n-r} p^r$.

$\therefore$ Probability of getting 2 successes

$$= P(2) = {}^4C_2 q^2 p^2$$

$$= 6\left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right)^2 = \frac{25}{216}.$$

3. There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item ?

[Solution. Refer Q. 14 ; Ex. 13(*g*)]

4. Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that

(*i*) all the five cards are spades ?

(*ii*) only 3 cards are spades ?

(*iii*) none is a spade ? *(H.P.B. 2017, 16; Kashmir B. 2016)*

Solution : Here p = P (spade card) $= \frac{13}{52} = \frac{1}{4}$.

$$\therefore \quad q = 1 - \frac{1}{4} = \frac{3}{4} \text{ and } n = 5.$$

(*i*) P (all five cards are spades) = P (5)

$$= {}^5C_5 q^0 p^5 = (1)(1)\left(\frac{1}{4}\right)^5 = \left(\frac{1}{4}\right)^5.$$

(*ii*) P (only 3 cards are spades) = P (3)

$$= {}^5C_3 q^2 p^3 = 10\left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^3$$

$$= 90 \times \left(\frac{1}{4}\right)^5.$$

(*iii*) P (none is spade) = P (0) $= {}^5C_0 q^5 p^0$

$$= (1)\left(\frac{3}{4}\right)^5 (1) = \left(\frac{3}{4}\right)^5.$$

5. The probability that a bulb produced by a factory will fuse after 150 days of use is 0·05. Find the probability that out of 5 such bulbs :

(*i*) none

(*ii*) not more than one

(*iii*) more than one

(*iv*) at least one

will fuse after 150 days of use. *(Karnatka B. 2017)*

Solution : Here $p = 0{\cdot}05 = \dfrac{5}{100} = \dfrac{1}{20}$

and $q = 1 - \dfrac{1}{20} = \dfrac{19}{20}$, $n = 5$.

Using $P(r) = {}^nC_r q^{n-r} p^r$, we have :

(*i*) $P(0) = {}^nC_0 q^n p^0 = 1 \times q^5 \times 1$

$$= q^5 = \left(\frac{19}{20}\right)^5.$$

(*ii*) $P(0) + P(1) = {}^nC_0 q^n p^0 + {}^nC_1 q^{n-1} p$

$= 1 \times q^5 \times 1 + {}^5C_1 q^4 p$

$$= \left(\frac{19}{20}\right)^5 + 5\left(\frac{19}{20}\right)^4\left(\frac{1}{20}\right)$$

$$= \left(\frac{19}{20}\right)^4\left(\frac{19}{20} + \frac{5}{20}\right) = \left(\frac{19}{20}\right)^4\left(\frac{24}{20}\right) = \frac{6}{5}\left(\frac{19}{20}\right)^4.$$

(*iii*) $P(2) + P(3) + P(4) + P(5)$

$$= 1 - (P(0) + P(1)) = 1 - \frac{6}{5}\left(\frac{19}{20}\right)^4$$

[*Using part (ii)*]

(*iv*) $P(1) + P(2) + \ldots\ldots\ldots\ldots + P(5)$

$$= 1 - P(0) = 1 - \left(\frac{19}{20}\right)^5. \qquad [\textit{Using part (i)}]$$

6. A bag consists of 10 balls each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0 ?

[Solution. Refer Q.12 ; Ex. 13(*g*)]

7. In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true'; if it falls tails, he answers 'false'. Find the probability that he answers at least 12 questions correctly.

Solution : Here $p = \dfrac{1}{2}$, $q = \dfrac{1}{2}$ and $n = 20$.

$\therefore$ Reqd. probability $= P(12) + P(13) + \ldots\ldots\ldots + P(20)$

$$= {}^{20}C_{12}\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^{12} + \ldots\ldots\ldots + {}^{20}C_{20}\left(\frac{1}{2}\right)^{20}$$

$$= \left(\frac{1}{2}\right)^{20}\left[{}^{20}C_{12} + {}^{20}C_{13} + \ldots\ldots\ldots + {}^{20}C_{20}\right]$$

$$= {}^{20}C_{12}\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^{12} = {}^{20}C_{12}\left(\frac{1}{2}\right)^{20}.$$

8. Suppose X has a binomial distribution $B\left(6, \dfrac{1}{2}\right)$. Show that X = 3 is the most likely outcome.

(Hint : $P(X = 3)$ is the maximum among all $P(x_i)$, $x_i = 0, 1, 2, 3, 4, 5, 6$.

[Solution. Refer 1(*b*) ; Ex. 13(*g*)]

9. On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answer just by guessing ?

[Solution. Refer Q.21 ; Ex. 13(*g*)]

10. A person buys a lottery ticket m 50 lotteries, in each of which his chance of winning a prize is $\dfrac{1}{100}$. What is the probability that he will win a prize (*a*) at least once (*b*) exactly once (*c*) least twice ?

Solution : Here $n = 50$ and $p = \dfrac{1}{100}$.

$$\therefore \quad q = 1 - p = 1 - \frac{1}{100} = \frac{99}{100}.$$

(*a*) P (the person wins a prize at least once)

$= 1 - P(0) = 1 - {}^{50}C_0 q^{50} p^0$

$$= 1 - (1)\left(\frac{99}{100}\right)^{50} = 1 - \left(\frac{99}{100}\right)^{50}.$$

(*b*) P (the person winning a prize exactly once)

$$= P(1) = {}^{50}C_1 q^{49} p^1 = 50\left(\frac{99}{100}\right)^{99}\left(\frac{1}{100}\right)$$

$$= \frac{1}{2}\left(\frac{99}{100}\right)^{49}.$$

(*c*) P (the person wins a prize at least twice)

$= 1 - (P(0) + P(1))$

$$= 1 - \left({}^{50}C_0 q^{50} p^0 + {}^{50}C_1 q^{49} p^1\right)$$

$= 1 - ((1) q^{50} (1) + 50 q^{49} p) = 1 - q^{49}(q + 50p)$

$$= 1 - \left(\frac{99}{100}\right)^{49}\left(\frac{99}{100} + 50 \times \frac{1}{100}\right)$$

$$= 1 - \frac{149}{100}\left(\frac{99}{100}\right)^{49}.$$

11. Find the probability of getting 5 exactly twice in 7 throws of a die.

12. Find the probability of throwing at most 2 sixes in 6 throws of a single die.

[Solutions : (11-12) Refer Q. 4. Ex. 13(*g*)].

13. It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective ? *(H.P.B. 2017)*

Solution : Here p = P (defective article) = 10%

$$= \frac{10}{100} = \frac{1}{10}.$$

$$\therefore \quad q = 1 - \frac{1}{10} = \frac{9}{10} \text{ and } n = 12.$$

$\therefore$ P (9 defective) $= {}^{12}C_9\, q^3 p^9$

$$= {}^{12}C_3 \left(\frac{9}{10}\right)^3 \left(\frac{1}{10}\right)^9$$

$$= \frac{12 \times 11 \times 10}{1 \times 2 \times 3} \times \frac{9^3}{10^{12}} = \frac{22 \times 9^3}{10^{11}}.$$

In each of the following, choose the correct answer :

14. In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is :

(A) 10^{-1} (B) $\left(\frac{1}{2}\right)^5$ (C) $\left(\frac{9}{10}\right)^5$ (D) $\frac{9}{10}$. **[Ans. (C)]**

15. The probability that a student is not a swimmer is $\frac{1}{5}$. Then the probability that out of five students, four are swimmers is :

(A) ${}^5C_4\left(\frac{4}{5}\right)^4 \frac{1}{5}$ (B) $\left(\frac{4}{5}\right)^4 \frac{1}{5}$ (C) ${}^5C_1 \frac{1}{5}\left(\frac{4}{5}\right)^4$

(D) None of these **[Ans. (A)]**

Miscellaneous Exercise on Chapter 13

1. A and B are two events such that P (A) $\neq$ 0. Find P (B / A), if :

(*i*) A is a subset of B (*ii*) A $\cap$ B = ϕ.

[**Solution.** Refer Q.1. ; Rev. Ex]

2. A couple has two children,

(*i*) Find the probability that both children are males, if it is known that at least one of the children is male.

(*ii*) Find the probability that both children are females, if it is known that the elder child is a female.

Solution : Here S, the sample space = {BB, BG,GB, GG},

where B ≡ Boy and G ≡ Girl

and first letter ≡ elder child;

second letter ≡ younger child.

(*i*) Let E : 'Both children are males'

i.e. E = {B B}

and F = 'At least one of the children is a male'

i.e. F = {BG, GB, BB}.

$\therefore$ E $\cap$ F = {BB} = E.

$$\therefore \text{Reqd. probability} = P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{P(E)}{P(F)}$$

$$= \frac{1/4}{3/4} = \frac{1}{3}.$$

(*ii*) Let E : 'Both children are females'

i.e. E = {G G}

and F : 'Elder child is female'

i.e. F = {GB, GG}

$\therefore$ E $\cap$ F = {G G} = E.

$$\therefore \text{Reqd. probability} = P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{P(E)}{P(F)}$$

$$= \frac{1/4}{2/4} = \frac{1}{2}.$$

3. Suppose that 5% of men and 0·25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male ? Assume that there are equal number of males and females.

[**Solution.** Refer Q.5 ; Rev. Ex.]

4. Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed ?

Solution. Here $p = \frac{90}{100} = \frac{9}{10}$

and $q = 1 - p = 1 - \frac{9}{10} = \frac{1}{10}$

and $n = 10$.

$\therefore$ Reqd. Probability = P (X $\leq$ 6)

= 1 – P (7 $\leq$ X $\leq$ 10)

$$= 1 - \sum_{r=7}^{10} {}^{10}C_r \left(\frac{9}{10}\right)^r \left(\frac{1}{10}\right)^{10-r}$$

$$= 1 - \sum_{r=7}^{10} {}^{10}C_r\, (0{\cdot}9)^r\, (0{\cdot}1)^{10-r}.$$

5. An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that :

(*i*) all will bear 'X' mark

(*ii*) not more than 2 will bear 'Y' mark

(*iii*) at least one ball will bear 'Y' mark

(*iv*) the number of balls with 'X' mark and 'Y' mark will be equal.

Solution : p, P (mark X) $= \frac{10}{25} = \frac{2}{5}$

q, P (mark Y)$= \frac{15}{25} = \frac{3}{5}$ and $n = 6$.

(*i*) P (all bear 'X' mark) $= {}^6C_6 q^0 p^6$

$= (1)(1)\left(\frac{2}{5}\right)^6 = \left(\frac{2}{5}\right)^6$.

(*ii*) Not more than 2 bear 'Y' mark means at least 4 bear 'X' mark.

$\therefore$ Reqd. probability = P (4) + P (5) + P (6)

$= {}^6C_4 q^2 p^4 + {}^6C_5 q^1 p^5 + {}^6C_6 q^0 p^6$

$= 15\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^4 + 6\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)^5 + (1)(1)\left(\frac{2}{5}\right)^6$

$= \left(\frac{2}{5}\right)^4\left[\frac{135}{25} + \frac{36}{25} + \frac{4}{25}\right] = \frac{175}{25}\left(\frac{2}{5}\right)^4$

$= 7\left(\frac{2}{5}\right)^4$.

(*iii*) P (at least one will bear 'Y' mark)

= P (at most 5 will bear 'X' mark)

$= 1 - P(6) = 1 - {}^6C_6 q^0 p^6$

$= 1 - (1)(1)\left(\frac{2}{5}\right)^6 = 1 - \left(\frac{2}{5}\right)^6$.

(*iv*) P (No. of balls with 'X' mark = No. of balls with 'Y' mark)

$= P(3) = {}^6C_3 q^3 p^3$

$= 20\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^3$

$= 20\left(\frac{6}{25}\right)^3 = \frac{864}{3125}$.

6. In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles ?

[**Solution.** Refer Q. 18 ; Ex 13(*g*)].

7. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

Solution : Probability of getting a six $= \frac{1}{6}$

Probability of not getting a six $= 1 - \frac{1}{6} = \frac{5}{6}$.

$\therefore$ P (Obtaining the third six in sixth throw)

= {P (Obtaining two sixes in the first five throws)} $\times \frac{1}{6}$

$= {}^5C_2\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)^2 \times \frac{1}{6} = \frac{625}{23328}$.

8. If a leap year is selected at random, what is the chance that it will contain 53 Tuesdays ?

[**Solution.** Refer Ex 2 ; Page 13/3]

9. An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be atleast 4 successes.

[**Solution.** Refer Q.17 ; Rev. Ex.]

10. How many times must a man toss a fair coin so that the probability of having at least one head is more than 90% ?

[**Solution.** Refer Q. 18 ; Rev. Ex.]

11. In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/loses.

Solution : Probability of getting a six $= \frac{1}{6} (= p)$

$\therefore \quad q = 1 - \frac{1}{6} = \frac{5}{6}$.

(I) When the man gets a six in first throw.

Here P (getting a six) $= \frac{1}{6}$.

(II) When the man does not get a six in first throw, but he gets a six in the second throw.

Here probability $= \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$.

(III) When the man does not get a six in first two throw but he gets a six in the third throw.

Here probability $= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$

$= \frac{25}{216}$.

(IV) Probability when the man does not get a six in any of three throws $= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$.

(I) When the man gets a six in the first throw, he gets ₹ 1.

(II) When the man gets a six in the second throw, he gets ₹ (1 – 1) = ₹ 0.

(III) When the man gets a six in the third throw, he gets (₹ (– 1 – 1 + 1) = ₹ (– 1)

i.e. the man loses ₹ 1.

∴ Expected value

$$= \left(\frac{1}{6}\right)(1) + \left(\frac{5}{36}\right)(0) + \left(\frac{25}{216}\right)(-1)$$

$$= \frac{1}{6} - \frac{25}{216} = \frac{36-25}{216} = ₹\ \frac{11}{216}.$$

12. Suppose we have four boxes A, B, C and D containing coloured marbles as given below :

Box	**Marble colour**		
	Red	**White**	**Black**
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A ? box B ? box C ?

[**Solution.** Refer Q.3. ; Rev. Ex.]

13. Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga ?

Solution. Let the events be :

E_1 : Patient follows meditation and Yoga

E_2 : Patient uses drug

and A : Patient suffers a heart attack.

$$\therefore \quad P(E_1) = P(E_2) = \frac{1}{2}.$$

$$\text{Also}\ P(A/E_1) = \frac{40}{100}\left(1 - \frac{30}{100}\right) = \frac{28}{100}$$

$$\text{and}\ P(A/E_2) = \frac{40}{100}\left(1 - \frac{25}{100}\right) = \frac{30}{100}.$$

By *Bayes' Theorem,*

$$P(E_1/A) = \frac{P(E_1).P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{28}{100}\right)}{\left(\frac{1}{2}\right)\left(\frac{28}{100}\right) + \left(\frac{1}{2}\right)\left(\frac{30}{100}\right)}$$

$$= \frac{28}{28+30} = \frac{28}{58} = \frac{14}{29}.$$

14. If each element of a second order determinant is either zero of one, what is the probability that the value of the determinate is positive ? (Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability $\frac{1}{2}$).

[**Solution.** Refer Q.7. ; Rev. Ex.]

15. An electronic assembly consists of two subsystems, say A and B. From previous testing procedures, the following probabilities are assumed to be known :

P (A fails) = 0·2

P (B fails alone) = 0·15

P (A and B fail) = 0·15

Evaluate the following probabilities :

(*i*) P (A fails/B has failed) (*ii*) P (A fails alone)

[**Solution.** Refer Q.8. ; Rev. Ex.]

16. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

Solution : Let the events be :

E_1 : Red ball is transferred from Bag I to Bag II

E_2 : Black ball is transferred from Bag I to Bag II

and A : Red ball is drawn from Bag I.

$$\therefore \quad P(E_1) = \frac{3}{7},\ P(E_2) = \frac{4}{7}$$

$$\text{Also} \quad P(A/E_1) = \frac{4+1}{(4+1)+5} = \frac{5}{10} = \frac{1}{2}$$

$$\text{and} \quad P(A/E_2) = \frac{4}{4+(5+1)} = \frac{4}{10}.$$

By *Bayes' Theorem,*

$P(E_2/A)$

$$= \frac{P(E_2).P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\left(\frac{4}{7}\right)\left(\frac{4}{10}\right)}{\left(\frac{3}{7}\right)\left(\frac{1}{2}\right) + \left(\frac{4}{7}\right)\left(\frac{4}{10}\right)} = \frac{16}{70} \times \frac{70}{15+16} = \frac{16}{31}.$$

Choose the correct answer in each of the following :

17. If A and B are two events such that $P(A) \neq 0$ and $P(B/A) = 1$, then :

(A) $A \subset B$ (B) $B \subset A$ (C) $B = \phi$ (D) $A = \phi$

[Ans. (A)]

18. If $P(A / B) > P(A)$, then which of the following is correct :

(A) $P(B/A) < P(B)$ (B) $P(A \cap B) < P(A) . P(B)$

(C) $P(B/A) > P(B)$ (D) $P(B/A) = P(B)$.

[Ans. (C)]

19. If A and B are any two events such that

$P(A) + P(B) - P(A \text{ and } B) = P(A)$, then

(A) $P(B/A) = 1$ (B) $P(A/B) = 1$

(C) $P(B/A) = 0$ (D) $P(A/B) = 0$ **[Ans. (B)]**

Questions From NCERT Exemplar

Example 1. A and B are two candidates seeking admission in a college. The probability that A is selected is 0·7 and the probability that exactly one of them is selected is 0·6. Find the probability that B is selected.

Solution. Let 'p' be the probability that B is selected.

By the question, P (exactly one of A and B is selected) = 0·6

$\Rightarrow$ P(A is selected, B is rejected ; B is selected, A is rejected) = 0·6

$\Rightarrow P(A \cap B') + P(A' \cap B) = 0{\cdot}6$

$\Rightarrow P(A)\, P(B') + P(A')\, P(B) = 0{\cdot}6$

$\Rightarrow (0{\cdot}7)\,(1 - p) + (0{\cdot}3)\, p = 0{\cdot}6$

$\Rightarrow p = 0{\cdot}25$.

Hence, the probability that B is selected = 0·25.

Example 2. The probability of simultaneous occurrence of at least one of two events A and B is p. If p is the probability that exactly one of A, B occurs is q, then prove that :

$$\mathbf{P(A') + P(B') = 2 - 2p + q.}$$

Solution. Since P(exactly one of A, B occurs) = q, *[Given]*

$\therefore P(A \cup B) - P(A \cap B) = q$

$\Rightarrow \quad p - P(A \cap B) = q$

$\Rightarrow \quad P(A \cap B) = p - q$

$\Rightarrow \quad 1 - P(A' \cup B') = p - q$

$\Rightarrow \quad P(A' \cup B') = 1 - p + q$

$\Rightarrow P(A') + P(B') - P(A' \cap B') = 1 - p + q$

$\Rightarrow P(A') + P(B') = (1 - p + q) + P(A' \cap B')$

$= (1 - p + q) + (1 - P(A \cup B))$

$= (1 - p + q) + (1 - p)$

$= 2 - 2p + q$, which is proved.

Example 3. 10% of the bulbs produced in a factory are of red colour and 2% are red and defective. If one bulb is picked up at random, determine the probability of its being defective if it is red.

Solution. Let the events be as :

A : Bulb is red

B : Bulb is defective.

$$\therefore P(A) = \frac{10}{100} = \frac{1}{10}, P(A \cap B) = \frac{2}{100} = \frac{1}{50}.$$

$$\text{Now } P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/50}{1/10} = \frac{10}{50} = \frac{1}{5}.$$

Hence, the probability that the bulb being defective, if it is red = $\frac{1}{5}$.

Example 4. Three machines E_1, E_2, E_3 in a certain factory produce 50%, 25% and 25% respectively of the total daily output of electric tubes. It is known that 4% of the tubes produced by each of machines E_1 and E_2 are defective, and that 5% of those produced on E_3 are defective. If one tube is picked up at random from a day's production, calculate the probability that it is defective.

Solution. Let D be the event that the picked up tube is defective. Let A_1, A_2 and A_3 be the events that the tube is produced on machines E_1, E_2 and E_3 respectively.

$$\text{Now } P(D) = P(A_1)\, P(D/A_1) + P(A_2)\, P(D/A_2) + P(A_3)\, P(D / A_3) \quad ...(1)$$

$$P(A_1) = \frac{50}{100} = \frac{1}{2}, P(A_2) = \frac{25}{100} = \frac{1}{4} \text{ and}$$

$$P(A_3) = \frac{25}{100} = \frac{1}{4}.$$

$$\text{And } P(D/A_1) = P(D/A_2) = \frac{4}{100} = \frac{1}{25}$$

$$P(D/A_3) = \frac{5}{100} = \frac{1}{20}.$$

Putting in (1), we get :

$$P(D) = \frac{1}{2} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{20}$$

$$= \frac{1}{50} + \frac{1}{100} + \frac{1}{80} = \frac{8+4+5}{400} = \frac{17}{400} = 0{\cdot}0425.$$

Example 5. Determine variance and standard deviation of the number of heads in three tosses of a coin.

Solution. Let 'X' denote the number of heads tossed.

Here 'X' takes values 0, 1, 2, 3.

$\therefore$ Sample space, S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.

$\therefore$ P(X = 0) = P(no head) = P (TTT) = $\frac{1}{8}$

P(X = 1) = P(one head) = P (HTT, THT, TTH) = $\frac{3}{8}$

P(X = 2) = P(two heads) = P (HHT, HTH, THH) = $\frac{3}{8}$

and P(X = 3) = P(three heads) = P (HHH) = $\frac{1}{8}$.

Thus probability distribution of X is :

X :	0	1	2	3
P(X) :	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Now Mean (μ) = $\Sigma x_i p_i$

$$= 0\times\frac{1}{8}+1\times\frac{3}{8}+2\times\frac{3}{8}+3\times\frac{1}{8}$$

$$= \frac{3}{8}+\frac{6}{8}+\frac{3}{8}=\frac{12}{8}=\frac{3}{2}$$

and Variance (σ^2) = $\Sigma x_i^2 p_i - \mu^2$

$$= \left(0^2\times\frac{1}{8}+1^2\times\frac{3}{8}+2^2\times\frac{3}{8}+3^2\times\frac{1}{8}\right)-\left(\frac{3}{2}\right)^2$$

$$= \left(\frac{3}{8}+\frac{3}{2}+\frac{9}{8}\right)-\frac{9}{4} = \frac{3+12+9}{8}-\frac{9}{4}=3-\frac{9}{4}=\frac{3}{4}.$$

Also Standard Deviation (σ) = $\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$.

Exercise

1. A committee of 4 students is selected at random from a group consisting of 8 boys and 4 girls. Given that there is at least one girl in the committee, calculate the probability that there are exactly 2 girls in the committee.

2. A bag contains 5 red marbles and 3 black marbles. Three marbles are drawn one by one without replacement. What is the probability that at least one of three marbles drawn be black if the first marble is red ?

3. A and B throw a pair of dice alternately. A wins the game if he gets a total of 6 and B wins if she gets a total of 7. If A starts the game, find the probability of winning the game by A in third row of pair of dice.

4. Four balls are to be drawn without replacement from a box containing 8 red and 4 white balls. If X denotes the number of red balls drawn, find the probability distribution of X.

5. Find the probability that in 10 throws of a fair die a score, which is a multiple of 3, will be obtained in at least 8 of the throws.

Answers

1. $\frac{168}{425}$. **2.** $\frac{25}{56}$. **3.** $\frac{775}{7776}$.

4.

X :	0	1	2	3	4
P(X) :	1/495	32/495	168/495	224/495	70/495

5. $\frac{201}{3^{10}}$.

Revision Exercise

1. A and B are two events such that P (A) $\neq$ 0. Find P (B/A) if :

(*i*) A is a subset of B (*ii*) $A \cap B = \phi$. *(N.C.E.R.T.)*

2. Coloured balls are distributed in four boxes as shown in the following table :

Box	Colour			
	Black	White	Red	Blue
I	3	4	5	6
II	2	2	2	2
III	1	2	3	1
IV	4	3	1	5

A box is selected at random and then a ball is randomly drawn from the selected box. The colour of the ball is black, what is the probability that ball is drawn from the box III ? *(N.C.E.R.T.)*

Solution. Let A, E_1, E_2, E_3 and E_4 be the events, defined as below :

A : Black ball is selected

E_1 : Box I is selected

E_2 : Box II is selected

E_3 : Box III is selected

and E_4 : Box IV is selected.

$\therefore \quad P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{1}{4}$

[$\because$ *Boxes are chosen at random*]

Also, $P(A/E_1) = \frac{3}{18}, P(A/E_2) = \frac{2}{8}, P(A/E_3) = \frac{1}{7}$

and $P(A/E_4) = \frac{4}{13}$.

By *Bayes' Theorem,*

P (Box III is selected if it is known that ball drawn is black)

$$= P(E_3/A)$$

$$= \frac{P(E_3).P(A/E_3)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3) + P(E_4)\ P(A/E_4)}$$

$$= \frac{\left(\frac{1}{4}\right)\left(\frac{1}{7}\right)}{\left(\frac{1}{4}\right)\left(\frac{3}{18}\right)+\left(\frac{1}{4}\right)\left(\frac{2}{8}\right)+\left(\frac{1}{4}\right)\left(\frac{1}{7}\right)+\left(\frac{1}{4}\right)\left(\frac{4}{13}\right)}$$

$$= 0{\cdot}165.$$

3. Suppose we have four boxes A, B , C and D containing coloured marbles as given below :

Box	Marble colour		
	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A ? box B ? box C ? *(N.C.E.R.T.)*

4. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from bag I to bag II and then a ball is drawn from bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black. *(N.C.E.R.T.)*

5. Suppose that 5% of men and 0·25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male ?

Assume that there are equal number of males and females. *(N.C.E.R.T.)*

6. Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its change by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga ?

7. If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive ? (Assume that individual entries of the determinant are chosen independently, each value being assumed with probability $\frac{1}{2}$). *(N.C.E.R.T.)*

8. An electronic assembly consists of two sub-systems; say A and B. From previous testing procedures, the following probabilities are assumed to be known :

P (A fails) = 0·2, P (B fails alone) = 0·15

P (A and B fail) = 0·15.

Evaluate the following probabilities :

(i) P (A fails / B has failed) (ii) P (A fails alone).

(N.C.E.R.T.)

Solution. Let $\overline{A}$ and $\overline{B}$ denote the events : A fails and B fails respectively.

Thus we have :

$$P\text{ (A fails)} = P(\overline{A}) = 0{\cdot}2$$

$$P\text{ (A and B fail)} = P(\overline{A} \cap \overline{B}) = 0{\cdot}15$$

$$P\text{ (B fails alone)} = P(\overline{B}) - P(\overline{A} \cap \overline{B}) = 0{\cdot}15.$$

Now $P(\overline{B}) - 0{\cdot}15 = 0{\cdot}15 \Rightarrow P(\overline{B}) = 0{\cdot}30.$

(*i*) $P(\overline{A}/\overline{B}) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})} = \frac{0{\cdot}15}{0{\cdot}30} = \frac{1}{2} = 0{\cdot}5.$

(*ii*) $P\text{ (A fails alone)} = P(\overline{A}) - P(\overline{A} \cap \overline{B})$

$$= 0{\cdot}2 - 0{\cdot}15 = 0{\cdot}05.$$

9. A box contains 16 bulbs out of which 4 bulbs are defective. 3 bulbs are drawn one by one from the box without replacement. Find the probability distribution of the number of defective bulbs. *(Type : Mizoram B. 2017)*

10. A box contains 13 bulbs out of which 5 bulbs are defective. 3 bulbs are drawn one by one from the box without replacement. Find the probability distribution of the number of defective bulbs.

11. In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins / loses. *(N.C.E.R.T.)*

12. (*i*) A die is thrown 7 times. If getting an "even number" is "success", find the probability of getting at least 6 successes.

(*ii*) A die is thrown 8 times. If getting an "even number" is a "success", find the probability of getting at least 7 successes.

13. A die is thrown 3 times. Getting a multiple of 3 is considered a success. Find the probability of at least 2 successes.

14. Six coins are tossed simultaneously. Find the probability of getting (*i*) 3 head (*ii*) no head (*iii*) at least one head. *(Assam B. 2017)*

15. Suppose that a radio tube inserted into a certain type of set has a probability 2 of functioning more than 500 hours. If we test 4 tubes, what is the probability that exactly k of these function for more than 500 hours, where $k = 0, 1, 2, 3$ and 4. Also draw a histogram for this distribution.

16. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die. *(N.C.E.R.T.)*

17. An experiment succeeds twice as often as it fails. Find the probability that in the next six trials there will be at least 4 successes. *(N.C.E.R.T.)*

18. How many times must a man toss a fair coin so that the probability of having at least one head is more than 90% ? *(N.C.E.R.T.)*

19. Fit a binomial distribution to the following data :

X :	0	1	2	3	4
f :	28	62	46	10	4

Solution. We have : $n = 4$ and $N = \Sigma f = 150$.

For fitting binomial distribution, we require the values of p and q.

We know, for binomial distribution,

$$np = \text{mean of the distribution} = \overline{X} \qquad .....(1)$$

X	f	fX
0	28	0
1	62	62
2	46	92
3	10	30
4	4	16
	150	200

$$\overline{X} = \frac{\Sigma fX}{\Sigma f}$$

$$\Rightarrow \quad \overline{X} = \frac{200}{150}$$

$$\Rightarrow \quad \overline{X} = \frac{4}{3}.$$

Putting the value of $\overline{X}$ in (1),

$$np = \frac{4}{3} \Rightarrow 4p = \frac{4}{3}$$

$$\Rightarrow \quad p = \frac{1}{3}.$$

Thus $\quad q = 1 - \frac{1}{3} = \frac{2}{3}.$

The expected binomial probabilities are given by :

$$p(X = r) = {}^nC_r\, p^r q^{n-r}$$

$$= {}^4C_r \left(\frac{1}{3}\right)^r \cdot \left(\frac{2}{3}\right)^{4-r} \qquad ....(2)$$

Putting $r = 0, 1, 2, 3$ and 4 in (2); we get the expected binomial probabilities as given in the following table :

X	*P(X)*	*Expected Frequency* $N \times P(X)$
0	${}^4C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^4 = \frac{16}{81}$	$150 \times \frac{16}{81} = 29{\cdot}63 = 30$
1	${}^4C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^3 = \frac{32}{81}$	$150 \times \frac{32}{81} = 59{\cdot}26 = 59$
2	${}^4C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2 = \frac{24}{81}$	$150 \times \frac{24}{81} = 44{\cdot}44 = 44$
3	${}^4C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^1 = \frac{8}{81}$	$150 \times \frac{8}{81} = 14{\cdot}81 = 15$
4	${}^4C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^0 = \frac{1}{81}$	$150 \times \frac{1}{81} = 1{\cdot}85 = 2$
	Total = 1	Total = 150

Hence, the fitted binomial distribution is :

X :	0	1	2	3	4	Total
f :	30	59	44	15	2	150.

20. A man takes a step forward with probability 0·4 and backward with probability 0·6. Find the probability that at the end of 11 steps, he is one step away from the starting point.

Solution. Probability of forward step

$$= p = 0{\cdot}4 = \frac{2}{5}.$$

Probability of backward step

$$= q = 0{\cdot}6 = \frac{3}{5}.$$

Probability of one step forward at the end of 11 steps

$$= {}^{11}C_6\, q^5 p^6 = {}^{11}C_6 \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^6.$$

Probability of one step backward at the end of 11 steps

$$= {}^{11}C_5\, q^6 p^5 = {}^{11}C_5 \left(\frac{3}{5}\right)^6 \left(\frac{2}{5}\right)^5.$$

∴ Reqd. probability

$$= {}^{11}C_6 \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^6 + {}^{11}C_5 \left(\frac{3}{5}\right)^6 \left(\frac{2}{5}\right)^5$$

$$= {}^{11}C_6 \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^5 \left(\frac{2}{5}+\frac{3}{5}\right) = {}^{11}C_6 \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^5.$$

21. Out of (2n + 1) tickets consecutively numbered, three are drawn at random. Find the probability that the numbers on them are in A.P.

Solution. If the smallest number is 1, the group of three numbers in A.P. are 1, 2, 3 ; 1, 3, 5 ; 1, 4, 7 ; ......; 1, $n + 1$, $2n + 1$. These are n in number.

If the smallest number is 2, the possible groupings are 2, 3, 4 ; 2, 4, 6 ; 2, 5, 8 ;........; 2, $n + 1$; $2n$. These are $(n - 1)$ in number.

If the smallest number is 3, the possible groupings are 3, 4, 5 ; 3, 5, 7 ; 3, 6, 9 ;......; 3, $n + 2$, $2n + 1$. These are $(n - 2)$ in number.

And so on.

Thus, M, the no. of favourable cases

$$= n + 2\,(n - 1) + 2\,(n - 2) + \ldots\ldots + 2\,(1)$$

$$= 2\,(1 + 2 + 3 + \ldots\ldots + (n - 1)) + n$$

$$= 2 \cdot \frac{(n-1)\,n}{2} + n = n^2 - n + n + n = n^2.$$

And N, the total no. of ways

$$= {}^{2n+1}C_3 = \frac{(2n+1)\,(2n)\,(2n-1)}{3!} = \frac{n\,(4n^2-1)}{3}.$$

Hence, the reqd. probability

$$= \frac{M}{N} = \frac{n^2}{\dfrac{n\,(4n^2-1)}{3}} = \frac{3n}{4n^2-1}.$$

Answers

1. (*i*) 1 (*ii*) 0. **3.** $\frac{1}{15}, \frac{2}{5}, \frac{8}{15}$. **4.** $\frac{16}{31}$.

5. $\frac{20}{21}$. **6.** $\frac{14}{29}$. **7.** $\frac{3}{16}$.

9. $P(0) = \frac{11}{28}, P(1) = \frac{33}{70}, P(2) = \frac{9}{70}, P(3) = \frac{1}{140}$.

10. $P(0) = \frac{28}{143}, P(1) = \frac{210}{429}, P(2) = \frac{120}{429}, P(3) = \frac{5}{143}$.

11. ₹ $\frac{91}{54}$. **12.** (*i*) $\frac{1}{16}$ (*ii*) $\frac{9}{256}$. **13.** $\frac{7}{27}$. **14.** ......

15. P(0) = ·4096, P(1) = ·4096, P(2) = ·1536, P(3) = ·0256, P(4) = ·0016.

16. $\frac{625}{23328}$. **17.** $\frac{31}{9}\left(\frac{2}{3}\right)^4$. **18.** $n \geq 4$.

CHECK YOUR UNDERSTANDING

1. If A and B are mutually exclusive, then $P(A \cap B)$ is equal to .... **(Fill in the Blanks)** *(Jammu B. 2015 W)* **[Ans.** ϕ**]**
2. Let E and F be events with :
   $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$.
   Are E and F independent ? **[Ans.** No]
3. If A and B are two independent events such that
   $P(A) = \frac{5}{13}$; $P(B) = \frac{2}{13}$, then P (AB) is equal to ......
   [Ans. $\frac{10}{169}$]
4. A pair of dice is tossed once and X denotes the sum of numbers that appear on the two dice, then $P(X \leq 4) =$ .... *(Kashmir B. 2015)*
   [Ans. $\frac{1}{6}$]
5. A dice is tossed twice. Find the probability of getting an odd number at least once. **[Ans.** $\frac{7}{8}$]
6. Write the formula of Bayes' Theroem with its conditions. *(H.B. 2016)*
7. The mean of the number of heads in two tosses of a coin is ..... **[Ans.** 1]
8. Obtain the Binomial Probability Distribution, if $n = 6$, $p = \frac{1}{5}$. **[Ans.** $\left(\frac{4}{5}+\frac{1}{5}\right)^6$]
9. Suppose X has a Probablity Distribution $B\left(6, \frac{1}{2}\right)$, find which value of X is most likely outcome. **[Ans.** X = 3]
10. If the Mean and Variance of a Binomial Distribution are 9 and 6 respectively, find the number of trials. **[Ans.** 27]

SUMMARY

PROBABILITY

Probability

Random Experiment and Probability

Conditional Probability

DEFINITIONS AND IMPORTANT RESULTS

1. DEFINITIONS

(*i*) **Random Experiment of Trial.** The performance of an experiment is called trial.

(*ii*) **Event.** The possible outcomes of a trial are called events.

(*iii*) **Equally likely Events.** The events are said to be equally likely if there is no reason to expect any one in preference to any other.

(*iv*) **Exhaustive Events.** It is the total number of all possible outcomes of any trial.

(*v*) **Mutually Exclusive Events.** Two or more events are said to be mutually exclusive if they cannot happen simultaneously in a trial.

(*vi*) **Favourable Events.** The cases which ensure the occurrence of the events are called favourable.

(*vii*) **Sample Space.** The set of all possible outcomes of an experiment is called a sample space.

(*viii*) Probability of occurrences of event A, denoted by P (A), is defined as :

$$P(A) = \frac{\text{No. of favourable cases}}{\text{No. of exhaustive cases}} = \frac{n(A)}{n(S)}.$$

2. THEOREMS

(*i*) In a random experiment, if S be the sample space and A an event, then :

(I) $P(A) \geq 0$ (II) $P(\phi) = 0$ (III) $P(S) = 1$.

(*ii*) If A and B are mutually exclusive events, then $P(A \cap B) = 0$.

(*iii*) If A and B are two mutually exclusive events, then $P(A) + P(B) = 1$.

(*iv*) If A and B are mutually exclusive events, then : $P(A \cup B) = P(A) + P(B)$.

(*v*) For any two events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(*vi*) For each event A, $P(\overline{A}) = 1 - P(A)$, where $(\overline{A})$ is the complementary event.

(*vii*) $0 \leq P(A) \leq 1$.

3. MORE DEFINITIONS

(*i*) **Compound Event.** The simultaneous happening of two or more events is called a compound event if they occur in connection with each other.

(*ii*) **Conditional Probability.** Let A and B be two events associated with the same sample space, then :

$$P(A/B) = \frac{\text{No. of elementary events favourable to B which are also favourable to A}}{\text{No. of elementary events favourable to B}}.$$

Theorem. $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

and $P(B/A) = \dfrac{P(A \cap B)}{P(A)}$.

(*iii*) **Independent Events.** Two events are said to be independent if the occurrence if one does not depend upon the occurrence of the other.

Theorem. $P(A \cap B) = P(A)\,P(B)$ when A, B are independent.

4. If $A_1, A_2, \ldots, A_r$ be r events, then the probability when at least one event happens $= 1 - P(\overline{A}_1)\,P(\overline{A}_2) \ldots P(\overline{A}_r)$.

5. BAYES' FORMULA

If $E_1, E_2, \ldots, E_n$ are mutually exclusive and exhaustive events and A is any event that occurs with $E_1, E_2, \ldots, E_n$, then :

$P(E_1/A) =$

$$\frac{P(E_1)P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + \ldots\ldots + P(E_n)\,P(A/E_n)}.$$

6. Mean and Variance of Random Variable.

Mean $(\mu) = \sum x_i p_i$.

Variance $(\sigma^2) = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2$.

7. Binomial Distribution :

$(q + p)^n$.

(*i*) Formula : $P(r) = {}^nC_r\, q^{n-r} p^r$.

(*ii*) Recurrence Formula :

$$P(r+1) = \frac{(n-r)}{(r+1)} \cdot \frac{p}{q} \cdot P(r).$$

(*iii*) Mean and Variance.

Mean = np and Variance = npq

S.D. $= \sqrt{npq}$.

MULTIPLE CHOICE QUESTIONS

For Board Examinations

1. If P (E) denotes probability of occurrence of event E, then :
(A) P (E) ∈ [– 1, 1] (B) P (E) ∈ (1, ∞)
(C) P (E) ∈ (0, 1) (D) P (E) ∈ [0, 1].
(P.B. 2018)

2. If A and B are independent events, then :
(A) P (A ∩ B) = P (A).P (B)
(B) P (A ∪ B) = P (A).P (B)
(C) P (A ∩ B) = P (A) + P (B)
(D) P (A ∪ B) = P (A) + P (B). *(H.P.B. 2018)*

3. The probability of obtaining an even prime number on each die, when a pair of dice is rolled, equal to :
(A) 0 (B) $\frac{1}{3}$
(C) $\frac{1}{12}$ (D) $\frac{1}{36}$ *(H.P.B. 2018, 15, 14)*

4. If P (A) = 0.5, P (B) = 0.6 and P (A ∪ B) = 0.8, then P (B/A) is :
(A) $\frac{1}{2}$ (B) $\frac{3}{5}$
(C) $\frac{1}{3}$ (D) None of these.
(H.B. 2018)

5. If A and B are two independent events such that P (A ∪ B) = 0.60 and P (A) = 0.2, then P (B) is :
(A) 0.5 (B) 0.6
(C) 0.7 (D) None of these.
(H.B. 2018)

6. A card is drawn from a pack of 52 cards and then a second card is drawn without replacement. The probability that both cards drawn are queens is :
(A) $\frac{1}{17}$ (B) $\frac{1}{221}$
(C) $\frac{1}{13}$ (D) None of these.
(H.B. 2018)

7. Two events A and B are independent events if :
(A) A and B are mutually exclusive
(B) P (A′B′) = [1 – P (A)] [1 – P (B)]
(C) P (A) = P (B)
(D) P (A) = P (B) = 1. *(Jammu B. 2018)*

8. A pair of dice is thrown once, the probability of doublet is :
(A) $\frac{1}{6}$ (B) $\frac{1}{3}$
(C) $\frac{1}{2}$ (D) None of these.
(Jammu B. 2018)

9. If P (A) = $\frac{1}{2}$ and P (B) = 0, then P (A/B) is equal to :
(A) 0 (B) $\frac{1}{2}$
(C) Not defined (D) 1.
(Kashmir B. 2017; Jammu B. 2018)

10. If A and B are events such that P(A/B) = P(B/A), then :
(A) A ⊂ B but A ≠ B (B) A = B
(C) A ∩ B = ϕ (D) P (A) = P (B).
(Kashmir B. 2018)

11. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is :
(A) 0 (B) $\frac{1}{13}$
(C) $\frac{1}{12}$ (D) $\frac{1}{36}$. *(Kashmir B. 2018)*

12. If A and B are two independent events and P (A) = $\frac{1}{4}$, P (B) = $\frac{1}{3}$, then :
(A) P (A ∪ B) = $\frac{1}{5}$ (B) P (A ∪ B) = 1
(C) P (A ∪ B) = $\frac{1}{2}$ (D) P (A ∪ B) = $\frac{1}{3}$.
(Mizoram B. 2018)

13. A die is rolled. If the outcome is an odd number, then the probability that it is a prime is :
(A) $\frac{2}{3}$ (B) $\frac{3}{4}$
(C) $\frac{5}{12}$ (D) $\frac{1}{3}$. *(Mizoram B. 2018)*

14. If P (A) = $\frac{3}{5}$ and P (B) = $\frac{1}{5}$, then the value of P (A ∩ B) when A and B are independent events is :
(A) $\frac{3}{25}$ (B) $\frac{3}{28}$
(C) $\frac{2}{7}$ (D) $\frac{2}{11}$. *(Nagaland B. 2018)*

15. (*i*) If P(A) = $\frac{1}{2}$, P(B) = $\frac{3}{8}$ and P(A ∩ B) = $\frac{1}{5}$, then P(A/B) is equal to :
(A) $\frac{2}{5}$ (B) $\frac{8}{15}$
(C) $\frac{2}{3}$ (D) $\frac{5}{8}$. *(P.B. 2017)*

16. If P(A/B) > P(A), then :
(A) P(B/A) < P(B) (B) P(A ∩ B) < P(A).P(B)
(C) P(B/A) > P(B) (D) P(B/A) = P(B).
(H.P.B. 2017)

17. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is :
(A) $\frac{1}{3}$ (B) $\frac{1}{36}$
(C) 0 (D) $\frac{11}{12}$. *(H.B. 2017)*

18. If P(A) = 0 and P(B) = $\frac{1}{5}$, then P(B/A) is :
(A) $\frac{1}{2}$ (B) Not defined
(C) 0 (D) 1. *(H.B. 2017)*

19. If a fair coin is tossed ten times, the probability of getting exactly six heads is :
(A) $\frac{193}{512}$ (B) $\frac{290}{512}$
(C) $\frac{105}{512}$ (D) None of these.
(H.B. 2017)

20. In a single throw of two dice, the probability of getting a total of 8 is :
(A) $\frac{1}{36}$ (B) $\frac{5}{36}$
(C) $\frac{7}{36}$ (D) $\frac{1}{9}$. *(H.B. 2017)*

21. Let E and F be two events associated with the same random experiment.Then E and F are said to be independent if P(E∩F) is equal to :
(A) $\frac{P(E)}{P(F)}$ (B) P(E) + P(F)
(C) P(E) – P(F) (D) P(E) . P(F).
(Jammu B. 2017)

22. If A and B are two events such that P(A) = $\frac{1}{2}$, P(B) = 0, then P(A/B) is :
(A) 0 (B) $\frac{1}{2}$
(C) Not defined (D) 1. *(Kashmir B. 2016)*

23. Two events A and B will be independent if :
(A) A and B are mutually exclusive
(B) P (A′B′) = [1 – P (A)] [1 – P (B)]
(C) P (A) = P (B)
(D) P (A) + P (B) = 1. *(H.P.B. 2016)*

24. If P(E) = $\frac{11}{36}$, P(F) = $\frac{5}{36}$ and P(E ∩ F) = $\frac{2}{36}$, then the value of P(E/F) is :
(A) $\frac{11}{5}$ (B) $\frac{5}{11}$
(C) $\frac{2}{5}$ (D) $\frac{2}{11}$. *(H.B. 2016)*

25. Probability distribution of X is given below, then the value of K is :

X :	1	2	3	4
P(X) :	1/4	K	2K	K

(A) $\frac{1}{4}$ (B) $\frac{3}{16}$
(C) $\frac{1}{8}$ (D) None of these.
(H.B. 2016)

26. In a single throw of a pair of die, the probability of getting total of 3 or 4 is :
(A) $\frac{3}{36}$ (B) $\frac{4}{36}$
(C) $\frac{5}{36}$ (D) None of these.
(Jammu B. 2016)

RCQ Pocket

(Single Correct Answer Type)
(JEE-Main and Advanced)

27. If it is given that the events A and B are such that P (A) = $\frac{1}{4}$, P (A/B) = $\frac{1}{2}$ and P (B/A) = $\frac{2}{3}$, then P (B) is :
(A) $\frac{1}{2}$ (B) $\frac{1}{6}$
(C) $\frac{1}{36}$ (D) $\frac{2}{3}$. *(A.I.E.E.E. 2008)*

28. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then P (A ∪ B) is :
(A) $\frac{2}{5}$ (B) $\frac{3}{5}$
(C) 0 (D) 1. *(A.I.E.E.E. 2008)*

29. One ticket is selected at random from 50 tickets numbered 00, 01, 02, ... , 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals.
(A) $\frac{1}{7}$ (B) $\frac{5}{14}$
(C) $\frac{1}{50}$ (D) $\frac{1}{14}$.
(A.I.E.E.E. 2009)

30. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is :

(A) $\frac{1}{3}$ (B) $\frac{2}{7}$

(C) $\frac{1}{21}$ (D) $\frac{2}{23}$.

(A.I.E.E.E. 2010)

31. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is :

(A) $P(C/D) = P(C)$ (B) $P(C/D) \geq P(C)$

(C) $P(C/D) < P(C)$ (D) $P(C/D) = \frac{P(D)}{P(C)}$.

(A.I.E.E.E. 2011)

32. Consider 5 independent Bernoulli's trials, each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval :

(A) $\left(\frac{1}{2}, \frac{3}{4}\right]$ (B) $\left(\frac{3}{4}, \frac{11}{12}\right]$

(C) $\left[0, \frac{1}{2}\right]$ (D) $\left(\frac{11}{12}, 1\right]$.

(A.I.E.E.E. 2011)

33. Let A, B, C be pairwise independent events with $P(C) > 0$ and $P(A \cap B \cap C) = 0$. Then $P(A^c \cap B^c / C)$ is :

(A) $P(A) - P(B^c)$ (B) $P(A^c) + P(B^c)$

(C) $P(A^c) - P(B^c)$ (D) $P(A^c) - P(B)$.

(A.I.E.E.E. 2011 S)

34. Three numbers are chosen at random without replacement from {1, 2, 3 ... 8}. The probability that their minimum is 3, given that their maximum is 6 is :

(A) $\frac{3}{8}$ (B) $\frac{1}{5}$

(C) $\frac{1}{4}$ (D) $\frac{2}{5}$. *(A.I.E.E.E. 2012)*

35. Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then the probability that the problem is solved by at least one of them is :

(A) $\frac{235}{256}$ (B) $\frac{21}{256}$

(C) $\frac{3}{256}$ (D) $\frac{253}{256}$.

(J.E.E. (Advanced) 2013)

36. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then the events A and B are :

(A) equally likely but not independent

(B) independent but not equally likely

(C) independent and equally likely

(D) mutually exclusive and independent.

(J.E.E. (Main) 2014)

37. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is :

(A) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (B) $\frac{55}{3}\left(\frac{2}{3}\right)^{10}$

(C) $220\left(\frac{1}{3}\right)^{12}$ (D) $22\left(\frac{11}{3}\right)^{11}$.

(J.E.E. (Main) 2015)

38. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up 4. E_2 is the event that die B shows up 2 and E_3 the event that the sum of the numbers on both dice is odd, then which of the following statements is 'NOT True' ?

(A) E_1 and E_2 are not independent

(B) E_1 and E_3 are independent

(C) E_1, E_2 and E_3 are independent

(D) E_1 and E_2 are independent.

(A.I.E.E.E. 2016)

39. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of number of green balls drawn is :

(A) 4 (B) $\frac{6}{25}$

(C) $\frac{12}{5}$ (D) 6.

(J.E.E. (Main) 2017)

40. If two different numbers are taken from set {0, 1, 2,....,10}, then the probability that their sum as well as absolute difference are both multiples of 4, is :

(A) $\frac{14}{45}$ (B) $\frac{7}{55}$

(C) $\frac{6}{55}$ (D) $\frac{12}{55}$.

(J.E.E. (Main) 2017)

41. For three events A, B and C,

P(Exactly one of A or B occurs)

= P(Exactly one of B or C occurs)

= P(Exactly one of C or A occurs) = $\frac{1}{4}$

and P(All three events occur simultaneously) = $\frac{1}{16}$.

Then the probability that at least one of the events occurs is :

(A) $\frac{3}{16}$ (B) $\frac{7}{32}$

(C) $\frac{7}{16}$ (D) $\frac{7}{64}$.

(J.E.E. (Main) 2017)

42. Three randomly chosen non-negative integers x, y and z are found to satisfy the equation $x + y + z = 0$. Then the probability that z is even, is :

(A) $\frac{5}{11}$ (B) $\frac{1}{2}$

(C) $\frac{36}{55}$ (D) $\frac{6}{11}$.

(J.E.E. (Advanced) 2017)

43. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from this bag, then the probability that this drawn ball is red, is :

(A) $\frac{3}{10}$ (B) $\frac{2}{5}$

(C) $\frac{1}{5}$ (D) $\frac{3}{4}$.

(J.E.E. (Main) 2018)

Answers

1. (D)	**2.** (A)	**3.** (D)	**4.** (B)	**5.** (A)	**6.** (B)	**7.** (B)	**8.** (A)	**9.** (C)	**10.** (D)
11. (D)	**12.** (C)	**13.** (A)	**14.** (A)	**15.** (B)	**16.** (C)	**17.** (B)	**18.** (B)	**19.** (C)	**20.** (B)
21. (D)	**22.** (C)	**23.** (B)	**24.** (C)	**25.** (B)	**26.** (C)	**27.** (C)	**28.** (D)	**29.** (D)	**30.** (B)
31. (B)	**32.** (C)	**33.** (D)	**34.** (B)	**35.** (A)	**36.** (B)	**37.** (A)	**38.** (C)	**39.** (C)	**40.** (C)
41. (C)	**42.** (D)	**43.** (B).							

Hints/Solutions

RCQ Pocket

27. (C) By definition, $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow \quad P(B)P(A/B) = P(A \cap B) \quad ...(1)$

Similarly $P(A)P(B/A) = P(B \cap A) \quad ...(2)$

From (1) and (2),

$P(A)P(B/A) = P(B)P(A/B)$

$[\because P(A \cap B) = P(B \cap A)]$

$\Rightarrow \quad \frac{1}{4}\cdot\frac{2}{3} = P(B)\cdot\frac{1}{2}$

$\Rightarrow \quad P(B) = \frac{1}{4}\cdot\frac{2}{3}\cdot 2 = \frac{1}{3}$.

28. (D) Here A = {4, 5, 6} and B = {1, 2, 3, 4}.

$\therefore \quad A \cup B = \{1, 2, 3, 4, 5, 6\} = S$.

$\therefore \quad P(A \cup B) = 1$.

29. (D) Let the events be :

A : Event when sum of digits is 8 (08, 17, 26, 35, 44)

$\therefore \quad n(A) = 5$.

B : Event when the product of digits is zero

(00, 01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 20, 30, 40)

$\therefore \quad n(B) = 14$

and $\quad n(A \cap B) = 1$.

$\therefore$ Reqd probability $= P(A/B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{1/50}{14/50} = \frac{1}{14}$.

30. (B) No. of exhaustive cases $= {}^9C_3 = \frac{9 \times 8 \times 7}{3 \times 2 \times 1} = 84$.

No. of ways of selecting exactly one ball of each colour

$= {}^3C_1 \times {}^4C_1 \times {}^2C_1$

$= 3 \times 4 \times 2 = 24$.

$\therefore$ Reqd. probability $= \frac{24}{84} = \frac{2}{7}$.

31. (B) $P(C/D) = \frac{P(C \cap D)}{P(D)}$

$= \frac{P(C)}{P(D)}$. $[\because C \subset D]$

Hence, $P(C/D) \geq P(C)$. $[\because 0 < P(D) \leq 1]$

32. (C) P (At least one failure) = 1 – P (No failure)

$= 1 - p^5$.

$\therefore \quad 1 - p^5 \geq \frac{31}{32} \Rightarrow p^5 \leq 1 - \frac{31}{32}$

$\Rightarrow \quad p^5 \leq \left(\frac{1}{2}\right)^5 \Rightarrow p \leq \frac{1}{2}$.

But $p \geq 0$.

$\therefore \; p \in \left[0, \frac{1}{2}\right]$.

33. (D) $P(A^c \cap B^c/C) = \frac{P((A^c \cap B^c) \cap C)}{P(C)}$

$= \frac{P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}{P(C)}$

$= \frac{P(C) - P(A)P(C) - P(B)P(C) - 0}{P(C)}$

$= 1 - P(A) - P(B)$

$= P(A^c) - P(B)$.

34. (B) Let the events be ({1, 2, 3, ... , 8})

A: Maximum of three numbers is 6

B : Minimum of three numbers is 3.

$\therefore \; P(A) = \frac{^5C_2}{^8C_3}$ (numbers < 6 are 5)

$P(B) = \frac{^5C_2}{^8C_3}$ (numbers > 3 are 5)

$P(A \cap B) = \frac{^2C_1}{^8C_3}$.

$\therefore$ Reqd. probability $= P(B/A)$

$= \frac{P(A \cap B)}{P(A)} \; \frac{^2C_1}{^5C_2} = \frac{2}{10} = \frac{1}{5}$.

35. (A) P (Problem is solved by at least one person)

= 1 – P (Problem is solved by none of the persons)

$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{3}{4}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{8}\right)$

$= 1 - \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{7}{8}\right)$

$= 1 - \frac{21}{256} = \frac{235}{256}$.

36. (B) $P(\overline{A \cup B}) = \frac{1}{6}, \; P(A \cup B) = \frac{5}{6}, \; P(A) = \frac{3}{4}$.

Now $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 1 - \frac{1}{6} = \frac{5}{6}$

$\Rightarrow \quad \frac{3}{4} + P(B) - \frac{1}{4} = \frac{5}{6}$

$\Rightarrow \quad P(B) = \frac{5}{6} + \frac{1}{4} - \frac{3}{4} = \frac{1}{3}$.

And $P(A \cap B) = P(A)\,P(B)$ $\left[\because \frac{1}{4} = \frac{3}{4} \times \frac{1}{3}\right]$

Hence, the result.

37. (A) Here $n(S) = 3^{12}$ and $n(E) = {^{12}C_3} \,.\, 2^9$.

$\therefore \; P(E) = \frac{n(E)}{n(S)} = \frac{^{12}C_3 . 2^9}{3^{12}} = \frac{55}{3}\left(\frac{2}{3}\right)^{11}$.

38. (C) $P(E_1) = \frac{6}{36} = \frac{1}{6}, \; P(E_2) = \frac{6}{36} = \frac{1}{6}$

and $P(E_3) = \frac{18}{36} = \frac{1}{2}$.

$\therefore \quad P(E_1 \cap E_2) = \frac{1}{6.6} = P(E_1)\,P(E_2)$

$P(E_1 \cap E_3) = \frac{1 \cdot 3}{6.6} = P(E_1)\,P(E_3)$

$P(E_2 \cap E_3) = \frac{1 \cdot 3}{6.6} = P(E_2)\,P(E_3)$.

Also $P(E_1 \cap E_2 \cap E_3) = 0$.

$\therefore \; E_1, \; E_2$ and E_3 are not independent.

39. (C) Here $n = 10, p = \frac{15}{25}$ and $q = \frac{10}{25}$.

$\therefore$ Var (X) $= npq$

$$= 10 \times \frac{15}{25} \times \frac{10}{25}$$

$$= 10 \times \frac{3}{5} \times \frac{2}{5} = \frac{12}{5}.$$

40. (C) Let A = {0, 1, 2, 3,.....,10}.

$\therefore$ $n(S) = {}^{11}C_2 = 55$.

Let E be the given event.

$\therefore$ E = {(0, 4), (0, 8), (2, 6), (2, 10), (4, 8), (6, 10) }.

$\therefore$ $n(E) = 6$.

Hence, $P(E) = \frac{n(E)}{n(S)} = \frac{6}{55}$.

41. (C) P(Exactly one of A or B occurs)

$= P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$ ...(1)

Similarly $P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$ ...(2)

and $P(C) + P(A) - 2P(C \cap A) = \frac{1}{4}$ ...(3)

Adding (1), (2), (3),

$2 [(P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)] = \frac{3}{4}$

$\Rightarrow P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)] = \frac{3}{8}$.

$\therefore P(A \cup B \cup C) = \Sigma P(A) - \Sigma P(A \cap B) + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$.

42. (D) Let $z = 2k$, where $k = 0, 1, 2, 3, 4, 5$.

$\therefore x + y = 10 - 2k$.

Number of non-regular integral solutions

$$= \sum_{k=0}^{5} {}^{11-2k}C_1 = \Sigma 11 - 2k = 36$$

Total number of cases $= {}^{10+3-1}C_3 = 66$.

Hence, the required probability $= \frac{36}{66} = \frac{6}{11}$.

43. (B) Let E_1 : Event that first ball is red

E_2 : Event that first ball is black

and E_3 : Event that second ball is red.

Now, $P(E) = P(E_1)\ P(E/E_1) + P(E_2)\ P(E/E_2)$

$$= \frac{4}{10} \times \frac{6}{12} + \frac{6}{10} \times \frac{4}{12}$$

$$= \frac{4}{120} = \frac{2}{5}.$$

CHAPTER TEST 13

Time Allowed : 1 Hour **Max. Marks : 34**

Notes : **1.** *All questions are compulsory.*

2. *Marks have been indicated against each question.*

1. If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, evaluate P(A / B). **(1)**

2. Let E and F be events with :

$$P(E) = \frac{3}{5}, P(F) = \frac{3}{10} \text{ and } P(E \cap F) = \frac{1}{5}.$$

Are E and F independent ? **(1)**

3. A coin is tossed 7 times. What is the probability that head appears an odd number of times ? **(2)**

4. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are black ? **(2)**

5. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that :

(*i*) the problem is solved (*ii*) exactly one of them solves the problem. **(4)**

6. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at radom with replacement. Find the probability distribution of the number of defective bulbs. **(4)**

7. Find the probability of the number of doublets in three throws of a pair of dice. **(4)**

8. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least one defective egg. **(4)**

9. Assume that the chances of a patient having a heart attack is 40%. Assume that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces the chance by 25%. At a time a patient can choose any one of two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga. **(6)**

10. The random variable 'X' has a probability distribution P (X) of the following form, where k is some number :

$$P(X) = \begin{cases} k, & \text{if } X = 0 \\ 2k, & \text{if } X = 1 \\ 3k, & \text{if } X = 2 \\ 0, & \text{if otherwise} \end{cases}.$$

(*a*) Determine the value of *k*.

(*b*) Find $P(X < 2)$, $P(X \leq 2)$, $P(X \geq 2)$. **(6)**

Answers

1. $\frac{4}{9}$. **2.** No. **3.** $\frac{1}{2}$. **4.** $\frac{3}{7}$.

5. (*i*) $\frac{2}{3}$ (*ii*) $\frac{1}{2}$. **6.** $P(0) = \frac{256}{625}, P(1) = \frac{256}{625}, P(2) = \frac{96}{625}, P(3) = \frac{16}{625}, P(4) = \frac{1}{625}$.

7. $P(0) = \frac{125}{216}, P(1) = \frac{75}{216}, P(2) = \frac{15}{216}, P(3) = \frac{1}{216}$. **8.** $1 - \frac{9^{10}}{10^{10}}$. **9.** $\frac{14}{29}$.