Shortcuts and Important Results to Remember

1 When two dice are thrown, the number of ways of getting a total r (sum of numbers on upper faces), is

(i) $r-1$, if $2 \le r \le 7$

(ii) $13-r$, if $8 \le r \le 12$

2 When three dice are thrown, the number of ways of getting a total r (sum of numbers on upper faces), is

(i) ${}^{r-1}C_2$, if $3 \le r \le 8$

(ii) 25, if $r = 9$

(iii) 27, if $r = 10, 11$

(iv) 25, if $r = 12$

(v) ${}^{20-r}C_2$, if $13 \le r \le 18$

3 The product of k consecutive positive integers is divisible by $k\,!$.

4 Number of zeroes in $n\,! = E_5(n\,!)$

5 n straight lines are drawn in the plane such that no two lines are parallel and no three lines are concurrent. Then, the number of parts into which these lines divides the plane is equal to $\dfrac{(n^2+n+2)}{2}$.

6 ${}^nC_r$ is divisible by n only, if n is a prime number $(1 \le r \le n-1)$.

7 The number of diagonals in n-gon (n sides closed polygon) is $\dfrac{n(n-3)}{2}$.

8 In n-gon no three diagonals are concurrent, then the total number of points of intersection of diagonals interior to the polygon is ${}^nC_4$.

9 Consider a polygon of n sides, then number of triangles in which no side is common with that of the polygon are $\dfrac{1}{6}n\,(n-4)\,(n-5)$.

10 If m parallel lines in a plane are intersected by a family of other n parallel lines. The total number of parallelograms so formed $= {}^mC_2 \cdot {}^nC_2 = \dfrac{mn(m-1)(n-1)}{4}$

11 Highest power of prime p in ${}^nC_r$, since

$$ {}^nC_r = \frac{n\,!}{r\,!\,(n-r)\,!} $$

If $\quad H_p(n\,!) = \alpha,$

$\quad H_p(r\,!) = \beta$

and $\quad H_p\{(n-r)\,!\} = \gamma$

Then, $\quad H_p({}^nC_r) = \alpha - (\beta + \gamma)$

12 Highest power of prime p in ${}^nP_r$, since

$$ {}^nP_r = \frac{n\,!}{(n-r)\,!} $$

If $H_p(n\,!) = \lambda$, $H_p\{(n-r)\,!\} = \mu$. Then, $H_p({}^nP_r) = \lambda - \mu$

13 If there are n rows. Ist row has m_1 squares, IInd row has m_2 squares, IIIrd row has m_3 squares and so on. If we placed λ X's in the squares such that each row contains atleast one X. Then the number of ways = Coefficient of x^λ in

$$ ({}^{m_1}C_1 x + {}^{m_1}C_2\, x^2 + \ldots + {}^{m_1}C_{m_1} x^{m_1}) $$
$$ \times ({}^{m_2}C_1 x + {}^{m_2}C_2\, x^2 + {}^{m_2}C_3 x^3 + \ldots + {}^{m_2}C_{m_2}\, x^{m_2}) \times $$
$$ ({}^{m_3}C_1 x + {}^{m_3}C_2\, x^2 + \ldots + {}^{m_3}C_{m_3} x^{m_3}) \times \ldots $$

If $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{n}$, $\forall\, x, y, n \in N$

$\Rightarrow \quad (x-n)\,(y-n) = n^2$

$\therefore \quad x = n + \lambda,$

$\quad y = n + \dfrac{n^2}{\lambda},$

where λ is divisor of n^2.

Then, number of integral solutions (x, y) is equal to number of divisors of n^2.

If $n = 3$, $n^2 = 9 = 3^2$, the equation has 3 solutions.

$(x, y) = (4, 12), (6, 6), (12, 4)$