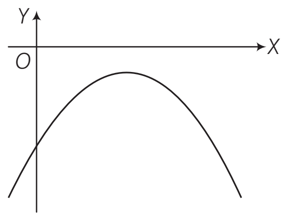
Theory of Equations Exercise 1 :
Single Option Correct Type Questions

- This section contains **30 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct

1. If a, b, c are real and $a \neq b$, the roots of the equation $2(a-b)x^2 - 11(a+b+c)x - 3(a-b) = 0$ are
(a) real and equal (b) real and unequal
(c) purely imaginary (d) None of these

2. The graph of a quadratic polynomial $y = ax^2 + bx + c$; $a, b, c \in R$ is as shown.



Which one of the following is not correct?
(a) $b^2 - 4ac < 0$ (b) $\frac{c}{a} < 0$
(c) c is negative
(d) Abscissa corresponding to the vertex is $\left(-\frac{b}{2a}\right)$

3. There is only one real value of 'a' for which the quadratic equation $ax^2 + (a+3)x + a - 3 = 0$ has two positive integral solutions. The product of these two solutions is
(a) 9 (b) 8 (c) 6 (d) 12

4. If for all real values of a one root of the equation $x^2 - 3ax + f(a) = 0$ is double of the other, $f(x)$ is equal to
(a) $2x$ (b) x^2 (c) $2x^2$ (d) $2\sqrt{x}$

5. A quadratic equation the product of whose roots x_1 and x_2 is equal to 4 and satisfying the relation $\frac{x_1}{x_1 - 1} + \frac{x_2}{x_2 - 1} = 2$, is
(a) $x^2 - 2x + 4 = 0$ (b) $x^2 - 4x + 4 = 0$
(c) $x^2 + 2x + 4 = 0$ (d) $x^2 + 4x + 4 = 0$

6. If both roots of the quadratic equation $x^2 - 2ax + a^2 - 1 = 0$ lie in $(-2, 2)$, which one of the following can be $[a]$? (where $[\cdot]$ denotes the greatest integer function)
(a) -1 (b) 1 (c) 2 (d) 3

7. If $(-2, 7)$ is the highest point on the graph of $y = -2x^2 - 4ax + \lambda$, then λ equals
(a) 31 (b) 11 (c) -1 (d) $-\frac{1}{3}$

8. If the roots of the quadratic equation $(4p - p^2 - 5)x^2 - (2p - 1)x + 3p = 0$ lie on either side of unity, the number of integral values of p is
(a) 1 (b) 2 (c) 3 (d) 4

9. Solution set of the equation $3^{2x^2} - 2 \cdot 3^{x^2 + x + 6} + 3^{2(x+6)} = 0$ is
(a) $\{-3, 2\}$ (b) $\{6, -1\}$ (c) $\{-2, 3\}$ (d) $\{1, -6\}$

10. Consider two quadratic expressions $f(x) = ax^2 + bx + c$ and $g(x) = ax^2 + px + q$ $(a, b, c, p, q \in R, b \neq p)$ such that their discriminants are equal. If $f(x) = g(x)$ has a root $x = \alpha$, then
(a) α will be AM of the roots of $f(x) = 0$ and $g(x) = 0$
(b) α will be AM of the roots of $f(x) = 0$
(c) α will be AM of the roots of $f(x) = 0$ or $g(x) = 0$
(d) α will be AM of the roots of $g(x) = 0$

11. If x_1 and x_2 are the arithmetic and harmonic means of the roots of the equation $ax^2 + bx + c = 0$, the quadratic equation whose roots are x_1 and x_2, is
(a) $abx^2 + (b^2 + ac)x + bc = 0$
(b) $2abx^2 + (b^2 + 4ac)x + 2bc = 0$
(c) $2abx^2 + (b^2 + ac)x + bc = 0$
(d) None of the above

12. $f(x)$ is a cubic polynomial $x^3 + ax^2 + bx + c$ such that $f(x) = 0$ has three distinct integral roots and $f(g(x)) = 0$ does not have real roots, where $g(x) = x^2 + 2x - 5$, the minimum value of $a + b + c$ is
(a) 504 (b) 532 (c) 719 (d) 764

13. The value of the positive integer n for which the quadratic equation $\sum_{k=1}^{n}(x + k - 1)(x + k) = 10n$ has solutions α and $\alpha + 1$ for some α, is
(a) 7 (b) 11 (c) 17 (d) 25

14. If one root of the equation $x^2 - \lambda x + 12 = 0$ is even prime, while $x^2 + \lambda x + \mu = 0$ has equal roots, then μ is
(a) 8 (b) 16 (c) 24 (d) 32

15. Number of real roots of the equation $\sqrt{x} + \sqrt{x - \sqrt{(1 - x)}} = 1$ is
(a) 0 (b) 1 (c) 2 (d) 3

16. The value of $\sqrt{7 + \sqrt{7 - \sqrt{7 + \sqrt{7 - \dots}}}}$ upto ∞ is
(a) 5 (b) 4
(c) 3 (d) 2

17. For any real x, the expression $2(k-x)[x+\sqrt{x^2+k^2}]$ cannot exceed

(a) k^2 (b) $2k^2$

(c) $3k^2$ (d) None of these

18. Given that, for all $x \in R$, the expression $\frac{x^2-2x+4}{x^2+2x+4}$ lies between $\frac{1}{3}$ and 3, the values between which the expression $\frac{9\cdot 3^{2x}+6\cdot 3^x+4}{9\cdot 3^{2x}-6\cdot 3^x+4}$ lies, are

(a) -3 and 1 (b) $\frac{3}{2}$ and 2

(c) -1 and 1 (d) 0 and 2

19. Let α, β, γ be the roots of the equation $(x-a)(x-b)(x-c)=d, d \neq 0$, the roots of the equation $(x-\alpha)(x-\beta)(x-\gamma)+d=0$ are

(a) a, b, d (b) b, c, d

(c) a, b, c (d) $a+d, b+d, c+d$

20. If one root of the equation $ix^2-2(1+i)x+2-i=0$ is $(3-i)$, where $i=\sqrt{-1}$, the other root is

(a) $3+i$ (b) $3+\sqrt{-1}$

(c) $-1+i$ (d) $-1-i$

21. The number of solutions of $|[x]-2x|=4$, where $[x]$ denotes the greatest integer $\leq x$ is

(a) infinite (b) 4 (c) 3 (d) 2

22. If x^2+x+1 is a factor of ax^3+bx^2+cx+d, the real root of $ax^3+bx^2+cx+d=0$ is

(a) $-\frac{d}{a}$ (b) $\frac{d}{a}$ (c) $\frac{a}{d}$ (d) None of these

23. The value of x which satisfy the equation $\sqrt{(5x^2-8x+3)}-\sqrt{(5x^2-9x+4)}=\sqrt{(2x^2-2x)}-\sqrt{(2x^2-3x+1)}$, is

(a) 3 (b) 2

(c) 1 (d) 0

24. The roots of the equation

$$(a+\sqrt{b})^{x^2-15}+(a-\sqrt{b})^{x^2-15}=2a,$$

where $a^2-b=1$, are

(a) $\pm 2, \pm\sqrt{3}$ (b) $\pm 4, \pm\sqrt{14}$

(c) $\pm 3, \pm\sqrt{5}$ (d) $\pm 6, \pm\sqrt{20}$

25. The number of pairs (x, y) which will satisfy the equation

$x^2-xy+y^2=4(x+y-4)$, is

(a) 1 (b) 2

(c) 4 (d) None of these

26. The number of positive integral solutions of $x^4-y^4=3789108$ is

(a) 0 (b) 1 (c) 2 (d) 4

27. The value of 'a' for which the equation $x^3+ax+1=0$ and $x^4+ax^2+1=0$, have a common root, is

(a) $a=2$ (b) $a=-2$

(c) $a=0$ (d) None of these

28. The necessary and sufficient condition for the equation $(1-a^2)x^2+2ax-1=0$ to have roots lying in the interval $(0, 1)$, is

(a) $a>0$ (b) $a<0$

(c) $a>2$ (d) None of these

29. Solution set of $x-\sqrt{1-|x|}<0$, is

(a) $\left[-1, \frac{-1+\sqrt{5}}{2}\right)$ (b) $[-1, 1]$

(c) $\left[-1, \frac{-1+\sqrt{5}}{2}\right]$ (d) $\left(-1, \frac{-1+\sqrt{5}}{2}\right)$

30. If the quadratic equations $ax^2+2cx+b=0$ and $ax^2+2bx+c=0 (b \neq c)$ have a common root, $a+4b+4c$, is equal to

(a) -2 (b) -1

(c) 0 (d) 1

Theory of Equations Exercise 2 :
More than One Correct Option Type Questions

- This section contains **15 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **MORE THAN ONE** may be correct.

31. If $0<a<b<c$ and the roots α, β of the equation $ax^2+bx+c=0$ are non-real complex numbers, then

(a) $|\alpha|=|\beta|$ (b) $|\alpha|>1$

(c) $|\beta|<1$ (d) None of these

32. If A, G and H are the arithmetic mean, geometric mean and harmonic mean between unequal positive integers. Then, the equation $Ax^2-|G|x-H=0$ has

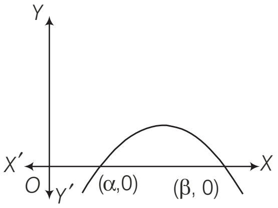
(a) both roots are fractions

(b) atleast one root which is negative fraction

(c) exactly one positive root

(d) atleast one root which is an integer

33. The adjoining graph of $y = ax^2 + bx + c$ shows that



(a) $a < 0$
(b) $b^2 < 4ac$
(c) $c > 0$
(d) a and b are of opposite signs

34. If the equation $ax^2 + bx + c = 0\,(a > 0)$ has two roots α and β such that $\alpha < -2$ and $\beta > 2$, then

(a) $b^2 - 4ac > 0$ (b) $c < 0$
(c) $a + |b| + c < 0$ (d) $4a + 2|b| + c < 0$

35. If $b^2 \geq 4ac$ for the equation $ax^4 + bx^2 + c = 0$, then all the roots of the equation will be real, if

(a) $b > 0,\ a < 0,\ c > 0$ (b) $b < 0,\ a > 0,\ c > 0$
(c) $b > 0,\ a > 0,\ c > 0$ (d) $b > 0,\ a < 0,\ c < 0$

36. If roots of the equation $x^3 + bx^2 + cx - 1 = 0$ from an increasing GP, then

(a) $b + c = 0$
(b) $b \in (-\infty, -3)$
(c) one of the roots is 1
(d) one root is smaller than one and one root is more than one

37. Let $f(x) = ax^2 + bx + c$, where $a, b, c \in R$, $a \neq 0$. Suppose $|f(x)| \leq 1$, $\forall\ x \in [0, 1]$, then

(a) $|a| \leq 8$ (b) $|b| \leq 8$
(c) $|c| \leq 1$ (d) $|a| + |b| + |c| \leq 17$

38. $\cos\alpha$ is a root of the equation $25x^2 + 5x - 12 = 0$, $-1 < x < 0$, the value of $\sin 2\alpha$ is

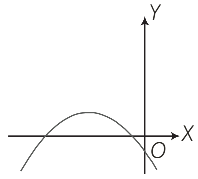
(a) $\frac{24}{25}$ (b) $-\frac{12}{25}$
(c) $-\frac{24}{25}$ (d) $\frac{20}{25}$

39. If $a, b, c \in R\,(a \neq 0)$ and $a + 2b + 4c = 0$, then equation $ax^2 + bx + c = 0$ has

(a) atleast one positive root
(b) atleast one non-integral root
(c) both integral roots
(d) no irrational root

40. For which of the following graphs of the quadratic expression $f(x) = ax^2 + bx + c$, the product of abc is negative

(a)



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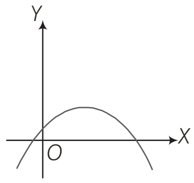

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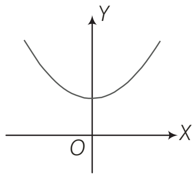
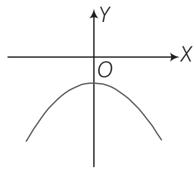


(d)



41. If $a, b \in R$ and $ax^2 + bx + 6 = 0$, $a \neq 0$ does not have two distinct real roots, the

(a) minimum possible value of $3a + b$ is -2
(b) minimum possible value of $3a + b$ is 2
(c) minimum possible value of $6a + b$ is -1
(d) minimum possible value of $6a + b$ is 1

42. If $x^3 + 3x^2 - 9x + \lambda$ is of the form $(x - \alpha)^2(x - \beta)$, then λ is equal to

(a) 27 (b) -27
(c) 5 (d) -5

43. If $ax^2 + (b - c)x + a - b - c = 0$ has unequal real roots for all $c \in R$, then

(a) $b < 0 < a$ (b) $a < 0 < b$
(c) $b < a < 0$ (d) $b > a > 0$

44. If the equation whose roots are the squares of the roots of the cubic $x^3 - ax^2 + bx - 1 = 0$ is identical with the given cubic equation, then

(a) $a = b = 0$
(b) $a = 0,\ b = 3$
(c) $a = b = 3$
(d) a, b are roots of $x^2 + x + 2 = 0$

45. If the equation $ax^2 + bx + c = 0\,(a > 0)$ has two real roots α and β such that $\alpha < -2$ and $\beta > 2$, which of the following statements is/are true?

(a) $4a - 2|b| + c < 0$
(b) $9a - 3|b| + c < 0$
(c) $a - |b| + c < 0$
(d) $c < 0,\ b^2 - 4ac > 0$

Theory of Equations Exercise 3 :
Passage Based Questions

- This section contains **6 passages**. Based upon each of the passage **3 multiple choice questions** have to be answered. Each of these questions has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Passage I
(Q. Nos. 46 to 48)

If G and L are the greatest and least values of the expression $\dfrac{2x^2-3x+2}{2x^2+3x+2}$, $x \in R$ *respectively.*

46. The least value of $G^{100}+L^{100}$ is

(a) 2^{100} (b) 3^{100} (c) 7^{100} (d) None of these

47. G and L are the roots of the equation

(a) $5x^2-26x+5=0$ (b) $7x^2-50x+7=0$

(c) $9x^2-82x+9=0$ (d) $11x^2-122x+11=0$

48. If $L^2<\lambda<G^2$, $\lambda \in N$, the sum of all values of λ is

(a) 1035 (b) 1081 (c) 1225 (d) 1176

Passage II
(Q. Nos. 49 to 51)

If roots of the equation $x^4-12x^3+cx^2+dx+81=0$ *are positive.*

49. The value of c is

(a) -27 (b) 27 (c) -54 (d) 54

50. The value of d is

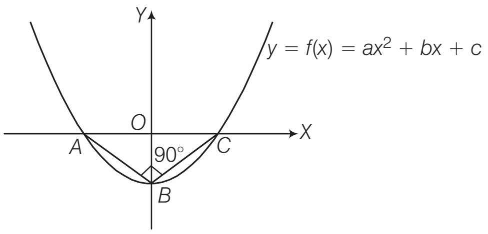
(a) -27 (b) -54 (c) -81 (d) -108

51. Root of the equation $2cx+d=0$, is

(a) -1 (b) $-\frac{1}{2}$ (c) 1 (d) $\frac{1}{2}$

Passage II
(Q. Nos. 52 to 54)

In the given figure vertices of ΔABC *lie on* $y=f(x)=ax^2+bx+c$. *The* ΔABC *is right angled isosceles triangle whose hypotenuse* $AC=4\sqrt{2}$ *units.*



52. $y=f(x)$ is given by

(a) $y=x^2-8$ (b) $y=\dfrac{x^2}{2\sqrt{2}}-2\sqrt{2}$

(c) $y=x^2-4$ (d) $y=\dfrac{x^2}{2}-\sqrt{2}$

53. Minimum value of $y=f(x)$ is

(a) $-4\sqrt{2}$ (b) $-2\sqrt{2}$

(c) 0 (d) $2\sqrt{2}$

54. Number of integral value of λ for which $\frac{\lambda}{2}$ lies between the roots of $f(x)=0$, is

(a) 9 (b) 10 (c) 11 (d) 12

Passage III
(Q. Nos. 55 to 57)

Let $f(x)=x^2+bx+c$ *and* $g(x)=x^2+b_1x+c_1$.

Let the real roots of $f(x)=0$ *be* α, β *and real roots of* $g(x)=0$ *be* $\alpha+k, \beta+k$ *for same constant* k. *The least value of* $f(x)$ *is* $-\frac{1}{4}$ *and least value of* $g(x)$ *occurs at* $x=\frac{7}{2}$.

55. The value of b_1 is

(a) -8 (b) -7 (c) -6 (d) 5

56. The least value of $g(x)$ is

(a) -1 (b) $-\frac{1}{2}$ (c) $-\frac{1}{3}$ (d) $-\frac{1}{4}$

57. The roots of $f(x)=0$ are

(a) 3, 4 (b) -3, 4

(c) -3, -4 (d) 3, -4

Passage IV
(Q. Nos. 58 to 60)

If $ax^2-bx+c=0$ *have two distinct roots lying in the interval* $(0,1)$; $a, b, c \in N$.

58. The least value of a is

(a) 3 (b) 4

(c) 5 (d) 6

59. The least value of b is

(a) 5 (b) 6

(c) 7 (d) 8

60. The least value of $\log_5 abc$ is

(a) 1 (b) 2

(c) 3 (d) 4

Passage V

(Q. Nos. 61 to 63)

If $2x^3 + ax^2 + bx + 4 = 0$ (*a and b are positive real numbers) has three real roots.*

61. The minimum value of a^3 is

(a) 108 (b) 216
(c) 432 (d) 864

62. The minimum value of b^3 is

(a) 432 (b) 864
(c) 1728 (d) None of these

63. The minimum value of $(a+b)^3$ is

(a) 1728 (b) 3456
(c) 6912 (d) 864

Passage VI

(Q. Nos. 64 to 66)

If $\alpha, \beta, \gamma, \delta$ *are the roots of the equation* $x^4 + Ax^3 + Bx^2 + Cx + D = 0$ *such that* $\alpha\beta = \gamma\delta = k$ *and* A, B, C, D *are the roots of* $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ *such that* $A + B = 0$.

64. The value of $\frac{C}{A}$ is

(a) $-\frac{k}{2}$ (b) $-k$ (c) $\frac{k}{2}$ (d) k

65. The value of $(\alpha+\beta)(\gamma+\delta)$ in terms of B and k is

(a) $B - 2k$ (b) $B - k$ (c) $B + k$ (d) $B + 2k$

66. The correct statement is

(a) $C^2 = AD$ (b) $C^2 = A^2D$ (c) $C^2 = AD^2$ (d) $C^2 = (AD)^2$

Theory of Equations Exercise 4 :

Single Integer Answer Type Questions

- This section contains **10 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

67. The sum of all the real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$ is

68. The harmonic mean of the roots of the equation $(5+\sqrt{2})x^2 - (4+\sqrt{5})x + 8 + 2\sqrt{5} = 0$ is

69. If product of the real roots of the equation,

$x^2 - ax + 30 = 2\sqrt{(x^2 - ax + 45)}, a > 0,$

is λ and minimum value of sum of roots of the equation is μ. The value of (μ) (where $(\cdot)$ denotes the least integer function) is

70. The minimum value of $\dfrac{\left(x+\frac{1}{x}\right)^6 - \left(x^6 + \frac{1}{x^6}\right) - 2}{\left(x+\frac{1}{x}\right)^3 + x^3 + \frac{1}{x^3}}$ is (for $x > 0$)

71. Let a, b, c, d are distinct real numbers and a, b are the roots of the quadratic equation $x^2 - 2cx - 5d = 0$. If c and d are the roots of the quadratic equation $x^2 - 2ax - 5b = 0$, the sum of the digits of numerical values of $a + b + c + d$ is

72. If the maximum and minimum values of $y = \dfrac{x^2 - 3x + c}{x^2 + 3x + c}$ are 7 and $\frac{1}{7}$ respectively, the value of c is

73. Number of solutions of the equation $\sqrt{x^2} - \sqrt{(x-1)^2} + \sqrt{(x-2)^2} = \sqrt{5}$ is

74. If α and β are the complex roots of the equation $(1+i)x^2 + (1-i)x - 2i = 0$, where $i = \sqrt{-1}$, the value of $|\alpha - \beta|^2$ is

75. If α, β be the roots of the equation $4x^2 - 16x + c = 0, c \in R$ such that $1 < \alpha < 2$ and $2 < \beta < 3$, then the number of integral values of c, are

76. Let r, s and t be the roots of the equation $8x^3 + 1001x + 2008 = 0$ and if $99\lambda = (r+s)^3 + (s+t)^3 + (t+r)^3$, the value of $[\lambda]$ is (where $[\cdot]$ denotes the greatest integer function)

Theory of Equations Exercise 5 :
Matching Type Questions

- This section contains **4 questions**. Questions 78 and 80 have three statements (A, B and C) given in **Column I** and four statements (p, q, r and s) in **Column II** and questions 77 and 79 have three statements (A, B and C) given in **Column I** and five statements (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II**.

77. Column I contains rational algebraic expressions and Column II contains possible integers which lie in their range. Match the entries of Column I with one or more entries of the elements of Column II.

	Column I		Column II
(A)	$y = \dfrac{x^2 - 2x + 4}{x^2 + 2x + 4}, x \in R$	(p)	-2
(B)	$y = \dfrac{2x^2 + 4x + 1}{x^2 + 4x + 2}, x \in R$	(q)	-1
(C)	$y = \dfrac{x^2 - 3x + 4}{x - 3}, x \in R$	(r)	2
		(s)	3
		(t)	8

78.

	Column I		Column II
(A)	If a, b, c, d are four non-zero real numbers such that $(d + a - b)^2 + (d + b - c)^2 = 0$ and the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are real and equal, then	(p)	$a + b + c = 0$
(B)	If the equation $ax^2 + bx + c = 0$ and $x^3 - 3x^2 + 3x - 1 = 0$ have a common real root, then	(q)	a, b, c are in AP
(C)	Let a, b, c be positive real numbers such that the expression $bx^2 + (\sqrt{(a + c)^2 + 4b^2})x + (a + c)$ is non-negative, $\forall x \in R$, then	(r)	a, b, c are in GP
		(s)	a, b, c are in HP

79. Column I contains rational algebraic expressions and Column II contains possible integers of a.

	Column I		Column II
(A)	$y = \dfrac{ax^2 + 3x - 4}{3x - 4x^2 + a}, x \in R$ and $y \in R$	(p)	0
(B)	$y = \dfrac{ax^2 + x - 2}{a + x - 2x^2}, x \in R$ and $y \in R$	(q)	1
(C)	$y = \dfrac{x^2 + 2x + a}{x^2 + 4x + 3a}, x \in R$ and $y \in R$	(r)	3
		(s)	5
		(t)	7

80.

	Column I		Column II
(A)	The equation $x^3 - 6x^2 + 9x + \lambda = 0$ have exactly one root is (1, 3), then $\|[\lambda + 1]\|$ is (where $[\cdot]$ denotes the greatest integer function)	(p)	0
(B)	If $-3 < \dfrac{x^2 - \lambda x - 2}{x^2 + x + 1} < 2, \forall x \in R$, then $\|[\lambda]\|$ is (where $[\cdot]$ denotes the greatest integer function)	(q)	1
(C)	If $x^2 + \lambda x + 1 = 0$ and $(b - c)x^2 + (c - a)x + (a - b) = 0$ have both the roots common, then $\|[\lambda - 1]\|$, (where $[\cdot]$ denotes the greatest integer function)	(r)	2
		(s)	3

Theory of Equations Exercise 6 :
Statement I and II Type Questions

- **Directions** (Q. Nos. 81 to 87) are Assertion-Reason type questions. Each of these questions contains two statements:
Statement-1 (Assertion) and **Statement-2** (Reason) Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice as given below.
(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(c) Statement1 is true, Statement-2 is false
(d) Statement-1 is false, Statement-2 is true

81. **Statement-1** If the equation $(4p-3)x^2 + (4q-3)x + r = 0$ is satisfied by $x = a$, $x = b$ and $x = c$ (where a, b, c are distinct), then $p = q = \frac{3}{4}$ and $r = 0$.

Statement-2 If the quadratic equation $ax^2 + bx + c = 0$ has three distinct roots, then a, b and c are must be zero.

82. **Statement-1** The equation $x^2 + (2m+1)x + (2n+1) = 0$, where $m, n \in I$, cannot have any rational roots.

Statement-2 The quantity $(2m+1)^2 - 4(2n+1)$, where $m, n \in I$, can never be perfect square.

83. **Statement-1** In the equation $ax^2 + 3x + 5 = 0$, if one root is reciprocal of the other, then a is equal to 5.

Statement-2 Product of the roots is 1.

84. **Statement-1** If one root of $Ax^3 + Bx^2 + Cx + D = 0$, $A \neq 0$, is the arithmetic mean of the other two roots, then the relation $2B^3 + k_1 ABC + k_2 A^2 D = 0$ holds good and then $(k_2 - k_1)$ is a perfect square.

Statement-2 If a, b, c are in AP, then b is the arithmetic mean of a and c.

85. **Statement-1** If x, y, z be real variables satisfying $x + y + z = 6$ and $xy + yz + zx = 8$, the range of variables x, y and z are identical.

Statement-2 $x + y + z = 6$ and $xy + yz + zx = 8$ remains same, if x, y, z interchange their positions.

86. **Statement-1** $ax^3 + bx + c = 0$, where $a, b, c \in R$ cannot have 3 non-negative real roots.

Statement-2 Sum of roots is equal to zero.

87. **Statement-1** The quadratic polynomial $y = ax^2 + bx + c\,(a \neq 0$ and $a, b, c \in R)$ is symmetric about the line $2ax + b = 0$.

Statement-2 Parabola is symmetric about its axis of symmetry.

Theory of Equations Exercise 7 :
Subjective Type Questions

- In this section, there are **24 subjective** questions.

88. For what values of m, the equation $(1+m)x^2 - 2(1+3m)x + (1+8m) = 0$ has $(m \in R)$
(i) both roots are imaginary?
(ii) both roots are equal?
(iii) both roots are real and distinct?
(iv) both roots are positive?
(v) both roots are negative?
(vi) roots are opposite in sign?
(vii) roots are equal in magnitude but opposite in sign?
(viii) atleast one root is positive?
(ix) atleast one root is negative?
(x) roots are in the ratio $2:3$?

89. For what values of m, then equation $2x^2 - 2(2m+1)x + m(m+1) = 0$ has $(m \in R)$
(i) both roots are smaller tha 2?
(ii) both roots are greater than 2?
(iii) both roots lie in the interval $(2, 3)$?
(iv) exactly one root lie in the interval $(2, 3)$?
(v) one root is smaller than 1 and the other root is greater than 1?
(vi) one root is greater than 3 and the other root is smaller than 2?
(vii) atleast one root lies in the interval $(2, 3)$?
(viii) atleast one root is greater than 2?
(ix) atleast one root is smaller than 2?
(x) roots α and β, such that both 2 and 3 lie between α and β?

90. If r is the ratio of the roots of the equation $ax^2+bx+c=0$, show that $\frac{(r+1)^2}{r}=\frac{b^2}{ac}$.

91. If the roots of the equation $\frac{1}{x+p}+\frac{1}{x+q}=\frac{1}{r}$ are equal in magnitude but opposite in sign, show that $p+q=2r$ and that the product of the roots is equal to $\left(-\frac{p^2+q^2}{2}\right)$.

92. If one root of the quadratic equation $ax^2+bx+c=0$ is equal to the nth power of the other, then show that $(ac^n)^{\frac{1}{n+1}}+(a^nc)^{\frac{1}{n+1}}+b=0$.

93. If α, β are the roots of the equation $ax^2+bx+c=0$ and γ, δ those of equation $lx^2+mx+n=0$, then find the equation whose roots are $\alpha\gamma+\beta\delta$ and $\alpha\delta+\beta\gamma$.

94. Show that the roots of the equation

$$(a^2-bc)x^2+2(b^2-ac)x+c^2-ab=0$$

are equal, if either $b=0$ or $a^3+b^3+c^3-3abc=0$.

95. If the equation $x^2-px+q=0$ and $x^2-ax+b=0$ have a common root and the other root of the second equation is the reciprocal of the other root of the first, then prove that $(q-b)^2=bq(p-a)^2$.

96. If the equation $x^2-2px+q=0$ has two equal roots, then the equation $(1+y)x^2-2(p+y)x+(q+y)=0$ will have its roots real and distinct only, when y is negative and p is not unity.

97. Solve the equation $x^{\log_x (x+3)^2}=16$.

98. Solve the equation

$$(2+\sqrt{3})^{x^2-2x+1}+(2-\sqrt{3})^{x^2-2x-1}=\frac{101}{10(2-\sqrt{3})}.$$

99. Solve the equation $x^2+\left(\frac{x}{x-1}\right)^2=8$.

100. Solve the equation

$$\sqrt{(x+8)+2\sqrt{(x+7)}}+\sqrt{(x+1)-\sqrt{(x+7)}}=4.$$

101. Find all values of a for which the inequation $4^{x^2}+2(2a+1)2^{x^2}+4a^2-3>0$ is satisfied for any x.

102. Solve the inequation $\log_{x^2+2x-3}\left(\frac{|x+4|-|x|}{x-1}\right)>0$.

103. Solve the system $|x^2-2x|+y=1, x^2+|y|=1$.

104. If α, β, γ are the roots of the cubic $x^3-px^2+qx-r=0$. Find the equations whose roots are

(i) $\beta\gamma+\frac{1}{\alpha}, \gamma\alpha+\frac{1}{\beta}, \alpha\beta+\frac{1}{\gamma}$

(ii) $(\beta+\gamma-\alpha), (\gamma+\alpha-\beta), (\alpha+\beta-\gamma)$

Also, find the value of $(\beta+\gamma-\alpha)(\gamma+\alpha-\beta)(\alpha+\beta-\gamma)$.

105. If $A_1, A_2, A_3, \dots, A_n, a_1, a_2, a_3, \dots, a_n, a, b, c \in R$, show that the roots of the equation

$$\frac{A_1^2}{x-a_1}+\frac{A_2^2}{x-a_2}+\frac{A_3^2}{x-a_3}+\dots+\frac{A_n^2}{x-a_n}=ab^2+c^2x+ac$$ are real.

106. For what values of the parameter a the equation $x^4+2ax^3+x^2+2ax+1=0$ has atleast two distinct negative roots?

107. If $[x]$ is the integral part of a real number x. Then solve $[2x]-[x+1]=2x$.

108. Prove that for any value of a, the inequation $(a^2+3)x^2+(a+2)x-6<0$ is true for atleast one negative x.

109. How many real solutions of the equation $6x^2-77[x]+147=0$, where $[x]$ is the integral part of x?

110. If α, β are the roots of the equation $x^2-2x-a^2+1=0$ and γ, δ are the roots of the equation $x^2-2(a+1)x+a(a-1)=0$, such that $\alpha, \beta \in (\gamma, \delta)$, find the value of '$a$'.

111. If the equation $x^4+px^3+qx^2+rx+5=0$ has four positive real roots, find the minimum value of pr.

Theory of Equations Exercise 8 :
Questions Asked in Previous 13 Years' Exam

- This section contains questions asked in **IIT-JEE, AIEEE, JEE Main & JEE Advanced** from year **2005** to year **2017**.

112. If α, β are the roots of $ax^2+bx+c=0$ and $\alpha+\beta$, $\alpha^2+\beta^2, \alpha^3+\beta^3$ are in GP, where $\Delta=b^2-4ac$, then **[IIT-JEE 2005, 3M]**

(a) $\Delta \neq 0$ (b) $b\Delta=0$ (c) $cb \neq 0$ (d) $c\Delta=0$

113. If S is a set of $P(x)$ is polynomial of degree ≤ 2 such that $P(0)=0, P(1)=1, P'(x)>0, \forall x \in (0,1)$, then **[IIT-JEE 2005, 3M]**

(a) $S=0$

(b) $S=ax+(1-a)x^2, \forall a \in (0, \infty)$

(c) $S=ax+(1-a)x^2, \forall a \in R$

(d) $S=ax+(1-a)x^2, \forall a \in (0, 2)$

114. If the roots of $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c$ is **[AIEEE 2005, 3M]**

(a) 1 (b) 2
(c) 3 (d) 4

115. If the equation $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x = 0$, $a_1 \neq 0$, $n \geq 2$, has a positive root $x = \alpha$, then the equation $na_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \dots + a_1 = 0$ has a positive root, which is **[AIEEE 2005, 3M]**

(a) greater than or equal to α
(b) equal to α
(c) greater than α
(d) smaller than α

116. If both the roots of the quadratic equation

$$x^2 - 2kx + k^2 + k - 5 = 0$$

are less than 5, k lies in the interval **[AIEEE 2005, 3M]**

(a) $(-\infty, 4)$ (b) $[4, 5]$
(c) $(5, 6)$ (d) $(6, \infty)$

117. Let a and b be the roots of equation $x^2 - 10cx - 11d = 0$ and those of $x^2 - 10ax - 11b = 0$ are c and d, the value of $a + b + c + d$, when $a \neq b \neq c \neq d$, is **IIT-JEE 2006, 6M]**

118. Let a, b, c be the sides of a triangle. No two of them are equal and $\lambda \in R$. If the roots of the equation $x^2 + 2(a+b+c)x + 3\lambda(ab+bc+ca) = 0$ are real, then **[IIT-JEE 2006, 3M]**

(a) $\lambda < \frac{4}{3}$ (b) $\lambda < \frac{5}{3}$
(c) $\lambda \in \left(\frac{1}{3}, \frac{5}{3}\right)$ (d) $\lambda \in \left(\frac{4}{3}, \frac{5}{3}\right)$

119. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4, lie in the interval **[AIEEE 2006, 3M]**

(a) $-2 < m < 0$ (b) $m > 3$
(c) $-1 < m < 3$ (d) $1 < m < 4$

120. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30°$ and $\tan 15°$, respectively, the value of $2 + q - p$ is

(a) 2 (b) 3 **[AIEEE 2006, 3M]**
(c) 0 (d) 1

121. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$. The value of r is **[IIT-JEE 2007, 3M]**

(a) $\frac{2}{9}(p-q)(2q-p)$ (b) $\frac{2}{9}(q-p)(2p-q)$
(c) $\frac{2}{9}(q-2p)(2q-p)$ (d) $\frac{2}{9}(2p-q)(2q-p)$

122. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, the set of possible values of a, is **[AIEEE 2007, 3M]**

(a) $(-3, 3)$ (b) $(-3, \infty)$
(c) $(3, \infty)$ (d) $(-\infty, -3)$

123. Let a, b, c, p, q be real numbers. Suppose α, β are roots of the equation $x^2 + 2px + q = 0$ and $\alpha, \frac{1}{\beta}$ are the roots of the equation $ax^2 + 2bx + c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$.

Statement-1 $(p^2 - q)(b^2 - ac) \geq 0$ and

Statement-2 $b \neq pa$ or $c \neq qa$ **[IIT-JEE 2008, 3M]**

(a) Statement-1 is true, Statement-2, is true; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(c) Statement-1 is true, Statement-2 is false
(d) Statement-1 is false, Statement-2 is true

124. The quadratic equation $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. The common root is **[AIEEE 2008, 3M]**

(a) 4 (b) 3 (c) 2 (d) 1

125. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have? **[AIEEE 2008, 3M]**

(a) 1 (b) 3 (c) 5 (d) 7

126. Suppose the cubic $x^3 - px + q = 0$ has three distinct real roots, where $p > 0$ and $q < 0$. Which one of the following holds? **[AIEEE 2008, 3M]**

(a) The cubic has minima at $\left(-\sqrt{\frac{p}{3}}\right)$ and maxima at $\sqrt{\frac{p}{3}}$

(b) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $\left(-\sqrt{\frac{p}{3}}\right)$

(c) The cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $\left(-\sqrt{\frac{p}{3}}\right)$

(d) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $\left(-\sqrt{\frac{p}{3}}\right)$

127. The smallest value of k, for which both roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have value at least 4, is **[IIT-JEE 2009, 4M]**

(a) 6 (b) 4 (c) 2 (d) 0

128. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$, is **[AIEEE 2009, 4M]**

(a) less than $(-4ab)$ (b) greater than $4ab$
(c) less than $4ab$ (d) greater than $(-4ab)$

129. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq -q$. If α and β are non-zero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots, is **[IIT-JEE 2010, 3M]**

(a) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$
(b) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$
(c) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$
(d) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

130. Consider the polynomial $f(x) = 1 + 2x + 3x^2 + 4x^3$. Let s be the sum of all distinct real roots of $f(x)$ and let $t = |s|$, real number s lies in the interval **[IIT-JEE 2010, 3M]**

(a) $\left(-\frac{1}{4}, 0\right)$ (b) $\left(-11, \frac{3}{4}\right)$ (c) $\left(-\frac{3}{4}, -\frac{1}{2}\right)$ (d) $\left(0, \frac{1}{4}\right)$

131. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, the value of $\frac{a_{10} - 2a_8}{2a_9}$ is
[IIT-JEE 2011, 3 and JEE Main 2015,4M]

(a) 1 (b) 2 (c) 3 (d) 4

132. A value of b for which the equations

$$x^2 + bx - 1 = 0 \quad x^2 + x + b = 0$$

have one root in common, is **[IIT-JEE 2011, 3M]**

(a) $-\sqrt{2}$ (b) $-i\sqrt{3}, i = \sqrt{-1}$
(c) $i\sqrt{5}, i = \sqrt{-1}$ (d) $\sqrt{2}$

133. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is **[IIT-JEE 2011, 4M]**

134. Let for $a \neq a_1 \neq 0$, $f(x) = ax^2 + bx + c$, $g(x) = a_1x^2 + b_1x + c_1$ and $p(x) = f(x) - g(x)$. If $p(x) = 0$ only for $x = (-1)$ and $p(-2) = 2$, the value of $p(2)$ is
[AIEEE 2011, 4M]

(a) 18 (b) 3 (c) 9 (d) 6

135. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots $(4, 3)$. Rahul made a mistake in writing down coefficient of x to get roots $(3, 2)$. The correct roots of equation are **[AIEEE 2011, 4M]**

(a) $-4, -3$ (b) $6, 1$ (c) $4, 3$ (d) $-6, -1$

136. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation

$$(\sqrt[3]{(1+a)} - 1)x^2 + (\sqrt{(1+a)} - 1)x + (\sqrt[6]{(1+a)} - 1) = 0,$$

where $a > -1$, then $\lim_{a \to 0^+} \alpha(a)$ and $\lim_{a \to 0^+} \beta(a)$, are
[IIT-JEE 2012, 3M]

(a) $\left(-\frac{5}{2}\right)$ and 1 (b) $\left(-\frac{1}{2}\right)$ and (-1)
(c) $\left(-\frac{7}{2}\right)$ and 2 (d) $\left(-\frac{9}{2}\right)$ and 3

137. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has **[AIEEE 2012, 4M]**

(a) exactly one real root
(b) exactly four real roots
(c) infinite number of real roots
(d) no real roots

138. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in R$ have a common root, then $a : b : c$ is
[JEE Main 2013, 4M]

(a) 3 : 2 : 1 (b) 1 : 3 : 2 (c) 3 : 1 : 2 (d) 1 : 2 : 3

139. If $a \in R$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where $[\cdot]$ denotes the greatest integer function) has no integral solution, then all possible values of a lie in the interval
[JEE Main 2014, 4M]

(a) $(-2, -1)$
(b) $(-\infty, -2) \cup (2, \infty)$
(c) $(-1, 0) \cup (0, 1)$
(d) $(1, 2)$

140. Let α, β be the roots of the equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in AP and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, the value of $|\alpha - \beta|$, is **[JEE Main 2014, 4M]**

(a) $\frac{\sqrt{34}}{9}$ (b) $\frac{2\sqrt{13}}{9}$
(c) $\frac{\sqrt{61}}{9}$ (d) $\frac{2\sqrt{17}}{9}$

141. Let $a \in R$ and let $f : R \to R$ be given by $f(x) = x^5 - 5x + a$. Then, **[JEE Advanced 2014, 3M]**

(a) $f(x)$ has three real roots, if $a > 4$
(b) $f(x)$ has only one real root, if $a > 4$
(c) $f(x)$ has three real roots, if $a < -4$
(d) $f(x)$ has three real roots, if $-4 < a < 4$

142. The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then, $p(p(x)) = 0$ has
[JEE Advanced 2014, 3M]

(a) only purely imaginary roots
(b) all real roots
(c) two real and two purely imaginary roots
(d) neither real nor purely imaginary roots

143. Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$.

Which of the following intervals is (are) a subset(s) of S?
[JEE Advanced 2015, 4M]

(a) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$ (b) $\left(-\frac{1}{\sqrt{5}}, 0\right)$
(c) $\left(0, \frac{1}{\sqrt{5}}\right)$ (d) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

144. The sum of all real values of x satisfying the equation

$(x^2-5x+5)^{x^2+4x-60}=1$ is **[JEE Main 2016, 4M]**

(a) 6 (b) 5 (c) 3 (d) −4

145. Let $-\dfrac{\pi}{6}<\theta<-\dfrac{\pi}{12}$. Suppose α_1 and β_1 are the roots of equation $x^2-2x\sec\theta+1=0$ and α_2 and β_2 are the roots of the equation $x^2+2x\tan\theta-1=0$. If $\alpha_1>\beta_1$ and $\alpha_2>\beta_2$, then $\alpha_1+\beta_2$ equals **[JEE Advanced 2016, 3M]**

(a) $2(\sec\theta-\tan\theta)$ (b) $2\sec\theta$

(c) $-2\tan\theta$ (d) 0

146. If for a positive integer n, the quadratic equation $x(x+1)+(x+1)(x+2)\ldots+(x+\overline{n-1})(x+n)=10n$ has two consecutive integral solutions, then n is equal to **[JEE Main 2017, 4M]**

(a) 11 (b) 12

(c) 9 (d) 10

Answers

Chapter Exercises

1.(b) **2.** (b) **3.** (b) **4.** (c) **5.** (a) **6.** (a)
7.(c) **8.** (b) **9.** (c) **10.** (a) **11.** (b) **12.** (c)
13.(b) **14.** (b) **15.** (b) **16.** (c) **17.** (b) **18.** (b)
19.(c) **20.** (d) **21.** (b) **22.** (a) **23.** (c) **24.** (b)
25.(a) **26.** (a) **27.** (b) **28.** (c) **29.** (a) **30.** (c)
31.(a,b) **32.**(b,c) **33.**(a,d) **34.**(a,b,c,d) **35.** (b,d) **36.** (a,b,c,d)
37.(a, b,c,d) **38.** (a,c) **39.** (a,b) **40.** (a,b,c,d) **41.** (a,c) **42.** (b,c)
43.(c,d) **44.**(a,c,d) **45.**(a,c,d)
46.(d) **47.** (b) **48.** (d) **49.**(d) **50.**(d) **51.** (c)
52.(b) **53.** (b) **54.** (c) **55.** (b) **56.** (d) **57.** (a)
58.(c) **59.** (a) **60.** (b) **61.** (c) **62.** (b) **63.** (c)
64.(d) **65.** (a) **66.** (b) **67.** (4) **68.** (4) **69.** (9)
70. (6) **71.** (3) **72.** (4) **73.** (2) **74.** (5) **75.**(3)
76.(7) **77.** (A) $\rightarrow$ (r,s), (B) $\rightarrow$ (p,q,r,s,t), (C) $\rightarrow$ (p,q,t)
78. (A) $\rightarrow$ (q,r,s), (B) $\rightarrow$ (p), (C) $\rightarrow$ (q)
79. (A) $\rightarrow$ (q,r,s,t), (B) $\rightarrow$ (q,r), (C) $\rightarrow$ (p,q)
80.(A) $\rightarrow$ (p,q,r,s), (B) $\rightarrow$ (p,q), (C) $\rightarrow$ (s) **81.**(d) **82.** (a)
83. (a) **84.** (a) **85.** (a) **86.** (d) **87.**(a)

88. (i) $m\in(0,3)$ (ii) $m=0,3$

(iii) $m\in(-\infty,0)\cup(3,\infty)$ (iv) $m\in(-\infty,-1)\cup[3,\infty)$

(v) $m\in\phi$ (vi) $m\in(-1,-1/8)$

(vii) $m=-1/3$ (viii) $m\in(-\infty,-1)\cup(-1,-1/8)\cup[3,\infty)$

(ix) $m\in(-1,-1/8)$ (x) $m=\dfrac{81\pm\sqrt{6625}}{32}$

89. (i) $m\in\left(-\infty,\dfrac{7-\sqrt{33}}{2}\right)$ (ii) $m\in\left(\dfrac{7+\sqrt{33}}{2},\infty\right)$ (iii) $m\in\phi$

(iv) $m\in\left(\dfrac{7-\sqrt{33}}{2},\dfrac{11-\sqrt{73}}{2}\right)\cup\left(\dfrac{7+\sqrt{33}}{2},\dfrac{11+\sqrt{73}}{2}\right)$

(v) $m\in(0,3)$ (vi) $m\in\left(\dfrac{7-\sqrt{33}}{2},\dfrac{7+\sqrt{33}}{2}\right)$

(vii) $m\in\left(\dfrac{7-\sqrt{33}}{2},\dfrac{11-\sqrt{73}}{2}\right)\cup\left(\dfrac{7+\sqrt{33}}{2},\dfrac{11+\sqrt{73}}{2}\right)$

(viii) $m\in\left(\dfrac{7-\sqrt{33}}{2},\dfrac{7+\sqrt{33}}{2}\right)\cup\left(\dfrac{7+\sqrt{33}}{2},\infty\right)$

(ix) $m\in\left(-\infty,\dfrac{7-\sqrt{33}}{2}\right)\cup\left(\dfrac{7-\sqrt{33}}{2},\dfrac{7+\sqrt{33}}{2}\right)$

(x) $m\in\left(\dfrac{11-\sqrt{73}}{2},\dfrac{7+\sqrt{33}}{2}\right)$

93. $a^2l^2x^2-ablmx+(b^2-2ac)ln+(m^2-2ln)ac=0$

97. $x\in\phi$

98. $x_1=1+\sqrt{1+\log_{2+\sqrt{3}}10}$, $x_2=1-\sqrt{1+\log_{2+\sqrt{3}}10}$

99. $x_1=2$, $x_2=-1+\sqrt{3}$ and $x_3=-1-\sqrt{3}$

100. $x_1=2$

101. $a\in(-\infty,-1)\cup\left(\dfrac{\sqrt{3}}{2},\infty\right)$

102. $x\in(-1-\sqrt{5},-3)\cup(\sqrt{5}-1,5)$

103. The pairs (0, 1), (1, 0), $\left(\dfrac{1-\sqrt{5}}{2},\dfrac{1-\sqrt{5}}{2}\right)$ are solutions of the original system of equations.

104. (i) $ry^3-q(r+1)y^2+p(r+1)^2y-(r+1)^3=0$

(ii) $y^3-py^2+(4q-p^2)y+(8r-4pq+p^3)=0$ and $4pq-p^3-8r$

106. $a\in\left(\dfrac{3}{4},\infty\right)$ **107.** $x_1=-1$, $x_2=-1/2$ **109.** Four

110. $a\in\left(-\dfrac{1}{4},1\right)$ **111.** 80 **112.** (d) **113.** (d) **114.** (a)

115. (d) **116.** (a) **117.** 1210 **118.**(a) **119.** (c) **120.** (b)
121. (d) **122.** (a) **123.** (b) **124.** (c) **125.** (a) **126.** (d)
127. (c) **128.** (d) **129.** (b) **130.** (c) **131.** (c) **132.** (b)
133.(2) **134.** (a) **135.** (b) **136.** (b) **137.** (d) **138.** (d)
139. (c) **140.** (b) **141.**(b,d) **142.** (d) **143.** (a, d)
144. (c) **145.** (c) **146.** (a)

Solutions

1. We have,

$$2(a-b)x^2 - 11(a+b+c)x - 3(a-b) = 0$$

$\therefore \quad D = \{-11(a+b+c)\}^2 - 4\cdot 2(a-b)\cdot(-3)(a-b)$

$$= 121(a+b+c)^2 + 24(a-b)^2 > 0$$

Therefore, the roots are real and unequal.

2. Here, $a < 0$

Cut-off Y-axis, $x = 0$

$\Rightarrow \quad y = c < 0$ [from graph]

$\therefore \quad c < 0$

x-coordinate of vertex > 0

$\Rightarrow \quad -\dfrac{b}{2a} > 0$

$\Rightarrow \quad \dfrac{b}{a} < 0$

But $\quad a < 0$

$\therefore \quad b > 0$

and y-coordinate of vertex < 0

$\Rightarrow \quad -\dfrac{D}{4a} < 0 \Rightarrow \dfrac{D}{4a} > 0$

$\therefore \quad D < 0$ $[\because a < 0]$

i.e. $\quad b^2 - 4ac < 0$

$\therefore \quad \dfrac{c}{a} > 0$ $[\because c < 0, a < 0]$

3. Sum of the roots $= -\dfrac{(a+3)}{a} = I^+$ [let]

$\therefore \quad a = \left(-\dfrac{3}{I^+ + 1}\right)$ ...(i)

Product of the roots $= \alpha\beta = \dfrac{a-3}{a} = I^+ + 2$ ...(ii)

and $\quad D = (a+3)^2 - 4a(a+3)$

$$= \frac{9}{(I^+ + 1)^2}\{(I^+ - 2)^2 - 12\} \quad \text{[from Eq. (i)]}$$

D must be perfect square, then $I^+ = 6$

From Eq. (ii),

Product of the roots $= I^+ + 2 = 6 + 2 = 8$

4. Let α be one root of

$$x^2 - 3ax + f(a) = 0$$

$\Rightarrow \quad \alpha + 2\alpha = 3a \Rightarrow 3\alpha = 3a$

$\Rightarrow \quad \alpha = a$ ...(i)

and $\quad \alpha \cdot 2\alpha = f(a)$

$\Rightarrow \quad f(a) = 2\alpha^2 = 2a^2$ [using Eq. (i)]

$\Rightarrow \quad f(x) = 2x^2$

5. $\because \quad x_1 x_2 = 4$ ...(i)

and $\quad \dfrac{x_1}{x_1 - 1} + \dfrac{x_2}{x_2 - 1} = 2$

$\Rightarrow \quad 2x_1x_2 - x_1 - x_2 = 2(x_1x_2 - x_1 - x_2 + 1)$

$\Rightarrow \quad 8 - x_1 - x_2 = 2(4 - x_1 - x_2 + 1)$ [from Eq. (i)]

or $\quad x_1 + x_2 = 2$ ...(ii)

From Eqs. (i) and (ii), required equation is

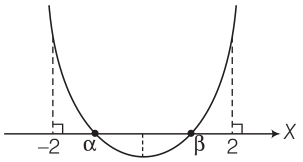
$$x^2 - (x_1 + x_2)x + x_1x_2 = 0$$

or $\quad x^2 - 2x + 4 = 0$

6. Let $f(x) = x^2 - 2ax + a^2 - 1$

Now, four cases arise:

Case **I** $D \ge 0$



$\Rightarrow \quad (-2a)^2 - 4\cdot 1(a^2 - 1) \ge 0$

$\Rightarrow \quad 4 \ge 0$

$\therefore \quad a \in R$

Case **II** $f(-2) > 0$

$\Rightarrow \quad 4 + 4a + a^2 - 1 > 0$

$\Rightarrow \quad a^2 + 4a + 3 > 0$

$\Rightarrow \quad (a+1)(a+3) > 0$

$\therefore \quad a \in (-\infty, -3) \cup (-1, \infty)$

***Case* III** $f(2) > 0$

$\Rightarrow \quad 4 - 4a + a^2 - 1 > 0$

$\Rightarrow \quad a^2 - 4a + 3 > 0$

$\Rightarrow \quad (a-1)(a-3) > 0$

$\therefore \quad a \in (-\infty, 1) \cup (3, \infty)$

Case **IV** $-2 < x$-coordinate of vertex < 2

$\Rightarrow \quad -2 < 2a < 2$

$\therefore \quad a \in (-1, 1)$

Combining all cases, we get $a \in (-1, 1)$

Hence, $\quad [a] = -1, 0$

7. We have, $-\left(\dfrac{-4a}{2(-2)}\right) = -2$

$\Rightarrow \quad a = 2$

$\therefore \quad y = -2x^2 - 8x + \lambda$ ...(i)

Since, Eq. (i) passes through points $(-2, 7)$

$\therefore \quad 7 = -2(-2)^2 - 8(-2) + \lambda$

$\Rightarrow \quad 7 = -8 + 16 + \lambda$

$\therefore \quad \lambda = -1$

8. Since, the coefficient of $n^2 = (4p - p^2 - 5) < 0$

Therefore, the graph is open downward.

According to the question, 1 must lie between the roots.

Hence, $$f(1) > 0$$
$$\Rightarrow \quad 4p - p^2 - 5 - 2p + 1 + 3p > 0$$
$$\Rightarrow \quad -p^2 + 5p - 4 > 0$$
$$\Rightarrow \quad p^2 - 5p + 4 < 0$$
$$\Rightarrow \quad (p-4)(p-1) < 0$$
$$\Rightarrow \quad 1 < p < 4$$
$$\therefore \quad p = 2, 3$$
Hence, number of integral values of p is 2.

9. We have, $3^{2x^2} - 2 \cdot 3^{x^2 + x + 6} + 3^{2(x+6)} = 0$
$$\Rightarrow \quad (3^{x^2} - 3^{x+6})^2 = 0$$
$$\Rightarrow \quad 3^{x^2} - 3^{x+6} = 0$$
$$\Rightarrow \quad 3^{x^2} = 3^{x+6} \Rightarrow x^2 = x + 6$$
$$\Rightarrow \quad x^2 - x - 6 = 0$$
$$\Rightarrow \quad (x-3)(x+2) = 0$$
$$\therefore \quad x = \{-2, 3\}$$

10. Given, $b^2 - 4ac = p^2 - 4aq$ ...(i)

and $$f(x) = g(x)$$
$$\Rightarrow \quad ax^2 + bx + c = ax^2 + px + q$$
$$\Rightarrow \quad (b-p)x = q - c$$
$$\therefore \quad x = \frac{q-c}{b-p} = \alpha \quad \text{[given] ...(ii)}$$
From Eq. (i), we get
$$(b+p)(b-p) + 4a(q-c) = 0$$
$$\Rightarrow \quad (b+p)(b-p) + 4a\alpha(b-p) = 0 \quad \text{[from Eq. (ii)]}$$
or $$\alpha = -\frac{(b+p)}{4a} \quad [\because b \neq p]$$
$$= \frac{\left(-\frac{b}{a}\right) + \left(-\frac{p}{a}\right)}{4}$$
$$= \frac{\begin{bmatrix} \text{Sum of the roots of } (f(x) = 0) \\ + \text{ Sum of the roots of } (g(x) = 0) \end{bmatrix}}{4}$$
$= \text{AM of the roots of } f(x) = 0$

and $g(x) = 0$

11. Let α and β be the roots of $ax^2 + bx + c = 0$.
$$\therefore \quad x_1 = \frac{\alpha + \beta}{2} = -\frac{b}{2a}$$
and $$x_2 = \frac{2\alpha\beta}{\alpha + \beta} = \frac{2 \cdot \frac{c}{a}}{-\frac{b}{a}} = -\frac{2c}{b}$$
$\therefore$ The required equation is
$$x^2 - \left[\left(-\frac{b}{2a}\right) + \left(-\frac{2c}{b}\right)\right]x + \frac{2bc}{2ab} = 0$$
i.e. $$2abx^2 + (b^2 + 4ac)x + 2bc = 0$$

12. Let α_1, α_2 and α_3 be the roots of $f(x) = 0$, such that
$$\alpha_1 < \alpha_2 < \alpha_3$$
and $g(x)$ can take all values from $[-6, \infty)$.
$$\because \quad g(x) = (x+1)^2 - 6 \geq -6$$
$$\therefore \quad \alpha_3 \leq -7, \alpha_2 \leq -8, \alpha_1 \leq -9$$
$$\therefore \quad a + b + c \geq 719$$
$\therefore$ Minimum value of $a + b + c$ is 719.
$$\because \quad \alpha_1 + \alpha_2 + \alpha_3 = -a$$
$$\Rightarrow \quad -a \leq -24$$
$$\Rightarrow \quad a \geq 24$$
$$\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_3\alpha_1 = b$$
$$\Rightarrow \quad b \geq 191$$
and $$\alpha_1\alpha_2\alpha_3 = -c$$
$$\Rightarrow \quad -c \leq -504$$
$$\Rightarrow \quad c \geq 504$$
$$\therefore \quad a + b + c \geq 719$$
Hence, minimum value of $a + b + c$ is 719.

13. $\because \sum_{k=1}^{n} (x + k - 1)(x + k) = 10n$
$$\Rightarrow \quad \sum_{k=1}^{n} x^2 + x(2k-1) + (k-1)k = 10n$$
$$\Rightarrow \quad nx^2 + x(1 + 3 + 5 + \ldots + (2n-1))$$
$$+ (0 + 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + (n-1)n) = 10n$$
$$\Rightarrow \quad nx^2 + x \cdot \frac{n}{2}(1 + 2n - 1) + \left(\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}\right) = 10n$$
$$\Rightarrow \quad nx^2 + n^2x + \frac{n(n^2-1)}{3} = 10n$$
$$\Rightarrow \quad x^2 + nx + \frac{(n^2 - 31)}{3} = 0 \quad \text{[dividing by } n]$$
$$\because \quad (\alpha + 1) - \alpha = \frac{\sqrt{D}}{1}$$
$$1 = \sqrt{D}$$
$$\Rightarrow \quad D = 1$$
$$\Rightarrow \quad n^2 - 4 \cdot 1 \cdot \frac{(n^2 - 31)}{3} = 1$$
$$\Rightarrow \quad 3n^2 - 4n^2 + 124 = 3$$
$$\Rightarrow \quad n^2 = 121$$
$$\therefore \quad n = 11$$

14. Since, 2 is only even prime.
Therefore, we have
$$2^2 + \lambda \cdot 2 + 12 = 0$$
$$\Rightarrow \quad \lambda = 8$$
$$\therefore \quad x^2 + \lambda x + \mu = 0$$
$$\Rightarrow \quad x^2 + 8x + \mu = 0 \quad \text{...(i)}$$
But Eq. (i) has equal roots.
$$\therefore \quad D = 0$$
$$\Rightarrow \quad 8^2 - 4 \cdot 1 \cdot \mu = 0$$
$$\Rightarrow \quad \mu = 16$$

15. We have, $\sqrt{x}+\sqrt{x-\sqrt{(1-x)}}=1$

$\Rightarrow \quad \sqrt{x-\sqrt{1-x}}=1-\sqrt{x}$

On squaring both sides, we get

$$x-\sqrt{1-x}=1+x-2\sqrt{x}$$

$\Rightarrow \quad -\sqrt{1-x}=1-2\sqrt{x}$

Again, squaring on both sides, we get

$$1-x=1+4x-4\sqrt{x}$$

$$4\sqrt{x}=5x$$

$\Rightarrow \quad \sqrt{x}=\frac{4}{5}$ [on squaring both sides]

$\Rightarrow \quad x=\frac{16}{25}$

Hence, the number of real solutions is 1.

16. Let $x=\sqrt{7+\sqrt{7-\sqrt{7+\sqrt{7-\ldots\infty}}}}$

$\Rightarrow \quad x=\sqrt{7+\sqrt{7-x}}$ [on squaring both sides]

$\Rightarrow \quad x^2-7=\sqrt{7-x}$

$\Rightarrow \quad (x^2-7)^2=7-x$ [again, squaring on both sides]

$\Rightarrow \quad x^4-14x^2+x+42=0$

$\Rightarrow \quad (x-3)(x^3+3x^2-5x-14)=0$

$\Rightarrow \quad (x-3)(x+2)(x^2+x-7)=0$

$\Rightarrow \quad x=3,-2,\frac{-1\pm\sqrt{29}}{2}$

$\therefore \quad x=3$ $[\because x>\sqrt{7}]$

17. Let $y=2(k-x)(x+\sqrt{(x^2+k^2)}$

$\Rightarrow \quad y-2(k-x)x=2(k-x)\sqrt{(x^2+k^2)}$

On squaring both sides, we get

$\Rightarrow \quad y^2+4(k-x)^2x^2-4xy(k-x)=4(k-x)^2(x^2+k^2)$

$\Rightarrow \quad y^2-4xy(k-x)=4(k-x)^2k^2$

$\Rightarrow \quad 4(k^2-y)x^2-4(2k^3-ky)x-y^2+4k^4=0$

Since, x is real.

$\therefore \quad D\geq 0$

$\Rightarrow \quad 16(2k^3-ky)^2-4\cdot 4(k^2-y)(4k^4-y^2)\geq 0$

[using, $b^2-4ac\geq 0$]

$\Rightarrow \quad 4k^6+k^2y^2-4k^4y-(-k^2y^2+4k^6+y^3-4yk^4)\geq 0$

$\Rightarrow \quad 2k^2y^2-y^3\geq 0$

$\Rightarrow \quad y^2(y-2k^2)\leq 0$

$\therefore \quad y\leq 2k^2$

18. We have, $\frac{1}{3}<\frac{x^2-2x+4}{x^2+2x+4}<3, \forall\ x\in R$

$$\frac{1}{3}<\frac{x^2+2x+4}{x^2-2x+4}<3, \forall\ x\in R$$

Let $\quad y=\frac{9\cdot 3^{2x}+6\cdot 3^x+4}{9\cdot 3^{2x}-6\cdot 3^x+4}=\frac{(3^{x+1})^2+2\cdot 3^{x+1}+4}{(3^{x+1})^2-2\cdot 3^{x+1}+4}$

$=\frac{t^2+2t+4}{t^2-2t+4}$, where $t=3^{x+1}$

$\Rightarrow \quad (y-1)t^2-2(y+1)t+4(y-1)=0$

By the given condition, for every $t\in R$,

$\frac{1}{3}<y<3$ ...(i)

But $\quad t=3^{x+1}>0$

We have, product of the roots $=4>0$, which is true.

And sum of the roots $=\frac{2(y+1)}{(y-1)}>0$

$\Rightarrow \quad \frac{y+1}{y-1}>0$

$\therefore \quad y\in(-\infty,-1)\cup(1,\infty)$ ...(ii)

From Eqs. (i) and (ii), we get

$$1<y<3$$

19. Since α, β and γ are the roots of

$$(x-a)(x-b)(x-c)=d$$

$\Rightarrow \quad (x-a)(x-b)(x-c)-d=(x-\alpha)(x-\beta)(x-\gamma)$

$\Rightarrow \quad (x-\alpha)(x-\beta)(x-\gamma)+d=(x-a)(x-b)(x-c)$

$\Rightarrow$ a, b and c are the roots of

$$(x-\alpha)(x-\beta)(x-\gamma)+d=0$$

20. Since, all the coefficients of given equation are not real.

Therefore, other root $\neq 3+i$.

Let other root be α.

Then, sum of the roots $=\frac{2(1+i)}{i}$

$\Rightarrow \quad \alpha+3-i=\frac{2(1+i)}{i}$

$\Rightarrow \quad \alpha+3-i=2-2i$

$\therefore \quad \alpha=-1-i$

21. We have, $|[x]-2x|=4$

$\Rightarrow \quad |[x]-2([x]+\{x\})|=4$

$\Rightarrow \quad |[x]+2\{x\}|=4$

which is possible only when

$$2\{x\}=0, 1$$

If $\{x\}=0$, then $[x]=\pm 4$ and then $x=-4, 4$ and if $\{x\}=\frac{1}{2}$, then

$$[x]+1=\pm 4$$

$\Rightarrow \quad [x]=3,-5$

$\therefore \quad x=3+\frac{1}{2}$ and $-5+\frac{1}{2}$

$\Rightarrow \quad x=\frac{7}{2},-\frac{9}{2} \Rightarrow x=-4,-\frac{9}{2},\frac{7}{2}, 4$

22. We know that, x^2+x+1 is a factor of ax^3+bx^2+cx+d.

Hence, roots of $x^2+x+1=0$ are also roots of $ax^3+bx^2+cx+d=0$. Since, ω and ω^2

$\left(\text{where } \omega = -\frac{1}{2} + \frac{3i}{2}\right)$ are two complex roots of $x^2 + x + 1 = 0$.

Therefore, ω and ω^2 are two complex roots of $ax^3 + bx^2 + cx + d = 0$.

We know that, a cubic equation has atleast one real root. Let real root be α. Then,

$$\alpha \cdot \omega \cdot \omega^2 = -\frac{d}{a} \Rightarrow \alpha = -\frac{d}{a}$$

23. We have, $\sqrt{(5x^2 - 8x + 3)} - \sqrt{(5x^2 - 9x + 4)}$

$$= \sqrt{(2x^2 - 2x)} - \sqrt{(2x^2 - 3x + 1)}$$

$$\Rightarrow \sqrt{(5x-3)(x-1)} - \sqrt{(5x-4)(x-1)}$$

$$= \sqrt{2x(x-1)} - \sqrt{(2x-1)(x-1)}$$

$$\Rightarrow \sqrt{x-1}\,(\sqrt{5x-3} - \sqrt{5x-4}) = \sqrt{x-1}\,(\sqrt{2x} - \sqrt{2x-1})$$

$$\Rightarrow \sqrt{x-1} = 0$$

$$\Rightarrow x = 1$$

24. We have, $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b = 1$ [given]

$$\therefore (a + \sqrt{b})^{x^2 - 15} + (a - \sqrt{b})^{x^2 - 15} = 2a$$

$$\Rightarrow (a + \sqrt{b})^{x^2 - 15} + \frac{1}{(a + \sqrt{b})^{x^2 - 15}} = 2a$$

Let $y = (a + \sqrt{b})^{x^2 - 15}$

$$\Rightarrow y + \frac{1}{y} = 2a \Rightarrow y^2 - 2ay + 1 = 0$$

$$\Rightarrow y = \frac{2a \pm \sqrt{4a^2 - 4}}{2} = a \pm \sqrt{a^2 - 1}$$

$$\therefore y = a \pm \sqrt{b} = (a + \sqrt{b})^{\pm 1} \quad [\because a^2 - b = 1]$$

$$\Rightarrow (a + \sqrt{b})^{x^2 - 15} = (a + \sqrt{b})^{\pm 1}$$

$$\therefore x^2 - 15 = \pm 1$$

$$\Rightarrow x^2 = 15 \pm 1 \Rightarrow x^2 = 16, 14$$

$$\Rightarrow x = \pm 4, \pm \sqrt{14}$$

25. We have, $x^2 - xy + y^2 = 4(x + y - 4)$

$$\Rightarrow x^2 - x(y + 4) + y^2 - 4y + 16 = 0$$

$$\because x \in R$$

$$\therefore (-(y + 4))^2 - 4 \cdot 1 \cdot (y^2 - 4y + 16) \geq 0$$

[using, $b^2 - 4ac \geq 0$]

$$\Rightarrow y^2 + 8y + 16 - 4y^2 + 16y - 64 \geq 0$$

$$\Rightarrow 3y^2 - 24y + 48 \leq 0$$

$$\Rightarrow y^2 - 8y + 16 \leq 0 \Rightarrow (y - 4)^2 \leq 0$$

$$\therefore (y - 4)^2 = 0$$

$$\therefore y = 4$$

Then, $x^2 - 4x + 16 = 4(x + 4 - 4)$

$$x^2 - 8x + 16 = 0$$

$$(x - 4)^2 = 0$$

$$x = 4$$

Number of pairs is 1 i.e., (4, 4).

26. Since, 3789108 is an even integer. Therefore, $x^4 - y^4$ is also an even integer. So, either both x and y are even integers or both of them are odd integers.

Now, $x^4 - y^4 = (x - y)(x + y)(x^2 + y^2)$

$\Rightarrow x - y,\ x + y,\ x^2 + y^2$ must be even integers.

Therefore, $(x - y)(x + y)(x^2 + y^2)$ must be divisible by 8. But 3789108 is not divisible by 8. Hence, the given equation has no solution.

$\therefore$ Number of solutions = 0

27. We have, $x^3 + ax + 1 = 0$

or $x^4 + ax^2 + x = 0$ ...(i)

and $x^4 + ax^2 + 1 = 0$ ...(ii)

From Eqs. (i) and (ii), we get

$$x - 1 = 0$$

$$\Rightarrow x = 1$$

which is a common root.

$$\therefore 1 + a + 1 = 0$$

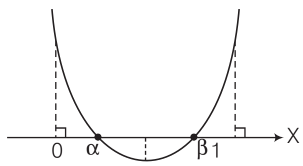
$$\Rightarrow a = -2$$

28. $\because (1 - a^2)\, x^2 + 2ax - 1 = 0$

$$1 - a^2 \neq 0$$

$$x^2 + \left(\frac{2a}{1 - a^2}\right)x - \left(\frac{1}{1 - a^2}\right) = 0$$

Let $f(x) = x^2 + \left(\frac{2a}{1 - a^2}\right)x - \left(\frac{1}{(1 - a^2)}\right)$



The following cases arise:

***Case* I** $D \geq 0$

$$\left(\frac{2a}{1 - a^2}\right)^2 - 4 \cdot 1 \cdot \left(\frac{-1}{1 - a^2}\right) \geq 0$$

$$\Rightarrow \frac{4a^2}{(1 - a^2)^2} + \frac{4}{(1 - a^2)} \geq 0$$

$$\Rightarrow \frac{4a^2 + 4 - 4a^2}{(1 - a^2)^2} \geq 0$$

$$\Rightarrow \frac{4}{(1 - a^2)^2} \geq 0 \quad \text{[always true]}$$

***Case* II** $f(0) > 0$

$$\Rightarrow \frac{-1}{(1 - a^2)} > 0 \Rightarrow \frac{1}{1 - a^2} < 0$$

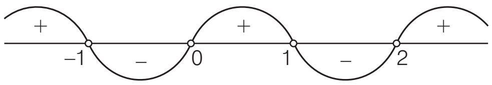
$$\Rightarrow 1 - a^2 < 0$$

$$\therefore a \in (-\infty, -1) \cup (1, \infty)$$

***Case* III** $f(1) > 0$

$$\Rightarrow 1 + \frac{2a}{(1 - a^2)} - \frac{1}{(1 - a^2)} > 0$$

$$\Rightarrow \quad \frac{1-a^2+2a-1}{(1-a^2)} > 0 \Rightarrow \frac{a^2-2a}{1-a^2} < 0$$



$$\Rightarrow \quad \frac{a(a-2)}{(a+1)(a-1)} > 0$$

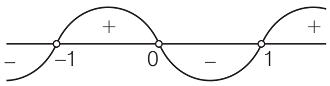
$$\therefore \quad a \in (-\infty, -1) \cup (0, 1) \cup (2, \infty)$$

***Case* IV** $0 < x$-coordinate of vertex < 1

$$\Rightarrow \quad 0 < -\frac{2a}{2(1-a^2)} < 1 \Rightarrow 0 < \frac{a}{a^2-1} < 1$$

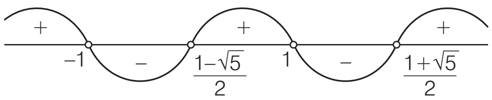
$$\Rightarrow \quad 0 < \frac{a}{(a+1)(a-1)} \text{ and } 1 - \frac{a}{a^2-1} > 0$$

$$\Rightarrow \quad \frac{a}{(a+1)(a-1)} > 0$$



$$\Rightarrow \quad a \in (-1, 0) \cup (1, \infty)$$

and
$$\frac{\left(a - \frac{1+\sqrt{5}}{2}\right)\left(a - \frac{1-\sqrt{5}}{2}\right)}{(a+1)(a-1)} > 0$$



and $a \in (-\infty, -1) \cup \left(\frac{1-\sqrt{5}}{2}, 1\right) \cup \left(\frac{1+\sqrt{5}}{2}, \infty\right)$

$$\therefore \quad a \in \left(\frac{1-\sqrt{5}}{2}, 0\right) \cup \left(\frac{1+\sqrt{5}}{2}, \infty\right)$$

Combining all cases, we get

$$a > 2$$

29. We have, $\quad x - \sqrt{1-|x|} < 0 \qquad$ ...(i)

which is defined only when

$$1 - |x| \ge 0$$

$$\Rightarrow \quad |x| \le 1$$

$$\Rightarrow \quad x \in [-1, 1]$$

Now, from Eq. (i), we get

$$x < \sqrt{1-|x|}$$

***Case* I** If $x \ge 0$, i.e., $0 \le x \le 1$

$$x - \sqrt{(1-|x|)} < 0$$

$$\Rightarrow \quad x < \sqrt{(1-x)}$$

On squaring both sides, we get

$$x^2 + x - 1 < 0$$

$$\Rightarrow \quad \frac{-1-\sqrt{5}}{2} < x < \frac{-1+\sqrt{5}}{2}$$

But $\quad x \ge 0$

$$\therefore \quad x \in \left[0, \frac{-1+\sqrt{5}}{2}\right)$$

***Case* II** If $x < 0$, i.e., $-1 \le x < 0$

$$x - \sqrt{(1+x)} < 0$$

$$\Rightarrow \quad x < \sqrt{1+x} \qquad \text{[always true]}$$

$$x \in [-1, 0)$$

Combining both cases, we get

$$x \in \left[-1, \frac{-1+\sqrt{5}}{2}\right)$$

30. We have, $(a \cdot 2b - 2c \cdot a)(2c \cdot c - b \cdot 2b) = (ba - ca)^2$

$$\Rightarrow \quad 2a(b-c) \cdot 2(c^2 - b^2) = a^2(b-c)^2$$

$$\Rightarrow \quad 4a(c-b)(c+b) = a^2(b-c) \qquad [\because b \ne c]$$

$$\Rightarrow \quad 4a(c+b) = -a^2$$

$$\Rightarrow \quad a + 4b + 4c = 0$$

31. $0 < a < b < c, \alpha + \beta = \left(-\frac{b}{a}\right)$ and $\alpha\beta = \frac{c}{a}$

For non-real complex roots,

$$b^2 - 4ac < 0$$

$$\Rightarrow \quad \frac{b^2}{a^2} - \frac{4c}{a} < 0$$

$$\Rightarrow \quad (\alpha+\beta)^2 - 4\alpha\beta < 0$$

$$\Rightarrow \quad (\alpha-\beta)^2 < 0$$

$$\because \quad 0 < a < b < c$$

$\therefore$ Roots are conjugate, then $|\alpha| = |\beta|$

But $\quad \alpha\beta = \dfrac{c}{a}$

$$|\alpha\beta| = \left|\frac{c}{a}\right| > 1 \qquad \left[\because a < c, \therefore \frac{c}{a} > 1\right]$$

$$\Rightarrow \quad |\alpha||\beta| > 1$$

$$\Rightarrow \quad |\alpha|^2 > 1 \text{ or } |\alpha| > 1$$

32. Given equation is

$$Ax^2 - |G|x - H = 0 \qquad \text{...(i)}$$

$$\therefore \quad \text{Discriminant} = (-|G|)^2 - 4A(-H)$$
$$= G^2 + 4AH$$
$$= G^2 + 4G^2 \qquad [\because G^2 = AH]$$
$$= 5G^2 > 0$$

$\therefore$ Roots of Eq. (i) are real and distinct.

$$\because \quad A = \frac{a+b}{2} > 0, G = \sqrt{ab} > 0, H = \frac{2ab}{a+b} > 0$$

[$\because$ a and b are two unequal positive integers]

Let α and β be the roots of Eq. (i). Then,

$$\alpha + \beta = \frac{|G|}{A} > 0$$

and $\quad \alpha\beta = -\dfrac{H}{A} < 0$

and $\quad \alpha - \beta = \dfrac{\sqrt{D}}{A} = \dfrac{G\sqrt{5}}{A} > 0$

$\therefore \quad \alpha = \frac{|G| + G\sqrt{5}}{2A} > 0$

and $\quad \beta = \frac{|G| - G\sqrt{5}}{2A} < 0$

Exactly one positive root and atleast one root which is negative fraction.

33. It is clear from graph that the equation $y = ax^2 + bx + c = 0$ has two real and distinct roots. Therefore,

$$b^2 - 4ac > 0 \quad \text{...(i)}$$

$\because$ Parabola open downwards.

$\therefore \quad a < 0$

and $y = ax^2 + bx + c$ cuts-off Y-axis at, $x = 0$.

$\therefore \quad y = c < 0$

$\Rightarrow \quad c < 0$

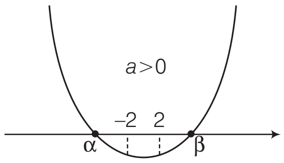
and x-coordinate of vertex > 0

$\Rightarrow \quad -\frac{b}{2a} > 0 \Rightarrow \frac{b}{a} < 0$

$\Rightarrow \quad b > 0 \quad [\because a < 0]$

It is clear that a and b are of opposite signs.

34. Let $y = ax^2 + bx + c$



Consider the following cases:

***Case* I** $D > 0$

$\Rightarrow \quad b^2 - 4ac > 0$

***Case* II** $af(-2) < 0$

$\Rightarrow \quad a(4a - 2b + c) < 0$

$\Rightarrow \quad 4a - 2b + c < 0$

***Case* III** $af(2) > 0$

$\Rightarrow \quad a(4a + 2b + c) > 0$

$\Rightarrow \quad 4a + 2b + c > 0$

Combining *Case* II and *Case* III, we get

$$4a + 2|b| + c < 0$$

Also, at $x = 0$, $\quad y < 0 \Rightarrow c < 0$

Also, since for $-2 < x < 2$,

$$y < 0$$

$\Rightarrow \quad ax^2 + bx + c < 0$

For $x = 1$, $\quad a + b + c < 0 \quad$ ...(i)

and for $x = -1$, $\quad a - b + c < 0 \quad$ ...(ii)

Combining Eqs. (i) and (ii), we get

$$a + |b| + c < 0$$

35. Put $x^2 = y$.

Then, the given equation can be written as

$$f(y) = ay^2 + by + c = 0 \quad \text{...(i)}$$

The given equation will have four real roots, i.e. Eq. (i) has two non-negative roots.

Then, $\quad -\frac{b}{a} \geq 0$

$af(0) \geq 0$

and $\quad b^2 - 4ac \geq 0 \quad$ [given]

$\Rightarrow \quad \frac{b}{a} \leq 0$

$ac \geq 0$

$\Rightarrow \quad a > 0, b < 0, c > 0$

or $\quad a < 0, b > 0, c < 0$

36. Let the roots be $\frac{a}{r}$, a and ar, where $a > 0$, $r > 1$

$\therefore$ Product of the roots $= 1$

$\Rightarrow \quad \frac{a}{r} \cdot a \cdot ar = 1$

$\Rightarrow \quad a^3 = 1$

$\therefore \quad a = 1 \quad$ [one root is 1]

Now, roots are $\frac{1}{r}$, 1 and r. Then,

$$\frac{1}{r} + 1 + r = -b$$

$\Rightarrow \quad \frac{1}{r} + r = -b - 1 \quad$ ...(i)

$\because \quad r + \frac{1}{r} > 2$

$\Rightarrow \quad -b - 1 > 2$

$\Rightarrow \quad b < -3 \quad$ [from Eq. (i)]

or $\quad b \in (-\infty, -3)$

Also, $\quad \frac{1}{r} \cdot 1 + 1 \cdot r + r \cdot \frac{1}{r} = c$

$\Rightarrow \quad \frac{1}{r} + r + 1 = c = -b \quad$ [from Eq. (i)]

$\therefore \quad b + c = 0$

Now, first root $= \frac{1}{r} < 1 \quad$ [$\because$ one root is smaller than one]

Second root $= 1$

Third root $= r > 1 \quad$ [$\because$ one root is greater than one]

37. We have, $\quad f(x) = ax^2 + bx + c$

$a, b, c \in R \quad [\because a \neq 0]$

On putting $x = 0, 1, \frac{1}{2}$, we get

$$|c| \leq 1$$

$$|a + b + c| \leq 1$$

and $\quad \left|\frac{1}{4}a + \frac{1}{2}b + c\right| \leq 1$

$\Rightarrow \quad -1 \leq c \leq 1,$

$-1 \leq a + b + c \leq 1$

and $\quad -4 \leq a + 2b + 4c \leq 4$

$\Rightarrow \quad -4 \leq 4a + 4b + 4c \leq 4$

and $\quad -4 \leq -a - 2b - 4c \leq 4$

On adding, we get

$$-8 \le 3a + 2b \le 8$$

Also, $\quad -8 \le a + 2b \le 8$

$\therefore \quad -16 \le 2a \le 16$

$\Rightarrow \quad |a| \le 8$

$\because \quad -1 \le -c \le 1, -8 \le -a \le 8$

We get, $\quad -16 \le 2b \le 16$

$\Rightarrow \quad |b| \le 8$

$\therefore \quad |a| + |b| + |c| \le 17$

38. $\because \quad x = \dfrac{-5 \pm \sqrt{25 + 1200}}{50} = \dfrac{-5 \pm 35}{50} = \dfrac{30}{50}, \dfrac{-40}{50}$

or $\quad \cos\alpha = \dfrac{3}{5}, \dfrac{-4}{5}$

But $\quad -1 < x < 0$

$\therefore \quad \cos\alpha = -\dfrac{4}{5}$ [lies in II and III quadrants]

$\therefore \quad \sin\alpha = \dfrac{3}{5}$ [lies in II quadrant]

$\therefore \quad \sin\alpha = -\dfrac{3}{5}$ [lies in III quadrant]

$\therefore \quad \sin 2\alpha = 2 \cdot \sin\alpha \cdot \cos\alpha = -\dfrac{24}{25}$

[lies in II quadrant]

$\therefore \quad \sin 2\alpha = 2 \cdot \sin\alpha \cdot \cos\alpha = \dfrac{24}{25}$ [lies in III quadrant]

39. $\because a + 2b + 4c = 0$

$\therefore \quad a\left(\dfrac{1}{2}\right)^2 + b\left(\dfrac{1}{2}\right) + c = 0$

It is clear that one root is $\dfrac{1}{2}$.

Let other root be α. Then, $\alpha + \dfrac{1}{2} = -\dfrac{b}{a}$

$\Rightarrow \quad \alpha = -\dfrac{1}{2} - \dfrac{b}{a}$

which depends upon a and b.

40. $\because$ Cut-off Y-axis, put $x = 0$, i.e. $f(0) = c$

Option (a) $a < 0, c < 0, -\dfrac{b}{2a} < 0$

or $\quad a < 0, c < 0, b < 0$

$\therefore \quad abc < 0$

Option (b) $\quad a < 0, c > 0, -\dfrac{b}{2a} > 0$

or $\quad a < 0, c > 0, b > 0$

$\therefore \quad abc < 0$

Option (c) $a > 0, c > 0, -\dfrac{b}{2a} > 0$

or $\quad a > 0, c > 0, b < 0$

$\therefore \quad abc < 0$

Option (d) $a < 0, c < 0, -\dfrac{b}{2a} < 0$

or $\quad a < 0, c < 0, b < 0$

$\therefore \quad abc < 0$

41. Here, $D \le 0$

and $\quad f(x) \ge 0, \forall x \in R$

$\therefore \quad f(3) \ge 0$

$\Rightarrow \quad 9a + 3b + 6 \ge 0$

or $\quad 3a + b \ge -2$

$\Rightarrow$ Minimum value of $3a + b$ is -2.

and $\quad f(6) \ge 0$

$\Rightarrow \quad 36a + 6b + 6 \ge 0$

$\Rightarrow \quad 6a + b \ge -1$

$\Rightarrow$ Minimum value of $6a + b$ is -1.

42. Since, $f(x) = x^3 + 3x^2 - 9x + \lambda = (x - \alpha)^2 (x - \beta)$

$\therefore \alpha$ is a double root.

$\therefore f'(x) = 0$ has also one root α.

i.e. $3x^2 + 6x - 9 = 0$ has one root α.

$\therefore \quad x^2 + 2x - 3 = 0 \quad$ or $\quad (x + 3)(x - 1) = 0$

has the root α which can either -3 or 1.

If $\alpha = 1$, then $f(1) = 0$ gives $\lambda - 5 = 0 \Rightarrow \lambda = 5$.

If $\alpha = -3$, then $f(-3) = 0$ gives

$$-27 + 27 + 27 + \lambda = 0$$

$\Rightarrow \quad \lambda = -27$

43. We have, $\quad D = (b - c)^2 - 4a(a - b - c) > 0$

$\Rightarrow \quad b^2 + c^2 - 2bc - 4a^2 + 4ab + 4ac > 0$

$\Rightarrow \quad c^2 + (4a - 2b)c - 4a^2 + 4ab + b^2 > 0, \forall c \in R$

Since, $c \in R$, so we have

$$(4a - 2b)^2 - 4(-4a^2 + 4ab + b^2) < 0$$

$\Rightarrow \quad 4a^2 - 4ab + b^2 + 4a^2 - 4ab - b^2 < 0$

$\Rightarrow \quad a(a - b) < 0$

If $a > 0$, then $a - b < 0$

i.e. $\quad 0 < a < b$

or $\quad b > a > 0$

If $a < 0$, then $a - b > 0$

i.e. $\quad 0 > a > b$

or $\quad b < a < 0$

44. We have, $\quad x^3 - ax^2 + bx - 1 = 0 \quad$ ...(i)

Then, $\quad \alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$$= a^2 - 2b$$

$\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 = (\alpha\beta + \beta\gamma + \gamma\alpha)^2$

$\quad - 2\alpha\beta\gamma(\alpha + \beta + \gamma) = b^2 - 2a$

and $\quad \alpha^2\beta^2\gamma^2 = 1$

Therefore, the equation whose roots are α^2, β^2 and γ^2, is

$$x^3 - (a^2 - 2b)x^2 + (b^2 - 2a)x - 1 = 0 \quad \text{...(ii)}$$

Since, Eqs. (i) and (ii) are indentical, therefore

$$a^2 - 2b = a \quad \text{and} \quad b^2 - 2a = b$$

Eliminating b, we have

$$\frac{(a^2-a)^2}{4}-2a=\frac{a^2-a}{2}$$

$\Rightarrow \quad a\{a(a-1)^2-8-2(a-1)\}=0$

$\Rightarrow \quad a(a^3-2a^2-a-6)=0$

$\Rightarrow \quad a(a-3)(a^2+a+2)=0$

$\Rightarrow \quad a=0$ or $a=3$ or $a^2+a+2=0$

$\Rightarrow \quad b=0$ or $b=3$

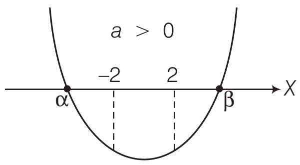
or $\quad b^2+b+2=0$

$\therefore \quad a=b=0$

or $\quad a=b=3$

or a and b are roots of $x^2+x+2=0$.

45. Here, $D>0$



$$b^2-4ac>0$$

or $\quad b^2>4ac \quad$ ...(i)

and $\quad f(0)<0$

$\Rightarrow \quad c<0 \quad$ ...(ii)

$f(1)<0$

$\Rightarrow \quad a+b+c<0 \quad$ ...(iii)

$f(-1)<0$

$\Rightarrow \quad a-b+c<0 \quad$ ...(iv)

$f(2)<0$

$\Rightarrow \quad 4a+2b+c<0 \quad$ ...(v)

$f(-2)<0$

$\Rightarrow \quad 4a-2b+c<0 \quad$ ...(vi)

From Eqs. (i) and (ii), we get

$$c<0,\ b^2-4ac>0$$

From Eqs. (iii) and (iv), we get

$$a-|b|+c<0$$

and from Eqs. (v) and (vi), we get

$$4a-2|b|+c<0$$

Solutions (Q. Nos. 46 to 48)

Let $\quad y=\dfrac{2x^2-3x+2}{2x^2+3x+2}$

$\Rightarrow \quad 2x^2y+3xy+2y=2x^2-3x+2$

$\Rightarrow 2(y-1)x^2+3(y+1)x+2(y-1)=0$

As $x\in R$

$\therefore \quad D\ge 0$

$\Rightarrow \quad 9(y+1)^2-4\cdot 2(y-1)\cdot 2(y-1)\ge 0$

$\Rightarrow \quad 9(y+1)^2-16(y-1)^2\ge 0$

$\Rightarrow \quad (3y+3)^2-(4y-4)^2\ge 0$

$\Rightarrow \quad (7y-1)(7-y)\ge 0$

$\Rightarrow \quad (7y-1)(y-7)\le 0$

$\therefore \quad \dfrac{1}{7}\le y\le 7$

$\therefore \quad G=7$ and $L=\dfrac{1}{7}$

$\therefore \quad GL=1$

Now, $\quad \dfrac{G^{100}+L^{100}}{2}\ge (GL)^{100} \Rightarrow \dfrac{G^{100}+L^{100}}{2}\ge 1$

$\Rightarrow \quad G^{100}+L^{100}\ge 2$

46. Least value of $G^{100}+L^{100}$ is 2.

47. The quadratic equation having roots G and L, is

$$x^2-(G+L)x+GL=0$$

$\Rightarrow \quad x^2-\dfrac{50}{7}x+1=0$

$\Rightarrow \quad 7x^2-50x+7=0$

48. We have, $\quad L^2<\lambda<G^2$

$\Rightarrow \quad \left(\dfrac{1}{7}\right)^2<\lambda<7^2$

$\Rightarrow \quad \dfrac{1}{49}<\lambda<49$

$\Rightarrow \quad \lambda=1,2,3,\ldots,48$ as $\lambda\in N$

$\therefore$ Sum of all values of $\lambda=1+2+3+\ldots+48=\dfrac{48\times 49}{2}=1176$

Solutions (Q. Nos. 49 to 51)

Let roots be $\alpha,\beta,\gamma,\delta>0$.

$\therefore \quad \alpha+\beta+\gamma+\delta=12$

$(\alpha+\beta)(\gamma+\delta)+\alpha\beta+\gamma\delta=c$

$\alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta)=-d$

$\alpha\beta\gamma\delta=81$

$\because \quad AM=\dfrac{\alpha+\beta+\gamma+\delta}{4}=3$

and $\quad GM=(\alpha\beta\gamma\delta)^{1/4}=(81)^{1/4}=3$

$\therefore \quad AM=GM$

$\Rightarrow \quad \alpha=\beta=\gamma=\delta=3$

49. $c=(\alpha+\beta)(\gamma+\delta)+\alpha\beta+\gamma\delta$

$=(3+3)(3+3)+3\cdot 3+3\cdot 3=36+18=54$

50. $\because \alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta)=-d$

$\therefore \quad d=-\{3\cdot 3\cdot(3+3)+3\cdot 3\cdot(3+3)\}=-108$

51. Required root $=-\dfrac{d}{2c}=-\dfrac{(-108)}{2\times 54}=1$

Solutions (Q. Nos. 52 to 54)

Given that, $\quad AC=4\sqrt{2}$ units

$\therefore \quad AB=BC=\dfrac{AC}{\sqrt{2}}=4$ units

and $\quad OB=\sqrt{(BC)^2-(OC)^2}$

$=\sqrt{(4)^2-(2\sqrt{2})^2} \qquad \left[\because OC=\dfrac{AC}{2}\right]$

$=2\sqrt{2}$ units

$\therefore$ Vertices are $A \equiv (-2\sqrt{2}, 0)$,

$B \equiv (0, -2\sqrt{2})$

and $C \equiv (2\sqrt{2}, 0)$

52. Since, $y = f(x) = ax^2 + bx + c$ passes through A, B and C, then

$$0 = 8a - 2\sqrt{2}b + c - 2\sqrt{2} = c$$

and $$0 = 8a + 2\sqrt{2}b + c$$

We get, $b = 0, a = \frac{1}{2\sqrt{2}}$ and $c = -2\sqrt{2}$

$$\therefore \quad y = f(x) = \frac{x^2}{2\sqrt{2}} - 2\sqrt{2}$$

53. Minimum value of $y = \frac{x^2}{2\sqrt{2}} - 2\sqrt{2}$ is at $x = 0$.

$$\therefore \quad (y)_{\min} = -2\sqrt{2}$$

54. $f(x) = 0$

$$\Rightarrow \quad \frac{x^2}{2\sqrt{2}} - 2\sqrt{2} = 0 \Rightarrow x = \pm 2\sqrt{2}$$

Given, $$-2\sqrt{2} < \frac{\lambda}{2} < 2\sqrt{2}$$

or $$-4\sqrt{2} < \lambda < 4\sqrt{2}$$

$\therefore$ Initial values of λ are

$-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$.

$\therefore$ Number of integral values is 11.

Solutions (Q. Nos. 55 to 57)

We have, $$(\alpha - \beta) = (\alpha + k) - (\beta + k)$$

$$\Rightarrow \quad \frac{\sqrt{b^2 - 4c}}{1} = \frac{\sqrt{b_1^2 - 4c_1}}{1}$$

$$\Rightarrow \quad b^2 - 4c = b_1^2 - 4c_1 \quad \text{...(i)}$$

Given, least value of $f(x) = -\frac{1}{4} - \frac{(b^2 - 4c)}{4 \times 1} = -\frac{1}{4}$

$$\Rightarrow \quad b^2 - 4c = 1$$

$$\therefore \quad b^2 - 4c = 1 = b_1^2 - 4c_1 \quad \text{[from Eq. (i)] ...(ii)}$$

Also, given least value of $g(x)$ occurs at $x = \frac{7}{2}$.

$$\therefore \quad -\frac{b_1}{2 \times 1} = \frac{7}{2}$$

$$\therefore \quad b_1 = -7$$

55. $b_1 = -7$

56. Least value of $g(x) = -\frac{b_1^2 - 4c_1}{4 \times 1} = -\frac{1}{4}$ [from Eq. (ii)]

57. $\because g(x) = 0$

$$\therefore \quad x^2 + b_1 x + c_1 = 0$$

$$\Rightarrow \quad x = \frac{-b_1 \pm \sqrt{b_1^2 - 4c_1}}{2}$$

$$= \frac{7 \pm 1}{2} = 3, 4$$

$\therefore$ Roots of $g(x) = 0$ are 3, 4.

Solutions (Q. Nos. 58 to 60)

Let $f(x) = ax^2 - bx + c$ has two distinct roots α and β. Then, $f(x) = a(x - \alpha)(x - \beta)$. Since, $f(0)$ and $f(1)$ are of same sign.

Therefore, $$c(a - b + c) > 0$$

$$\Rightarrow \quad c(a - b + c) \geq 1$$

$$\therefore \quad a^2 \alpha\beta (1 - \alpha)(1 - \beta) \geq 1$$

But $$\alpha(1 - \alpha) = \frac{1}{4} - \left(\frac{1}{2} - \alpha\right)^2 \leq \frac{1}{4}$$

$$\therefore \quad a^2 \alpha\beta (1 - \alpha)(1 - \beta) < \frac{a^2}{16}$$

$$\Rightarrow \quad \frac{a^2}{16} > 1 \Rightarrow a > 4 \quad [\because \alpha \neq \beta]$$

$$\Rightarrow \quad a \geq 5 \text{ as } a \in I$$

Also, $$b^2 - 4ac \geq 0$$

$$\Rightarrow \quad b^2 \geq 4ac \geq 20$$

$$\Rightarrow \quad b \geq 5$$

Next, $a \geq 5, b \geq 5$, we get $c \geq 1$

$$\therefore \quad abc \geq 25$$

$$\therefore \quad \log_5 abc \geq \log_5 25 = 2$$

58. Least value of a is 5.

59. Least value of b is 5.

60. Least value of $\log_b abc$ is 2.

Solutions (Q. Nos. 61 to 63)

Let α, β and γ be the roots of $2x^3 + ax^2 + bx + 4 = 0$.

$$\therefore \quad \alpha + \beta + \gamma = -\frac{a}{2}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{b}{2} \text{ and } \alpha\beta\gamma = -2$$

61. $\because$ AM $\geq$ GM

$$\therefore \quad \frac{(-\alpha) + (-\beta) + (-\gamma)}{3} \geq \{(-\alpha)(-\beta)(-\gamma)\}^{1/3}$$

$$\Rightarrow \quad \frac{\frac{a}{2}}{3} \geq (2)^{1/3}$$

$$\therefore \quad a \geq 6(2)^{1/3} \quad \text{...(i)}$$

or $$a^3 \geq 432$$

Hence, minimum value of a^3 is 432.

62. $\because$ AM $\geq$ GM

$$\therefore \quad \frac{(-\alpha)(-\beta) + (-\beta)(-\gamma) + (-\gamma)(-\alpha)}{3}$$

$$\geq \{(-\alpha)(-\beta)(-\beta)(-\gamma)(-\gamma)(-\alpha)\}^{1/3}$$

$$\Rightarrow \quad \frac{b/2}{3} \geq (4)^{1/3}$$

$$\Rightarrow \quad b \geq 6(4)^{1/3} \quad \text{...(ii)}$$

or $$b^3 \geq 864$$

Hence, minimum value of b^3 is 864.

63. From Eqs. (i) and (ii), we get

$$ab \geq 6(2)^{1/3} \cdot 6(4)^{1/3}$$

$\Rightarrow \quad ab \geq 36 \times 2$

$\because \quad \dfrac{a+b}{2} \geq \sqrt{ab} \geq 6\sqrt{2} \Rightarrow \dfrac{a+b}{2} \geq 6\sqrt{2}$

$\therefore \quad a+b \geq 12\sqrt{2}$

or $\quad (a+b)^3 \geq 3456\sqrt{2}$

Hence, minimum value of $(a+b)^3$ is $3456\sqrt{2}$.

Solutions (Q. Nos. 64 to 66)

$\therefore \quad \alpha+\beta+\gamma+\delta = -A \quad ...(i)$

$(\alpha+\beta)(\gamma+\delta)+\alpha\beta+\gamma\delta = B \quad ...(ii)$

$\alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta) = -C \quad ...(iii)$

and $\quad \alpha\beta\gamma\delta = D \quad ...(iv)$

64. $\because \quad \dfrac{C}{A} = \dfrac{\alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta)}{\alpha+\beta+\gamma+\delta}$

$= \dfrac{k(\gamma+\delta)+k(\alpha+\beta)}{\alpha+\beta+\gamma+\delta} \quad [\because \alpha\beta = \gamma\delta = k]$

$= k \quad ...(v)$

65. From Eq. (ii), we get

$(\alpha+\beta)(\gamma+\delta) = B-(\alpha\beta+\gamma\delta) = B-2k \quad [\because \alpha\beta=\gamma\delta=k]$

66. From Eq. (iv), we get

$\alpha\beta\gamma\delta = D$

$\Rightarrow \quad k \cdot k = D \quad [\because \alpha\beta=\gamma\delta=k]$

$\Rightarrow \quad \left(\dfrac{C}{A}\right)^2 = D \quad$ [from Eq. (v)]

$\therefore \quad C^2 = A^2 D$

67. The given equation is $|x-2|^2+|x-2|-2=0$.

There are two cases:

***Case* I** If $x \geq 2$, then $(x-2)^2+x-2-2=0$

$\Rightarrow \quad x^2-3x=0$

$\Rightarrow \quad x(x-3)=0$

$\Rightarrow \quad x=0, 3$

Here, 0 is not possible.

$\therefore \quad x=3$

***Case* II** If $x<2$, then

$(x-2)^2-x+2-2=0$

$\Rightarrow \quad x^2-5x+4=0$

$\Rightarrow \quad (x-1)(x-4)=0$

$\Rightarrow \quad x=1, 4$

Here, 4 is not possible.

$\therefore \quad x=1$

$\therefore$ The sum of roots $=1+3=4$

Aliter

Let $|x-2|=y$.

Then, we get $\quad y^2+y-2=0$

$\Rightarrow \quad (y-1)(y+2)=0 \Rightarrow y=1, -2$

But -2 is not possible.

Hence, $\quad |x-2|=1 \Rightarrow x=1, 3$

$\therefore$ Sum of the roots $=1+3=4$

68. We have,

$(5+\sqrt{2})x^2-(4+\sqrt{5})x+8+2\sqrt{5}=0$

$\therefore$ Sum of the roots $= \dfrac{4+\sqrt{5}}{5+\sqrt{2}}$

and product of the roots $= \dfrac{8+2\sqrt{5}}{5+\sqrt{2}}$

$\therefore$ The harmonic mean of the roots

$= \dfrac{2 \times \text{Product of the roots}}{\text{Sum of the roots}} = \dfrac{2 \times (8+2\sqrt{5})}{(4+\sqrt{5})} = 4$

69. Let $x^2-ax+30=y$

$\therefore \quad y = 2\sqrt{y+15} \quad ...(i)$

$\Rightarrow \quad y^2-4y-60=0$

$\Rightarrow \quad (y-10)(y+6)=0$

$\therefore \quad y=10, -6$

$\Rightarrow \quad y=10, y \neq -6 \quad [\because y>0]$

Now, $\quad x^2-ax+30=10$

$\Rightarrow \quad x^2-ax+20=0$

Given, $\quad \alpha\beta = \lambda = 20$

$\therefore \quad \dfrac{\alpha+\beta}{2} \geq \sqrt{\alpha\beta} = \sqrt{20}$

$\Rightarrow \quad \alpha+\beta \geq 2\sqrt{20}$

or $\quad \mu = 4\sqrt{5}$

$\therefore$ Minimum value of μ is $4\sqrt{5}$.

i.e., $\quad \mu = 4\sqrt{5} = 8.9 \Rightarrow (\mu) = 9$

70. $\because N^r = \left(x+\dfrac{1}{x}\right)^6 - \left(x^6+\dfrac{1}{x^6}\right) - 2$

$= \left(x+\dfrac{1}{x}\right)^6 - \left(x^3+\dfrac{1}{x^3}\right)^2 = \left(\left(x+\dfrac{1}{x}\right)^3 + \left(x^3+\dfrac{1}{x^3}\right)\right)$
$\left(\left(x+\dfrac{1}{x}\right)^3 - \left(x^3+\dfrac{1}{x^3}\right)\right)$

$= D^r \cdot \left(3\left(x+\dfrac{1}{x}\right)\right)$

$\therefore \quad \dfrac{N^r}{D^r} = 3\left(x+\dfrac{1}{x}\right) \geq 6$

Hence, minimum value of $\dfrac{N^r}{D^r}$ is 6.

71. $\quad a+b=2c \quad ...(i)$

$ab=-5d \quad ...(ii)$

$c+d=2a \quad ...(iii)$

$cd=-5b ...(iv)$

From Eqs. (i) and (iii), we get

$a+b+c+d=2(a+c)$

$\therefore \quad a+c=b+d \quad ...(v)$

From Eqs. (i) and (iii), we get

$$b - d = 3(c - a) \qquad ...(vi)$$

Also, a is a root of $x^2 - 2cx - 5d = 0$

$\therefore$ $$a^2 - 2ac - 5d = 0 \qquad ...(vii)$$

And c is a root of

$$c^2 - 2ac - 5b = 0 \qquad ...(viii)$$

From Eqs. (vii) and (viii), we get

$$a^2 - c^2 - 5(d - b) = 0$$

$\Rightarrow$ $(a + c)(a - c) + 5(b - d) = 0$

$\Rightarrow$ $(a + c)(a - c) + 15(c - a) = 0$ [from Eq. (vi)]

$\Rightarrow$ $(a - c)(a + c - 15) = 0$

$\therefore$ $a + c = 15, a - c \neq 0$

From Eq. (v), we get $b + d = 15$

$\therefore$ $a + b + c + d = a + c + b + d = 15 + 15 = 30$

$\Rightarrow$ Sum of digits of $a + b + c + d = 3 + 0 = 3$

72. $\because$ $y = \dfrac{x^2 - 3x + c}{x^2 + 3x + c}$

$\Rightarrow$ $x^2(y - 1) + 3x(y + 1) + c(y - 1) = 0$

$\because$ $x \in R$

$\therefore$ $9(y + 1)^2 - 4c(y - 1)^2 \geq 0$

$(2\sqrt{c}y - 2\sqrt{c})^2 - (3y + 3)^2 \leq 0$

$\Rightarrow$ $\{(2\sqrt{c} + 3)y - (2\sqrt{c} - 3)\}\{(2\sqrt{c} - 3)y - (2\sqrt{c} + 3)\} \leq 0$

or $$\frac{2\sqrt{c} - 3}{2\sqrt{c} + 3} \leq y \leq \frac{2\sqrt{c} + 3}{2\sqrt{c} - 3}$$

But given, $$\frac{2\sqrt{c} + 3}{2\sqrt{c} - 3} = 7$$

$\Rightarrow$ $2\sqrt{c} + 3 = 14\sqrt{c} - 21$

or $12\sqrt{c} = 24$ or $\sqrt{c} = 2$

$\therefore$ $c = 4$

73. We have, $\sqrt{x^2} - \sqrt{(x - 1)^2} + \sqrt{(x - 2)^2} = \sqrt{5}$

$\Rightarrow$ $|x| - |x - 1| + |x - 2| = \sqrt{5}$

***Case* I** If $x < 0$, then

$$-x + (x - 1) - (x - 2) = \sqrt{5}$$

$$x = 1 - \sqrt{5}$$

***Case* II** If $0 \leq x < 1$, then

$$x + (x - 1) - (x - 2) = \sqrt{5}$$

$\Rightarrow$ $x = \sqrt{5} - 1$, which is not possible.

***Case* III** If $1 \leq x < 2$, then

$$x - (x - 1) - (x - 2) = \sqrt{5}$$

$\Rightarrow$ $x = 3 - \sqrt{5}$, which is not possible.

***Case* IV** If $x > 2$, then

$$x - (x - 1) + (x - 2) = \sqrt{5}$$

$\Rightarrow$ $x = 1 + \sqrt{5}$

Hence, number of solutions is 2.

74. $\because$ $(1 + i)x^2 + (1 - i)x - 2i = 0$

$\Rightarrow$ $x^2 + \dfrac{(1 - i)}{(1 + i)}x - \dfrac{2i}{(1 + i)} = 0$

$\Rightarrow$ $x^2 - ix - (1 + i) = 0$

$\therefore$ $\alpha + \beta = i$, and $\alpha\beta = -(1 + i)$

$\therefore$ $\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} = \sqrt{i^2 + 4(1 + i)} = \sqrt{(3 + 4i)}$

$|\alpha - \beta| = \sqrt{\sqrt{9 + 16}} = \sqrt{5}$

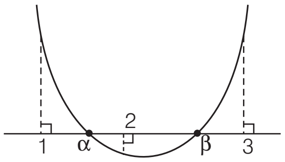
$\therefore$ $|\alpha - \beta|^2 = 5$

75. $\because$ $4x^2 - 16x + c = 0$

$\Rightarrow$ $x^2 - 4x + \dfrac{c}{4} = 0$

Let $f(x) = x^2 - 4x + \dfrac{c}{4}$

Then, the following cases arises:



***Case* I** $D > 0$

$\Rightarrow$ $16 - c > 0$

$\therefore$ $c < 16$

***Case* II** $f(1) > 0$

$\Rightarrow$ $1 - 4 + \dfrac{c}{4} > 0$

$\Rightarrow$ $\dfrac{c}{4} > 3$

$\therefore$ $c > 12$

***Case* III** $f(2) < 0$

$\Rightarrow$ $4 - 8 + \dfrac{c}{4} < 0$

$\Rightarrow$ $\dfrac{c}{4} < 4$

$\therefore$ $c < 16$

***Case* IV** $f(3) > 0$

$\Rightarrow$ $9 - 12 + \dfrac{c}{4} > 0$

$\Rightarrow$ $\dfrac{c}{4} > 3$

$\Rightarrow$ $c > 12$

Combining all cases, we get

$$12 < c < 16$$

Thus, integral values of c are 13, 14 and 15.

Hence, number of integral values of c is 3.

76. We have, $$r + s + t = 0 \qquad ...(i)$$

$$rs + st + tr = \frac{1001}{8} \qquad ...(ii)$$

and $$rst = -\frac{2008}{8} = -251 \qquad ...(iii)$$

Now, $(r+s)^3+(s+t)^3+(t+r)^3=(-t)^3+(-r)^3+(-s)^3$

$[\because r+s+t=0]$

$$=-(t^3+r^3+s^3)=-3\,rst \quad [\because r+s+t=0]$$

$$=-3(-251)=753$$

Now, $\quad 99\,\lambda=(r+s)^3+(s+t)^3+(t+r)^3=753$

$\therefore \quad \lambda=\dfrac{753}{99}=7.6$

$\therefore \quad [\lambda]=7$

77. A $\rightarrow$ (r,s); B $\rightarrow$ (p,q,r,s,t); C $\rightarrow$ (p,q,t)

(A) We have, $y=\dfrac{x^2-2x+4}{x^2+2x+4}$

$\Rightarrow \quad x^2(y-1)+2(y+1)x+4(y-1)=0$

As $x\in R$, we get

$$D\geq 0$$

$\Rightarrow \quad 4(y+1)^2-16(y-1)^2\geq 0$

$\Rightarrow \quad 3y^2-10y+3\leq 0$

$\Rightarrow \quad (y-3)(3y-1)\leq 0$

$\Rightarrow \quad \dfrac{1}{3}\leq y\leq 3$

(B) We have, $y=\dfrac{2x^2+4x+1}{x^2+4x+2}$

$\Rightarrow \quad x^2(y-2)+4(y-1)\,x+2y-1=0$

As $x\in R$, we get

$$D\geq 0$$

$\Rightarrow \quad 16(y-1)^2-4(y-2)(2y-1)\geq 0$

$\Rightarrow \quad 4(y-1)^2-(y-2)(2y-1)\geq 0$

$\Rightarrow \quad 2y^2-3y+2\geq 0$

$\Rightarrow \quad y^2-\dfrac{3}{2}y+1\geq 0$

$\Rightarrow \quad \left(y-\dfrac{3}{4}\right)^2+\dfrac{7}{16}\geq 0$

$\therefore \quad y\in R$

(C) We have, $\quad y=\dfrac{x^2-3x+4}{x-3}$

$\Rightarrow \quad x^2-(3+y)x+3y+4=0$

As $x\in R$, we get

$$D\geq 0 \Rightarrow (3+y)^2-4(3y+4)\geq 0$$

$\Rightarrow \quad y^2-6y-7\geq 0 \Rightarrow (y+1)(y-7)\geq 0$

$\Rightarrow \quad y\in(-\infty,-1]\cup[7,\infty)$

78. A $\rightarrow$ (q,r,s); B $\rightarrow$ (p); C $\rightarrow$ (q)

(A) $\because (d+a-b)^2+(d+b-c)^2=0$

which is possible only when

$$d+a-b=0,\ d+b-c=0$$

$\Rightarrow \quad b-a=c-b$

$\Rightarrow \quad 2b=a+c$

$\therefore$ a, b and c are in AP. ...(i)

$\because \quad a(b-c)+b(c-a)+c(a-b)=0$

$\therefore$ $x=1$ is a root of

$a(b-c)x^2+b(c-a)x+c(a-b)=0$ ...(ii)

Given, roots [Eq. (ii)] are equal.

$\therefore \quad 1\times 1=\dfrac{c(a-b)}{a(b-c)}$

$\Rightarrow \quad a(b-c)=c(a-b)$

or $\quad b=\dfrac{2ac}{a+c}$

$\therefore$ a, b and c are in HP. ...(iii)

From Eqs. (i) and (ii), we get

$$a=b=c$$

$\therefore$ a, b and c are in AP, GP and HP.

(B) $\because \quad x^3-3x^2+3x-1=0$

$\Rightarrow \quad (x-1)^3=0$

$\therefore \quad x=1,1,1$

$\Rightarrow$ Common root, $x=1$

$\therefore \quad a(1)^2+b(1)+c=0$

$\Rightarrow \quad a+b+c=0$

(C) Given, $\quad bx^2+\left(\sqrt{(a+c)^2+4b^2}\right)x+(a+c)\geq 0$

$\therefore \quad D\leq 0$

$\Rightarrow \quad (a+c)^2+4b^2-4b(a+c)\leq 0$

$\Rightarrow \quad (a+c-2b)^2\leq 0$

or $\quad (a+c-2b)^2=0$

$\therefore \quad a+c=2b$

Hence a, b and c are in AP.

79. A $\rightarrow$ (q,r,s,t); B $\rightarrow$ (q,r); C $\rightarrow$ (p,q)

(A) We have, $\quad y=\dfrac{ax^2+3x-4}{3x-4x^2+a}$

$\Rightarrow \quad x^2(a+4y)+3(1-y)x-(ay+4)=0$

As $x\in R$, we get

$$D\geq 0$$

$\Rightarrow \quad 9(1-y)^2+4(a+4y)(ay+4)\geq 0$

$\Rightarrow (9+16a)\,y^2+(4a^2+46)y+(9+16a)\geq 0,\ \forall\ y\in R$

$\Rightarrow$ If $9+16a>0$, then $D\leq 0$

Now, $\quad D\leq 0$

$\Rightarrow \quad (4a^2+46)^2-4(9+16a)^2\leq 0$

$\Rightarrow \quad 4[(2a^2+23)^2-(9+16a)^2]\leq 0$

$\Rightarrow \quad [(2a^2+23)+(9+16a)][(2a^2+23)-(9+16a)]\leq 0$

$\Rightarrow \quad (2a^2+16a+32)(2a^2-16a+14)\leq 0$

$\Rightarrow \quad 4(a+4)^2(a^2-8a+7)\leq 0$

$\Rightarrow \quad a^2-8a+7\leq 0$

$\Rightarrow \quad (a-1)(a-7)\leq 0$

$\Rightarrow \quad 1\leq a\leq 7$

$\therefore \quad 9+16a>0$ and $1\leq a\leq 7$

$\Rightarrow \quad 1\leq a\leq 7$

(B) We have, $\quad y = \dfrac{ax^2 + x - 2}{a + x - 2x^2}$

$\Rightarrow \quad x^2(a + 2y) + x(1 - y) - (2 + ay) = 0$

As $x \in R$, we get

$$D \geq 0$$

$\Rightarrow \quad (1 - y)^2 + 4(2 + ay)(a + 2y) \geq 0$

$\Rightarrow \quad (1 + 8a)y^2 + (4a^2 + 14)y + (1 + 8a) \geq 0$

$\Rightarrow$ If $1 + 8a > 0$, then $D \leq 0$

$\Rightarrow \quad (4a^2 + 14)^2 - 4(1 + 8a)^2 \leq 0$

$\Rightarrow \quad 4[(2a^2 + 7)^2 - (1 + 8a)^2] \leq 0$

$\Rightarrow \quad [(2a^2 + 7) + (1 + 8a)][(2a^2 + 7) - (1 + 8a)] \leq 0$

$\Rightarrow \quad (2a^2 + 8a + 8)(2a^2 - 8a + 6) \leq 0$

$\Rightarrow \quad 4(a + 2)^2(a^2 - 4a + 3) \leq 0$

$\Rightarrow \quad a^2 - 4a + 3 \leq 0$

$\Rightarrow \quad (a - 1)(a - 3) \leq 0$

$\Rightarrow \quad 1 \leq a \leq 3$

Thus, $1 + 8a > 0$ and $1 \leq a \leq 3$

$\Rightarrow \quad 1 \leq a \leq 3$

(C) We have, $\quad y = \dfrac{x^2 + 2x + a}{x^2 + 4x + 3a}$

$\Rightarrow \quad x^2(y - 1) + 2(2y - 1)x + a(3y - 1) = 0$

As $x \in R$, we get

$$D \geq 0$$

$\Rightarrow \quad 4(2y - 1)^2 - 4(y - 1)a(3y - 1) \geq 0$

$\Rightarrow \quad (4 - 3a)y^2 - (4 - 4a)y + (1 - a) \geq 0$

$\Rightarrow$ If $4 - 3a > 0$, then $D \leq 0$

$\Rightarrow \quad (4 - 4a)^2 - 4(4 - 3a)(1 - a) \leq 0$

$\Rightarrow \quad 4(2 - 2a)^2 - 4(4 - 3a)(1 - a) \leq 0$

$\Rightarrow \quad 4 + 4a^2 - 8a - (4 - 7a + 3a^2) \leq 0$

$\Rightarrow \quad a^2 - a \leq 0$

$\Rightarrow \quad a(a - 1) \leq 0$

$\Rightarrow \quad 0 \leq a \leq 1$

80. A $\rightarrow$ (p,q,r,s); B $\rightarrow$ (p,q); C $\rightarrow$ (s)

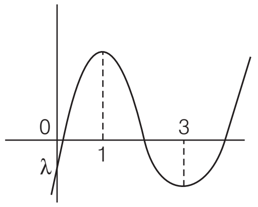
(A) Let $\quad y = f(x) = x^3 - 6x^2 + 9x + \lambda$

$f'(x) = 3x^2 - 12x + 9 = 0$

$\therefore \quad x = 1, 3$

$f''(x) = 6x - 12$

$f''(1) < 0$ and $f''(3) > 0$



Also, $\quad f(0) < 0 \Rightarrow \lambda < 0 \quad$ ...(i)

$f(1) > 0$

$\Rightarrow \quad 1 - 6 + 9 + \lambda > 0$

$\Rightarrow \quad \lambda > -4 \quad$ ...(ii)

and $\quad f(3) < 0$

$\Rightarrow \quad 27 - 54 + 27 + \lambda < 0$

$\Rightarrow \quad \lambda < 0 \quad$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$$-4 < \lambda < 0$$

$\Rightarrow \quad -3 < \lambda + 1 < 1$

$\therefore \quad [\lambda + 1] = -3, -2, -1, 0$

$\therefore \quad |[\lambda + 1]| = 3, 2, 1, 0$

(B) $\because \quad x^2 + x + 1 > 0, \forall x \in R$

Given, $\quad -3 < \dfrac{x^2 - \lambda x - 2}{x^2 + x + 1} < 2$

$\Rightarrow \quad -3x^2 - 3x - 3 < x^2 - \lambda x - 2 < 2x^2 + 2x + 2$

$\Rightarrow \quad 4x^2 - (\lambda - 3)x + 1 > 0$

and $\quad x^2 + (\lambda + 2)x + 4 > 0$

$\therefore \quad (\lambda - 3)^2 - 4 \cdot 4 \cdot 1 < 0$

and $\quad (\lambda + 2)^2 - 4 \cdot 1 \cdot 4 < 0$

$\Rightarrow \quad (\lambda - 3)^2 - 4^2 < 0$

and $\quad (\lambda + 2)^2 - 4^2 < 0$

$\Rightarrow \quad -4 < \lambda - 3 < 4$

and $\quad -4 < \lambda + 2 < 4$

or $\quad -1 < \lambda < 7$

and $\quad -6 < \lambda < 2$

We get, $\quad -1 < \lambda < 2$

$\therefore \quad [\lambda] = -1, 0, 1$

$\Rightarrow \quad |[\lambda]| = 0, 1$

(C) $\because \quad (b - c) + (c - a) + (a - b) = 0$

$\therefore$ $x = 1$ is a root of

$$(b - c)x^2 + (c - a)x + (a - b) = 0$$

Also, $x = 1$ satisfies

$$x^2 + \lambda x + 1 = 0$$

$\Rightarrow \quad 1 + \lambda + 1 = 0$

$\therefore \quad \lambda = -2$

Now, $\quad \lambda - 1 = -3$

$[\lambda - 1] = -3$

$\Rightarrow \quad |[\lambda - 1]| = 3$

81. If quadratic equation $ax^2 + bx + c = 0$ is satisfied by more than two values of x, then it must be an identity.

Therefore, $\quad a = b = c = 0$

$\therefore$ Statement-2 is true.

But in Statement-1,

$$4p - 3 = 4q - 3 = r = 0$$

Then, $\quad p = q = \dfrac{3}{4}, r = 0$

which is false.

Since, at one value of p or q or r, all coefficients at a time $\neq 0$.

$\therefore$ Statement-1 is false.

82. We have, $x^2 + (2m+1)x + (2n+1) = 0$ ...(i)

$m, n \in I$

$\therefore \quad D = b^2 - 4ac$

$= (2m+1)^2 - 4(2n+1)$

is never be a perfect square.

Therefore, the roots of Eq. (i) can never be integers. Hence, the roots of Eq. (i) cannot have any rational root as $a = 1, b, c \in I$.

Hence, both statements are true and Statements −2 is a correct explanation of Statement-1.

83. Let α be one root of equation $ax^2 + 3x + 5 = 0$. Therefore,

$$\alpha \cdot \frac{1}{\alpha} = \frac{5}{a}$$

$$\Rightarrow \quad 1 = \frac{5}{a}$$

$$\Rightarrow \quad a = 5$$

Hence, both the statements are true and Statement-2 is the correct explanation of Statement-1.

84. Let roots of $Ax^3 + Bx^2 + Cx + D = 0$ ...(i)

are $\alpha - \beta, \alpha, \alpha + \beta$ (in AP).

Then, $(\alpha - \beta) + \alpha + (\alpha + \beta) = -\frac{B}{A}$

$\Rightarrow \quad \alpha = -\frac{B}{3A}$, which is a root of Eq. (i).

Then, $A\alpha^3 + B\alpha^2 + C\alpha + D = 0$

$$\Rightarrow A\left(-\frac{B}{3A}\right)^3 + B\left(-\frac{B}{3A}\right)^2 + C\left(-\frac{B}{3A}\right) + D = 0$$

$$\Rightarrow \quad -\frac{B^3}{27A^2} + \frac{B^3}{9A^2} - \frac{BC}{3A} + D = 0$$

$$\Rightarrow \quad 2B^3 - 9ABC + 27A^2D = 0$$

Now, comparing with $2B^3 + k_1ABC + k_2A^2D = 0$, we get

$$k_1 = -9, k_2 = 27$$

$$\therefore \quad k_2 - k_1 = 27 - (-9) = 36 = 6^2$$

Hence, both statements are true and Statement-2 is a correct explanation of Statement-1.

85. $\because x, y, z \in R$

$x + y + z = 6$ ...(i)

and $xy + yz + zx = 8$ ...(ii)

$\Rightarrow \quad xy + (x+y)\{6 - (x+y)\} = 8$ [from Eq. (i)]

$\Rightarrow \quad xy + 6x + 6y - (x^2 + 2xy + y^2) = 8$

or $\quad y^2 + (x-6)y + x^2 - 6x + 8 = 0$

$\therefore \quad (x-6)^2 - 4 \cdot 1 \cdot (x^2 - 6x + 8) \geq 0, \forall y \in R$

$\Rightarrow \quad -3x^2 + 12x + 4 \geq 0$ or $3x^2 - 12x - 4 \leq 0$

or $\quad 2 - \frac{4}{\sqrt{3}} \leq x \leq 2 + \frac{4}{\sqrt{3}}$

or $\quad x \in \left[2 - \frac{4}{\sqrt{3}}, 2 + \frac{4}{\sqrt{3}}\right]$

Similarly, $\quad y \in \left[2 - \frac{4}{\sqrt{3}}, 2 + \frac{4}{\sqrt{3}}\right]$

and $\quad z \in \left[2 - \frac{4}{\sqrt{3}}, 2 + \frac{4}{\sqrt{3}}\right]$

Since, Eqs. (i) and (ii) remains same, if x, y, z interchange their positions.

Hence, both statements are true and Statement-2 is a correct explanation of Statement-1.

86. Let $y = ax^3 + bx + c$

$$\therefore \quad \frac{dy}{dx} = 3ax^2 + b$$

For maximum or minimum $\frac{dy}{dx} = 0$, we get

$$x = \pm\sqrt{-\frac{b}{3a}}$$

Case **I** If $a > 0, b > 0$, then $\frac{dy}{dx} > 0$

In this case, function is increasing, so it has exactly one root

Case **II** If $a < 0, b < 0$, then $\frac{dy}{dx} < 0$

In this case, function is decreasing, so it has exactly one root.

Case **III** $a > 0, b < 0$ or $a < 0, b > 0$, then $y = ax^3 + bx + c$ is maximum at one point and minimum at other point.

Hence, all roots can never be non-negative.

$\therefore$Statement-1 is false. But

$$\text{Sum of roots} = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = 0$$

i.e., Statement-2 is true.

87. Statement-2 is obviously true.

But $\quad y = ax^2 + bx + c$

$$y = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$$

$$= a\left\{\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a^2}\right\} \quad [\text{where, } D = b^2 - 4ac]$$

$$\Rightarrow \quad \left(x + \frac{b}{2a}\right)^2 = \frac{1}{a}\left(y + \frac{D}{4a}\right)$$

Let $\quad x + \frac{b}{2a} = X$ and $y + \frac{D}{4a} = Y$.

$$\therefore \quad X^2 = \frac{1}{a}Y$$

Equation of axis, $X = 0$ i.e. $x + \frac{b}{2a} = 0$

or $\quad 2ax + b = 0$

Hence, $y = ax^2 + bx + c$ is symmetric about the line $2ax + b = 0$.

$\therefore$ Both statements are true and Statement-2 is a correct explanation of Statement-1.

88. $\because (1+m)x^2 - 2(1+3m)x + (1+8m) = 0$

$\therefore \quad D = 4(1+3m)^2 - 4(1+m)(1+8m) = 4m(m-3)$

(i) Both roots are imaginary.

$\therefore \quad D < 0$

$\Rightarrow \quad 4m(m-3) < 0$

$\Rightarrow \quad 0 < m < 3$

or $\quad m \in (0, 3)$

(ii) Both roots are equal.

$\therefore \quad D = 0$

$\Rightarrow \quad 4m(m-3) = 0$

$\Rightarrow \quad m = 0, 3$

(iii) Both roots are real and distinct.

$\therefore \quad D > 0$

$\Rightarrow \quad 4m(m-3) > 0$

$\Rightarrow \quad m < 0$ or $m > 3$

$\therefore \quad m \in (-\infty, 0) \cup (3, \infty)$

(iv) Both roots are positive.

***Case* I** Sum of the roots > 0

$$\Rightarrow \quad \frac{2(1+3m)}{(1+m)} > 0$$

$$\Rightarrow \quad m \in (-\infty, -1) \cup \left(-\frac{1}{3}, \infty\right)$$

***Case* II** Product of the roots > 0

$$\Rightarrow \quad \frac{(1+8m)}{(1+m)} > 0$$

$$m \in (-\infty, -1) \cup \left(-\frac{1}{8}, \infty\right)$$

***Case* III** $\quad D \geq 0$

$\Rightarrow \quad 4m(m-3) \geq 0$

$m \in (-\infty, 0] \cup [3, \infty)$

Combining all Cases, we get

$m \in (-\infty, -1) \cup [3, \infty)$

(v) Both roots are negative.

Consider the following cases:

***Case* I** Sum of the roots < 0 $\Rightarrow \dfrac{2(1+3m)}{(1+m)} < 0$

$$\Rightarrow \quad m \in \left(-1, -\frac{1}{3}\right)$$

***Case* II** Product of the roots > 0 $\Rightarrow \dfrac{(1+8m)}{(1+m)} > 0$

$$\Rightarrow \quad m \in (-\infty, 1) \cup \left(-\frac{1}{8}, \infty\right)$$

***Case* III** $D \geq 0$

$4m(m-3) \geq 0 \Rightarrow m \in (-\infty, 0] \cup [3, \infty)$

Combining all cases, we get

$m \in \phi$

(vi) Roots are opposite in sign, then

***Case* I** Consider the following cases:

Product of the roots < 0

$$\Rightarrow \quad \frac{(1+8m)}{(1+m)} < 0$$

$$m \in \left(-1, -\frac{1}{8}\right)$$

***Case* II** $\quad D > 0 \Rightarrow 4m(m-3) > 0$

$\Rightarrow \quad m \in (-\infty, 0) \cup (3, \infty)$

Combining all cases, we get

$$m \in \left(-1, -\frac{1}{8}\right)$$

(vii) Roots are equal in magnitude but opposite in sign, then Consider the following cases:

***Case* I** Sum of the roots = 0

$$\Rightarrow \quad \frac{2(1+3m)}{(1+m)} = 0$$

$$\Rightarrow \quad m = -\frac{1}{3}, m \neq 1$$

***Case* II** $\quad D > 0 \Rightarrow 4m(m-3) > 0$

$\Rightarrow \quad m \in (-\infty, 0) \cup (3, \infty)$

Combining all cases, we get

$$m = -\frac{1}{3}$$

(viii) Atleast one root is positive, then either one root is positive or both roots are positive.

i.e. $\quad (d) \cup (f)$

or $\quad m \in (-\infty, -1) \cup \left(-1, -\dfrac{1}{8}\right) \cup [3, \infty)$

(ix) Atleast one root is negative, then either one root is negative or both roots are negative.

i.e. $\quad (e) \cup (f)$ or $m \in \left(-1, -\dfrac{1}{8}\right)$

(x) Let roots are 2α are 3α. Then,

Consider the following cases:

***Case* I** Sum of the roots $= 2\alpha + 3\alpha = \dfrac{2(1+3m)}{(1+m)}$

$$\Rightarrow \quad \alpha = \frac{2(1+3m)}{5(1+m)}$$

***Case* II** Product of the roots $= 2\alpha \cdot 3\alpha = \dfrac{(1+8m)}{(1+m)}$

$$\Rightarrow \quad 6\alpha^2 = \frac{(1+8m)}{(1+m)}$$

From Eqs. (i) and (ii), we get

$$6\left\{\frac{2(1+3m)}{5(1+m)}\right\}^2 = \frac{(1+8m)}{(1+m)}$$

$\Rightarrow \quad 24(1+3m)^2 = 25(1+8m)(1+m)$

$\Rightarrow \quad 24(9m^2+6m+1) = 25(8m^2+9m+1)$

$16m^2 - 81m - 1 = 0$

$$\text{or} \quad m = \frac{81 \pm \sqrt{(-81)^2 + 64}}{32}$$

$$\Rightarrow \quad m = \frac{81 \pm \sqrt{6625}}{32}$$

89. $\because 2x^2 - 2(2m+1)x + m(m+1) = 0 \quad [\because m \in R]$

$\therefore \quad D = [-2(2m+1)]^2 - 8m(m+1) \quad [D = b^2 - 4ac]$

$= 4\{(2m+1)^2 - 2m(m+1)\}$

$= 4(2m^2 + 2m + 1)$

$$= 8\left(m^2 + m + \frac{1}{2}\right) = 8\left\{\left(m + \frac{1}{2}\right)^2 + \frac{1}{4}\right\} > 0$$

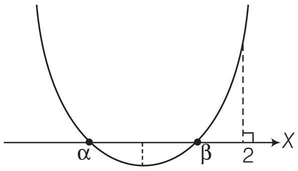
or $\qquad D > 0, \forall\, m \in R \qquad$ ...(i)

x-coordinate of vertex $= -\dfrac{b}{2a} = \dfrac{2(2m+1)}{4} = \left(m + \dfrac{1}{2}\right) \qquad$ ...(ii)

and let

$$f(x) = x^2 - (2m+1)x + \frac{1}{2}m(m+1) \qquad \text{...(iii)}$$

(i) Both roots are smaller than 2.



Consider the following cases:

***Case* I** $D \geq 0$

$\therefore \qquad m \in R \qquad$ [from Eq. (i)]

***Case* II** x-coordinate of vertex < 2.

$\Rightarrow \qquad m + \dfrac{1}{2} < 2 \qquad$ [from Eq. (ii)]

or $\qquad m < \dfrac{3}{2}$

***Case* III** $f(2) > 0$

$\Rightarrow \qquad 4 - (2m+1)2 + \dfrac{1}{2}m(m+1) > 0$

$\Rightarrow \qquad m^2 - 7m + 4 > 0$

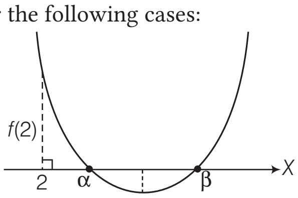
$\therefore \qquad m \in \left(-\infty, \dfrac{7-\sqrt{33}}{2}\right) \cup \left(\dfrac{7+\sqrt{33}}{2}, \infty\right)$

Combining all cases, we get

$$m \in \left(-\infty, \frac{7-\sqrt{33}}{2}\right)$$

(ii) Both roots are greater than 2.

Consider the following cases:



***Case* I** $D \geq 0$

$\therefore \qquad m \in R \qquad$ [from Eq. (i)]

***Case* II** x-coordinate of vertex > 2

$\Rightarrow \qquad m + \dfrac{1}{2} > 2 \qquad$ [from Eq. (ii)]

$\therefore \qquad m > \dfrac{3}{2}$

***Case* III** $f(2) > 0$

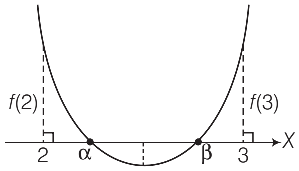
$m \in \left(-\infty, \dfrac{7-\sqrt{33}}{2}\right) \cup \left(7 + \dfrac{\sqrt{33}}{2}, \infty\right) \qquad$ [from part (a)]

Combining all cases, we get

$$m \in \left(\frac{7+\sqrt{33}}{2}, \infty\right)$$

(iii) Both roots lie in the interval (2, 3).

Consider the following cases:



***Case* I** $D \geq 0$

$\therefore \qquad m \in R \qquad$ [from Eq. (i)]

***Case* II** $f(2) > 0$

$\therefore \qquad m \in \left(-\infty, \dfrac{7-\sqrt{33}}{2}\right) \cup \left(\dfrac{7+\sqrt{33}}{2}, \infty\right) \qquad$ [from part (a)]

***Case* III** $f(3) > 0$

$\Rightarrow \qquad 9 - 3(2m+1) + \dfrac{1}{2}m(m+1) > 0$

or $\qquad m^2 - 11m + 12 > 0$

$\therefore \qquad m \in \left(-\infty, \dfrac{11-\sqrt{73}}{2}\right) \cup \left(\dfrac{11+\sqrt{73}}{2}, \infty\right)$

***Case* IV** $2 < x$-coordinate of vertex < 3

$\Rightarrow \qquad 2 < m + \dfrac{1}{2} < 3$

or $\qquad \dfrac{3}{2} < m < \dfrac{5}{2}$ or $m \in \left(\dfrac{3}{2}, \dfrac{5}{2}\right)$

Combining all cases, we get

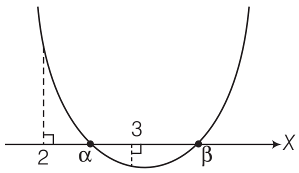
$$m \in \phi$$

(iv) Exactly one root lie in the interval (2,3) .

Consider the following cases:

***Case* I** $D > 0$

$\therefore \qquad m \in R \qquad$ [from Eq. (i)]

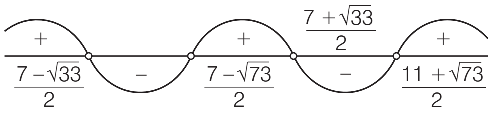


***Case* II** $f(2)\, f(3) < 0$

$$\left(4 - 2(2m+1) + \frac{1}{2}m(m+1)\right)\left(9 - 3(2m+1) + \frac{1}{2}m(m+1)\right) < 0$$

$\Rightarrow \qquad (m^2 - 7m + 4)(m^2 - 11m + 12) < 0$

$\Rightarrow \qquad \left(m - \dfrac{7-\sqrt{33}}{2}\right)\left(m - \dfrac{7+\sqrt{33}}{2}\right)\left(m - \dfrac{11-\sqrt{73}}{2}\right)\left(m - \dfrac{11+\sqrt{73}}{2}\right) < 0$



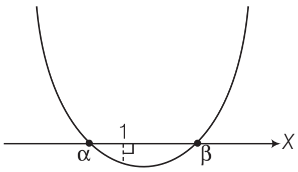
$\therefore \quad m \in \left(\frac{7-\sqrt{33}}{2}, \frac{11-\sqrt{73}}{2}\right) \cup \left(\frac{7+\sqrt{33}}{2}, \frac{11+\sqrt{73}}{2}\right)$

Combining all cases, we get

$$m \in \left(\frac{7-\sqrt{33}}{2}, \frac{11-\sqrt{73}}{2}\right) \cup \left(\frac{7+\sqrt{33}}{2}, \frac{11+\sqrt{73}}{2}\right)$$

(v) One root is smaller than 1 and the other root is greater than 1.

Consider the following cases:



***Case* I** $D > 0$

$\therefore \qquad m \in R \qquad$ [from Eq. (i)]

***Case* II** $f(1) < 0$

$\Rightarrow \qquad 1-(2m+1)+\frac{1}{2}m(m+1) < 0 \qquad$ [from Eq. (iii)]

$\Rightarrow \qquad m^2 - 3m < 0$

$\Rightarrow \qquad m(m-3) < 0$

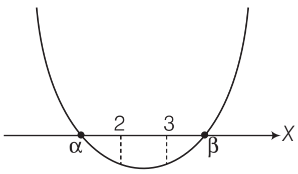
$\therefore \qquad m \in (0, 3)$

Combining both cases, we get

$$m \in (0, 3)$$

(vi) One root is greater than 3 and the other root is smaller than 2.

Consider the following cases:



***Case* I** $D > 0$

$\therefore \qquad m \in R \qquad$ [from Eq. (i)]

***Case* II** $f(2) < 0$

$\Rightarrow \qquad m^2 - 7m + 4 < 0$

$\therefore \qquad \frac{7-\sqrt{33}}{2} < m < \frac{7+\sqrt{33}}{2}$

$\therefore \qquad m \in \left(\frac{7-\sqrt{33}}{2}, \frac{7+\sqrt{33}}{2}\right)$

***Case* III** $f(3) < 0$

$\Rightarrow \qquad m^2 - 11m + 12 < 0$

$\therefore \qquad \frac{11-\sqrt{73}}{2} < m < \frac{11+\sqrt{73}}{2}$

$\therefore \qquad m \in \left(\frac{11-\sqrt{73}}{2}, \frac{11+\sqrt{73}}{2}\right)$

Combining all cases, we get

$$m \in \left(\frac{7-\sqrt{33}}{2}, \frac{7+\sqrt{33}}{2}\right)$$

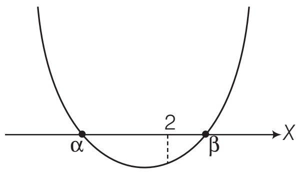
(vii) Atleast one root lies in the interval (2, 3).

i.e. $\qquad (d) \cup (c)$

$\therefore \quad m \in \left(\frac{7-\sqrt{33}}{2}, \frac{11-\sqrt{73}}{2}\right) \cup \left(\frac{7+\sqrt{33}}{2}, \frac{11+\sqrt{73}}{2}\right)$

(viii) Atleast one root is greater than 2.

i.e. (Exactly one root is greater than 2) ∪ (Both roots are greater than 2)



or(Exactly one root is greater than 2) ∪ (b) ...(I)

Consider the following cases:

***Case* I** $D > 0$

$\therefore \qquad m \in R \qquad$ [from Eq. (i)]

***Case* II** $f(2) < 0$

$\Rightarrow \qquad m^2 - 7m + 4 < 0$

$\therefore \qquad m \in \left(\frac{7-\sqrt{33}}{2}, \frac{7+\sqrt{33}}{2}\right)$

Combining both cases, we get

$$m \in \left(\frac{7-\sqrt{33}}{2}, \frac{7+\sqrt{33}}{2}\right) \qquad ...(II)$$

Finally from Eqs. (I) and (II), we get

$$m \in \left(\frac{7-\sqrt{33}}{2}, \frac{7+\sqrt{33}}{2}\right) \cup \left(\frac{7+\sqrt{33}}{2}, \infty\right)$$

(ix) Atleast one root is smaller than 2.

i.e. (Exactly one root is smaller than 2) ∪(Both roots are smaller than 2)

or $\qquad$ (h) (II) ∪ (a)

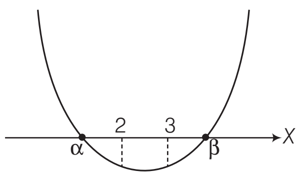
We get, $\quad m \in \left(-\infty, \frac{7-\sqrt{33}}{2}\right) \cup \left(\frac{7-\sqrt{33}}{2}, \frac{7+\sqrt{33}}{2}\right)$

(x) Both 2 and 3 lie between α and β.

Consider the following cases:

***Case* I** $D > 0$

$\therefore \qquad m \in R \qquad$ [from Eq. (i)]



***Case* II** $f(2) < 0$

$$\Rightarrow \quad m^2 - 7m + 4 < 0$$

$$\therefore \quad m \in \left(\frac{7-\sqrt{33}}{2}, \frac{7+\sqrt{33}}{2}\right)$$

***Case* III** $f(3) < 0$

$$\Rightarrow \quad m^2 - 11m + 12 < 0$$

$$\therefore \quad m \in \left(\frac{11-\sqrt{73}}{2}, \frac{11+\sqrt{73}}{2}\right)$$

Combining all cases, we get

$$m \in \left(\frac{11-\sqrt{73}}{2}, \frac{7+\sqrt{33}}{2}\right)$$

90. $\because \quad \dfrac{\alpha}{\beta} = r$

$$\Rightarrow \quad \frac{\alpha+\beta}{\alpha-\beta} = \frac{r+1}{r-1}$$

[using componendo and dividendo method]

$$\Rightarrow \quad \frac{-b/a}{\frac{\sqrt{D}}{a}} = \frac{r+1}{r-1} \Rightarrow b(1-r) = (1+r)\sqrt{D}$$

On squaring both sides, we get

$$\Rightarrow \quad b^2(1-r)^2 = (1+r)^2(b^2 - 4ac)$$

or $\quad (1+r)^2 \cdot 4ac = b^2(4r)$ or $\dfrac{(1+r)^2}{r} = \dfrac{b^2}{ac}$

91. We have, $\quad \dfrac{1}{x+p} + \dfrac{1}{x+q} = \dfrac{1}{r}$

$$\Rightarrow \quad \frac{(x+q)+(x+p)}{x^2+(p+q)x+pq} = \frac{1}{r}$$

$$\Rightarrow \quad x^2 + (p+q-2r)x + pq - (p+q)r = 0$$

Now, since the roots are equal in magnitudes, but opposite in sign. Therefore,

Sum of the roots = 0

$$\Rightarrow \quad p + q - 2r = 0$$

$$\Rightarrow \quad p + q = 2r \qquad \text{...(i)}$$

and product of the roots $= pq - (p+q)r$

$$= pq - (p+q)\left(\frac{p+q}{2}\right) \qquad \text{[from Eq. (i)]}$$

$$= \frac{2pq - p^2 - q^2 - 2pq}{2}$$

$$= -\frac{p^2+q^2}{2}$$

92. Let α be one root of the equation $ax^2 + bx + c = 0$.

Then, other root be α^n.

$$\therefore \quad \alpha + \alpha^n = -\frac{b}{a} \qquad \text{...(i)}$$

and $\quad \alpha \cdot \alpha^n = \dfrac{c}{a}$

$$\Rightarrow \quad \alpha^{n+1} = \frac{c}{a}$$

$$\Rightarrow \quad \alpha = \left(\frac{c}{a}\right)^{\frac{1}{n+1}}$$

$\therefore$ From Eq. (i), we get

$$\left(\frac{c}{a}\right)^{\frac{1}{n+1}} + \left(\frac{c}{a}\right)^{\frac{n}{n+1}} = -\frac{b}{a}$$

$$\Rightarrow \quad (c)^{\frac{1}{n+1}} \cdot a^{-\frac{1}{n+1}+1} + (c^n)^{\frac{1}{n+1}} \cdot a^{-\frac{n}{n+1}+1} + b = 0$$

$$\Rightarrow \quad c^{\frac{1}{n+1}} \cdot a^{\frac{n}{n+1}} + (c^n)^{\frac{1}{n+1}} \cdot a^{\frac{1}{n+1}} + b = 0$$

$$\Rightarrow \quad (a^n c)^{\frac{1}{n+1}} + (c^n a)^{\frac{1}{n+1}} + b = 0$$

93. We have, $\quad \alpha + \beta = -\dfrac{b}{a}$

$$\alpha\beta = \frac{c}{a} \Rightarrow \gamma + \delta = -\frac{m}{l} \text{ and } \gamma\delta = \frac{n}{l}$$

Now, sum of the roots

$$= (\alpha\gamma + \beta\delta) + (\alpha\delta + \beta\gamma) = (\alpha + \beta)\gamma + (\alpha + \beta)\delta$$

$$= (\alpha + \beta)(\gamma + \delta)$$

$$= \left(-\frac{b}{a}\right)\left(-\frac{m}{l}\right) = \frac{mb}{al}$$

and product of the roots

$$= (\alpha\gamma + \beta\delta)(\alpha\delta + \beta\gamma)$$

$$= (\alpha^2 + \beta^2)\gamma\delta + \alpha\beta(\gamma^2 + \delta^2)$$

$$= \{(\alpha+\beta)^2 - 2\alpha\beta\}\gamma\delta + \alpha\beta\{(\gamma+\delta)^2 - 2\gamma\delta\}$$

$$= \left\{\left(-\frac{b}{a}\right)^2 - \frac{2c}{a}\right\}\frac{n}{l} + \frac{c}{a}\left\{\left(-\frac{m}{l}\right)^2 - \frac{2n}{l}\right\}$$

$$= \left\{\frac{b^2 - 2ac}{a^2}\right\}\frac{n}{l} + \frac{c}{a}\left\{\frac{m^2 - 2nl}{l^2}\right\} = \frac{(b^2 - 2ac)\,ln + (m^2 - 2nl)\,ac}{a^2 l^2}$$

$\therefore$ Required equation is

$$x^2 - \left(\frac{mb}{al}\right)x + \frac{(b^2 - 2ac)\,ln + (m^2 - 2nl)\,ac}{a^2l^2} = 0$$

$$\Rightarrow \quad a^2l^2x^2 - mbalx + (b^2 - 2ac)\,ln + (m^2 - 2nl)\,ac = 0$$

94. Since, the roots are equal.

$$\therefore \quad D = 0$$

$$\Rightarrow \quad 4(b^2 - ac)^2 - 4(a^2 - bc)(c^2 - ab) = 0$$

$$\Rightarrow \quad (b^2 - ac)^2 - (a^2 - bc)(c^2 - ab) = 0$$

$$\Rightarrow \quad b(a^3 + b^3 + c^3 - 3abc) = 0$$

$$\Rightarrow \quad b = 0 \text{ or } a^3 + b^3 + c^3 - 3abc = 0$$

95. Let α and β be the roots of $x^2 - px + q = 0$. Then,

$$\alpha + \beta = p \qquad \text{...(i)}$$

$$\alpha\beta = q \qquad \text{...(ii)}$$

And α and $\dfrac{1}{\beta}$ be the roots of $x^2 - ax + b = 0$. Then,

$$\alpha + \frac{1}{\beta} = a \qquad \text{...(iii)}$$

$$\frac{\alpha}{\beta} = b \qquad \text{... (iv)}$$

Now, $\quad \text{LHS} = (q-b)^2$

$$= \left(\alpha\beta - \frac{\alpha}{\beta}\right)^2 \quad \text{[from Eqs. (ii) and (iv)]}$$

$$= \alpha^2\left(\beta - \frac{1}{\beta}\right)^2 = \alpha^2\left[(\alpha+\beta) - \left(\alpha + \frac{1}{\beta}\right)\right]^2$$

$$= \alpha^2(p-a)^2 \text{ [from Eqs. (i) and (iii)]}$$

$$= \alpha\beta \cdot \frac{\alpha}{\beta}(p-a)^2$$

$$= bq\,(p-a)^2 \quad \text{[from Eqs. (ii) and (iv)]}$$

$$= \text{RHS}$$

96. Since, roots of $x^2 - 2px + q = 0$ are equal.

$\therefore \quad D = 0$

i.e., $\quad (-2p)^2 - 4q = 0$ or $p^2 = q \quad$ ...(i)

Now, $(1+y)\,x^2 - 2\,(p+y)\,x + (q+y) = 0$

$\therefore$ Discriminant $= 4\,(p+y)^2 - 4\,(1+y)\,(q+y)$

$$= 4\,(p^2 + 2py + y^2 - q - y - qy - y^2)$$

$$= 4\,[(2p - q - 1)\,y + p^2 - q]$$

$$= 4\,[(2p - p^2 - 1)\,y + 0] \quad \text{[from Eq. (i)]}$$

$$= -4\,(p-1)^2 y$$

$$> 0 \quad [\because y < 0 \text{ and } p \neq 1]$$

Hence, roots of $(1+y)\,x^2 - 2\,(p+y)\,x + (q+y) = 0$ are real and distinct.

97. $x^{\log_x (x+3)^2} = 16 \quad$ ...(i)

Equation is defined, when

$$x > 0,\ x \neq 1,\ x \neq -3,$$

Then, $\quad (x+3)^2 = 4^2 \quad$ [by property]

$\Rightarrow \quad x + 3 = \pm 4$

$\therefore \quad x = 1$ and $x = -7$

But $\quad x \neq 1,\ x \neq -7$

i.e. no solution.

$\therefore \quad x \in \phi$

98. $\because (2+\sqrt{3})^{x^2-2x+1} + (2-\sqrt{3})^{x^2-2x-1} = \dfrac{101}{10\,(2-\sqrt{3})}$

$$\Rightarrow \quad (2+\sqrt{3})^{x^2-2x} \cdot (2+\sqrt{3})\,(2-\sqrt{3}) + (2-\sqrt{3})^{x^2-2x-1} \cdot (2-\sqrt{3}) = \frac{101}{10}$$

$$\Rightarrow \quad (2+\sqrt{3})^{x^2-2x} + (2-\sqrt{3})^{x^2-2x} = \frac{101}{10}$$

or $\quad (2+\sqrt{3})^{x^2-2x} + \dfrac{1}{(2+\sqrt{3})^{x^2-2x}} = \dfrac{101}{10} \quad$ ...(i)

$$\left[\because 2 - \sqrt{3} = \frac{1}{2+\sqrt{3}}\right]$$

Let $(2+\sqrt{3})^{x^2-2x} = \lambda$, then Eq. (i) reduces to

$$\lambda + \frac{1}{\lambda} = \frac{101}{10}$$

$\Rightarrow \quad 10\lambda^2 - 101\lambda + 10 = 0$

or $\quad (\lambda - 10)\,(10\lambda - 1) = 0$

$\therefore \quad \lambda = 10, \dfrac{1}{10}$

$\Rightarrow \quad (2+\sqrt{3})^{x^2-2x} = 10, 10^{-1}$

$\Rightarrow \quad x^2 - 2x = \log_{2+\sqrt{3}} 10, -\log_{2+\sqrt{3}} 10$

$\Rightarrow \quad (x-1)^2 = 1 + \log_{2+\sqrt{3}} 10,\ 1 - \log_{2+\sqrt{3}} 10$

$\therefore \quad (x-1)^2 = 1 + \log_{2+\sqrt{3}} 10$

$$[\because (x-1)^2 \neq 1 - \log_{2+\sqrt{3}} 10]$$

$\Rightarrow \quad x = 1 \pm \sqrt{(1 + \log_{2+\sqrt{3}} 10)}$

$\Rightarrow \quad x_1 = 1 + \sqrt{(1 + \log_{2+\sqrt{3}} 10)}$

$\quad x_2 = 1 - \sqrt{(1 + \log_{2+\sqrt{3}} 10)}$

99. We have, $\quad x^2 + \left(\dfrac{x}{x-1}\right)^2 = 8$

$$\Rightarrow \quad \left(x + \frac{x}{x-1}\right)^2 - 2 \cdot x \cdot \frac{x}{(x-1)} = 8$$

$$\Rightarrow \quad \left(\frac{x^2}{x-1}\right)^2 - 2\left(\frac{x^2}{x-1}\right) - 8 = 0 \quad \text{...(i)}$$

Let $y = \dfrac{x^2}{x-1}$. Then, Eq. (i) reduces to

$$y^2 - 2y - 8 = 0$$

$\Rightarrow \quad (y-4)\,(y+2) = 0$

$\therefore \quad y = 4, -2$

If $y = 4$, then $\quad 4 = \dfrac{x^2}{x-1}$

or $\quad x^2 - 4x + 4 = 0$

or $\quad (x-2)^2 = 0$

or $\quad x = 2$

$\therefore \quad x_1 = 2$

and if $y = -2$, then $\quad -2 = \dfrac{x^2}{x-1}$

or $\quad x^2 + 2x - 2 = 0$

$\therefore \quad x = \dfrac{-2 \pm \sqrt{(4+8)}}{2}$

$\Rightarrow \quad x = -1 \pm \sqrt{3}$

$\therefore \quad x_2 = -1 + \sqrt{3},\ x_3 = -1 - \sqrt{3}$

100. We have, $\sqrt{x + 8 + 2\sqrt{(x+7)}} + \sqrt{(x+1) - \sqrt{(x+7)}} = 4 \quad$ ...(i)

Let $\quad \sqrt{(x+7)} = \lambda \quad$ ...(ii)

or $\quad x = \lambda^2 - 7$

Then, Eq. (i) reduces to

$$\sqrt{(\lambda^2 - 7 + 8 + 2\lambda)} + \sqrt{(\lambda^2 - 7 + 1 - \lambda)} = 4$$

$\Rightarrow \quad (\lambda + 1) + \sqrt{(\lambda^2 - \lambda - 6)} = 4$

or $\quad \sqrt{(\lambda^2 - \lambda - 6)} = 3 - \lambda$

On squaring both sides, we get

$$\lambda^2 - \lambda - 6 = 9 + \lambda^2 - 6\lambda$$

$\Rightarrow \quad 5\lambda = 15$

$\therefore \quad \lambda = 3$

$\Rightarrow \quad \sqrt{(x+7)} = 3$ [from Eq. (ii)]

or $\quad x + 7 = 9$

$\therefore \quad x = 2$

and $x = 2$ satisfies Eq. (i).

Hence, $x_1 = 2$

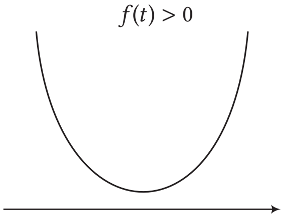
101. We have, $\quad 4^{x^2} + 2(2a+1)2^{x^2} + 4a^2 - 3 > 0$ ...(i)

Putting $t = 2^{x^2}$ in the Eq. (i), we get

$$t^2 + 2(2a+1)t + 4a^2 - 3 > 0$$

Let $\quad f(t) = t^2 + 2(2a+1)t + 4a^2 - 3$ $\quad [\because t > 0, \therefore 2^{x^2} > 0]$

$\because \quad f(t) > 0$



Consider the following cases:

***Case* I** Sum of the roots > 0

$$-2\frac{(2a+1)}{1} > 0$$

$\therefore \quad a \in \left(-\infty, -\frac{1}{2}\right)$

***Case* II** Product of the roots > 0

$\Rightarrow \quad \frac{4a^2 - 3}{1} > 0$

or $\quad a^2 > \frac{3}{4}$

or $\quad a \in \left(-\infty, -\frac{\sqrt{3}}{2}\right) \cup \left(\frac{\sqrt{3}}{2}, \infty\right)$

***Case* III** $\quad D < 0$

$\Rightarrow \quad 4(2a+1)^2 - 4 \cdot 1 \cdot (4a^2 - 3) < 0$

$\Rightarrow \quad 4a + 4 < 0$

$\therefore \quad a < -1$

or $\quad a \in (-\infty, -1)$

Combining all cases, we get

$$a \in (-\infty, -1) \cup \left(\frac{\sqrt{3}}{2}, \infty\right)$$

102. We have, $\quad \log_{x^2+2x-3}\left(\frac{|x+4| - |x|}{x-1}\right) > 0$

The given inequation is valid for

$$\frac{|x+4| - |x|}{(x-1)} > 0$$

and $\quad x^2 + 2x - 3 > 0, \neq 1$ ...(i)

Now, consider the following cases:

***Case* I** If $0 < x^2 + 2x - 3 < 1$

$\Rightarrow \quad 4 < x^2 + 2x + 1 < 5$

$\Rightarrow \quad 4 < (x+1)^2 < 5$

$\Rightarrow \quad -\sqrt{5} < (x+1) < -2 \quad$ or $\quad 2 < x + 1 < \sqrt{5}$

$\Rightarrow \quad -\sqrt{5} - 1 < x < -3$ or $1 < x < \sqrt{5} - 1$

$\therefore \quad x \in (-\sqrt{5} - 1, -3) \cup (1, \sqrt{5} - 1)$ ...(ii)

Then, $\quad \frac{|x+4| - |x|}{(x-1)} < 1$

Now, $x < -4$, then $\frac{-(x+4) + x}{(x-1)} < 1$

$\Rightarrow \quad 1 + \frac{4}{x-1} > 0$

$\Rightarrow \quad \frac{(x+3)}{(x-1)} > 0$

$\therefore \quad x \in (-\infty, -3) \cup (1, \infty)$

$\Rightarrow \quad x \in (-\infty, -4)$ $\quad [\because x < -4]$ ...(iii)

$-4 \le x < 0$, then $\frac{x+4+x}{(x-1)} - 1 < 0$

$\Rightarrow \quad \frac{(x+5)}{(x-1)} < 0$

$\therefore \quad x \in (-5, 1)$

$\Rightarrow \quad x \in [-4, 0)$ $\quad [\because -4 \le x < 0]$ ...(iv)

and $\quad x \ge 0$, then $\frac{(x+4) - x}{(x-1)} < 1$

$\Rightarrow \quad 1 - \frac{4}{x-1} > 0$

$\Rightarrow \quad \frac{(x-5)}{(x-1)} > 0$

$\therefore \quad x \in (-\infty, 1) \cup (5, \infty)$

$\Rightarrow \quad x \in [0, 1) \cup (5, \infty)$ $\quad [\because x \ge 0]$ ...(v)

From Eqs. (iii), (iv) and (v), we get

$$x \in (-\infty, 1) \cup (5, \infty) \quad \text{...(vi)}$$

Now, common values in Eqs. (ii) and (iv) is

$$x \in (-\sqrt{5} - 1, -3) \quad \text{...(vii)}$$

***Case* II** If $\quad x^2 + 2x - 3 > 1$

$\Rightarrow \quad x^2 + 2x + 1 > 5 \Rightarrow (x+1)^2 > 5$

$\Rightarrow \quad x + 1 < -\sqrt{5}$

or $\quad x + 1 > \sqrt{5}$

$\therefore \quad x \in (-\infty, -1 - \sqrt{5}) \cup (\sqrt{5} - 1, \infty)$ ...(viii)

Then, $\quad \frac{|x+4| - |x|}{(x-1)} > 1$

Now, $\quad x < -4$, then $\frac{-4}{x-1} > 1$

$\Rightarrow \quad 1 + \frac{4}{x-1} < 0$

$\Rightarrow \quad \frac{x+3}{x-1} < 0$

$\therefore \quad x \in (-3, 1)$

which is false. $[\because x < -4]$

$-4 \le x < 0$, then $\dfrac{2x+4}{(x-1)} - 1 > 0$

$\Rightarrow \quad \dfrac{(x+5)}{(x-1)} > 0$

$\therefore \quad x \in (-\infty, -5) \cup (1, \infty)$

which is false. $[\because -4 \le x < 0]$

and $\quad x \ge 0$, then $\dfrac{4}{x-1} > 1$

$\Rightarrow \quad 1 - \dfrac{4}{x-1} < 0$

$\Rightarrow \quad \dfrac{x-5}{x-1} < 0$

$\therefore \quad x \in (1, 5)$ ...(ix)

which is false. $[\because x \ge 0]$

Now, common values in Eq. (viii) and (ix) is

$\therefore \quad x \in (\sqrt{5} - 1, 5)$ ...(x)

Combining Eqs. (viii) and (x), we get

$$x \in (-\sqrt{5} - 1, -3) \cup (\sqrt{5} - 1, 5)$$

103. Let $y \ge 0$, then $|y| = y$

and then given system reduces to

$$|x^2 - 2x| + y = 1 \quad \text{...(i)}$$

and $\quad x^2 + y = 1$ ...(ii)

From Eqs. (i) and (ii), we get

$$x^2 = |x^2 - 2x|$$

$\Rightarrow \quad x^2 = |x||x-2|$

Now, $\quad x < 0, \quad 0 \le x < 2, \quad x \ge 2$

$$x^2 = x(x-2), \quad x^2 = -x(x-2)$$

$$x^2 = x(x-2)$$

$\therefore \quad x = 0$

$\Rightarrow \quad x(x + x - 2) = 0$

$\therefore \quad x = 0$

fail $\therefore x = 0, 1$ fail

$\Rightarrow \quad x = 0, 1$, then $y = 1, 0$

$\therefore$Solutions are $(0, 1)$ and $(1, 0)$.

If $y < 0$ then $|y| = -y$ and then given system reduces to

$$|x^2 - 2x| + y = 1 \quad \text{...(iii)}$$

and $\quad x^2 - y = 1$ ...(iv)

From Eqs. (iii) and (iv), we get

$$|x^2 - 2x| + x^2 = 2$$

$\Rightarrow \quad |x||x-2| + x^2 = 2$

Now, $\quad x < 0, \quad 0 \le x < 2, \quad x \ge 2$

$$x(x-2) + x^2 = 2$$

$$-x(x-2) + x^2 = 2$$

$$x(x-2) + x^2 = 2$$

$\Rightarrow \quad 2x^2 - 2x - 2 = 0 \quad \Rightarrow 2x = 2$

$\Rightarrow \quad x^2 - x - 1 = 0$

$\Rightarrow \quad x^2 - x - 1 = 0$

$\therefore \quad x = 1$

$\therefore \quad x = \dfrac{1 \pm \sqrt{5}}{2}$

$\therefore \quad x = \dfrac{1 \pm \sqrt{5}}{2}$ fail

$\Rightarrow \quad x = \dfrac{1 - \sqrt{5}}{2}$ $[\because x < 0]$

$\Rightarrow \quad x = \dfrac{1-\sqrt{5}}{2}, 1$, then $y = \dfrac{1-\sqrt{5}}{2}, 0$

$\therefore$ Solutions are $\left(\dfrac{1-\sqrt{5}}{2}, \dfrac{1-\sqrt{5}}{2}\right)$ and $(1, 0)$.

Hence, all pairs $(0, 1)$, $(1, 0)$ and $\left(\dfrac{1-\sqrt{5}}{2}, \dfrac{1-\sqrt{5}}{2}\right)$ are solutions of the original system of equations.

104. Given, α, β and γ are the roots of the cubic equation

$$x^3 - px^2 + qx - r = 0 \quad \text{...(i)}$$

$\therefore \quad \alpha + \beta + \gamma = p, \ \alpha\beta + \beta\gamma + \gamma\alpha = q, \ \alpha\beta\gamma = r$

(i) Let $\quad y = \beta\gamma + \dfrac{1}{\alpha}$

$\Rightarrow \quad y = \dfrac{\alpha\beta\gamma + 1}{\alpha} = \dfrac{r+1}{\alpha}$

$\therefore \quad \alpha = \dfrac{r+1}{y}$

From Eq. (i), we get

$$\alpha^3 - p\alpha^2 + q\alpha - r = 0$$

$\Rightarrow \quad \dfrac{(r+1)^3}{y^3} - \dfrac{p(r+1)^2}{y^2} + \dfrac{q(r+1)}{y} - r = 0$

or $\quad ry^3 - q(r+1)y^2 + p(r+1)^2 y - (r+1)^3 = 0$

(ii) Let $y = \beta + \gamma - \alpha = (\alpha + \beta + \gamma) - 2\alpha = p - 2\alpha$

$\therefore \quad \alpha = \dfrac{p-y}{2}$

From Eq. (i), we get

$$\alpha^3 - p\alpha^2 + q\alpha - r = 0$$

$\Rightarrow \quad \dfrac{(p-y)^3}{8} - \dfrac{p(p-y)^2}{4} + \dfrac{q(p-y)}{2} - r = 0$

or $\quad y^3 - py^2 + (4q - p^2)y + (8r - 4pq + p^3) = 0$

Also product of roots $= -(8r - 4pq + p^3)$

105. Assume $\alpha + i\beta$ is a complex root of the given equation, then conjugate of this root, i.e. $\alpha - i\beta$ is also root of this equation. On putting $x = \alpha + i\beta$ and $x = \alpha - i\beta$ in the given equation, we get

$$\frac{A_1^2}{\alpha + i\beta - a_1} + \frac{A_2^2}{\alpha + i\beta - a_2} + \frac{A_3^2}{\alpha + i\beta - a_3} + \ldots + \frac{A_n^2}{\alpha + i\beta - a_n}$$

$$= ab^2 + c^2(\alpha + i\beta) + ac \quad \text{...(i)}$$

and $\dfrac{A_1^2}{\alpha - i\beta - a_1} + \dfrac{A_2^2}{\alpha - i\beta - a_2} + \dfrac{A_3^2}{\alpha - i\beta - a_3} + \ldots + \dfrac{A_n^2}{\alpha - i\beta - a_n}$

$$= ab^2 + c^2(\alpha - i\beta) + ac \quad \text{...(ii)}$$

On subtracting Eq. (i) from Eq. (ii), we get

$$2i\beta\left[\frac{A_1^2}{(\alpha-a_1)^2+\beta^2}+\frac{A_2^2}{(\alpha-a_2)^2+\beta^2}+\frac{A_3^2}{(\alpha-a_3)^2+\beta^2}+\ldots+\frac{A_n^2}{(\alpha-a_n)^2+\beta^2}+c^2\right]=0$$

The expression in bracket $\neq 0$

$\therefore \quad 2i\beta = 0 \Rightarrow \beta = 0$

Hence, all roots of the given equation are real.

106. Given equation is

$$x^4 + 2ax^3 + x^2 + 2ax + 1 = 0 \quad \ldots\text{(i)}$$

On dividing by x^2, we get

$$x^2 + 2ax + 1 + \frac{2a}{x} + \frac{1}{x^2} = 0$$

$\Rightarrow \quad \left(x^2 + \frac{1}{x^2}\right) + 2a\left(x + \frac{1}{x}\right) + 1 = 0$

$\Rightarrow \quad \left(x + \frac{1}{x}\right)^2 - 2 + 2a\left(x + \frac{1}{x}\right) + 1 = 0$

or $\quad \left(x + \frac{1}{x}\right)^2 + 2a\left(x + \frac{1}{x}\right) - 1 = 0$

or $\quad y^2 + 2ay - 1 = 0$, where $y = x + \frac{1}{x}$

$\therefore \quad y = \frac{-2a \pm \sqrt{(4a^2+4)}}{2} = -a \pm \sqrt{(a^2+1)}$

Taking '+' sign, we get

$$y = -a + \sqrt{(a^2+1)}$$

$\Rightarrow \quad x + \frac{1}{x} = -a + \sqrt{(a^2+1)}$

or $\quad x^2 + (a - \sqrt{(a^2+1)})\, x + 1 = 0 \quad \ldots\text{(ii)}$

Taking '–' sign, we get $\quad y = -a - \sqrt{(a^2+1)}$

$\Rightarrow \quad x + \frac{1}{x} = -a - \sqrt{(a^2+1)}$

or $\quad x^2 + (a + \sqrt{(a^2+1)})\, x + 1 = 0 \quad \ldots\text{(iii)}$

Let α, β be the roots of Eq. (ii) and γ, δ be the roots of Eq. (iii).

Then, $\quad \alpha + \beta = \sqrt{(a^2+1)} - a$

and $\quad \alpha\beta = 1$

and $\quad \gamma + \delta = -\sqrt{(a^2+1)} - a$

and $\quad \gamma\delta = 1$

Clearly, $\alpha + \beta > 0$ and $\alpha\beta > 0$

$\therefore$Either α, β will be imaginary or both real and positive according to the Eq. (i) has atleast two distinct negative roots. Therefore, both γ and δ must be negative. Therefore,

(i) $\gamma\delta > 0$, which is true as $\gamma\delta = 1$.

(ii) $\quad \gamma + \delta < 0$

$\Rightarrow \quad -(a + \sqrt{(a^2+1)}) < 0$

$\Rightarrow \quad a + \sqrt{(a^2+1)} > 0$, which is true for all a.

$\therefore \quad a \in R$

(iii) $\quad D > 0$

$\therefore \quad (a + \sqrt{(a^2+1)})^2 - 4 > 0$

$\Rightarrow \quad (a + \sqrt{(a^2+1)} + 2)(a + \sqrt{(a^2+1)} - 2) > 0$

$\because \quad a + \sqrt{(a^2+1)} + 2 > 0$

$\therefore \quad a + \sqrt{(a^2+1)} - 2 > 0$

$\Rightarrow \quad \sqrt{(a^2+1)} > 2 - a$

$$\begin{cases} a \geq 2 \\ \text{or } a^2 + 1 > (2-a)^2, \text{ if } a < 2 \end{cases}$$

$\Rightarrow \quad \begin{cases} a \geq 2 \\ \text{or } a > \frac{3}{4}, \text{ if } a < 2 \end{cases}$

$\Rightarrow \quad \begin{cases} a \geq 2 \\ \text{or } \frac{3}{4} < a < 2 \end{cases}$

Hence, $\quad \frac{3}{4} < a < \infty$ or $\quad a \in \left(\frac{3}{4}, \infty\right)$

107. We have, $\quad [2x] - [x+1] = 2x$

Since, $\quad$ LHS = Integer

$\therefore \quad$ RHS $= 2x =$ Integer

$\Rightarrow \quad [2x] = 2x$

Now, $\quad -[x+1] = 0$

$\Rightarrow \quad [x+1] = 0$

or $\quad 0 \leq x + 1 < 1$

or $\quad -1 \leq x < 0$

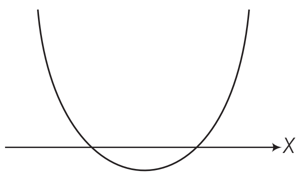
or $\quad -2 \leq 2x < 0$

$\therefore \quad 2x = -2, -1$

or $\quad x = -1, -\frac{1}{2}$

or $\quad x_1 = -1, x_2 = -\frac{1}{2}$

108. We have, $\quad (a^2+3)x^2 + (a+2)x - 6 < 0$



Let $\quad f(x) = (a^2+3)\,x^2 + (a+2)\,x - 6$

$\because \quad (a^2+3) > 0$ and $f(x) < 0$

$\therefore \quad D > 0$

$\Rightarrow (a+2)^2 + 24\,(a^2+3) > 0$ is true for all $a \in R$.

109. We have, $\quad 6x^2 - 77[x] + 147 = 0$

$\Rightarrow \quad \frac{6x^2 + 147}{77} = [x]$

$\Rightarrow \quad (0.078)\,x^2 = [x] - 1.9$

$\because \quad (0.078)\,x^2 > 0 \Rightarrow x^2 > 0$

$\therefore \quad [x] - 1.9 > 0$

or $\quad [x] > 1.9$

$\therefore \quad [x] = 2, 3, 4, 5, \ldots$

If $\quad [x] = 2$, i.e. $2 \le x < 3$

Then, $\quad x^2 = \frac{2 - 1.9}{0.078} = 1.28$

$\therefore \quad x = 1.13$ [fail]

If $\quad [x] = 3$, i.e. $3 \le x < 4$

Then, $\quad x^2 = \frac{3 - 1.9}{0.078} = 14.1$

$\therefore \quad x = 3.75$ [true]

If $\quad [x] = 4$, i.e. $4 \le x < 5$

Then, $\quad x^2 = \frac{4 - 1.9}{0.078} = 26.9$

$\therefore \quad x = 5.18$ [fail]

If $\quad [x] = 5$, i.e. $5 \le x < 6$

Then, $\quad x^2 = \frac{5 - 1.9}{0.078} = 39.7$

$\therefore \quad x = 6.3$ [fail]

If $\quad [x] = 6$, i.e. $6 \le x < 7$

Then, $\quad x^2 = \frac{6 - 1.9}{0.078} = \frac{4.1}{0.078} = 52.56$

$\therefore \quad x = 7.25$ [fail]

If $\quad [x] = 7$, i.e. $7 \le x < 8$

Then, $\quad x^2 = \frac{7 - 1.9}{0.078} = \frac{5.1}{0.078} = 65.38$

$\therefore \quad x = 8.08$ [fail]

If $\quad [x] = 8$, i.e. $8 \le x < 9$

Then, $\quad x^2 = \frac{8 - 1.9}{0.078} = \frac{6.1}{0.078} = 78.2$

$\therefore \quad x = 8.8$ [true]

If $\quad [x] = 9$, i.e. $9 \le x < 10$

Then, $\quad x^2 = \frac{9 - 1.9}{0.078} = \frac{7.1}{0.078} = 91.03$

$\therefore \quad x = 9.5$ [true]

If $\quad [x] = 10$, i.e. $10 \le x < 11$

Then, $\quad x^2 = \frac{10 - 1.9}{0.078} = \frac{8.1}{0.078} = 103.8$

$\therefore \quad x = 10.2$ [true]

If $\quad [x] = 11$, i.e. $11 \le x < 12$

Then, $\quad x^2 = \frac{11 - 1.9}{0.078}$

$$= \frac{9.1}{0.078} = 116.7$$

$\therefore \quad x = 10.8$ [fail]

Other values are fail.

Hence, number of solutions is four.

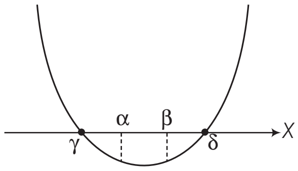
110. Since, the given equation is

$$x^2 - 2x - a^2 + 1 = 0$$

$\Rightarrow \quad (x - 1)^2 = a^2$

$\therefore \quad x - 1 \ne a$ or $x = 1 \pm a$

$\Rightarrow \quad \alpha = 1 + a$ and $\beta = 1 - a$



Let $f(x) = x^2 - 2(a + 1)\,x + a\,(a - 1)$, thus the following conditions hold good:

Consider the following cases:

***Case* I** $\quad D > 0$

$\Rightarrow \quad 4\,(a + 1)^2 - 4a\,(a - 1) > 0$

$\Rightarrow \quad 3a + 1 > 0$

$\therefore \quad a > -\frac{1}{3}$

***Case* II** $\quad f(\alpha) < 0$

$\Rightarrow \quad f(1 + a) < 0$

$\Rightarrow \quad (1 + a)^2 - 2\,(1 + a)\,(1 + a) + a\,(a - 1) < 0$

$\Rightarrow \quad -(1 + a)^2 + a\,(a - 1) < 0$

$\Rightarrow \quad -3a - 1 < 0$

$\Rightarrow \quad a > -\frac{1}{3}$

***Case* III** $\quad f(s) = 0$

$\Rightarrow \quad f(1 - a) < 0$

$\Rightarrow \quad (1 - a)^2 - 2\,(a + 1)\,(1 - a) + a\,(a - 1) < 0$

$\Rightarrow \quad (4a + 1)\,(a - 1) < 0$

$\therefore \quad -\frac{1}{4} < a < 1$

Combining all cases we get

$$a \in \left(-\frac{1}{4}, 1\right)$$

111. $pr = (-p)\,(-r)$

$$= (\alpha + \beta + \gamma + \delta)\,(\alpha\beta\gamma + \alpha\beta\delta + \gamma\delta\alpha + \gamma\delta\beta)$$

$$= \alpha^2\beta\gamma + \alpha^2\beta\delta + \alpha^2\gamma\delta + \alpha\beta\gamma\delta + \beta^2\gamma\alpha$$

$$+ \beta^2\alpha\delta + \alpha\beta\gamma\delta + \beta^2\gamma\delta + \gamma^2\alpha\beta + \alpha\beta\gamma\delta$$

$$+ \gamma^2\delta\alpha + \gamma^2\delta\beta + \alpha\beta\gamma\delta + \alpha\beta\delta^2 + \gamma\alpha\delta^2 + \gamma\beta\delta^2$$

$\because \quad$ AM $\ge$ GM

$\Rightarrow \quad \frac{pr}{16} \ge (\alpha^{16}\beta^{16}\gamma^{16}\delta^{16})^{1/6} = \alpha\beta\gamma\delta = 5$

$\Rightarrow \quad \frac{pr}{16} \ge 5$

or $\quad pr \ge 80$

$\therefore \quad$ Minimum value of pr is 80.

112. $(\alpha^2 + \beta^2)^2 = (\alpha + \beta)\,(\alpha^3 + \beta^3)$

$\Rightarrow \quad \{(\alpha + \beta)^2 - 2\alpha\beta\}^2 = (\alpha + \beta)\,\{(\alpha + \beta)^3 - 3\alpha\beta\,(\alpha + \beta)\}$

$\Rightarrow \quad \left(\frac{b^2}{a^2} - \frac{2c}{a}\right)^2 = \left(-\frac{b}{a}\right)\left(\frac{-b^3}{a^3} + \frac{3bc}{a^2}\right)$

$\Rightarrow \quad \left(\frac{b^2 - 2ac}{a^2}\right)^2 = \left(\frac{-b}{a}\right)\left(\frac{-b^3 + 3abc}{a^3}\right)$

$\Rightarrow \quad 4a^2c^2 = acb^2$

$\Rightarrow \quad ac(b^2 - 4ac) = 0$

As $\quad a \neq 0$

$\Rightarrow \quad c\Delta = 0$

113. Let $P(x) = bx^2 + ax + c$

As $\quad P(0) = 0$

$\Rightarrow \quad c = 0$

As $\quad P(1) = 1$

$\Rightarrow \quad a + b = 1$

$P(x) = ax + (1 - a)x^2$

Now, $\quad P'(x) = a + 2(1 - a)x$

As $\quad P'(x) > 0$ for $x \in (0, 1)$

Only option (d) satisfies above condition.

114. Let the roots are α and $\alpha + 1$, where $\alpha \in I$.

Then, sum of the roots $= 2\alpha + 1 = b$

Product of the roots $= \alpha(\alpha + 1) = c$

Now, $\quad b^2 - 4c = (2\alpha + 1)^2 - 4\alpha(\alpha + 1)$

$= 4\alpha^2 + 1 + 4\alpha - 4\alpha^2 - 4\alpha = 1$

$\therefore \quad b^2 - 4c = 1$

115. Let $\quad f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x,$

$f(0) = 0; f(\alpha) = 0$

$\Rightarrow \quad f'(x) = 0$ has atleast one root between $(0, \alpha)$.

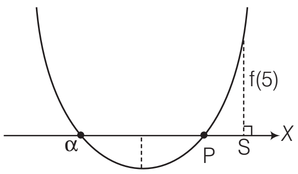
i.e. Equation

$na_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \ldots + a_1 = 0$

has a positive root smaller than α.

116. Let $f(x) = x^2 - 2kx + k^2 + k - 5$

Consider the following cases:



***Case* I** $\quad D \geq 0$

$\Rightarrow \quad 4k^2 - 4.1(k^2 + k - 5) \geq 0$

$\Rightarrow \quad -4(k - 5) \geq 0$

$\Rightarrow \quad k - 5 \leq 0$

$\Rightarrow \quad k \leq 5 \quad$ or $\quad k \in (-\infty, 5]$

***Case* II** x-Coordinate of vertex $x < 5$

$\Rightarrow \quad \dfrac{2k}{2} < 5$

$\Rightarrow \quad k < 5$ or $k \in (-\infty, 5)$

***Case* III** $\quad f(5) > 0$

$\Rightarrow \quad 25 - 10k + k^2 + k - 5 > 0$

$\Rightarrow \quad k^2 - 9k + 20 > 0$

$\Rightarrow \quad (k - 4)(k - 5) > 0$ or $k \in (-\infty, 4) \cup (5, \infty)$

Combining all cases, we get

$k \in (-\infty, 4)$

117. We have, $\quad a + b = 10c, ab = -11d$

and $\quad c + d = 10a, cd = -11b$

$\therefore \quad a + b + c + d = 10(a + c)$

and $\quad abcd = 121\, bd$

$\Rightarrow \quad b + d = 9(a + c)$

and $\quad ac = 121$

Next, $\quad a^2 - 10ac - 11d = 0$

and $\quad c^2 - 10ac - 11b = 0$

$\Rightarrow \quad a^2 + c^2 - 20\,ac - 11(b + d) = 0$

$\Rightarrow \quad (a + c)^2 - 22 \times 121 - 99(a + c) = 0$

$\Rightarrow \quad a + c = 121$ or -22

If $a + c = -22 \Rightarrow a = c$, rejecting these values, we have

$a + c = 121$

$\therefore \quad a + b + c + d = 10(a + c) = 1210$

118. $\quad D \geq 0$

$4(a + b + c)^2 - 12\lambda(ab + bc + ca) \geq 0$

$(a^2 + b^2 + c^2) - (3\lambda - 2)(ab + bc + ca) \geq 0$

$\therefore \quad (3\lambda - 2) \leq \dfrac{(a^2 + b^2 + c^2)}{(ab + bc + ca)}$

Since, $\quad |a - b| < c$

$\Rightarrow \quad a^2 + b^2 - 2ab < c^2 \quad$ (i)

$|b - c| < a$

$\Rightarrow \quad b^2 + c^2 - 2bc < a^2 \quad$ ...(ii)

$|c - a| < b$

$\Rightarrow \quad c^2 + a^2 - 2ca < b^2 \quad$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$\dfrac{a^2 + b^2 + c^2}{ab + bc + ca} < 2 \quad$ ...(iv)

From Eqs. (i) and (iv), we get

$3\lambda - 2 < 2 \Rightarrow \lambda < \dfrac{4}{3}$

119. $\because \quad x^2 - 2mx + m^2 - 1 = 0$

$\Rightarrow \quad (x - m)^2 = 1$

$\therefore \quad x - m = \pm 1$ or $x = m - 1, m + 1$

According to the question,

$m - 1 > -2, m + 1 > -2$

$\Rightarrow \quad m > -1, m > -3$

Then, $\quad m > -1 \quad$ ...(i)

and $\quad m - 1 < 4, m + 1 < 4$

$\Rightarrow \quad m < 5, m < 3$ and $m < 3 \quad$ ...(ii)

From Eqs. (i) and (ii), we get $-1 < m < 3$

120. $x^2 + px + q = 0$

Sum of the roots $= \tan 30° + \tan 15° = -p$

Product of the roots $= \tan 30° \cdot \tan 15° = q$

$\tan 45° = \tan(30° + 15°) = \dfrac{\tan 30° + \tan 15°}{1 - \tan 30° \cdot \tan 15°}$

$$\Rightarrow \quad 1 = \frac{-p}{1-q} \Rightarrow -p = 1 - q$$

$$\Rightarrow \quad q - p = 1$$

$$\therefore \quad 2 + q - p = 3$$

121. The equation $x^2 - px + r = 0$ has roots (α, β) and the equation $x^2 - qx + r$ has roots $\left(\frac{\alpha}{2}, 2\beta\right)$.

$$\Rightarrow \quad r = \alpha\beta \text{ and } \alpha + \beta = p \text{ and } \frac{\alpha}{2} + 2\beta = q$$

$$\Rightarrow \quad \beta = \frac{2q - p}{3} \text{ and } \alpha = \frac{2(2p - q)}{3}$$

$$\Rightarrow \quad \alpha\beta = r = \frac{2}{9}(2q - p)(2p - q)$$

122. $$\alpha + \beta = -a$$

$$|\alpha - \beta| < \sqrt{5} \Rightarrow (\alpha - \beta)^2 < 5$$

$$\Rightarrow \quad a^2 - 4 < 5 \Rightarrow a \in (-3, 3)$$

123. Suppose roots are imaginary, then $\beta = \overline{\alpha}$

and $$\frac{1}{\beta} = \overline{\alpha}$$

$$\Rightarrow \quad \beta = -\frac{1}{\beta} \qquad \text{[not possible]}$$

$\Rightarrow$ Roots are real $\Rightarrow (p^2 - q)(b^2 - ac) \geq 0$

$\Rightarrow$ Statement -1 is true.

$$-\frac{2b}{a} = \alpha + \frac{1}{\beta}$$

and $$\frac{\alpha}{\beta} = \frac{c}{a}, \alpha + \beta = -2p, \alpha\beta = q$$

If $\beta = 1$, then $\alpha = q$

$$\Rightarrow \quad c = qa \qquad \text{[not possible]}$$

Also, $$\alpha + 1 = \frac{-2b}{a}$$

$$\Rightarrow \quad -2p = \frac{-2b}{a}$$

$$\Rightarrow \quad b = ap \qquad \text{[not possible]}$$

$\Rightarrow$ Statement -2 is true but it is not the correct explanation of Statement-1.

124. Let $\alpha, 4\beta$ be roots of $x^2 - 6x + a = 0$ and $\alpha, 3\beta$ be the roots of $x^2 - cx + 6 = 0$.

Then, $$\alpha + 4\beta = 6 \text{ and } 4\alpha\beta = a \qquad \text{...(i)}$$

$$\alpha + 3\beta = c \text{ and } 3\alpha\beta = 6 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$a = 8, \alpha\beta = 2$$

Now, first equation becomes

$$x^2 - 6x + 8 = 0$$

$$\Rightarrow \quad x = 2, 4$$

If $$\alpha = 2, 4\beta = 4, \text{ then } 3\beta = 3$$

If $$\alpha = 4, 4\beta = 2, \text{ then } 3\beta = \frac{3}{2} \qquad \text{[non-integer]}$$

$\therefore$ Common root is $x = 2$.

125. Let $f(x) = x^7 + 14x^5 + 16x^3 + 30x - 560$

$$\therefore \quad f'(x) = 7x^6 + 70x^4 + 48x^2 + 30 > 0, \forall x \in R$$

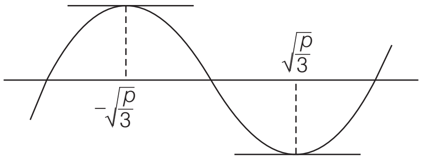
$\Rightarrow f(x)$ is an increasing function, for all $x \in R$

Hence, number of real solutions is 1.

126. Let $f(x) = x^3 - px + q$

$$\therefore \quad f'(x) = 3x^2 - p$$

$$\Rightarrow \quad f''(x) = 6x$$



For maxima or minima, $f'(x) = 0$

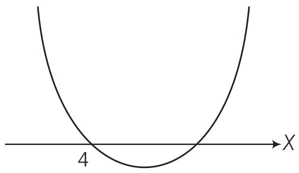
$$\therefore \quad x = \pm\sqrt{\frac{p}{3}}$$

$$\Rightarrow \quad f''\left(\sqrt{\frac{p}{3}}\right) = 6\sqrt{\left(\frac{p}{3}\right)} > 0$$

and $$f''\left(-\sqrt{\frac{p}{3}}\right) = -6\sqrt{\frac{p}{3}} < 0$$

Hence, given cubic minima at $x = \sqrt{\frac{p}{3}}$ and maxima at $x = -\sqrt{\frac{p}{3}}$.

127. Let $f(x) = x^2 - 8kx + 16(k^2 - k + 1)$



$$\therefore \quad D > 0$$

$$\Rightarrow \quad 64k^2 - 4 \cdot 16(k^2 - k + 1) > 0$$

$$\Rightarrow \quad k > 1 \qquad \text{...(i)}$$

$$\Rightarrow \quad \frac{-b}{2a} > 4 \Rightarrow \frac{8k}{2} > 4$$

$$\Rightarrow \quad k > 1 \qquad \text{...(ii)}$$

and $$f(4) \geq 0$$

$$\Rightarrow \quad 16 - 32k + 16(k^2 - k + 1) \geq 0$$

$$\Rightarrow \quad k^2 - 3k + 2 \geq 0$$

$$\Rightarrow \quad (k - 1)(k - 2) \geq 0$$

$$\Rightarrow \quad k \leq 1 \text{ or } k \geq 2 \qquad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii), we get

$$k \geq 2$$

$$k_{\min} = 2$$

128. Since, roots of $bx^2 + cx + a = 0$ are imaginary.

$$\therefore \quad c^2 - 4ab < 0$$

$$\Rightarrow \quad -c^2 > -4ab \qquad \text{...(i)}$$

Let $f(x) = 3b^2x^2 + 6bcx + 2c^2$

Since, $3b^2 > 0$

and $D = (6bc)^2 - 4(3b^2)(2c^2) = 12b^2c^2$

$\therefore$ Minimum value of $f(x) = -\dfrac{D}{4a} = -\dfrac{12b^2c^2}{4(3b^2)} = -c^2 > -4ab$

129. $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta} = \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$ ...(i)

and given, $\alpha^3 + \beta^3 = q, \alpha + \beta = -p$

$\Rightarrow (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = q$

$\Rightarrow -p^3 + 3p\alpha\beta = q$

or $\alpha\beta = \dfrac{q + p^3}{3p}$

$\therefore$ From Eq. (i), we get

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{p^2 - \dfrac{2(q+p^3)}{3p}}{\dfrac{(q+p^3)}{3p}} = \frac{p^3 - 2q}{(q+p^3)}$$

and product of the roots $= \dfrac{\alpha}{\beta} \cdot \dfrac{\beta}{\alpha} = 1$

$\therefore$ Required equation is $x^2 - \left(\dfrac{p^3 - 2q}{q + p^3}\right)x + 1 = 0$

or $(q + p^3)x^2 - (p^3 - 2q)x + (q + p^3) = 0$

130. Since, $f'(x) = 12x^2 + 6x + 2$

Here, $D = 6^2 - 4 \cdot 12 \cdot 2 = 36 - 96 = -60 < 0$

$\therefore f'(x) > 0, \forall x \in R$

$\Rightarrow$ Only one real root for $f(x) = 0$

Also, $f(0) = 1, f(-1) = -2$

$\Rightarrow$ Root must lie in $(-1, 0)$.

Taking average of 0 and (-1), $f\left(-\dfrac{1}{2}\right) = \dfrac{1}{4}$

$\Rightarrow$ Root must lie in $\left(-1, -\dfrac{1}{2}\right)$.

Similarly, $f\left(-\dfrac{3}{4}\right) = -\dfrac{1}{2}$

$\Rightarrow$ Root must lie in $\left(-\dfrac{3}{4}, -\dfrac{1}{2}\right)$.

131. $\because \alpha^2 - 6\alpha - 2 = 0 \Rightarrow \alpha^2 - 2 = 6\alpha$ ...(i)

and $\beta^2 - 6\beta - 2 = 0 \Rightarrow \beta^2 - 2 = 6\beta$ ...(ii)

$$\therefore \frac{a_{10} - 2a_8}{2a_9} = \frac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8 \cdot 6\alpha - \beta^8 \cdot 6\beta}{2(\alpha^9 - \beta^9)} \quad \text{[from Eqs. (i) and (ii)]}$$

$$= \frac{6(\alpha^9 - \beta^9)}{2(\alpha^9 - \beta^9)} = 3$$

132. Let α be the common root.

Then, $\alpha^2 + b\alpha - 1 = 0$ and $\alpha^2 + \alpha + b = 0$

$$\Rightarrow \begin{vmatrix} 1 & b \\ 1 & 1 \end{vmatrix} \times \begin{vmatrix} b & -1 \\ 1 & b \end{vmatrix} = \begin{vmatrix} -1 & 1 \\ b & 1 \end{vmatrix}^2$$

$\Rightarrow (1 - b)(b^2 + 1) = (-1 - b)^2$

$\Rightarrow b^3 + 3b = 0$

$\therefore b = 0, i\sqrt{3}, -i\sqrt{3}$, where $i = \sqrt{-1}$.

133. Let $f(x) = x^4 - 4x^3 + 12x^2 + x - 1$

$\therefore f'(x) = 4x^3 - 12x^2 + 24x + 1$

$\Rightarrow f''(x) = 12x^2 - 24x + 24$

$= 12(x^2 - 2x + 2)$

$= 12[(x-1)^2 + 1] > 0$

i.e. $f''(x)$ has no real roots.

Hence, $f(x)$ has maximum two distinct real roots, where $f(0) = -1$.

134. Given, $p(x) = f(x) - g(x)$

$\Rightarrow p(x) = (a - a_1)x^2 + (b - b_1)x + (c - c_1)$

It is clear that $p(x) = 0$ has both equal roots -1, then

$$-1 - 1 = -\frac{(b - b_1)}{(a - a_1)}$$

and $-1 \times -1 = \dfrac{c - c_1}{a - a_1}$

$\Rightarrow b - b_1 = 2(a - a_1)$ and $c - c_1 = (a - a_1)$ ...(i)

Also given, $p(-2) = 2$

$\Rightarrow 4(a - a_1) - 2(b - b_1) + (c - c_1) = 2$ ...(ii)

From Eqs. (i) and (ii), we get

$4(a - a_1) - 4(a - a_1) + (a - a_1) = 2$

$\therefore (a - a_1) = 2$ ...(iii)

$\Rightarrow b - b_1 = 4$ and $c - c_1 = 2$ [from Eq. (i)] ...(iv)

Now, $p(2) = 4(a - a_1) + 2(b - b_1) + (c - c_1)$

$= 8 + 8 + 2 = 18$ [from Eqs. (iii) and (iv)]

135. Let the quadratic equation be

$$ax^2 + bx + c = 0$$

Sachin made a mistake in writing down constant term.

$\therefore$ Sum of the roots is correct.

i.e. $\alpha + \beta = 7$

Rahul made a mistake in writing down coefficient of x.

$\therefore$ Product of the roots is correct.

i.e. $\alpha\beta = 6$

$\Rightarrow$ Correct quadratic equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow x^2 - 7x + 6 = 0$

$\Rightarrow (x - 6)(x - 1) = 0 \Rightarrow x = 6, 1$

Hence, correct roots are 1 and 6.

136. Let $a + 1 = h^6$

$\therefore (h^2 - 1)x^2 + (h^3 - 1)x + (h - 1) = 0$

$$\Rightarrow \quad \left(\frac{h^2-1}{h-1}\right)x^2 + \left(\frac{h^3-1}{h-1}\right)x + 1 = 0$$

As $a \to 0$, then $h \to 1$

$$\lim_{h\to 1}\left(\frac{h^2-1}{h-1}\right)x^2 + \lim_{h\to 1}\left(\frac{h^3-1}{h-1}\right)x + 1 = 0$$

$$\Rightarrow \quad 2x^2 + 3x + 1 = 0$$

$$\Rightarrow \quad 2x^2 + 2x + x + 1 = 0$$

$$\Rightarrow \quad (2x+1)(x+1) = 0$$

$$\therefore \quad x = -1 \text{ and } x = -\frac{1}{2}$$

137. Let $e^{\sin x} = t$ ...(i)

Then, the given equation can be written as

$$t - \frac{1}{t} - 4 = 0 \Rightarrow t^2 - 4t - 1 = 0$$

$$\therefore \quad t = \frac{4 \pm \sqrt{(16+4)}}{2}$$

$$\Rightarrow \quad e^{\sin x} = (2+\sqrt{5}) \qquad [\because e^{\sin x} > 0, \therefore \text{taking + ve sign}]$$

$$\Rightarrow \quad \sin x = \log_e(2+\sqrt{5}) \qquad \text{...(ii)}$$

$$\because \quad (2+\sqrt{5}) > e \qquad [\because e = 2.71828\ldots]$$

$$\Rightarrow \quad \log_e(2+\sqrt{5}) > 1 \qquad \text{...(iii)}$$

From Eqs. (ii) and (iii), we get

$$\sin x > 1 \qquad \text{[which is impossible]}$$

Hence, no real root exists.

138. Given equations are

$$ax^2 + bx + c = 0 \qquad \text{...(i)}$$

and

$$x^2 + 2x + 3 = 0 \qquad \text{...(ii)}$$

Clearly, roots of Eq. (ii) are imaginary, since Eqs. (i) and (ii) have a common root, therefore common root must be imaginary and hence both roots will be common. Therefore, Eqs. (i) and (ii) are identical.

$$\therefore \quad \frac{a}{1} = \frac{b}{2} = \frac{c}{3} \text{ or } a : b : c = 1 : 2 : 3$$

139. $\because x - [x] = \{x\}$ [fractional part of x]

For no integral solution, $\{x\} \neq 0$

$$\therefore \quad a \neq 0 \qquad \text{...(i)}$$

The given equation can be written as

$$3\{x\}^2 - 2\{x\} - a^2 = 0$$

$$\Rightarrow \quad \{x\} = \frac{2 \pm \sqrt{(4+12a^2)}}{6} = \frac{1+\sqrt{(1+3a^2)}}{3} \qquad [\because 0 < \{x\} < 1]$$

$$\Rightarrow \quad 0 < \frac{1+\sqrt{(1+3a^2)}}{3} < 1 \Rightarrow \sqrt{(1+3a^2)} < 2$$

$$\Rightarrow \quad a^2 < 1 \Rightarrow -1 < a < 1 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$a \in (-1, 0) \cup (0, 1)$$

140. $\because \dfrac{1}{\alpha} + \dfrac{1}{\beta} = 4 \Rightarrow \dfrac{\alpha+\beta}{\alpha\beta} = 4$

$$\Rightarrow \quad \frac{-\frac{q}{p}}{\frac{r}{p}} = 4$$

$$\Rightarrow \quad q = -4r \qquad \text{... (i)}$$

Also, given p, q, r are in AP.

$$\therefore \quad 2q = p + r$$

$$\Rightarrow \quad p = -9r \qquad \text{[from Eq. (i)] ...(ii)}$$

$$\text{Now, } |\alpha - \beta| = \frac{\sqrt{D}}{|a|} \qquad \left[\because \text{ for } ax^2+bx+c=0, \alpha-\beta = \frac{\sqrt{D}}{a}\right]$$

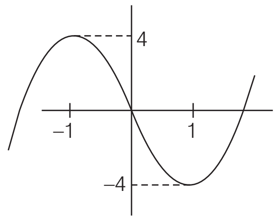
$$= \frac{\sqrt{(q^2-4pr)}}{|p|}$$

$$= \frac{\sqrt{(16r^2+36r^2)}}{9|r|} = \frac{\sqrt{52}\,|r|}{9|r|} \qquad \text{[from Eqs. (i) and (ii)]}$$

$$= \frac{2\sqrt{13}}{9}$$

141. $f(x) = x^5 - 5x$ and $g(x) = -a$

$$\therefore \quad f'(x) = 5x^4 - 5$$



$$= 5(x^2+1)(x-1)(x+1)$$

Clearly, $f(x) = g(x)$ has one real root, if $a > 4$ and three real roots, if $|a| < 4$.

142. Since, $b = 0$ for $p(x) = ax^2 + bx + c$, as roots are pure imaginary.

$\Rightarrow x = \pm\sqrt{\dfrac{(-c \pm i\sqrt{c})}{a}}$, which are clearly neither pure real nor pure imaginary, as $c \neq 0$.

143. $\because \alpha x^2 - x + \alpha = 0$ has distinct real roots.

$$\therefore \quad D > 0$$

$$\Rightarrow \quad 1 - 4\alpha^2 > 0 \Rightarrow \alpha \in \left(-\frac{1}{2}, \frac{1}{2}\right) \qquad \text{...(i)}$$

$$\text{Also,} \quad |x_1 - x_2| < 1 \Rightarrow |x_1 - x_2|^2 < 1$$

$$\Rightarrow \quad \frac{D}{a^2} < 1 \Rightarrow \frac{1-4\alpha^2}{\alpha^2} < 1 \Rightarrow \alpha^2 > \frac{1}{5}$$

$$\Rightarrow \quad \alpha \in \left(-\infty, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right) \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$S = \left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$$

144. $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$

Case I

$x^2 - 5x + 5 = 1$ and $x^2 + 4x - 60$ can be any real number

$\Rightarrow \quad x = 1, 4$

Case II

$x^2 - 5x + 5 = -1$ and $x^2 + 4x - 60$ has to be an even number

$\Rightarrow \quad x = 2, 3$

For $x = 3$, $x^2 + 4x - 60$ is odd, $\therefore\ x \neq 3$

Hence, $\quad x = 2$

Case III $x^2 - 5x + 5$ can be any real number and

$$x^2 + 4x - 60 = 0$$

$\Rightarrow \quad x = -10, 6$

$\Rightarrow$ Sum of all values of $x = 1 + 4 + 2 - 10 + 6 = 3$

145. $\because\ x^2 - 2x \sec\theta\, a + 1 = 0 \ \Rightarrow\ x = \sec\theta \pm \tan\theta$

and $\quad -\frac{\pi}{6} < \theta < -\frac{\pi}{12}$

$\Rightarrow \quad \sec\left(-\frac{\pi}{6}\right) > \sec\theta > \sec\left(-\frac{\pi}{12}\right)$

or $\quad \sec\left(\frac{\pi}{6}\right) > \sec\theta > \sec\left(\frac{\pi}{12}\right)$

and $\quad \tan\left(-\frac{\pi}{6}\right) < \tan\theta < \tan\left(-\frac{\pi}{12}\right)$

$\Rightarrow \quad -\tan\left(\frac{\pi}{6}\right) < \tan\theta < -\tan\left(\frac{\pi}{12}\right)$

or $\quad \tan\left(\frac{\pi}{6}\right) > -\tan\theta > \tan\left(\frac{\pi}{12}\right)$

$\because\ \alpha_1, \beta_1$ are roots of $x^2 - 2x\sec\theta + 1 = 0$ and $\alpha_1 > \beta_1$

$\therefore\ \alpha_1 = \sec\theta - \tan\theta$ and $\beta_1 = \sec\theta + \tan\theta$

$\Rightarrow\ \alpha_2, \beta_2$ are roots of $x^2 + 2x\tan\theta - 1 = 0$

and $\quad \alpha_2 > \beta_2$

$\therefore \quad \alpha_2 = -\tan\theta + \sec\theta$

and $\quad \beta_2 = -\tan\theta - \sec\theta$

Hence, $\quad \alpha_1 + \beta_2 = -2\tan\theta$

146. $\because\ x(x+1) + (x+1)(x+2) + \ldots. + (x + \overline{n-1})(x+n) = 10n$

$\Rightarrow\ nx^2 + x(1 + 3 + 5 + \ldots + (2n-1)) + (1 \,.\, 2 + 2 \,.\, 3 + \ldots + (n-1) \,.\, n) = 10n$

or $\ nx^2 + n^2x + \frac{1}{3}(n-1)\,n\,(n+1) = 10n$

or $\quad 3x^2 + 3nx + (n^2 - 1) = 30 \qquad (\because\ n \neq 0)$

or $\quad 3x^2 + 3nx + (n^2 - 31) = 0$

$\because \quad |\alpha - \beta| = 1$

or $\quad (\alpha - \beta)^2 = 1$

or $\quad \frac{D}{a^2} = 1$

or $\quad D = a^2$

or $\quad 9n^2 - 12 \,.\, (n^2 - 31) = 9$

or $\quad n^2 = 121$

$\therefore \quad n = 11$