4. QUADRATIC EQUATIONS

1. A child purchases a number of notebooks for ₹ 80. If he had purchased four more notebooks for the same amount from the other bookseller, then a notebook would have cost ₹ 1 less. Represent the above situation in the form of a quadratic equation.

   [Ans : $x^2 + 4x - 320 = 0$**]**

2. If one root of the equation $4x^2 - 8kx - 9 = 0$ is negative of the other, then find the value of k.

   [Ans : $k = 0$**]**

3. Solve for x : $x^2 - px + pq - qx = 0$

   [Ans : $x = p$ (or) $x = q$**]**

4. Solve the following quadratic equation for x : $x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0.$

[Ans : $x = \frac{-a}{a+b}$ (or) $x = \frac{-(a+b)}{a}$**]**

5. Divide 29 into two such parts so that the sum of their squares is 425.

[Ans : 16 and 13**]**

6. A certain number of soldiers stand for a parade, in such a way that the number of soldiers in each row is 2 more than the total number of rows. If the number of rows are doubled and the number of soldiers in each row is reduced by 7, the total number of soldiers is increased by 160. Find the original number of soldiers in each row.

[Sol. : Let the original number of soldiers in each row be x.

So, the number of rows = x – 2.

Thus, total number of soldiers = $x(x - 2) = x^2 - 2x$.

If the number of rows are doubled, the number of rows will be 2x – 4.

Now, the number of soldiers in each row = x –7.

So, new total number of soldiers = $(2x - 4)(x - 7)$

$= 2x^2 - 18x + 28.$

As per given condition, we get

$2x^2 - 18x + 28 = x^2 - 2x + 160$

$\Rightarrow x^2 - 16x - 132 = 0$

$\Rightarrow x^2 - 22x + 6x - 132 = 0$

$\Rightarrow x(x - 22) + 6(x - 22) = 0$

$\Rightarrow (x - 22)(x + 6) = 0$

$\Rightarrow (x - 22) = 0$ or $(x + 6) = 0$

$\Rightarrow x = 22$ or $x = -6.$

So, x can be 22, neglecting the negative value of x = – 6.

Thus, original number of soldiers in each row is 22.**]**

7. Solve for x : $\left(\frac{2x}{x-5}\right)^2 + \frac{10x}{(x-5)} - 24 = 0, x \neq 5.$

[Hint : Let $\frac{2x}{x-5} = y$, then the equation becomes $y^2 + 5y - 24 = 0$

By solving, $y = -8$ (or) $y = 3$

The required solution is x = 4 (or) x = 15**]**

8. Find the roots of the following quadratic equation : $4x^2 - 4px + (p^2 - q^2) = 0.$

[Ans : $x = \frac{p+q}{2}$ (or) $x = \frac{p-q}{2}$**]**

9. Find the roots of the quadratic equation : $\frac{1}{3}x^2 - \sqrt{11}x + 1 = 0$

[Ans : $x = \frac{3\sqrt{11} + \sqrt{87}}{2}$ (or) $x = \frac{3\sqrt{11} - \sqrt{87}}{2}$ **]**

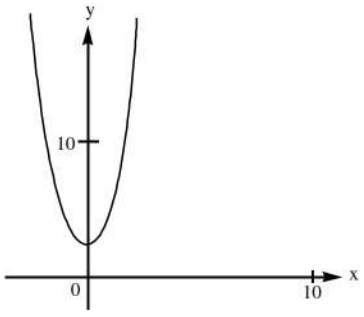
10. Five years ago, a woman's age (in years) was the square of her son's age. Ten years later from now, her age will be twice that of her son's age. Find :
a) The age of the son five years ago.
b) The present age of the woman.
[Ans : a) The age of the son five years ago = 5 years
b) The present age of the woman = 30 years**]**

11. If the roots of the quadratic equation $(a-b)x^2 + (b-c)x + (c-a) = 0$ are equal, prove that $2a = b + c$.

12. For what value of a, the quadratic equation $x^2 - ax + 1 = 0$ does not have real roots ?
[Ans : For $a > -2$ and $a < 2$ i.e., $-2 < a < 2$**]**

13. Three students were asked how they would verify their solution of a quadratic equation. $(x - 2)(x - 5) = 0$. Shown below are their responces.
Student 1 said, "In the first bracket, x must equal 2, and in the second bracket, x must equal 5. So, $(2 - 2)(5 - 5) = 0$".
Student 2 said, "In the first bracket, x must equal 2, but in the second bracket, x can have any real number value. For example, $(2 - 2)(3 - 5) = 0$ or $(2 - 2)(10 - 5) = 0$".
Student 3 said, "Both brackets should always have the same x value. So, x is either 2 or 5 in brackets. For examples, $(2 - 2)(2 - 5) = 0$ and $(5 - 2)(5 - 5) = 0$".
Whose responce is correct?
a) only student 1
b) only student 3
c) only students 1 and 2
d) all students - 1, 2 and 3
[Ans : (b)**]**

14. Shown below is the graph of a quadratic polynomial, $y(x) = px^2 + qx + r$.



Find the nature of the roots of the quadratic equation, $px^2 + qx + r = 0$. Give a reason for your answer.
[Sol. : Since the quadratic equation, $px^2 + qx + r = 0$, has no real roots since the given graph does not intersect the x - axis.**]**

15. Arpit was asked to represent the following statements in the form of a quadratic equation :
"The sum of the squares of two positive integers is 225. The square of the larger number is 16 times the smaller number".
If Arpit write the equation correctly, what could he have written?
[Ans : $x^2 + 16x - 225 = 0$**]**

16. Ayush used the quadratic formula to solve a quadratic equation in y to get : $y = \frac{7 \pm \sqrt{169}}{10}$

Write a quadratic equation Ayush could have been solving. Show your steps.
[Ans : $5y^2 - 7y - 6 = 0$**]**